

Basics of magnetohydrodynamics and solar wind / magnetosphere interaction

Diffusion and frozen flux

Assumptions:

- magnetised plasma;
- collisionless plasma (collision frequencies are much less than gyrofrequency, but not fully negligible);
- “cold” or “warm” plasma (particle energies substantially below relativistic, ~10s - 100s of eV).

From Faraday’s law and generalised Ohm’s law, eliminating \mathbf{E} field:

$$\frac{\partial \mathbf{B}}{\partial t} = \nabla \times (\mathbf{v} \times \mathbf{B} - \mathbf{j}/\sigma_0)$$

Using $\nabla \cdot \mathbf{B} = 0$ and neglecting displacement currents (as usual in plasma physics):

$$\frac{\partial \mathbf{B}}{\partial t} = \boxed{\nabla \times (\mathbf{v} \times \mathbf{B})} + \boxed{\frac{1}{\mu_0 \sigma_0} \nabla^2 \mathbf{B}}$$

Advection
term

Diffusion
term

Magnetic diffusion

Assumptions:

- plasma at rest (or moving with constant velocity);
- conductivity is finite.

$$\frac{\partial \mathbf{B}}{\partial t} = D_m \nabla^2 \mathbf{B} \quad \leftarrow \text{only diffusion term left}$$

with the magnetic diffusion coefficient: $D_m = (\mu_0 \sigma_0)^{-1}$

Solution is given by: $B = B_0 \exp(\pm t/\tau_d)$

with the magnetic diffusion time: $\tau_d = \mu_0 \sigma_0 L_B^2$

Consider typical solar wind:

Density $\sim 5 \text{ cm}^{-3}$

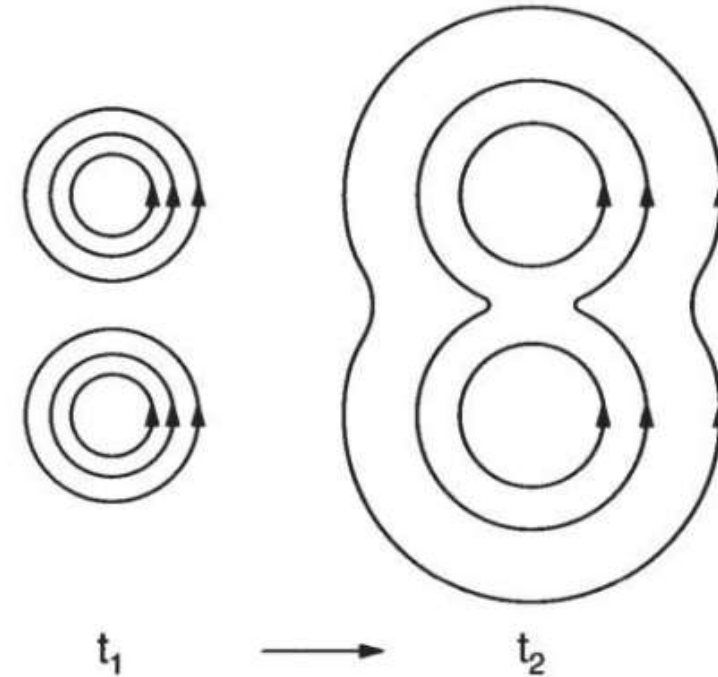
Temperature $\sim 50 \text{ eV}$

$$\sigma_0 = \frac{n_e e^2}{m_e v_c}$$

For the travel time from Sun to Earth ($\sim 3 \text{ days}$)

$$L_B \sim 10^3 \text{ m}$$

In solar wind the magnetic diffusion is negligible!



Frozen-in condition

Assumption:

- collisionless plasma with infinite conductivity ($\sigma \rightarrow \infty$)

$$\frac{\partial \mathbf{B}}{\partial t} = \nabla \times (\mathbf{v} \times \mathbf{B})$$

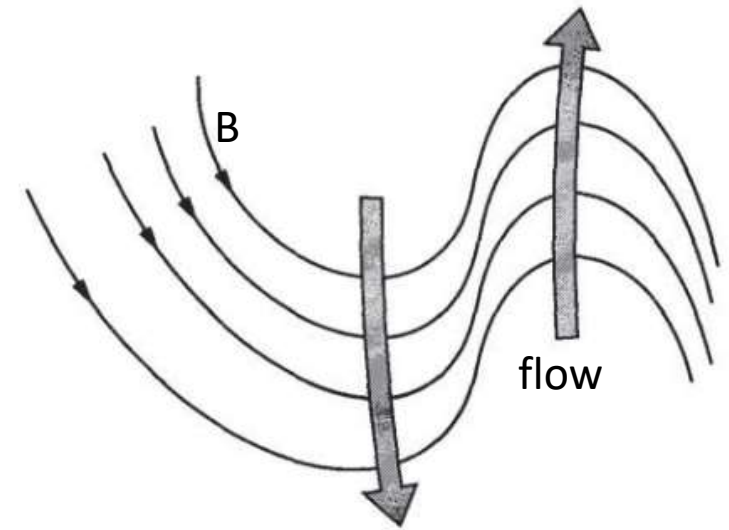
or equivalent

$$\mathbf{E} + \mathbf{v} \times \mathbf{B} = 0$$

Hydromagnetic theorem:

