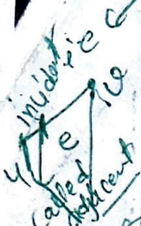
Basic Terminology of vertices & edges of Undirected graphs

Def 1: Two vertices (used for vertices) u and v in an undirected graph G are called **adjacent** (or neighbours) in G if u and v are endpoints of an edge e of G. Such an edge e is called incident with the vertices u and v and e is said to connect u and v. [Two vertices are adjacent if they are connected by an edge.]



used for edge

Def 2. The degree of a vertex in an undirected graph is the number of edges incident with it, except that a loop at a vertex contributes twice to the degree of that vertex

The degree of the vertex v is denoted by $\deg(v)$

$$\boxed{\text{Total degree of a vertex} = \text{incident edges} + 2*(\text{self loops})}$$

Def 3: A vertex of degree zero is called isolated

Def 4: A vertex is pendant if and only if it has degree one.

Theorem:- The Handshaking Theorem:- Let $G=(V,E)$ be an undirected graph with m edges. Then,

$$2m = \sum_{v \in V} \deg(v) = \text{Total degree of a graph}$$

vertices

see example in paper (CA)

Theorem:- An undirected graph has an even number of vertices of odd degree.

vertices of odd degree vertices are always even (Number of odd degree vertices in undirected graph)

Proof: $2m = \sum_{v \in V_1} \deg(v) + \sum_{v \in V_2} \deg(v)$ where V_1 and V_2 be set of vertices even and odd degree respectively.

$\Rightarrow$ i.e. $2m = \sum$ even degree Vertices $+ \sum$ odd deg. vertices

$\Rightarrow \sum$ odd deg vertices $= 2m - \sum$ even deg. vertices

$e = 30$

Def: When (u, v) is an edge of the graph G with directed edges, u is said to be adjacent to v" and "v is said to be adjacent from u". The vertex u is called initial vertex of (u, v) and v is called the terminal or end vertex of (u, v). The initial vertex and terminal vertex of a loop are the same.

Def^n:- Indegree and out-degree of vertex v →

In a graph with directed edges, the indegree of a vertex v, denoted by $\deg^-(v)$, is the number of edges with v as their terminal vertex. The out-degree of v, denoted by $\deg^+(v)$ is the number of edges with v as their initial vertex.

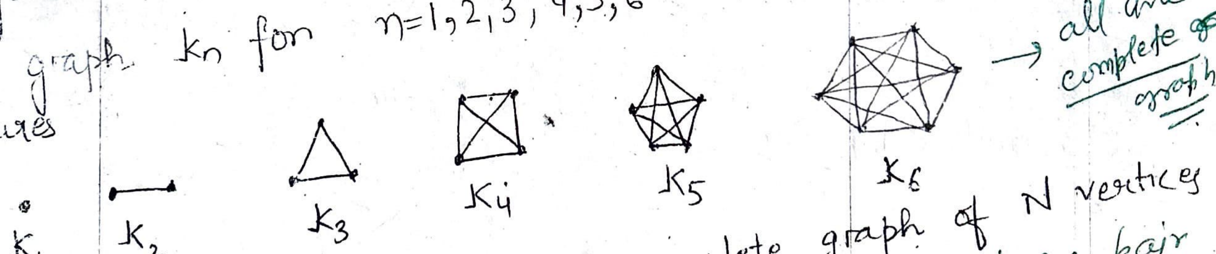
$\deg(v) = \deg^-(v) + \deg^+(v)$

Note:- A loop at a vertex contributes 1 to both the in-degree and the out-degree of this vertex.

Some special simple graphs

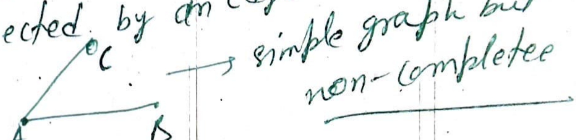
1. Complete graphs: A complete graph on n vertices, denoted by K_n, is a simple graph that contains exactly one edge between each pair of distinct vertices.

The graph K_n for $n = 1, 2, 3, 4, 5, 6$ are displayed in following figures



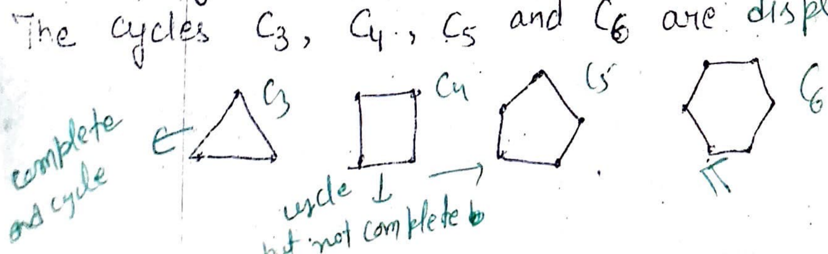
Total number of edges in a complete graph of N vertices $= \frac{n(n-1)}{2}$

