Home exercise #2

Solve the following differential equation with the help of Green's function method

$$(\frac{d^2}{dr^2} - a^2)\phi = -4\pi r\rho \qquad \rho(r) = \frac{1}{8\pi}e^{-r}$$

with the following boundary conditions

$$\phi(r=0) = \phi(r \to \infty) = 0$$

a is a constant.

The solution to the homogeneous differential equation is

$$\phi^h(r) = \frac{1}{2a}(e^{ar_<} - e^{-ar_<})e^{-ar_>}$$

$$\phi^h_<(r_<) = \frac{1}{\sqrt{2a}}(e^{ar_<} - e^{-ar_<}) \qquad \phi^h_>(r>) = -\frac{1}{\sqrt{2a}}e^{-ar_>}$$

Use the following equation to get the numerical solution of the differential equation

$$\phi(r) = \phi^h_>(r) \int_0^r \phi^h_<(r')S(r')dr' + \phi^h_<(r) \int_r^\infty \phi^h_>(r')S(r')dr'$$

Use an accurate method for numerical integration (e.g., Bode's/Boole's rule) for the above integrals. Bode's/Boole's integration method is the following.

$$\int_{x_0}^{x_4} f(x)dx \approx \frac{2h}{45}[7f_0 + 32f_1 + 12f_2 + 32f_3 + 7f_4]$$

Remember: N must be a multiple of 4 in order to apply Bode's/Boole's method.
N is the total number of points for integration.
Compare your numerical solution to the following analytical solution.

$$\phi(r) = (\frac{1}{1-a^2})^2(e^{-ar} - e^{-r}[1 + \frac{1}{2}(1-a^2)r])$$

Consider a=4 and r_{max}=30 for integration.

Write a small report where the code(s), the plots and a small discussion are included. Upload the code separately. Only one upload per group in studentportalen is required. Don't forget to include your names and group number in the report. The deadline is midnight of 15th February, 2019.