Bitcoin Mechanics

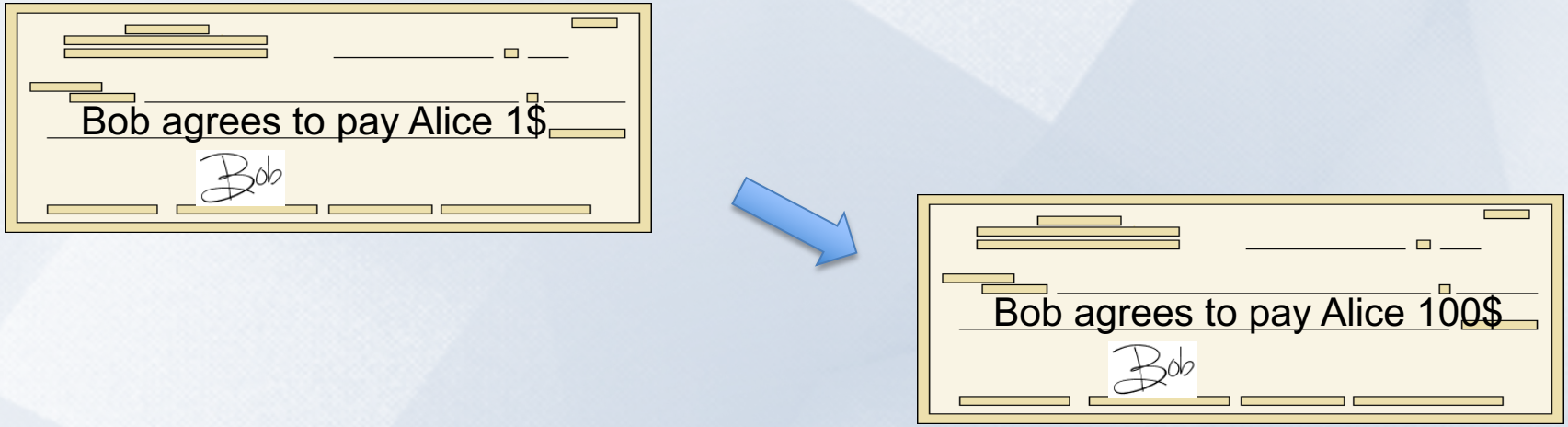
Yu Feng
University of California, Santa Barbara

UCSB

Slides adapted from Stanford CS251

Digital Signatures

Physical signatures: bind transaction to author

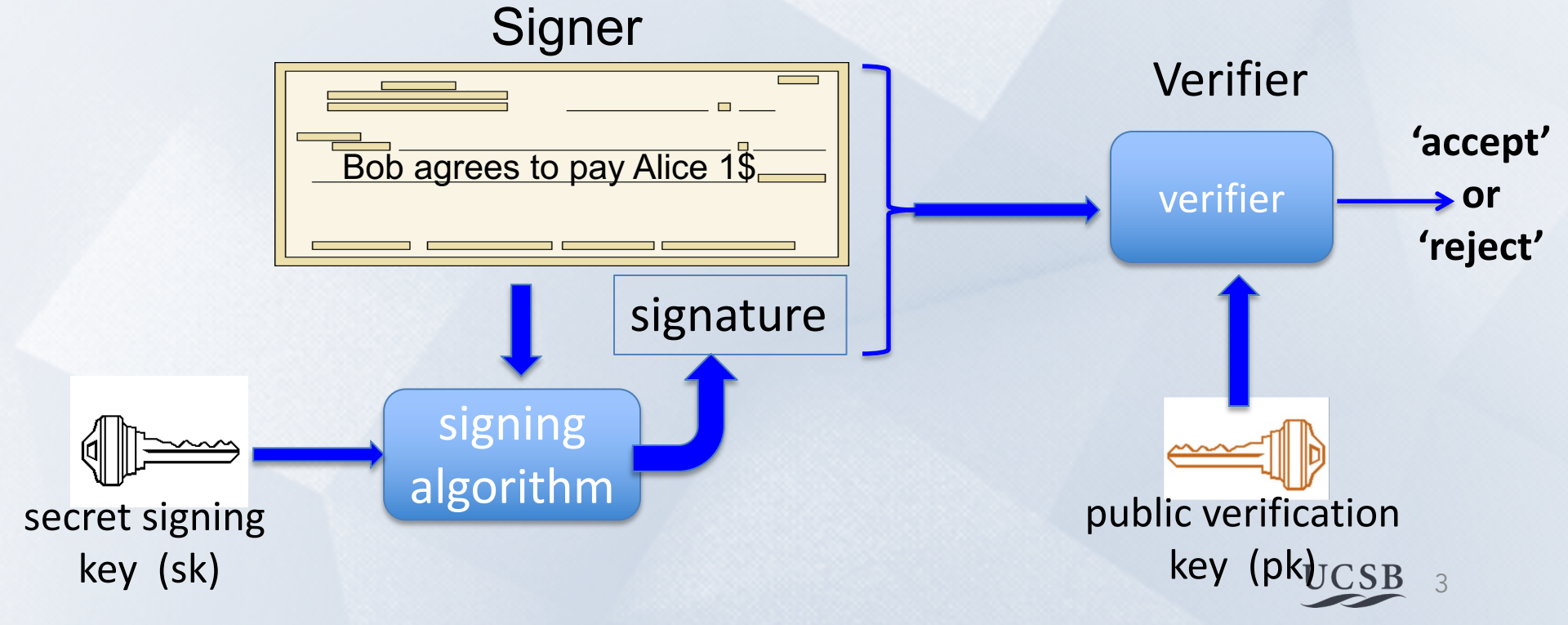


Problem in the digital world:

anyone can copy Bob's signature from one doc to another

Digital signatures

Solution: make signature depend on document



Digital signatures: syntax

Def: a signature scheme is a triple of algorithms:

- **Gen**(): outputs a key pair (pk, sk)
- **Sign**(sk, msg) outputs sig. **σ**
- **Verify**(pk, msg, **σ**) outputs 'accept' or 'reject'

Secure signatures: (informal)

Adversary who sees signatures **on many messages** of his choice,

cannot forge a signature on a new message.

Families of signature schemes

1. RSA signatures (old … not used in blockchains):
   - long sigs and public keys (≥256 bytes), fast to verify
2. Discrete-log signatures: Schnorr and ECDSA (Bitcoin, Ethereum)
   - short sigs (48 or 64 bytes) and public key (32 bytes)
3. BLS signatures: 48 bytes, aggregatable, easy threshold
   (Ethereum 2.0, Chia, Dfinity)

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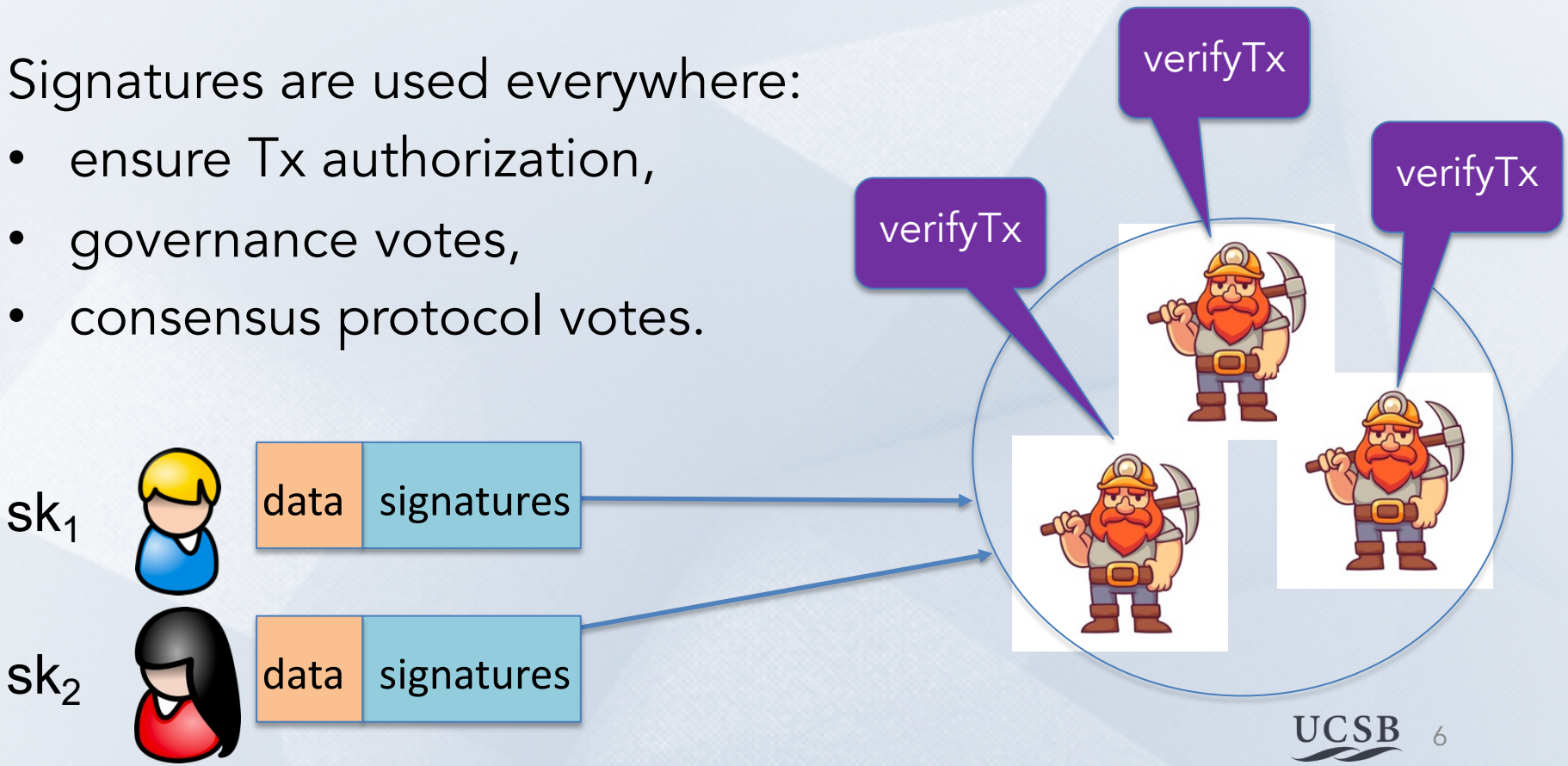
4. Post-quantum signatures: long (≥600 bytes)

Signatures on the blockchain

Signatures are used everywhere:

- ensure Tx authorization,
- governance votes,
- consensus protocol votes.



