Isomorphism:-

Two simple graphs are isomorphic, if there is a one to one correspondence between vertices of the two graphs that preserves the adjacency relations

#. For isomorphism first check
- no. of vertices
- no. of edges
- degree sequence

b/w two vertices are same?

If yes then build mapping and corresponding adjac matrices.

Examples:

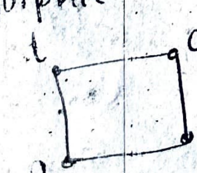
Non isomorphic ∵ vertices not sa

Non isomorphic ∵ edges count not same

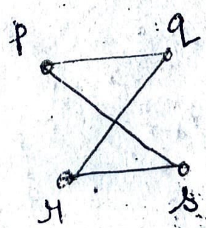
Non isomorphic ∵ degree seque not equal.

Eg. Show that the graphs $G_1=(V,E)$ and $H=(W,F)$ are isomorphic

G:

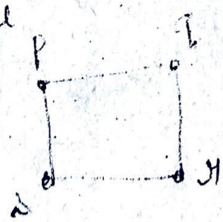


H



Sol^n:- Reconstruct the complicated figure in form of sim figure

H:



Now mapping

$f(a) = s$

$f(b) = r$

$f(c) = q$

$f(d) = p$

$$A_G: \begin{array}{c|cccc} & a & b & c & d \\ \hline a & 0 & 1 & 0 & 1 \\ b & 1 & 0 & 1 & 0 \\ c & 0 & 1 & 0 & 1 \\ d & 1 & 0 & 1 & 0 \end{array}$$

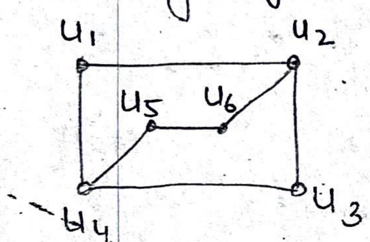
$$A_H \begin{array}{c|ccc} r & 0 & 1 & \cancel{1} \\ s & & & \\ q & & & \\ p & & & \end{array}$$

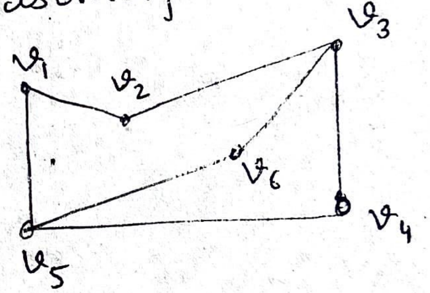
w.r.t. to ordering
a, b, c, d

$$H: \begin{array}{c|cccc} & s & r & q & p \\ \hline s & 0 & 1 & 0 & 1 \\ r & 1 & 0 & 1 & 0 \\ q & 0 & 1 & 0 & 1 \\ p & 1 & 0 & 1 & 0 \end{array}$$

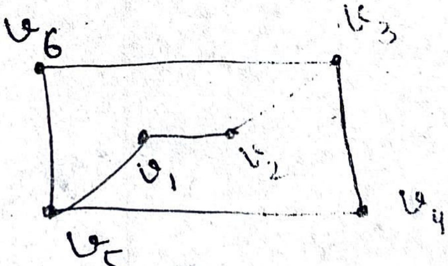
Both the adjacency matrices are same. ∴ the obtained mapping proves that G and H are isomorphic.

Example: Determine whether the graphs G and H displayed in following figures are isomorphic





Solution



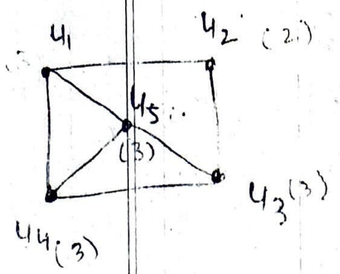
$f(u_1) = v_6$
$f(u_2) = v_3$
$f(u_3) = v_4$
$f(u_4) = v_5$
$f(u_5) = v_1$
$f(u_6) = v_2$

