An Accurate Markov Model for Slotted CSMA/CA Algorithm in IEEE 802.15.4 Networks

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Abstract— **In this letter we propose an Markov model for slotted CSMA/CA algorithm working in a non-acknowledgement mode, specified in IEEE 802.15.4 standard. Both saturation throughput and energy consumption are modeled as functions of backoff window size, number of contending devices and frame length. Simulations show that the proposed model can achieve a very high accuracy (less than 1% mismatch) if compared to all existing models (bigger than 10% mismatch).**

Index Terms— **IEEE 802.15.4, Slotted CSMA/CA, MAC.**

I. Introduction

IEEE 802.15.4 standard defines physical layer and medium access control (MAC) layer specifications for short-rang radio data communication devices with low-data-rate and low power [1]. It can be used in a wide range of applications, e.g., wireless sensor networks and home networking. Two types of channel access algorithms are specified in the standard: unslotted and slotted CSMA/CA algorithms [1]. Slotted CSMA/CA is used only in the beaconed mode [1].

Recently, there are increasing research interests on analytically modeling slotted CSMA/CA algorithm based on Markov chain. A simplified but inaccurate Markov model was proposed in [2] with approximated distribution for random backoff slots. An embedded Markov model was proposed in [3], but it is incomplete. Markov models are developed with an assumption about independent channel accessing probability [4] [5]. An improved model was proposed in [6] with an assumption of independent channel sensing probability. However, the above models are inaccurate as well. A new Markov model was proposed in [7] with an assumption that the results of clear channel assessment (CCA) are dependent on when the CCA is performed. Unfortunately, it is observed from detailed analysis that the state transitions in the Markov chain is incorrectly modeled.

In this letter, we proposed an Markov model modified from [7] for 802.15.4 slotted CSMA/CA algorithm with saturated traffic. Both throughput and energy consumption can be accurately analyzed by the model. The impacts of MAC parameters, number of network devices and frame length are also investigated. The model achieves a relatively high accuracy if compared to all existing proposed models.

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II. Slotted CSMA/CA Algorithm

In slotted CSMA/CA algorithm [1], each device maintains three variables for each transmission attempt: NB, W and CW. NB is the backoff stage, representing the number of times that CSMA/CA algorithm is required to backoff while attempting the current transmission. W is the backoff window used to determine the number of backoff slots a device shall wait before accessing a channel. CW is the contention window length, defining the number of backoff periods that need to be clear for CCA before transmission. CW is initialized to two before each transmission attempt and reset to two each time the channel is assessed to be busy.

Before each new transmission attempt, NB is initialized to zero and W is initialized to W_0. A backoff counter is set with a value chosen from a random number in the range $[0, W_0 - 1]$. The backoff counter value decrements at every slot without sensing the channel. Once the backoff counter reaches zero, it performs the first CCA (denoted by CCA1) at the beginning of the backoff slots. If the channel is idle in CCA1, then CW is decremented by one and the second CCA (denoted by CCA2) is performed in the next backoff slot. If the channel is idle again during CCA2, it transmits the frame in the next backoff slot. If the channel is busy during either CCA1 or CCA2, CW should be reset to two, NB increases by one, and W is doubled until it reaches the maximal backoff window W_x. If NB is less than or equal to an allowed number of retries (denoted by m), the above backoff and CCA processes are repeated. If NB is greater than m, CSMA/CA algorithm terminates.

III. Analytical Model

Let us consider a single hop 802.15.4 WPAN operating in the beaconed mode. We assume N devices in the network, excluding the coordinator device. Each device has saturated uplink traffic to send to the coordinator in a non-acknowledgment mode. Channel access is organized by the coordinator with a superframe structure [1]. To focus our attention on CSMA/CA analysis, we will consider a superframe consisting of only contention access period (CAP).

Our model will use the similar assumptions made in [7] that the probability of channel being sensed busy is related to when CCA is performed. Consider the operation of MAC in a device as a renewal process, which starts with a data frame transmission (with fixed length of L slots) and followed by an idle period. The duration of the idle period depends on transmission activity and backoff situations of the devices. The maximal number of slots in an idle period is W_x plus two (for CCA1 and CCA2).

