STAT 344 Group Project

2023-11-07

Group Members

- Lucas Qin (44870194) - Group Leader
  - wrote the majority of code for sampling and analysis
  - provided technical support to group members when needed
  - wrote part of the report
  - (part 2) read the research paper and edited summary
- Isabel Wilson (86591872)
  - made a csv file of departments for use when assigning courses to strata
  - help code stratified sampling
  - wrote part of the report
  - (part 2) read the research paper and edited summary
- Frederick Wang (57974032)
  - initialized Git repository for project and maintained it
  - performed sample size calculations
  - fixed bugs in R calculation expressions
  - (part 2) read the research paper and wrote the summary for it

Project Part 1

Objectives and Background

Past UBC grade distributions are available online. On this public GitHub repo, course grade data are sorted into folders by year (from 1996 to 2021) and session (summer or winter). Each folder contains one table for each department, each of which contains the class size, class average, and the number of students who received a grade from 90-100, 80-90, 70-80 (etc) for every course offered in that department in that session. (More information about the dataset's structure can be found in the Appendix.)

These public grade distributions have long been used by students as a resource for making course decisions. Students are eager to know past course averages, as this allows them to make informed decisions regarding their timetables and instructors. They're also eager to see how their grades compare to their peers. In recent years, there has been some controversy regarding grade inflation at schools across North America; the UBC grades dataset allows for comparison of past and present grades. In light of recent grade inflation, is an A (80%) still a rare grade? For this project, our objective is to determine (1) the mean grade and (2) the proportion of students with a final grade of above 90% across all courses during the most recently available (at the time) 2021 winter session. This is important because knowing this will allow students to compare their grades to the global averages. We chose the mean grade because it is the most common population grade metric (e.g., GPA uses the mean; course averages are listed on students' transcripts), and we chose the proportion above 90% because this is something that most motivated students are interested in. Professors can also refer to this mean when scaling their class averages.

Target Population, Parameters of Interest, and Sampling Details

We downloaded the 2021 winter session data set from this GitHub repo. We chose 2021W because (at the time of choosing) it was the most recent academic session available on GitHub. So, the target population was all courses in the 2021W session. Note that we filtered the dataset such that, for courses with multiple sections, we retained only the row labelled "overall" (i.e., the overall number of students, the overall average, etc). The parameters of interest were the mean course average and the population proportion of students with grades above 90%.

To determine the sample size required, we used a guess of the standard error of the sample. Since we didn't have access to the population standard deviation to guess the standard error of the sample, we decided to use our personal transcript standard deviations to estimate the sample standard error. We found our transcript standard deviations to be around 7 percent, so we decided to guess the sample standard error to be 7 as well. This was not a perfect estimation (e.g., the grades from the dataset were from multiple students, and ours are from individuals), but it was adequate for this guess. Using this guess, we calculated a sample size for the mean and the proportion and chose the larger of the two ("n") as our actual sample size.

We obtained an SRS for the average by randomly choosing "n" courses from the dataset. To calculate the sample mean, we used the mean function in R; to calculate the sample proportion above 90%, we divided the number of course averages above 90% by n.

When choosing strata, we reasoned that there were far too many departments (probably more than 80) for us to stratify by department; so, we grouped departments into strata by field: arts, science, business, engineering, and other. Using a CSV file containing a table of departments and strata, we grouped courses (via their department) into strata (arts, science, business, engineering, other). We thought that the predicted variance would be the same for each stratum. We chose proportional allocation because proportional allocation is the optimal allocation when the predicted variance is the same for each stratum, and the cost of sampling from each stratum is the same.

Estimates, Standard Errors, and Confidence Intervals

From a simple random sample of 353 observations, we have estimated the mean of all course averages to be 81.90%, with a standard error of 0.37%. A 95% confidence interval for this estimate is [81.17%, 82.63%]. This means that we are 95% confident that the population mean of all course averages is between 81.17% and 82.63%. We assumed that the sample size (353) is large enough for the CLT to hold. The advantage of this method is that it is very simple to implement and very easy to intuitively understand the results. The disadvantage is that it has a relatively high standard error, and so is not very precise.

