

THE ASSET ALLOCATION DECISION

Study Session 12

EXAM FOCUS

There is nothing difficult here, but the material is important because it is likely to be tested and it is the foundation for the portfolio construction material at Level 2 and especially Level 3. You should be ready to explain the what and why of an investment policy statement and know the objectives (risk and return)

and the constraints: liquidity, legal, time horizon, tax treatment, and unique circumstances. Know the four common return objectives, why the objectives part of the investment policy statement should include risk objectives, and (in broad strokes) the factors that influence risk tolerance.

LOS 52.a: Describe the steps in the portfolio management process and explain the reasons for a policy statement.

There are four general steps in the portfolio management process:

1. *Write a policy statement* that specifies the investor's goals and constraints and itemize the risks the investor is willing to take to meet these goals.
2. *Develop an investment strategy* designed to satisfy the investor's policy statement based on an analysis of the current financial and economic conditions.
3. *Implement the plan* by constructing the portfolio and allocating the investor's assets across countries, asset classes, and securities based on current and future forecasts of economic conditions.
4. *Monitor and update* the investor's needs and market conditions. Rebalance the investor's portfolio as needed. Rebalancing refers to shifting assets when the account allocations to different asset classes deviate significantly from the strategic asset allocation specified.

The **policy statement** is the framework that provides structure to the investment process. It forces investors to understand their own needs and constraints and to articulate them within the construct of realistic goals. The policy statement helps investors understand the risks and costs of investing and guides the actions of the portfolio manager. In essence, the purpose of the policy statement is to impose investment discipline on the client and the portfolio manager.

Performance cannot be judged without an objective standard. The policy statement should state the standards by which the portfolio's performance will be judged and specify the benchmark that represents the investor's risk preferences. The portfolio performance should be measured relative to the stated benchmark and not simply by the portfolio's raw returns.

LOS 52.b: Explain why investment objectives should be expressed in terms of both risk and return and list the factors that may affect an investor's risk tolerance.

Investment objectives must be stated in terms of both risk and return.

Return objectives may be stated in absolute terms (dollar amounts) or percentages. Return considerations also cover capital preservation, capital appreciation, current income needs, and total returns.

Specifying investment goals in terms of just return may expose an investor to inappropriate, high-risk investment strategies. Also, return-only objectives can lead to unacceptable behavior on the part of investment managers, such as excessive trading to generate commissions (churning).

Risk tolerance is a function of the investor's psychological makeup and personal factors such as age, family situation, existing wealth, insurance coverage, current cash reserves, and income.

LOS 52.c: Describe the return objectives of capital preservation, capital appreciation, current income, and total return and describe the investment constraints of liquidity, time horizon, tax concerns, legal and regulatory factors, and unique needs and preferences.

Return Objectives

Capital preservation is the objective of earning a return on an investment that is at least equal to the inflation rate with little or no chance of loss. The concern here is the maintenance of purchasing power. To achieve this objective, the nominal rate of return must at least equal the inflation rate. This is an appropriate goal when the funds will be needed in the near future.

Capital appreciation is the objective of earning a nominal return that exceeds the rate of inflation over some period of time. Achieving this goal means that the purchasing power of the initial investment increases over time, usually through capital gains. This is an appropriate goal when the need for the funds is further in the future, such as for retirement.

Current income is the objective when the primary purpose of an account is to produce income as opposed to capital appreciation. The current income objective is usually appropriate when an investor wants or needs to supplement other sources of income to meet living expenses or some other planned spending need, as in retirement.

Total return is the objective of having a portfolio grow in value to meet a future need through both capital gains and the reinvestment of current income. The total return objective is riskier than the income objective but less risky than the capital appreciation objective. This would be an appropriate objective for an investor with a longer-term investment horizon but only moderate risk tolerance.

Investment Constraints

Liquidity refers to the ability to quickly convert investments into cash at a price close to their fair market value. Liquidity, from the investor's view, is the potential need for ready cash. This may necessitate selling assets at unfavorable terms if adequate liquidity is not provided in the portfolio.

Time horizon (investment horizon) refers to the time between making an investment and needing the funds. There is a relationship between an investor's time horizon, liquidity needs, and ability to handle risk. Since losses are harder to overcome in a short time frame, investors with shorter time horizons usually prefer lower-risk investments.

Tax concerns play an important role in investment planning because after-tax returns are what investors should be concerned with. The tax codes in the U.S., as in most other countries, are complex. For instance, in the U.S., interest and dividends were, until recently, taxed at the investor's marginal tax rate, while capital gains were taxed at another rate. Taxes on unrealized capital gains can be deferred indefinitely, and estate taxes must be considered. Other tax-related issues include the following:

- There is a trade-off between taxes and diversification needs. The decision to sell some stock to diversify one's portfolio by reinvesting the proceeds in other assets must be balanced against the resulting tax liability.
- Some sources of income are tax exempt at the federal and state levels. For example, high-income individuals are motivated to invest in municipal bonds because the interest income is tax free.

- The investor must also consider tax-deferred investment opportunities such as IRAs, 401(k) and 403(b) plans, and various life insurance contracts.
- Young investors will want to put as much as possible into tax-deferred plans. The only drawback is the loss of liquidity.
- For older retirees, the need for tax-deferred investments may decrease. Also, taxable income may now offer higher after-tax returns than tax-exempt income. If a retirement account contains a lot of an old employer's stock, diversification becomes more important than taxes.

Legal and regulatory factors are more of a concern to institutional investors than individuals, but the investment strategies of both may be restricted due to these constraints.

Unique needs and preferences are constraints that investors may have that address special needs or place special restrictions on investment strategies for personal or socially conscious reasons. This is a catch-all constraint category for those "special" circumstances that don't fit neatly into one of the other constraint areas.

LOS 52.d: Describe the importance of asset allocation, in terms of the percentage of a portfolio's return that can be explained by the target asset allocation and list reasons for the differences in the average asset allocation among citizens of different countries.

Several studies support the idea that approximately 90% of a portfolio's returns can be explained by its target asset allocations. The clear implication of this result is that differences in returns among asset classes are much more important than differences in security selection in determining overall portfolio returns. For actively managed funds, actual portfolio returns are slightly less than those that would have been achieved if the manager strictly maintained the target asset allocation. This illustrates the real difficulty of improving returns by varying from target allocations (market timing) or by selecting undervalued securities in very efficiently priced markets.

Average asset allocations across countries differ for reasons related to demographics, social factors, legal constraints, and taxation. Countries with younger populations tend to have greater average allocations to equities. Some countries have legal restrictions on the percentage of equities that various institutions can hold. The existence of a strong government pension program in Germany tends to decrease the equity holdings of workers and private pension plans since less growth is needed for retirement funding needs. The German society also has an historical aversion to financial risk, and equity ownership is not typical for its citizens. Differences in historical inflation rates are correlated with differences in equity allocations across countries as well. Countries with higher historical inflation rates tend to have greater allocations to equities.

KEY CONCEPTS

1. The four-step portfolio management (investment) process:
 - Write a policy statement.
 - Develop an investment strategy.
 - Implement the plan by constructing the portfolio and allocating the assets.
 - Monitor, update, and rebalance the portfolio.
2. An investment policy statement provides investment discipline by requiring investors to articulate their needs, goals, and risk tolerance, ensuring that goals are realistic, and providing an objective measure of portfolio performance.
3. Investment objectives should be expressed in terms of both risk and return so that meeting the return objective does not expose the investor to more risk than he is prepared to tolerate.
4. Risk tolerance depends on an investor's psychological profile and other personal factors, including family situation, wealth, income, age, and insurance coverage.
5. Common return objectives are:
 - Capital preservation—minimizing the risk of loss in real terms.
 - Capital appreciation—managing real growth in the portfolio to meet some future need.
 - Income—meeting specified spending needs.
 - Total return—growing a portfolio through capital appreciation and reinvested income.

6. Investment constraints include:
 - Liquidity—for cash spending needs (anticipated or unexpected).
 - Time horizon—when funds will be needed.
 - Tax—the tax treatments of various accounts, and the investor's marginal tax bracket.
 - Legal—restrictions on investments in retirement, personal, and trust accounts.
 - Unique needs—constraints because of investor preferences or other factors not already considered.
7. Target allocations to different asset classes can explain approximately 90% of the differences in portfolio returns over time.
8. Differences in average asset allocations across countries exist due to differences in social factors, demographics, legal constraints, tax laws, and historical inflation rates.

CONCEPT CHECKERS: THE ASSET ALLOCATION DECISION

1. Which of the following is NOT an example of a portfolio constraint?
 - A. Tax concerns.
 - B. Liquidity needs.
 - C. Total return requirement of 15%.
 - D. Legal and regulatory requirements.
2. Which of the following statements about investment policy statements is TRUE?
 - A. For some investors, specifying an investment goal in terms of return alone is appropriate.
 - B. An investment policy statement should have objectives and constraints.
 - C. Risk is an important investment policy constraint.
 - D. Taxes are the most important legal constraint.
3. The approximate percentage of a portfolio's returns that can be explained by differences in target portfolio asset allocations is:
 - A. 10%.
 - B. 50%.
 - C. 90%.
 - D. 100%.
4. The return objective of an investor who is relatively risk averse yet has a long time horizon and little need for liquidity would *most likely* be described as:
 - A. capital preservation.
 - B. capital appreciation.
 - C. total return.
 - D. long-term appreciation.
5. In determining the appropriate asset allocation for a client's investment account, the manager should:
 - A. consider only the investor's risk tolerance.
 - B. consider the investor's risk tolerance and future needs, but not market conditions.
 - C. not consider market conditions but should consider the taxable status of the account.
 - D. rely on forecasts of future economic conditions.

ANSWERS – CONCEPT CHECKERS: THE ASSET ALLOCATION DECISION

1. C Return objectives are part of a policy statement's objectives, not constraints.
2. B The policy statement should specify objectives and constraints. Return should always be considered with risk. Taxes are a separate constraint, and setting a risk tolerance is an investment policy objective, not constraint.
3. C Studies suggest that approximately 90% of a portfolio's returns can be explained by target asset allocation choices.
4. C A total return strategy is appropriate for an investor with a longer-term investment horizon who is very risk tolerant. The inclusion of a significant allocation to income producing securities such as bonds and high-dividend stocks makes this a less risky strategy than that for an objective of capital appreciation.
5. D An advisor's forecasts of expected returns from and expected volatility (risk) of investing indifferent asset classes are an important part of determining an appropriate asset allocation.

The following is a review of the Portfolio Management principles designed to address the learning outcome statements set forth by CFA Institute®. This topic is also covered in:

AN INTRODUCTION TO PORTFOLIO MANAGEMENT

Study Session 12

EXAM FOCUS

This topic review looks at Markowitz portfolio theory and optimal portfolio choice. The major result is the development of the efficient frontier. Understanding the relationship between portfolio risk and correlation is the key to understanding modern portfolio theory. Be able to discuss diversification, correlation, indifference curves, expected return, and the efficient

frontier. A sound grasp of the portfolio theory presented here is essential to an understanding of the capital market theory covered in our next topic review. Additionally, portfolio theory is the heart of much of the portfolio management material essential for the Level 3 exam.

