# TDT4171 Artificial Intelligence Methods
# Assignment 1

Fredrik Veidahl Aagaard

February 2019

# 1 5-card Poker Hands

## a)

There are $\binom{52}{5} = 259860$ different 5-card poker hands possible

## b)

The probability of drawing any one of these hands is then $\binom{52}{5}^{-1} \approx 0.000385\%$.

## c)

There are only 4 hands that gives a royal flush (10 through A in the 4 different suits). Thus the probability of getting a royal flush is $\binom{4}{1}\binom{52}{5}^{-1} \approx 0.000154\%$.

When finding the probability for four-of-a-kind we first see that there are 13 different numbers that can be chosen $\binom{13}{1}$. When one of the numbers are chosen, the number of ways we can draw the 3 other cards of that number is $\binom{4}{4}$. Then the last card can be chosen from the remaining 48 cards $\binom{48}{1}$. Thus the probability of drawing four-of-a-kind is $\binom{13}{1}\binom{4}{4}\binom{48}{1}\binom{52}{5}^{-1} \approx 0.0240\%$

## 2 Bayesian Network Construction

### 1.

$p(A, B, C, D) = p(A \mid B, C)p(B \mid D)p(C \mid D)p(D)$

| P(D) | P(¬D) |
|---|---|
| 0.7 | 0.3 |

| D | P(B\|D) | P(¬B\|D) |
|---|---|---|
| t | 0.6 | 0.4 |
| f | 0.35 | 0.65 |

| D | P(C\|D) | P(¬C\|D) |
|---|---|---|
| t | 0.5 | 0.5 |
| f | 0.2 | 0.8 |

| B | C | P(A\|B,C) | P(¬A\|B,C) |
|---|---|---|---|
| t | t | 0.9 | 0.1 |
| t | f | 0.45 | 0.55 |
| f | t | 0.4 | 0.6 |
| f | f | 0.05 | 0.95 |

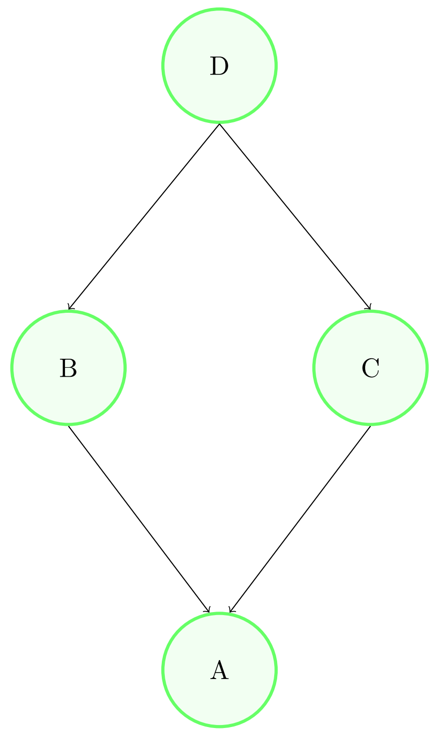


Figure 1: Bayesian Network and conditional probability tables for 1

**2.**

$p(F, T, A, S, L, R) = p(F)p(T)p(A \mid F, T)p(S \mid F)p(L \mid A)p(R \mid L)$

| P(F) | P(¬F) |
|---|---|
| 0.005 | 0.995 |

| P(T) | P(¬T) |
|---|---|
| 0.0015 | 0.9985 |

| F | T | P(A\|F,T) | P(¬A\|F,T) |
|---|---|---|---|
| t | t | 0.5 | 0.5 |
| t | f | 0.7 | 0.3 |
| f | t | 0.99 | 0.01 |
| f | f | 0.001 | 0.999 |

