

3D Printing the Trefoil



and its Pages.

2015 UGA CURO Symposium

Fred Hohman

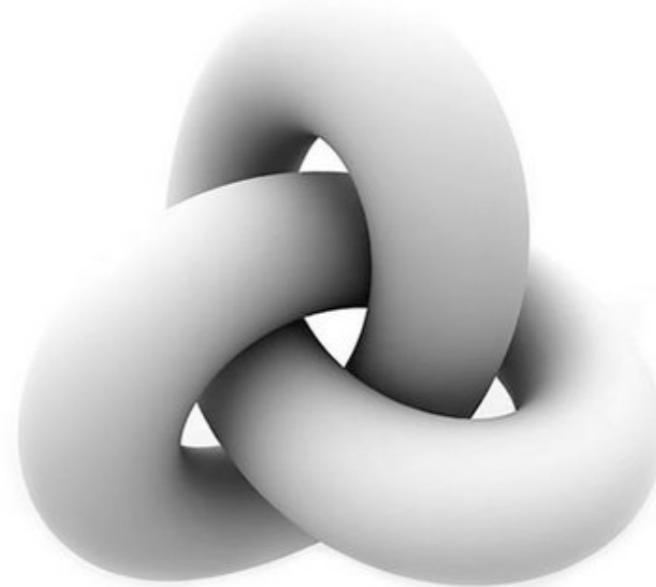
Dr. David Gay, Department of Mathematics

Topics.

- The Question
- Knot Examples
- 3D Mesh Manipulation
- 3D Printing
- Photo Gallery
- Results
- Final Remarks and Questions

The Question.

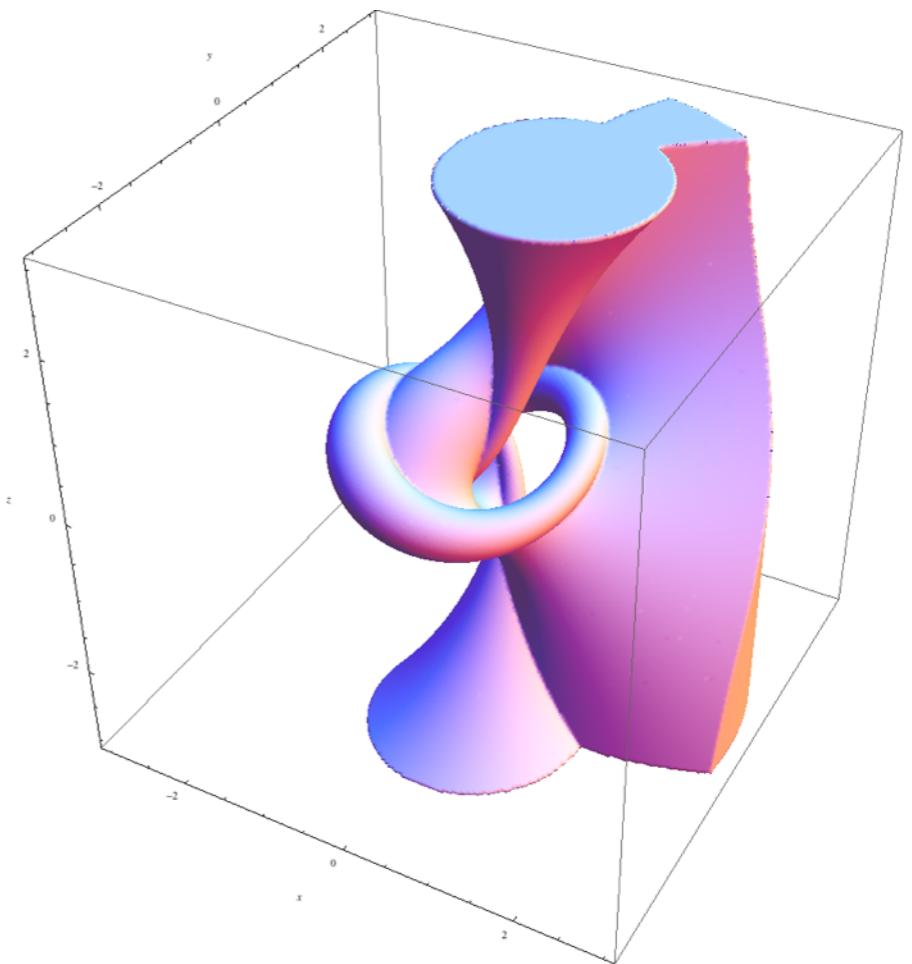
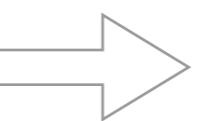
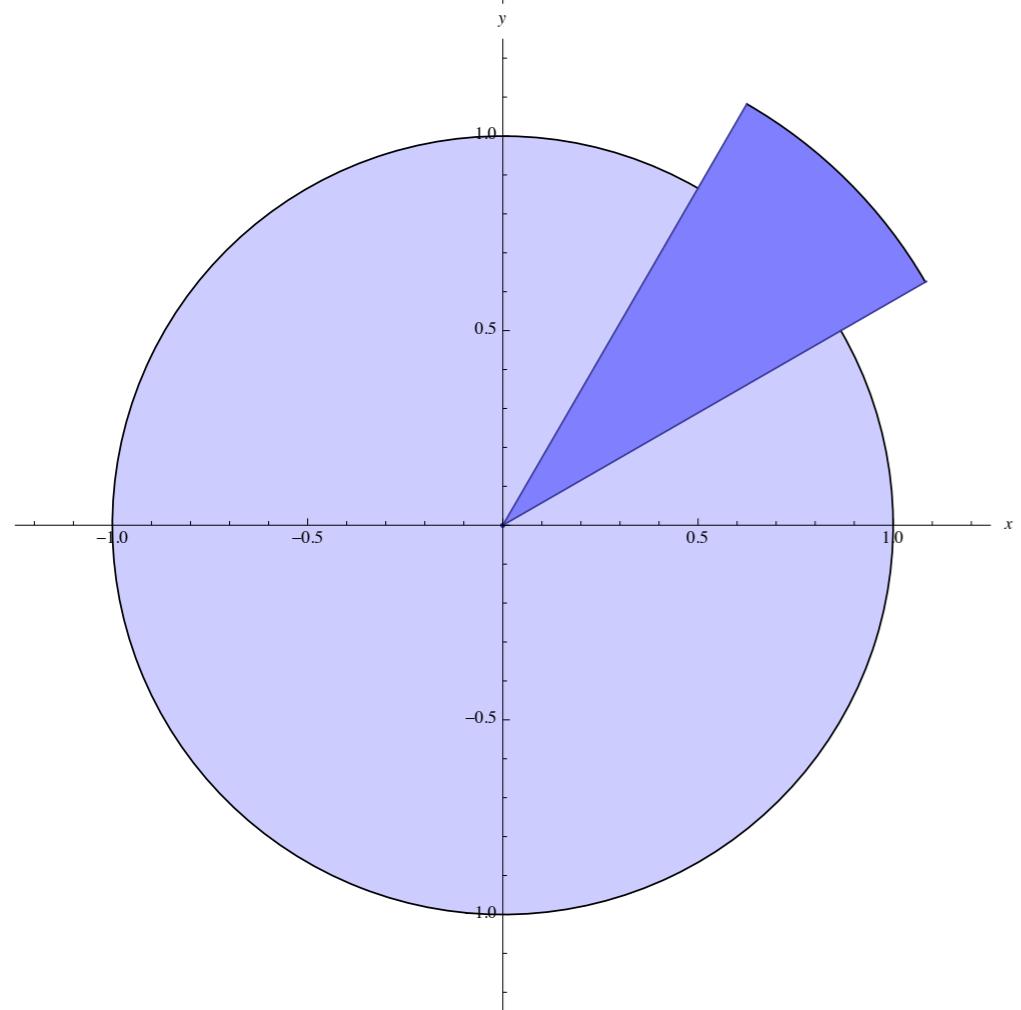