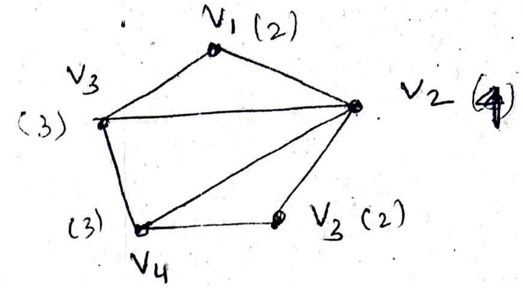
$$\begin{array}{c|cccccc} & u_1 & u_2 & u_3 & u_4 & u_5 & u_6 \\ \hline u_1 & 0 & 1 & 0 & 1 & 0 & 0 \\ u_2 & 1 & 0 & 1 & 0 & 0 & 1 \\ u_3 & 0 & 1 & 0 & 1 & 0 & 0 \\ u_4 & 1 & 0 & 1 & 0 & 1 & 0 \\ u_5 & 0 & 0 & 0 & 1 & 0 & 1 \\ u_6 & 0 & 1 & 0 & 0 & 1 & 0 \end{array}$$

62, 64
3, 8, 11, 15, 19, 20 ...

49, 68, (...

Eg 36



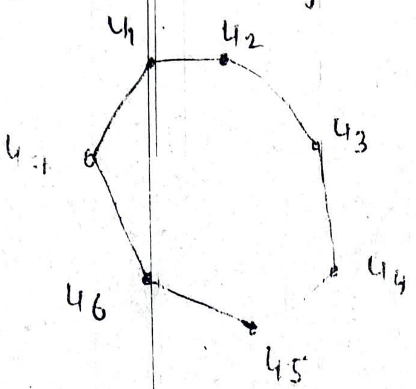


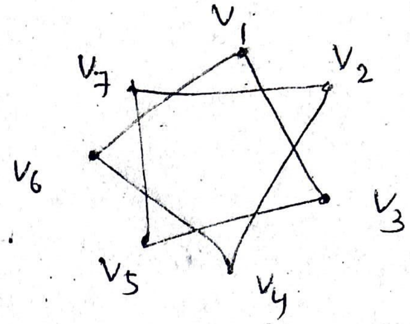
Re~~construct U~~ in form of V

degree sequence is not so

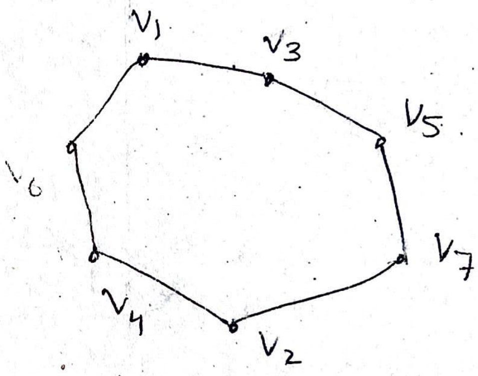
∴ Not an isomorph graphs

Eg 37





Reconstruct V in form of U



So mapping is

$f(u_1) = V_1$

$f(u_2) = V_3$

$f(u_3) = V_5$

$f(u_4) = V_7$

$f(u_5) = V_2$

$f(u_6) = V_4$

$f(u_7) = V_6$

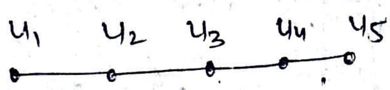
~~$f(u_8) =$~~

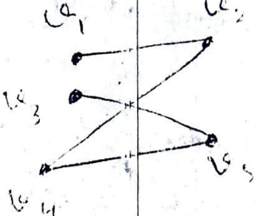
Construct their adjacency ma and show that they are equal according to ordering of mapp which further proofs that graphs are isomorphic.

$$\begin{array}{c|ccccc}
v_3 & 1 & & 1 & & 1 \\
v_4 & & 1 & & 1 & \\
v_5 & 1 & & 1 & & 1 \\
v_1 & & & & 1 & 1 \\
v_2 & & 1 & & & 1
\end{array}$$

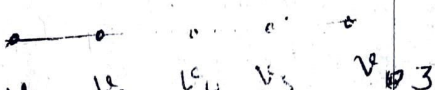
Both the adjacent matrices are same. Thus; defined mapping shows that graphs are isomorphic

