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Determination of Total Electron Content in the Ionosphere Using GPS Technology

Miloš MARKOVIĆ[1*]

[1] University College of Applied Science of Civil Engineering and Geodesy, Belgrade, Serbia

Abstract. One of major limitations to achieve accuracy when using single frequency GPS receivers is the problem of delays in signal propagation through the ionosphere. The problem is very pronounced during periods of high solar activity, when various processes in ionosphere occur. This paper shows one way to model the ionosphere, which opens the possibility of calculating delays, which can then be applied as a correction during measurement processing. On the other hand, thanks to the knowledge of geodesy and global positioning system, the scientists involved in the ionosphere problem get a very powerful tool that can be used to form a two- or three-dimensional model of the ionosphere. One such two-dimensional model is presented in the paper.

Keywords: total electron content – TEC, two-frequency model of the ionosphere, ionospheric delay, single frequency GPS

[*] Miloš Marković> milmarkovic85@gmail.com

1 Introduction

Ionosphere, as one of the Earth's atmosphere layers, is a changing environment, both spatially and temporally. It extends to a height of about 50 to 1000 km and is the basic protection of the Earth and life on Earth from the dangerous influence of the Sun and the cosmos itself. Studies of spatial and temporal changes, shapes, characteristic regions and the frequent disturbances of the very ionosphere have been rampant in the past two decades due to the effects of the Sun, especially with the development of the Global Positioning System. Ionospheric research also very attracts the GPS community since the phenomenon of ionospheric signal delay is one of the major sources of error for GPS measurements. On the other hand the application of GPS technology allows scientists to gain insight into the shape and behaviour of the ionosphere. By all accounts the benefit is mutual.

The signals beamed from satellites must pass through the ionosphere on their way to Earth. Free electrons, as the most massive particles in the ionosphere affect the propagation of the signal, changing their speed, direction and shape of the signal path (Figure 1). Positioning error that occurs due to this effect is called the ionospheric delay [5].

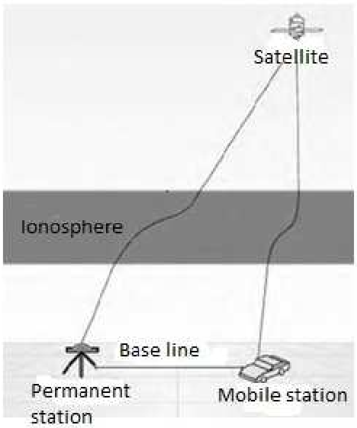


Figure 1. Appearance of signal path while passing through the ionosphere (Ya'acob, Abdullah, Ismail 2009.)

The parameter that most affects the propagation of *GPS* signal is called total electron content, or abbreviated *TEC*. Knowing the parameters of *TEC*, estimation of errors and calculation of corrections can be made.

This paper is based on the use of dual-frequency techniques for the collecting and mapping of *TEC* on the territory of the Republic of Serbia.

The results of this work are the numerical values of *TEC* and their cartographic representations, as well as spatial and temporal variations of *TEC* on the hourly level. Using data on these maps, the movement of electrons can be predicted and the necessary corrections in the *GPS* measurements can be calculated.

2 Theory and Ionosphere Modelling Techniques

The ionosphere is one of the layers of the atmosphere, containing ions, electrically charged particles. The temperature rises to an altitude of 400 km. The upper limit is not clearly defined, because at altitudes over 1000 km of the electron density gradually decreases, so it is difficult to determine precisely the transition from the ionosphere to the plasma sphere. Ionosphere itself is divided into four smaller layers D, E, F1 and F2, each with special characteristics. Figure 2 shows the layers of the ionosphere, as well as the change of the electron density with a change in height [2].