| F | P(S\|F) | P(¬S\|F) |
|---|---|---|
| t | 0.95 | 0.05 |
| f | 0.02 | 0.98 |

| A | P(L\|A) | P(¬L\|A) |
|---|---|---|
| t | 0.98 | 0.02 |
| f | 0.1 | 0.9 |

| L | P(R\|L) | P(¬R\|L) |
|---|---|---|
| t | 0.65 | 0.35 |
| f | 0.05 | 0.95 |

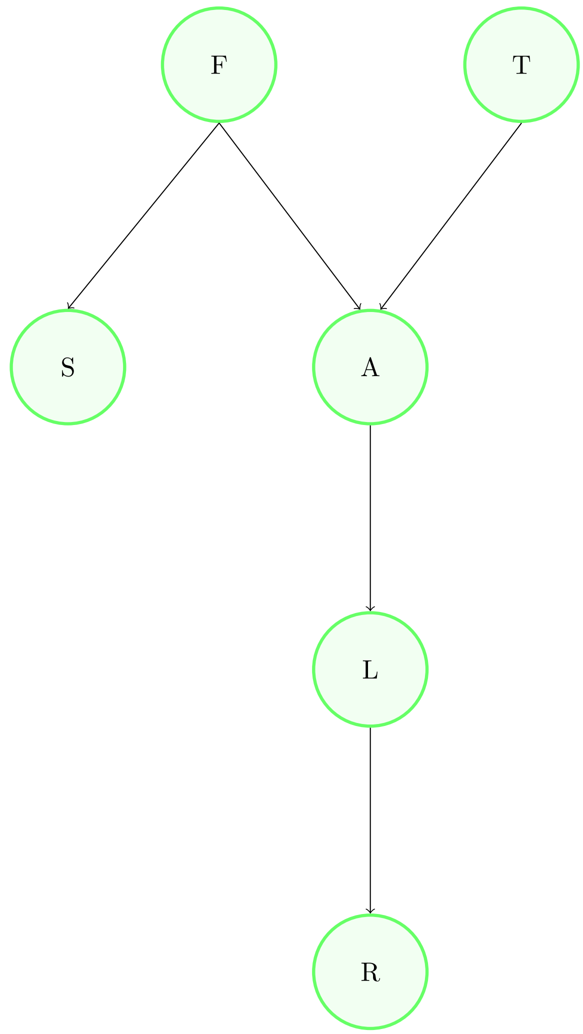


Figure 2: Bayesian Network and conditional probability tables for 2.

**3.**

p(RT,RY,FW,UUT,CS)=p(RY)p(CS)p(RT|RY,CS)p(FW|RT,RY)p(UUT|RT,CS)

| P(RY) | P(¬RY) |
|---|---|
| 0.3 | 0.7 |

| P(CS) | P(¬CS) |
|---|---|
| 0.6 | 0.4 |

| RY | CS | P(RT\|RY,CS) | P(¬RT\|RY,CS) |
|---|---|---|---|
| t | t | 0.65 | 0.35 |
| t | f | 0.02 | 0.98 |
| f | t | 0.3 | 0.7 |
| f | f | 0.01 | 0.99 |

| RT | RY | P(FW\|RT,RY) | P(¬FW\|RT,RY) |
|---|---|---|---|
| t | t | 0.99 | 0.01 |
| t | f | 0.97 | 0.03 |
| f | t | 0.3 | 0.7 |
| f | f | 0.1 | 0.9 |

| RT | CS | P(UUT\|RT,CS) | P(¬UUT\|RT,CS) |
|---|---|---|---|
| t | t | 0.99 | 0.01 |
| t | f | 0.85 | 0.15 |
| f | t | 0.8 | 0.2 |
| f | f | 0.001 | 0.999 |

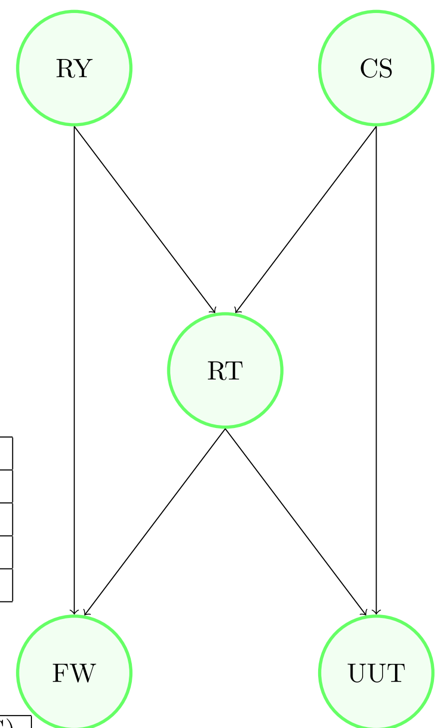


Figure 3: Bayesian Network and conditional probability tables for 3.

Using:
RY = RainYesterday
CS = CloudSky
RT = RainToday
FW = FloorWet
UUT = UseUmbrellaToday

When constructing the conditional probability tables for the first task I have just chosen some values for the probabilities. It is hard to get an intuitive sense of the different probabilities as the nodes are just letters.

