Comparable & Incomparable :- The elements a & b of the Poset $(S, \leq)$ are called comparable if either $a \leq b$ or $b \leq a$.

When a & b are the elements of S such that neither $a \leq b$ nor $b \leq a$ then it is called as incomparable. Represented by $\boxed{a/b = \frac{a}{b}}$ & $\boxed{a|b = \frac{b}{a}}$.

Example, In the Poset $(Z^+, |)$ are the integers

(i) 3 & 9 comparable? → Yes

(ii) 5 & 7 comparable? → No

Totally ordered :- If $(\overset{\text{set}}{S}, \overset{\text{Relation}}{\leq})$ Poset & every 2 elements of S are comparable then S is called as totally ordered / linearly ordered set. A totally ordered set is also called a $\boxed{\text{chain}}$.

Types of Relation

Here $aRa \Rightarrow (a,a) \in R$
$a \not R a \Rightarrow (a,a) \notin R$

Reflexive if $aRa \ \forall a \in A$

Symmetric if aRb, then bRa; $\forall a, b \in A$

transitive if aRb and bRc, then $aRc \ \forall a, b, c \in R$

Irreflexive if $a \not R a \ \forall a \in A$

Antisymmetric if aRb and bRa, then $a = b$
OR
aRb, then $b \not R a$, unless $a = b$

Not reflexive if $a \not R a$ for some $a \in A$

Asymmetric if $b \not R a$ when aRb for some $a, b \in R$

~~$A = \{1, 2, 3, 4\}$~~
~~$R = \{(a,b) \mid a \mid b\}$~~

Counting no. of relations

- No. of relations on set A with n elements $= 2^{n^2}$
- → No. of reflexive relation $= 2^{n(n-1)}$
- → " " Symmetric relation $= 2^{\frac{n(n+1)}{2}}$
- → " " Antisymmetric " $= 2^n \cdot 3^{\frac{n(n-1)}{2}}$
- → " " Asymmetric " $= 3^{\frac{n(n-1)}{2}}$
- → " " irreflexive " $= 2^{n(n-1)}$

If $|A| = m$, $|B| = n$, then
total no of relation from A to B $= 2^{m \times n}$

Recurrence Relation (Unit-2)

Q: Find the coefficient of x^{10}, in the power series of ~~these~~ following function

(a) $(1+x^5+x^{10}+x^{15}+\cdots)^3$

Ans

$$G(x) = (1+x^5+x^{10}+x^{15}+\cdots)^3$$

$$= \left(\frac{1}{1-x^5}\right)^3 \quad \{\because \text{ G.P with } a=1,\ r=x^5\}$$

$$= \frac{1}{(1-x^5)^3} = (1-x^5)^{-3}$$

$$= 1+3x^5+\frac{(-3)(-4)}{2!}(x^5)^2+\frac{(-3)(-4)(-5)}{6!}(x^5)^3+\cdots$$

{using binomial

$$(1+x)^n = 1+nx+\frac{n(n-1)}{2!}x^2+\frac{n(n-1)(n-2)}{3!}x^3+\cdots$$

$$= 1+3x^5+\frac{(-3)(-4)}{2}x^{10}+\frac{(-3)(-4)(-5)}{6!}x^{15}$$

Now Coefficient of $x^{10} = \frac{(-3)(\cancel{-4})^{2}}{\cancel{2}} = \underline{\underline{6}}$.

b) $(x^3+x^4+x^5+x^6+\cdots)^3$

$$= \left(\frac{x^3}{1-x}\right)^3 \quad \{\text{G.P with } a=x^3,\ r=x\}$$

$$= \frac{x^9}{(1-x)^3} = x^9(1-x)^{-3} = x^9(1+3x+6x^2+\cdots) = x^9+3x^{10}+6x^{11}+\cdots$$

Now Coeff of $x^{10} = \underline{\underline{3}}$.