In summary …

Digital signatures: (Gen, Sign, Verify)

Gen() → (pk, sk),

Sign(sk, m) → **σ**, Verify(pk, m, **σ**) → accept/reject

signing key

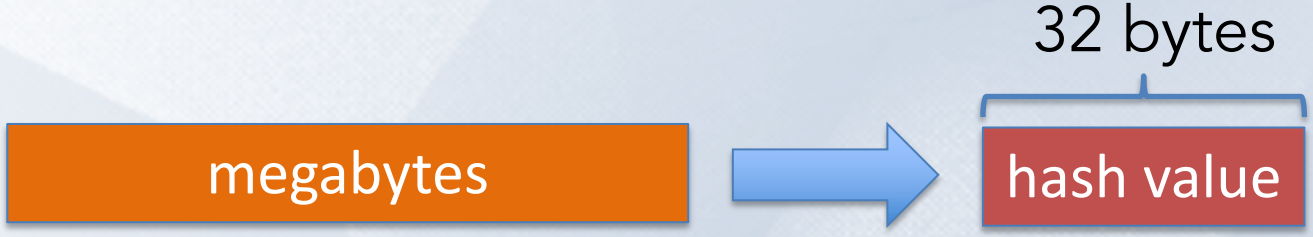
verification key

Cryptography Background

(1) cryptographic hash functions

An efficiently computable function $H: M \rightarrow T$

where $|M| \gg |T|$



$T = \{0,1\}^{256}$

Collision resistance

Def: a **collision** for $H: M \to T$ is pair $x \neq y \in M$ s.t. $H(x) = H(y)$

$|M| \gg |T|$ implies that many collisions exist

Def: a function $H: M \to T$ is **collision resistant** if it is "hard" to find even a single collision for H (we say H is a CRF)

Example: **SHA256**: $\{x : \text{len}(x) < 2^{64} \text{ bytes}\} \to \{0,1\}^{256}$

(output is 32 bytes)

Application: committing to data on a blockchain

Alice has a large file m. She posts $h = H(m)$ (32 bytes)

Bob reads h. Later he learns m' s.t. $H(m') = h$

H is a CRF $\Rightarrow$ Bob is convinced that $m' = m$

(otherwise, m and m' are a collision for H)

We say that $h = H(m)$ is a **binding commitment** to m

(note: not hiding, h may leak information about m)

Committing to a list (of transactions)

Alice has $S = (m_1, m_2, \ldots, m_n)$

32 bytes

Goal:

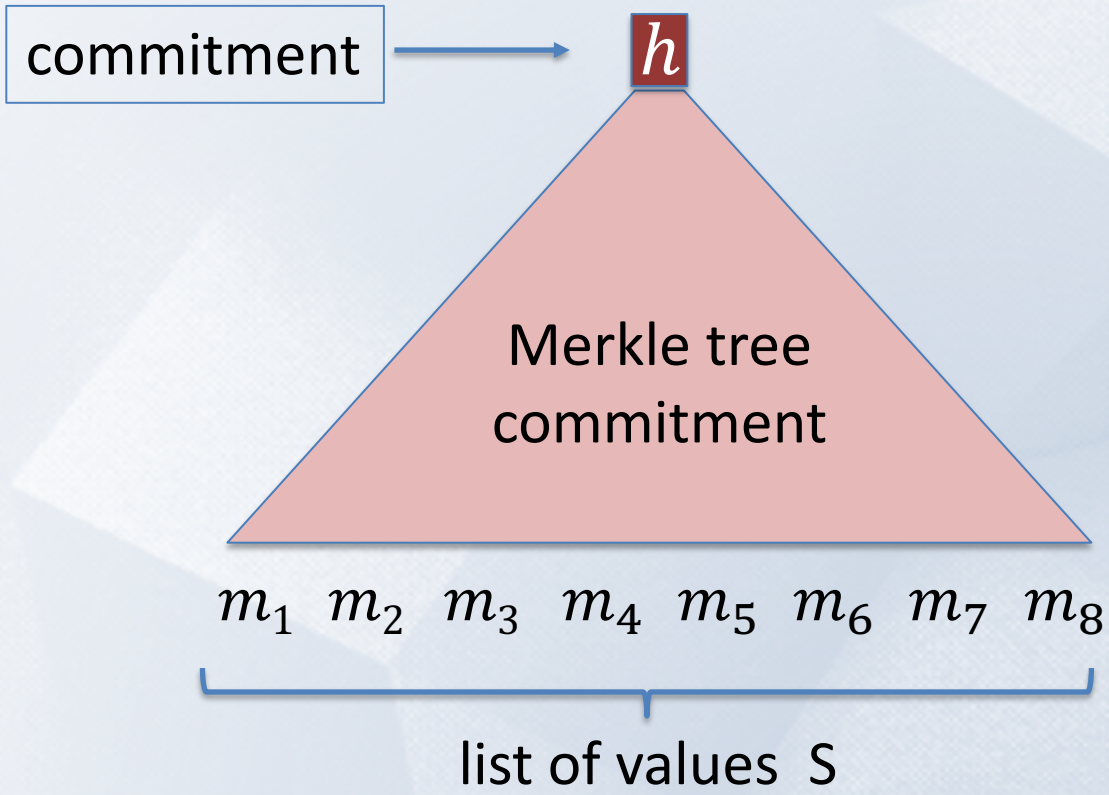
- Alice posts a short binding commitment to S, $h = \text{commit}(S)$
- Bob reads h. Given $(m_i,\ \text{proof}\ \pi_i)$ can check that $S[i] = m_i$

  Bob runs $\text{verify}(h, i, m_i, \pi_i) \rightarrow$ accept/reject

security: adv. cannot find (S, i, m, π) s.t. $m \neq S[i]$ and

$\text{verify}(h, i, m, \pi) = \text{accept}$ where $h = \text{commit}(S)$

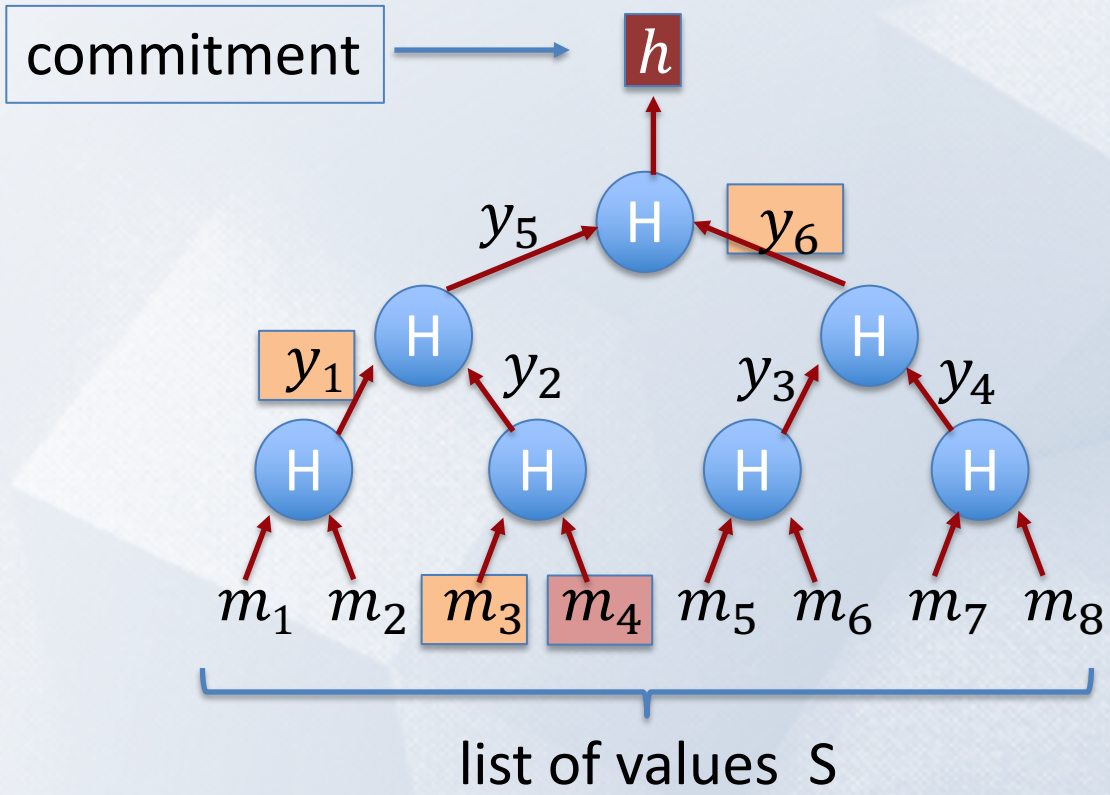
Merkle tree (Merkle 1989)



