```go
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package main

// O(n^2) time | O(n) space
func NumberOfBinaryTreeTopologies(n int) int {
  return helper(n, map[int]int{0: 1})
}

func helper(n int, cache map[int]int) int {
  if val, found := cache[n]; found {
    return val
  }
  numberOfTrees := 0
  for leftTreeSize := 0; leftTreeSize < n; leftTreeSize++ {
    rightTreeSize := n - 1 - leftTreeSize
    numberOfLeftTrees := helper(leftTreeSize, cache)
    numberOfRightTrees := helper(rightTreeSize, cache)
    numberOfTrees += numberOfLeftTrees * numberOfRightTrees
  }
  cache[n] = numberOfTrees
  return numberOfTrees
}

```