Forward

- Long forward A = Long Spot A + Long A Bond + Short B Bond = Short forward B
- e.g. Buy forward USD(w/JPY) = Buy Spot USD(w/JPY) + Buy US Bond(Lend out) + Sell JP Bond(Borrow From)
- $F_{t,T}(B\ per\ A) = S_t(B\ per\ A) * \frac{Yield_B}{Yield_A}$
- lower bound when cost exist = replicating short = sell at forward bid
- upper bound when cost exist = replicating long = buy at forward ask
- For Asset pay discrete yield(e.g. dividends) D: $F_{t,T} = [S_t - \sum_i PV(D_i)]e^{r(T-t)}$

Future

- Cash Settlement future/ non-deliverable forwards/ non-physical underlyings
- f = F when: (1) carry cost random **and** (2) carry cost highly correlated with underlying. $F > f\ iff\ \rho(S_t, (r_t - d_t)) < 0$
- extra case: spot increase then r decrease: reinvestment of future cashflow is hurted
- Hedge future and Forwards given carry cost: Suppose f(T-1)>F(T-1) or Suppose f(T-1)F(T-2). Use So f(T-1)=F(T-1) to show that too lets you construct an arbitrage. Then so on T-3, T-4...
- $\Delta_F = 1$, $\Delta_f > 1$(Long)

Swap

- deliver A and receiving B cannot be different than long a series of forward B (Quoted in A) and financed with lending
- $\sum_1^T F_{0,t}^{(dollar\ per\ B)} B_{0,t} = s(A\ per\ B) \sum_1^T F_{0,t}^{(dollar\ per\ A)} B_{0,t} => s(A\ per\ B) = \frac{\sum_1^T F_{0,t}^{(dollar\ per\ B)} B_{0,t}}{\sum_1^T F_{0,t}^{(dollar\ per\ A)} B_{0,t}}$
  - e.g. deliver gold and receiving cash: $s(gold\ per\ dollar) = \frac{\sum_1^T F_{0,t}^{(dollar\ per\ dollar)} B_{0,t}}{\sum_1^T F_{0,t}^{(dollar\ per\ gold)} B_{0,t}} => s(dollar\ per\ gold) = \frac{\sum_1^T F_{0,t}^{(dollar\ per\ gold)} B_{0,t}}{\sum_1^T F_{0,t}^{(dollar\ per\ dollar)} B_{0,t}}$
  - e.g. deliver JPY and receiving USD (Change of home country): $s(JPY\ per\ dollar) = \frac{\sum_1^T F^{(JPY\ per\ dollar)} B_{0,t}}{\sum_1^T B_{0,t}} = S_0(JPY\ per\ dollar) \frac{\sum_1^T e^{-r^{USD} t}}{\sum_1^T e^{-r^{JPY} t}}$

- When Losing, "hedged" swap position have no change in expiring payoff during counter party default/ bankruptcy
- Transaction cost: $s^{ask}(dollar\ per\ sth) = \frac{\sum_1^T B_{0,t}^{ask/lend} F_{0,t}^{ask}}{\sum_1^T B_{0,t}^{bid/borrow}}$, change side and reverse get bid
- Value of old swap in new price: $V(X, X^*) = (X^* - X)\sum_1^T B_{0,t}$ (floating receiver for a IRS)
- Rate Market:
  - Gov Debt, T-Bill/T-note(10yr-,0-coupon); T-Bond(10yr+, w/ coupon) and it's STRIP
  - Repo: Lend you money, taking bonds as collateral, you pay rate (typically 3m TB)
  - EuroDollar/ LIBOR
- IRS: Fixed side value $= sN\sum_1^T B_{0,t}$, Floating side value $= N(1 - B_{0,T})$, $s = \frac{1-B_{0,T}}{\sum_1^T B_{0,t}} = \frac{\sum_1^T B_{0,t} R_{0,t-1,t}^f}{\sum_1^T B_{0,t}}$
- IRS from total bond review: Value of IRS must be the difference between two assets
- forward rate $R_{0,t_1,t_2}^f = \frac{B_{0,t_1}}{B_{0,t_2}} - 1 => e^{-r_{0,t_2}t_2}(1 + R_{0,t_1,t_2}^f) = e^{-r_{0,t_1}t_1}$
- OIS: fixed receiver pay compound all O/N interest rate, fixed receivor gets $s - [\prod_0^{T-1}(1 + r_{t,t+1}^{Fed\ Fund}) - 1]$
- Swap Spread = swap - rf ytm, e.g. $s^{ED} - y_T^{TB} = \frac{\sum_1^T [R_{0,t-1,t}^{f,ED} - R_{0,t-1,t}^{f,TB}] B_{0,t}^{TB}}{\sum_1^T B_{0,t}^{TB}}$
- Asset Swap: swap payout and pricipal difference at expiring
- Total-return Swap(TRS): periodically exchange realized return, then rebalancing to unit value
- Credit Default Swap(CDS):
  - you pay me cf from long X coupon bond at T; I pay you cf from long rf bond at T; both at par; exchange difference at maturity; if defaults, terminal payoff is: face value of riskless bond - recovery value of defaulted bond
  - fee should be the same as credit spread.