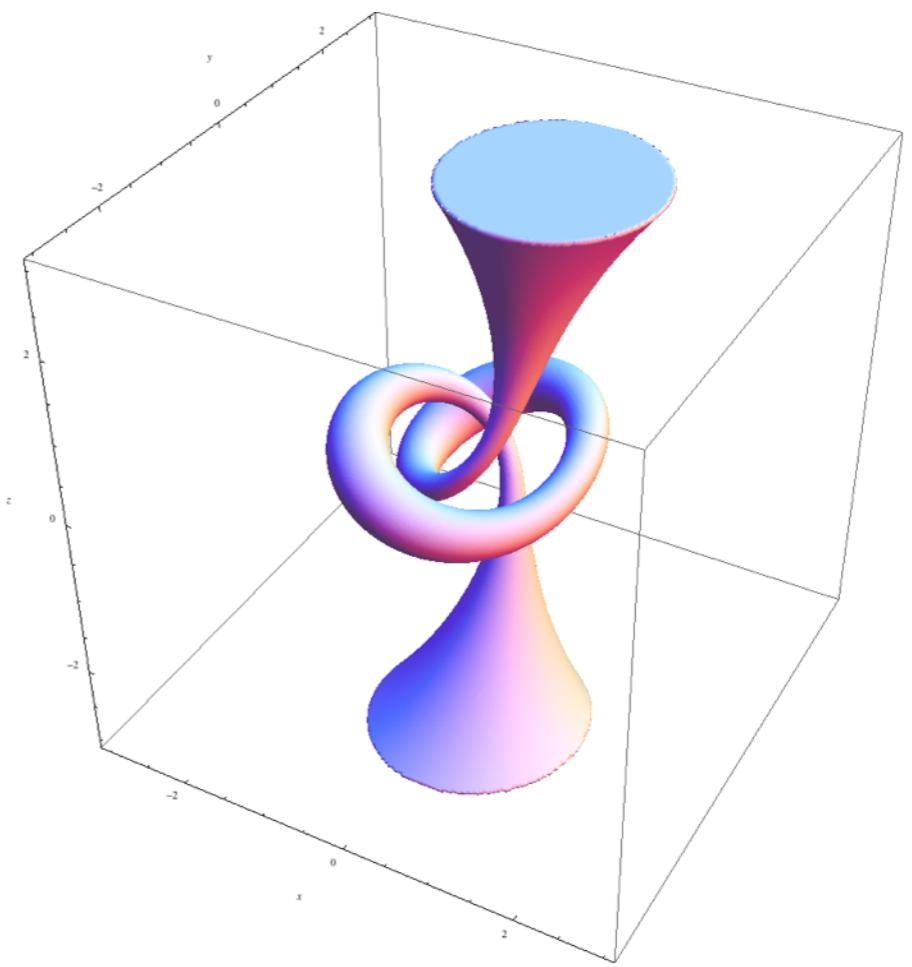
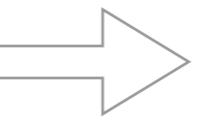
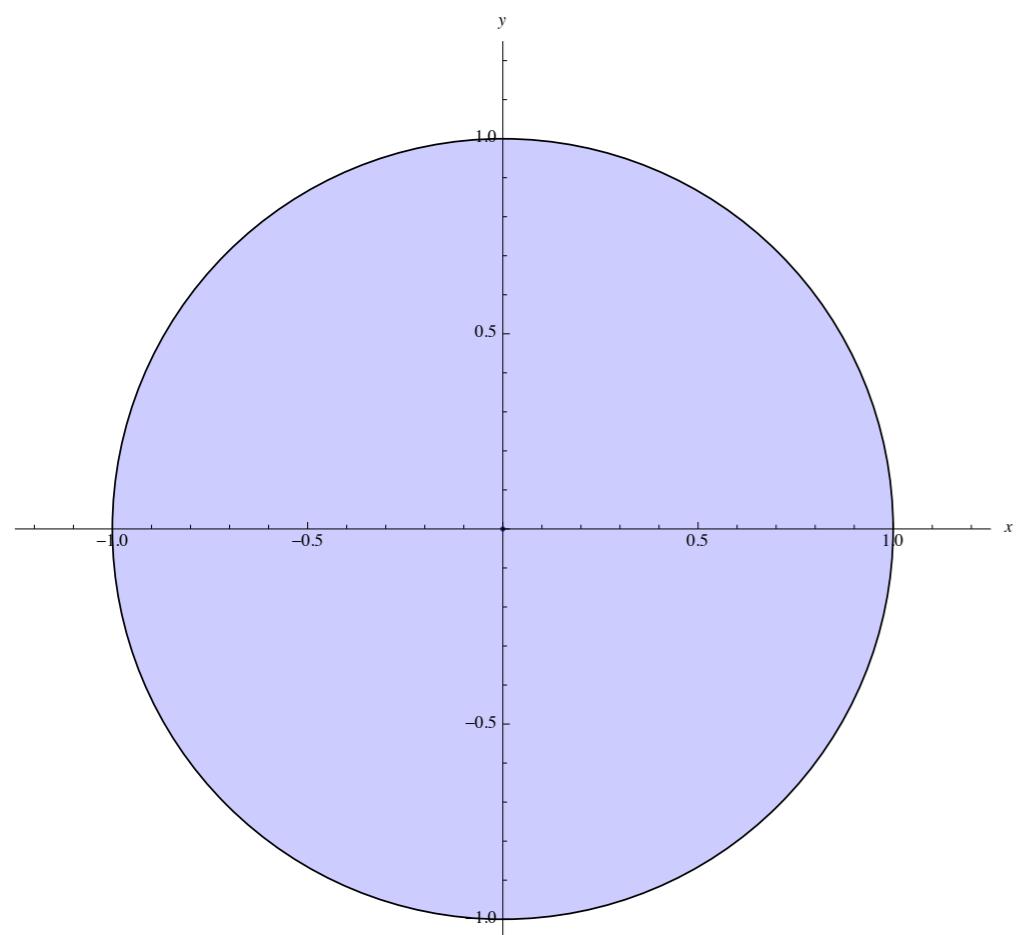
- Given a **complicated** shape defined **mathematically**, what is the best way to create a **puzzle** from the shape demonstrating its **properties**?
- Construction of object.
 1. Take the inverse image of sets in complex plane
 2. Stereographic project into 3D from 4D
- In other words, composition of two functions.