[* A simple graph for which there is at least one pair of distinct vertex not connected by an edge is called noncomplete Ex



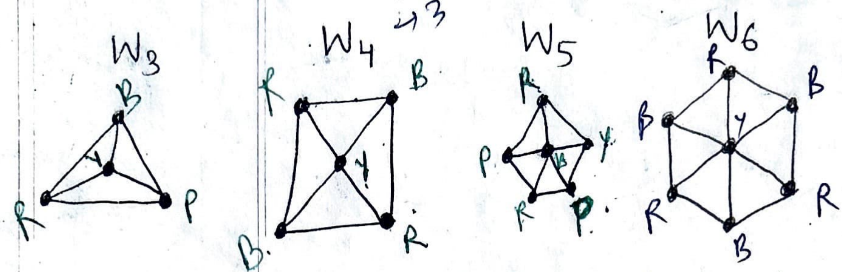
Cycles: A cycle C_n, $n \geq 3$ consists of n vertices $v_1, v_2, \ldots v_n$ and edges $\{v_1, v_2\}, \{v_2, v_3\}, \ldots \{v_{n-1}, v_n\}$ and $\{v_n, v_1\}$.

The cycles C_3, C_4, C_5 and C_6 are displayed in following figures



Wheel: We obtain a wheel W_n when we add an additional vertex to a cycle C_n, for $n \geq 3$, and connect this new vertex to each of the n vertices in C_n, by new edges.

A wheel graph is a graph formed by connecting a single universal vertex to all vertex of a cy...



4. Regular graph:- A graph is called regular graph if degree of each vertex is equal. A graph is called k-regular if degree of each vertex in the graph is k.

Properties: (Cycle). (i.e jaha se start kiya wahi closed k'ya). equal

- A cycle in which only the first and last vertices are equal
- All edges in cycle are distinct.
- The number of vertices in C_n equals to the number of edges and every vertex has degree 2 i.e., every vertex has exactly two edges incident with it
- A cycle is a closed walk in which all vertices are distinct but the first and last vertices are same.

(Regular)

- A complete graph with N vertices is (N-1) regular.
- For a k-regular graph, if k is odd, then the number of vertex of the graph must be even.
- Cycle C_n is always 2-Regular   degree d and $\frac{4 \cdot 2}{2} = 4$ $\frac{6 \cdot 3}{2} = 9$ $\frac{N * d}{2}$
- Number of edge of a k-regular graph ~~with~~ of N vertices =

Proof:- Let the number of edges of a k-regular graph with vertices be E.

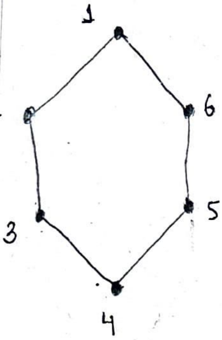
From Handshaking theorem, we know,

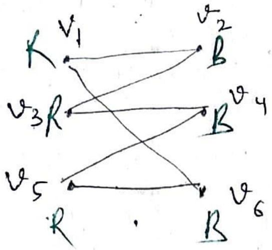
... degree of all the vertices = 2 * E

its vertex set V can be partitioned into two disjoint sets V_1 and V_2 such that every edge in a graph connects a vertex in V_1 and a vertex in V_2 (so that no edge in G connects either two vertices in V_1 or two vertices in V_2).
When this condition holds, we call the pair (V_1, V_2) a bipartition of a vertex set V of G.

Eg: G_6 is bipartite but K_3 is not bipartite.

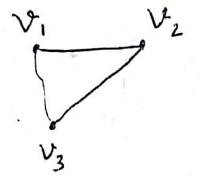
G_6:





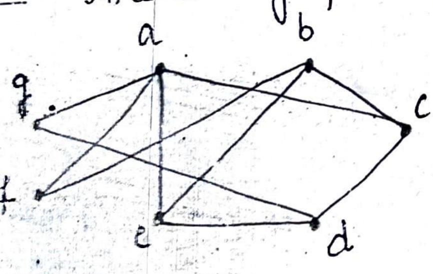
break into two vertices
[1 is connected with 2, so it will be in opposite set of vertices]

K_3:

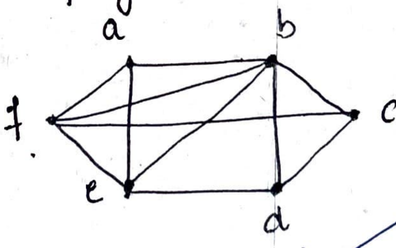


1 is connected with 2,
2 is connected with 3,
but 3 is also connected with 1, so can't place in same set of vertices. $\therefore K_3$ is not bipartite.

...ion: Are the graphs G and H displayed as follows bipartite?



