Laboration 1: The k Maximum Sum Subsequences problem

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1 Initial Filtering

Consider sequence $X = \langle x_1, x_2, \ldots x_n \rangle$, of integer numbers. The aim of the lab is to first remove all duplicate elements from X, except for the first occurrence of the element. Call the new sequence of elements $Y = \langle y_1, y_2, \ldots y_{n'} \rangle$. In the sequence Y, we have that $y_i \neq y_j \forall i, j \in [1, n'], i \neq j$. That is, all elements of Y are unique.

2 k Maximum Sum Subsequences

Now, the problem is to find the k continuous subsequences of Y of maximum sum. The largest subsequence of a sequence Z is a sequence $\langle z_i, z_{i+1}, \ldots, z_j \rangle$ such that the sum $z_i + z_{i+1} + \ldots + z_j$ is maximized over all $i \leq j$. In this problem, however, we are to find the k largest subsequences. Those subsequences can overlap.

3 Solving the problem

Define a function `kmaxsubunique` that takes two arguments: The list X (first argument) and the value k (second argument). The function should return a list of triples (v, i, j), where v is the sum of the subsequence starting at i and ending at j. The list should be ordered with the triple representing the largest subsequence first, then the second largest and so on, up to the k:th largest subsequence.

3.1 Implementation

It is probably convenient to begin with the filtering step, and then compute the sum of all possible subsequence, whereafter a search is performed to find the k largest. You are not allowed to use the built-in function `nub`.

3.2 Example

If $Z = \langle 1, 4, 2, -1, 3 \rangle$ and $k = 3$, the solution would then be [(9, 1, 5), (8, 2, 5) , (7, 1, 3)], because (1+4+2+(-1)+3) is the largest subsequence sum, (4+2+(-1)+3) is the second largest and (1+4+2) is the third largest.