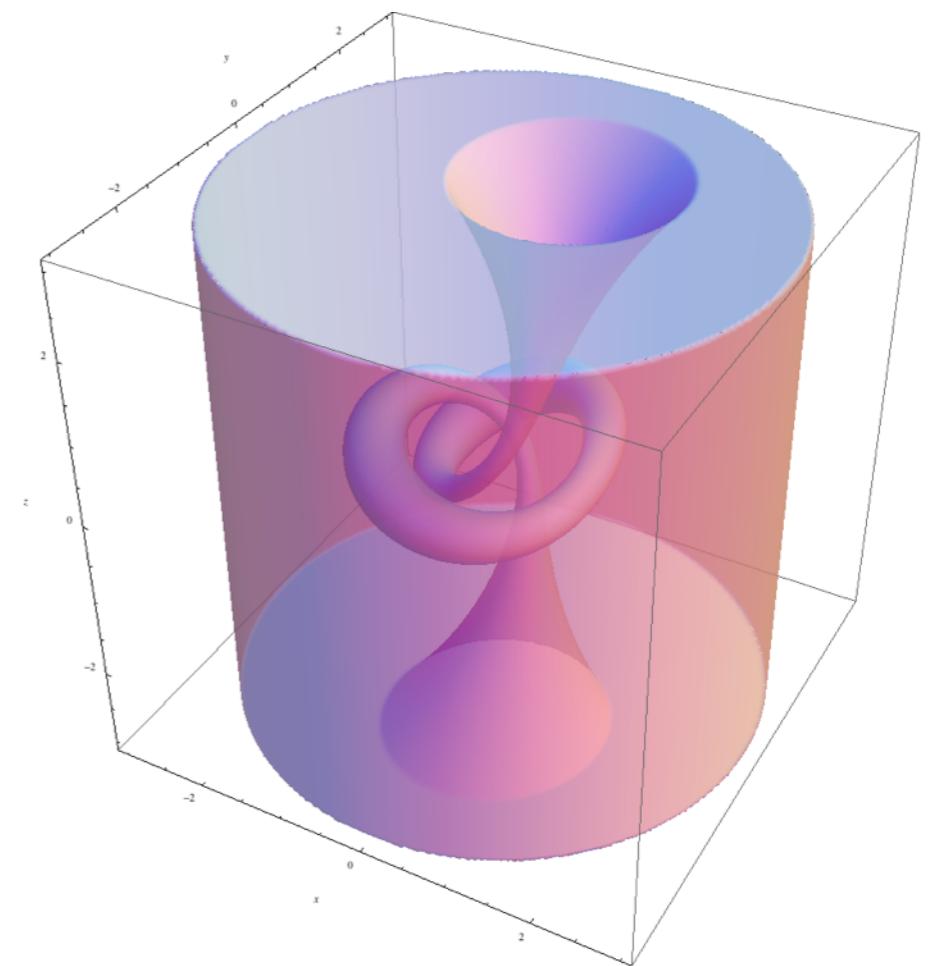
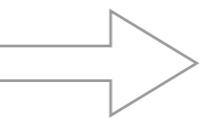
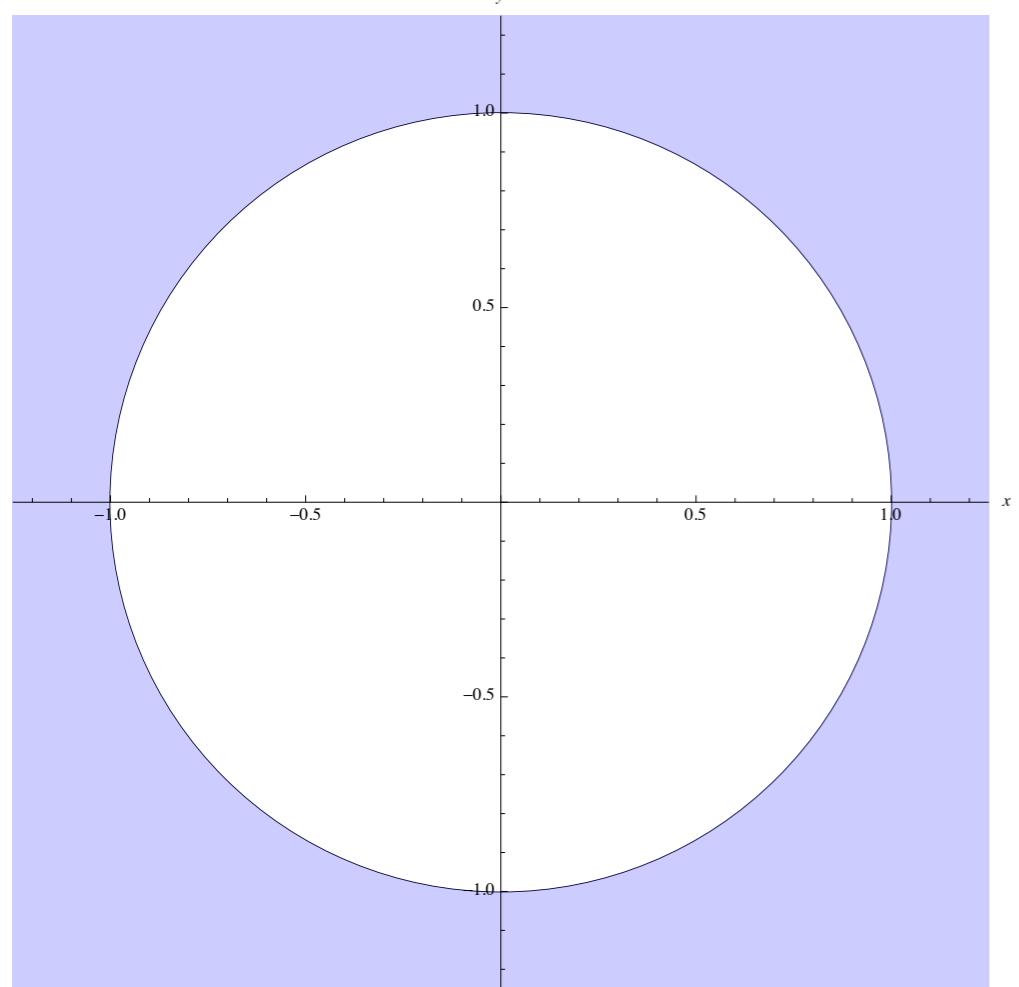
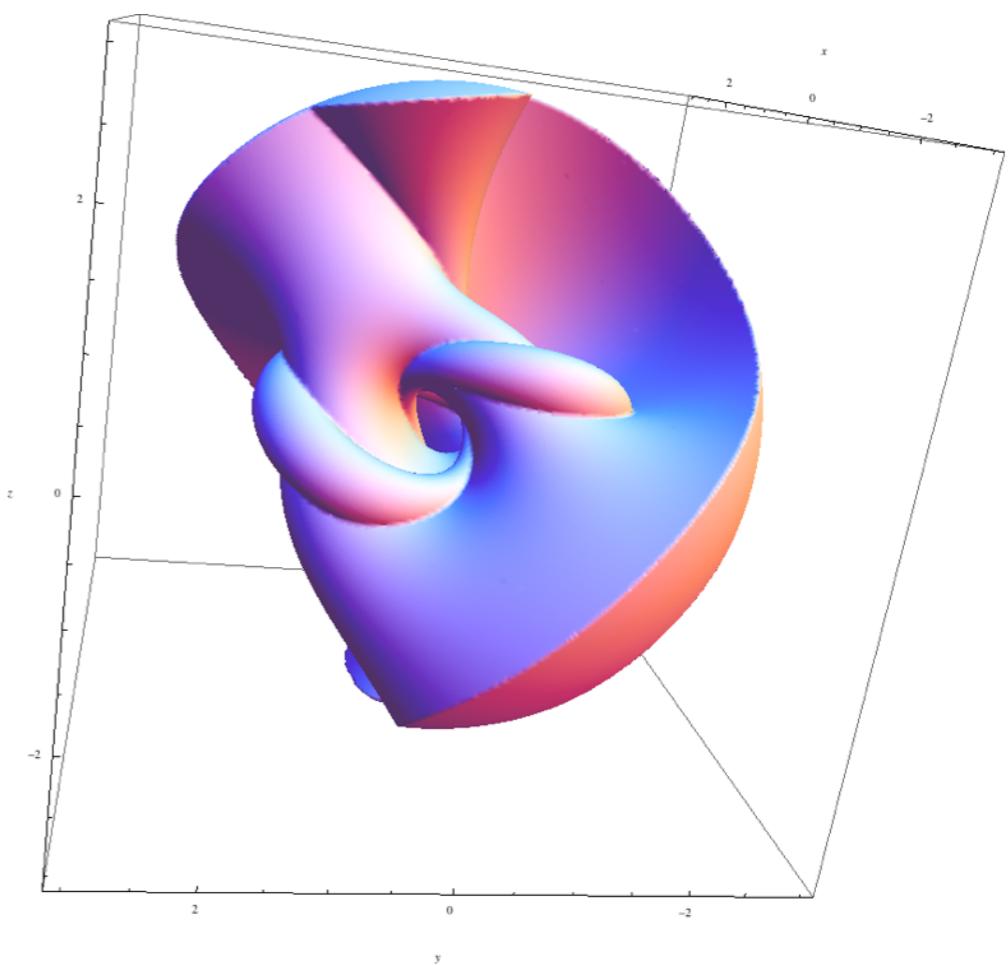
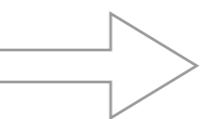
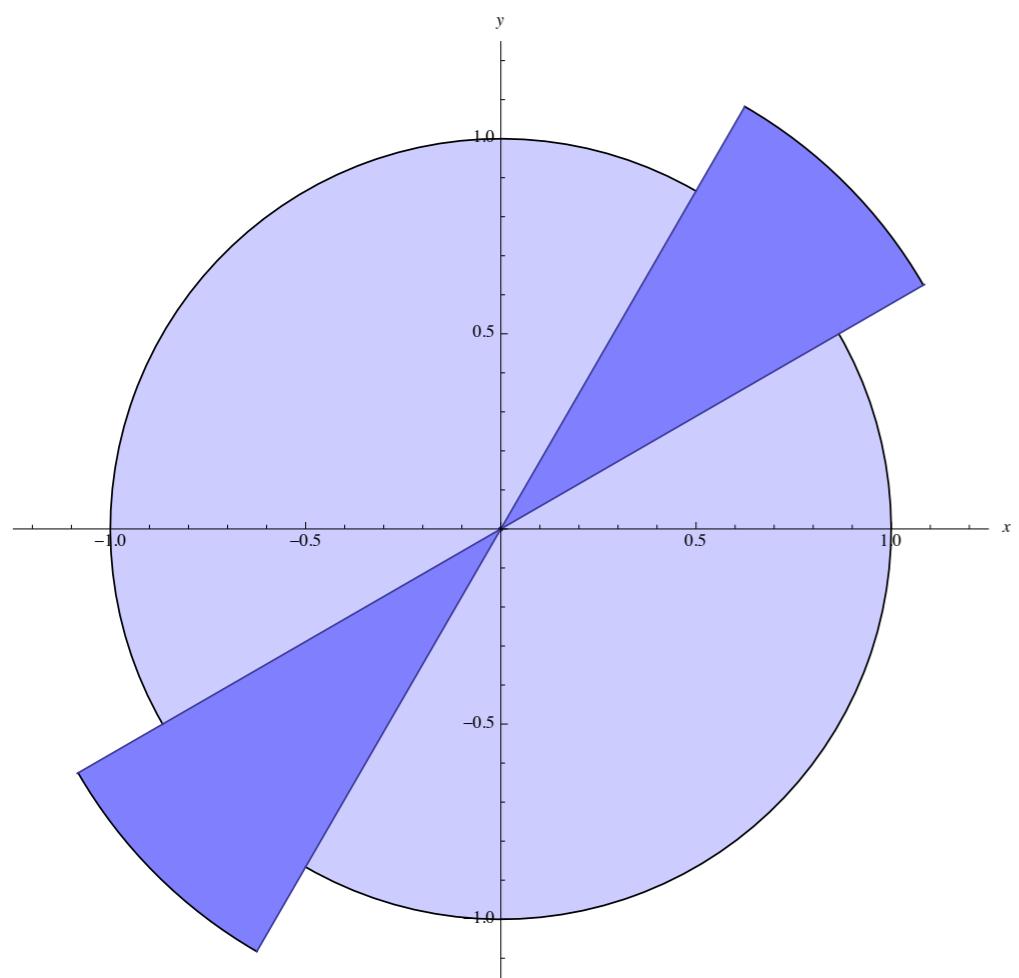