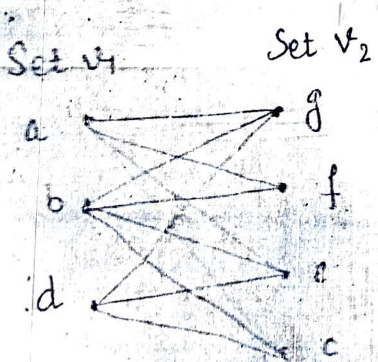
G



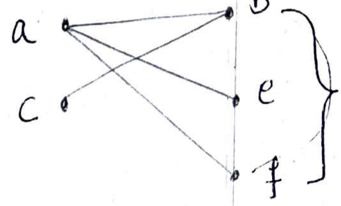
H

check
NM

...ou:-



$\therefore$ G is partite



but b and f are in same set

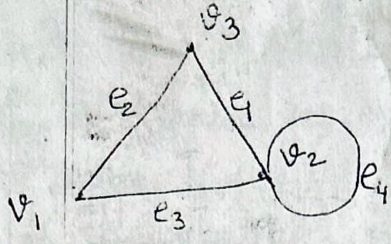
$\therefore$ H is not bipartite

GRAPH

Definition: A graph $G = (V, E)$ consists of V, a nonempty set of vertices (or nodes) and E, set of edges.

Each edge has either one or two vertices associated with it, called its endpoints.

Eg:



$V = \{v_1, v_2, v_3\}$

$E = \{e_1, e_2, e_3, e_4\}$

Remark :- ① The set of vertices V of a graph G may be infinite
A graph with an infinite vertex set or an infinite number
of edges is called an infinite graph

② A graph with a finite vertex set and a finite edg
set is called a finite graph.

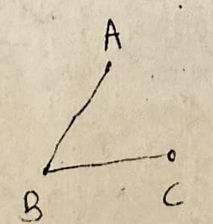
Example: ① Telecommunication of the whole world is example of
Infinite graph

② computer network can be modelled using a graph in which the vertices of the graph represent the data centers and the edges represent communication links.

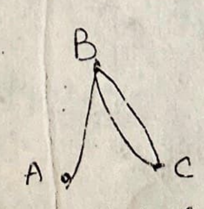
Self loop:- An edge having same vertices at its end points.

Simple graph:- A simple graph is a graph that does not have more than one edge between any two vertices and no edge starts and ends at the same vertex. In other words, a graph does not have any loop or multiple edges.

Eg.



Simple graph



Not a simple graph

Multigraphs: Graphs that may have multiple edges connecting the same vertices are called multigraphs.

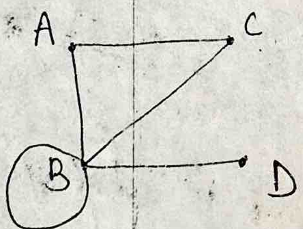
Pseudograph:- A pseudograph is a non-simple graph in both loops and multiple edges are allo

Null graph: A graph whose edge set is empty. In otherw a graph with vertices without edges.

Directed graph:- A directed graph (or diagraph) (V, E) consis of a non-empty set of vertices V and a set of directe edges E. Each directed edge associated with the ordere pair~~(u,v)~~ of vertices. The directed edge associated with ordered pair (u,v) is said to start at u & end at v.

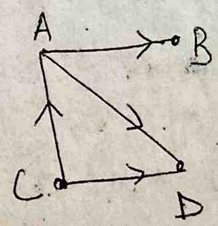
* A directed graph that may have multiple directed ed from a vertex to a second vertex. Such graphs are calle directed multigraphs

Eg:

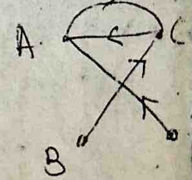


Pseudograph
(Undirected graph)

Null Graph



Directed Graph



Directe Multigr

* Mixed graph: A graph with both directed and undirecte edges.

Graph Terminology:-

Type	Edges	Multiple edges	Loops
Simple	Undirected	×	×
Multi	"	✓	×
Pseudo	"	✓	✓
Simple directed	Directed	×	×
Directed graph	"	✓	✓
Mixed	Both	✓	✓

Basic Terminology of vertices/edges of Undirected graphs.

Two vertices u and v in an undirected graph G are called adjacent (or neighbours) in G if u and v are endpoints of an edge e of G. Such an edge e is called incident with the vertices u and v and e is sa[id] to connect u and v.

Def^n 2. The degree of a vertex in an undirected graph is th[e] number of edges incident with it, except that a loop at a vertex contributes twice to the degree of that ve[rtex].