In infinitely conductive plasma, the total magnetic induction encircled by a closed loop remains unchanged

-> magnetic field lines are “frozen” into the plasma flow



Frozen magnetic field lines

Magnetic merging and reconnection

Back to the case of finite conductivity (both advection and diffusion terms present)

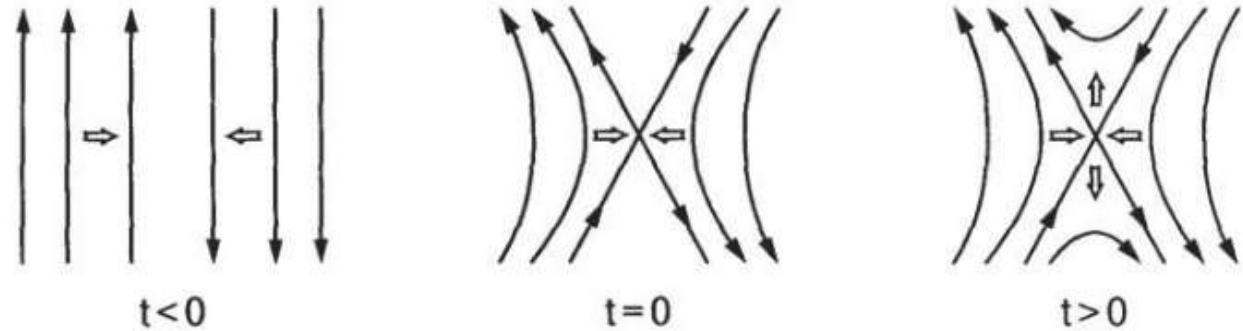
$$\frac{\partial \mathbf{B}}{\partial t} = \nabla \times (\mathbf{v} \times \mathbf{B}) + \frac{1}{\mu_0 \sigma_0} \nabla^2 \mathbf{B}$$