Theory.

- **Stereographic Projection:**
Mapping from the sphere in 3D
to the plane in 2D
- Now generalize: project from
the “sphere” in 4D ($\mathbb{C}^2 = \mathbb{R}^4$)
onto \mathbb{R}^3
- **Open-book decomposition:**
imagine our knot is the spine of
book, what does a page look
like inside?



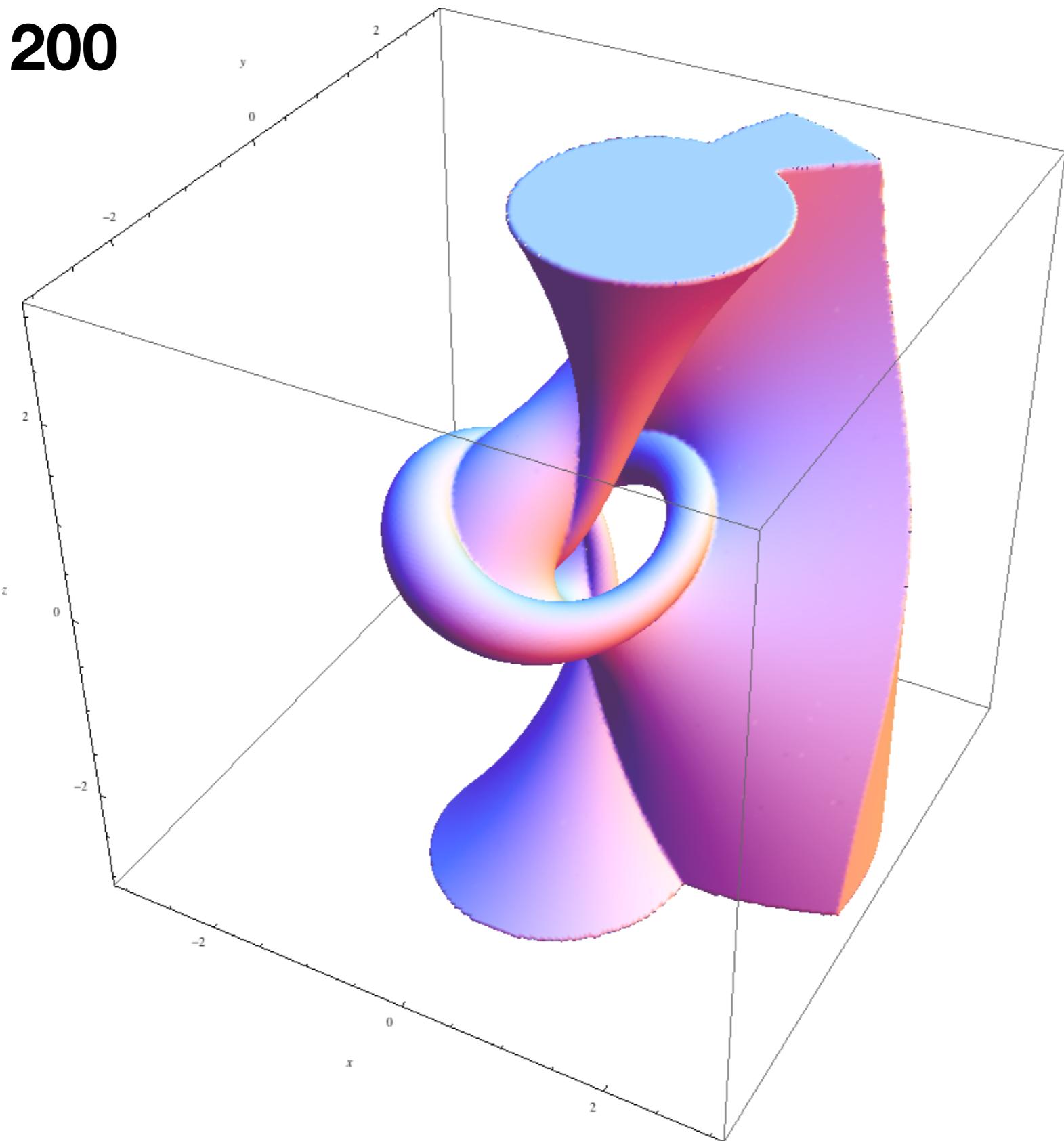




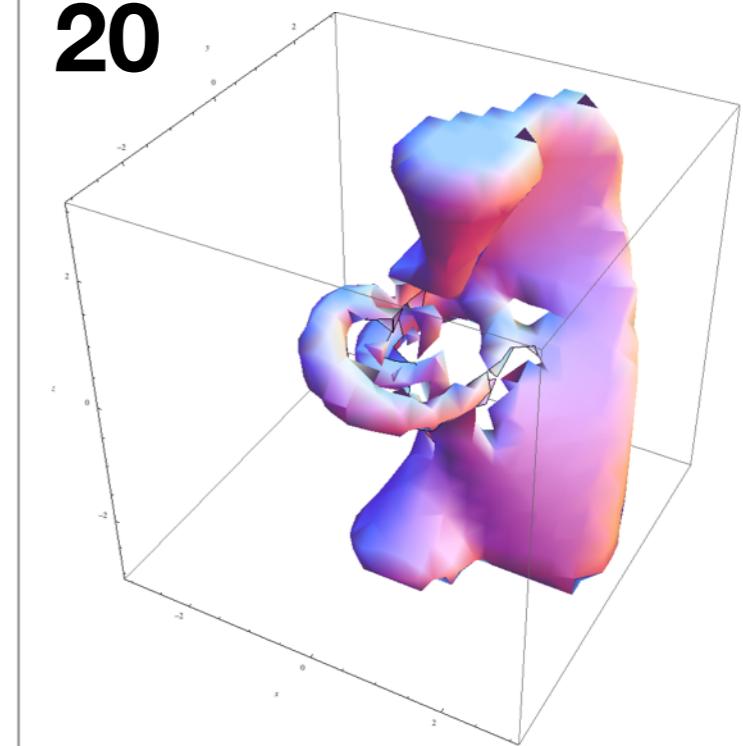
Generating 3D Models.

- **ST**ereo**L**ithography files (.stl) describe surface geometry of 3D objects
- Software: Mathematica, Blender, Meshlab, CAD, etc.
- **Resolution:** quality of model
 - more triangles + more vertices = higher resolution
 - Achieved in Mathematica via PlotPoints

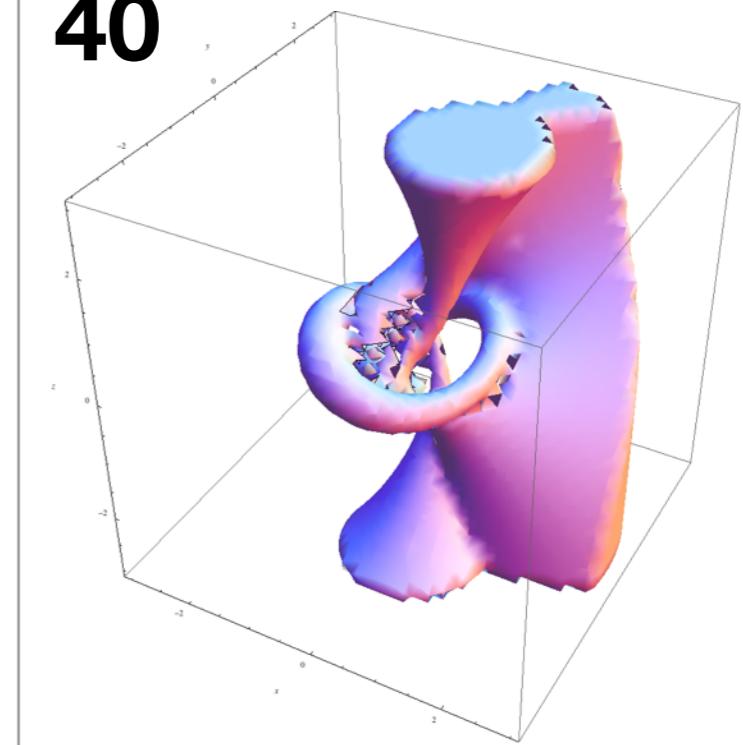
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20