From the same simple random sample of 353 observations, we have estimated the proportion of course averages above 90% to be 0.156, with a standard error of 0.018. A 95% confidence interval for this estimate is [0.120, 0.192]. This means that we are 95% confident that the population proportion of course averages above 90% is between 0.120 and 0.192. We assumed that the sample size (353), the estimated proportion (0.156), and one minus the estimated proportion (0.844) are all large enough for the normal approximation of the binomial distribution to hold. The advantages and disadvantages of this method are the same as the SRS for the mean (above).

From a stratified sample of 353 observations, stratifying by field, and using proportional allocation, we have estimated the mean of all course averages to be 82.68%, with a standard error of 0.35%. A 95% confidence interval for this estimate is [81.99%, 83.36%]. This means that we are 95% confident that the population mean of all course averages is between 81.99% and 83.36%. We assumed that the sample sizes for each stratum were large enough for the CLT to hold. The advantage of this method is that it has less standard error than an SRS (since variation from stratified sampling is only the within-strata variation). The disadvantage of this method is that it requires more effort to sample, as we need to collect data from each stratum.

From the same stratified sample of 353 observations, we have estimated the proportion of course averages above 90% to be 0.116, with a standard error of 0.016. A 95% confidence interval for this estimate is [0.084, 0.148]. This means that we are 95% confident that the population proportion of course averages above 90% is between 0.084 and 0.148. We assumed that the sample sizes, estimated proportions, and one minus the estimated proportions for each stratum are large enough for the normal approximation of the binomial distribution to hold. The advantages and disadvantages of this method are the same as the stratified sample for the mean (above).

Final Conclusions and Discussion

In 2021W, the estimated mean of the mean course average was (by either SRS or proportional stratified sampling) an A- grade. The estimated proportion of students above 90% was (by SRS) 0.156, with a standard error of 0.018, or (by stratified sampling) 0.116, with a standard error of 0.016.

A limitation of the conclusions is that it's based on random samples of the data, which is susceptible to the effects of randomness. Since we did not sample every observation in the population, the estimates and standard errors we obtained are subject to random variation. These conclusions cannot be reliably generalized to other populations, such as course averages at other institutions (since they may have different school-wide standards for grading), or even past or future years at UBC (since the standards for grading at UBC have always been evolving throughout the years).

STAT 344 Group Project - Appendix: Data

We collected the data from online. The full data can be found here.

Here are 10 randomly selected observations from the data (after some initial pre-processing), to give a representative idea of what the data looks like:

```
##    Subject
## 1     ANTH
## 2     CPSC
## 3     ASTR
## 4     PPGA
## 5     ITST
## 6     PSYC
## 7     ASIA
## 8     FREN
## 9     MECH
## 10    LING
##                                                                     Title
## 1                                          Ethnography of Special Areas
## 2                                            Topics in Computer Systems
## 3                                                    Galactic Astronomy
## 4                                       Special Topics in Public Policy
## 5                                        Introduction to Italian Cinema
## 6                      Introduction to Biological and Cognitive Psychology
## 7                                     Introduction to Religions in Asia
## 8                                                        Women's Writing
## 9  Emerging Topics in Mechatronics, Manufacturing, Controls, and Automation
## 10                                  Linguistic Problems in a Special Area
##    Enrolled      Avg   Std.dev High Low X.50 X50.54 X55.59 X60.63 X64.67 X68.71
## 1        33 83.57576 12.922921   97  39    1      0      1      2      0      0
## 2         8 90.37500  3.739270   99  87    0      0      0      0      0      0
## 3         8 89.37500  5.527529   98  83    0      0      0      0      0      0
## 4        16 84.81250  1.869715   87  81    0      0      0      0      0      0
## 5        14 83.50000 15.776077   97  33    1      0      0      0      0      0
## 6      2122 71.86616 15.120886  100   0  120    118    150    151    176    234
## 7       101 66.33663 25.453989   95   0   22      1      2      5      2     10
## 8         9 78.44444 12.032364   90  52    0      1      0      0      0      0
## 9        13 83.07692  6.664102   91  71    0      0      0      0      0      2
## 10       10 87.20000  2.043961   91  86    0      0      0      0      0      0
##    X72.75 X76.79 X80.84 X85.89 X90.100 CourseNum
## 1       2      3      3      8      13      303D
## 2       0      0      0      5       3      538L
## 3       0      0      1      4       3      505B
## 4       0      0      6     10       0      591I
## 5       0      2      2      3       6       234
## 6     222    202    308    236     205       101
## 7       9     12     16     13       9       110
```

```
## 8        3      0      1      3       1      419
## 9        0      1      4      4       2     540A
## 10       0      0      0      8       2     530G
```