Options

- Basic Arbitrage: (1) option value > 0 (2)$C^A > C^E, P^A > P^E$ (3) $C \leq S_t, P \leq K$ (4) $P^E \leq KB_{t,T}$ (5) $A \geq max$ (6) $A_{T_2} \geq A_{T_1}\ if\ T_2 > T_1$ (7) $div = 0, C \geq max[S - KB_{t,T}, 0], C^E_{T_2} \geq C^E_{T_1}\ if\ T_2 > T_1$ (8) $P^E \geq max[KB_{t,T} - S, 0]$ (9) $div = 0, C^E = C^A$ (10) C^A optimal exercising: right before ex-date (11) P^A not optimal exercising before ex-date
- Put-Call Parity:
  - $S_t = C^E - P^E + KB_{t,T}$ or $C^E - P^E = S_t - PV(D) - KB_{t,T}$
  - $S - K \leq C^A - P^A \leq S - B_{t,T}K$ or $S - PV(D) - K \leq C^A - P^A \leq S - B_{t,T}K$
- Butterfly and Risk Neutral: $\frac{1}{\delta K}$ share of butterfly price b_k, is the risk-neutral prob q times $B_{t,T}$
- Breeden and Litzenberger Theorem: one can identifying RN prob by watching curvature.
- Binomial when we have div or foreign: $q \equiv \frac{\frac{\bar{r_d}}{\bar{r_f}} - d}{u-d} \approx \frac{e^{(r_d - r_f)\Delta t} - d}{u-d} = \frac{e^{(r-div)\Delta t} - d}{u-d}$ conventional $u = e^{\sigma\sqrt{\Delta t}}$
- Black PDE General setting:
  - Geometric Brownian: $\frac{dS}{S} = \mu dt + \sigma dW$ or in general $dS = a(S,t)dt + b(S,t)dW$ a: drift, b: diffusion, a/S: expected, b/S volatility(std dev)
  - Ito and Delta hedge:

$$\begin{aligned}
&dC = C_S dS + C_t dt + \tfrac{b^2}{2} C_{SS} dt \\
&d\Pi = dC - C_S dS + rIdt = C_t dt + \tfrac{b^2}{2} C_{SS} dt + rIdt = r\Pi dt \\
&r(C - C_S S + I)dt = C_t dt + \tfrac{b^2}{2} C_{SS} dt + rIdt \\
&C_t + rSC_S + \tfrac{b^2}{2} C_{SS} - rC = 0 \\
&C_t + rSC_S + \tfrac{1}{2}\sigma^2 S^2 C_{SS} - rC = 0
\end{aligned}$$

  - Based on Black Schole formula: $I = Ke^{-r(T-t)N(d_2)} => N(d_2)$ is the precentage of PV of K we need to borrow to replicate portfolio Π
- Black Schole's "additional" assumption: (1) continue transaction is possible (2) no price jump (3) r σ constant (4) d = 0
- Black Schole Greek

Greeks	Call	Put	Call Sign	Put Sign
Δ	$N(d_1)$	$-N(-d_1)$	l+ s-	l- s+
Γ	$\frac{1}{S\sigma\sqrt{T-t}}\phi(d_1)$	Same	l+ s-	l+ s-

Greeks	Call	Put	Call Sign	Put Sign
v	$S\sqrt{T-t}\phi(d_1)$	Same	l+ s-	l+ s-
Θ	$\frac{S\sigma}{2\sqrt{T-t}}\phi(d_1) rKe^{-r(T-t)}N(d_2)$	$\frac{S\sigma}{2\sqrt{T-t}}\phi(d_1) rKe^{-r(T-t)}N(-d_2)$	l- s+	l- s+
ρ	$K(T-t)e^{-r(T-t)}N(D_2)$	$-K(T-t)e^{-r(T-t)}N(-D_2)$	l+ s-	l- s+