When researching bayesian networks I stumbled across a presentation (1) with a network which is the same as the one in the second task, except with words instead of letters (F=Fire, T=Tampering, S=Smoke, A=Alarm, L=Leaving, R=Report). This gives a more intuitive feel for the probabilities, but I have not used the same as the ones given in the presentation.

Thus in the second and third task, I have chosen probabilities that I think might represent the given scenarios.

Also since the variables in the tasks are binary we see that the rows in the tables sum to 1. Here we could have ommited the last column $P(\neg X \mid X(parents))$ as this is simply $(1 - P(X \mid X(parents))$.

## 3 Bayesian Network Application

When we start out by selecting a door, the probability of a given door containing the prize is $\frac{1}{3}$. This means that when we have chosen a door, the probability that the prize is behind one of the other doors is $\frac{2}{3}$. When the official then opens one of the doors the probability is still $\frac{2}{3}$ that the prize is behind one of them, only now we know that the prize can't be behind the door he opened.

Thus the probability that the prize is behind the door he didn't open (and that we didn't pick) is $\frac{2}{3}$ and we should switch our choise.

Figure 4 shows the bayesian network for the scenario described. We see that OpenedByOfficial is dependant on MyChoise and ContainsPrize. And we know this must be true as OpenedByOfficial can not be the door we have chosen, nor can it be the door containing the prize.

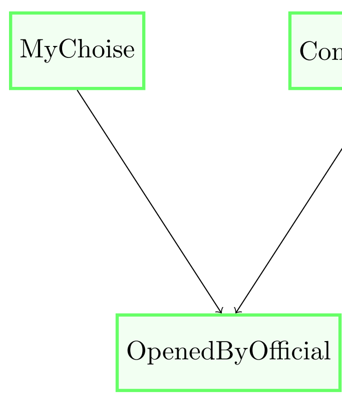


Figure 4: Bayesian network for doors

We can also see what is intuitively stated above by looking at the conditional probability tables. Table 1 gives the probabilities for which door we pick (we assume equal probability of picking each door), and Table 2 gives the probabilities for which door has the prize behind it (also assume equal probability the prize is behind each door). This is the a priori knowledge we have.

Then when we pick a door, the official must open one of the two other doors. Table 3 gives probabilites for which door the official opens given which door we chose and which door the prize is behind.

Let's say we chose door A. Then the official must open either door B or C. Let's say he picks door B. After he opens door B, revealing no prize, the new conditional probability table (the a posteriori knowledge) for ContainsPrize is given by Table 4. Here we see the same conclusion from the explanation given above: That there is a $\frac{2}{3}$ chance the prize is behind the door we didn't pick (door C in this case). We see that we should indeed switch to door C to have the highest chance of winning!

| P(MyChoise(A)) | P(MyChoise(B)) | P(MyChoise(C)) |
|---|---|---|
| $\frac{1}{3}$ | $\frac{1}{3}$ | $\frac{1}{3}$ |

Table 1: Conditional probability table for MyChoise before picking a door

| P(ContainsPrize(A)) | P(ContainsPrize(B)) | P(ContainsPrize(C)) |
|---|---|---|
| $\frac{1}{3}$ | $\frac{1}{3}$ | $\frac{1}{3}$ |

Table 2: Conditional probability table for ContainsPrize before official opens a door

| MyChoise | | A | | | B | | | C | | |
|---|---|---|---|---|---|---|---|---|---|---|
| ContainsPrize | | A | B | C | A | B | C | A | B | C |
| OpenedByOfficial | A | 0 | 0 | 0 | 0 | 0.5 | 1 | 0 | 1 | 0.5 |
| | B | 0.5 | 0 | 1 | 0 | 0 | 0 | 1 | 0 | 0.5 |
| | C | 0.5 | 1 | 0 | 1 | 0.5 | 0 | 0 | 0 | 0 |

Table 3: Conditional probability table for OpenedByOfficial before official opens a door

| P(ContainsPrize(A)) | P(ContainsPrize(B)) | P(ContainsPrize(C)) |
|---|---|---|
| $\frac{1}{3}$ | 0 | $\frac{2}{3}$ |

Table 4: Conditional probability table for ContainsPrize after official opens door B

# References

[1] U. D. of Computer Science, “Reasoning under uncertainty: Bayesian networks intro,” 2011. [Online]. Available: https://www.cs.ubc.ca/ hutter/teaching/cpsc322/6-Uncertainty4.pdf