40



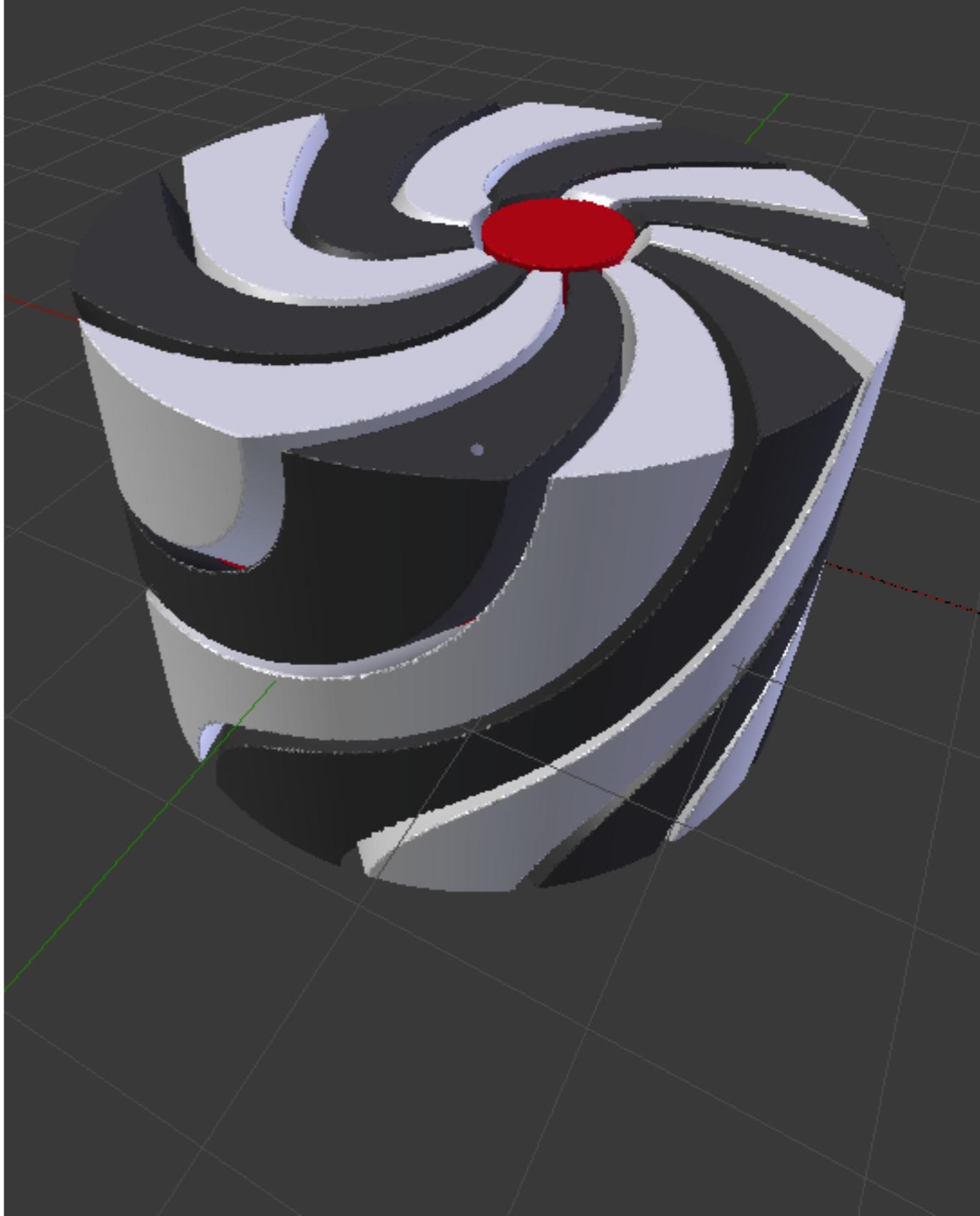
Generating 3D Models—Trefoil Knot + Page.
Created in Mathematica.

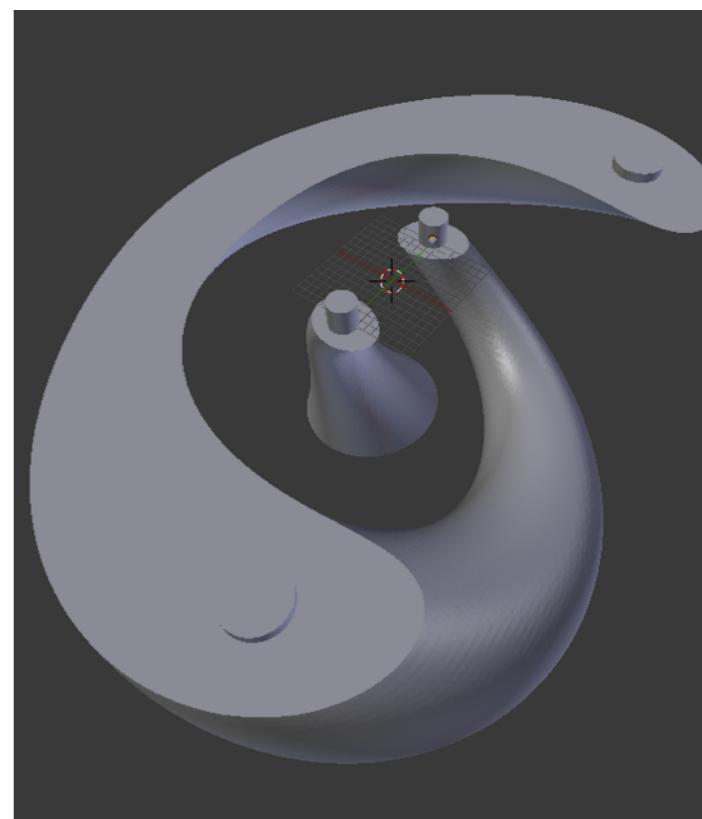
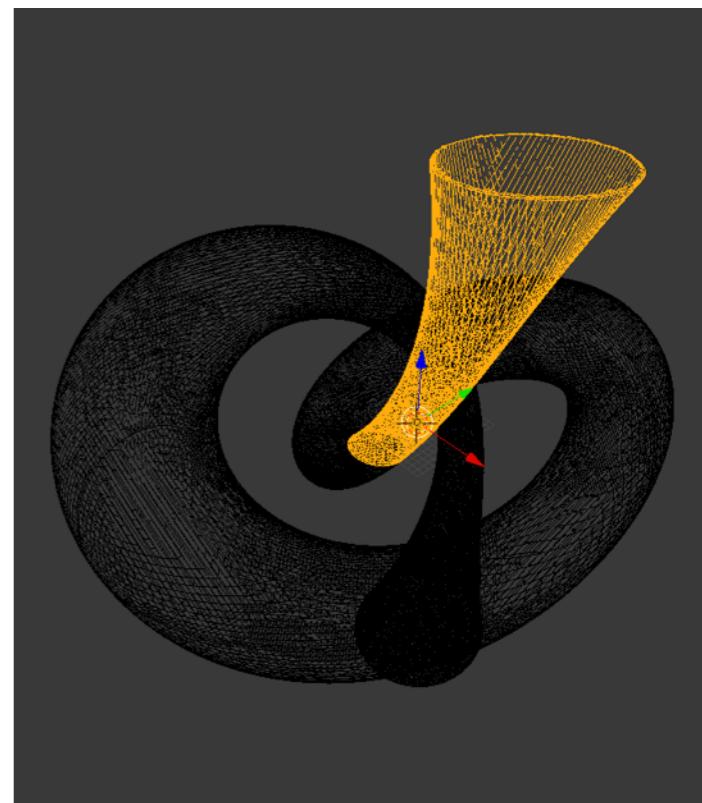
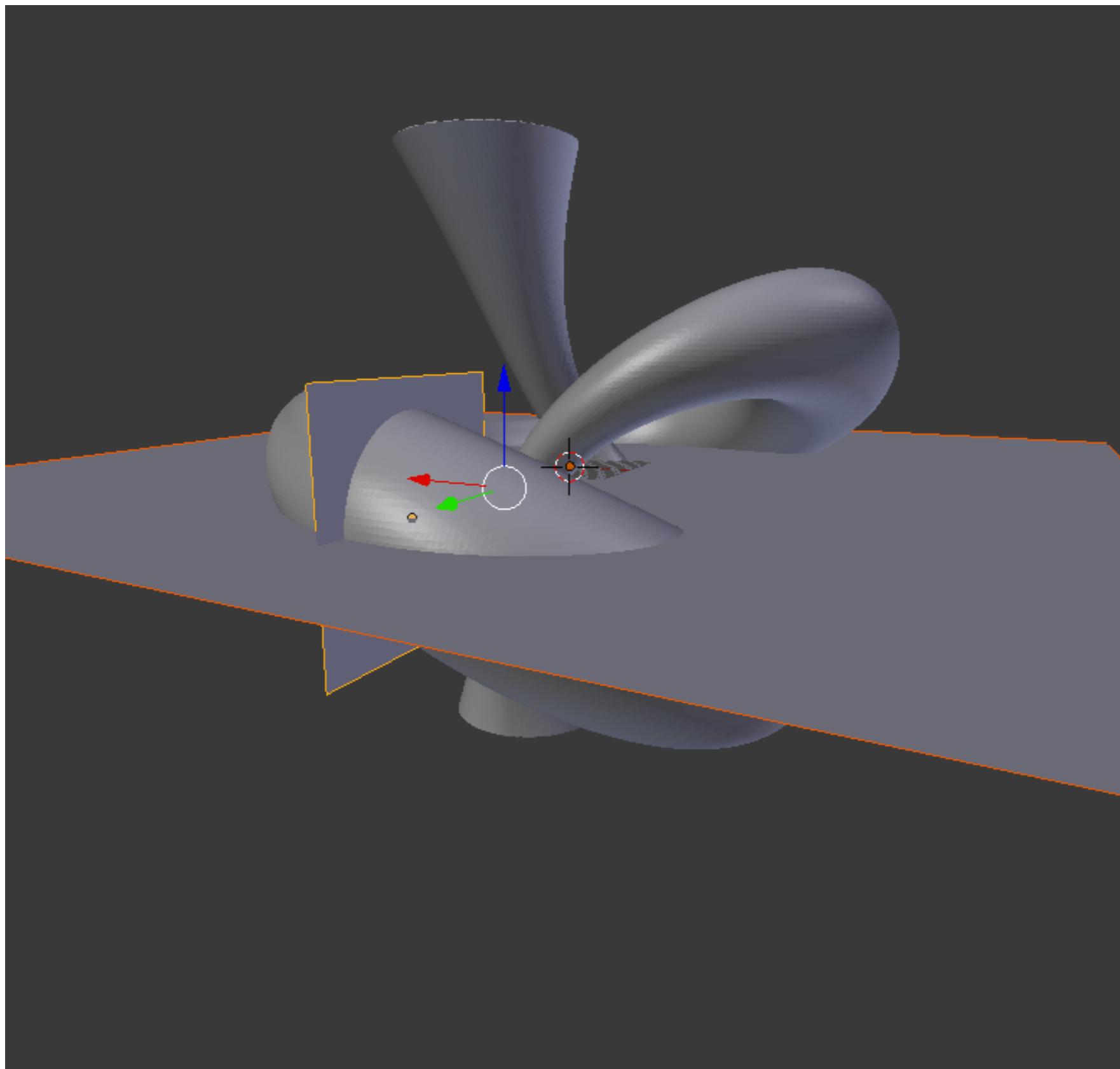
3D Mesh Manipulation.