Let p_k denote the probability of a transmission from devices other than a tagged device starting after exactly the kth idle slots since the last transmission, and define $q_k = 1 - p_k$, where $k \in [0, W_x{+}1]$. To calculate transmission probability of a device in a general backoff slot in the renewal process, an

Markov chain is constructed for each device. It consists of the following states, which are associated with channel activities in a slot:

- Busy state: denoted by $B_{i,j,l}$, during which at least one device other than the tagged device transmits the lth part of a frame of L slots, with the backoff stage and backoff counter of the tagged device being i and j, respectively, where $i \in [0, m]$, $j \in [0, W_i]$, and $l \in$ [2,L]. W_i is the minimum of $2^i W_0$ and W_x.
- Backoff state: denoted by $K_{i,j,k}$, during which the tagged device goes backoff with a backoff counter being j at backoff stage i, after k idle slots since the last transmission, where $i \in [0, m]$, $j \in [0, W_i]$, and $k \in$ [0,W_i +1].
- Sensing state: denoted by $C_{i,k}$, during which the tagged device performs CCA2 at the ith backoff stage, after k idle slots since the last transmission, where $i \in [0, m]$ and $k \in$ [0,W_i +1].
- Initial transmission state: denoted by $X_{i,k}$, during which the tagged device starts to transmit a frame at backoff stage i, after k idle slots since the last transmission.
- Transmission state: denoted by T_l, during which the tagged device transmits the lth part of a frame, where $l \in [2, L]$. The first part is transmitted in the state $X_{i,k}$.

As an example, an Markov chain with m=0 is shown in Fig.1. It can be easily extended to the cases of $m > 0$. As some states will never be visited from any other states, those states are not shown in Fig.1 and their steady state probabilities will be zero.

Let $\overline{M}$ denote the steady state probability of an Markov state M. For the busy states $B_{0,j,2}$ with $j \in [0, W_0 - 1]$, two types of events will cause transitions into them from other states. One is that the tagged device senses channel busy in CCA1 or CCA2 at the mth backoff stage. Under saturated traffic conditions, the device will use CSMA/CA to retransmit a frame or transmit a new frame. The other event is that the channel becomes busy when the tagged device is in its backoff states with its backoff stage being zero. The balance equations for states $B_{0,j,2}$ with $j \in [0, W_0 - 1]$ are obtained as follows:

$$\overline{B}_{0,j,2} = \frac{1}{W_0}\left[\sum_{k=2}^{W_0-1} p_k \overline{K}_{0,j+1,k} + \sum_{k=2}^{W_m} p_k(\overline{K}_{m,0,k} + \overline{C}_{m,k})\right] \tag{1}$$

One of the problems in the Markov model of [7] is that the busy states start with some other devices transmitting the first part of an L slots frame, which is incorrect as the first part of the frame is transmitted in one of the backoff states.

Similarly, the steady state balance equations for the busy states $B_{i,j,2}$ is obtained for $i \in [1, m]$ and $j \in [0, W_i - 1]$ by:

$$\overline{B}_{i,j,2} = \frac{1}{W_i}\left[\sum_{k=2}^{W_i-1} p_k \overline{K}_{i,j+1,k} + \sum_{k=2}^{W_{i-1}} p_k(\overline{K}_{i-1,0,k} + \overline{C}_{i-1,k})\right] \tag{2}$$

The transitions to the other busy states $B_{i,j,l}$ can happen in two cases: (1) backoff counter decrements by one in the busy states of the same backoff stage; or (2) the device performs CCA1 in the busy states. The balance equations can be expressed as (where $j \in [0, W_i - 1]$ and $l \in [3, L]$):

$$\overline{B}_{i,j,l} = \begin{cases} \overline{B}_{0,j+1,l-1} + \overline{B}_{m,0,l-1}/W_0, & i = 0 \\ \overline{B}_{i,j+1,l-1} + \overline{B}_{i-1,0,l-1}/W_i, & i \in [1, m] \end{cases} \tag{3}$$

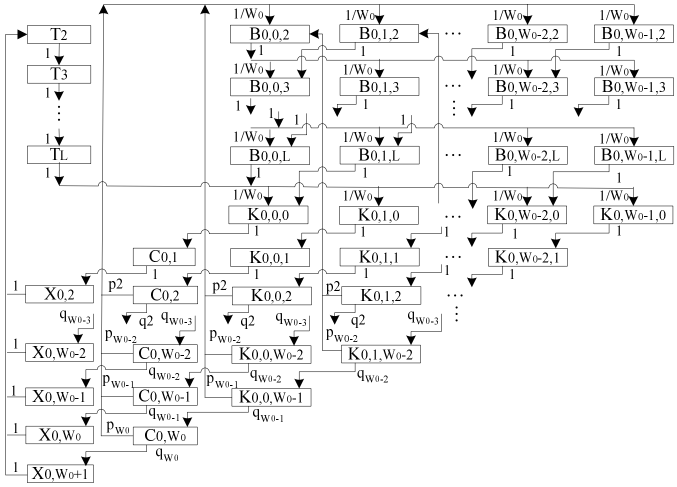

Fig. 1. Markov chain model for the slotted CSMA/CA algorithm in non-acknowledgment mode with m=0.