The degree of the vertex v is denoted by deg(v)

$$\boxed{\text{Total degree of a vertex} = \text{incident edges} + 2 * (\text{self loops})}$$

Def^n 3: A vertex of degree zero is called isolated

Def^n 4. A vertex is pendent if and only if it has degree on[e]

Theorem:- The Handshaking Theorem:- Let $G=(V,E)$ be an undirec[ted] graph with m edges. Then,

$$2m = \sum_{v \in V} \deg(v) = \text{Total degree of a gra[ph]}$$

Theorem:- An undirected graph has an even number of vertices of odd degree.

(Number of odd degree vertices are always even in undirected graph)

Proof $2m = \sum_{v \in V_1} \deg(v) + \sum_{v \in V_2} \deg(v)$ where V_1 and V_2 be set of vertices even and odd degr[ee] respectively.

$\Rightarrow$ i.e. $2m = \sum$ even degree Vertices $+ \sum$ odd deg. vertices

$\Rightarrow \sum$ odd deg. vertices $= 2m - \sum$ even deg. vertices

Def: When (u, v) is an edge of the graph G with directed edges
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adjacent from v". The vertex u is called initial vertex
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denoted by $\deg^-(v)$, is the number of edges with v as their
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The graph K_n for $n = 1, 2, 3, 4, 5, 6$ are displayed in following
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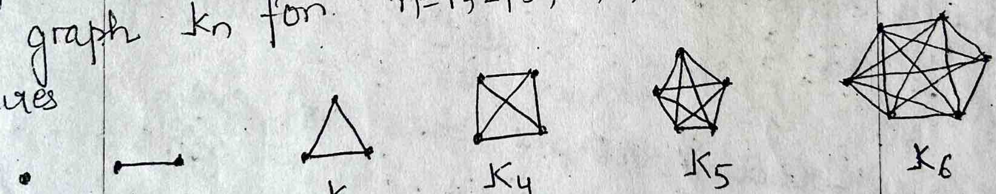
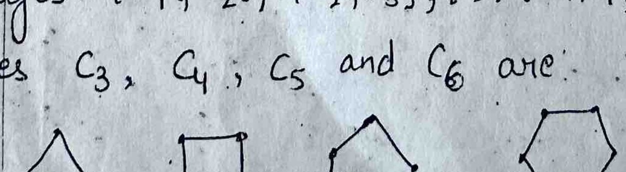


K_1 K_2 K_3 K_4 K_5 K_6

Total number of edges in a complete graph of N vertices
$= \frac{n(n-1)}{2}$

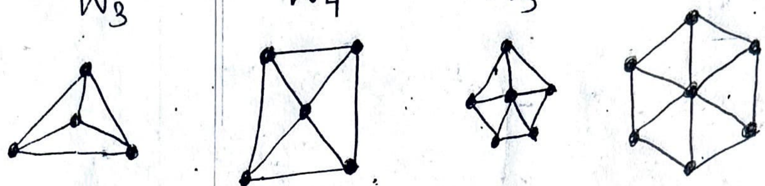
2. Cycles: A cycle C_n, $n \geq 3$ consists of n vertices $v_1, v_2, \ldots v_n$
and edges $\{v_1, v_2\}, \{v_2, v_3\}, \ldots \{v_{n-1}, v_n\}$ and $\{v_n, v_1\}$.
The cycles C_3, C_4, C_5 and C_6 are:



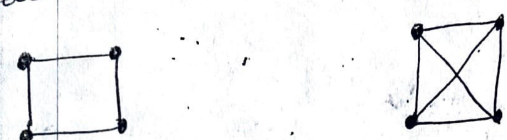
Wheel: We obtain a Wheel W_n when we add an additional vertex to a cycle C_n, for $n \geq 3$, and connect this new vertex to each of the n vertices in C_n, by new edges.

A wheel graph is graph formed by connecting a single universal vertex to all vertex of a c...

W_3 W_4 W_5 W_6



4. **Regular graph:-** A graph is called regular graph if degree of each vertex is equal. A graph is called k-regular if degree of each vertex in the graph is k.



2 - regular 3 regular.

Properties: (Cycle)

- A cycle in which only the first and last vertices are e... distinct.
- All edges in cycle are
- The number of vertices in C_n equals to the number of edges and every vertex has degree 2 i.e., every vert... has exactly two edges incident with it.
- A cycle is a closed walk in which all vertices are di... but the first and last vertices are same.

(Regular)

- A complete graph with N vertices is (N-1) regular.
- For a k-regular graph; if K is odd, then the number of ve... of the graph must be even.
- Cycle C_n is always 2- Regular
- Number of edge of a k- regular graph with N vertices $= \frac{N*}{2}$ $= N \cdot \frac{K}{2}$

Proof :- Let the number of edges of a k- regular graph with vertices be E.