Goal:

- commit to list S of size n
- Later prove $S[i] = m_i$

Merkle tree (Merkle 1989) [simplified]



Goal:

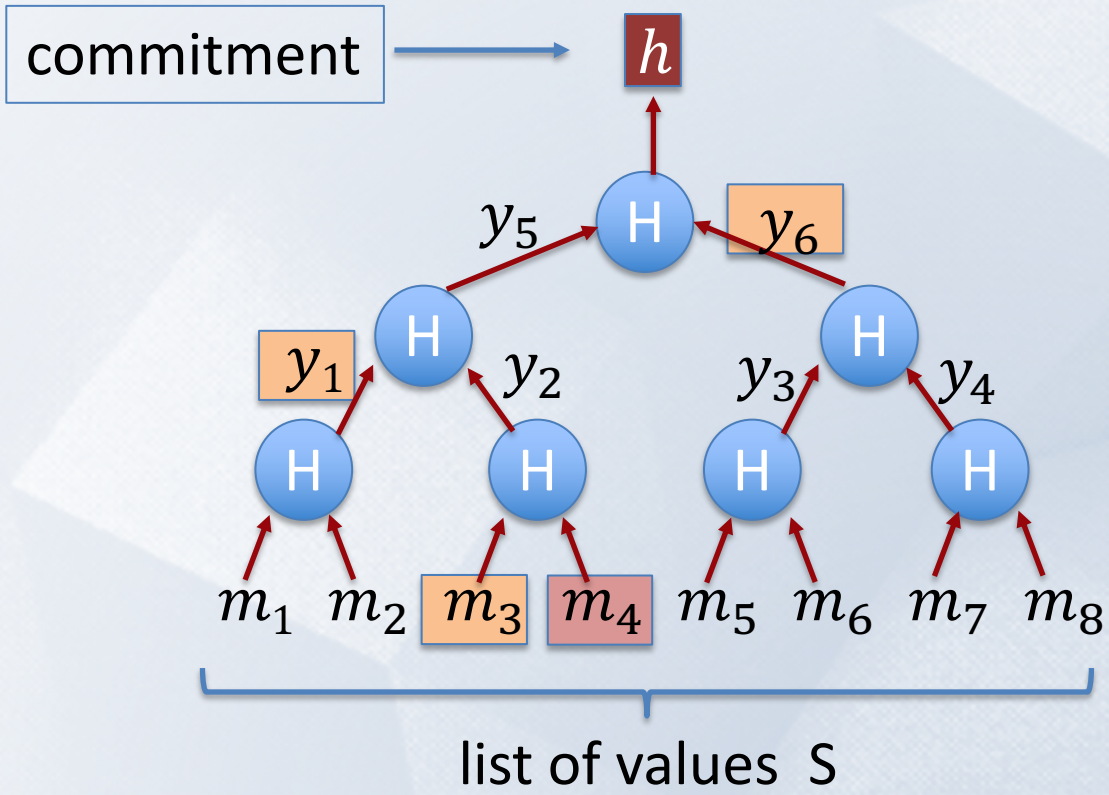
- commit to list S of size n
- Later prove $S[i] = m_i$

To prove $S[4] = m_4$,

proof $\pi = (m_3, y_1, y_6)$

length of proof: $\log_2 n$

Merkle tree (Merkle 1989) [simplified]



To prove $S[4] = m_4$,

proof $\pi = (m_3, y_1, y_6)$

Bob does:

$y_2 \leftarrow H(m_3, m_4)$

$y_5 \leftarrow H(y_1, y_2)$

$h' \leftarrow H(y_5, y_6)$

accept if $h = h'$

Merkle tree (Merkle 1989)

Thm: For a given n: if H is a CRF then

adv. cannot find (S, i, m, π) s.t. $|S| = n$, $m \neq S[i]$,

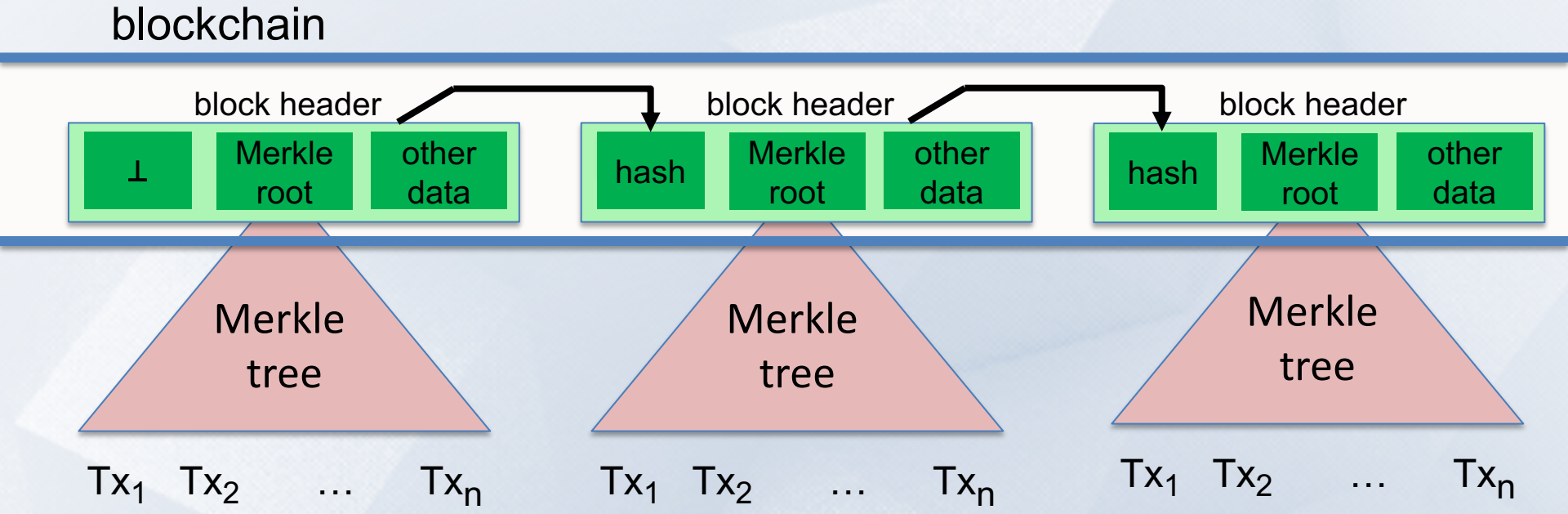
$$h = \text{commit}(S), \text{ and } \text{verify}(h, i, m, \pi) = \text{accept}$$

(to prove, prove the contra-positive)

How is this useful? To post a block of transactions S on chain suffices to only write commit(S) to chain. Keeps chain small.

⇒ Later, can prove contents of every Tx.

Abstract block chain



Merkle proofs are used to prove that a Tx is “on the block chain”

Another application: proof of work

Goal: computational problem that
- takes time $\Omega(D)$ to solve, but
- solution takes time O(1) to verify

(D is called the **difficulty**)

How? $H: X \times Y \to \{0,1,2,\ldots,2^n - 1\}$ e.g. $n = 256$

- puzzle: input $x \in X$, output $y \in Y$ s.t. $H(x,y) < 2^n/D$
- verify(x,y): accept if $H(x,y) < 2^n/D$

Another application: proof of work

Thm: if H is a "random function" then the best algorithm requires D evaluations of H in expectation.

Note: this is a parallel algorithm

⇒ the more machines I have, the faster I solve the puzzle.

Proof of work is used in some consensus protocols (e.g., Bitcoin)