Example: 34





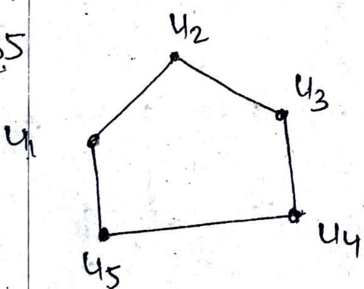
Reconstruct V graph in form of U

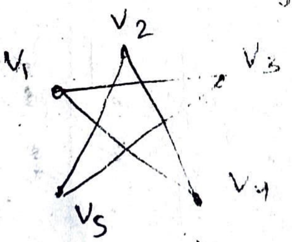


∴ Mapping is
$f(u_1) = v_1$
$f(u_2) = v_2$
$f(u_3) = v_4$
$f(u_4) = v_5$
$f(u_5) = v_3$

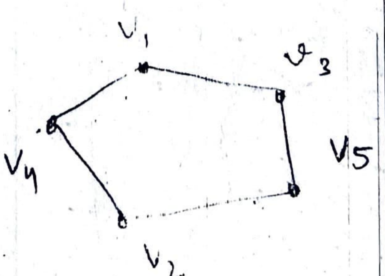
Now check its adjacency matrices are equal?

Example 35





∴ reconstruct like U



∴ Mapping is.
$f(u_1) = v_4$
$f(u_2) = v_1$
$f(u_3) = v_3$
$f(u_4) = v_5$
$f(u_5) = v_2$

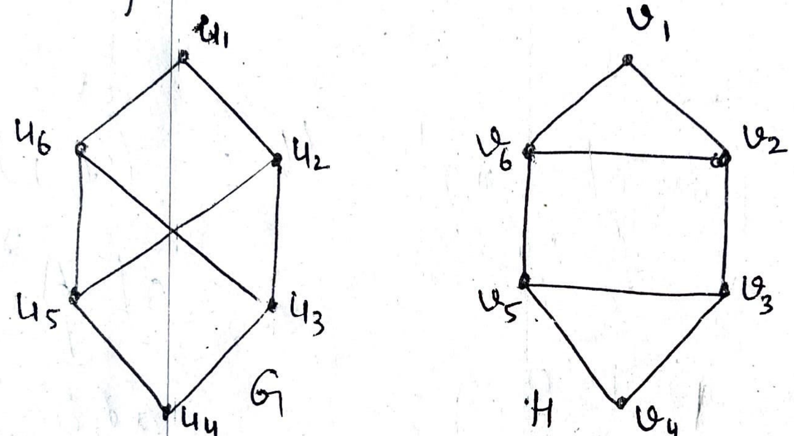
Now Draw their adjacency matrices.

Paths and Isomorphism

Paths and circuits can help determine whether tu graphs are isomorphic.
For eg, the existence of a simple circuit of a particul length is a useful invariant that can be used to show that two graphs are not isomorphic.

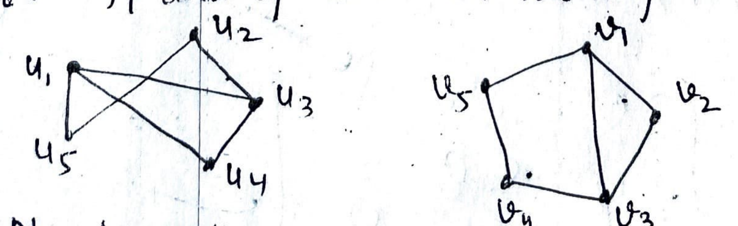
⊙ A useful isomorphic invariant for simple graphs is the existence of a simple circuit of length k, where k is a positive integer greater than 2.

Example:- Determine whether the graph G and H are isomorphic?



Solⁿ: Both G and H have six vertices and eight edges.
Degree sequence $(3,3,3,3,2,2)$ is same.
So three invariants - no. of vertices, edges & degree sequence agrees for the two graphs.
However, H has a simple circuit of length three, $v_1 v_2 v_6 v_1$ whereas G has no simple circuit of length three.
∴ H and G are not isomorphic.