or in dimensionless form

$$\frac{B}{\tau} = \frac{V B}{L_B} + \frac{B}{\tau_d}$$

Magnetic Reynolds number

$$R_m = \mu_0 \sigma_0 L_B V$$



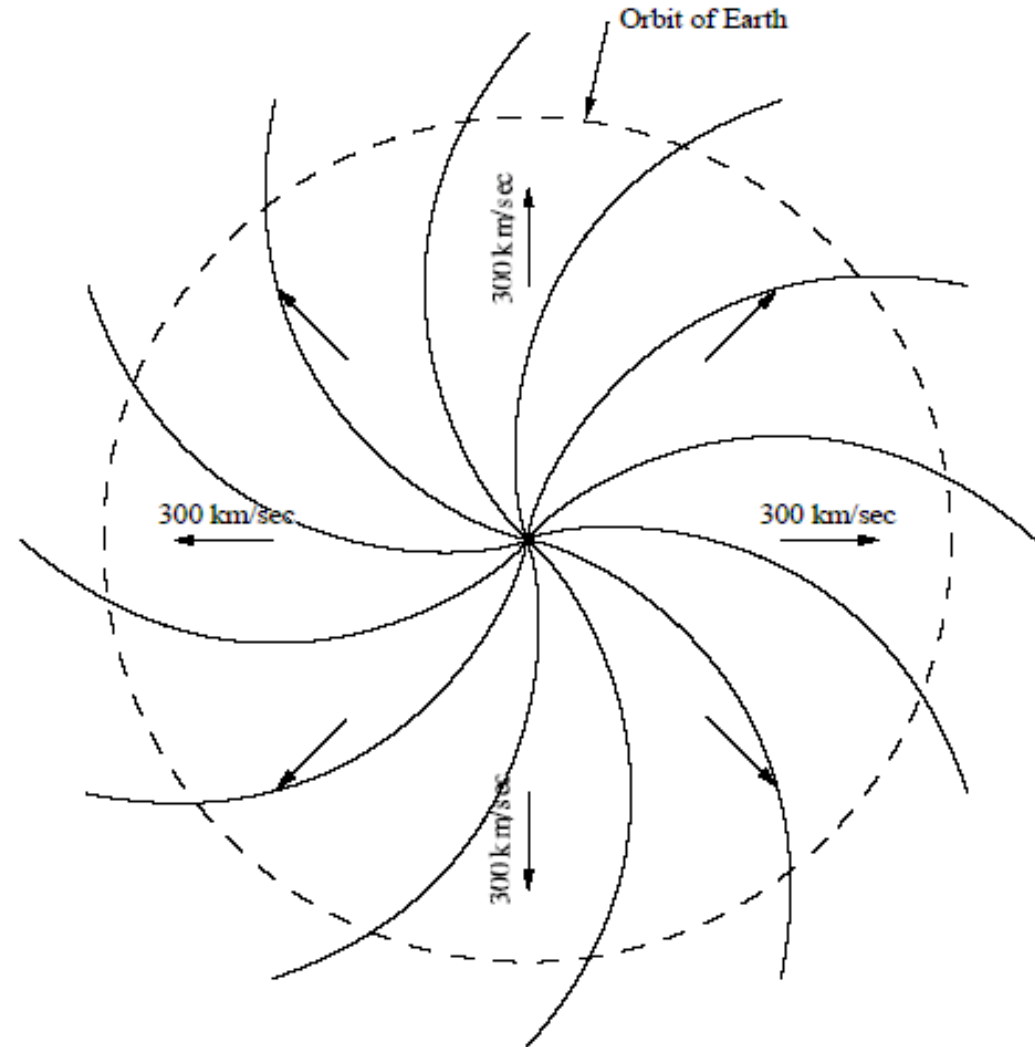
Magnetic field line merging

For $R_m \gg 1$: diffusion is negligible (in solar wind $R_m \approx 10^{17}$)

For $R_m \sim 1$: diffusion is substantial and can dominate -> **merging/reconnection**

Interplanetary magnetic field (IMF): Parker spiral

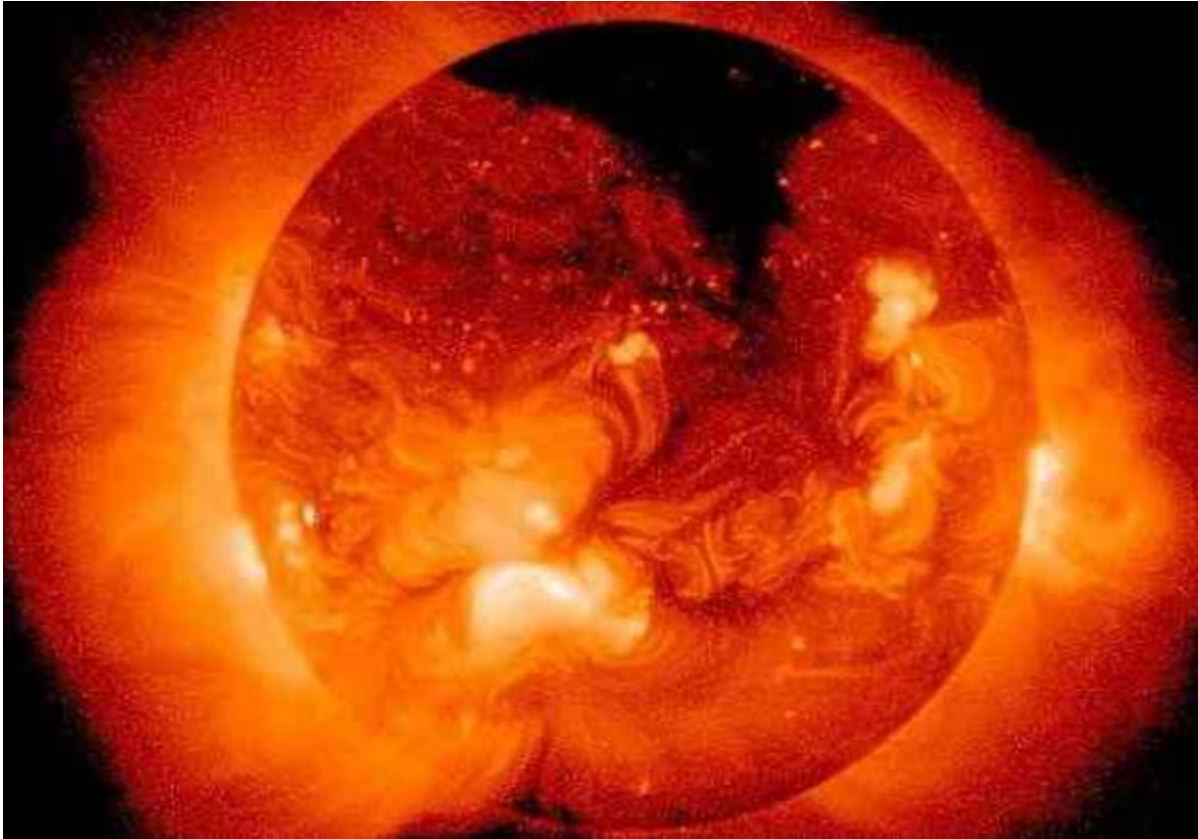
- Solar rotation drags out frozen solar wind magnetic field forming **Parker spiral** (after Gene Parker)
- Winding angle depends on wind speed, but on average
- $\sim 45^\circ$ at Earth orbit
- $\sim 90^\circ$ 10 AU