Bitcoin uses $H(x, y) = \mathrm{SHA256}(\mathrm{SHA256}(x.y))$

Bitcoin mechanics

UCSB

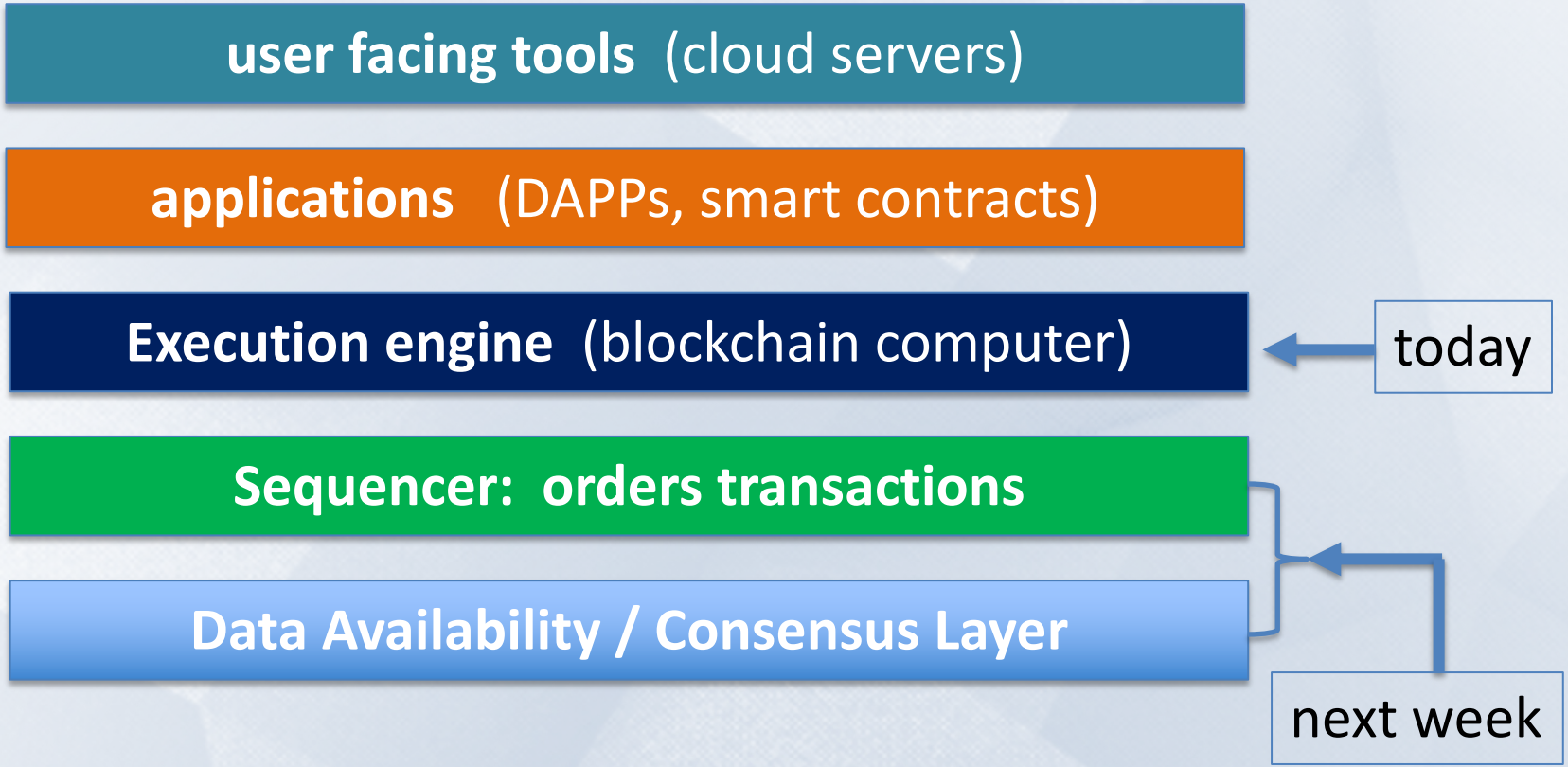
This lecture: Bitcoin mechanics

Oct. 2008: paper by Satoshi Nakamoto
Jan. 2009: Bitcoin network launched

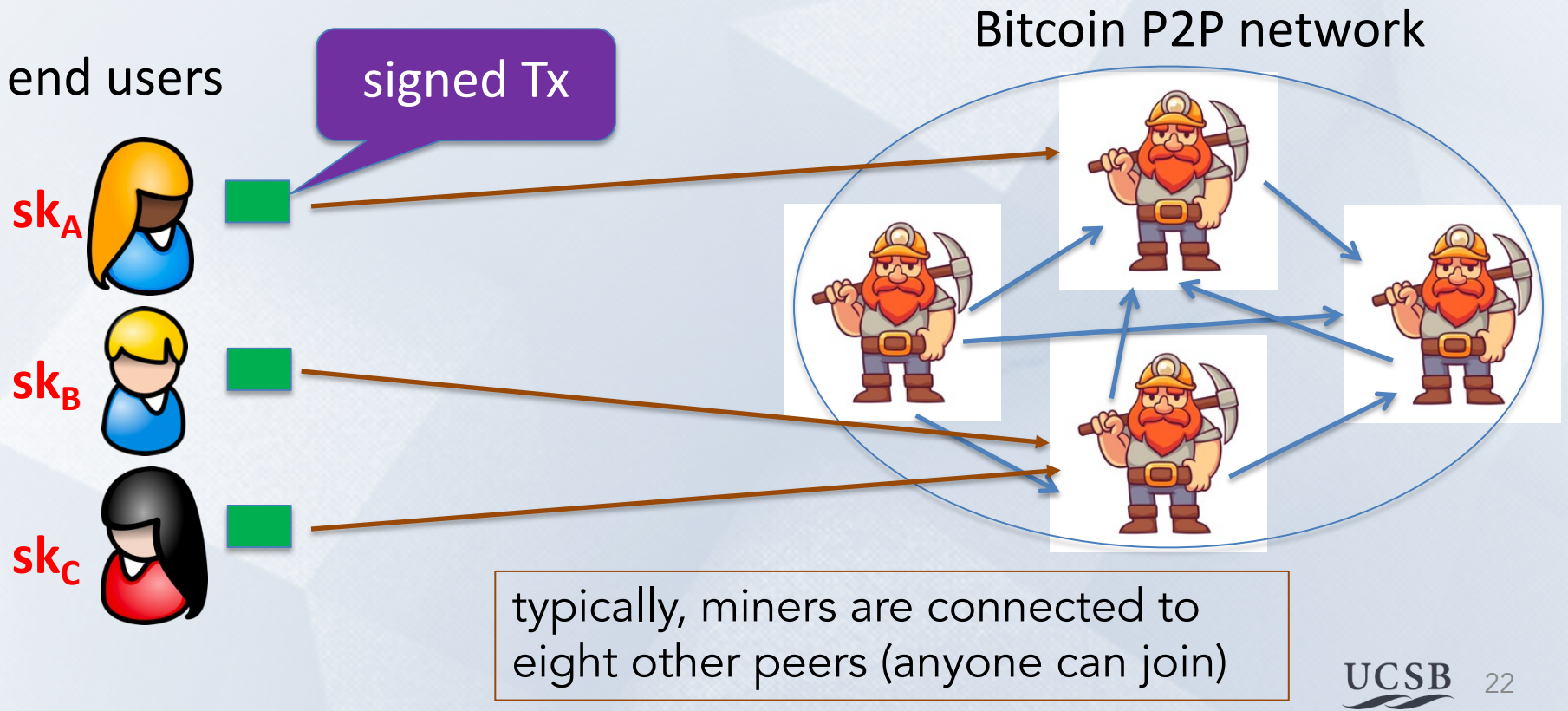
Total market value:

Apr. 2024: $1.36T

This lecture: Bitcoin mechanics



First: overview of the Bitcoin consensus layer



typically, miners are connected to eight other peers (anyone can join)

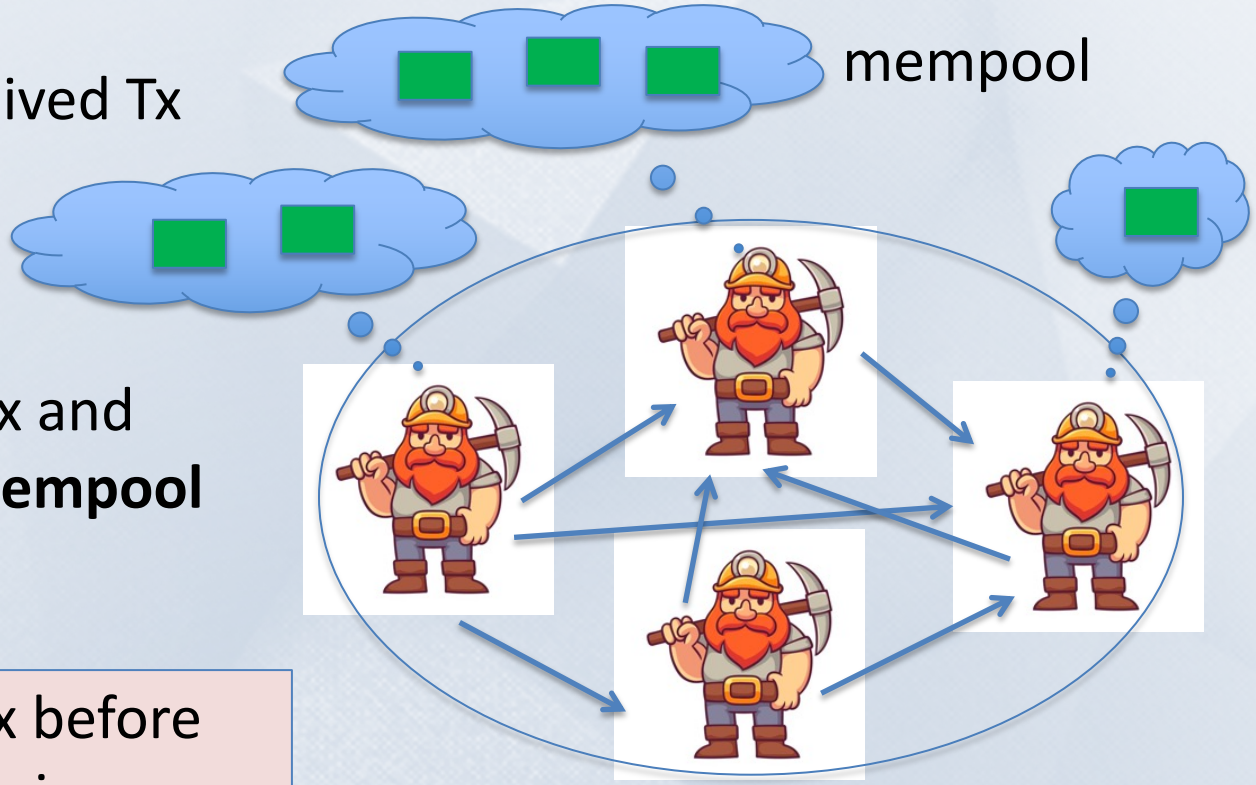
First: overview of the Bitcoin consensus layer

miners broadcast received Tx to the P2P network

every miner:

validates received Tx and stores them in its **mempool** (unconfirmed Tx)