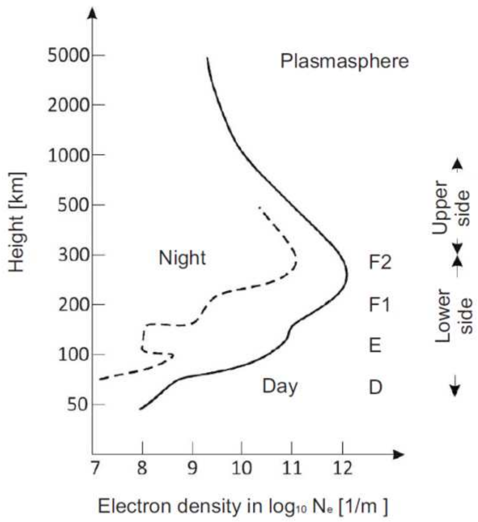


Figure 2. Vertical profile of the ionosphere (Komjathy, 1997.)

The main source of energy that forms the ionosphere is the electromagnetic (EM) radiation from the Sun and in the form of solar extreme - ultraviolet (EUV) radiation and X radiation. Collisions of particles of EUV beam, called photons, with the atoms and molecules of gas in the atmosphere, can create enough energy for the occurrence of photonisation, forming positively charged ions (cations) and negatively charged free electrons. Another source of ionization is the cosmic radiation, which consists of highly charged particles from space, with the level of energy so high that can penetrate to the Earth's surface.

The opposite process in the ionosphere is called recombination, where the cations and free electrons recombine and form neutral atoms and molecules. Another interesting phenomenon, is the merging of free electrons with neutral atoms and molecules, which is how a negatively charged ions (anions) are created. However, anions and cations have very little effect on the propagation of EM waves, due to the relatively large mass and inability to oscillate when exposed to EM wave. Therefore, this paper mainly made reference to the effect of free electrons on the propagation of EM waves [2].

2.1 The Effects of the Ionosphere on the Propagation of Electromagnetic Waves

When radio waves, such as those emitted from *GPS* satellites pass through the ionized path, there are two effects: the trajectory of the beam is bent and the signal comes to a destination with a delay. The free electrons in the ionosphere are the "culprits" for this phenomenon, due to the effect called refraction. Refraction of beam is defined by Snell-Descartes law.

However, the behavior of waves in the ionosphere cannot be described with this relatively simple equation only. To adequately describe the behavior of radio waves passing through the ionosphere, it must be borne in mind that the ionosphere is only partially ionized, spherically stratified plasma with a broad spectrum of unevenly spaced irregularities, which extends along the uneven magnetic field, which is distorted in itself due to the disorder that arises as a result of the occurrence of solar winds.

In the case of the ionosphere, the refractive index is a complex quantity, which Edward Appleton has described in his magnetic-ionospheric theory. He demonstrated first that when going through the magnetized plasma, a plane polarized wave splits into two circularly polarized waves rotating in opposite directions. Hartree suggested that the Lorentz polarization should apply in the theorem, so the formula for calculating the complex refractive index was named Appleton-Hartree formula [2].

Appleton-Hartree formula refers to an environment that is electrically neutral, and does not result in any charge in space and with an equal number of electrons and cations, which extends along a constant magnetic field, and the effect of cations on the wave is negligible.

Let a plane EM wave travel along the x axis of orthogonal coordinate system as shown in Figure 3 and a uniform external magnetic field lie in *x-y* plane forming an angle Θ with the direction of wave propagation.

The complex index of refraction given by Appleton-Hartree formula reads:

$$n^2 = 1 - \frac{X}{(1-iZ)-\left[\frac{Y_T^2}{2(1-X-iZ)}\right]\pm\left[\frac{Y_T^4}{4(1-X-iZ)^2}+Y_L^2\right]^{\frac{1}{2}}}, \quad (1)$$

or generally:

$$n^2 = 1 - F(f, f_N, f_H, f_c, \Theta), \quad (2)$$

where n is the complex refractive index (μ-$i\chi$), with real part μ and imaginary part χ.

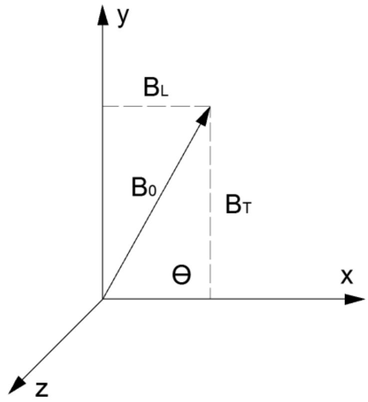


Figure 3. The orthogonal coordinate system (Komjathy, 1997.)

