```java
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import java.util.*;

class Program {
  // O(n^2) time | O(n) space
  public static int numberOfBinaryTreeTopologies(int n) {
    List<Integer> cache = new ArrayList<Integer>();
    cache.add(1);
    for (int m = 1; m < n + 1; m++) {
      int numberOfTrees = 0;
      for (int leftTreeSize = 0; leftTreeSize < m; leftTreeSize++) {
        int rightTreeSize = m - 1 - leftTreeSize;
        int numberOfLeftTrees = cache.get(leftTreeSize);
        int numberOfRightTrees = cache.get(rightTreeSize);
        numberOfTrees += numberOfLeftTrees * numberOfRightTrees;
      }
      cache.add(numberOfTrees);
    }
    return cache.get(n);
  }
}

```