There are three types of transitions into backoff states $K_{0,j,0}$: (1) after the device finishes transmission of a frame; (2) after the backoff count decrements by one from the busy state $B_{0,j+1,L}$; and (3) after the device senses channel busy in busy state $B_{m,0,L}$. The balance equations for $K_{0,j,0}$ with $j \in [0, W_0 - 1]$ are obtained by:

$$\overline{K}_{0,j,0} = \overline{B}_{0,j+1,L} + (\overline{B}_{m,0,L} + \overline{T}_L)/W_0 \tag{4}$$

The balance equations for $K_{i,j,0}$ with $i \in [1, m]$ and $j \in [0, W_0 - 1]$ are obtained by:

$$\overline{K}_{i,j,0} = \overline{B}_{i,j+1,L} + \overline{B}_{i-1,0,L}/W_i \tag{5}$$

Similarly, the equations for the other backoff states $K_{i,j,k}$ with $i \in [0, m]$, $j \in [0, W_i - 1]$ and $k \in [1, W_i + 1]$ can be obtained by:

$$\overline{K}_{i,j,k} = \begin{cases} \overline{K}_{i,j+1,k-1}, & k \in [1, 2] \\ (1 - p_{k-1})\overline{K}_{i,j+1,k-1}, & 3 \le k \le W_m \end{cases} \tag{6}$$

The sensing states $C_{i,k}$ with $i \in [0, m]$ and $k \in [1, W_i]$ are directly transited from $K_{i,0,k-1}$, with their balance equations as:

$$\overline{C}_{i,k} = \begin{cases} \overline{K}_{i,0,k-1}, & k \in [1, 2] \\ (1 - p_{k-1})\overline{K}_{i,0,k-1}, & k \in [3, W_i] \end{cases} \tag{7}$$

The initial transmission states $X_{i,k}$ with $i \in [0, m]$ and $k \in [2, W_i + 1]$ are directly transited from $C_{i,k-1}$, with their equations as:

$$\overline{X}_{i,k} = \begin{cases} \overline{C}_{i,0,k-1}, & k = 2 \\ (1 - p_{k-1})\overline{C}_{i,0,k-1}, & k \in [3, W_i + 1] \end{cases} \tag{8}$$

The balance equations for transmission states T_l, $l \in [2, L]$, are obtained by:

$$\overline{T}_l = \begin{cases} \displaystyle\sum_{i=0}^{m} \sum_{k=2}^{W_i+1} \overline{X}_{i,k}, & l = 2 \\ \overline{T}_{l-1}, & l \in [3, L] \end{cases} \tag{9}$$

The transmission probability τ_k that the tagged device transmits after exactly k idle slots since the last transmission

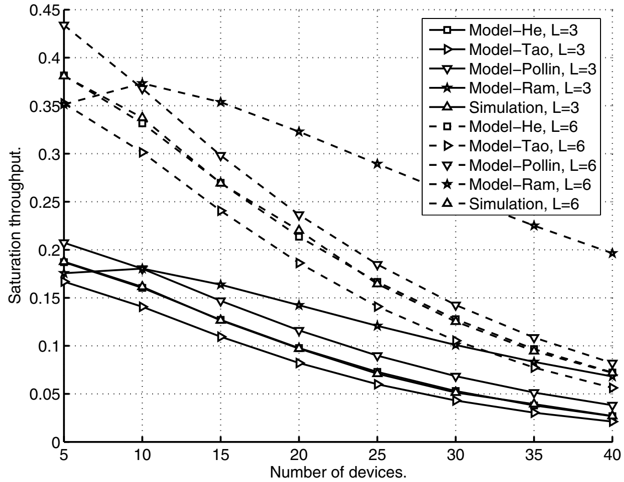

Fig. 2. Saturation throughput of slotted CSMA/CA algorithm, L=3 and L=6.