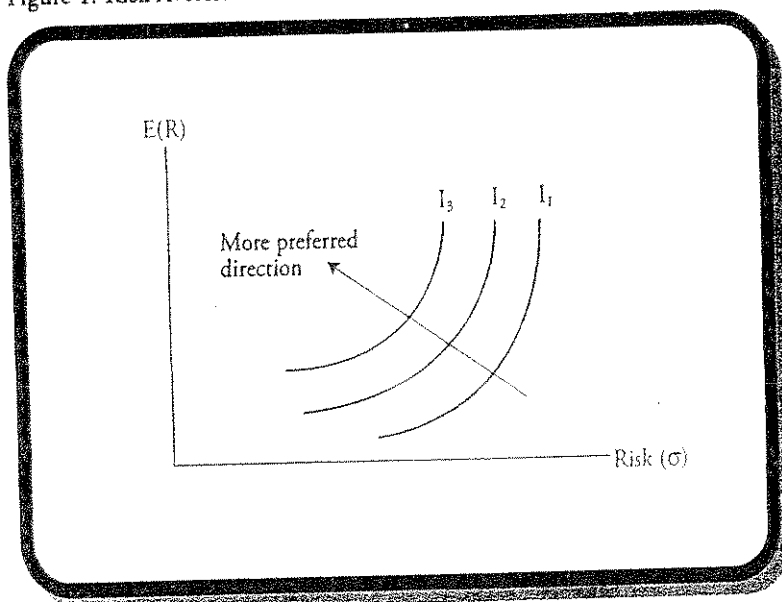
LOS 53.a: Define risk aversion and discuss evidence that suggests that individuals are generally risk averse.

Risk aversion refers to the fact that individuals prefer less risk to more risk. Risk-averse investors:

- Prefer lower to higher risk for a given level of expected returns.
- Will only accept a riskier investment if they are compensated in the form of greater expected return.

In Figure 1, we examine the concept of risk aversion using indifference curves.

Figure 1: Risk Aversion



The curved lines, I_1 , I_2 , and I_3 , represent *indifference curves* because all investments (combinations of risk and expected return) that lie along each curve are equally preferred. Because we have a “good” (expected return) and a “bad” (risk), a higher or more preferred indifference curve lies in the northwest direction (more expected return and less risk). Focusing on indifference curve I_1 , a risk-averse investor whose preferences are represented by these

curves will be equally happy with, or indifferent among, any risk/return combinations on this curve. Notice that as risk increases, a risk-averse investor demands an increasingly higher rate of return as compensation. While an investor would be equally happy with any point on I_1 , she prefers all risk/return combinations on I_2 to any combination on I_1 . In reality, there are an infinite number of indifference curves, and the indifference curves for any given investor can never cross.

The fact that most individuals buy some sort of insurance, whether auto, health, or homeowners, indicates that they are generally risk averse. Interestingly, however, an individual may exhibit risk-averse tendencies in one area and not in others. For example, many people buy auto insurance to protect against the costs associated with auto accidents but will not buy health insurance or will buy lottery tickets or participate in other forms of gambling.

LOS 53.b: List the basic assumptions behind the Markowitz Portfolio Theory.

In the investment framework he developed, Nobel laureate Harry Markowitz made the following assumptions about investor behavior:

- *Returns distribution.* Investors look at each investment opportunity as a probability distribution of expected returns over a given investment horizon.
- *Utility maximization.* Investors maximize their expected utility over a given investment horizon, and their indifference curves exhibit diminishing marginal utility of wealth (i.e., they are convex).
- *Risk is variability.* Investors measure risk as the variance (standard deviation) of expected returns.
- *Risk/return.* Investors make all investment decisions by considering only the risk and return of an investment opportunity. This means that their utility (indifference) curves are a function of the expected return (mean) and the variance of the returns distribution they envision for each investment.
- *Risk aversion.* Given two investments with equal expected returns, investors prefer the one with the lower risk. Likewise, given two investments with equal risk, investors prefer the one with the greater expected return.

Professor's Note: Make sure you understand each of the Markowitz assumptions—it will make asset pricing models easier to grasp.

LOS 53.c: Compute the expected return for an individual investment and for a portfolio.

Professor's Note: It's not obvious whether the exam will require that you describe and calculate expected returns using expectational (probabilistic) data or historical data, so we will do it both ways here and throughout this review wherever appropriate, just to be safe.

Expected Return for an Individual Investment

The *expected rate of return* from *expectational data* for a single risky asset can be calculated as:

$$E(R) = \sum_{i=1}^n P_i R_i = P_1 R_1 + P_2 R_2 + \dots + P_n R_n \quad (\text{using expectational returns})$$

where:

P_i = probability that state i will occur

R_i = asset return if the economy is in state i

The expected return, based on expectational data, is simply the weighted mean of the distribution of all possible returns.

The *expected rate of return from historical data* for a single risky asset can be calculated as:

$$E(R) = \bar{R} = \frac{\sum_{t=1}^n R_t}{n} = \frac{(R_1 + R_2 + \dots + R_n)}{n}$$

where:

R_t = the return in time period t

n = the number of time periods (using historical returns)

Professor's Note: The expected return with historical data is simply the average return over n years.

Example: Calculating expected return from expectational data

The first three columns of Figure 2 contain the probability of outcomes (states of the world) and the returns for a security in each state of the world. Calculate the expected return on the security.

Answer:

The computation of expected return is illustrated in the fourth column of Figure 2.

Figure 2: Computing Expected Return

State of the World	Probability (P_i)	Return (R_i)	Expected Return ($P_i R_i$)
Expansion	0.25	5.0%	$(0.25)(5.0) = 1.25\%$
Normal	0.50	15.0%	$(0.50)(15.0) = 7.50\%$
Recession	0.25	25.0%	$(0.25)(25.0) = 6.25\%$
			$E(R) = \sum_{i=1}^3 P_i R_i = 15.00\%$

Example: Calculating expected return from historical data

Assume that the returns on a stock over the first six months of the year are +10%, -15%, +20%, +25%, -30%, and +20%. Compute the expected (average) return.

Answer:

$$\bar{R} = \frac{0.10 - 0.15 + 0.20 + 0.25 - 0.30 + 0.20}{6} = 0.05 = 5.0\%$$

Expected Return for a Portfolio of Risky Assets

The expected return on a portfolio of assets is simply the weighted average of the returns on the individual assets, using their portfolio weights. Thus, for a two-asset portfolio, the expected return is:

$$E(R_p) = w_1 E(R_1) + w_2 E(R_2)$$

where:

$E(R_1)$ = expected return on asset 1

$E(R_2)$ = expected return on asset 2

w_1 = percentage of the total portfolio value invested in asset 1

w_2 = percentage of the total portfolio value invested in asset 2

LOS 53.d: Compute the variance and standard deviation for an individual investment.

Professor's Note: It's not obvious whether the exam will require you to calculate the variance and standard deviation of returns using expectational (probabilistic) data or historical data, so again we will do it both ways.

In finance, the variance and standard deviation of expected returns are common measures of investment risk. Both of these related measures determine the variability of a distribution of returns about its mean.

The variance and standard deviation of rates of return from *expectational data* for an individual investment are calculated as:

$$\text{variance} = \sigma^2 = \sum_{i=1}^n P_i [R_i - E(R)]^2$$

$$\text{standard deviation} = \sigma = \sqrt{\sigma^2}$$

where:

R_i = return in state of the world i

P_i = probability of state i occurring

$E(R)$ = expected return

Example: Calculating variance with expectational data

The returns expectations from the previous example are reproduced in the first three columns of Figure 3. Using this expectational data, calculate the variance and standard deviation of expected returns. Recall that the expected return is 15%.

Answer:

Based on the computations illustrated in Figure 3, the variance and standard deviation are 0.0050 and 7.07%, respectively.

Figure 3: Variance and Standard Deviation Computation

State i	Probability P_i	Return R_i	Expected Return $E(R)$	$[(R_i) - E(R)]^2$	$P_i[(R_i) - E(R)]^2$
Expansion	0.25	0.05	0.15	0.01	$(0.25)(0.01) = 0.0025$
Normal	0.50	0.15	0.15	0.00	$(0.50)(0.00) = 0.0000$
Recession	0.25	0.25	0.15	0.01	$(0.25)(0.01) = 0.0025$
Variance = $\sum P_i[(R_i) - E(R)]^2 = 0.0025 + 0.0000 + 0.0025 = 0.0050$ Standard deviation = $(0.0050)^{1/2} = 0.0707 = 7.07\%$					

The variance and standard deviation of returns from *historical data* are calculated as:

$$\text{variance} = \sigma^2 = \frac{\sum_{t=1}^n (R_t - \bar{R})^2}{n}$$

$$\text{standard deviation} = \sigma = \sqrt{\sigma^2}$$

where:

R_t = return in period t

\bar{R} = average return (expected return)

n = number of returns

Example: Calculating variance with historical data

The historical returns from the previous example are reported in the first column of Figure 4. Compute the variance and standard deviation.

Answer:

As shown in Figure 4, the computations of the variance and standard deviation result in 0.0417 and 20.41%, respectively.

Figure 4: Variance and Standard Deviation Using Historical Data

R_t	$(R_t - \bar{R})$	$(R_t - \bar{R})^2$
+0.1000	+0.0500	0.0025
-0.1500	-0.2000	0.0400
+0.2000	+0.1500	0.0225
+0.2500	+0.2000	0.0400
-0.3000	-0.3500	0.1225
+0.2000	+0.1500	0.0225
$\bar{R} = 0.0500$		$\Sigma = 0.2500$
$\sigma^2 = 0.2500/6 = 0.0417$		
$\sigma = 0.2041 = 20.41\%$		

LOS 53.e: Compute the covariance of rates of return, and show how it is related to the correlation coefficient.

Covariance measures the extent to which two variables move together over time. A positive covariance means that the variables (e.g., rates of return on two stocks) tend to move together. Negative covariance means that the two variables tend to move in opposite directions. A covariance of zero means there is no relationship between the two variables.

The covariance between two assets computed from expectational data is equal to:

$$\text{Cov}_{1,2} = \sum_{i=1}^n \{P_i [R_{i,1} - E(R_1)] [R_{i,2} - E(R_2)]\}$$

where:

$R_{t,1}$ = return on asset 1 in state i

$R_{t,2}$ = return on asset 2 in state i

P_i = probability of state i occurring

$E(R_1)$ = expected return on asset 1

$E(R_2)$ = expected return on asset 2

Example: Calculating covariance with expectational data

Calculate the covariance between Asset 1 and Asset 2 with the returns distribution described in the first three columns of Figure 5.

Answer:

First, we must compute the expected return for each of the assets as follows:

$$E(R_1) = \sum_{i=1}^n P_i R_{i,1} = 0.25(0.05) + 0.50(0.15) + 0.25(0.25) = 0.0125 + 0.0750 + 0.0625 = 0.15$$

$$E(R_2) = \sum_{i=1}^n P_i R_{i,2} = 0.25(0.32) + 0.50(0.14) + 0.25(0.04) = 0.08 + 0.07 + 0.01 = 0.16$$

Armed with the asset's expected returns, we can compute the covariance following the procedure illustrated in Figure 5.

Figure 5: Computing Covariance

P_i	$R_{i,1}$	$R_{i,2}$	$(R_{i,1}) - E(R_1)$	$(R_{i,2}) - E(R_2)$	$P_i[(R_{i,1}) - E(R_1)][(R_{i,2}) - E(R_2)]$
0.25	0.05	0.32	-0.10	+0.16	-0.004
0.50	0.15	0.14	+0.00	-0.02	0.000
0.25	0.25	0.04	+0.10	-0.12	-0.003
$Cov_{1,2} = \sum P_i[(R_{i,1}) - E(R_1)][(R_{i,2}) - E(R_2)] = -0.007$					

The covariance between two asset returns using historical data is computed as:

$$Cov_{1,2} = \frac{\sum_{t=1}^n \{ [R_{t,1} - \bar{R}_1][R_{t,2} - \bar{R}_2] \}}{n}$$

where:

$R_{i,1}$ = return on asset 1 in period t

$R_{i,2}$ = return on asset 2 in period t

\bar{R}_1 = mean return on asset 1

\bar{R}_2 = mean return on asset 2

n = number of returns

Professor's Note: There is a similar formula for the covariance in the quantitative methods material that has $n - 1$ in the denominator. The difference is that the formula we are working with here is a population measure, whereas in the quant material, $n - 1$ is used because the covariance there is a sample statistic.