Ex2:



Solⁿ: No. of vertices - 5 in G & H
No of edges - 6 in G as well as H
Degree seqⁿ of G : $[3,3,2,2,2]$ same as H
Both have a simple circuit of length 3, 4 and 5.

Dijkstra's Algorithm :-

It is used to find the shortest path between two vertices in a connected weighted simple graph.

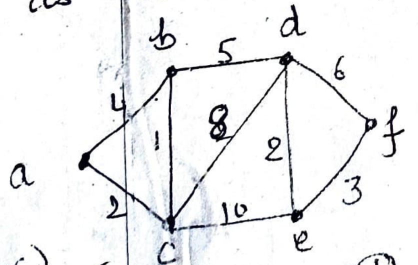
Remark:- ① All edges weights must be non-negative
② Remove self loop and parallel edge
③ Applicable on weighted graph

Note:- If source vertex is given then start from that vertex, otherwise, start from any vertex.

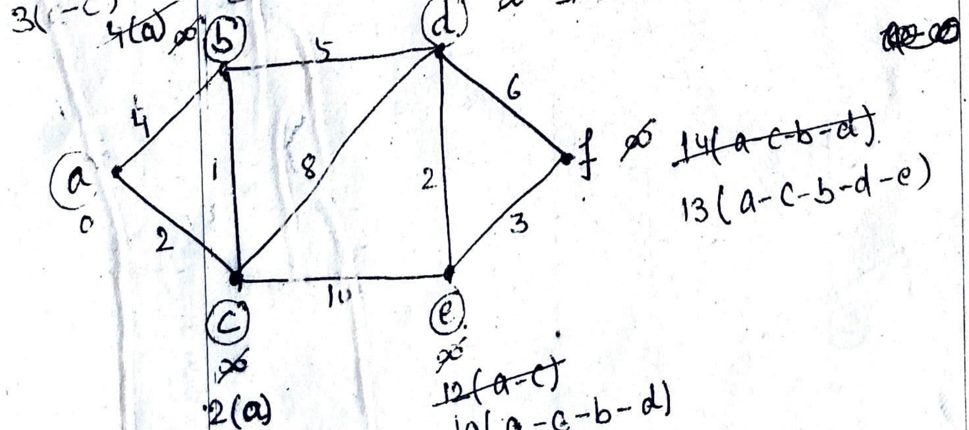
Procedure:-

1. Begins by labelling '0' to source vertex and '∞' to other vertices.
2. Calculate the distance (d) of adjacent vertices and update it '∞' to 'd'.
3. Visit to vertex with smallest distance and mark it as current vertex and repeat step 2.

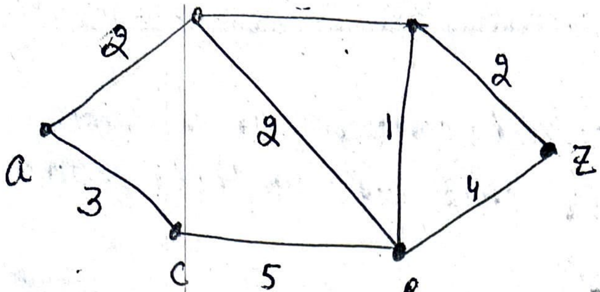
Example:- find shortest path b/w vertices a and f.