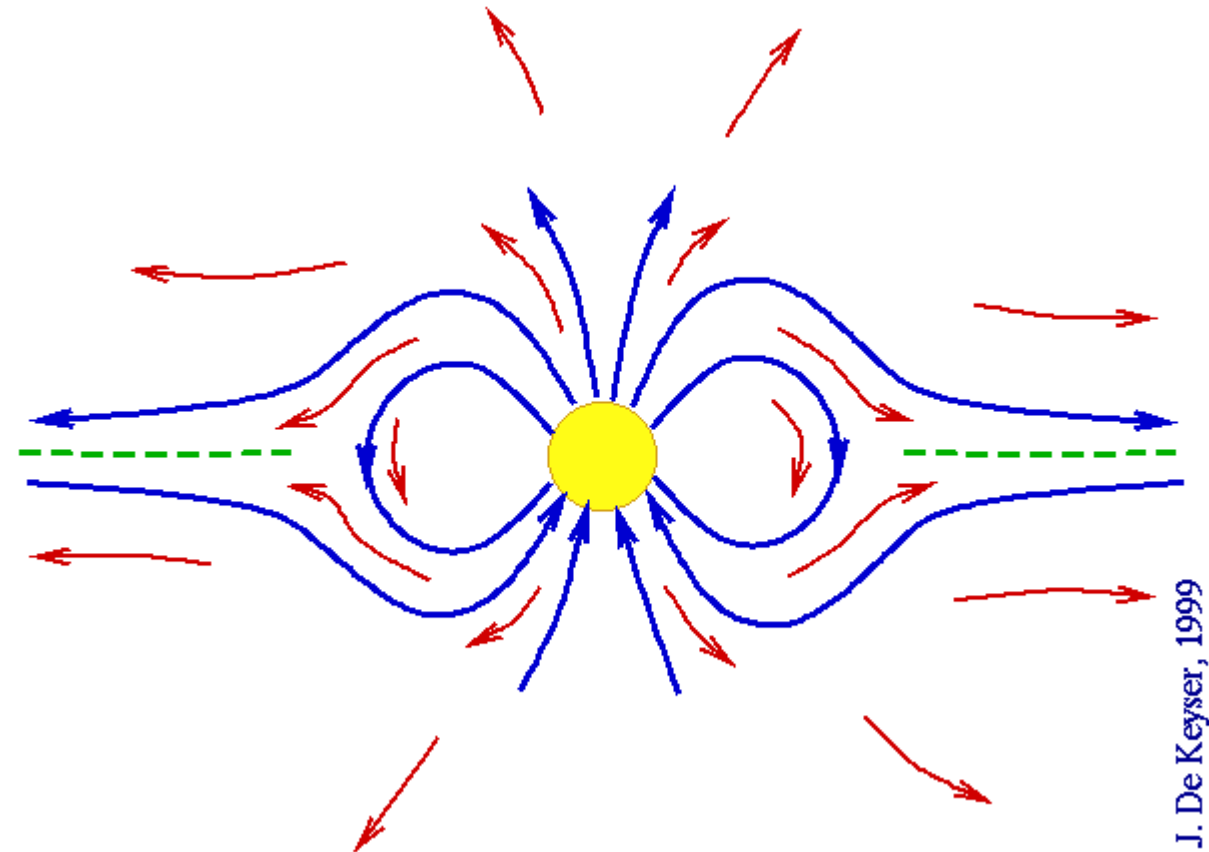
Interplanetary magnetic field (IMF): meridian view

Two distinct regions of the solar corona:

- Equatorial latitudes.
- Polar latitudes: coronal holes.