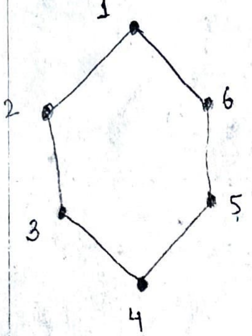
From Handshaking theorem, we know,

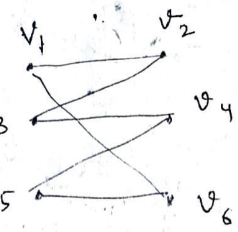
Sum of degree of all the vertices $= 2*E$

Bipartite graphs :- A simple graph G is called bipartite if its vertex set V can be partitioned into two disjoint sets V_1 and V_2 such that every edge in a graph connects a vertex in V_1 and a vertex in V_2 (so that no edge in G connects either two vertices in V_1 or two vertices in V_2). When this condition holds, we call the pair (V_1, V_2) a bipartition of a vertex set V of G.

Eg: C_6 is bipartite but K_3 is not bipartite.

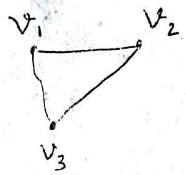
C_6:





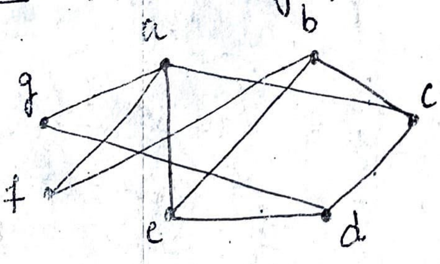
break into two vertices [1 is connected with 2, so it will be in opposite set of vertices]

K_3:

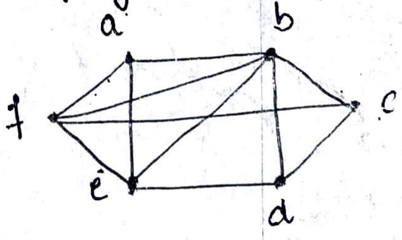


1 is connected with 2, 2 is connected with 3, but 3 is also connected with 1, so can't place in same set of vertices. ∴ K_3 is not bipartite.

Question: Are the graphs G and H displayed as follows bipartite?

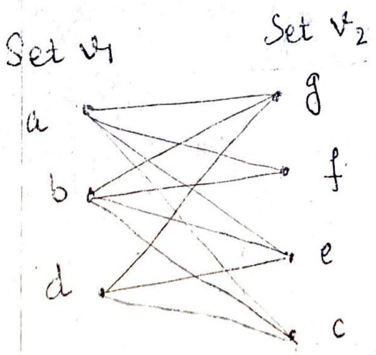


G

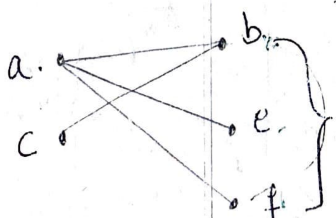


H

Solution:-



∴ G is partite



but b and f are in same set

∴ H is not bipartite

6. Complete Bipartite Graphs :-

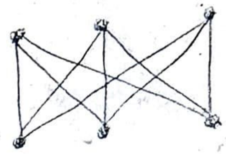
A complete bipartite graph $K_{m,n}$ is a graph that has its vertex set partitioned into two subsets of m and n vertices respectively with an edge between two vertices if and only if one vertex is in the first subset and the other vertex is in the second subset.

Draw K_3 & K_2,

Examples:-

It is a simple type of bipartite graph where every vertex of one set is connected to ~~connect~~ every vertex of other set.

$K_{2,3}$



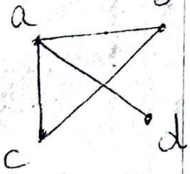
$K_{3,3}$

10.3 Representing Graphs and Graph Isomorphism.

Suppose that $G=(V,E)$ is a simple graph where $|V|=n$. Suppose that the vertices of G are listed arbitrarily as $v_1, v_2, \ldots$ with respect to this listing of the vertices, The adjacent matrix A (A_G) of G is the $n \times n$ zero-one matrix with 1 as its (i,j)th entry when v_i and v_j are adjacent, and 0 as its (i,j)th entry when v_i and v_j are not adjacent.

$$A = [a_{ij}] = \begin{cases} 1 & \text{if } \{v_i, v_j\} \text{ is an edge of } G. \\ 0 & \text{Otherwise.} \end{cases}$$

Eg.

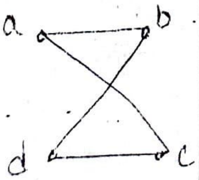


$$A = \begin{array}{c} \\ a \\ b \\ c \\ d \end{array} \begin{array}{c} \begin{array}{cccc} a & b & c & d \end{array} \\ \begin{bmatrix} 0 & 1 & 1 & 1 \\ 1 & 0 & 1 & 0 \\ 1 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix} \end{array}$$

is the adjacent matrix with respect to the order of vertices a, b, c, d.