1. Cut in half
 2. Account for connected pages
6 and 7
 3. Cut trefoil knot to fit inside
pages
 4. Add numbers to each half
page
 5. Add holes to each piece
-

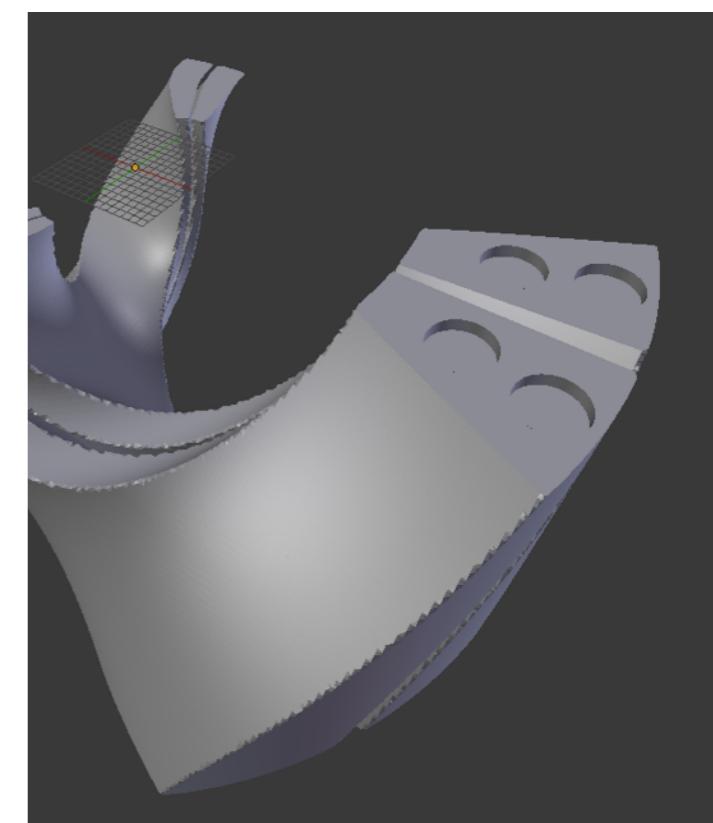
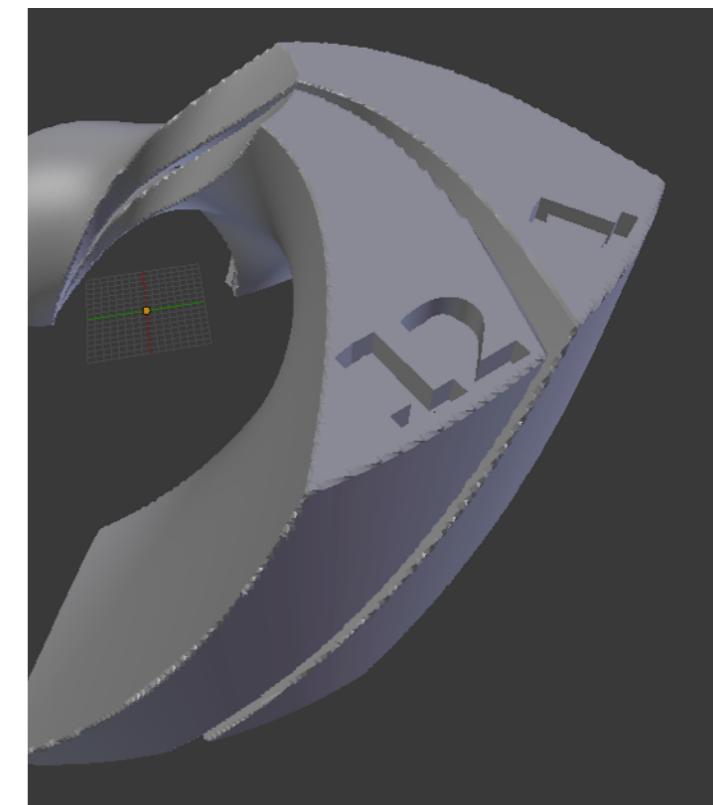
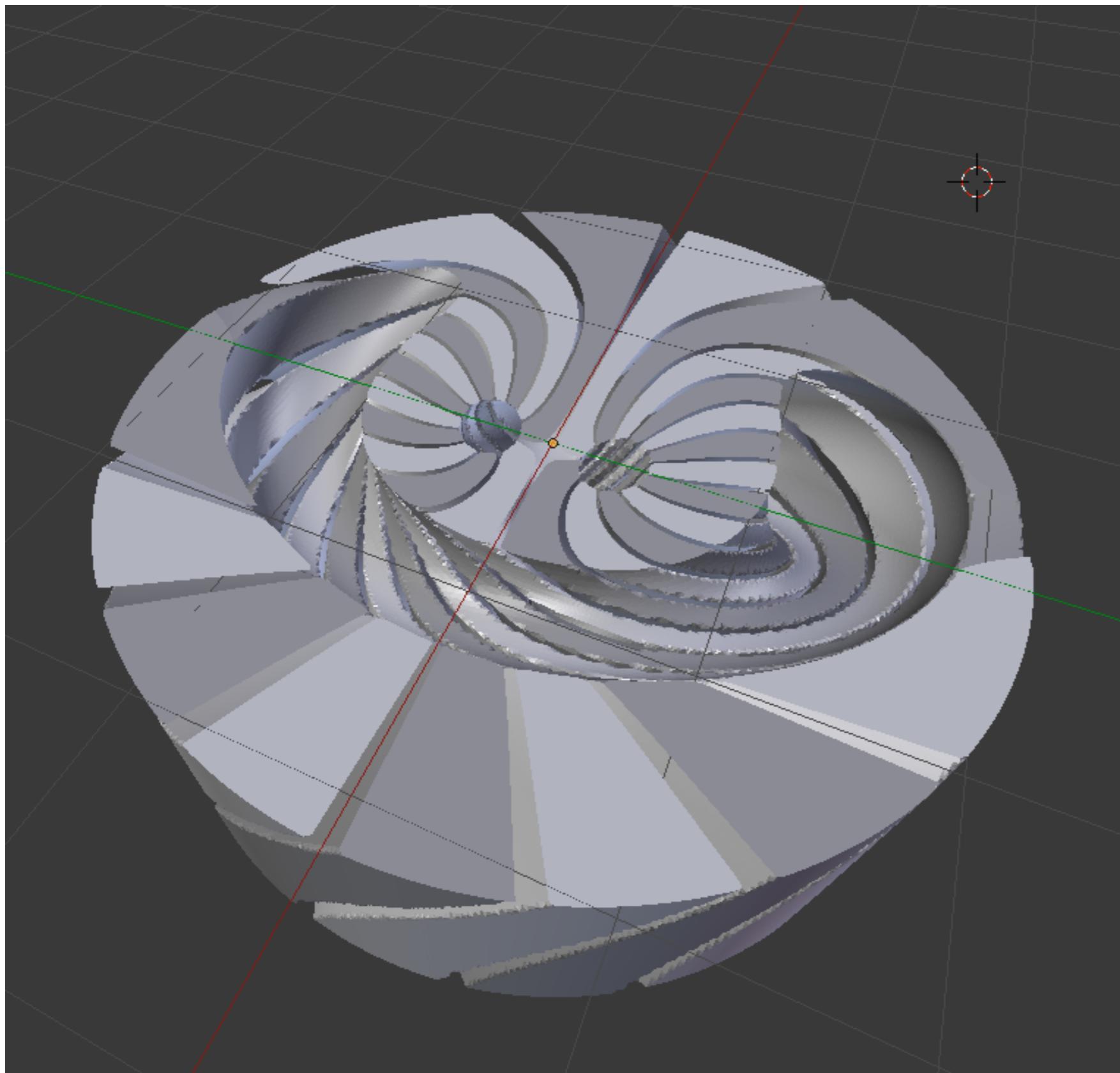
12+12+2+2+6 = 34 pieces.

24+24+6+6+10 = 70 holes = 70 magnets.