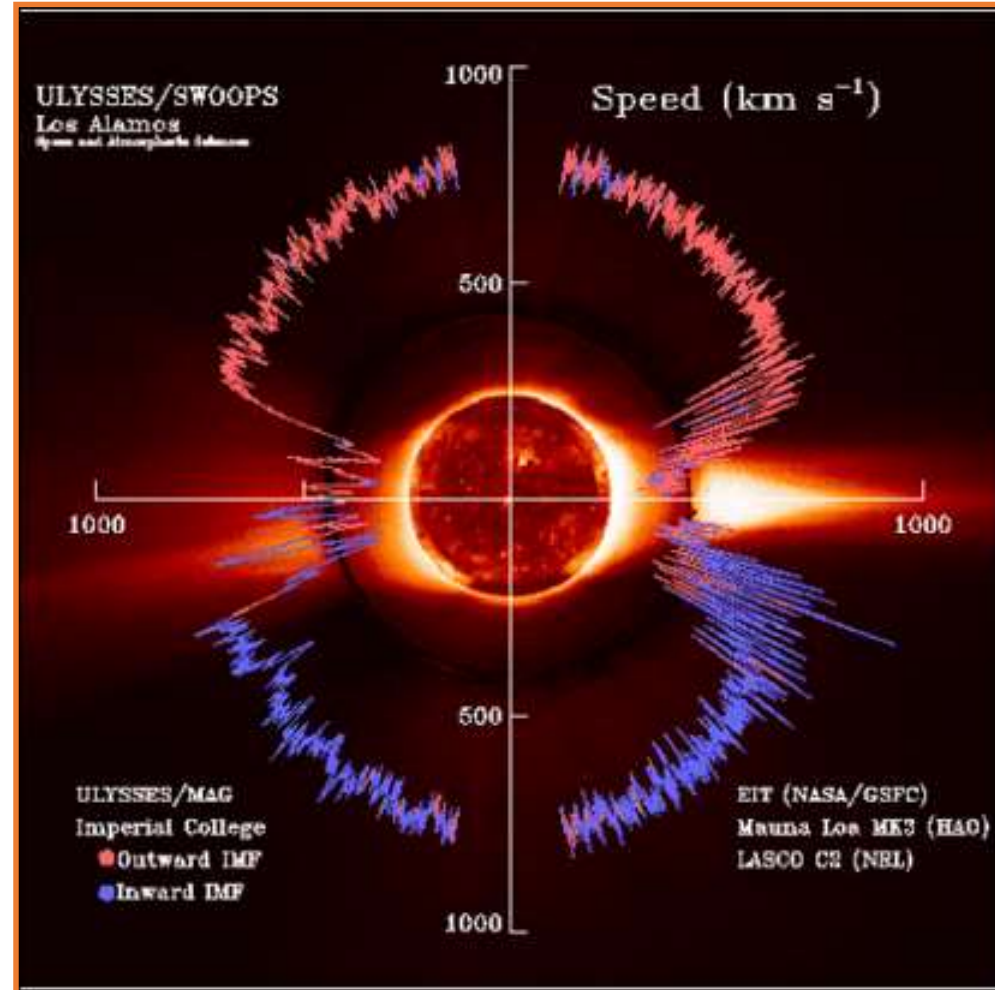
SOLAR-A probe image, courtesy of NASA



J. De Keyser, 1999

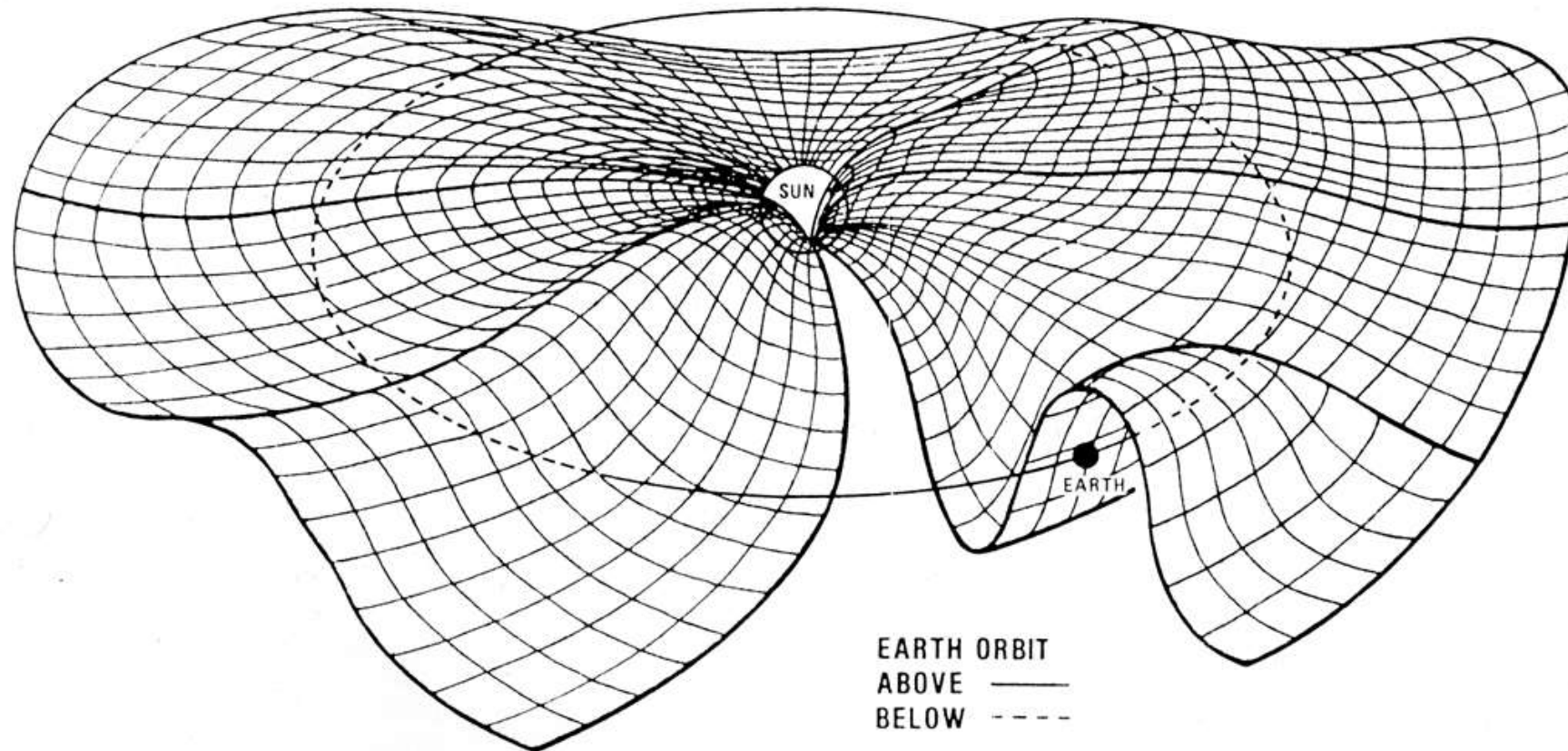
Solar wind variability with latitude: fast and slow wind