Eg. Use an adjacency matrix to represent the pseudograph shown on following figures

(i)



Adjacency matrix with ~~repres~~ respect to the ordering a, b, c, d.

$$\begin{bmatrix} 0 & 1 & 1 & 0 \\ 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 \end{bmatrix}$$

(ii)

Adjacent matrix with respect to the ordering

$$\begin{bmatrix} 0 & 3 & 0 & 2 \\ 3 & 0 & 1 & 1 \\ \ldots \end{bmatrix}$$

1. Note that ... the ordering chosen for the vertexes. Hence, there may be as many as $n!$ different adjacency matrixes for a graph with n vertices. (Note that an adjacency matrix of a graph is based on the ordering chosen ——)

So, adjacency matrix is not unique.

2. ~~The adjacency matrix of a simple~~ graph is symmetric. Moreover, the principle diagonal entries are zero because, a simple graph has no loops.

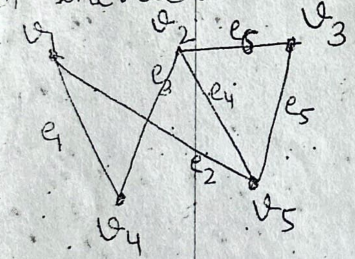
3. For simple graph, Adjacent matrix is zero-one matrix
For non-simple graph, " " is no longer zero-one matrix.

Incidence Matrix:- Another way to represent the graph.

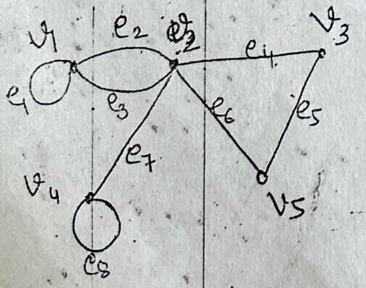
Let $G=(V,E)$ be an undirected graph. Suppose that $v_1, v_2, \dots, v_n$ are the vertices and $e_1, e_2, \dots, e_m$ are the edges of G. Then the incidence matrix with respect to this ordering of V and E is the $n \times m$ matrix

$$M = [m_{ij}] = \begin{cases} 1 & \text{when edge } e_j \text{ is incident with } v_i \\ 0 & \text{otherwise.} \end{cases}$$

Example - Represent the graph shown in following figures with an incidence matrix



$$\begin{array}{c|c} & \begin{array}{cccccc} e_1 & e_2 & e_3 & e_4 & e_5 & e_6 \end{array} \\ \begin{array}{c} v_1 \\ v_2 \\ v_3 \\ v_4 \\ v_5 \end{array} & \begin{bmatrix} 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & 1 \\ 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & 1 & 0 \end{bmatrix} \end{array}$$

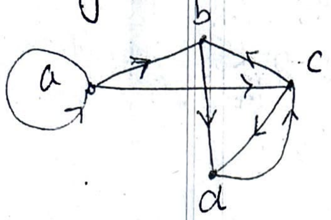


$$\begin{array}{c|c} & \begin{array}{cccccccc} e_1 & e_2 & e_3 & e_4 & e_5 & e_6 & e_7 & e_8 \end{array} \\ \begin{array}{c} v_1 \\ v_2 \\ v_3 \\ v_4 \\ v_5 \end{array} & \begin{bmatrix} 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 1 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 \end{bmatrix} \end{array}$$

Incidence matrices can also be used to represent multiple edges and loops (a) Same columns indicate multiple edges

(b) Only (Exactly) one entry equal to 1 indicates loops corresponding to the vertex.

Adjacency matrix for directed graph:



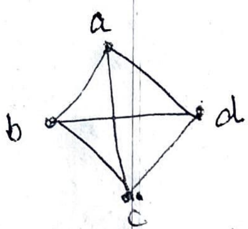
$$A = \begin{bmatrix} 1 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$ w.r.t. to ordering a, b, c, d

(In this case, Number of 1's = No's of ed

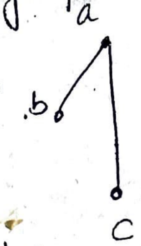
Subgraph:- A subgraph of a graph $G=(V,E)$ is a graph $H=(W,F)$, where $W \subseteq V$ and $F \subseteq E$. A subgraph H of is a proper subgraph of G if $H \neq G$.

Eg:

K_4

$G =$ 

its subgraph

$H_1 =$ 

$H_1 \neq G$

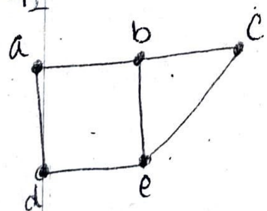
$\therefore$ proper subgraph

proper subgraph

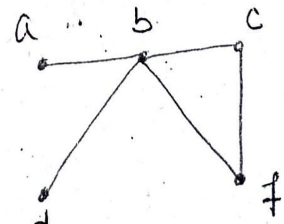
Union and intersection of Graph

The union of two simple graphs $G_1 = (V_1, E_1)$ and $G_2 = (V_2, E_2)$ is the simple graph with vertex set $V_1 \cup$ and edge set $E_1 \cup E_2$. The union of G_1 and G_2 is deno by $G_1 \cup G_2$.