STAT 344 Group Project - Appendix: R Code

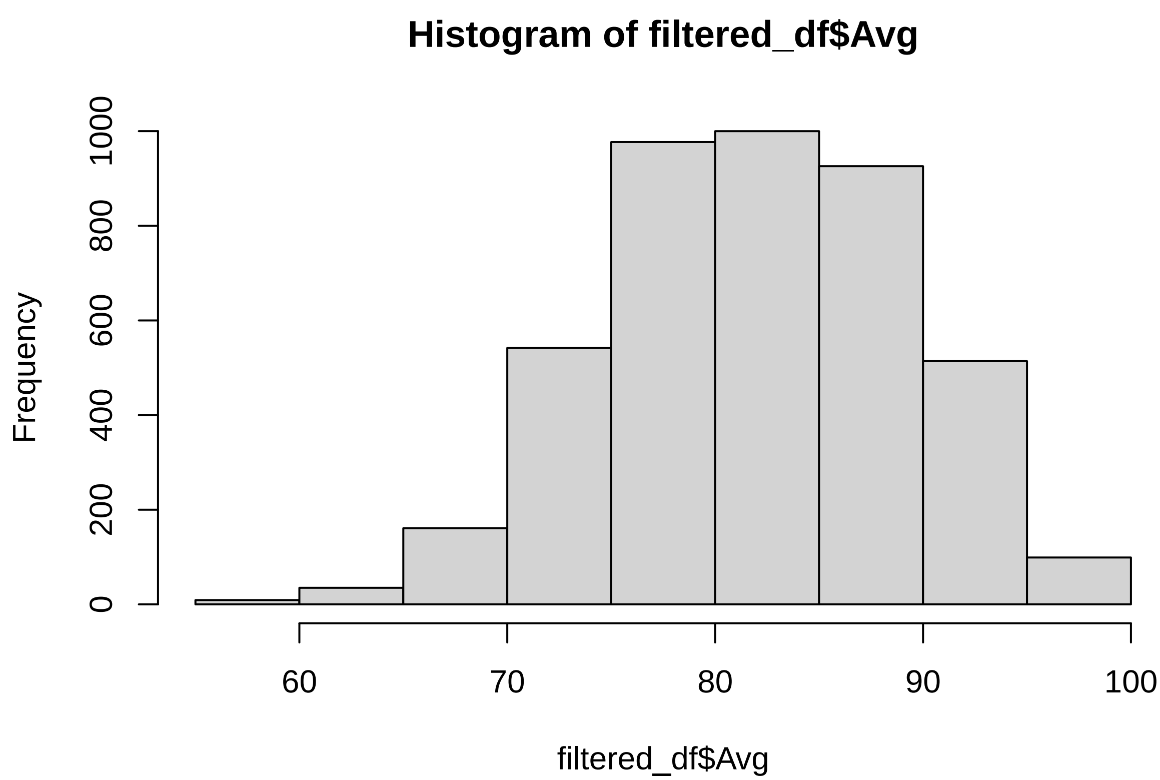
Reading in Data

```r
all_data_files = list.files("data")
all_df = read.csv(paste0("data/", all_data_files[1]))

for (i in 2 : length(all_data_files)) {
  all_df = rbind(all_df, read.csv(paste0("data/", all_data_files[i])))
}

filtered_df = all_df %>% filter(Section == "OVERALL") %>%
  mutate(CourseNum = ifelse(
    is.na(Detail), Course, paste0(Course, Detail)
  )) %>%
  select(-Campus, -Year, -Session, -Section, -Professor, -Course, -Detail)

N = nrow(filtered_df)
set.seed(2630)
filtered_df %>% slice(sample(N, 10))
```

```
##    Subject
## 1     ANTH
## 2     CPSC
## 3     ASTR
## 4     PPGA
## 5     ITST
## 6     PSYC
## 7     ASIA
## 8     FREN
## 9     MECH
## 10    LING
##                                                                    Title
## 1                                         Ethnography of Special Areas
## 2                                          Topics in Computer Systems
## 3                                                  Galactic Astronomy
## 4                                      Special Topics in Public Policy
## 5                                       Introduction to Italian Cinema
## 6                    Introduction to Biological and Cognitive Psychology
## 7                                   Introduction to Religions in Asia
## 8                                                     Women's Writing
## 9  Emerging Topics in Mechatronics, Manufacturing, Controls, and Automation
## 10                                  Linguistic Problems in a Special Area
##    Enrolled      Avg   Std.dev High Low X.50 X50.54 X55.59 X60.63 X64.67 X68.71
## 1        33 83.57576 12.922921   97  39    1      0      1      2      0      0
```

```
## 2          8 90.37500  3.739270   99   87    0       0       0       0       0       0
## 3          8 89.37500  5.527529   98   83    0       0       0       0       0       0
## 4         16 84.81250  1.869715   87   81    0       0       0       0       0       0
## 5         14 83.50000 15.776077   97   33    1       0       0       0       0       0
## 6       2122 71.86616 15.120886  100    0  120     118     150     151     176     234
## 7        101 66.33663 25.453989   95    0   22       1       2       5       2      10
## 8          9 78.44444 12.032364   90   52    0       1       0       0       0       0
## 9         13 83.07692  6.664102   91   71    0       0       0       0       0       2