Further applies:

$$X = \frac{\omega_N^2}{\omega^2} = \frac{f_N^2}{f^2}, \quad (3)$$

$$Y = \frac{\omega_H}{\omega} = \frac{f_H}{f}, \quad (4)$$

$$Y_L = \frac{\omega_L}{\omega} = \frac{f_L}{f},\ Y_T = \frac{\omega_T}{\omega} = \frac{f_T}{f}, \quad (5)$$

$$Z = \frac{\omega_c}{\omega} = \frac{f_c}{f}, \quad (6)$$

where:

f [Hz] - carrier frequency; ω [radian/s] – angular frequency of the carrier wave;

f_N [Hz] – frequency of the plasma; ω_N [radian/s] – angular frequency of the plasma, which is calculated according to the formula $\omega_N^2 = \frac{Ne^2}{\varepsilon_0 m}$, with density of electrons N [electron/m^3], charge e ($1.6\cdot10^{-19}$ C), dielectric permittivity of free space ε_0 ($8.8542\cdot10^{-12}$ F/m) and the electron mass m ($9.1095\cdot10^{-31}$ kg);

f_H [Hz] - gyro frequency of free electrons; ω_H [radian/s] – angular gyro frequency, which is calculated according to the formula $\omega_H = \frac{B_0|e|}{m}$, with magnetic induction B_0 [T = Wb/m^2];

f_L [Hz] - longitudinal gyro frequency; ω_L [radian/s] – longitudinal angular gyro frequency, which is calculated according to the formula $\omega_L =$

$\frac{B_0|e|}{m} cos\Theta$;

f_T [Hz] - transverse gyro frequency; ω_T [radian/s] – transverse angular gyro frequency, which is calculated according to the formula $\omega_T = \frac{B_0|e|}{m} sin\Theta$;

f_c [Hz] - frequency of collisions between electrons and heavy particles; ω_c – angular frequency of collisions;

Θ - angle between the signal and the vector magnetic field.

Equation (1) can be developed in endless series. If collisions ($Z \approx 0$) and the influence of magnetic fields ($\Theta \approx 0$) are neglected, then we can take only the first two members:

$$n = 1 - \frac{1}{2} \cdot \frac{f_N^2}{f^2}. \qquad (7)$$

Given, that for each point $f_N^2 = 80.6 \cdot N \text{ Hz}^2$ is valid (N is the density of electrons in electron/m^3), pure carrier wave can be calculated as follows:

$$n = 1 - 40.3 \cdot \frac{N}{f^2}. \qquad (8)$$

For the ionosphere it can be written:

$$n_p = 1 - 40.3 \cdot \frac{N}{f^2}, \qquad (9)$$

where n_p is phase refractive index, which is equivalent to equation (8), while the group refractive index is:

$$n_g = 1 + 40.3 \cdot \frac{N}{f^2}. \qquad (10)$$

In reality, it is not geometric, but electromagnetic distance that is generally measured between the satellite and the receiver. This distance can be written as:

$$S = \int_{Satellite}^{Reciever} nds. \qquad (11)$$

Substituting equation (9) in (11) we obtain:

$$S = \rho - 40.3 \cdot \frac{1}{f^2} \int_{Satellite}^{Reciever} Nds = \rho - 40.3 \cdot \frac{TEC}{f^2}, \qquad (12)$$

where *TEC* is total electron content, i.e. integrated electron density along the signal path given in the *TEC* units (1 *TECU* = $10^{16} \frac{1}{m^2}$), and ρ is the right distance. The equivalent expression for the modulated signal reads:

$$S = \rho + 40.3 \cdot \frac{TEC}{f^2}. \qquad (13)$$

From the last two equations it follows that during the passage through the ionosphere, phase of the carrier wave will accelerate (12) (the distance S is shorter than the actual distance ρ), and the modulated signal will be delayed (13) (the distance S is longer than the actual distance ρ).