Example: Calculating covariance with historical data

Calculate the covariance for the returns of Stock 1 and Stock 2 given the six months of historical returns presented in the first three columns of Figure 6.

Answer:

The covariance calculation is demonstrated in the right side of Figure 6.

Figure 6: Calculating Covariance From Historical Returns

Year	Stock 1	Stock 2	$(R_t - \bar{R}_1)$	$(R_t - \bar{R}_2)$	$(R_t - \bar{R}_1)(R_t - \bar{R}_2)$
1998	+0.10	+0.20	+0.05	+0.10	+0.005
1999	-0.15	-0.20	-0.20	-0.30	+0.060
2000	+0.20	-0.10	+0.15	-0.20	-0.030
2001	+0.25	+0.30	+0.20	+0.20	+0.040
2002	-0.30	-0.20	-0.35	-0.30	+0.105
2003	+0.20	+0.60	+0.15	+0.50	+0.075
	$\bar{R}_1 = 0.05$	$\bar{R}_2 = 0.10$			$\Sigma = 0.255$ $\text{Cov} = 0.255/6 = 0.0425$

Correlation. The magnitude of the covariance depends on the magnitude of the individual stocks' standard deviations and the relationship between their co-movements. The covariance is an absolute measure and is measured in return units squared.

Covariance can be standardized by dividing by the product of the standard deviations of the two securities being compared. This standardized measure of co-movement is called *correlation* and is computed as:

$$\rho_{1,2} = \frac{\text{Cov}_{1,2}}{\sigma_1 \sigma_2}$$

$$\text{or } \text{Cov}_{1,2} = \rho_{1,2} \sigma_1 \sigma_2$$

Professor's Note: The calculation of correlation is the same whether we are using expectational or historical data.

The term $\rho_{1,2}$ is called the *correlation coefficient* between the returns of securities 1 and 2. The correlation coefficient has no units. It is a pure measure of the co-movement of the two stocks' returns and is bounded by -1 and +1.

How should you interpret the correlation coefficient?

- A correlation coefficient of +1 means that returns always move together in the same direction. They are perfectly positively correlated.
- A correlation coefficient of -1 means that returns always move in the exact opposite direction. They are perfectly negatively correlated.
- A correlation coefficient of zero means that there is no relationship between the two stocks' returns. They are uncorrelated. One way to interpret a correlation (or covariance) of zero is that, in any period, knowing the actual value of one variable tells you nothing about the other.

Example: Computing correlation

The covariance between the returns on two stocks is 0.0425. The standard deviations of stocks 1 and 2 are 0.2041 and 0.2944, respectively. Calculate and interpret the correlation between the two assets.

Answer:

$$\rho_{1,2} = \frac{0.0425}{0.2041 \times 0.2944} = 0.71$$

The returns from the two stocks are positively correlated, meaning they tend to move in the same direction at the same time. However, the correlation is not perfect because the correlation coefficient is less than one.

Example: Computing covariance

The correlation between the returns on two stocks is 0.56. The standard deviations of the returns from Stock 1 and Stock 2 are 0.1544 and 0.0892, respectively. Calculate and interpret the covariance between the two assets.

Answer:

$$\text{Cov}_{1,2} = 0.56 \times 0.1544 \times 0.0892 = 0.0077$$

The covariance between the returns from Stock 1 and Stock 2 shows that the two securities' returns tend to move together. However, the strength of this tendency cannot be measured using the covariance—we must rely on the correlation to provide us with an indication of the relative strength of the relationship.

LOS 53.f: List the components of the portfolio standard deviation formula, and explain which component is most important to consider when adding an investment to a portfolio.

Earlier in this review, we showed that the expected return for a portfolio of risky assets is the weighted average of the expected returns of the individual assets in the portfolio. This is not the case for the variance and standard deviation of a portfolio of risky assets. The variance and, by association, the standard deviation of a portfolio of two assets are *not* simple weighted averages of the asset variances (standard deviations). Portfolio variance (standard deviation) is not only a function of the variance (standard deviation) of the returns of the individual assets in the portfolio. It is also a function of the correlation (covariance) among the returns of the assets in the portfolio.

The general formula for the standard deviation for a portfolio of n risky assets is as follows:

$$\sigma_p = \sqrt{\sigma_p^2} = \sqrt{\sum_{i=1}^n w_i^2 \sigma_i^2 + \sum_{i=1}^n \sum_{\substack{j=1 \\ i \neq j}}^n w_i w_j \text{Cov}_{i,j}}$$

where:

σ_p^2 = portfolio variance

w_i = the market weight of asset i

σ_i^2 = variance of returns for asset i

$\text{Cov}_{i,j}$ = the covariance between the returns of assets i and j

For a portfolio of two risky assets this is equivalent to:

$$\sigma_p = \sqrt{w_1^2 \sigma_1^2 + w_2^2 \sigma_2^2 + 2w_1 w_2 \sigma_1 \sigma_2 \rho_{1,2}} \text{ or } \sqrt{w_1^2 \sigma_1^2 + w_2^2 \sigma_2^2 + 2w_1 w_2 \text{Cov}_{1,2}}$$

For a portfolio of three risky assets, the expanded form is:

$$\sigma_p = \sqrt{w_1^2\sigma_1^2 + w_2^2\sigma_2^2 + w_3^2\sigma_3^2 + 2w_1w_2\text{Cov}_{1,2} + 2w_1w_3\text{Cov}_{1,3} + 2w_2w_3\text{Cov}_{2,3}}$$

Note that in the first formula for a two-asset portfolio we have substituted $\sigma_1\sigma_2\rho_{1,2}$ for $\text{Cov}_{1,2}$ (using the definition of $\rho_{1,2}$) because the formula is often written this way as well to emphasize the role of correlation in portfolio risk.

The first part of the formula is intuitive—the risk of a portfolio of risky assets depends on the risk of the assets in the portfolio and how much of each asset is in the portfolio (the σ and w terms). The second part of the formula is there because the risk (standard deviation) of a portfolio of risky assets also depends on how the returns on the assets move in relation to each other (the covariance or correlation of their returns).

Note that if the asset returns are *negatively correlated*, the final term in the formula for a two-asset portfolio is negative and reduces the portfolio standard deviation. If the correlation is zero, the final term is zero, and the portfolio standard deviation is greater than when the correlation is negative. If the correlation is positive, the final term is positive, and portfolio standard deviation is greater still. The maximum portfolio standard deviation for a portfolio of two assets with given portfolio weights will result when the correlation coefficient is +1 (perfect positive correlation). When assets are perfectly positively correlated, there is no diversification benefit.

This is the key point of the Markowitz analysis and the point of this LOS. *The risk of a portfolio of risky assets depends on the asset weights and the standard deviations of the assets' returns, and crucially on the correlation (covariance) of the asset returns.*

Other things equal, the higher (lower) the correlation between asset returns, the higher (lower) the portfolio standard deviation.

PORTFOLIO RISK AND RETURN FOR A TWO-ASSET PORTFOLIO

Before we move on to the next LOS, let's take a minute to show graphically the risk-return combinations from varying the proportions of two risky assets and then to show how the graph of these combinations is affected by changes in the correlation coefficient for the returns on the two assets.

Figure 7 provides the risk and return characteristics for two stocks, Sparklin' and Caffeine Plus. Figure 8 shows the calculation of portfolio risk and expected return for portfolios with different proportions of each stock (calculated from the formula in the previous LOS).

Figure 7: Risk/Return Characteristics for Two Individual Assets

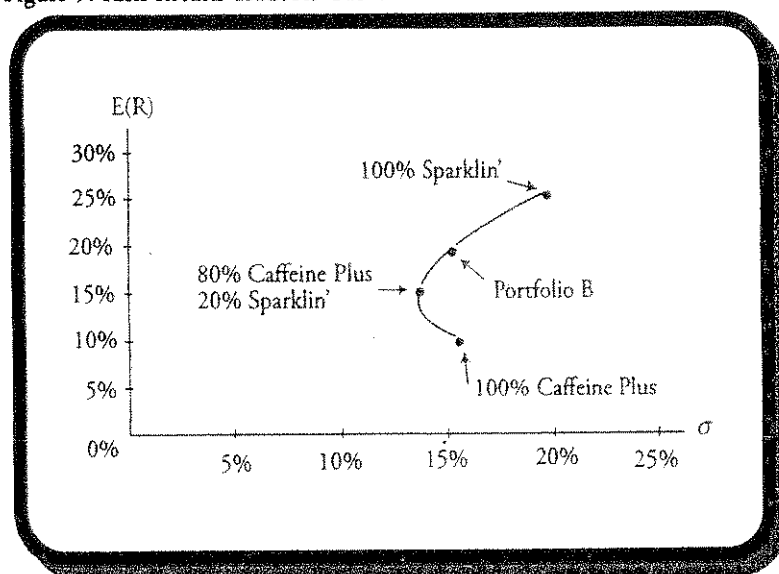
	<i>Caffeine Plus</i>	<i>Sparklin'</i>
Expected return (%)	11%	25%
Standard deviation (%)	15%	20%
Correlation	0.3	

Figure 8: Possible Combinations of Caffeine Plus and Sparklin'

$W_{\text{Caffeine Plus}}$	100%	80%	60%	40%	20%	0%
$W_{\text{Sparklin'}}$	0%	20%	40%	60%	80%	100%
$E(R_p)$	11.0%	13.8%	16.6%	19.4%	22.2%	25.0%
σ_p	15.0%	13.7%	13.7%	14.9%	17.1%	20.0%

The plot in Figure 9 represents all possible expected return and standard deviation combinations attainable by investing in varying amounts of Caffeine Plus and Sparklin'. We'll call it the *risk-return tradeoff curve*.

Figure 9: Risk-Return Tradeoff Curve



If you have all your investment in Caffeine Plus, your "portfolio" will have an expected return and standard deviation equal to that of Caffeine Plus, and you will be at one end of the curve (at the point labeled "100% Caffeine Plus"). As you increase your investment in Sparklin' to 20% and decrease your investment in Caffeine Plus to 80%, you will move up the risk-return tradeoff curve to the point where the expected return is 13.8% with a standard deviation of 13.7% (labeled "80% Caffeine Plus/20% Sparklin"). Moving along the curve (and changing the expected return and standard deviation of the portfolio) is a matter of changing your portfolio allocation between the two stocks.

We can create portfolios with the *same* risk level (i.e., same standard deviation) and *higher* expected returns by diversifying our investment portfolio across many stocks. We can even benefit by adding just Sparklin' to a portfolio of only Caffeine Plus stock. We can create a combination of Caffeine Plus and Sparklin' (portfolio B) that has the same standard deviation but a higher expected return. Risk-averse investors would always prefer that combination to Caffeine Plus by itself.