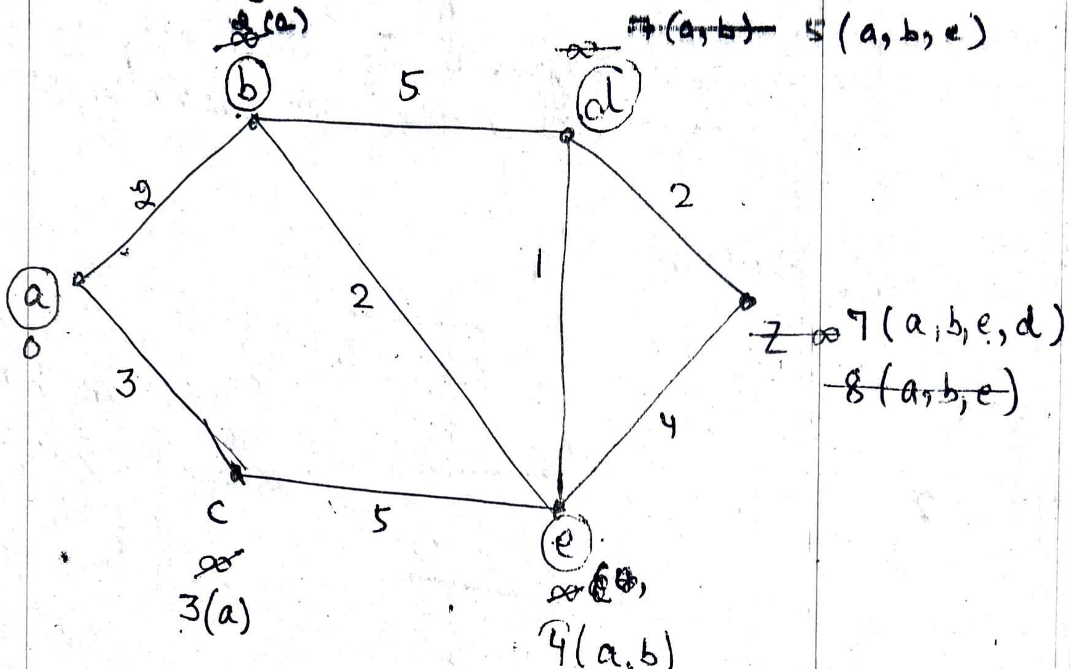
Sol^n:



The shortest path from a to z is a-c-b-d-e-z with length 13.