Given that the value of ρ is the true distance from the satellite to the receiver, the second part of equations (12) and (13) represents the error caused by signal propagation through the ionosphere, known as ionospheric signal delay, i.e.:

$$d_{jon} = (I_{A,f}^i) = 40.3 \cdot \frac{TEC}{f^2}. \qquad (14)$$

Since the frequencies of *GPS* signal are known, it follows that ionosphere signal delay is the only function of *TEC*. Hence it can be concluded that the knowledge of *TEC* and its characteristics allows the modelling of the ionosphere for scientific purposes (e.g. forecasting of solar storms on the basis of developments and changes in the number of electrons), the determination of error propagation of radio waves, etc [4].

2.2 Two-Frequency Model of Ionosphere

Because of the maximum variation of free electrons, F layer is considered as the main "culprit" for the error of signals propagation through the ionosphere. More than two-thirds of the total number of electrons is located in this layer. Therefore, in this model the calculated *TEC* values only apply to the F layer, namely the F2 layer.

Figure 4 shows the algorithm of creating the two-frequency model.

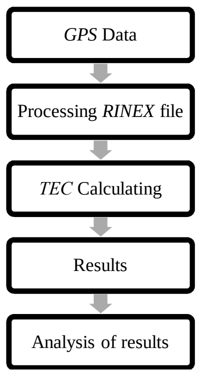


Figure 4. Algorithm of creating two-frequency model

2.2.1 Collection and Processing of GPS data

Satellite observations were conducted from the permanent stations throughout Serbia. Observed values are published in the form of RINEX 2.1 ASCII files.

2.2.2 TEC Calculating

By using the equation for the code pseudo range for two different frequencies f1 and f2, it can be written as follows [1]:

$$P^i_{A,1} = \rho^i_A + c_0\left(\delta t_A - \delta t^i\right) + T^i_A + I^i_{A,1} + \varepsilon^i_{A,1,P} \quad (15)$$

and

$$P^i_{A,2} = \rho^i_A + c_0\left(\delta t_A - \delta t^i\right) + T^i_A + I^i_{A,2} + \varepsilon^i_{A,2,P}. \quad (16)$$

where:
$P^i_{A,f}$ – code pseudo range (f = 1,2),
ρ^i_A – linear (geometric) distance,
c_0 – speed of light in vacuum,
δt_A and δt^i - errors of receiver and satellite clocks,
T^i_A – tropospheric delay,
$I^i_{A,f}$ – ionospheric delay and
$\varepsilon^i_{A,f,P}$ – measurement error.

From $I^i_{A,2} = \left(f_1^2 / f_2^2 \right) I^i_{A,1}$ and (14) it follows that:

$$P^i_{A,1} - P^i_{A,2} = I^i_{A,1} - I^i_{A,2} = 40.3TEC\left(\frac{1}{f_1^2 - f_2^2}\right), \quad (17)$$

where the final expression for calculating the *TEC* is obtained:

$$TEC = \frac{1}{40.3}\left(\frac{f_1 f_2}{f_1 - f_2}\right)(P_2 - P_1). \quad (18)$$

Expression (18) is the so-called "oblique" *TEC*, which is a measure of the total electron content in the ionosphere along the propagation direction of the beam.

2.2.3 TEC Mapping

Mapping can begin after the calculation of *TEC*. For the purposes of mapping, "oblique" *TEC* has to be transformed into "vertical" *TEC*. "Vertical" *TEC* is that which enables the mapping of *TEC* values. Figure 5 shows the transformation algorithm from "oblique" to the "vertical" *TEC*.

For the calculation of the intersection points, it must be assumed that the ionosphere is "compressed" in the infinitesimally thin layer around the Earth at an altitude of 350 km. At the intersection of the direction of propagation of the signal and ionospheric layer there is the "pierce point" (IPP - Ionospheric Pierce Point, Figure 6) [4].

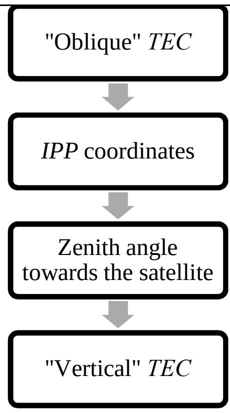


Figure 5. *TEC* transformation

Ionospheric layer can be approximated by a sphere in the territory of Serbia. The equation of the sphere is:

$$x^2 + y^2 + z^2 = (R + h)^2, \quad (19)$$

where:
x, y, z – coordinates of a point on the sphere,
R – Earth radius and
h - layer height.

