Math for Computer Science - Problem Set 1

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Problem 1

Let's prove by contradiction. Suppose that $\log_4 6$ is rational. Then, we can write $\log_4 6$ as a fraction $\frac{n}{d}$, where n and d are integers. Therefore, we have:

$$\log_4 6 = \frac{n}{d}$$

We can rearrange the equation, since $\log_a b \cdot x = \log_a b^x$:

$$\log_4 6 \cdot d = n$$

$$\log_4 6^d = n$$

From the definition of Log:

$$4^n = 6^d$$

$$2^{2n} = (2 \cdot 3)^d$$

$$2^{2n} = 2^d \cdot 3^d$$

$$2^{2n-d} = 3^d$$

Since an even number multiplied by an even number is always even, and an odd number multiplied by an odd number is alwasys odd, the condition does not meet, and proof by contradiction is finished.

Problem 2

$$n \leq 3^{n/3}$$

Hint: Verify (1) for $n \leq 4$ by explicit calculation.
For n = 1:

$$1 \leq 3^{1/3}$$

Verifies. For n = 2:

$$2 \leq 3^{2/3}$$
$$2 \leq 9^{1/3}$$
$$8^{1/3} \leq 9^{1/3}$$

Verifies. For n = 3:

$$3 \leq 3^{3/3}$$
$$3 \leq 3$$

Verifies. For n = 4:

$$4 \leq 3^{4/3}$$
$$2^2 \leq 9^{2/3}$$
$$8^{2/3} \leq 9^{2/3}$$

Verifies. Suppose there is a number where the inequality does not verify. Let's say the smallest number is n, so:

$$n > 3^{n/3}$$

However

$$n - 1 \leq 3^{(n-1)/3}$$
$$n \leq 3^{(n-1)/3} + 1$$

$$3^{n/3} < 3^{(n-1)/3} + 1$$

$$3^{n/3} < \frac{3^{n/3}}{3} + 1$$

$$3^{n/3} - \frac{3^{n/3}}{3} < 1$$

$$3^{n/3} \cdot (1 - \frac{1}{3}) < 1$$

$$3^{n/3} \cdot \frac{2}{3} < 1$$

$$3^{n/3} < \frac{3}{2}$$

Since n must be greater than 4, the inequality does not hold, which ends up finishing our proof by contradiction.

Problem 3

a.

$$(P \implies Q) \vee (Q \implies P)$$

P	Q	$(P \implies Q)$	$(Q \implies P)$	$(P \implies Q) \vee (Q \implies P)$
T	T	T	T	T
T	F	F	T	T
F	T	T	F	T
F	F	T	T	T

b.

P	Q	R
T	T	T
T	F	F
F	T	F
F	F	T

$$(P \wedge Q) \vee \neg(Q \vee P)$$

c.

P	$\neg P$	P is valid $\iff \neg P$
T	F	F
F	T	F

d. $S = \neg P_1 \vee \neg P_2 ...$