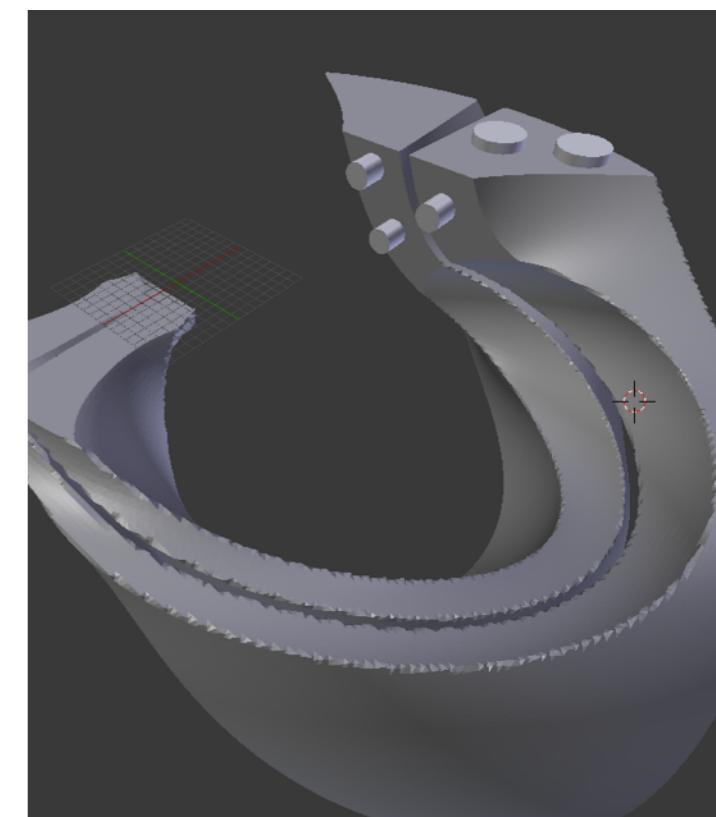
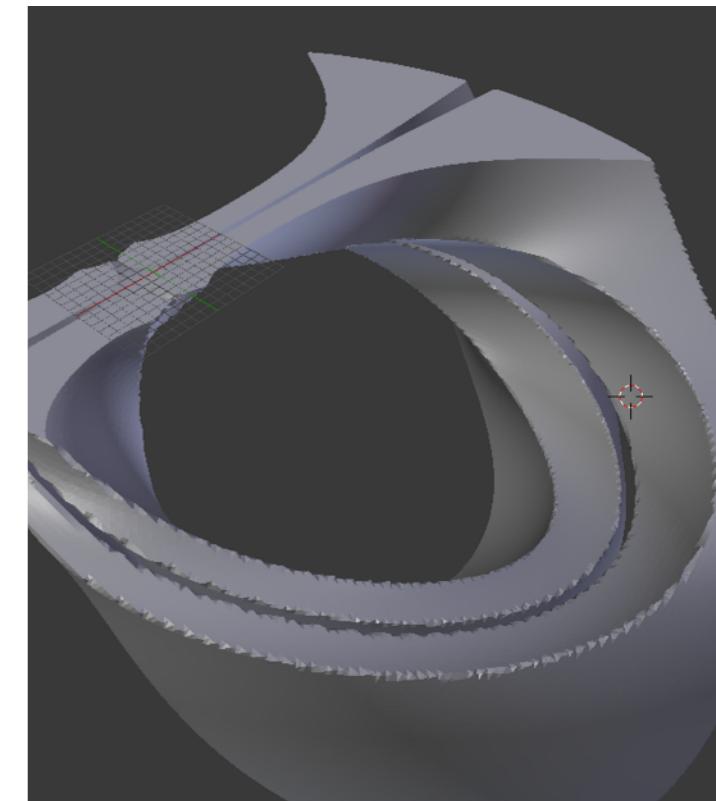
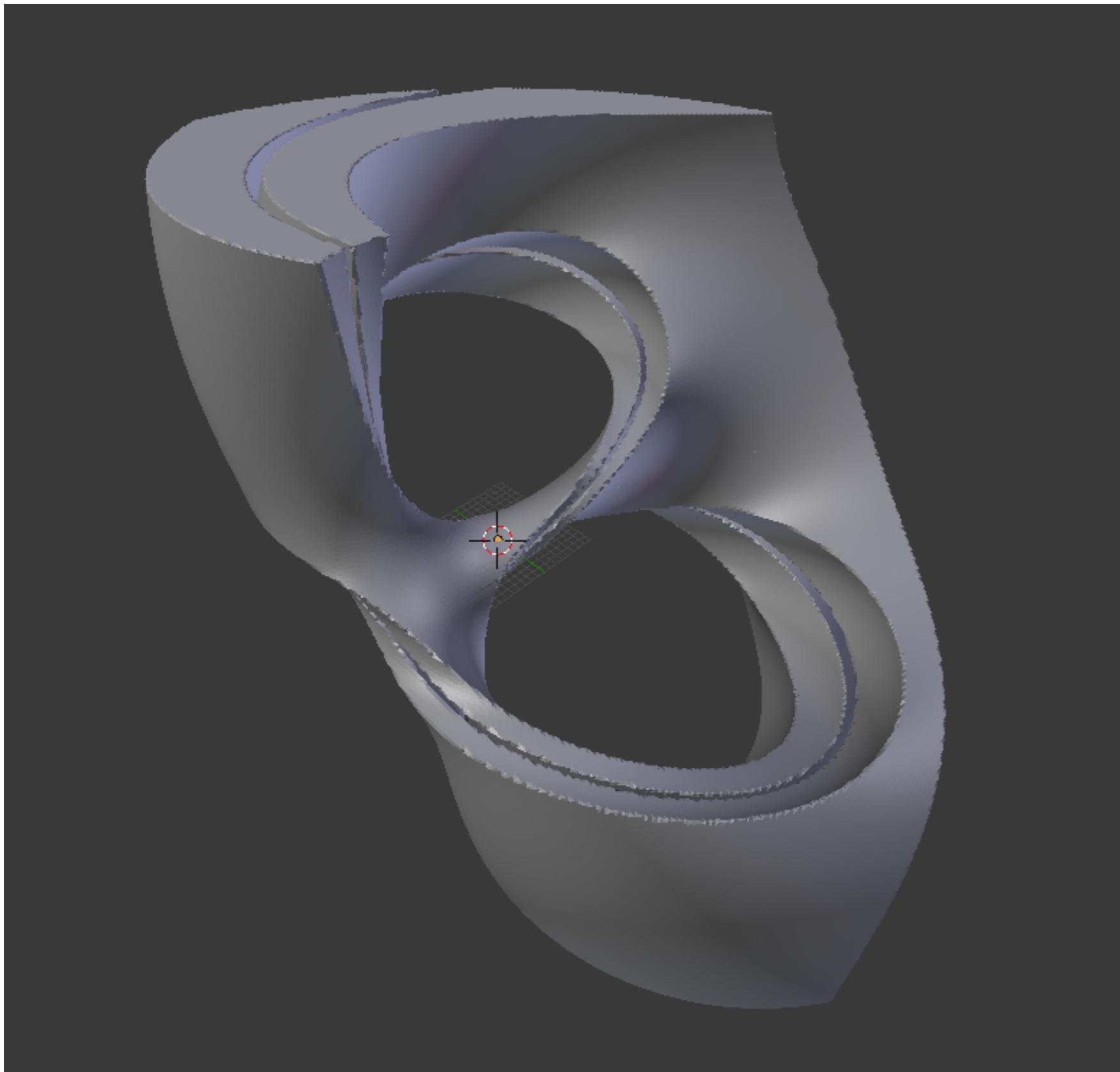




Blender Manipulations: Trefoil Knot.



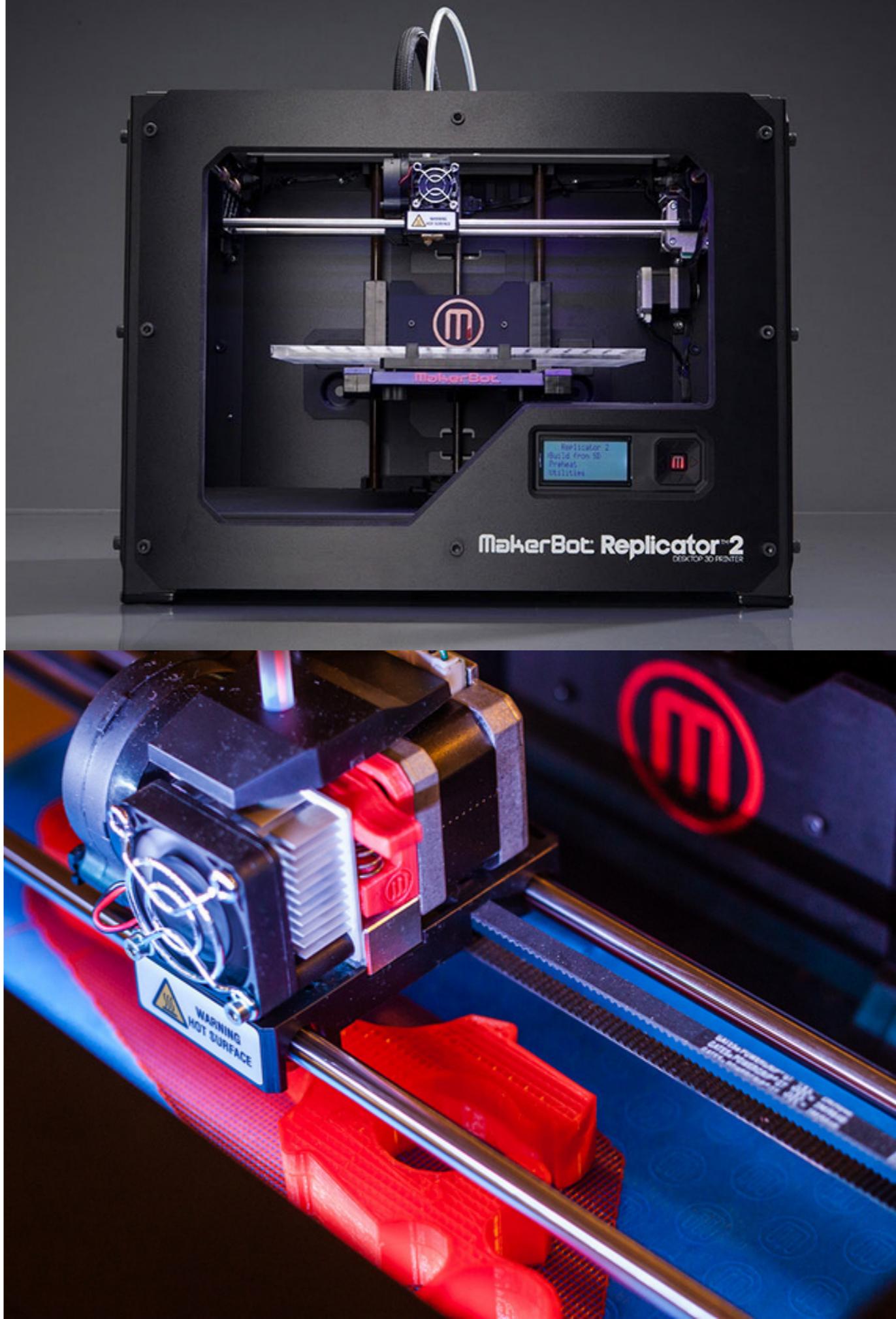
Blender Manipulations: 12 Pages.