- Lira-Peso Case Margarbe's Formula: $option = S_t^{Lira}e^{-r^{Lira}(T-t)}N(d_1) - S_t^{Peso}e^{-r^{Peso}(T-t)}N(d_2)$, where $d_1 = \frac{ln(\frac{S_t^{Lira}}{S_t^{Peso}})+(r^{Peso}-r^{Lira})(T-t)+\frac{1}{2}\sigma^2(T-t)}{\sigma\sqrt{T-t}}$, and $\sigma = \sqrt{\sigma_X^2+\sigma_Y^2-2\rho\sigma_X\sigma_Y}$ is the volatility of the ratio
- Lira-Peso Case II Future Option: $C = S_t B_{t,T^{Lira}}N(d_1) - KB_{t,T}^{Peso}N(d_2) = e^{r(T-t)}[FN(d_1)-KN(d_2)]$, where $d_1 = \frac{ln(\frac{F}{K})+\frac{1}{2}\sigma^2(T-t)}{\sigma\sqrt{T-t}}$, $e^{r(T-t)}$ for future, $e^{r(T'-t)}$ for Forward.
- Merton's Forward volatility: $\bar{\sigma_{12}} = \sqrt{\frac{(T_2-t)\bar{\sigma_2}^2-(T_1-t)\bar{\sigma_1}^2}{T_2-T_1}}$

From Midterm

- 2017Q3 OIS: $V(s) = B_{0,T}(1+s)-1 => s = R_{0,T}$, one year simple rate(EAR).
- 2017Q4 Bound: find best and worst scenario
- 2016Q2 $max[Se^{-r_f(T-t)} - Ke^{-r_d(T-t)}, 0]$ and $max[S-K, 0]$. Treat as a func of S, find K^*
- 2016Q4 Through Black Schole PDE back into ito's result

Volatility

- short-dated options Deeo OTM, not sensitive to volatility
- Heston Model: $d\sigma_t = \kappa(\sigma_0-\sigma_t)+s\sigma_t dW_t^\sigma$
- When κ is high and s is low, mean reverting faster and behave like constant vol.
- VS GARCH: GARCH is 1. discrete in time, 2. when take time to limit become non-stochastic
- Multi dimensional Ito: add $\frac{1}{2}b_Y^2\frac{\partial^2 C}{\partial Y^2}d<Y>_t$, $\frac{\partial C}{\partial Y}dY$, and a $b_Y b_\theta\frac{\partial C}{\partial Y\partial\theta}d<Y,\theta>_t$ to all other risk factors.

- Idea of hedging non-tradable risk factor(e.g. Heston): Creating a all-else neutral (e.g. BS delta-neutral) portfolio Π_C and, create another such portfolio Π_P, long 1 Π_C and short ratio $= \frac{Vol_C[Payoff]}{Vol_P[Payoff]}$ (e.g. $\frac{b_C \Pi_C}{b_P Pi_P}$)
- Doing hedge above, get (not RN) market price of risk factor $\lambda = \frac{a-r}{b}$, a kind of sharpe ratio (use sharpe ratio approach to estimate from hist data).
- Feymann-Kac: 1. risk-neutralize (traded asset drift to $r_f S$, non-traded asset drift to $a - \lambda b$) in PDE 2. calculate value w.r.t that drift 3. disc at r_f 4. take $E^*[DF_{t,T}C(X_T)]$
- Observation from Heston Model: 1. fatter tail of return dist 2. leptokurtic 3. Deep OTM option more valuable, Near-the-money option less valuable
- ρ positive: 1.less likely to go down after go down, higher likely to go up after go up 2.left low right high 3. OTM call more/ put less likely to payoff ITM 4. Smile skewed ll to ur 5. vice versa for ρ negative
- Problem with Multi-dimension: (Econometrics)1. stochastic model of each 2. parameters (Market Completeness)3. two securities you can use to hedge (risk X only, long and short) 4. Jumps negligible (Risk preference)5. λ
- Given GBM and using Feymann-Kac: $E[S_T] = S_0 e^{a^{S,RN}T}$
- Vasicek: $dr = \kappa(r^* - r)dt + bdW$ (RN, change to $\bar{r}$ if physical), r_T N, CIR: change b to $s\sqrt{r}$
- Vasicek way to calculate Term Strucuture: $B_{t,T}(r_T) = e^{G_0(t,T) - r_t G_1(t,T)}, G_1 = \frac{(1-e^{-\kappa(T-t)})}{\kappa}, G_0 = \frac{(G_1 - (T-t))(r^*\kappa^2 - b^2/2)}{\kappa^2} - \frac{(bG_1)^2}{4\kappa}$
- Error of MC: 1. Discretization (ϵ $\mathcal{O}(N^-1)$) 2. Not enough paths (ϵ $\mathcal{O}(M^{-\frac{1}{2}})$)
- Short Log forward get $L_0 = B_{0,T} - \frac{1}{2}\int_0^T \sigma_t^2 dt$, where $MFIV^2 = \frac{1}{T}E_0^Q[\int_0^T \sigma_t^2 dt]$
- Butterfly approach toward VIX: $L_0 = \sum_k log(\frac{k}{F})b(k) => MFIV^2 = \frac{2}{T}B_{0,T}^{-1}(\int_0^{F_{0,T}} \frac{p(k)}{k^2}dk + \int_{F_{0,T}}^{\infty} \frac{c(k)}{k^2}dk)$

