

# Verifying Absence of Side Channels using Cartesian Hoare Logic



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$$\exists H_1, H_2, L. R_P(H_1, L) \neq R_P(H_2, L)$$



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$$\forall H_1, H_2, L. |R_P(H_1, L) - R_P(H_2, L)| < \epsilon$$

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- To verify bounded non-interference, need technique for verifying  $k$ -safety

# Self-composition

- Simplest technique to verify  $k$ -safety is **self-composition (SC)**
- Given 2-safety property  $F$ , self composition sequentially composes two alpha-renamed copies of the program

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Verify(P, F) {  
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**Unfortunately, self-composition  
does not work well in practice**

# Why Self-composition Doesn't Work

- To see why SC doesn't work, consider this program

```
f(int[] secret,
  int k, int n) {

  i:=0;
  while(i<n){
    if(secret[i])
      foo(); //r+=c
    else
      bar(); //r+=c
    i:=i+k;
  }
}
```

# Why Self-composition Doesn't Work

- To see why SC doesn't work, consider this program
- With SC, we obtain this new program

```
i1:=0;
while(i1<n){
    if(secret1[i1])
        foo(); //r1+=c
    else
        bar(); //r1+=c
    i1:=i1+k;
}
i2:=0;
while(i2<n){
    if(secret2[i2])
        foo(); //r2+=c
    else
        bar(); //r2+=c
    i2:=i2+k;
}
assert(r1 - r2 < b);
```



# Why Self-composition Doesn't Work

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- With SC, we obtain this new program
- Unfortunately, need to know precise values of  $r1$ ,  $r2$  after each loop to prove assertion, but this is very difficult

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i1:=0;
while(i1<n){
    if(secret1[i1])
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    i1:=i1+k;
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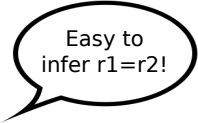
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Easy to  
infer  $r1=r2$ !

# Cartesian Hoare Logic (CHL)

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(Expand)	$\frac{\vdash \langle \Phi \rangle \oplus S^n \langle \Psi \rangle}{\vdash \ \Phi\  S \ \Psi\ }$
(Lift)	$\frac{\vdash \langle \Phi \rangle S \langle \Psi \rangle}{\vdash \langle \Phi \rangle S \langle \Psi \rangle}$
( $\exists$ -intro 1)	$\frac{\vdash \langle \Phi \rangle S; b \oplus \chi \langle \Psi \rangle}{\vdash \langle \Phi \rangle S \oplus \chi \langle \Psi \rangle}$
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(Assoc)	$\frac{\vdash \langle \Phi \rangle \chi_1 \oplus (\chi_2 \oplus \chi_3) \langle \Psi \rangle}{\vdash \langle \Phi \rangle (\chi_1 \oplus \chi_2) \oplus \chi_3 \langle \Psi \rangle}$
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(Step)	$\frac{\vdash \langle \Phi \rangle S_1 \langle \Phi' \rangle \vdash \langle \Phi' \rangle S_2 \oplus \chi \langle \Psi \rangle}{\vdash \langle \Phi \rangle S_1; S_2 \oplus \chi \langle \Psi \rangle}$
(Havoc)	$\frac{V = \text{vars}(\chi)}{\vdash \langle \Phi \rangle \chi \langle \exists V. \Phi \rangle}$
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- CHL proof rules allow synchronization between loops
- Easy to automate reasoning in CHL
- More scalable and precise than prior techniques

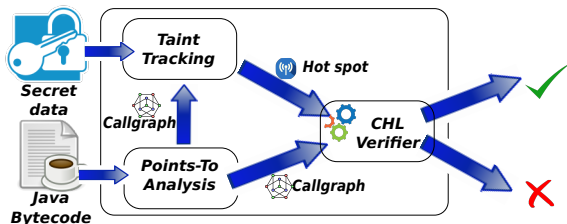
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- However, combines CHL verifier with taint analysis for better scalability





- Using this technique, we solved several benchmarks from engagements (e.g., GabFeed, SnapBuddy, TourPlanner)
- Furthermore, we were able to find vulnerabilities in real Java applications, such as Tomcat, Jetty, JBoss, and SpringSecurity
- In addition, we also used technique to analyze other k-safety properties (e.g. transitivity, associativity, symmetry etc.)

# Demo Time!