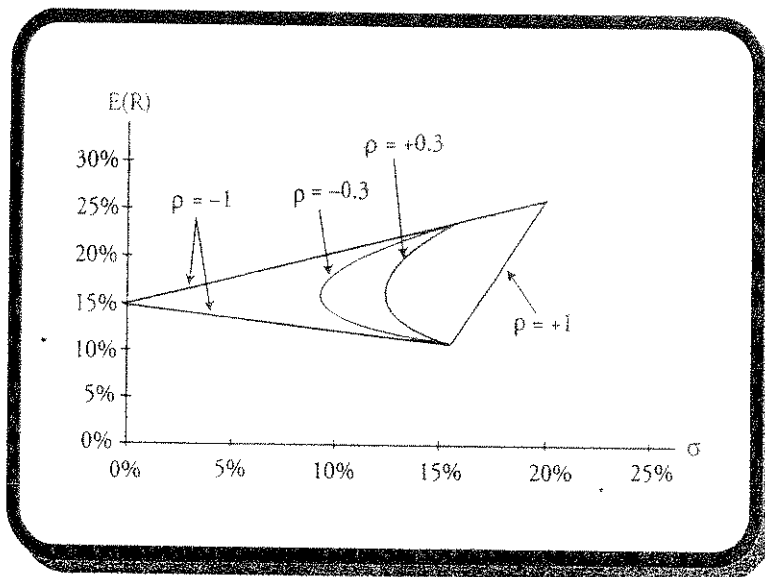
Let's take an analytical look at how diversification reduces risk by using the portfolio combinations in Figure 9. As indicated, the end points of this curve represent the risk/return combination from a 100% investment in either Sparklin' or Caffeine Plus. Notice that as Sparklin' is added to Caffeine Plus, the frontier "bulges" up and to the left (i.e., northwesterly, if you think of the plot as a map and north as up). This bulge is what creates the diversification benefits because portfolios with between 100% and 80% allocations to Caffeine Plus have both less risk and greater expected return than a portfolio of Caffeine Plus only.

The Special Role of Correlation

As the correlation between the two assets decreases, the benefits of diversification increase. That's because, as the correlation decreases, there is less of a tendency for stock returns to move together. The separate movements of each stock serve to reduce the volatility of the portfolio to a level that is less than that of its individual components.

Figure 10 illustrates the effects of correlation levels on diversification benefits. We've created the risk-return trade-off line for four different levels of correlation between the returns on the two stocks. Notice that the amount of bulge in the risk-return trade-off line is a function of the correlation between the two assets: the lower the correlation (closer to -1), the greater the bulge; the larger the correlation (closer to $+1$), the smaller is the bulge.

Figure 10: Effects of Correlation on Diversification Benefits



What does all this tell us? *The lower the correlation between the returns of the stocks in the portfolio, all else equal, the greater the diversification benefits.* This principle also applies to portfolios with many stocks, as we'll see next.

LOS 53.g: Describe the efficient frontier and explain the implications for incremental returns as an investor assumes more risk.

The calculations required to generate what we called the risk-return trade-off curve for a two-asset portfolio are not too difficult to do with a spreadsheet. However, the statistical input requirements to apply Markowitz portfolio theory in a large portfolio are significant. Specifically, we must estimate:

- The expected return for each asset available for investment.
- The standard deviation for each asset.
- The correlations between every pair of assets.

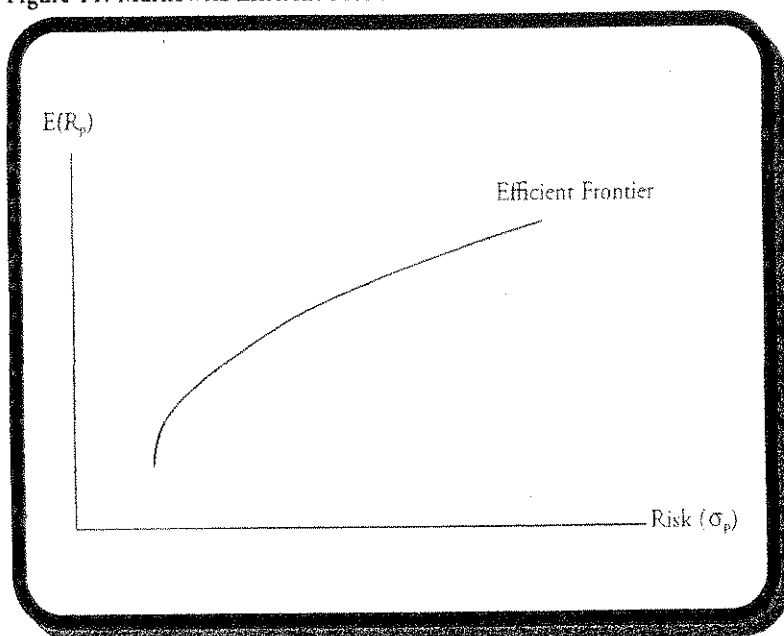
This need for correlations can be particularly onerous. For example, if the universe of potential securities includes 100 different stocks, then there are 4,950 pairwise correlation coefficients that must be estimated.

However, with enough computer power, we can generate the set of efficient portfolios from among all the possible combinations of all the assets available for investment. A portfolio is considered to be *efficient* if no other portfolio offers a higher expected return with the same (or lower) risk or if no other portfolio offers lower risk

with the same (or higher) return. The concept of efficient portfolios is a key concept in portfolio theory (and capital market theory, discussed in the next topic review).

The efficient frontier represents the set of portfolios that will give you the highest return at each level of risk (or, alternatively, the lowest risk for each level of return). The efficient frontier is portrayed graphically in Figure 11.

Figure 11: Markowitz Efficient Frontier

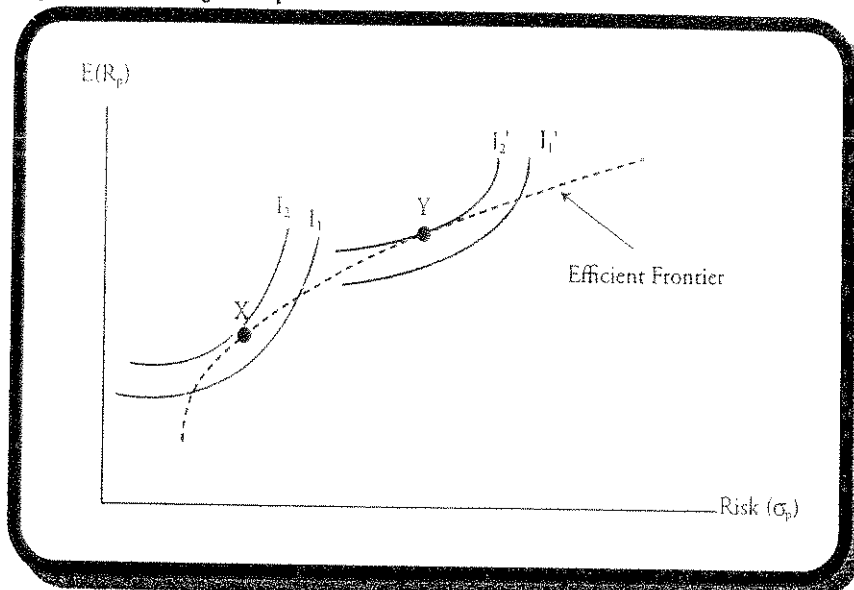


LOS 53.h: Define optimal portfolio and show how each investor may have a different optimal portfolio.

We can combine the concepts of the efficient frontier and indifference curve analysis to describe how a risk averse investor selects his optimal portfolio. Steep indifference curves for Investor A in Figure 12 (I_1 and I_2) indicate greater risk aversion than Investor B, who has relatively flat indifference curves (I'_1 and I'_2). The *optimal portfolio* for each investor is at the point where the investor's (highest) indifference curve is tangent to the efficient frontier. The optimal portfolio is the portfolio that is the most preferred of the possible portfolios (i.e., the one that lies on the highest indifference curve).

Investor A, the more risk-averse investor, has portfolio X as his most preferred portfolio, while Investor B, the less risk-averse investor, has portfolio Y as his most preferred portfolio. Investor B will expect more return than Investor A but is also willing to assume more risk than Investor A to get a higher expected return. The bottom line here is simple—the less risk-averse investor will have a most-preferred portfolio that is riskier, compared to the more risk-averse investor.

Figure 12: Locating the Optimal Portfolio



Professor's Note: The steeper the slope at the point of tangency, the greater the level of risk aversion because it takes more additional return to accept a unit of increased risk. If you think this was a lot of work to show that a less risk-averse investor chooses a riskier portfolio, you may be right, but we will use this analysis shortly to extend the model.

KEY CONCEPTS

1. A risk-averse investor prefers higher expected returns for the same level of expected risk and prefers lower risk for a given level of expected returns.
2. The Markowitz assumptions are:
 - Investors see a probability distribution of expected returns for every investment.
 - Investors maximize one-period expected utility, and their indifference curves exhibit diminishing marginal utility of wealth.
 - Investors measure risk as the variance (standard deviation) of expected returns.
 - Investors make all investment decisions only on the basis of risk and return.
 - Investors are risk averse.
3. The expected rate of return from expectational data for a single risky asset is:

$$E(R) = \sum_{i=1}^n P_i R_i$$

The expected rate of return on a single risky asset from historical data is:

$$\bar{R} = \frac{\sum_{t=1}^n R_t}{n}$$

4. The variance of rates of return from expectational data for an individual investment is calculated as:

$$\text{variance} = \sigma^2 = \sum_{i=1}^n \left\{ [R_i - E(R)]^2 \times P_i \right\}$$

From historical data, variance is calculated as:

$$\text{variance} = \sigma^2 = \frac{\sum_{t=1}^N (R_t - \bar{R})^2}{n}$$

5. The covariance from expectational data is calculated as:

$$\text{Cov}_{1,2} = \sum_{i=1}^n \{P_i [R_{i,1} - E(R_1)] [R_{i,2} - E(R_2)]\}$$

The covariance calculation using historical data is:

$$\text{Cov}_{1,2} = \frac{\sum_{t=1}^n \{[R_{t,1} - \bar{R}_1] [R_{t,2} - \bar{R}_2]\}}{n}$$

6. The correlation coefficient can take values from -1 to $+1$ and is a standardized measure of how two random variables change in relation to each other. It is calculated as:

$$\rho_{1,2} = \frac{\text{Cov}_{1,2}}{\sigma_1 \times \sigma_2} \text{ so that } \text{Cov}_{1,2} = \rho_{1,2} \sigma_1 \sigma_2$$

7. The variance of a portfolio of two assets is a function of the correlation between the returns of the two assets, the asset weights, and the standard deviations of the asset returns. It is calculated as:

$$\sigma_p^2 = w_1^2 \sigma_1^2 + w_2^2 \sigma_2^2 + 2w_1 w_2 \rho_{1,2} \sigma_1 \sigma_2$$

8. The efficient frontier represents the set of portfolios that will give you the highest return at each level of risk and shows the increases in expected return that an investor can expect in equilibrium for taking on more portfolio risk (standard deviation) in an efficient portfolio.
9. The optimal portfolio for an investor is at the point of where an investor's (highest) risk-return indifference curve is tangent to the efficient frontier.

CONCEPT CHECKERS: AN INTRODUCTION TO PORTFOLIO MANAGEMENT

Use the following data to answer Questions 1 through 3.

An investment has a 50% chance of a 20% return, a 25% chance of a 10% return, and a 25% chance of a -10% return.

1. What is the investment's expected return?
 - A. 5.0%.
 - B. 10.0%.
 - C. 12.5%.
 - D. 15.0%.
2. What is the investment's variance of returns?
 - A. 0.005.
 - B. 0.010.
 - C. 0.015.
 - D. 0.150.
3. What is the investment's standard deviation of returns?
 - A. 1.225%.
 - B. 1.500%.
 - C. 2.250%.
 - D. 12.250%.
4. Which of the following statements about covariance and correlation is FALSE?
 - A. Positive covariance means that asset returns move together.
 - B. A zero covariance implies there is no linear relationship between the two variables.
 - C. If two assets have perfect negative correlation, it is impossible to reduce the portfolio's overall variance.
 - D. The covariance of a two-stock portfolio is equal to the correlation coefficient times the standard deviation of one stock times the standard deviation of the other stock.