note: miners see all Tx before they are posted on chain

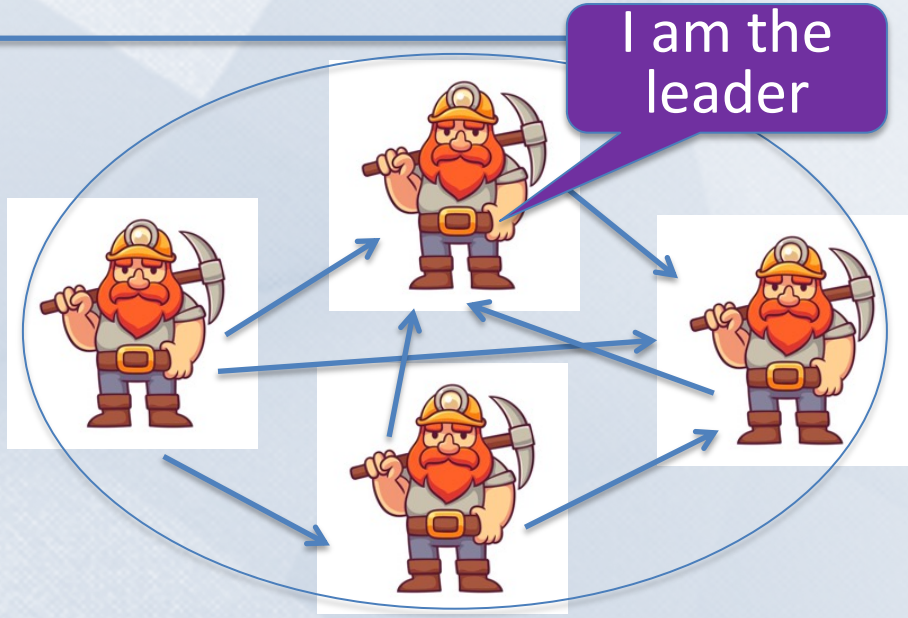


First: overview of the Bitcoin consensus layer

blockchain

Every ≈**10 minutes**:

- Each miner creates a candidate block from Tx in its mempool
- a "random" miner is selected (how: next week), and broadcasts its block to P2P network
- all miners validate new block



Bitcoin P2P network

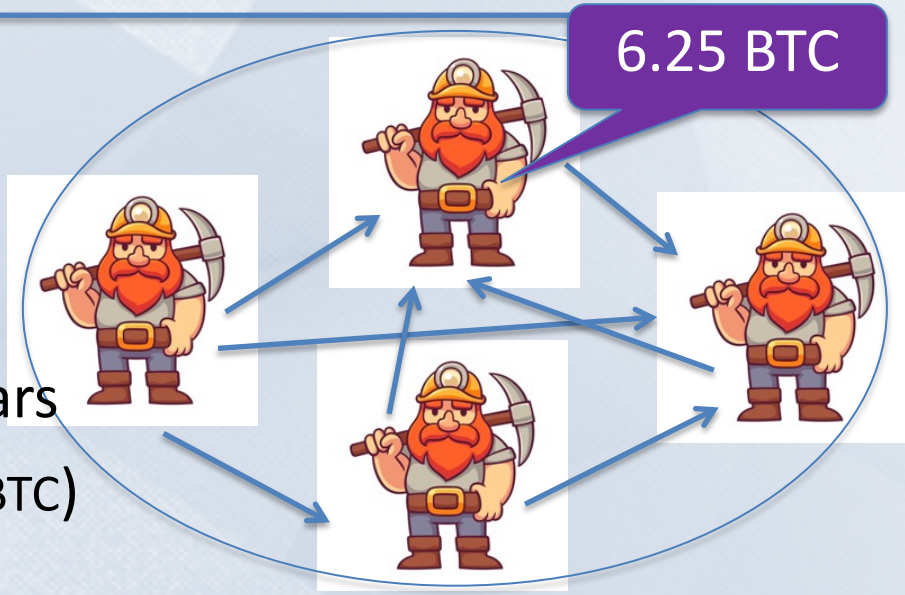
First: overview of the Bitcoin consensus layer

blockchain

Selected miner is paid 6.25 BTC
in **coinbase Tx** (first Tx in the block)

- only way new BTC is created
- block reward halves every four years

  ⇒ max 21M BTC (currently 19.6M BTC)



note: miner chooses order of Tx in block

Properties (very informal)

Next week:

Safety / Persistence:

- to remove a block, need to convince 51% of mining power *

Liveness:

- to block a Tx from being posted, need to convince 51% of mining power **

(some sub 50% censorship attacks, such as feather forks)

Bitcoin blockchain: a sequence of block headers, 80 bytes each

H

BH_1

version	(4 bytes)
prev	(32 bytes)
time	(4 bytes)
bits	(4 bytes)
nonce	(4 bytes)
Tx root	(32 bytes)

80 bytes

H

BH_2

prev

Tx root

H

BH_3

prev

Tx root

...

coinbase Tx

coinbase Tx

Bitcoin blockchain: a sequence of block headers, 80 bytes each

time: time miner assembled the block. Self reported.
(block rejected if too far in past or future)

bits: proof of work difficulty

nonce: proof of work solution } for choosing a leader (next week)

Merkle tree: payer can give a short proof that Tx is in the block

new block every ≈10 minutes.

An example

Tx data

Height	Mined	Miner	Size	#Tx
648494	17 minutes	Unknown	1,308,663 bytes	1855
648493	20 minutes	SlushPool	1,317,436 bytes	2826
648492	59 minutes	Unknown	1,186,609 bytes	1128
648491	1 hour	Unknown	1,310,554 bytes	2774
648490	1 hour	Unknown	1,145,491 bytes	2075
648489	1 hour	Poolin	1,359,224 bytes	2622

Block 648493

Timestamp	2020-09-15 17:25
Height	648493
Miner	SlushPool
Number of Transactions	2,826
Difficulty	17,345,997,805,929.09
Merkle root	350cbb917c918774c93e945b960a2b3ac1c8d448c2e67839223bbcf595baff89
Transaction Volume	11256.14250596 BTC
Block Reward	6.25000000 BTC
Fee Reward	0.89047154 BTC

(from coinbase Tx)

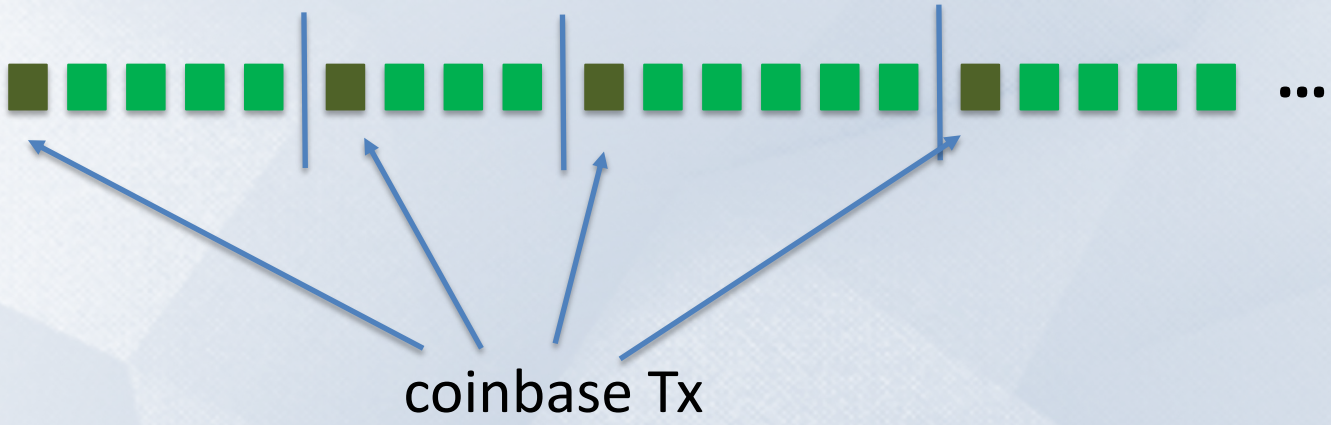
(D)

(adjusts every two weeks)

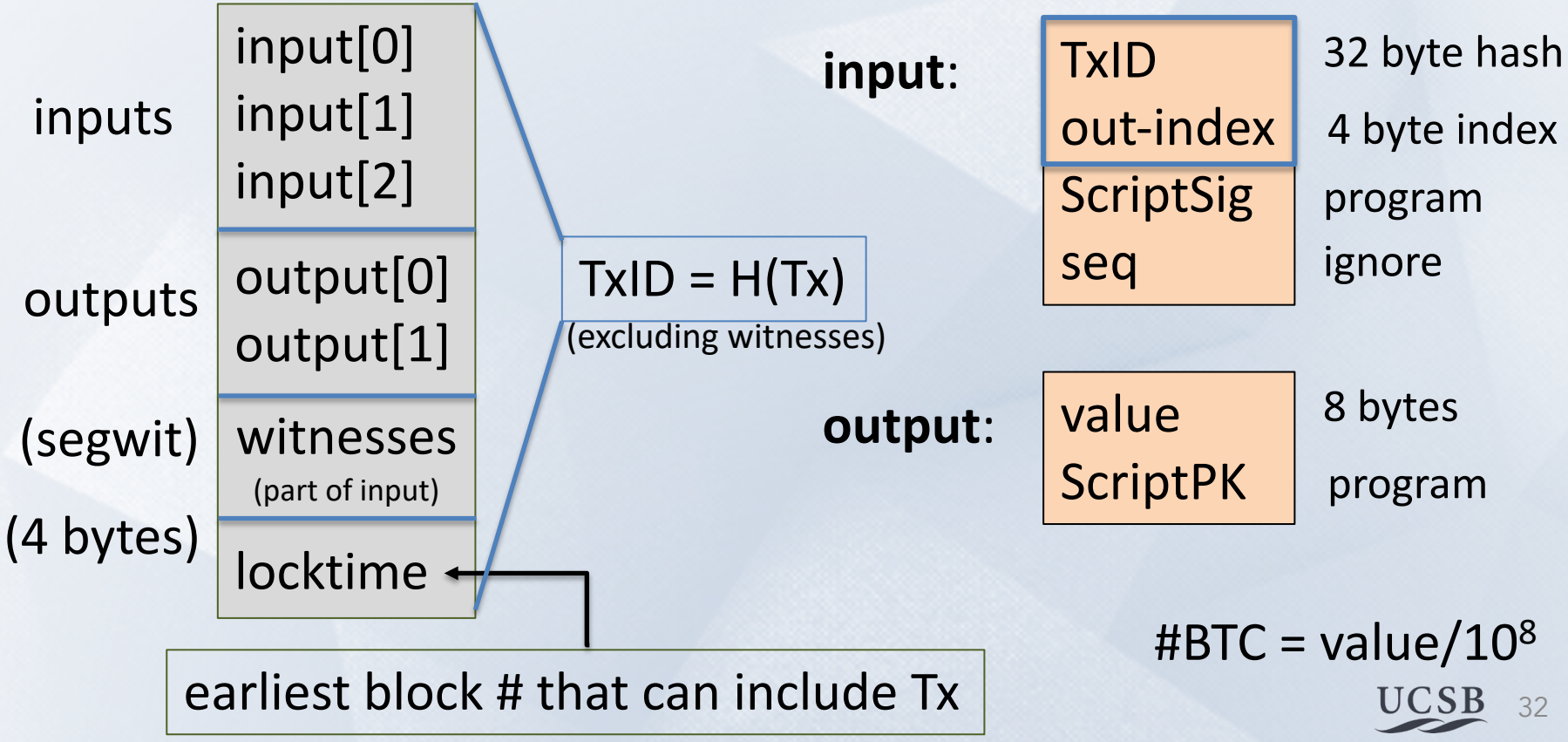
(Tx fees given to miner in coinbase Tx)

