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Sine and Cosine: from the unit circle

Your task: read the notes given to build intuition on the sine and cosine functions.

Trigonometric functions are surely some of the most important functions in all of mathematics. They are ***everywhere!*** Typically introduced within the context of right-angled triangles, many students perceive them as magical objects created by their maths teacher in another effort to torture them with laborious SOH-CAH-TOA homework sheets.

Here we'll explore a short and interesting definition of both functions that shed a bit more light on their properties. It goes like this.

1. Draw a unit circle (a circle with radius 1) centred at the origin $(0, 0)$.
2. Extend a line of any angle θ from the origin out to the circle.
3. The x-coordinate of this point is defined to be $\cos(\theta)$. Similarly, the y-coordinate of this point is defined to be $\sin(\theta)$.

This process is depicted in Figure 1. Ensure that the angle θ is subtended between the line and the positive x-axis.

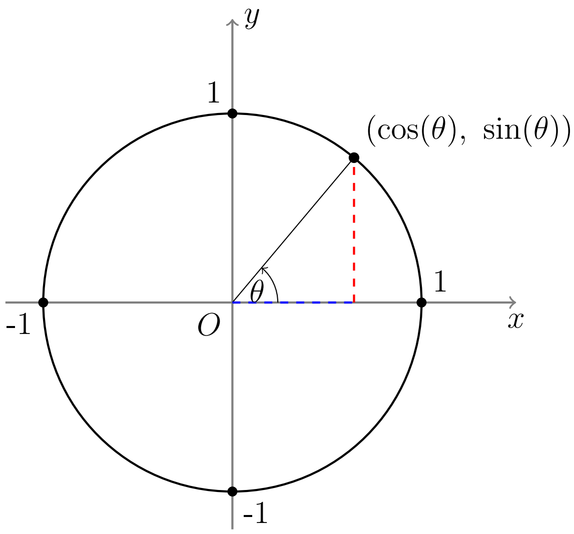

Figure 1: A unit circle centred at $(0, 0)$. An angle of size θ is formed between the positive x-axis and a line that has been extended out to the unit circle. The intersection of this line and the circle is precisely the point $(\cos(\theta),\ \sin(\theta))$.

These definitions of the sine and cosine functions make their properties much clearer. Take two, for example.

1. Both $\sin(\theta)$ and $\cos(\theta)$ are 2π-periodic (or $360°$-periodic) because they repeat themselves after one loop of a circle.
2. The identity $\sin^2(\theta) + \cos^2(\theta) \equiv 1$ holds by using Pythagoras' theorem on the right-angled triangle induced by any point (highlighted in dashed red and blue).

You can also define the tangent function from this diagram: it is the length of tangent line from the point on the unit circle to the x-axis. This is another way to justify why $\tan(\theta) \to \infty$ as $\theta \to \pi/2$ (or $90°$): try it for yourself!