

$$1. a) \quad w_{ji} := w_{ji} - \alpha \frac{\partial \mathcal{L}}{\partial w_{ji}}$$

$$\frac{\partial \mathcal{L}}{\partial w_{ji}} = \frac{\partial \mathcal{L}}{\partial z_j} \frac{\partial z_j}{\partial w_{ji}}$$

$$= \delta_j \cdot \frac{\partial}{\partial w_{ji}} \left( \sum_i w_{ji} x_i \right)$$

$$= \delta_j \cdot \frac{\partial}{\partial w_{ji}} \left( w_{ji} x_i + \sum_{l \neq i} w_{jl} x_l \right)$$

$$= \underline{\delta_j x_i}$$

$$\delta_j = \frac{\partial \mathcal{L}}{\partial z_j} = \sum_k \frac{\partial \mathcal{L}}{\partial z_k} \frac{\partial z_k}{\partial a_j} \frac{\partial a_j}{\partial z_j}$$

$$= \sum_k \delta_k \frac{\partial}{\partial a_j} \left( \sum_j w_{kj} a_j \right) \frac{\partial f(z_j)}{\partial z_j}$$

$$= \sum_k \delta_k \frac{\partial}{\partial a_j} \left( w_{kj} a_j + \sum_{m \neq j} w_{km} a_m \right) f'(z_j)$$

$$= \underline{f'(z_j) \sum_k w_{kj} \delta_k}$$

1.6) Hidden layer to output layer:

$$W_{kj} := W_{kj} - \alpha \delta_k a_j$$

$W$  is a  $64 \times 10$  matrix

$\alpha$  is a scalar

$\delta$  is a Batch-size  $\times 10$  matrix

$a$  is a Batch-size  $\times 64$  matrix

$$\underline{W := W - \alpha a^T \delta}$$

Input layer to ~~into~~ hidden layer:

$$W_{ji} := W_{ji} - \alpha \delta_j x_i$$

$W$  is a  $785 \times 64$  matrix

$\alpha$  is a scalar

$\delta$  is a Batch-size  $\times 64$  matrix

$X$  is a Batch-size  $\times 785$  matrix

$$\underline{W := W - \alpha X^T \delta}$$