Use the following data to answer Questions 5 and 6.

A portfolio was created by investing 25% of the funds in asset A (standard deviation = 15%) and the balance of the funds in asset B (standard deviation = 10%).

5. If the correlation coefficient is 0.75, what is the portfolio's standard deviation?
 - A. 11.2%.
 - B. 10.6%.
 - C. 12.4%.
 - D. 15.0%.
6. If the correlation coefficient is -0.75, what is the portfolio's standard deviation?
 - A. 2.8%.
 - B. 4.2%.
 - C. 5.3%.
 - D. 10.6%.

7. Which of the following statements about correlation is FALSE?
- Potential benefits from diversification arise when correlation is less than +1.
 - If the correlation coefficient were 0, a zero variance portfolio could be constructed.
 - If the correlation coefficient were -1, a zero variance portfolio could be constructed.
 - The lower the correlation coefficient, the greater the potential benefits from diversification.
8. A measure of how well the returns of two risky assets move together is the:
- range.
 - covariance.
 - semivariance.
 - standard deviation.
9. A portfolio manager adds a new stock to a portfolio that has the same standard deviation of returns as the existing portfolio but has a correlation coefficient with the existing portfolio that is less than +1. If the new stock is added, the portfolio's standard deviation will:
- decrease.
 - not change.
 - increase by the amount of the new stock's standard deviation.
 - increase by less than the amount of the new stock's standard deviation.
10. An investor currently owns Brown Co. and is thinking of adding either James Co. or Beta Co. to his holdings. All three stocks offer the same expected return and total risk. The covariance of returns between Brown Co. and James Co. is -0.5 and the covariance between Brown Co. and Beta Co. is +0.5. Which of choices below *best describes* the portfolio's risk? The portfolio's risk would:
- decline more if only Beta Co. is purchased.
 - decline more if only James Co. is purchased.
 - increase if only Beta Co. is purchased.
 - remain unchanged if both stocks are purchased.
11. Which of the following portfolios falls *below* the Markowitz efficient frontier?
- | Portfolio | Expected Return | Expected Standard Deviation |
|-----------|-----------------|-----------------------------|
| A. A | 7% | 14% |
| B. B | 9% | 26% |
| C. C | 12% | 22% |
| D. D | 15% | 30% |
12. In time 1, Stock A's return was 10% and Stock B's return was 15%. In time 2, Stock A's return was 6% and Stock B's return was 9%. What is the covariance of returns between A and B?
- 2.
 - 3.
 - 6.
 - 12.
13. The standard deviation of returns is 0.30 for Stock A and 0.20 for Stock B. The covariance between the returns of A and B is 0.006. The return correlation between A and B is:
- 0.10.
 - 0.20.
 - 0.30.
 - 0.35.

ANSWERS – CONCEPT CHECKERS: AN INTRODUCTION TO PORTFOLIO MANAGEMENT

1. B $(0.5 \times 0.2) + (0.25 \times 0.1) + (0.25 \times -0.1) = 0.1$, or 10%
2. C $[0.5(0.2 - 0.1)^2] + [0.25(0.1 - 0.1)^2] + [0.25(-0.1 - 0.1)^2] = 0.005 + 0 + 0.01 = 0.015$
3. D $\sqrt{0.015} = 0.1225 = 12.25\%$
4. C If two assets have perfect negative correlation, it is possible to reduce the overall risk to zero. Note that positive correlation means that assets move together, a zero correlation implies no relationship, and covariance is defined as the correlation coefficient times the standard deviation of the two stocks in a two-stock portfolio.
5. B $\sqrt{[(0.25)^2(0.15)^2] + [(0.75)^2(0.10)^2] + [2(0.25)(0.75)(0.15)(0.10)(0.75)]} = \sqrt{(0.001406) + (0.005625) + (0.004219)}$
 $= \sqrt{(0.01125)} = 0.106 = 10.6\%$
6. C $\sqrt{[(0.25)^2(0.15)^2] + [(0.75)^2(0.10)^2] + [2(0.25)(0.75)(0.15)(0.10)(-0.75)]} = \sqrt{(0.001406) + (0.005625) - (0.004219)}$
 $= \sqrt{(0.002812)} = 0.053 = 5.3\%$
7. B A zero-variance portfolio can only be constructed if the correlation coefficient between assets is negative. Note that benefits can arise from diversification when correlation is less than +1, and the lower the correlation, the greater the potential benefit.
8. B The covariance is defined as the co-movement of the returns of two assets, or how well the returns of two risky assets move together. Note that range, semivariance, and standard deviation are measures of dispersion and measure risk, not how assets move together.
9. A There are potential benefits from diversification anytime the correlation coefficient with the existing portfolio is less than one. Because the correlation coefficient of the asset being added with the existing portfolio is less than one, the overall risk of the portfolio should decrease, resulting in a lower standard deviation.
10. B The overall risk would decline if either asset were added to the portfolio because both assets have correlation coefficients of less than one. The risk would decline the most if James Co. were added because it has the lowest correlation coefficient.
11. B Portfolio B must be the portfolio that falls below the Markowitz efficient frontier because there is a portfolio (Portfolio C) that offers a higher return and lower risk.
12. C Mean return A = $\frac{10+6}{2} = 8\%$; mean return B = $\frac{15+9}{2} = 12\%$

$$\text{Cov}_{A,B} = \frac{(10-8)(15-12) + (6-8)(9-12)}{2} = 6$$
13. A Correlation = $0.006 / [(0.30)(0.20)] = 0.10$

AN INTRODUCTION TO ASSET PRICING MODELS

Study Session 12

EXAM FOCUS

This topic review picks up where our review of the work of Prof. Markowitz left off. Adding a riskless asset to the opportunity set transforms portfolio theory into capital market theory. Key concepts in this topic review include the development of the capital market line, the separation of risk into systematic and unsystematic components, and the development of the

capital asset pricing model. Be sure that you can discuss each of the major concepts and that you can value an asset using the capital asset pricing model. You should also be familiar with the difference between the necessary assumptions for these pricing models.

LOS 54.a: List the assumptions of the capital market theory.

The assumptions of capital market theory are:

- *Markowitz investors.* All investors use the Markowitz mean-variance framework to select securities. This means they want to select portfolios that lie along the efficient frontier, based on their utility functions.
- *Unlimited risk-free lending and borrowing.* Investors can borrow or lend any amount of money at the risk-free rate.
- *Homogeneous expectations.* This means that when investors look at a stock, they all see the same risk/return distribution.
- *One-period horizon.* All investors have the same one-period time horizon.
- *Divisible assets.* All investments are infinitely divisible.
- *Frictionless markets.* There are no taxes or transaction costs.
- *No inflation and constant interest.* There is no inflation, and interest rates do not change.
- *Equilibrium.* The capital markets are in equilibrium.

LOS 54.b: Explain what happens to the expected return, the standard deviation of returns, and possible risk-return combinations when a risk-free asset is combined with a portfolio of risky assets.

Markowitz's efficient frontier did not consider the existence of a risk-free asset. Adding a risk-free asset to the Markowitz portfolio construction process allows portfolio theory to develop into capital market theory. Here's the bottom line (no pun intended):

The introduction of a risk-free asset changes the Markowitz efficient frontier from a curve into a straight line called the capital market line (CML). Let's see how this conclusion is derived.

If you invest a portion of your total funds in a risky portfolio M and the remaining portion in the risk-free asset, the equation for the expected return of the resulting portfolio will be:

$$E(R_p) = (1 - w_M)RFR + w_ME(R_M) = RFR + w_M[E(R_M) - RFR]$$

where:

RFR = the risk-free rate

$E(R_M)$ = the expected return on portfolio M

w_M = percentage (weight) of the total portfolio value invested in portfolio M

$1 - w_M$ = the percentage (weight) of the total portfolio value invested in the risk-free asset

Professor's Note: You do not need to memorize the preceding formula! We are going to use it to derive the equation for the CAPM, but on the exam, you can always determine the $E(R_p)$ for a two-stock portfolio (even if it contains the risk-free asset) using the equation: $E(R_p) = w_{RFR}RFR + w_ME(R_M)$.

If you combine the risk-free asset with a risky portfolio, the equation for the expected standard deviation of the resulting portfolio will be the same as for a two-risky-asset portfolio:

$$\sigma_P = \sqrt{(1 - w_M)^2 \sigma_{RFR}^2 + w_M^2 \sigma_M^2 + 2(1 - w_M)w_M \sigma_{RFR} \sigma_M \rho_{RFR,M}}$$

where:

σ_{RFR} = standard deviation of the risk-free asset

σ_M = standard deviation of the expected returns on portfolio M

$\rho_{RFR,M}$ = correlation between the risk-free asset and portfolio M

When one of the assets is risk-free, the calculation is much easier! By definition, under the assumptions of portfolio theory and capital market theory, if an asset is risk-free, its return does not vary. Thus, its variance and standard deviation are zero. If an asset has no variance, its expected return doesn't move. If the risk-free rate, RFR, is constant, it can't co-vary with other assets. In other words, the risk-free rate is stationary. Thus, its correlation coefficient with all other assets is zero.

Since $\sigma_{RFR} = \rho_{RFR,M} = 0$, the equation for portfolio standard deviation simplifies to:

$$\sigma_P = w_M \sigma_M$$

If we put 40% of our portfolio assets in the risky portfolio and the remainder in the risk-free asset, the resulting portfolio has 40% of the standard deviation of the risky portfolio! The risk/return relationship is now linear.

Combining this with our expected return equation gives us the following linear equation for the expected portfolio return as a function of portfolio standard deviation:

$$E(R_p) = RFR + \sigma_P \left\{ \frac{[E(R_M) - RFR]}{\sigma_M} \right\}$$

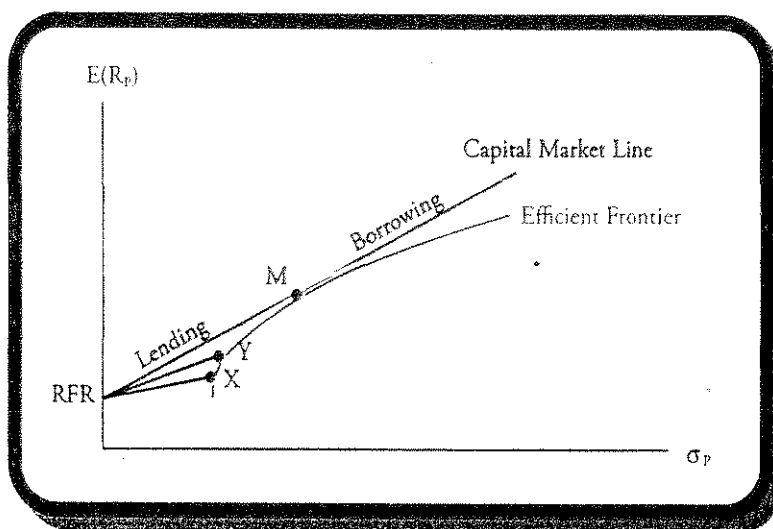
This is the equation for the capital market line (CML). The CML represents all possible portfolio allocations between the risk-free asset and a risky portfolio. The CML has an intercept of RFR and a slope equal to:

$$\frac{[E(R_M) - RFR]}{\sigma_M}$$

How do we select the optimal risky portfolio when a risk-free asset is also available? First, let's pick a risky portfolio (like portfolio X in Figure 1) that's on the Markowitz efficient frontier, since we know that these efficient portfolios dominate everything below them in terms of return offered for risk taken. Now, let's combine the risk-free asset with portfolio X. Remember, the risk/return relationship resulting from the combination of the risk-free asset and a risky portfolio is a straight line.