This lecture

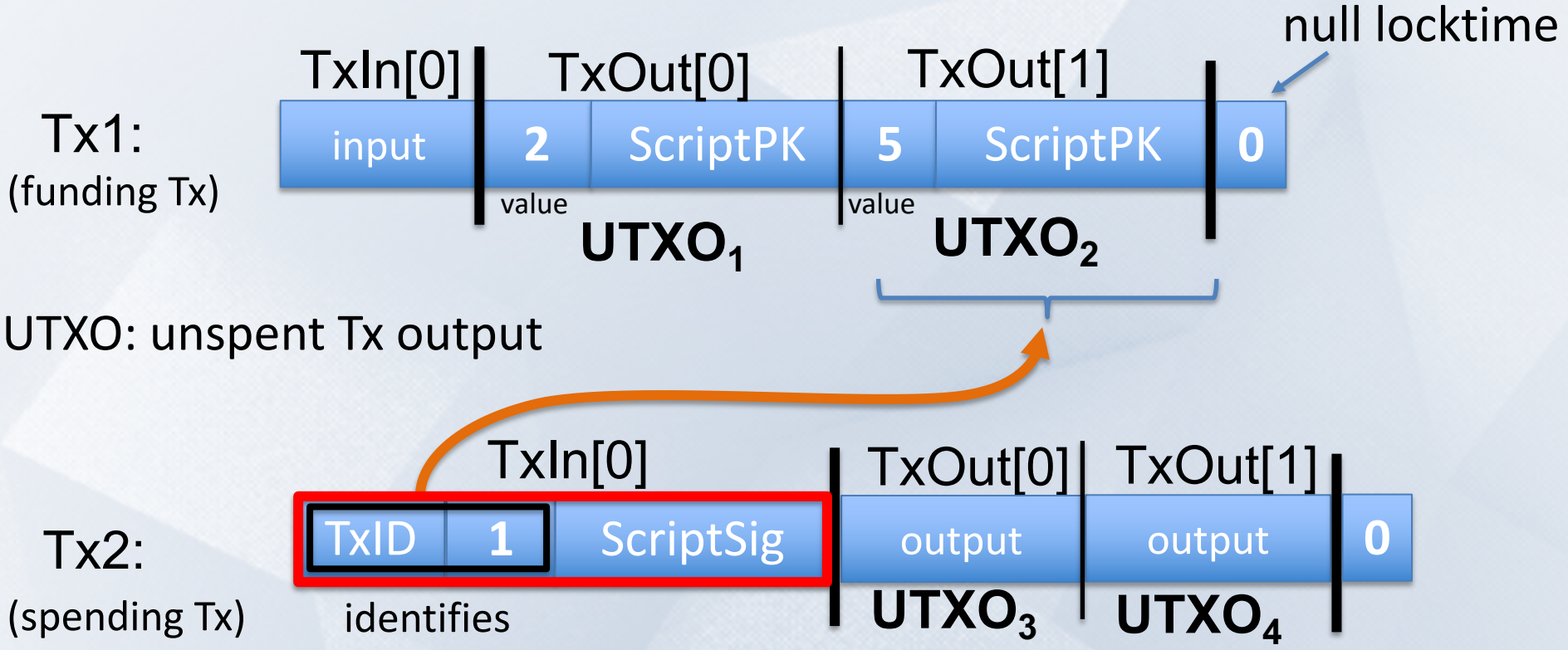
View the blockchain as a sequence of Tx (append-only)



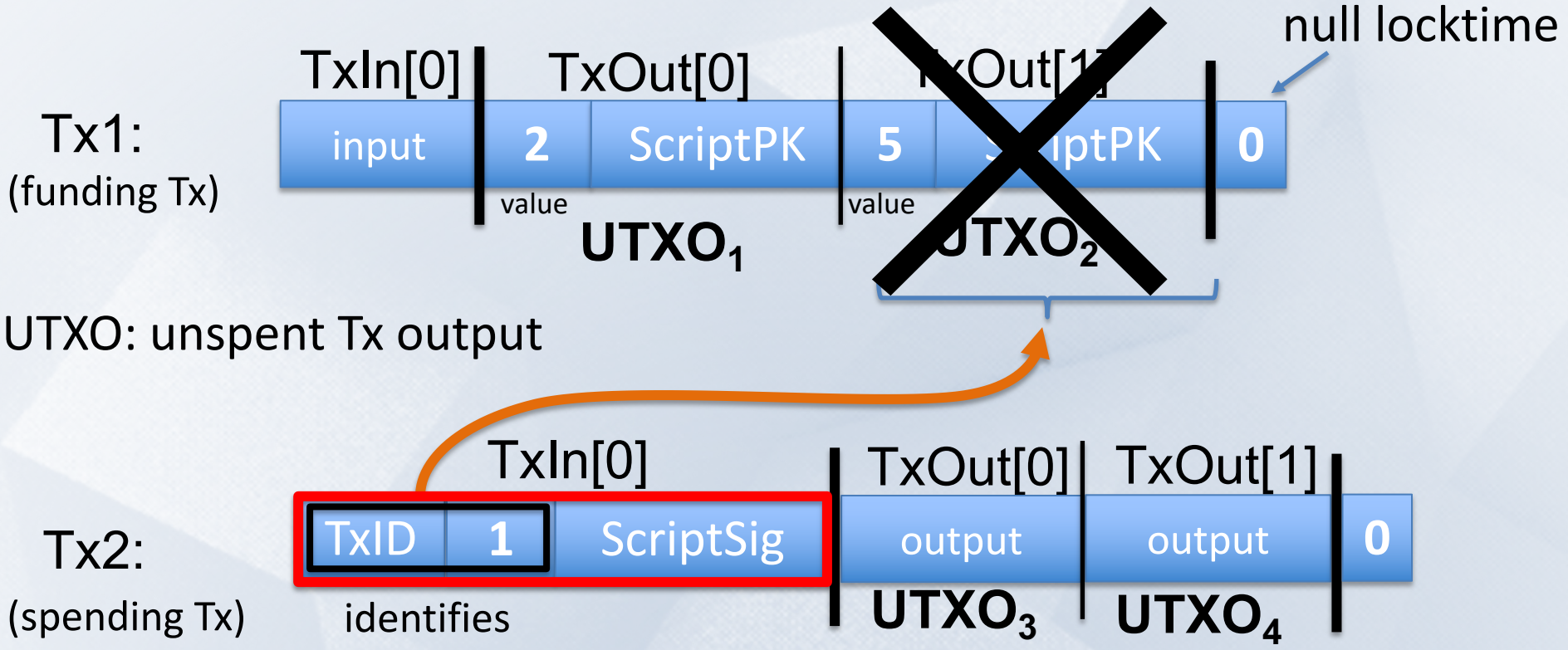
Tx structure (non-coinbase)



Example


null locktime
TxIn[0]
TxOut[0]
TxOut[1]
Tx1:
(funding Tx)
input
2
ScriptPK
5
ScriptPK
0
value
value
UTXO1
UTXO2
UTXO: unspent Tx output
TxIn[0]
TxOut[0]
TxOut[1]
Tx2:
(spending Tx)
TxID
1
ScriptSig
output
output
0
identifies
a UTXO
UTXO3
UTXO4

Example


Tx1:
(funding Tx)
TxIn[0]
input
TxOut[0]
2
value
ScriptPK
UTXO1
5
value
UTXO2
0
null locktime
UTXO: unspent Tx output
Tx2:
(spending Tx)
TxIn[0]
TxID
1
ScriptSig
identifies
a UTXO
TxOut[0]
output
UTXO3
TxOut[1]
output
UTXO4
0