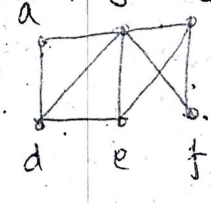
Example:-



G_1



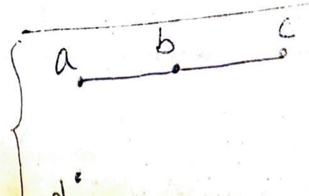
G_1



$G_1 \cup G_2$

The intersection of two simple graphs $G_1 = (V_1, E_1)$ and $G_2 = (V_2, E_2)$ is the simple graph with vertex set $V_1 \cap V_2$ and edge set $E_1 \cap E_2$. The intersection of G_1 G_2 is denoted by $G_1 \cap G_2$

Example:-

$G_1 \cap G_2 =$ 

The ~~complimentary graph~~

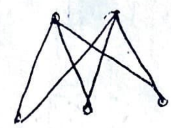
the same vertices as G. Two vertices are adjacent in $\overline{G}$ if and only if they are not adjacent in G.

(The complementary graph: $\overline{G}$ of a simple graph G has the same vertices as G. two vertices are adj. in $\overline{G}$)

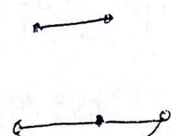
Eg: $\overline{K_n}$: The graph with n vertices and no edges

$\overline{K_{m,n}}$: The disjoint union of K_m and K_n

$K_{2,3}$:

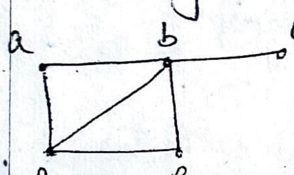


$\overline{K_{2,3}}$



Degree sequence :- A degree sequence of the graph is the sequence of the degrees of the vertices of the graph in non increasing order.

Eg:



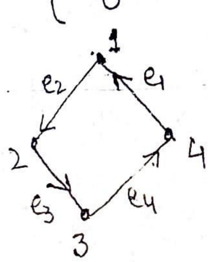
degree sequence of the graph G is $[4, 4, 4, 3, 2, 1, 0]$

Weighted graph: Graphs that have a number assigned to each edge is called weighted graph

Incidence Matrix of a directed graph :-

$$M = [m_{ij}] = \begin{cases} -1 & \text{if the ith vertex is an initial vertex} \\ +1 & \text{if the ith vertex is an terminal vertex} \\ 0 & \text{otherwise} \end{cases}$$

Eg: Graph:



$$M = \begin{array}{c} \\ 1 \\ 2 \\ 3 \\ 4 \end{array} \begin{array}{c} \begin{array}{cccc} e_1 & e_2 & e_3 & e_4 \end{array} \\ \begin{bmatrix} 1 & -1 & 0 & 0 \\ 0 & 1 & -1 & 0 \\ 0 & 0 & 1 & -1 \\ -1 & 0 & 0 & 1 \end{bmatrix} \end{array}$$

Connectivity :-

Path: A path is a sequence of edges that begins at a vertex of a graph and travels from vertex to vertex along edges of the graph.

- A path of length n from u to v in G is a sequence of n edges $e_1, e_2, \ldots e_n$ of G for which there exists a sequence $x_0 = u, x_1, x_2, \ldots, x_{n-1}, x_n = v$ of vertices such that e_i has end points x_{i-1} and x_i, for $i = 1, \ldots n$

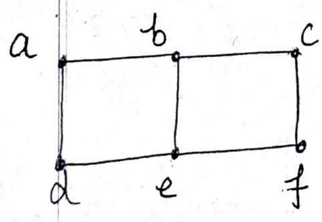
Note In a path, edges can repeat (Vertices can also repeat)

Circuit The path is circuit if it begins and ends on the same vertex, i.e., if $u = v$, and has length greater than zero.

Simple Path:- A path or circuit is simple if it does not contain the same edge more than once.

Length of path:- Total number of edges in a path.

Example:-



- Simple path of length 4: a b c f
- a d e c a is not path
- b, c, f, e, b is circuit of length
- a, b, e, d, a, b is not simple path

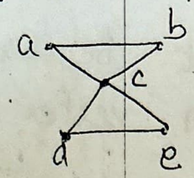
* Walk = Path
* Trail = simple path
* Circuit = closed path
* A circuit in a graph is also called as cycle in a graph if no vertex is repeated except end points.

Connected graph: An undirected graph is called connected if there is a path between every pair of distinct vertices of the graph.

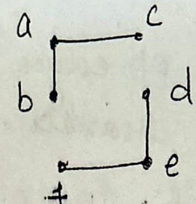
A graph that is not connected is called disconn

only if the graph of this network is connected.
(Any two computers in the network can communicate if and only if -

Eg.