Blender Manipulations: Pages 6 and 7.

3D Printing.

- MakerBot Replicator 2
- Controls:
 - **Raft and Supports:** on
 - Infill percentage (how solid/hollow the object is): 15%
 - Layer Height: 0.2mm
 - Extruder temperature: 230° C, ~446° F
 - Number of shells (boundary layers): 3



Trefoil Knot
+ 12 Pages.

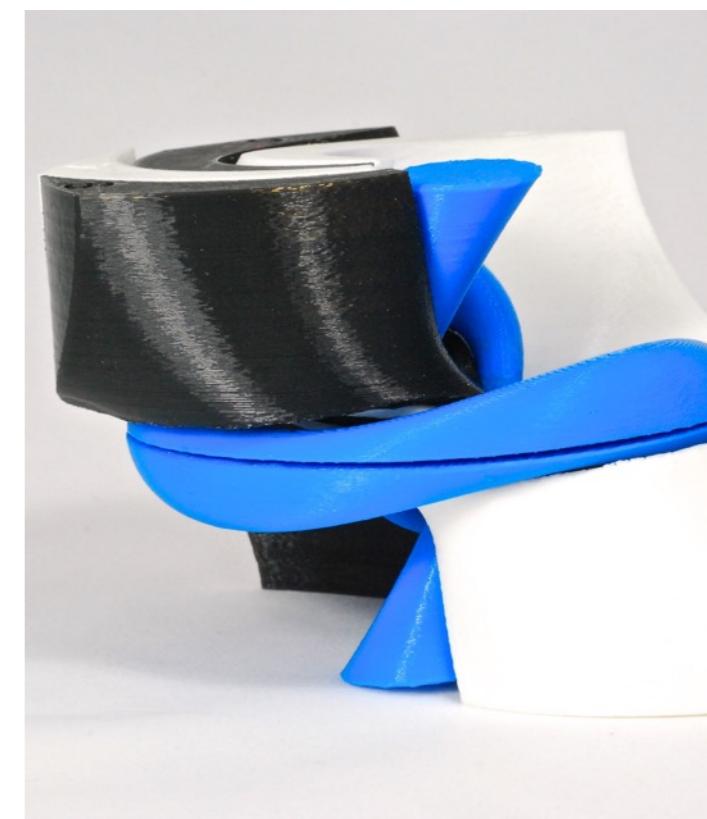




Trefoil Knot + 12 Pages.



Trefoil Knot + 4 Pages.



Trefoil Knot + 4 Pages.



Trefoil Knot + 6 Pages.



Trefoil Knot + 4 Pages.

Results.

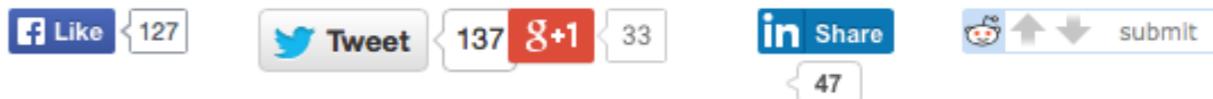
- Handful of models posted on **Thingiverse**
 - MakerBot's website for digital design file sharing
 - Currently has 6500+ views and 1000+ downloads (Mar. 31, 2015)
- **Wolfram Community**
 - Over 5,500+ views
- Written about by **3dprint.com**



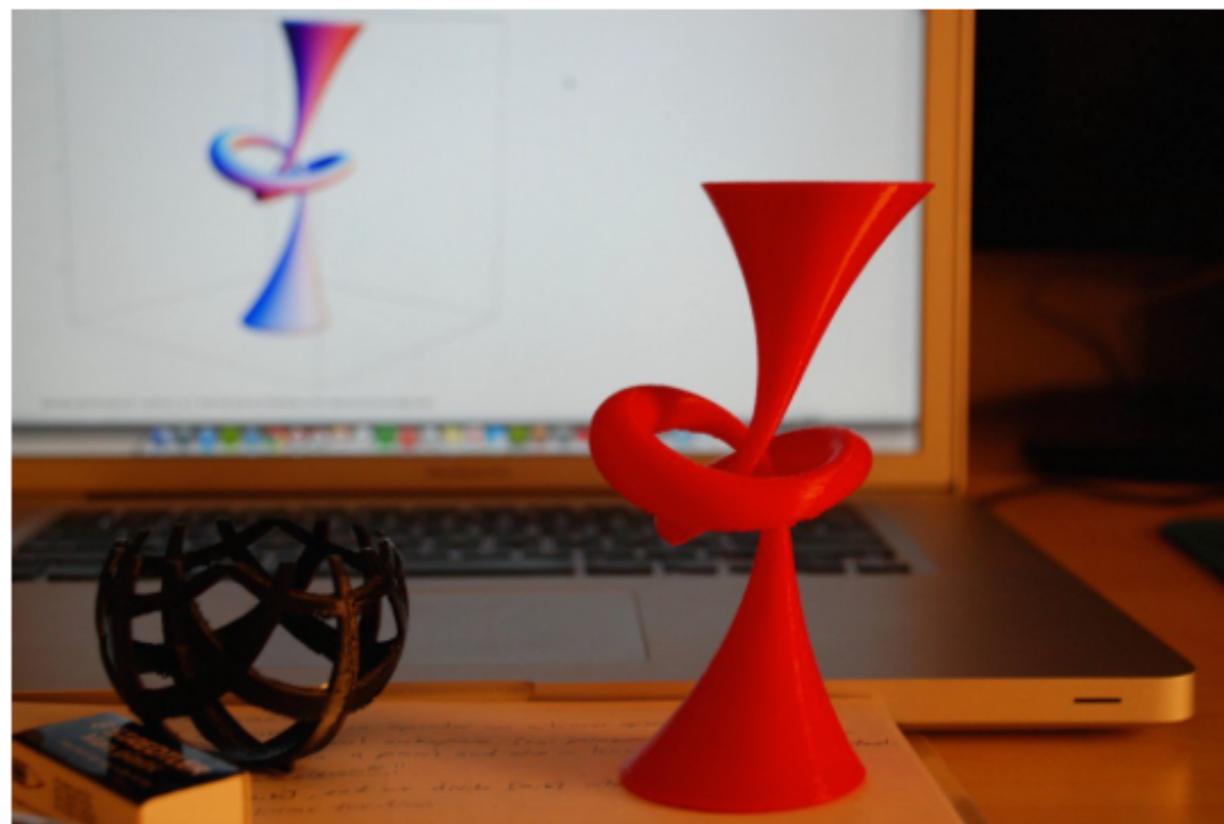
3D DESIGN / 3D PRINTING

Mathematics/Physics Student Creates 3D Printed Puzzle of Trefoil Knot, Catches Mathematical Community's Interest

BY SARAH ANDERSON · DECEMBER 5, 2014



Senior year of high school, I made room for drama in my class schedule by dropping from the track of honors math courses (Calculus? no, thanks!) I'd been taking since middle school. I went on to double-major in English and Theatre, which obviously my parents just loved. Since then, I've been working as an industry tech editor and writer for the better part of a decade, go figure.



Final Remarks and Questions.

- **Tinkercad**: free 3D modeling program that runs in the browser (Chrome)—no downloading required!
 - Short tutorial (~10 minutes)
 - Easiest way to begin to model
- **Thingiverse**

Final Remarks and Questions.

Questions?

Fred Hohman

fred.hohman@gmail.com

Dr. David Gay

dgay@math.uga.edu

References. (online)

- Mandalland Blogspot—Triangles
- Wikipedia—3D Printing
- Thingiverse—Stereographic Projection
- Makerbot
- Atomic Spin—Replicator 2 Issues

Full Photo Gallery available at
fredhohman.com/projects/trefoil-puzzle