## 10        10 87.20000  2.043961   91   86    0       0       0       0       0       0
##    X72.75 X76.79 X80.84 X85.89 X90.100 CourseNum
## 1       2      3      3      8      13      303D
## 2       0      0      0      5       3      538L
## 3       0      0      1      4       3      505B
## 4       0      0      6     10       0      591I
## 5       0      2      2      3       6       234
## 6     222    202    308    236     205       101
## 7       9     12     16     13       9       110
## 8       3      0      1      3       1       419
## 9       0      1      4      4       2      540A
## 10      0      0      0      8       2      530G
```

```
histogram = hist(filtered_df$Avg)
```




```
margin_of_error = 1 # desired width is 2%
sample_stdev_guess = 7 # intuitive guess since we don't have previous studies
```

```r
n1 = (1/(margin_of_error^2/(qnorm(0.975)^2*sample_stdev_guess^2) + 1/N)) %>% ceiling()
n1 # minimum sample size for the mean
```

```
## [1] 181
```

```r
margin_of_error = 0.05 # 2% width
conservative_squared_se = 0.5 * (1 - 0.5)
n2 = (1/(margin_of_error^2/(qnorm(0.975)^2*conservative_squared_se) + 1/N)) %>% ceiling()
n2 # minimum sample size for the proportion
```

```
## [1] 353
```

```r
n = max(n1, n2) # final decided sample size
n
```

```
## [1] 353
```

```r
set.seed(1)
srs = sample(filtered_df$Avg, n)
srs
```

```
##   [1] 74.56250 71.29213 91.22581 82.38462 75.32812 88.62500 90.33333 77.70563
##   [9] 93.63636 74.67647 78.22605 75.90698 82.68966 83.72727 83.57576 83.21176
##  [17] 75.77686 76.36039 86.11111 91.98361 62.62740 86.45455 76.51220 69.29897
##  [25] 84.25000 83.87619 81.00000 80.87143 93.33333 90.90000 75.72131 90.20833
##  [33] 78.67857 72.79200 83.06897 81.03704 75.95000 78.44578 66.02703 80.45238
##  [41] 81.35294 81.75000 85.73620 73.94118 87.06780 85.92683 86.07143 80.51515
##  [49] 91.38462 95.75000 79.85714 75.74747 84.56250 75.88889 91.58333 86.65000
##  [57] 76.47059 92.33333 83.11111 90.69565 71.75000 84.98291 79.82143 87.87500
##  [65] 79.30000 82.53191 82.73256 87.57143 89.92683 80.47619 95.19048 75.70968
##  [73] 88.13228 92.00000 74.70992 71.26263 82.66667 92.50000 77.78571 76.13793
##  [81] 87.58333 79.35435 86.69444 74.53982 93.10000 72.71951 93.66667 90.72727
##  [89] 80.63158 90.87500 69.23427 76.96667 69.82143 87.30000 68.88806 90.55556
##  [97] 80.44928 76.21818 72.16667 85.70588 85.88889 83.40441 93.02128 87.37500
## [105] 88.07317 85.54545 79.70513 81.17290 78.73077 