Connected graph



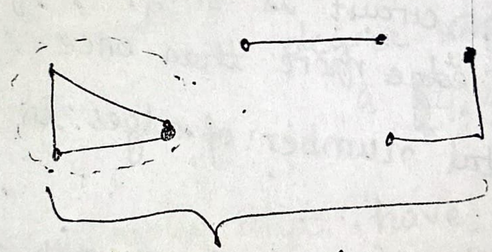
Disconnected graph (because there is no path from 'a' to 'd')

Theorem:-
There is a simple path between every pair of distinct vertices of a connected undirected graph.

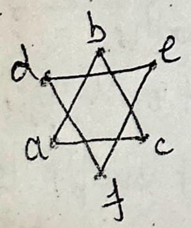
Connected components:- A connected component of a graph G is a connected subgraph of G that is not a proper subgraph another connected subgraph of G.

A graph G that is not connected has two or more connected components that are disjoint and have G as their union.

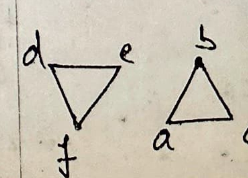
Eg:



H graph



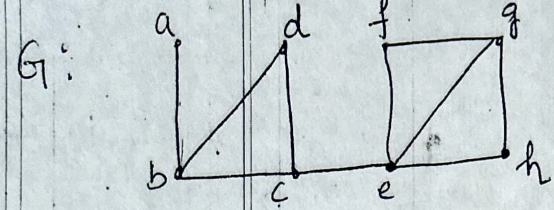
→ This is, disconnected graph and connected components are



Cut vertex:-
A single vertex whose removal disconnects a graph is called a cut vertex. It is also called articulation points

Cut edge:- An edge 'e' ∈ G is called a cut edge if its removal disconnects a graph.
It is also called as bridge.

Example: Find the cut vertices and cut edges in the graph G.



Cut vertices: b, c & e

Cut edges: {a,b} and {c,e

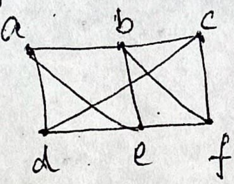
Remark:- Not all graphs have cut vertices; for eg: Complete graph K_n, where $n \geq 3$, has no cut vertices.

optional

* **Non-separable graphs:-** Connected graph without cut ver[rat]

Vertex cut or separating set:- A vertex subset V' of $G=(V,E)$ is a vertex cut or separating set, if $G - V'$ is dis conn [disconnecte]

Eg: In ~~above~~ following figure, the set {b,c,e} is a vertex cut.



Vertex connectivity:- The vertex connectivity of a noncom[plete] [noncomplete] graph G, denoted by $K(G)$ [Kappa of G], as the minimu[m] number of vertices in a vertex ~~set~~ cut.

Edge cut:- A set of edge E' is called an edge cut of G if the subgraph $G - E'$ is disconnected.

Edge connectivity:- The edge connectivity of a graph G denoted by $\lambda(G)$ ~~if the subgraph G-E' is disconnected~~ is the minimum number of edges in an edge cut of

Inequality for vertex connectivity and edge connectivity.

$$K(G) \leq \lambda(G) \leq \min_{v \in V} \deg(v)$$

Connectivity -

connectedness in Directed graphs

f^{n}:- A directed graph is **strongly connected** if there is a path from 'a' to 'b' and from 'b to 'a' whenever 'a' and 'b' are vertices in the graph (every pair)

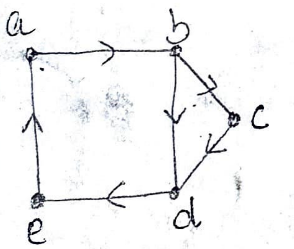
^{n}:- A directed graph is weakly connected if there is a path between every two vertices in the underlying undirected graph.

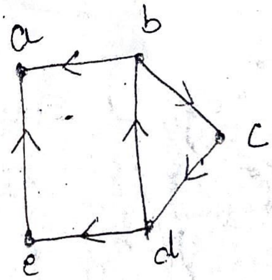
i.e., a directed graph is weakly connected if and only if there is always a path between two vertices when the directions of the edges are disregarded.

& clearly, any strongly connected directed graph is also weakly connected.

Are the directed graphs G and H strongly connected?

Are they weakly connected?

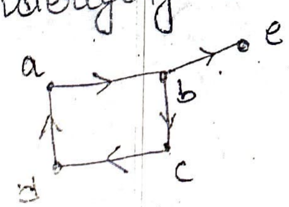




H

G

G is strongly connected. Hence, G is also weakly connected. H is not strongly connected because no directed path from a to b in this graph. However, H is weakly connected, because there is a path between any two vertices in the underlying undirected graph of H.



Not strongly connected because no path from e to b.