- Dipolar solar magnetic field during solar minimum.
- High latitudes dominated by high speed wind from polar coronal holes.
- At Low latitudes: fast and slow streams, very variable over solar cycle.
- Different magnetic polarity in each hemisphere due to solar dipole.



IMF in ecliptic plane: Heliospheric current sheet

As the Sun rotates the three dimensional current sheet becomes wavy (also called the Ballerina skirt model of the heliosphere).



IMF near Earth orbit

By the time IMF arrives to the Earth orbit, it has complex, dynamic structure with 3 components:

- radial (B_x);
- east-west (B_y);
- north-south (B_z).

Of the 3 IMF components, B_z has the most significance for solar wind / magnetosphere interaction.

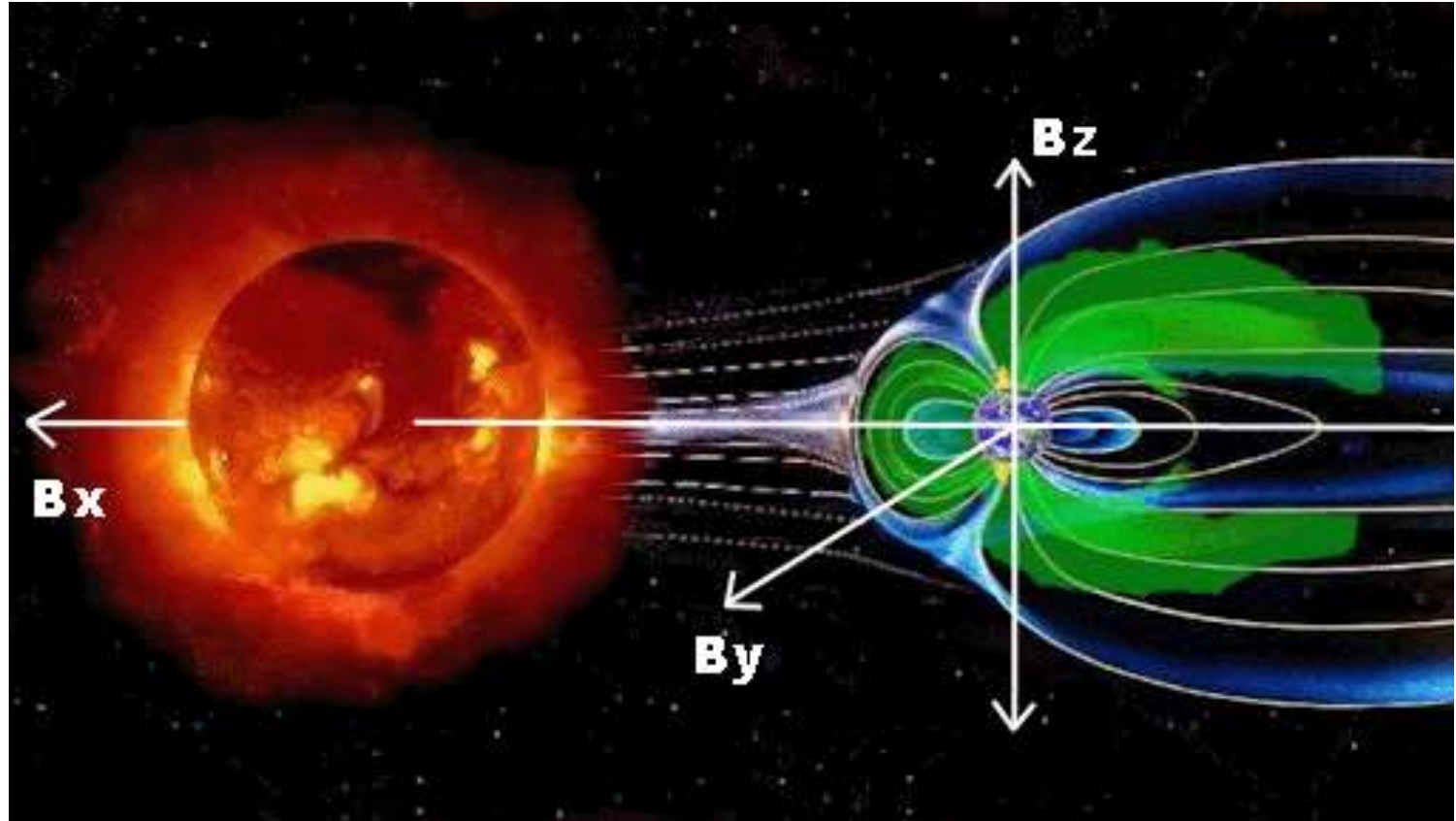
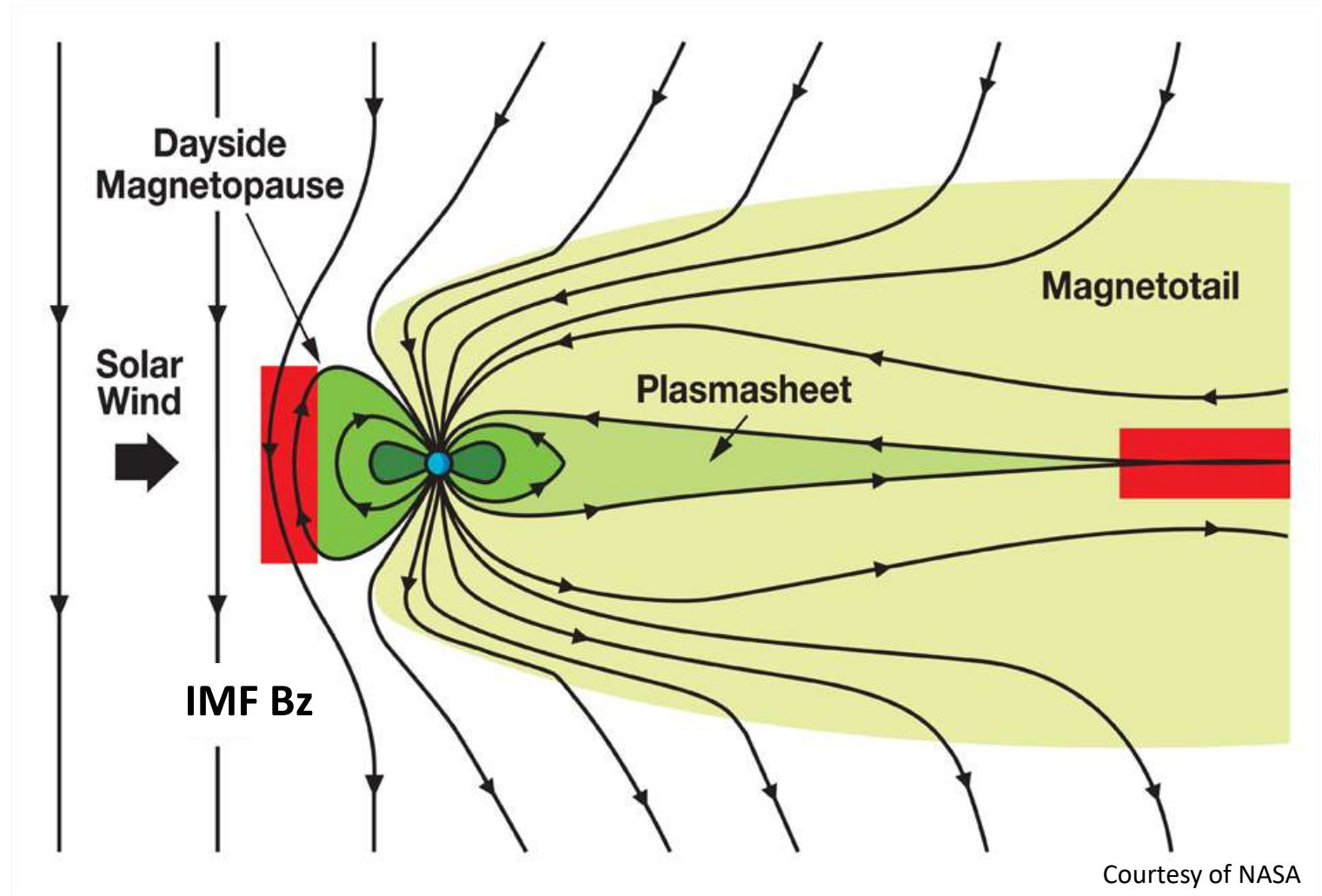


