

$$\begin{aligned}
 \# \text{ If } \frac{1}{\Delta^2} n^{(1)} &= \frac{1}{\Delta} \left( \frac{1}{\Delta} n^{(1)} \right) = \frac{1}{\Delta} \frac{(n^{(2)})}{2} \quad \left\{ \because \frac{1}{\Delta} \text{ as integration} \right. \\
 &\quad \left. \text{i.e. } \frac{1}{\Delta} (n^{(m)}) = \frac{n^{(m+1)}}{m+1} \right. \\
 &= \frac{n^{(3)}}{6} = \frac{n(n-1)(n-2)}{6} = \frac{(n^2-n)(n-2)}{6} = \frac{n^3 - 2n^2 - n^2 + 2n}{6} = \frac{n^3 - 3n^2 + 2n}{6}
 \end{aligned}$$

$$\# \frac{1}{\Delta} n^{(1)} = \frac{n^{(2)}}{2} = \frac{n(n-1)}{2} = \frac{n^2 - n}{2}$$

$$\# \Delta(n^{(1)}) = 1, \quad \Delta(n^{(2)}) = 2n^{(1)} \quad \left( \text{as differentiation} \right. \\
 \left. \text{i.e. } \Delta(n^{(m)}) = mn^{(m-1)} \right)$$

Generating function.

If  $G(x) = \frac{1}{(1-2x)(1-3x)}$ , the sequence, will be evaluated by partial fraction.

$$\text{i.e. } \frac{1}{(1-2x)(1-3x)} = \frac{A}{1-2x} + \frac{B}{1-3x} \quad (*)$$

$$= \frac{(1-3x)A + B(1-2x)}{(1-2x)(1-3x)}$$

$$\frac{1}{(1-2x)(1-3x)} = \frac{A - 3xA + B - 2Bx}{(1-2x)(1-3x)}$$

$$\Rightarrow 1 = (A+B) - 3Ax - 2Bx$$

$$\text{Equating Coeff. of } x \quad -3A - 2B = 0 \Rightarrow 3A + 2B = 0 \quad (1)$$

$$\text{(Const.)} \quad A + B = 1 \Rightarrow B = 1 - A \quad (2)$$

using in (1)

$$3A + 2(1-A) = 0 \Rightarrow 3A + 2 - 2A = 0$$

$$\Rightarrow A + 2 = 0 \Rightarrow A = -2$$

$\therefore$  from (2)

$$B = 1 - (-2) = 1 + 2 = 3$$

$$\therefore G(x) = -\frac{1}{2} \left( \frac{1}{1-2x} \right) + \frac{1}{3} \left( \frac{1}{1-3x} \right) \quad \{ \text{using } (*) \}$$

$$\Rightarrow \frac{G(x)}{a_n} = -\frac{1}{2}(2)^n + \frac{1}{3}(3)^n \quad \left\{ \because \text{for } \frac{1}{1-kx} \quad G(x) = \frac{1}{1-kx}; a_n = k^n \right\}$$

$$\Rightarrow a_n = \frac{1}{3}(3)^n - \frac{1}{2}(2)^n$$

$$\boxed{a_n = 3^{n-1} - 2^{n-1}}$$

\* If  $G(x) = \frac{1}{x^2 - 5x + 6}$  (factorise the term)

$$= \frac{1}{(x-3)(x-2)} \quad , \text{ then same as above}$$

\* If in numerator some term is there, then also same procedure.