Now, choose a risky portfolio that is above portfolio X on the efficient frontier, such as portfolio Y. Portfolios on the line from RFR to Y will be preferred to portfolios on the line from RFR to X because we get more return for a given amount of risk. Actually, we can keep getting better portfolios by moving up the efficient frontier. At point M you reach the best possible combination. Portfolio M is at the point where the risk-return tradeoff line is just tangent to the efficient frontier. The line from RFR to M represents portfolios that are preferred to all the portfolios on the "old" efficient frontier, except M.

Figure 1: Capital Market Lines



Investors at point RFR have 100% of their funds invested in the risk-free asset. Investors at point M have 100% of their funds invested in portfolio M. Between RFR and M, investors hold both the risk-free asset and portfolio M. This means investors are *lending* some of their funds at the risk-free rate (i.e., buying the risk-free asset) and investing the rest in portfolio M. To the right of M, investors hold more than 100% of portfolio M. This means they are *borrowing* funds to buy more of portfolio M. The *levered positions* represent a 100% investment in portfolio M and borrowing to buy even more of portfolio M.

The introduction of a risk-free asset changes the Markowitz efficient frontier into a straight line called the CML.

LOS 54.c: Identify the market portfolio, and describe the role of the market portfolio in the formation of the capital market line (CML).

All investors have to do to get the risk and return combination that suits them is to simply vary the proportion of their investment in the risky portfolio M and the risk-free asset. So, in the CML world, all investors will hold some combination of the risk-free asset and portfolio M. Since all investors will want to hold the same risky portfolio, risky portfolio M must be the market portfolio.

The market portfolio is the portfolio consisting of every risky asset; the weights on each asset are equal to the percentage of the market value of the asset to the market value of the entire market portfolio. For example, if the market value of a stock is \$100 million and the market value of the market portfolio is \$5 billion, that stock's weight in the market portfolio is 2% (\$100 million / \$5 billion).

Logic tells us that the market portfolio, which will be held by all investors, has to contain all the stocks, bonds, and risky assets in existence because all assets have to be held by someone. This portfolio theoretically includes all risky assets, so it is completely diversified.

LOS 54.d: Define systematic and unsystematic risk and explain why an investor should not expect to receive additional return for assuming unsystematic risk.

When you diversify across assets that are not perfectly correlated, the portfolio's risk is less than the weighted sum of the risks of the individual securities in the portfolio. The risk that disappears in the portfolio construction process is called the asset's *unsystematic risk* (also called *unique*, *diversifiable*, or *firm-specific risk*). Since the market portfolio contains *all* risky assets, it must represent the ultimate in diversification. All the risk that can be diversified away must be gone. The risk that is left cannot be diversified away, since there is nothing left to add to the portfolio. The risk that remains is called the *systematic risk* (also called *nondiversifiable risk* or *market risk*).

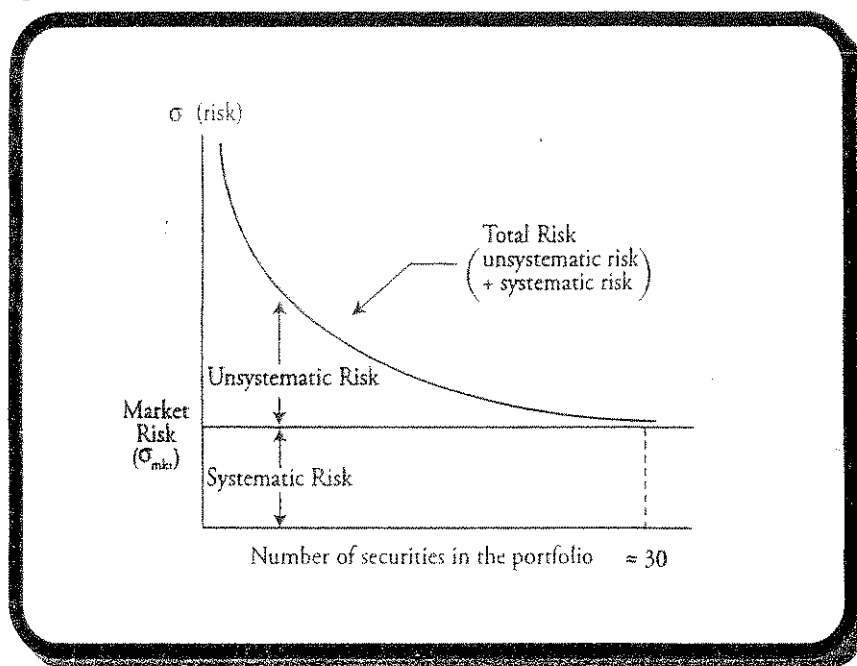
The concept of systematic risk applies to individual securities as well as to portfolios. Some securities are very sensitive to market changes. Typical examples of firms that are very sensitive to market movements are luxury goods manufacturers such as Ferrari automobiles and Harley Davidson motorcycles. Small changes in the market will lead to large changes in the value of luxury goods manufacturers. These firms have high systematic risk (i.e., they are very responsive to market, or systematic, changes). Other firms, such as utility companies, respond very little to changes in the overall market. These firms have very little systematic risk. Hence, total risk (as measured by standard deviation) can be broken down into its component parts: unsystematic risk and systematic risk. Mathematically:

$$\text{total risk} = \text{systematic risk} + \text{unsystematic risk}$$

Professor's Note: Know this concept!

Do you actually have to buy all the securities in the market to diversify away unsystematic risk? No. Academic studies have shown that as you increase the number of stocks in a portfolio, the portfolio's risk falls toward the level of market risk. One study showed that it only took about 12 to 18 stocks in a portfolio to achieve 90% of the maximum diversification possible. Another study indicated it took 30 securities. Whatever the number, it is significantly less than *all* the securities. Figure 2 provides a general representation of this concept. Note, in the figure, that once you get to 30 or so securities in a portfolio, the standard deviation remains constant. The remaining risk is systematic, or nondiversifiable, risk. We will develop this concept later when we discuss beta, a measure of systematic risk.

Figure 2: Risk vs. Number of Portfolio Assets



Systematic Risk is Relevant in Portfolios

One important conclusion of capital market theory is that equilibrium security returns depend on a stock's or a portfolio's systematic risk, not its total risk as measured by standard deviation. One of the assumptions of the model is that diversification is free. The reasoning is that investors will not be compensated for bearing risk that can be eliminated at no cost. If you think about the costs of a no-load index fund compared to buying individual stocks, diversification is actually very low cost if not actually free.

The implications of this conclusion are very important to asset pricing (expected returns). The riskiest stock, with risk measured as standard deviation of returns, does not necessarily have the greatest expected return. Consider a biotech stock with one new drug product that is in clinical trials to determine its effectiveness. If it turns out that the drug is effective and safe, stock returns will be quite high. If, on the other hand, the subjects in the clinical trials are killed or otherwise harmed by the drug, the stock will fall to approximately zero and returns will be quite poor. This describes a stock with high standard deviation of returns (i.e., high total risk).

The high risk of our biotech stock, however, is primarily from firm-specific factors, so its unsystematic risk is high. Since market factors such as economic growth rates have little to do with the eventual outcome for this stock, systematic risk is a small proportion of the total risk of the stock. Capital market theory says that the equilibrium return on this stock may be less than that of a stock with much less firm-specific risk but more sensitivity to the factors that drive the return of the overall market. An established manufacturer of machine tools may not be a very risky investment in terms of total risk, but may have a greater sensitivity to market (systematic) risk factors (e.g., GDP growth rates) than our biotech stock. Given this scenario, the stock with more total risk (the biotech stock) has less systematic risk and will therefore have a lower equilibrium rate of return according to capital market theory.

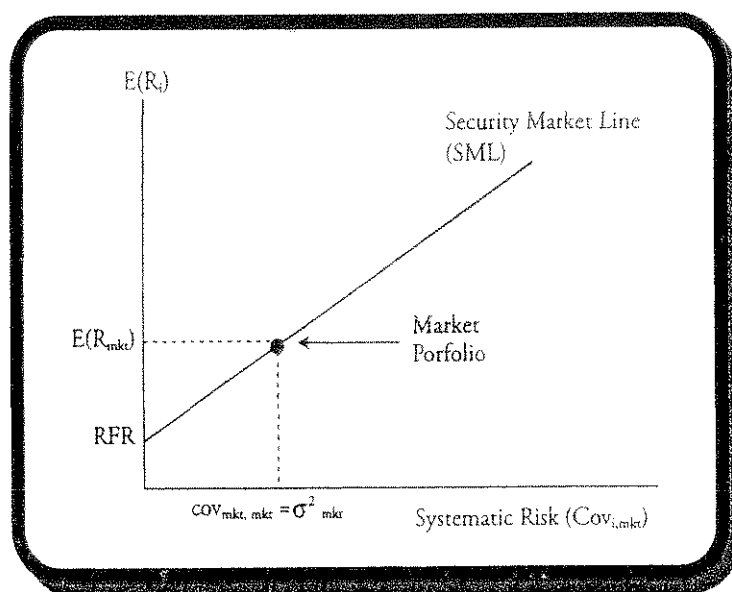
Note that holding many biotech firms in a portfolio will diversify away the firm-specific risk. Some will have blockbuster products and some will fail, but you can imagine that when 50 or 100 such stocks are combined into a portfolio, the uncertainty about the portfolio return is much less than the uncertainty about the return of a single biotech firm stock.

To sum up, unsystematic risk is not compensated in equilibrium because it can be eliminated for free through diversification. Systematic risk is measured by the contribution of a security to the risk of a well diversified portfolio and the expected equilibrium return (required return) on an individual security will depend on its systematic risk.

LOS 54.e: Describe the capital asset pricing model, diagram the security market line (SML), and define beta.

Given that the only relevant risk for an individual asset i is the covariance between the asset's returns and the return on the market, $Cov_{i,mkt}$, we can plot the relationship between risk and return for individual assets using $Cov_{i,mkt}$ as our measure of systematic risk. The resulting line, plotted in Figure 3, is one version of what is referred to as the security market line (SML).

Figure 3: Security Market Line



The equation of the SML is:

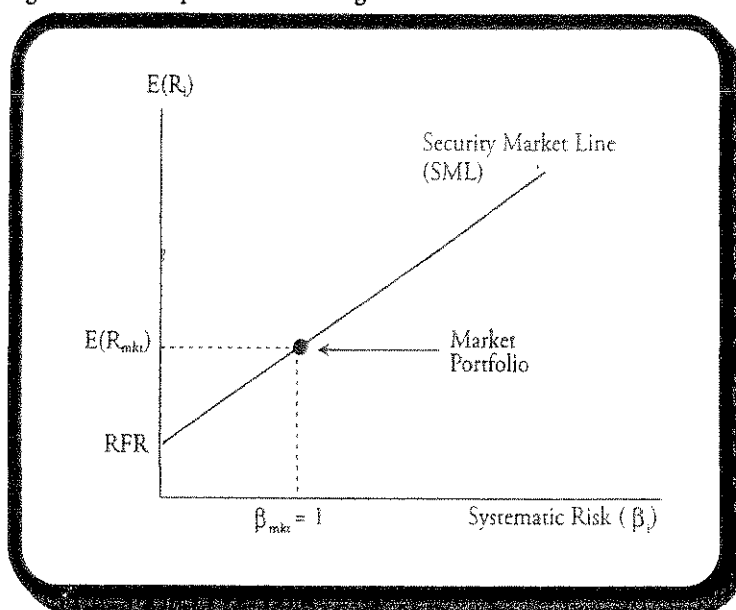
$$E(R_i) = RFR + \frac{E(R_{mkt}) - RFR}{\sigma^2_{mkt}} (Cov_{i,mkt})$$

which can be rearranged and stated as:

$$E(R_i) = RFR + \frac{Cov_{i,mkt}}{\sigma^2_{mkt}} [E(R_{mkt}) - RFR]$$

The line described by this last equation is presented in Figure 4, where we let the standardized covariance term, $\frac{Cov_{i,mkt}}{\sigma^2_{mkt}}$, be defined as beta, β_i . This is the most common means of describing the SML, and this relation between beta (systematic risk) and expected return is known as the capital asset pricing model (CAPM).