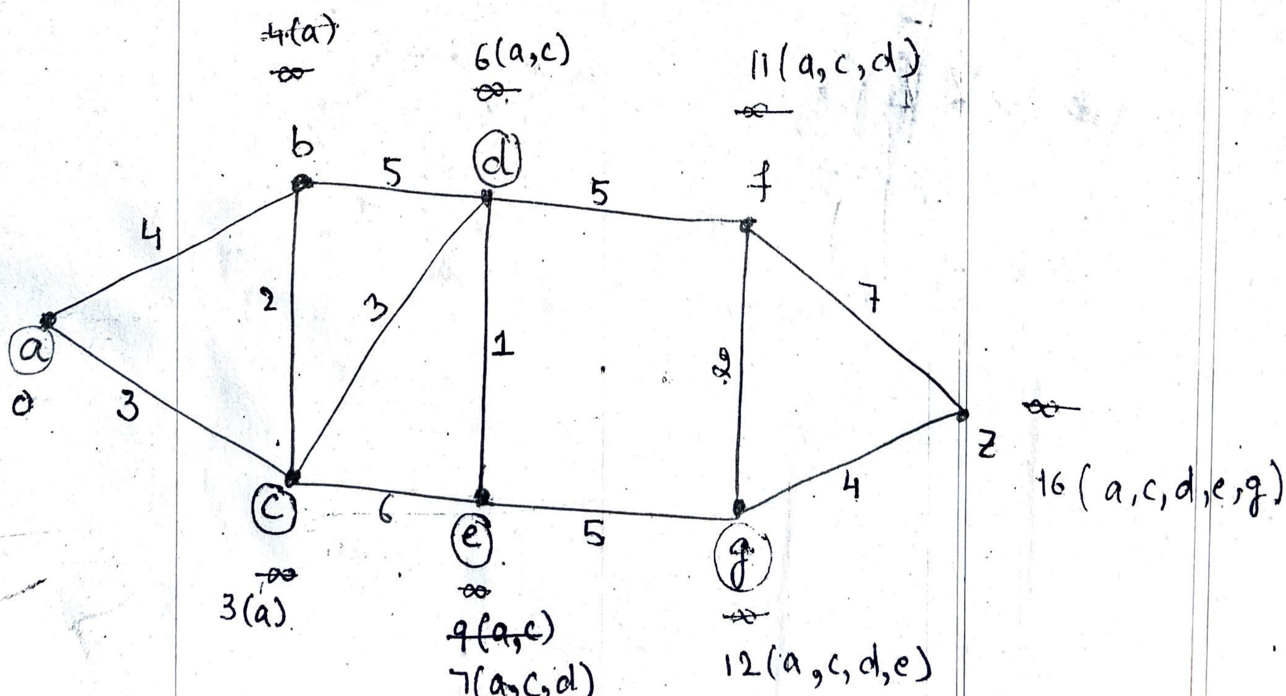
Sol^n



Shortest path from a to z is

a - b - e - d - z with weight 7

Example 3.



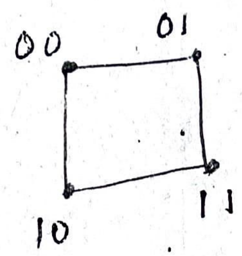
n-Cubes :- An n-dimensional hypercube or n-cube, denoted by Q_n, is a graph that has vertices representi[ng] the 2^n bit strings of length n.

Two vertices are adjacent if and only if the bit strin[gs] that they represent differ in exactly one bit positi[on].

Eg :- Q_n, $n = 1, 2, 3$

$n=1$ $\quad Q_1$:

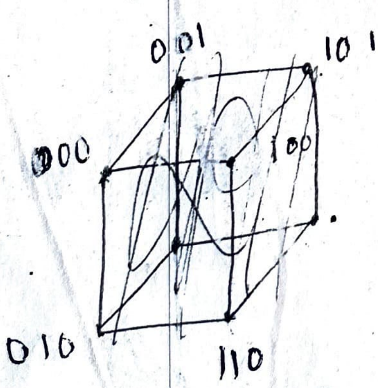
$n=2$ $\quad Q_2$

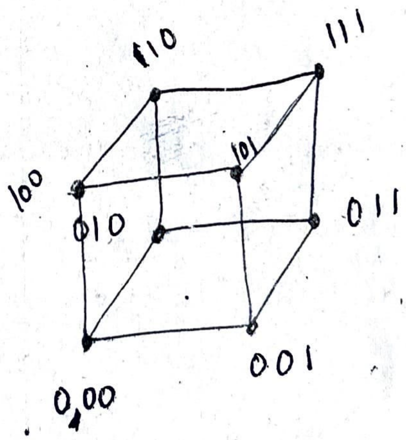


so $\Leftarrow$ Two bits possibilit[ies]

00 → diff is 1
01] diff is not 1
10
11
diff. is 1

$n=3$. $\quad Q_3$ possibilities
000, 001, 010, 100, 011, 110, 101, 111



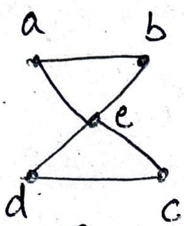


zyx

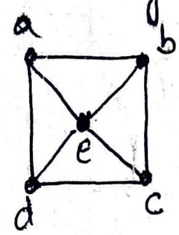
Euler path: An Euler path in G is a simple path containing every edge of G (No repeated edge).

Euler circuit:- An Euler circuit in a graph G is a simple circuit containing every edge of G.

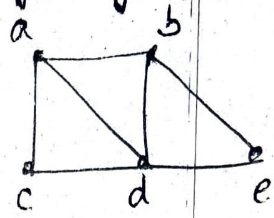
Example:-



G_1



G_2



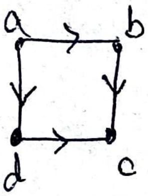
G_3

Solution:

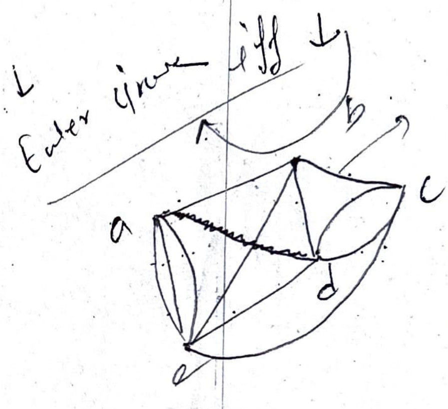
Euler path: — | — | a c d e b d a b

Euler circuit: a e c d e b a | — |

Example:



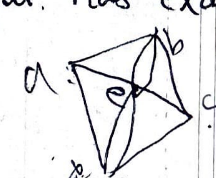
No Euler path
& No Euler circuit



Necessary and sufficient conditions for Euler circuits and paths.