80.45000 71.55556 79.50000
## [113] 77.26667 72.25989 84.60000 60.23077 91.27273 83.31250 87.26316 84.89062
## [121] 83.45238 88.54545 95.61538 88.64474 80.14545 81.80952 73.81690 78.90625
## [129] 74.73585 73.81767 67.89908 92.25000 81.10526 85.45161 88.30000 78.05882
## [137] 74.00000 75.92000 95.50000 94.14286 81.72308 73.29412 85.64286 84.80645
## [145] 88.00000 84.33333 88.55556 80.25000 88.65714 72.52941 69.92453 84.71429
## [153] 87.23077 79.77778 84.55556 89.83333 88.20000 78.57143 86.46667 72.50000
## [161] 82.13043 86.65217 76.41917 78.73077 78.76471 78.38095 82.10169 89.16667
## [169] 71.21429 75.18919 88.64706 72.67188 80.32967 76.21164 84.00000 77.33333
## [177] 90.36735 89.94737 91.66667 84.67188 74.38596 81.08537 78.59633 81.59259
## [185] 79.02386 84.95652 79.47917 87.65049 90.37037 87.00000 76.80000 93.50000
## [193] 84.66667 68.99180 73.65000 78.85106 74.17391 69.49717 69.74555 90.64815
## [201] 89.15385 82.28571 82.15686 81.13136 88.71429 84.28889 76.22222 89.04545
## [209] 71.17391 95.85417 88.80645 96.71429 78.74336 72.97500 92.88889 87.33333
## [217] 79.41463 84.81818 85.87097 76.30078 71.94118 81.04167 85.58621 71.67347
## [225] 92.20000 74.98361 96.75000 82.19444 85.10000 70.04762 92.64286 70.05714
## [233] 72.53333 81.22526 90.57143 81.58000 75.14815 74.10968 89.37037 87.14286
```

```
## [241] 84.40000 70.92500 83.96364 75.02899 89.50000 84.60000 66.71111 90.00000
## [249] 81.44444 89.18349 88.40000 84.57143 75.34545 86.24561 91.27500 80.83721
## [257] 80.72047 70.31959 90.20000 85.41176 72.18182 76.77778 73.60934 81.29249
## [265] 79.63804 80.84615 92.00000 85.37037 83.36559 82.62500 70.70492 81.16667
## [273] 71.16522 85.52632 66.55882 90.71429 93.35714 76.09091 85.66667 96.17647
## [281] 79.58824 88.09677 72.07143 89.00000 81.34783 94.00000 77.15385 85.56250
## [289] 79.92857 86.01852 83.00000 74.04167 72.73050 75.93220 80.81250 92.08696
## [297] 70.98392 88.60000 78.57642 72.31925 79.53333 78.87160 83.54902 73.92308
## [305] 75.66935 75.36538 95.75000 88.02353 86.66667 92.25000 91.00000 78.37500
## [313] 83.00000 67.23684 76.49123 82.36364 87.28571 84.84848 71.00000 77.00388
## [321] 86.85507 87.68182 77.37500 84.41667 84.33333 81.49287 88.93548 90.10000
## [329] 82.16667 58.03279 91.14815 71.92000 82.66667 75.73973 79.37838 77.55422
## [337] 88.00000 75.29787 67.70588 86.53333 94.26667 78.47619 82.44186 80.50000
## [345] 77.81720 76.87912 84.96392 85.30508 85.00000 80.04639 75.21212 94.65000
## [353] 91.93960
```

