TDT4171 Artificial Intelligence Methods
Assignment 2

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Part A

The scenario for the *Umbrella World* is that a security guard who is working in a basement is wondering wether it is today. He cannot observe the rain directly, but he can observe if his boss comes in with an umbrella or not.

We want to describe the *Umbrella world* as a HMM. For this HMM we see that:

- The set of unobserved variables is $\mathbf{X_t} = \{Rain_t\}$
- The set of observed variables is $\mathbf{E_t} = \{Umbrella_t\}$
- The dynamic model is given by $\mathbf{P}(\mathbf{X_t}|\mathbf{X_{t-1}}) = \begin{bmatrix} 0.7 & 0.3 \\ 0.3 & 0.7 \end{bmatrix}$

  and the observation model is given by $\mathbf{P}(\mathbf{E_t}|\mathbf{X_t}) = \begin{bmatrix} 0.9 & 0.1 \\ 0.2 & 0.8 \end{bmatrix}$

  (Found from Figure 15.2)
- There are a couple of key assumptions for this model
  - We make a Markov assumption by assuming that the current state only depends on a finite fixed number of previous states. Thus this is a Markov process and in this case it is a **first-order Markov process** since the current state only depends on the previous one.
  - We also assume that the changes in the states are caused by a stationary process (the laws governing the change in state do not change over time).
  - We make a **sensor Markov assumption** by assuming that evidence only depend on the current state

By making these assumptions we get a pretty simple model. In real life some of these assumptions are hard to justify. E.g. we assume that the probability of it raining on a specific day only depend on if it rained the previous day. One would think that if it had rained multiple days prior, this probability would change. Also we assume that this is a stationary process, which may hold in some domains, but may also be very innacurate in others.

Part B

I have chosen the prior probability of rain to be $\mathbf{P}(\mathbf{X_0}) = [0.5, 0.5]$ as they did in the book.

After implementing the FORWARD algorithm we can calculate probabilites of rain on a specific day given a sequence of observations. We have calculated the probability of rain on day 2 given 2 observations and day 5 given 5 observations to be $\mathbf{P}(\mathbf{X_2}|\mathbf{e_{1:2}}) = [0.883, 0.117]$, and $\mathbf{P}(\mathbf{X_5}|\mathbf{e_{1:5}}) = [0.867, 0.133]$ respectively.

k	$\mathbf{f_{1:k}}$
1	[0.818, 0.182]
2	[0.883, 0.117]
3	[0.191, 0.809]
4	[0.731, 0.269]
5	[0.867, 0.133]

Table 1: Normalized forward messages $\mathbf{f_{1:k}} \quad k \in \{1, 2, 3, 4, 5\}$

Part C

After implementing the FORWARD-BACKWARD algorithm we can calculate the probability of rain on day 1 given a sequence of observations. We have calculated the probability of rain on day 1 given 2 observations and 5 observations to be $\mathbf{P}(\mathbf{X_1}|\mathbf{e_{1:2}}) = [0.883, 0.117]$, and $\mathbf{P}(\mathbf{X_1}|\mathbf{e_{1:5}}) = [0.867, 0.133]$ respectively.

k	$\mathbf{b_{k+1:t}}$
1	[0.066, 0.046]
2	[0.091, 0.150]
3	[0.459, 0.244]
4	[0.690, 0.410]
5	[1.000, 1.000]

Table 2: Normalized backward messages $\mathbf{b_{k+1:t}} \quad k \in \{1, 2, 3, 4, 5\}$