If satellite coordinates $A\ (X_A, Y_A, Z_A)$ and two frequency receiver coordinates $i\ (x_i, y_i, z_i)$ are known, we can derive the spatial equation of the line through two points:

$$\frac{x - x_i}{X_A - x_i} = \frac{y - y_i}{Y_A - y_i} = \frac{z - z_i}{Z_A - z_i}. \quad (20)$$

By solving equations (19) and (20) IPP coordinates ($x \rightarrow x_p$, $y \rightarrow y_p$, $z \rightarrow z_p$) can be obtained.

Figure 6 shows the principle of determining the zenith distance on the pierce point, which can be calculated from the expression:

$$z = 180^o - \alpha. \quad (21)$$

The angle α can be calculated using the cosine rule:

$$\alpha = \cos^{-1}\left(\frac{|r_p|^2 + |r_A - r_p|^2 + |r_A|^2}{2 \cdot |r_p| \cdot |r_A - r_p|}\right), \quad (22)$$

where:
$|\vec{r_p}|$ – intensity of geocentric radius vector of intersection points $\vec{r}_p = (x_p, y_p, z_p)$ and
$|\vec{r_A}|$ – intensity of geocentric radius vector of satellite $\vec{r}_A = (X_A, Y_A, Z_A)$.

The transformation of "oblique" in the "vertical" *TEC* can now be carried out using the expression:

$$TECv = TECk \cdot \cos(z). \quad (23)$$

The last step is the transformation of Cartesian coordinates into geodetic coordinates. Specifically, this transformation is:

$$(x_p, y_p, z_p) \rightarrow (\Phi, \Lambda, h),$$

$$\Phi = \sin^{-1}\left(\frac{R+h}{z_p}\right), \tag{24}$$

$$\Lambda = \tan^{-1}\left(\frac{y_p}{x_p}\right) \tag{25}$$

and

$$h = 350 \text{ km}.$$

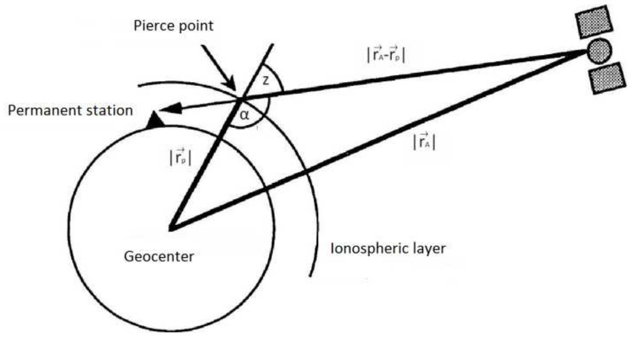


Figure 6. Zenith distance on pierce point (Webster, 1993.)

3 Processing and Mapping of Results

Ionospheric *GPS-TEC* measurements were conducted with the total of 29 AGROS (Active Geodetic Reference Base of Serbia) permanent stations. Observations started on 30/11/2008 at 00:00:00, and were completed on the same day at 23:59:59. The time interval between observations was one second.

The obtained data were analysed in three stages:

1. processing RINEX observational and navigation file;
2. creating ASCII files containing the coordinates and values of *TEC* on the pierce point and
3. creating *TEC* maps for each hour.

3.1 Processing of Rinex Files

The resulting RINEX files were processed in the software *GPS TEC* analysis, developed by Professor Gopi Krishna Seemala, from Boston University [3].