Figure 4: The Capital Asset Pricing Model



So, we can define beta, $\beta = \frac{\text{Cov}_{i,\text{mkt}}}{\sigma_{\text{mkt}}^2}$, as a standardized measure of systematic risk. Beta measures the sensitivity of a security's returns to changes in the market return.

Formally, the CAPM is stated as:

$$E(R_i) = \text{RFR} + \beta_i [E(R_{\text{mkt}}) - \text{RFR}]$$

The CAPM holds that, in equilibrium, the expected return on risky asset $E(R_i)$ is the risk-free rate (RFR) plus a beta-adjusted market risk premium, $\beta_i [E(R_{\text{mkt}}) - \text{RFR}]$. Beta measures systematic risk.

It is important that you recognize that the CML and SML are very different. Recall the equation of the CML:

$$E(R_p) = \text{RFR} + \sigma_p \left\{ \frac{[E(R_M) - \text{RFR}]}{\sigma_M} \right\}$$

The CML uses total risk $= \sigma_p$ on the X-axis. Hence, only efficient portfolios will plot on the CML. On the other hand, the SML uses beta (systematic risk) on the X-axis. So in a CAPM world, *all properly priced securities and portfolios of securities will plot on the SML*.

The CAPM is one of the most fundamental concepts in investment theory. The CAPM is an equilibrium model that predicts the expected return on a stock, given the expected return on the market, the stock's beta coefficient, and the risk-free rate.

Example: Capital asset pricing model

The expected return on the market is 15%, the risk-free rate is 8%, and the beta for stock A (β_A) is 1.2. Compute the rate of return that would be expected (required) on this stock.

Answer:

$$E(R_A) = 0.08 + 1.2 (0.15 - 0.08) = 0.164$$

Note: $\beta_A > 1$ so $E(R_A) > E(R_{mkt})$

Professor's Note: Know this calculation!

Example: Capital asset pricing model

The expected return on the market is 15%, the risk-free rate is 8%, and the beta for stock B (β_B) is 0.8. Compute the rate of return that would be expected (required) on this stock.

Answer:

$$E(R_B) = 0.08 + 0.8 (0.15 - 0.08) = 0.136$$

Note: Beta < 1 so $E(R_B) < E(R_{mkt})$

LOS 54.f: Calculate and interpret using the SML, the expected return on a security, and evaluate whether the security is undervalued, overvalued, or properly valued.

Let's clarify some terminology before we continue. The LOS asks for expected return based on the SML. You should also think of this as the "required return." There is another type of "expected return" which is based on *opinions* of the returns that can be earned on the stock given our future price and dividend forecasts. To keep this straight, we will refer to the expected return based on the theory of the CAPM as the *required* return, and the expected return based on perception and opinion as an *estimated* or *forecast* return.

In a CAPM world, all asset returns should fall on the SML. The SML tells us an asset's *required return* given its level of systematic risk (as measured by beta). The way we can use the CAPM to identify mispriced securities is to compare an asset's *estimated return* (given our forecasts of future prices and dividends) to the required return according to the SML. If the returns are not equal, the asset is either overvalued or undervalued and an appropriate trading strategy may be implemented.

- An asset with an estimated return greater than its required return from the SML is undervalued; we should buy it (return too high, price too low).
- An asset with an estimated return less than its required return from the SML is overvalued; we should sell it (return too low, price too high).
- An asset with an estimated return equal to its required return from the SML is properly valued; we're indifferent between buying or selling it.

Professor's Note: This is the most important LOS in this review. You are likely to see this material on the exam. Make sure you know it and nail the exam question!

Example: Identifying mispriced securities

Figure 5 contains information based on analyst's forecasts for three stocks. Assume a risk-free rate of 7% and a market return of 15%. Compute the expected and required return on each stock, determine whether each stock is undervalued, overvalued, or properly valued, and outline an appropriate trading strategy.

Figure 5: Forecast Data

Stock	Price Today	$E(\text{Price})$ in 1 Year	$E(\text{Dividend})$ in 1 Year	Beta
A	\$25	\$27	\$1.00	1.0
B	40	45	2.00	0.8
C	15	17	0.50	1.2

Answer:

Expected and required returns computations are shown in Figure 6.

Figure 6: Forecasts vs. Required Returns

Stock	Forecast Return	Required Return
A	$(\$27 - \$25 + \$1) / \$25 = 12.0\%$	$0.07 + (1.0)(0.15 - 0.07) = 15.0\%$
B	$(\$45 - \$40 + \$2) / \$40 = 17.5\%$	$0.07 + (0.8)(0.15 - 0.07) = 13.4\%$
C	$(\$17 - \$15 + \$0.5) / \$15 = 16.6\%$	$0.07 + (1.2)(0.15 - 0.07) = 16.6\%$

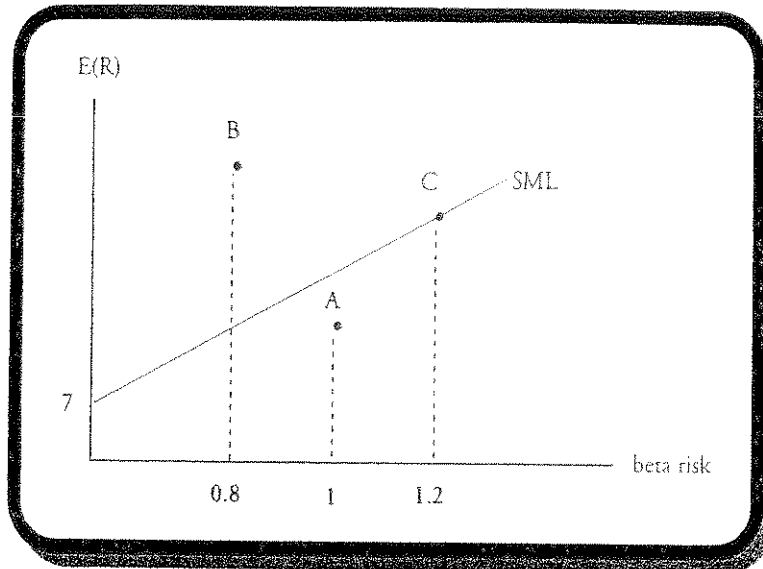
- Stock A is *overvalued*. It is expected to earn 12%, but based on its systematic risk it should earn 15%. It plots *below* the SML.
- Stock B is *undervalued*. It is expected to earn 17.5%, but based on its systematic risk it should earn 13.4%. It plots *above* the SML.
- Stock C is *properly valued*. It is expected to earn 16.6%, and based on its systematic risk it should earn 16.6%. It plots *on* the SML.

The appropriate trading strategy is:

- Short sell stock A.
- Buy stock B.
- Buy, sell, or ignore stock C.

We can do this same analysis graphically. The expected return/beta combinations of all three stocks are graphed in Figure 7 relative to the SML.

Figure 7: Identifying Mispriced Securities



Professor's Note: A trick to use when working these types of problems is: if the estimated return plots "over" the SML, the security is "under" valued. If the estimated return plots "under" the SML, the security is "over" valued.

Remember, all stocks should plot on the SML; any stock not plotting on the SML is mispriced. Notice that Stock A falls below the SML, Stock B lies above the SML, and Stock C is on the SML. If you plot a stock's expected return and it falls below the SML, the stock is overpriced. That is, the stock's expected return is too low given its systematic risk. If a stock plots above the SML, it is underpriced and is offering an expected return greater than required for its systematic risk. If it plots on the SML, the stock is properly priced.

Since the equation of the SML is the capital asset pricing model, you can determine if a stock is over- or underpriced graphically or mathematically. Your answers will always be the same.

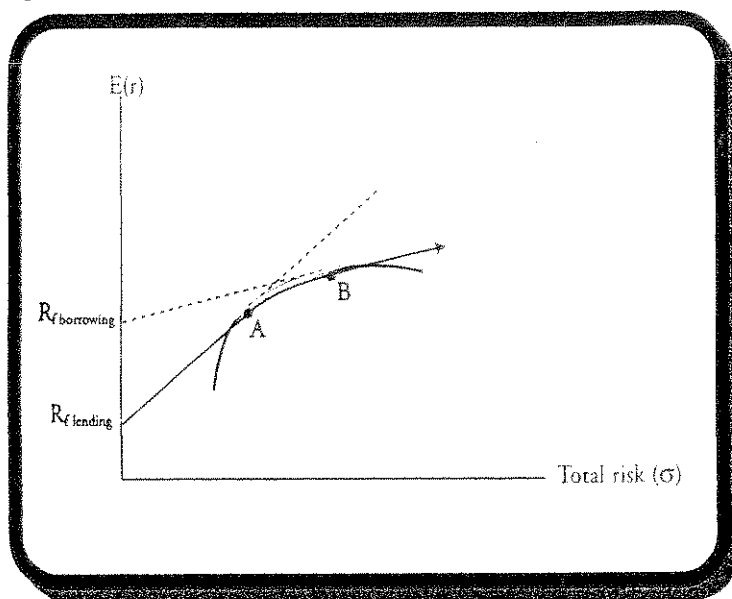
LOS 54.g: Describe the effect on the SML of relaxing each of its main underlying assumptions line.

The CAPM requires a number of assumptions, many of which do not reflect the true nature of the investment process. This section addresses the impact on the CAPM of relaxing some of the assumptions required in the derivation of the model.

Different borrowing and lending rates. One of the key assumptions of the CAPM is the ability of investors to lend and borrow at the risk-free rate. This assumption is what makes the CML straight. A straight CML allows risk to be separated into its systematic and unsystematic components. Without an equal lending and borrowing rate, you cannot determine a security's systematic risk, and, therefore, you cannot derive the SML. Without the SML, you cannot derive the CAPM.

Investors can lend all they want by buying investments at the risk-free rate, but investors must pay a premium over the risk-free rate to borrow. The graph in Figure 8 shows what this does to the CML. With unequal borrowing and lending rates, the CML follows the Markowitz efficient frontier (i.e., the no risk-free asset efficient frontier) between points A and B. Essentially, this puts a kink in the CML.

Figure 8: CAPM with Unequal Borrowing and Lending Rates



Without borrowing and lending at the same rate, can the validity of the CAPM be maintained? Yes, by the introduction of the zero-beta portfolio. The CAPM cannot be derived without equal borrowing and lending rates or some substitute for equal borrowing and lending rates. Fortunately, we have a substitute—the zero-beta model. The zero-beta version of the CAPM assumes that investors can find a *portfolio of securities with returns that are uncorrelated with market returns*. Since the portfolio is uncorrelated with the market, the portfolio will have a beta of zero, that is, no systematic risk.

As long as the expected return on the zero-beta portfolio is assumed to be greater than the risk-free lending rate, the resulting security market line will have a smaller risk premium (i.e., a flatter slope). With the introduction of a zero-beta portfolio with expected returns greater than those of the risk-free asset, we can still derive a linear relation between systematic risk and expected returns, a zero-beta CAPM. This relation can be expressed as:

$$E(R_{\text{stock}}) = E(R_{\text{zero beta portfolio}}) + (\text{Beta}_{\text{stock}}) [E(R_{\text{market}}) - E(R_{\text{zero beta portfolio}})]$$

Transaction costs. The no-transaction-costs assumption guarantees that all securities move to the SML. Why? Securities below the SML are overpriced, and securities above the SML are underpriced. Investors will buy the underpriced securities and sell the overpriced securities until no excess return opportunities exist. When all excess return opportunities have been eliminated, all securities will lie on the SML.