Credit

- Merton's Model: Equity is a long european Call with K = Debt FV => CFO increase volatility of stock to max shareholders benefit (hurting Bond holders), assuming 1.$dV = (\mu - \Pi)Vdt + \sigma_V VdW$ 2. no taxes and reorganization cost 3. won't do unanticipated financing 4. enforcable
- Modigliani-Miller Theorem: market value of firm is independent of its capital structure and only cash flows instaed.

- assuming earning ~ GBM, constant rate, and perpetual, $V(E) = \frac{E_t}{r-(\mu_E - \lambda_E\sigma_E)}$ => $\Pi = \frac{E}{V} = r - (\mu_E - \lambda_E\sigma_E)$. Then F, a claim on V, follow BS PDE with $r = r - \Pi$ and add a coupon rate Γ and has solution $F = FVe^{-r(T-t)}N(d_2) + VN(-d_1)$, yield spread $= y_t - r = -\frac{1}{t}log(N(d_2) + \frac{1}{d}N(-d_1))$

- Equity Model: assume S ~ CEV $dS = \mu Sdt + \omega\sqrt{S}dW$ then F follow BS PDE with S^2 => S in second derivative term, add a coupon rate Γ and change r to (r-div)

- Convertible Bond: Change BC on Stock to $F(S_t, t) \geq NS_t$ where $N = \frac{FV}{S_C}$, Manager convert whenever Bond Value is greater than that.

- Better model = 1. Jump 2. Future Financing 3. Restructring 4. Constraint

- CDS: long credit spread = protection buyer = pay fee to protection seller = short credit spread = pay (1-R) to buyer when credit events happen.

- CLN: Z =pricinpal=> Y, won't payback pricipal if X default still need to pay high coupon rate.

- Assuming no recovery, risky Bond $\hat{B_{t,T}}(V_T) = B_{t,T}(1-H(T;t))$, $H(T;t) = 1 - e^{\lambda^X(T-t)}$ => cdf of dying before T, h => pdf, f = λ => conditional probability (if constant = ϕ, or credit spread)

- CDS $\phi = \frac{\int_t^T \hat{B_{t,s}} f(s;t)ds}{\int_t^T \hat{B_{t,s}} ds}$, MtM value $V = (\phi_{new,s} - \phi)\int_s^T \hat{B_{s,u}} du$

- fetching RN probablity of default: $\phi_{t,t+1} = \frac{(1-R)B_{t,t+1}h(t+1;t)}{B_{t,t+1}(1-h(t+1;t)}$, then recursively solve all others.

- Credit Value Adjustment = Expected Discounted Unrecoverable Mark-to-Market Profit = $E^*[\int_t^T DF_{t,s}h(s;t)(R^Y - 1)max[V(Y), 0]ds]$

- If CVA is a loss on one leg, then Debit Value Adjustment is the same idae on the other leg. Generally: adj. value to protection leg = protection Leg value - fee leg value - CVA + DVA

- ABS, CDO and unhedgeable Repayment Risk(Timing risk): transfer! long tranch in CDO = long rf + short CDS on those tranch(or short CDX NA IG)