SRS for mean:

```r
sample_mean = mean(srs); sample_mean
```

```
## [1] 81.89962
```

```r
sample_se = sqrt((1-n/N)*sd(srs)^2/n); sample_se
```

```
## [1] 0.3723216
```

```r
ci_lb = sample_mean - qnorm(0.975)*sample_se
ci_ub = sample_mean + qnorm(0.975)*sample_se

conf_int = c(ci_lb, ci_ub)
c("Confidence Interval for Average UBC Grades Across All Classes in 2021",
  "Winter Using Simple Random Sample",
  conf_int)
```

```
## [1] "Confidence Interval for Average UBC Grades Across All Classes in 2021"
## [2] "Winter Using Simple Random Sample"
## [3] "81.169882922767"
## [4] "82.6293567986551"
```

SRS for proportion of grades above 90%:

```r
p_hat = length(srs[srs >= 90]) / n; p_hat
```

```
## [1] 0.1558074
```

```r
se_p_hat = sqrt((1 - n / N) * (p_hat * (1 - p_hat)) / n); se_p_hat
```

```
## [1] 0.01848665
```

```r
conf_int_p_hat = p_hat + c(-1, 1) * qnorm(0.975) * se_p_hat
c("Confidence Interval for Proportion of UBC Grades",
  "above 90% Across All Classes in 2021",
  "Winter Using Simple Random Sample",
  conf_int_p_hat)
```

```
## [1] "Confidence Interval for Proportion of UBC Grades"
## [2] "above 90% Across All Classes in 2021"
## [3] "Winter Using Simple Random Sample"
## [4] "0.119574204578259"
## [5] "0.192040526299928"
```

Preprocessing before stratifying:

```r
process_faculty = function(faculty) {
  if (faculty == "Faculty of Arts" |
      faculty == "Faculty of Education") {
    return ("arts")
  } else if (faculty == "Faculty of Science" |
             faculty == "Faculty of Medicine") {
    return ("science")
  } else if (faculty == "Faculty of Applied Science") {
    return ("engineering")
  } else if (faculty == "Faculty of Comm and Bus Admin" |
             faculty == "Vancouver School of Economics") {
    return ("business")
  } else {
    return ("other")
  }
}
```

```r
code2faculty = read.csv("summary.csv") %>%
  mutate(Faculty = Vectorize(process_faculty)(FacultyRaw)) %>%
  select(-Description, -FacultyRaw)
filtered_df = merge(code2faculty, filtered_df, by.x = "Subject")
```

```r
counts = filtered_df %>% group_by(Faculty) %>%
  summarize(counts = length(Faculty)) %>% arrange(desc(counts))
```

Stratified sampling (proportional allocation):

```r
stratas = counts$Faculty %>% as.vector()
num_stratas = length(stratas)
Nh = counts$counts
weights = Nh / N
nh = round(weights * n)

means = rep(0, num_stratas)
sd = rep(0, num_stratas)
props = rep(0, num_stratas)
```

```r
for (i in 1 : num_stratas) {
  subpopulation = filtered_df %>% filter(Faculty == stratas[i])
  sample = sample(subpopulation$Avg, nh[i])
  means[i] = mean(sample)
  sd[i] = sd(sample)
  props[i] = length(sample[sample >= 90]) / nh[i]
}

mean_average_str = sum(weights * means)
prop_str = sum(weights * props)

se = sd / sqrt(nh) * sqrt(1 - nh / Nh)
se_mean_average_str = sqrt(sum(weights^2 * se^2))

se_props_squared = props * (1 - props) / nh * (1 - nh / Nh)
se_prop_str = sqrt(sum(weights^2 * se_props_squared))

ci_mean_str =
  mean_average_str + c(-1, 1) * qnorm(0.975) * se_mean_average_str
ci_prop_str =
  prop_str + c(-1, 1) * qnorm(0.975) * se_prop_str
```

Project Part 2

One of the most practical skills of a statistician is statistical intuition, a sixth sense that comes into play and becomes a crucial part of directing the development of whatever statistical analysis or testing one may be conducting. This is the fundamental message of *The Emperor's New Tests* (Perlman and Wu, 1999). In the paper, Perlman and Wu devise a fascinating allegory which demonstrates the importance of one's wisdom and intuition in statistics. In the story, the well-established and tried-and-tested likelihood ratio test was challenged by a young and ambitious statistician's New Test. At first glance, on the surface, the New Test indeed seemed superior to the antiquated likelihood ratio test in multiple aspects and was adopted in the Imperial Court. Then came along the wise and scholarly Emperor who, upon inspection, found that the New Test was defective. It was found that the examples proposed by the young statistician to demonstrate his New Test were tailored to make the likelihood ratio test look inferior but is, in practice, not nearly sensitive enough to properly conduct hypothesis tests and produce coherent results. The Emperor immediately was able to notice something wrong with the results of the New Test by virtue of his statistical intuition and eventually dismissed the incorrect New Test. The main message of this story seems to be that one should always rely on one's intuition above anything else in statistics and science in general; if something doesn't seem right then it could very well be incorrect. Another message is that knowledge and experience in a field are superior to clever tricks. Lastly, the moral of the story is that one should never be blinded by one's goals and ambitions and seek to cheat and manipulate to obtain desired results. The ends should not justify the means in science and the best way forward is to focus on the process, guided by intuition.

References:

Perlman, Michael D., and Lang Wu. “The emperor’s new tests.” *Statistical Science*, vol. 14, no. 4, 1999, https://doi.org/10.1214/ss/1009212517.