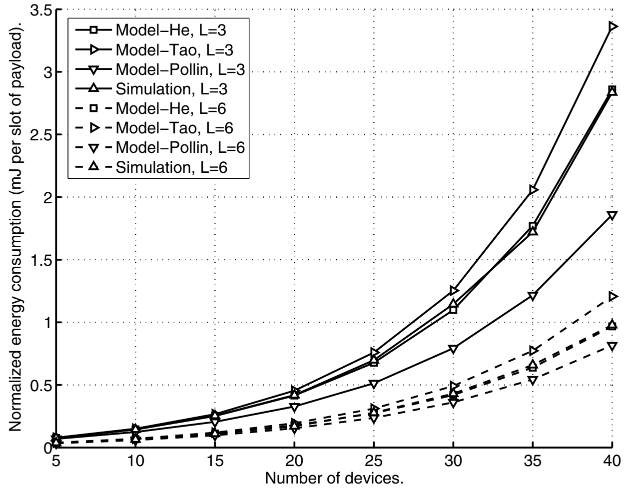

Fig. 3. Energy consumption of slotted CSMA/CA algorithm, L=3 and L=6.

in the channel can be computed by τ_k=0, for $k \in [0, 1]$, and for $k \in [2, W_x - 1]$

$$\tau_k = \sum_{i=0}^{m} \overline{X}_{i,k} / \sum_{i=0}^{m} \Big[\overline{X}_{i,k} + \overline{C}_{i,k} + \sum_{j=0}^{W_i-1} \overline{K}_{i,j,k} \Big] \qquad (10)$$

With the expressions for transmission probability τ_k, channel busy probability p_k for $k \in [0, W_x{+}1]$ is obtained by:

$$p_k = 1 - \big(1 - \tau_k\big)^{N-1} \qquad (11)$$

Since we have derived the balance equations for all steady state probabilities and expressions for p_k, $k \in [0, W_x{+}1]$, the Markov chain can be numerically solved. Assume the length of data payload in a frame is L_d slots. Define the saturation throughput S as the ratio of length of data payload (in backoff slots) to the average number of backoff slots used to successfully transmit a frame in the network. After the Markov chain is solved, S can be calculated by:

$$S = N L_d \sum_{i=0}^{m} \sum_{k=1}^{W_i} C_{i,k-1} \big(1 - p_{k-1}\big)\big(1 - p_k\big) \qquad (12)$$

To analyze energy consumption, we use normalized energy consumption (denoted by η_e), defined in [5] as the average energy consumed to transmit one slot of payload. The energy consumed to transmit a frame in a slot (denoted by E_t) and perform a CCA (denoted by E_c) in a slot is set to 0.01 mJ and 0.01135 mJ, respectively [5]. η_e is obtained by:

$$\eta_e = \frac{N}{S} \sum_{i=0}^{m} \{ \sum_{l=2}^{L} E_c B_{i,0,l} + \sum_{k=0}^{W_i+1} [E_c(K_{i,0,k} + C_{i,k} + L E_t X_{i,k}]\} \qquad (13)$$

IV. Numerical Results

A discrete event simulator is used to verify the accuracy of the proposed analytical model (denoted by "Model-He"), which is compared to those proposed in [7] (denoted by "Model-Tao"), in [6] ("Model-Pol") and in [2] ("Model-Ram"), all in non-acknowledgment mode. The MAC parameters are configured with default values: $W_0 = 2^3$, $W_x = 2^5$, and $m = 4$. We assume that the overhead of the header L_h in a frame is 1.5 slots in the non-acknowledgment mode, and $L = L_d + L_h$.

Fig. 2 shows the saturation throughput obtained by the analytical models and simulator for $L = 3$ and $L = 6$. Each simulation result was obtained from the average of 20 simulations, and a simulation terminates after transmitting 10^6 frames. It is observed that Model-He matches closely to the simulations results, with a mismatch only less than 1% on the average. The mismatches between the results obtained by any other analytical models and simulation results are larger than 10%. Fig. 3 compares the normalized energy consumption η_e of Model-He for $L = 3$ and $L = 6$, with extended Model-Tao and Model-Pol. It is observed that η_e exponentially increases with the number of devices N. With no surprise, Model-He has the highest accuracy. The inaccuracy of Model-Tao and Model-Pol increases with N. Similar throughput and energy performance are observed with other configurations of MAC parameters and frame lengths.

V. Conclusion

We have proposed an accurate Markov chain model for the slotted CSMA/CA algorithm of IEEE 802.15.4 standard. Both saturation throughput and energy consumption are modeled as the functions of MAC parameters and frame length. It is observed that the slotted CSMA/CA algorithm can be accurately modeled by the Markov chain, and the proposed model achieve the best accuracy among all existing analytical models.

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