Theorem:- A connected multigraph with atleast two vertices has an Euler circuit if and only if each of its vertices has even degree.

Theorem:- A connected multigraph has an Euler path but not an Euler circuit if and only if it has exactly two vertices of odd degree.

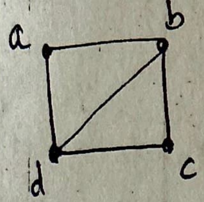


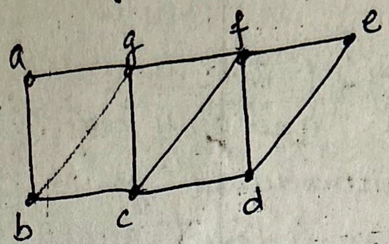
Remark:

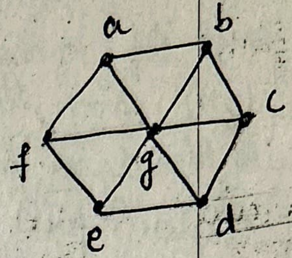
Note:- A graph can't have both Euler paths and Euler circuits

Eulerian graph: A graph which contain Euler circuit is called Eulerian graph.

Q. Which of the following graphs have an Euler path and Euler circuit?







G_1

Solution:- G_1 contains exactly two vertices of odd degree, namely 'b' and 'd'. Hence it has an Euler path that must have 'b' and 'd' as end points.

One such Euler path is d a b c d b.

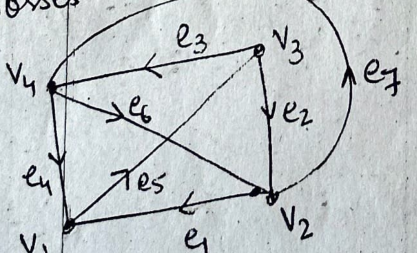
Similarily, G_2 has exactly two vertices of odd degree, na[...] b and d. So it has an Euler path that must ha[...] b and d as endpoints. One such Euler path is

b, a, g, f, e, d, c, f, d.

G_3 has no Euler path because it has six vertices of od[...] degree.

G_1, G_2 and G_3 doesn't have Euler circuit

Q Does the graph given below possesses an Euler circuit, [...] it posses then write the Euler circuit.



Sol^n: Total degree of vertice[...]

$deg(v_1) = 3$

$deg(v_2) = 4$

$deg(v_3) = 3$

$deg(v_4) = 4$

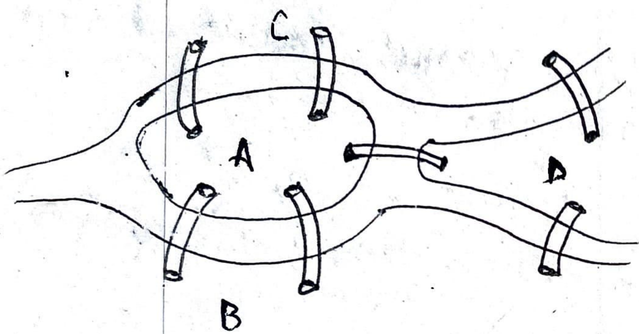
It cannot have Euler circuit because it has exactl[...] two vertices of odd degree. But it has Euler path i[...] given by

$v_3\ e_2\ v_2\ e_7\ v_4\ e_6\ v_2\ e_1\ v_1\ e_4\ v_4\ e_3\ v_3\ e_5\ v_1$.

