Fred Dickinson fredsterdickinson@gmail.com

Solving Quadratic Equations

*Your task: complete the problems below. Solutions are given at the end of the document; attempt **and check** all questions before verifying your answers.*

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1. Solve for x by factorising each equation.

(a) $x^2 - 4x - 19 = 2$,
(b) $3x^2 + 10x + 4 = x^2 + x$,
(c) $3x^2 - 6x + 3 = 0$,
(d) $5x^2 + 89x - 18 = 0$.

2. Solve for x by completing the square.

(a) $x^2 - 8x + 6 = 0$,
(b) $x^2 + 10x - 17 = 0$,
(c) $3x^2 - 21x + 31 = 0$.

3. Explain how you have checked your answers in questions 1(a-d) and 2(a-c).

4. What is the quadratic formula? And, in your own words, explain where it comes from.

5. Use the quadratic formula to solve 1(b).

Solutions. 1. In 1(a-b) remember to first set your right-hand side to zero. Otherwise we factorise as normal,

(a) $x^2 - 4x - 21 = 0 \iff (x-7)(x+3) = 0$, so $x = 7$ or $x = -3$;
(b) $2x^2 + 9x + 4 = 0 \iff (2x+1)(x+4) = 0$, so $x = -\frac{1}{2}$ or $x = -4$;
(c) $3x^2 - 6x + 3 = 0 \iff 3(x^2 - 2x + 1) = 0 \iff 3(x-1)^2 = 0$, so $x = 1$ only;
(d) $5x^2 + 89x - 18 = 0 \iff (5x-1)(x+18) = 0$, so $x = \frac{1}{5}$ or $x = -18$.

2. For (a), if $x^2 - 8x + 6 = 0$ then

$$\begin{aligned}(x-4)^2 - (-4)^2 + 6 &= 0\\ (x-4)^2 - 16 + 6 &= 0\\ (x-4)^2 &= 10\\ x - 4 &= \pm\sqrt{10}\end{aligned}$$

and therefore $x = 4 \pm \sqrt{10}$.

(b) If $x^2 + 10x - 17 = 0$ then

$$\begin{aligned}(x+5)^2 - (5)^2 - 17 &= 0\\ (x+5)^2 - 25 - 17 &= 0\\ (x+5)^2 &= 42\\ x + 5 &= \pm\sqrt{42}\end{aligned}$$

so $x = -5 \pm \sqrt{42}$.

(c) If $3x^2 - 21x + 31 = 0$, we need to factor out the leading coefficient of 3. The constant term $+31$ does not need to be included. So

$$\begin{aligned}3(x^2 - 7x) + 31 &= 0\\ 3\left[\left(x - \tfrac{7}{2}\right)^2 - \left(-\tfrac{7}{2}\right)^2\right] + 31 &= 0\\ 3\left[\left(x - \tfrac{7}{2}\right)^2 - \tfrac{49}{4}\right] + 31 &= 0.\end{aligned}$$

In these types of problem it common to have to deal with fractions. At this step you should think about moving terms to the right hand side,

$$\begin{aligned}3\left(x - \tfrac{7}{2}\right)^2 - \tfrac{147}{4} &= -31\\ 3\left(x - \tfrac{7}{2}\right)^2 &= \tfrac{23}{4}\\ \left(x - \tfrac{7}{2}\right)^2 &= \tfrac{23}{12}\\ x - \tfrac{7}{2} &= \pm\sqrt{\tfrac{23}{12}}\end{aligned}$$

so $x = \frac{7}{2} \pm \sqrt{\frac{23}{12}}$. You could simplify the surd if you like. This question is no different to the others in principle; it just had trickier algebra with fractions.

3. You can check your solutions for these problems by substituting in your x-values into the original equation. For example, in 1(b) we found $x = -4$ or $x = -\frac{1}{2}$. We could check $x = -4$ by calculating $2x^2 + 9x + 4 = 0$, i.e.

$$2(-4)^2 + 9(-4) + 4 = 2(16) - 36 + 4 = 32 - 36 + 4 = 0,$$

as it should be.

4. For a quadratic $ax^2 + bx + c$, the quadratic formula is

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}.$$

It is derived by completing the square on the general form of a quadratic above.

5. For 1(b), we have $2x^2 + 9x + 4 = 0$. So then

$$x = \frac{-9 \pm \sqrt{81 - 4(2)(4)}}{4} = \frac{-9 \pm \sqrt{49}}{4} = \frac{-9 \pm 7}{4}$$

so $x = \frac{-9+7}{4} = \frac{-2}{4} = -\frac{1}{2}$, or $x = \frac{-9-7}{4} = -4$. These are the same answers as before.