Validating Tx2

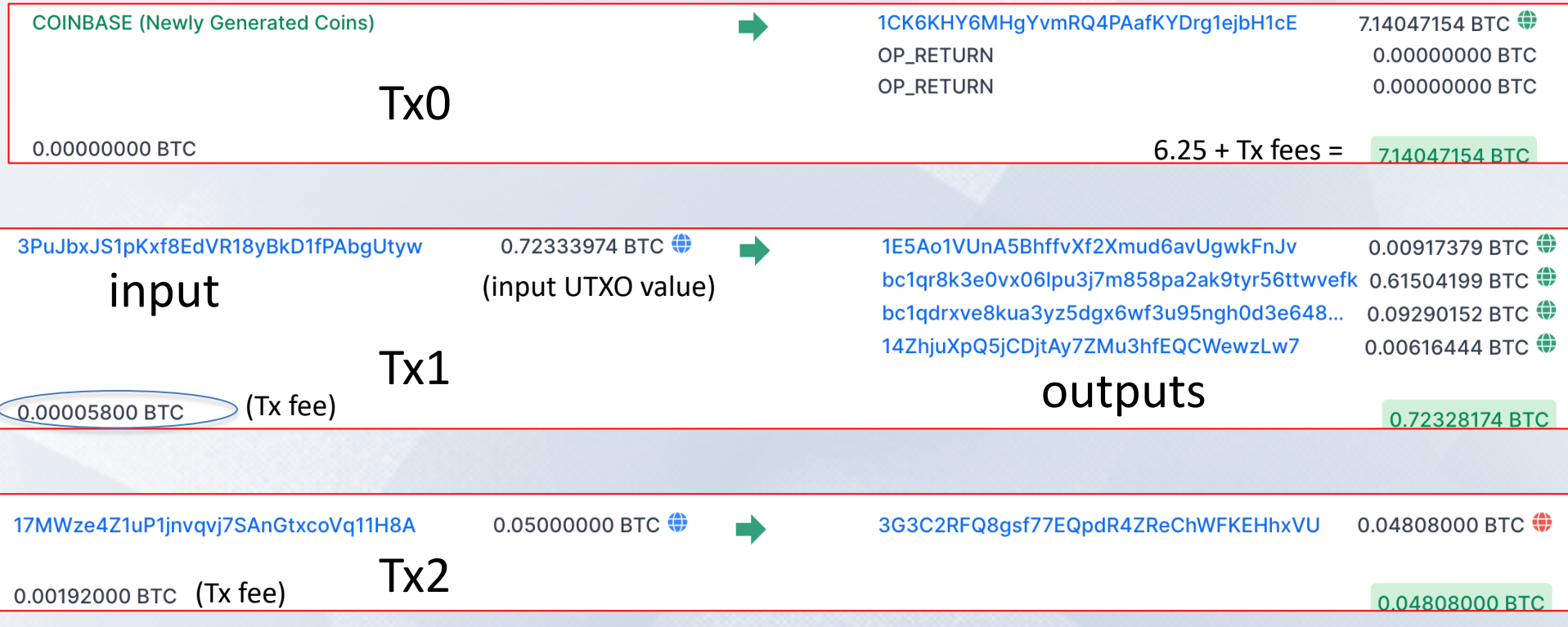
Miners check (for each input):

program from funding Tx: under what conditions can UTXO be spent

1. The program ScriptSig | ScriptPK returns true
2. TxID | index is in the current UTXO set
3. sum input values ≥ sum output values

After Tx2 is posted, miners remove $UTXO_2$ from UTXO set

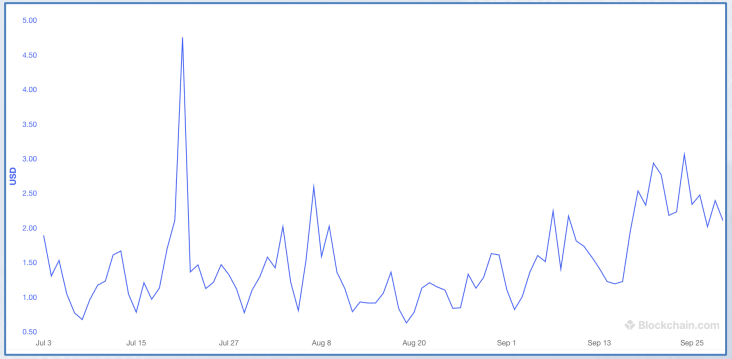
An example (block 648493) [2826 Tx]



sum of fees in block added to coinbase Tx

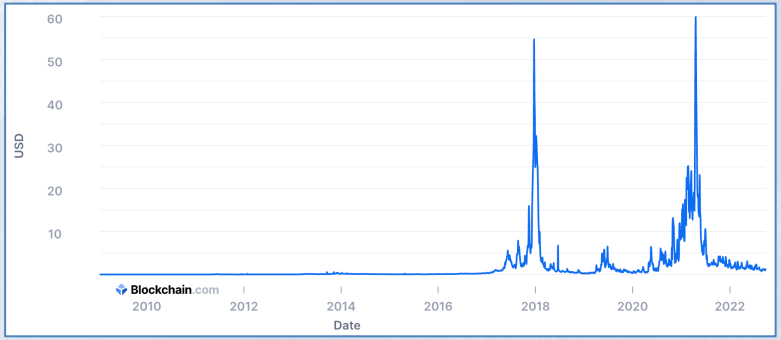
Tx fees

Bitcoin average Tx fees in USD (last 60 days, sep. 2023)

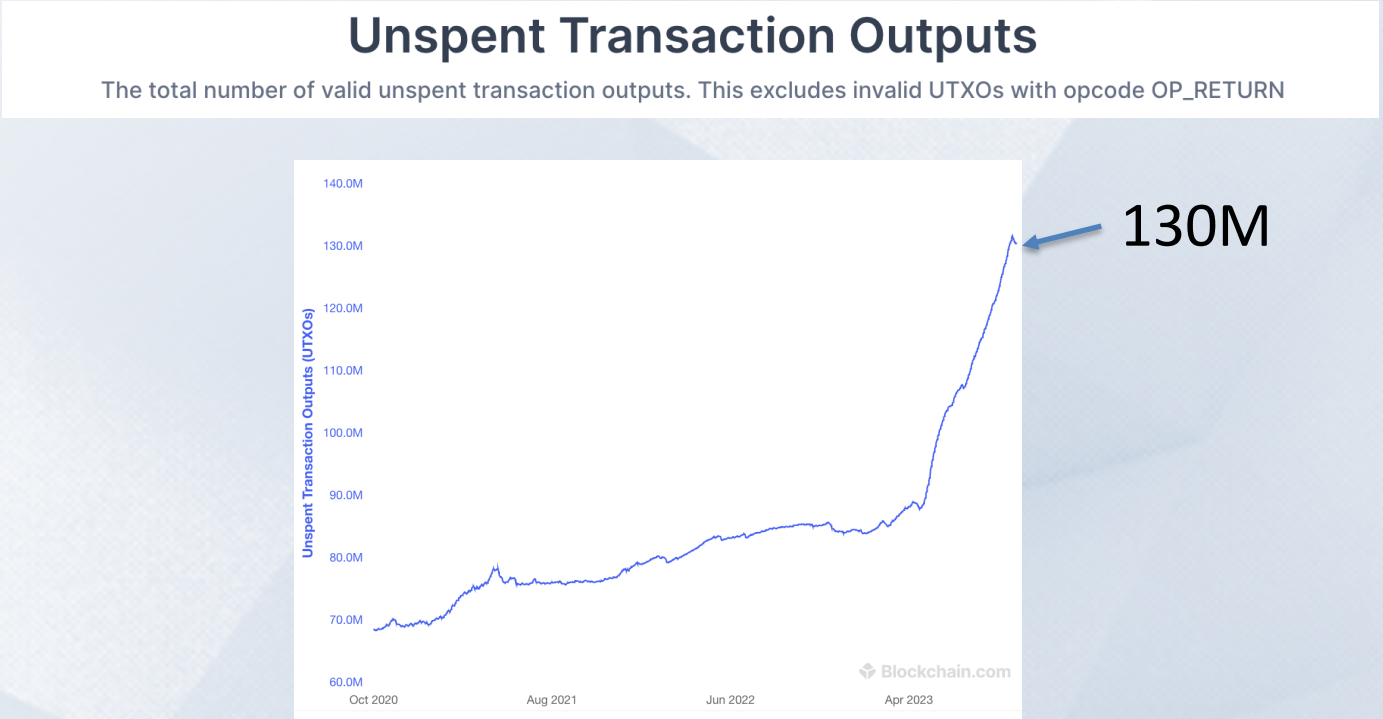


$2.11

Bitcoin average Tx fees in USD (all time)



All value in Bitcoin is held in UTXOs



Sep. 2023: miners need to store ≈130M UTXOs in memory

Focusing on Tx2: TxInp[0]

from UTXO
(Bitcoin script)

Value	0.05000000 BTC
Pkscript	OP_DUP OP_HASH160 45b21c8a0cb687d563342b6c729d31dab58e3a4e OP_EQUALVERIFY OP_CHECKSIG
Sigscript	304402205846cace0d73de82dfbdeba4d65b9856d7c1b1730eb401cf4906b2401a69bdc90220589d36d36be64e774c8796b96c011f29768191abeb7f56ba20ffb0351280860c01 03557c228b080703d52d72ead1bd93fc72f45c4ddb4c2b7a20c458e2d069c8dd9e

from TxInp[0]

Bitcoin Script

A stack machine. Not Turing Complete: no loops.