For input data RINEX Observation and Navigation files are entered. For navigation, data software automatically downloads the precise ephemeris from the server of IGS service. After entering the input data, the software offers the option of selecting the output data as follows:

1. CMN file: selecting this option form the file in ASCII format with the extension .cmn, in which the calculated values of *TEC* are printed. The obtained data are processed measurements of pseudo ranges observed towards all visible satellites throughout the day, with the observation interval of one second. Therefore, it is necessary to remove the values that do not relate to a full hour, because the *TEC* maps are created on hourly level. Using the MATLAB programming language each CMN file was successfully filtered, and the printing of individual time-text files (.txt) was the result. One hour file represents, in fact, the list of coordinates processed at all stations for a specific hour. The first column shows geodetic latitudes, the second geodetic longitudes and the third "vertical" *TEC* values. Each row is, in fact, the three-dimensional coordinate of pierce point;
2. STD file: selecting this option a file in ASCII format with the extension .std is formed, in which daily average values *TEC* are printed;
3. Bias file: selecting this option a file in ASCII format with the extension .bias is formed, in which the values of the impact of satellites and receivers that are removed during processing are printed;
4. TEC 24 Hrs Image: selecting this option *TEC* values calculated for 24 hours and for all satellites detected on the selected cell are plotted (Figure 7);

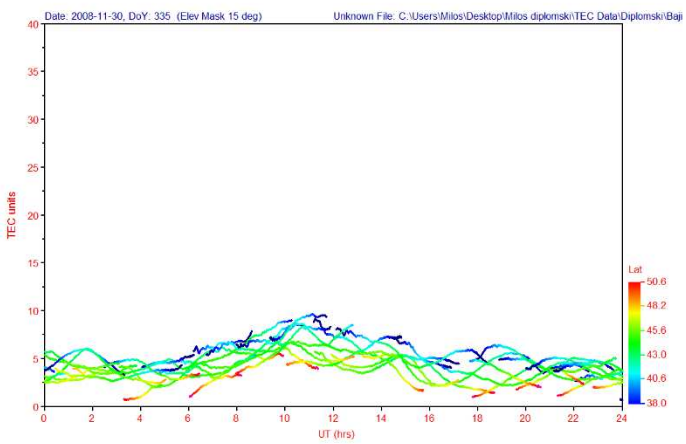


Figure 7. An example of the value of the total electron content for 24h, obtained from one of the permanent stations in Serbia

5. TEC PRN images: by selecting this option the PRN codes of satellites observed at a desired station as a function of time over which they were visible and inclination of satellites are plotted and
6. (UN) / Bias TEC image: selecting this option the *TEC* values before and after removing the influence of the satellite and the receiver are plotted (Figure 8).

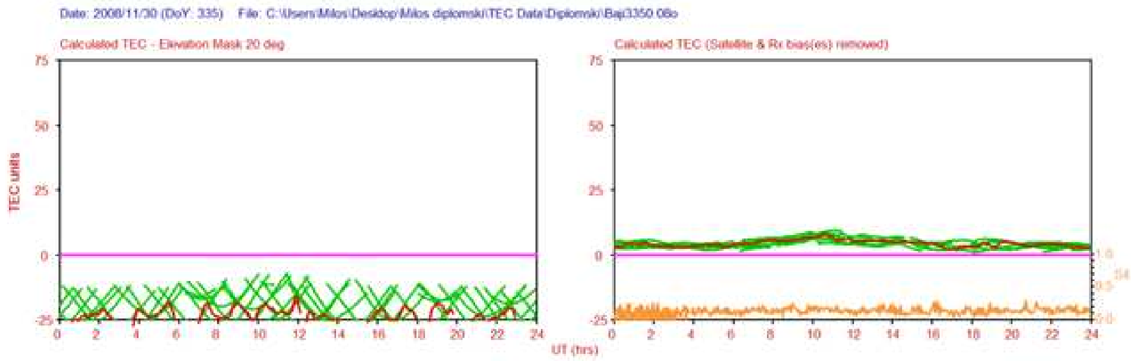


Figure 8. The values of the total electron content before and after removing the influence of the satellite and the receiver, the result from one permanent station

