Fred Dickinson fredsterdickinson@gmail.com

Introduction to Linear and Quadratic Equations.

Your task: read the summary on linear and quadratic equations. There is one short exercise at the end.

We use algebra to represent unknown variables. These could be almost anything you like: perhaps x represents the population of a certain species; the price of an ice cream; or the height of a ball above the ground.

We can form equations that describe how these variables must behave. Often, we want to *solve* these equations, i.e. find the value(s) of x that keep the equations satisfied.

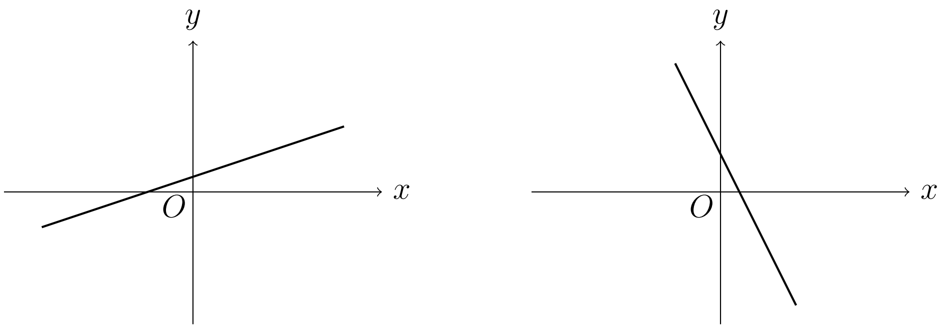
An equation is *linear* if the highest power of x is one. For example, $x + 4 = 0$ is linear, and so is $16 - x = 3$. On the other hand, the equation $x^3 - x + 4 = 0$ is *nonlinear.*

Linear equations are quite simple to solve. All that's needed is to re-arrange and make x *the subject* of the equation. For example, if

$$2x - 3 = x + 4,$$

then the solution to this equation is $x = 7$.

Linear equations form a **line** when plotted on standard axes.



(a) The line $y = \frac{1}{3}x + 1$. (b) The line $y = 3 - 2x$.

Figure 1: Two linear equations plotted on the x and y axes.

You might recall the general form of a line $y = mx + c$, where m and c are any numbers. This format covers any type of line you can draw, and equivalently, any type of linear equation. More on linear equations will be covered when we talk about line graphs in more detail.

It's great that linear equations are simple. Unfortunately, they're not particularly useful - not many scenarios lend themselves to only linear equations. The second-simplest type of equations are *quadratic*[1] *equations.*

Quadratic equations.

Quadratic equations are equations where the highest power of x is two. For example, $x^2 - 3 = 0$ is a quadratic equation, as is $3x^2 - 6x = x^2 + 4$.

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[1] Quadratic comes from the Latin 'quadratus', meaning square. It is unfortunate that we typically use *quad* to describe things of four, such as a quad bike or a quadrilateral (four sided shape).

These are more difficult to solve than linear equations. If you try to re-arrange and make x the subject of the equation, you'll run into problems. Take this attempt as an example;

$$\begin{aligned} x^2 - 6x + 9 &= 0 \\ x^2 - 6x &= -9 \\ x(x-6) &= -9 \\ x &= -\frac{9}{x-6} \ . \end{aligned}$$

Trying to solve the equation this way isn't working because we have x on both sides.

Instead, we use a method called *factorising* to solve quadratic equations. **An important and underappreciated idea here is as follows.** If you have two numbers that multiply each other so that their product is zero, say

$$a \times b = 0,$$

then we **must** have that a and/or b is zero. It is the **only** possible way in which two things can multiply to be zero.

We can apply this idea to quadratic equations. First, re-arrange your quadratic to be equal to zero. Then, factorise your quadratic so it is the product of two terms. These steps are carried out for an example quadratic $x^2 - 5x = x - 8$ below,

$$\begin{aligned} x^2 - 6x + 8 &= 0 \\ (x-4)(x-2) &= 0. \end{aligned}$$

So now the quadratic has been reduced to two *factors* $(x-4)$ and $(x-2)$, multiplying each other to be zero. Remember that the **only** way that this can happen is if

(a) the first factor $x - 4 = 0$,
(b) or the second factor $x - 2 = 0$.

So then we **must have** either $x = 4$ or $x = 2$. These are the *solutions* to the quadratic equation.

Exercise 1.1. *Check for yourself that $x = 4$ and $x = 2$ are solutions to the original quadratic equation $x^2 - 5x = x - 8$. Do so by substituting these values in to both sides of the equation and check they are equal.*

There are alternative methods to solve quadratic equations, namely *completing the square* and using the *quadratic formula.* These will be covered separately.