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On The Area of a Triangle

Your task: read the notes and attempt the exercises at the end.
Note: current figures are temporary and will be redrawn later.

The area of a triangle, 'half base times height', is surely one of the most well known formulas in all of mathematics. It's short, taught early, used frequently, and easy to reason geometrically.
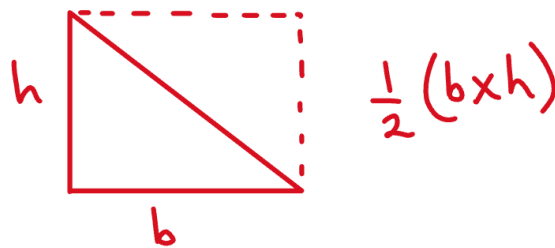

Figure 1: Extend a triangle to be exactly half of a rectangle to see area $= \frac{1}{2}(b \times h)$.

With just some quick trigonometry, this formula can be written

$$\triangle_{\text{area}} = \frac{1}{2}ab\sin(C)$$

where a, b, and c are the side lengths of the triangle, and C is the angle opposite c. This is useful because the 'height' of a triangle may not be given.

To justify my 'just quick trigonometry' claim, consider the following steps and Figure 2 (overleaf).

  (a) Draw *any* triangle.
  (b) Draw a fourth straight edge that splits any angle in half.
  (c) Label this angle A, and subsequently label the remaining angles B and C.
  (d) Respectively label the edges opposite each angle a, b, and c.
  (e) Use trigonometry with the triangle of angle C to find the length of the fourth edge.
  (f) Justify to yourself that this edge is the 'height' of the triangle if side a is the base. Hence the area $= \frac{1}{2}ab\sin(C)$.

Exercise 1.1. *Repeat these steps twice for triangles of different shape. Can you follow with a right-angled triangle?*

$$x = b \cdot \sin(C).$$

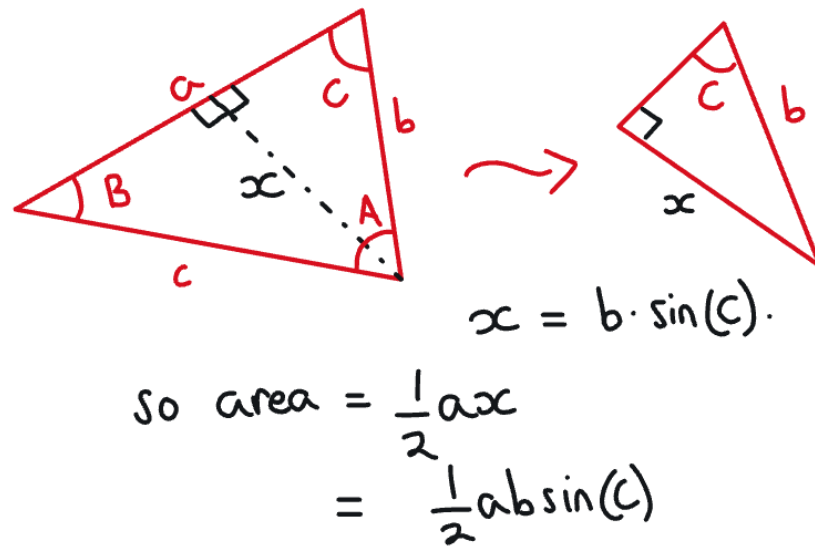$$\text{So area} = \frac{1}{2}ax$$

$$= \frac{1}{2}ab\sin(C)$$

Figure 2: A quick sketch outlining the steps (a)-(f) described.