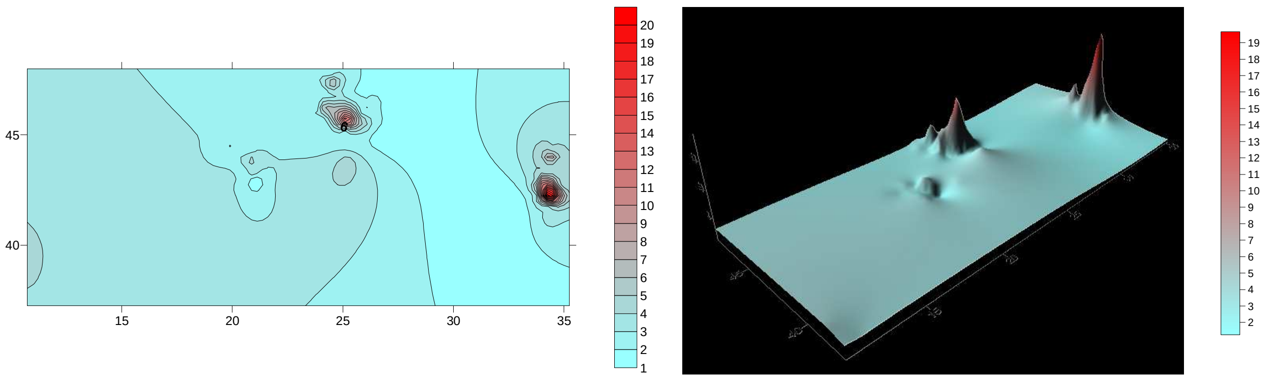
The calculated "oblique" *TEC* is polluted due to the influence of the receiver and the satellite. Therefore, software fixes the calculated value of "oblique" *TEC* using the formula:

$$TEC\hat{s} = TECs + B_{Rx} + B_{Rich} + B_{sat}. \quad (24)$$

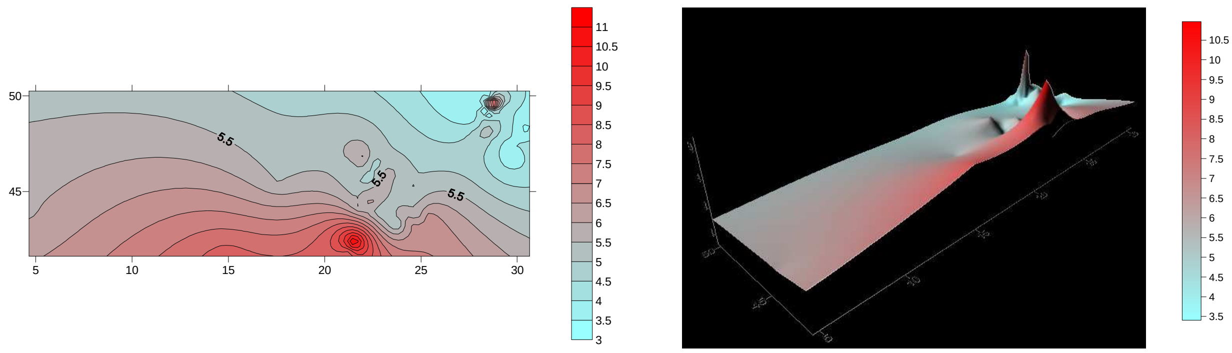
Values of influence from the expression (24) are given in an ASCII file with the extension .bias if the printing of that file is selected as the output data.

3.2 TEC Mapping Results

Mapping the obtained *TEC* values was performed using the software package SURFER. Data from the hour file were loaded into the program, in which a contour line was generated and a digital model of the ionosphere was formed as a function of *TEC*. Below are some examples of time *TEC* maps for UT = 04:00, UT = 11:00 and UT = 14:00. Using these maps the movement of electrons can be monitored and the value of the *TEC* can be read. By tracking the movement we can predict the expected geomagnetic period. With a good forecast of geomagnetic period and design of measurement performance time plan, it is possible to reduce the impact of ionospheric refraction.



