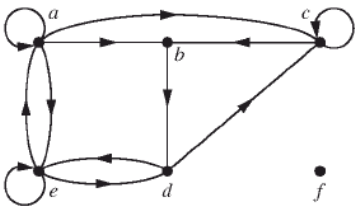
1. How many edges are there in a graph with 10 vertices each of degree six?

2.

EXAMPLE 4 Find the in-degree and out-degree of each vertex in the graph G with directed edges shown in Figure 2.

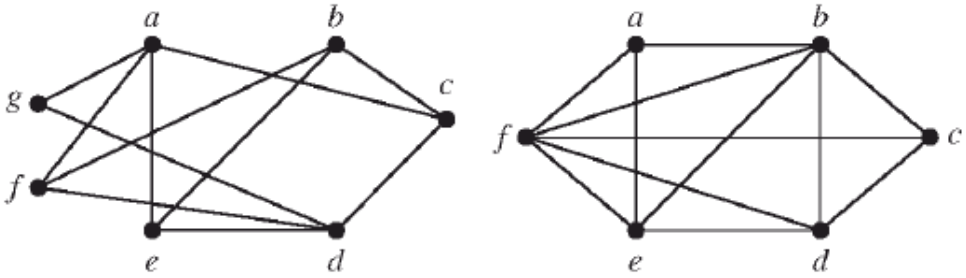


G

FIGURE 2 **The Directed Graph G.**

Solution: The in-degrees in G are $\deg^-(a) = 2$, $\deg^-(b) = 2$, $\deg^-(c) = 3$, $\deg^-(d) = 2$, $\deg^-(e) = 3$, and $\deg^-(f) = 0$. The out-degrees are $\deg^+(a) = 4$, $\deg^+(b) = 1$, $\deg^+(c) = 2$, $\deg^+(d) = 2$, $\deg^+(e) = 3$, and $\deg^+(f) = 0$. ◀

3. Are the graph G and H displayed in the following figure are bipartite?



G H

4. Use an adjacency matrix to represent the graph shown below?

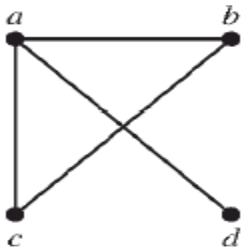


FIGURE 3

5. Draw a graph with the following adjacency matrix

$$\begin{bmatrix} 0 & 1 & 1 & 0 \\ 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 \end{bmatrix}$$

with respect to the ordering of vertices a, b, c, d.

6.

Use an adjacency matrix to represent the pseudograph shown in Figure 5.

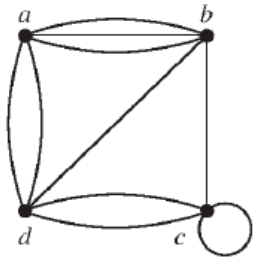


FIGURE 5

7. Represent the graph shown in fig below with an incidence matrix?

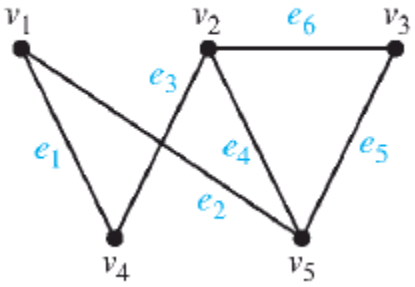
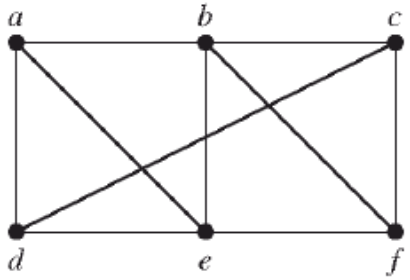


Figure 6

8. ***Definition***: A subset V' of the vertex set V of $G = (V, E)$ is **vertex cut,** or **separating set,** if $G - V'$ is disconnected. For example the set $\{b, c, e\}$ is a vertex cut with three vertices for the following graph.