However, with transaction costs, securities that are just slightly mispriced will not be brought back to the SML, because the transaction costs will be greater than the profit potential. This will allow a band of expected returns to exist around the SML. The width of the band is a function of the size of the transaction costs: the higher the costs the wider the band.

Heterogeneous expectations and planning periods. If investors have different risk and return expectations or project their expectations over different time horizons, each investor will have a unique view of the SML. The homogeneous expectations and single holding period assumptions are necessary to bring the multitude of individual security market lines together into one SML and one CML. If these assumptions are not valid, there will be many SMLs and CMLs. The composite graph would be a band of lines with the width of the band determined by the divergence of expectations and time horizons; the greater the divergence of expectations and planning periods, the wider the band. The impact of heterogeneous expectations and multiple planning periods on the CAPM is similar to the impact of transactions costs—the SML becomes a band rather than a line.

Taxes. The expected after-tax returns for taxable investors are usually much different from the pre-tax returns we used in developing the CAPM. Individual investors pay ordinary income tax on dividend income and capital gains tax on realized gains. Individual investors facing different marginal tax rates will have different after-tax return expectations, so their security market lines and capital market lines will be quite different.

KEY CONCEPTS

1. The assumptions of capital market theory are:
 - All investors use the Markowitz mean-variance framework to select securities.
 - Investors can borrow or lend any amount of money at the risk-free rate.
 - All investors have homogeneous expectations.
 - All investors have the same one-period time horizon.
 - All investments are infinitely divisible.
 - There are no taxes or transaction costs.
 - There is no inflation, and interest rates do not change.
 - Capital markets are in equilibrium.
2. The introduction of a risk-free asset changes the Markowitz efficient frontier from a curve into a straight line called the CML. The equation of the CML is:

$$E(R_p) = RFR + \sigma_p \times \left\{ \frac{[E(R_M) - RFR]}{\sigma_M} \right\}$$

where R_M and σ_M are the return and standard deviation of the market portfolio

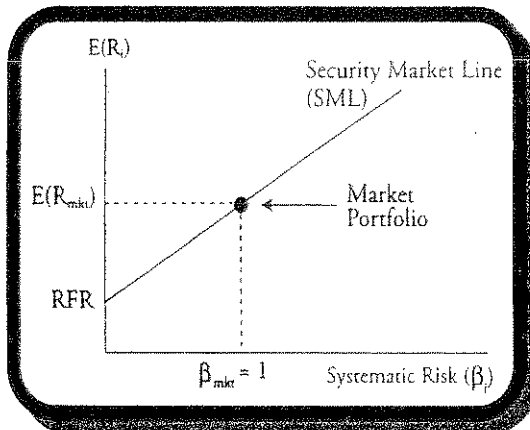
3. The market portfolio is the tangent point where the CML touches the Markowitz efficient frontier. The market portfolio consists of every risky asset; the weights on each asset are equal to the percentage of the market value of the asset to the market value of the entire market portfolio.
4. Total risk is equal to systematic risk plus unsystematic risk.
 - Market or systematic risk cannot be diversified away.
 - Unique or company risk is unsystematic and can be diversified away.
5. The equation of the SML shows the conclusion of the CAPM; expected security returns depend only on systematic risk as measured by beta:

$$E(R_i) = RFR + \beta_i [E(R_{mkt}) - RFR]$$
6. Beta (β) is a standardized measure of systematic risk. It is calculated as:

$$\beta_i = \frac{Cov_{i,mkt}}{\sigma_{mkt}^2} = \left(\frac{\sigma_i}{\sigma_{mkt}} \right) \times \rho_{i,mkt}$$

7. The SML will tell us assets' required returns given their level of systematic risk (as measured by beta). We can compare this to the assets' expected returns (given our forecasts of future prices and dividends) to identify undervalued assets and overvalued assets.

8. The graph of the SML is:



9. Relaxing the CAPM assumptions changes the model's implications.

- The CAPM cannot be derived without equal borrowing and lending rates, unless investors can create a zero-beta portfolio.
- The existence of transactions costs means that the SML is a band (with fairly tight upper and lower bounds on prices) rather than a line.
- The impact of heterogeneous expectations and multiple planning periods on the CAPM is similar to the impact of transactions costs—the SML becomes a band rather than a line.
- Individual investors facing different marginal tax rates will have different after-tax return expectations, so their security market lines (SML) and capital market lines (CML) will be quite different.

CONCEPT CHECKERS: AN INTRODUCTION TO ASSET PRICING MODELS

1. An investor put 60% of his money into a risky asset offering a 10% return with a standard deviation of returns of 8%, and he put the balance of his funds in the risk-free asset offering 5%. What is the expected return and standard deviation of his portfolio?

<u>Expected Return</u>	<u>Standard Deviation</u>
A. 6.0%	6.8%
B. 8.0%	8.0%
C. 8.0%	4.8%
D. 10.0%	6.6%
2. What is the risk measure associated with the capital market line (CML)?
 - A. Beta.
 - B. Covariance.
 - C. Market risk.
 - D. Standard deviation.
3. A portfolio to the right of the market portfolio on the CML is:
 - A. a lending portfolio.
 - B. a borrowing portfolio.
 - C. an inefficient portfolio.
 - D. an impossible portfolio.
4. As you increase the number of stocks in a portfolio, the systematic risk will:
 - A. remain constant.
 - B. increase at a decreasing rate.
 - C. decrease at a decreasing rate.
 - D. decrease at an increasing rate.
5. Total risk equals:
 - A. unique plus diversifiable risk.
 - B. market plus nondiversifiable risk.
 - C. systematic plus unsystematic risk.
 - D. systematic plus nondiversifiable risk.
6. What is the required rate of return for a stock with a beta of 1.2, when the risk-free rate is 6% and the market is offering 12%?
 - A. 6.0%.
 - B. 7.2%.
 - C. 12.0%.
 - D. 13.2%.
7. What is the required rate of return for a stock with a beta of 0.7, when the risk-free rate is 7% and the market is offering 14%?
 - A. 11.9%.
 - B. 14.0%.
 - C. 14.9%.
 - D. 16.8%.

8. The risk-free rate is 6% and the expected market return is 15%. An investor sees a stock with a beta of 1.2 selling for \$25 that will pay a \$1 dividend next year. If he thinks the stock will be selling for \$30 at year end, he thinks it is:
- A. overpriced, so buy it.
 - B. overpriced, so short it.
 - C. underpriced, so buy it.
 - D. underpriced, so short it.
9. A stock with a beta of 0.7 currently priced at \$50 is expected to increase in price to \$55 by year end and pay a \$1 dividend. The expected market return is 15%, and the risk-free rate is 8%. The stock is:
- A. overpriced, so do not buy it.
 - B. underpriced, so buy it.
 - C. properly priced, so buy it.
 - D. properly priced, so do not buy it.
10. The market is expected to return 15% next year and the risk-free rate is 7%. What is the expected rate of return on a stock with a beta of 1.3?
- A. 10.4%.
 - B. 16.3%.
 - C. 17.1%.
 - D. 17.4%.
11. The market is expected to return 12% next year and the risk free rate is 6%. What is the expected rate of return on a stock with a beta of 0.9?
- A. 10.8%.
 - B. 11.4%.
 - C. 13.0%.
 - D. 16.2%.
12. The covariance of the market's returns with the stock's returns is 0.005 and the standard deviation of the market's returns is 0.05. What is the stock's beta?
- A. 0.1.
 - B. 1.0.
 - C. 1.5.
 - D. 2.0.
13. The covariance of the market's returns with the stock's returns is 0.008. The standard deviation of the market's returns is 0.08 and the standard deviation of the stock's returns is 0.11. What is the correlation coefficient between the returns of the stock and returns of the market?
- A. 0.50.
 - B. 0.91.
 - C. 1.00.
 - D. 1.25.
14. Which of the following statements about the SML and the CML is FALSE?
- A. Securities that plot above the SML are undervalued.
 - B. Investors expect to be compensated for systematic risk.
 - C. The market portfolio consists of all the risky assets in the universe.
 - D. Securities that fall on the SML have no intrinsic value to the investor.

15. Susan Kinicki is an analyst. She is talking with a colleague, Charles Riker, about how to determine whether a security is undervalued or overvalued. After meeting with her supervisor, she meets Riker for lunch. During lunch, she makes the following statements:
- i) "I'm not recommending ONJ stock because the expected return is greater than the return I calculated using the CAPM."
 - ii) "Relaxing the standard assumptions of homogeneous expectations and zero transactions costs have the same effect on the SML—the SML becomes a band rather than a straight line."

Riker considers Kinicki's statements and then replies, "I agree with you about the ONJ stock; I don't intend to recommend it either. However, I thought that heterogeneous expectations and positive transactions costs affected the SML differently. I know that transactions costs result in a band, but I don't think introducing heterogeneous expectations has that effect."

Answer this question in context of the SML. Riker is correct with regard to:

- A. statement (i), but not statement (ii).
- B. statement (ii), but not statement (i).
- C. both statement (i) and (ii).
- D. neither statement (i) or (ii).

ANSWERS – CONCEPT CHECKERS: AN INTRODUCTION TO ASSET PRICING MODELS

1. C Expected return: $(0.60 \times 0.10) + (0.40 \times 0.05) = 0.08$, or 8.0%.

Standard deviation: $0.60 \times 0.08 = 0.048$, or 4.8%.

2. D Remember that the capital market line (CML) plots return against *total risk* which is measured by standard deviation.
3. B A portfolio to the right of a portfolio on the CML has more risk than what is offered by the market. Investors seeking to take on more risk will *borrow* at the risk-free rate to purchase more of the market portfolio.
4. A When you increase the number of stocks in a portfolio, *unsystematic risk* will decrease at a decreasing rate; however, *systematic risk* cannot be diversified away and will remain constant no matter how many assets are added to the portfolio.
5. C Because unsystematic risk can be diversified away, investors only expect to be compensated for taking on *systematic risk*.

6. D $6 + 1.2(12 - 6) = 13.2\%$

7. A $7 + 0.7(14 - 7) = 11.9\%$

8. C Required rate = $6 + 1.2(15 - 6) = 16.8\%$

Return on stock = $(30 - 25 + 1) / 25 = 24\%$

Based on risk, the stock plots above the SML and is underpriced, so buy it!

9. A Required rate = $8 + 0.7(15 - 8) = 12.9\%$

Return on stock = $(55 - 50 + 1) / 50 = 12\%$

The stock falls below the SML so it is *overpriced*.

10. D $7 + 1.3(15 - 7) = 17.4\%$

11. B $6 + 0.9(12 - 6) = 11.4\%$

12. D Beta = covariance / market variance

Market variance = $0.05^2 = 0.0025$

Beta = $0.005 / 0.0025 = 2.0$

13. B $\rho_{1,2} = \frac{\text{Cov}_{1,2}}{\sigma_1 \sigma_2} = \frac{0.008}{(0.08)(0.11)} = 0.909$

14. D Securities that fall on the SML are expected to earn the market rate of return and, therefore, do have intrinsic value to the investor (and may have diversification benefits). The other statements are true.

15. D Riker is incorrect to agree with Kinicki about ONJ stock—a stock with an expected return greater than that calculated with the CAPM is undervalued. He is incorrect with respect to statement ii) as well. Introducing heterogeneous expectations and positive transactions costs both make the SML relationship a band rather than a line.