Quick survey of op codes:

1. **OP_TRUE** (OP_1), **OP_2**, …, **OP_16**: push value onto stack

81 82 96

2. **OP_DUP**: push top of stack onto stack

118

Bitcoin Script

3. control:

99 OP_IF <statements> OP_ELSE <statements> OP_ENDIF

105 OP_VERIFY: abort fail if top = false

106 OP_RETURN: abort and fail

what is this for? ScriptPK = [OP_RETURN, <data>]

136 OP_EQVERIFY: pop, pop, abort fail if not equal

Bitcoin Script

4. arithmetic:

   OP_ADD, **OP_SUB**, **OP_AND**, …: pop two items, add, push

5. crypto:

   OP_SHA256: pop, hash, push

   OP_CHECKSIG: pop pk, pop sig, verify sig. on Tx, push 0 or 1

6. Time: **OP_CheckLockTimeVerify** (CLTV):
   fail if value at the top of stack > Tx locktime value.
   usage: UTXO can specify min-time when it can be spent

Example: a common script

<sig> <pk>	DUP HASH256 <pkhash> EQVERIFY CHECKSIG

stack:		
	empty	init
	<sig> <pk>	push values
	<sig> <pk> <pk>	DUP
	<sig> <pk> <hash>	HASH256
	<sig> <pk> <hash> <pkhash>	push value
	<sig> <pk>	EQVERIFY
	1	CHECKSIG

⇒ successful termination

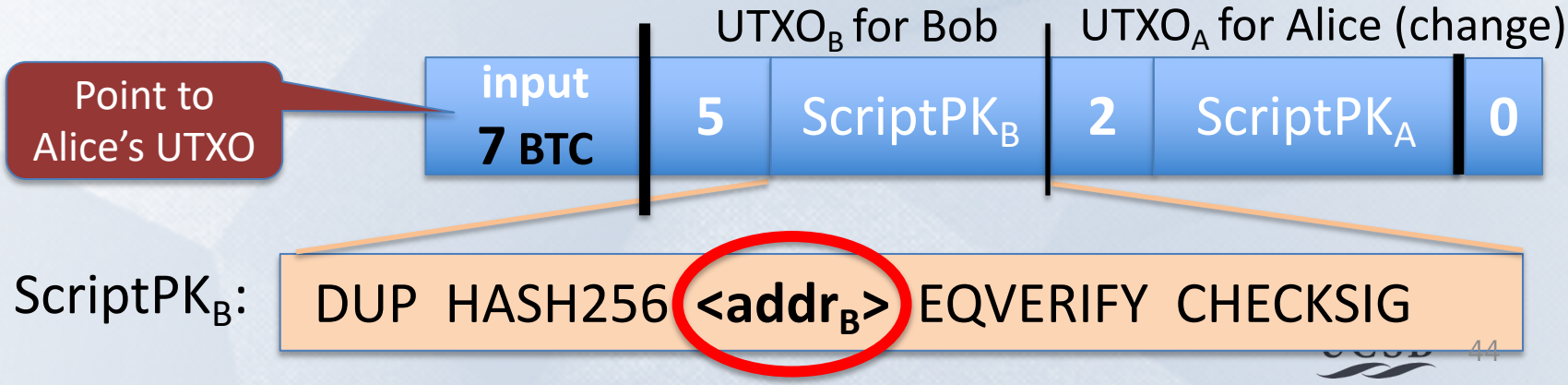
verify(pk, Tx, sig)

Transaction types: (1) P2PKH

pay to public key hash

Alice want to pay Bob 5 BTC:

- step 1: Bob generates sig key pair $(pk_B, sk_B) \leftarrow Gen()$
- step 2: Bob computes his Bitcoin address as $addr_B \leftarrow H(pk_B)$
- step 3: Bob sends $addr_B$ to Alice
- step 4: Alice posts Tx:



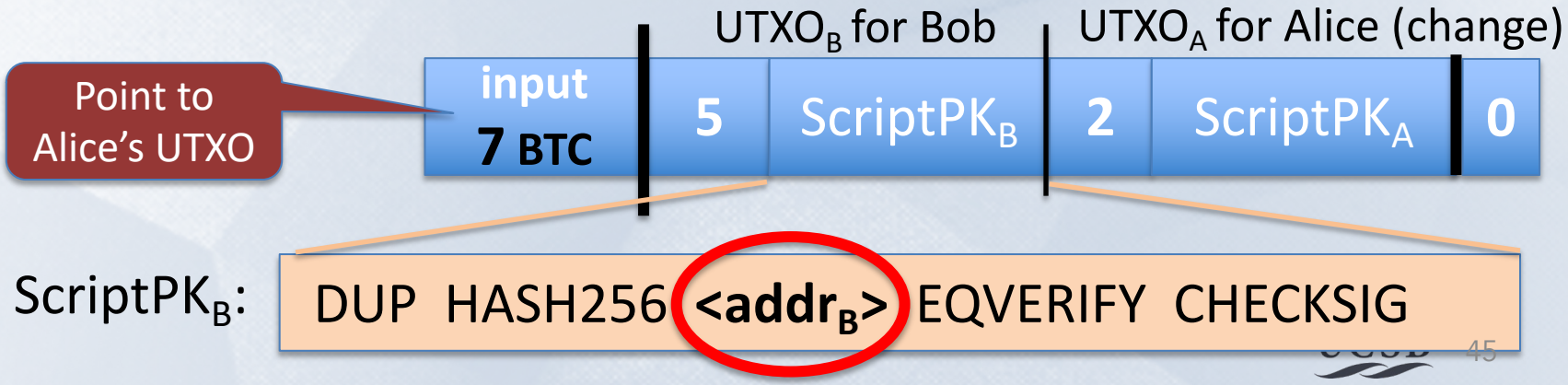
Transaction types: (1) P2PKH

pay to public key hash

"input" contains ScriptSig that authorizes spending Alice's UTXO

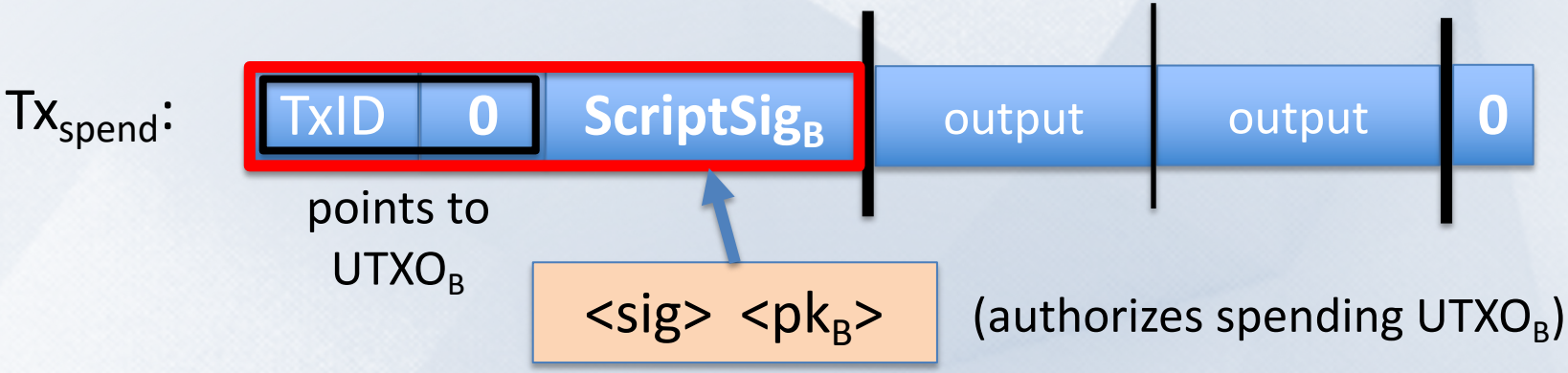
- example: ScriptSig contains Alice's signature on Tx

  $\Longrightarrow$ miners cannot change $ScriptPK_B$ (will invalidate Alice's signature)



Transaction types: (1) P2PKH

Later, when Bob wants to spend his UTXO: create a Tx_{spend}



<sig> = Sign(sk_B, Tx) where Tx = (Tx_{spend} excluding all ScriptSigs) (SIGHASH_ALL)

Miners validate that $ScriptSig_B$ | $ScriptPK_B$ returns true

P2PKH: comments

- Alice specifies recipient's pk in $UTXO_B$
- Recipient's pk is not revealed until UTXO is spent
  (some security against attacks on pk)
- Miner cannot change $<Addr_B>$ and steal funds:
  invalidates Alice's signature that created $UTXO_B$

Segregated Witness

ECDSA malleability:

Given (m, sig) anyone can create (m, sig') with sig ≠ sig'

⇒ miner can change sig in Tx and change TxID = SHA256(Tx)

⇒ Tx issuer cannot tell what TxID is, until Tx is posted

⇒ leads to problems and attacks

Segregated witness: signature is moved to witness field in Tx

TxID = Hash(Tx without witnesses)

Transaction types: (2) P2SH: pay to script hash

(pre SegWit in 2017)

Let's payer specify a redeem script (instead of just pkhash)

Usage: payee publishes hash(redeem script) ← Bitcoint addr.

payer sends funds to that address

ScriptPK in UTXO: HASH160 <H(redeem script)> EQUAL

ScriptSig to spend: $<sig_1>$ $<sig_2>$ … $<sig_n>$ <redeem script>

payer can specify complex conditions for when UTXO can be spent

P2SH

Miner verifies:

(1) <ScriptSig> ScriptPK = true ⟵ payee gave correct script

(2) ScriptSig = true ⟵ script is satisfied

Example P2SH: multisig

Goal: spending a UTXO requires t-out-of-n signatures

Redeem script for 2-out-of-3: (set by payer)

<2> <PK_1> <PK_2> <PK_3> <3> CHECKMULTISIG

hash gives P2SH address

ScriptSig to spend: (by payee) <0> <sig1> <sig3> <redeem script>

END OF LECTURE

Next lecture: interesting scripts,
wallets, and how to manage crypto assets

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