We define the ***vertex connectivity*** of a graph G, denoted by $\boldsymbol{\kappa(G)}$, as the minimum number of vertices in a vertex cut.

9. Find cut vertices and cut edges in the graph shown below?

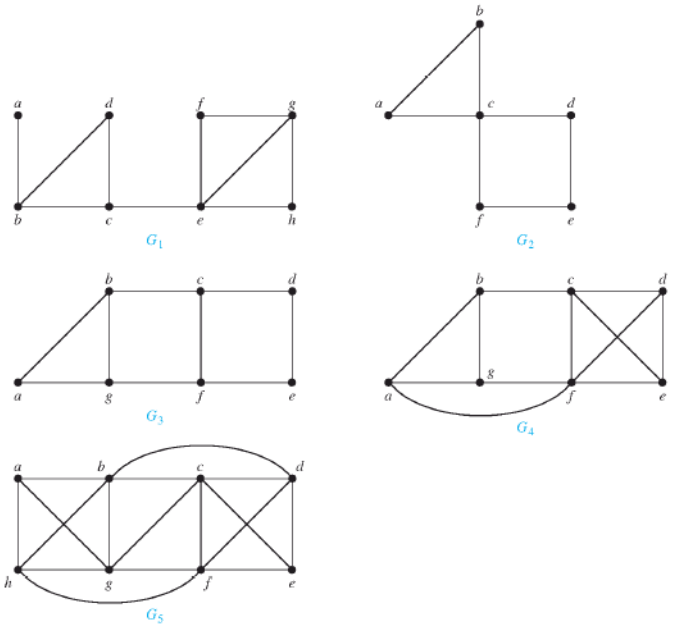
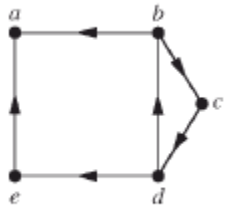


FIGURE 4 **Some Connected Graphs**

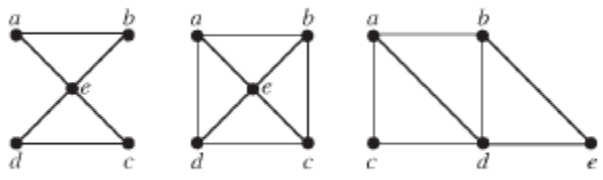
10. Find the vertex connectivity for each of the graphs in Figure 4. (See above figure)

11. Check whether the graph in the following figure is strongly connected? Is that weakly connected?

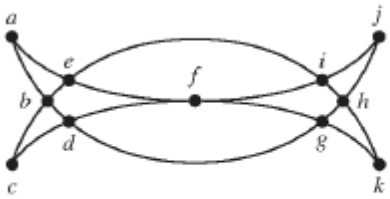


H

12. Which of the following graphs have Euler circuit? If it exists, then find it? Which of following have Euler path? If it exists, then find it?