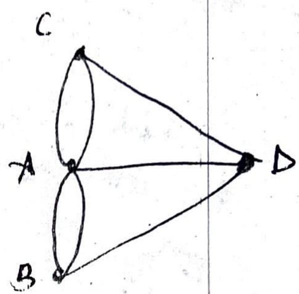
Seven bridges of Konigsberg

There was 7 bridges connecting 4 lands around the city of Konigsberg in Prussia. Was there any way to start from any of the land and go through each of the bridges once and only once?

Sol^n: Euler first introduced graph theory to solve this problem. He considered each of the lands as a node of a graph and each bridge in between as an edge in between. Now he calculated if there is any eulerian path in that graph then there is a solution otherwise not.



Seven bridges of Konigsberg



Multigraph Model of problem

Hamilton Paths and Circuit

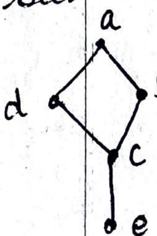
Hamilton Path:- A simple path in a graph G that passes t every vertex exactly once is called Hamilt path.

Hamilton circuit:- A simple circuit in a graph G that pa through every vertex exactly once is called Hamil circuit.

Hamiltonian graph:- A graph contains Hamiltonian circ

Important Points:-

1. A graph can have both Hamilton path as well as Ha circuit.
2. If a graph has Hamilton circuit then it also has Ha path but converse is not true.

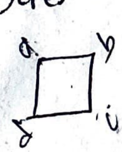


Hamilton path is e, c, d, a, b
But not having Hamilton circuit.

3. Only a connected graph can have Hamilton circuit / Pat
4. A graph with a vertex of degree one cannot have a H.C.

Conditions for the existence of Hamilton circuit

1. DIRAC's Theorem:- If G is a simple graph with n vertic with $n \geq 3$ such that the degree of every vertex in G is at least $\frac{n}{2}$, then G has a Hamilton circuit.



2. ORE'S Theorem:- If G is a simple graph with n vertices with $n \geq 3$ such that $\deg(u) + \deg(v) \geq n$ for every pai of nonadjacent vertices u and v in G, then G has a Hamilton circuit.

$2+2 \geq 4$

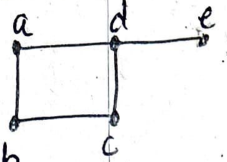
$4 \geq 4$

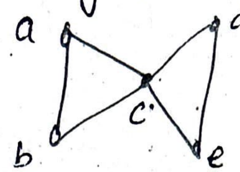
Example:

5. A Hamilton circuit cannot contain a smaller circuit wit it.

Doubt

ample:- Show that neither graph displayed

G: 

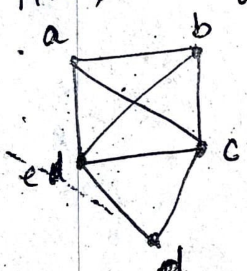
H: 

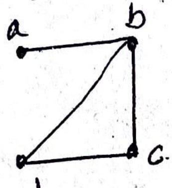
Solution:- There is no Hamilton circuit in G because G has a vertex of degree one.

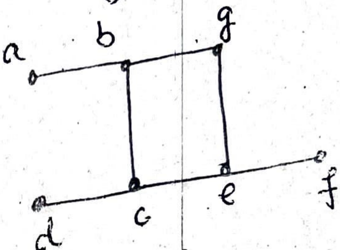
~~In H Graph,~~ ~~a smallest circuit with in it exist Therefore,~~ ~~H has no H.C.~~

In H, degrees of the vertices a, b, d, and e are all two, every edge incident with these vertices must be part of any Hamilton circuit. It is now easy to see that no H.C. can exist in H, for any H.C would have to contain four edges incident with c, which is impossible.

Which of the simple graph in following figure have a H.C, or if not, a H.P?







Sol^n: G_1 - has a H.C. a b c d e a

G_2: No H.C. (this can be seen by noting that any circuit containing every vertex must contain the edge $\{a,b\}$ twice), but it does have a H.P, namely a b c d.

G_3: G_3 has neithe H.C. nor H.P. because any path containing all vertices must contain one of the edge $\{a,b\}$, $\{e,f\}$ and $\{c,d\}$ more than once.

1, 8, ~~21~~, ~~20~~, 30, 55, 60, 65, 66, 73