Image from spaceweather.com

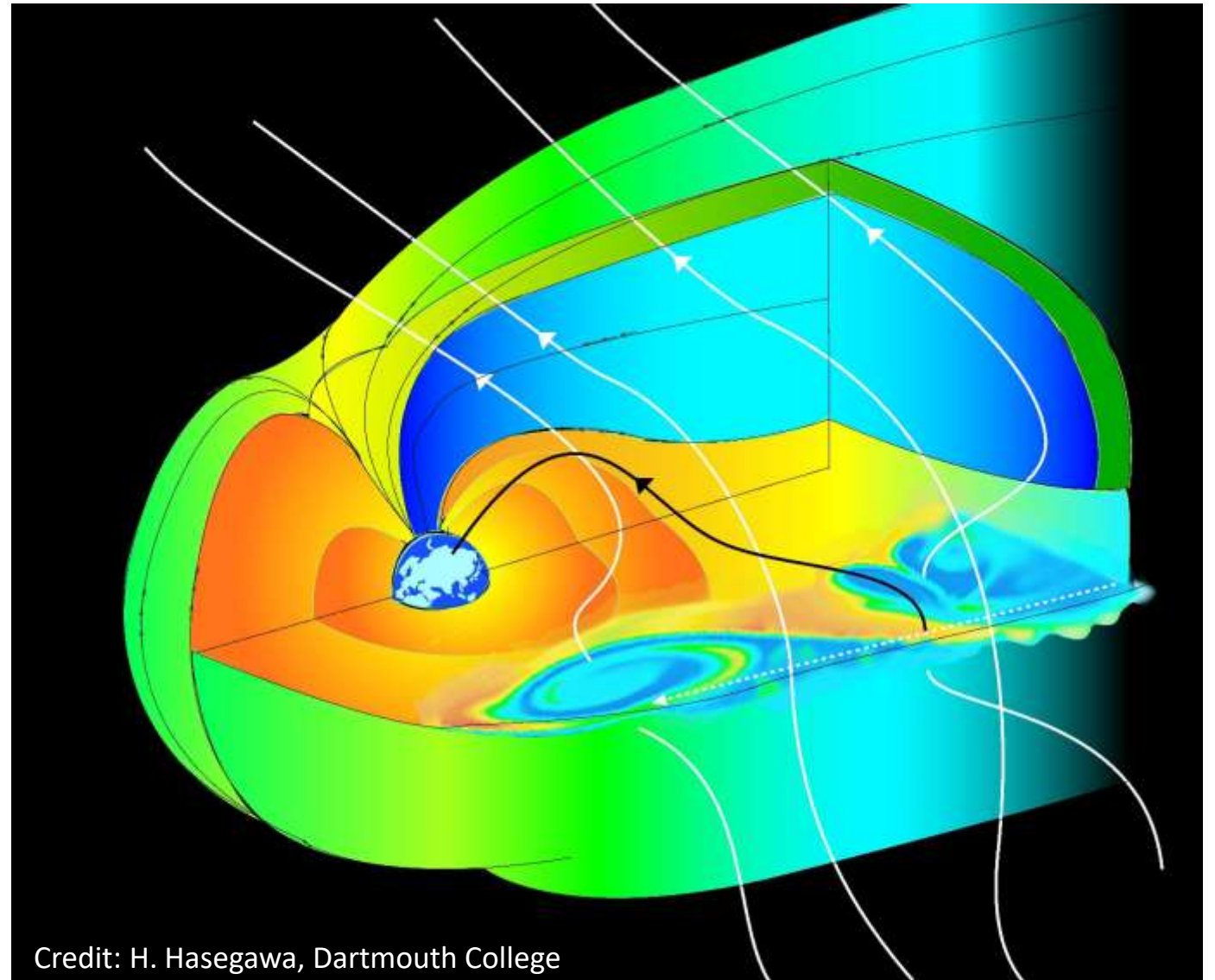
Earth's Magnetosphere

- Magnetosphere is the region of space where the Earth's own magnetic field dominates.
- Under southward IMF conditions, the merging of magnetic field lines is possible at the nose of magnetosphere (dayside reconnection).
- Closed magnetic field lines have both ends linked to the Earth; open field lines have one end linked to the solar wind.



Complexity of the dayside reconnection

- Under northward IMF (B_z positive), the dayside reconnection becomes complex with merging points in the north/south flanks of the magnetosphere.
- Further complexity arises from 3-dimensionality, and due to the excitement of large scale flow instabilities (e.g., KH vortices).



<https://omniweb.gsfc.nasa.gov/>

Interpreting solar wind data