UT = 04:00



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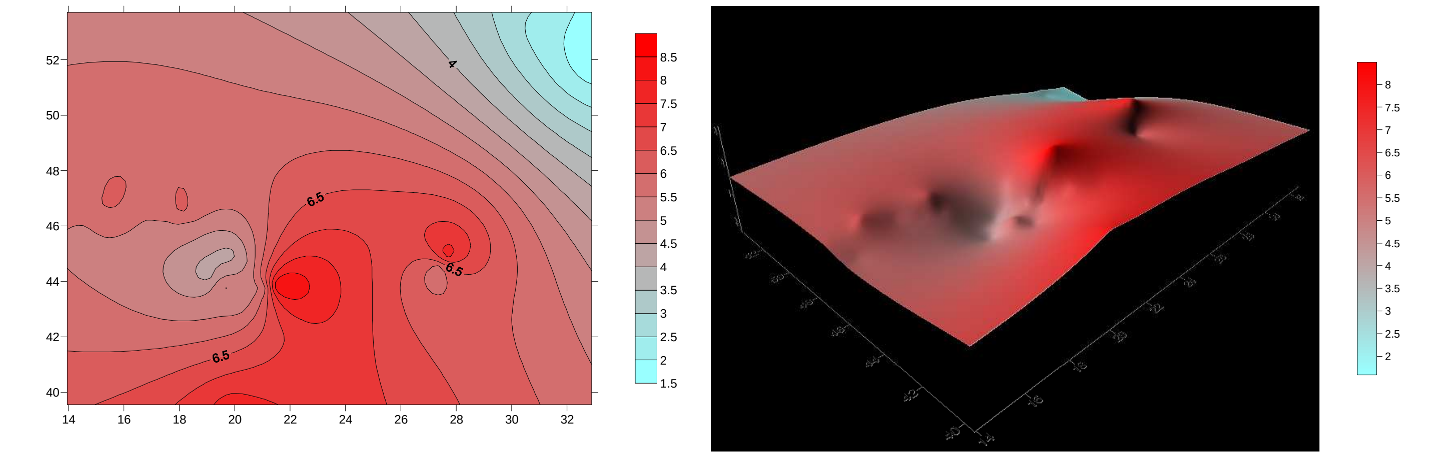

UT = 14:00

4 Conclusion

Space "weather" forecasting is relatively a new field in science. Scientists are studying the cosmic time in order to learn more about the physical and chemical processes that occur in the atmosphere and above it. For this purpose, a wide range of tools are used. Global Positioning System (*GPS*) is one of them. The emergence of this system has led to enormous technical progress in all fields related to geodesy and survey. *GPS* system allows users to determine the position in places where conventional surveying methods are unsuccessful.

In addition to geodesy or geophysics, *GPS* has a great significance in scientific research. Satellites orbiting the Earth at an altitude of 20.200 km, send signals that, on their way to the receiver, pass through the ionosphere which extends from 50 - 1000 km above the Earth's surface. The free electrons that are found in this region of the atmosphere, affect the propagation of the signal, changing the speed and direction of propagation of the signal. Due to the in homogeneity of the ionosphere, the direction of propagation of the *GPS* signal is curved. The influence of the ionosphere can cause positioning errors for users who need highly accurate measurements.

The ionospheric parameter that has the greatest impact on radio signals beamed from satellites is the total electron content (*TEC*). With the modelling of *TEC*, errors caused by ionospheric refraction can be detected and corrected. Propagation delay is proportional to the *TEC* along the signal path from the satellite to the receiver. *TEC* is defined as the integral of the electron density per square meter along the path through which the signal is transmitted. The collection of values of *TEC* with dual-frequency receiver, is one of the most important methods of testing the Earth's ionosphere.

This paper presents a model for collecting and processing the *TEC* called dual-frequency *GPS* model. We used observations of code pseudo ranges, which were later processed with the appropriate software. The experiment was conducted in three phases: data collection, processing and transformation into "vertical" *TEC* and the mapping of the results.

Variations of the total electron content during the day were displayed. The peak value of the *TEC* was between 10 and 12 AM, after which the value was declining. Taking into account this phenomenon and monitoring of the *TEC* maps, it is possible to forecast geomagnetic quiet period in which the measurements should be conducted. It is advisable to keep this in mind, because the error caused by the influence of the ionosphere is practically reduced to a minimum.

Based on the analysis it can be concluded that for the purposes of accurate measurements, the application of these methods is acceptable only if the phase and code measurements are combined. However, the aim of this study was not to achieve accuracy, but to show the possibilities that Global Positioning System offers. A reference was also made to the observations of satellites since the knowledge of the observation values allows the modelling of the processes occurring in the ionosphere.

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