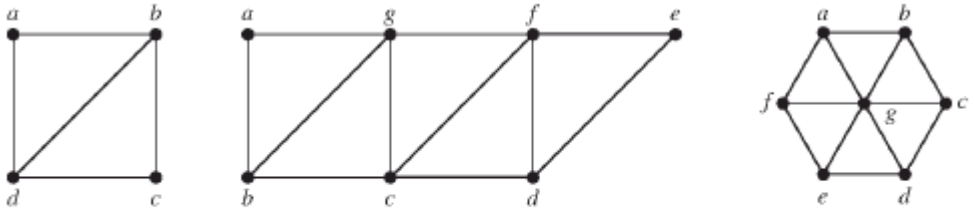


G_1 G_2 G_3



G

FIGURE 6 **Mohammed's Scimitars.**



G_1 G_2 G_3

FIGURE 7 **Three Undirected Graphs.**

13. Which of the simple graphs in Figure 10 have a Hamilton circuit or, if not, a Hamilton path?

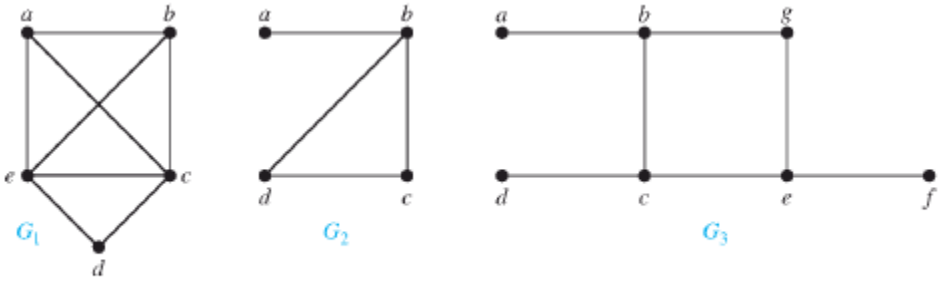


FIGURE 10 **Three Simple Graphs.**

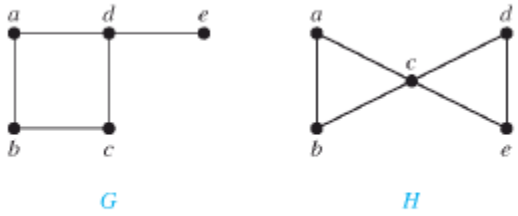


FIGURE 11 **Two Graphs That Do Not Have a Hamilton Circuit.**

14. For which values of $n, K_n, C_{n,}\ K_{m,n}$ and W_n are Euler graphs. Also find for which values of n these graphs will have Euler path?

15. For which values of $n, K_n, C_{n,}\ K_{m,n}$ and W_n are Hamiltonian graphs. Also find for which values of n these graphs will have Hamiltonian path?

16. Find the minimum numbers of colors required for coloring of following map?

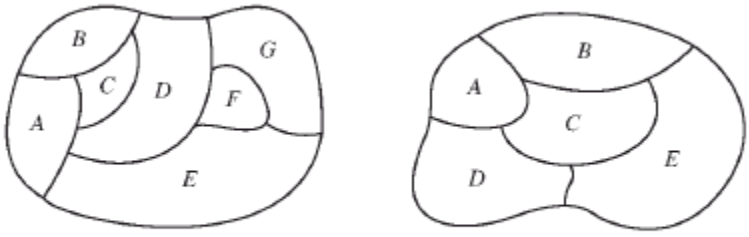
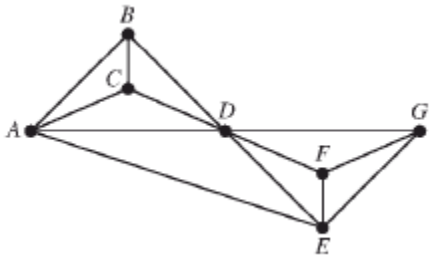
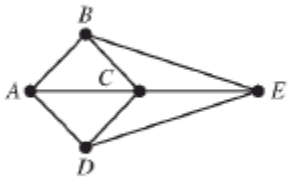
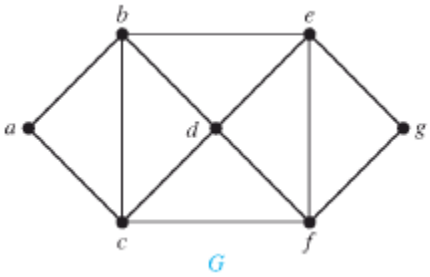


FIGURE 1 **Two Maps.**

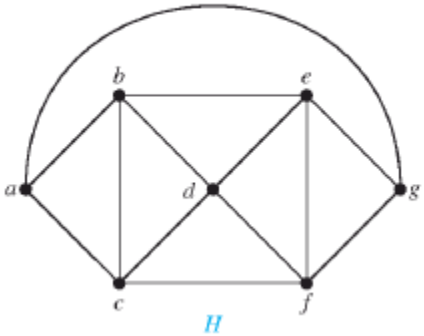
17. Find the chromatic number of the following graphs?







G



H

18. Find the chromatic number of K_n, C_n, W_n and of Q_3.