A-Level Maths: Calculating Derivatives

Author: Fred Dickinson

This question booklet contains small exercises on derivative calculation for A-level students. For solutions and marking email fredserdickinson@gmail.com. Good luck!

Give your solutions on a separate piece of paper or digital notebook. Attempt to answer all questions; if you intend to leave a question blank, make it clear that you are doing so.

1. Find the derivative $\frac{\mathrm{d}y}{\mathrm{d}x}$ for
   (a) the constant function $y = 6$, (1)
   (b) the line $y = 6x + 3$, (1)
   (c) the curve $y = 2x^4 - 3x^3 + \sqrt{x} + \frac{7}{x}$. (2)

2. Consider the trigonometric functions $\sin(x)$ and $\cos(x)$.
   (a) What is the derivative of $\sin(x)$? What is the derivative of $\cos(x)$? (1)
   (b) Let $f(x) = \sin(x)$. What is the fourth derivative $f^{(4)}(x)$? (1)
   (c) Find $f^{(2023)}(x)$, where $f^{(2023)}$ is the 2023rd derivative of f. (2)

   This question is soooooo last year, ugh.
   Hint: what is the nearest multiple of 4 to 2023? How do the derivatives of the sine function cycle?

3. Let a be some real positive real number.
   (a) Given $y = a^x$, find $\frac{\mathrm{d}y}{\mathrm{d}x}$. (2)
   (b) Suppose a is equal to Euler's constant e. Use (a) to evaluate the derivative $\frac{\mathrm{d}y}{\mathrm{d}x}$ in this case, giving your answer in its simplest form. (2)

4. Calculate $f'(t)$ given $f(t) = e^{4t+2}$. (2)

5. Given $y = (x^2 + 3)\sin(x)$, find $\frac{\mathrm{d}y}{\mathrm{d}x}$. (3)

6. Let $y = 5^t \sin(3t)$. Find $\frac{\mathrm{d}y}{\mathrm{d}t}$. (3)

7. A curve C is given implicitly by $2y = x^3 + \sin(y) + xe^y + 7$. Find the gradient of the curve at the point $(1, \pi)$, leaving your answer in terms of e. (5)

8. Let $f(y) = \ln(\tan(y))$. Is the value $f'(0)$ well defined? Justify your answer. (5)

9. Consider a curve defined with parametric equations

$$x = 2t + 1, \quad y = t^2 - 1.$$

   (a) Find the points where the curve crosses the coordinate axes. (2)

   (b) Find an expression for $\frac{\mathrm{d}y}{\mathrm{d}x}$ in terms of x. (3)

10. Consider a curve defined with parametric equations

$$x = \tan(\theta), \quad y = \cos(\theta), \qquad 0 < \theta < \pi.$$

   (a) Find $\frac{\mathrm{d}y}{\mathrm{d}x}$. (3)

   (b) Find the equation of the normal to the curve at $\theta = \frac{\pi}{2}$. (3)

END OF BOOKLET.