1. 已知函数表

x_i	0.2	0.3	0.4
$f(x_i)$	0.04	0.09	0.16

则一阶差商 $f[0.2,0.4]$ 为多少？

$$f[0.2,0.4] = \frac{0.16-0.04}{0.4-0.2} = 0.6$$

2. 设 $f(0) = 0$，$f(1) = 16$，$f(2) = 46$，则差商 $f[0,1]$、$f[0,1,2]$ 为多 $f(x)$ 的二次牛顿插值多项式怎样表达？

$$f[0,1] = \frac{16-0}{1-0} = 16 \qquad f[0,1,2] = \frac{f[0,2]-f[0,1]}{2-1} = \frac{\dfrac{46-0}{2-0} - \dfrac{16-0}{1-0}}{2-1} = 7$$

或： $$f[0,1,2] = \frac{f[1,2]-f[0,1]}{2-0} = \frac{\dfrac{46-16}{2-1} - \dfrac{16-0}{1-0}}{2-0} = 7$$

$$N_2(x) = f(x_0) + \sum_{k=1}^{2} f[x_0,x_1,...,x_k] \prod_{j=0}^{k-1} (x-x_j)$$

$$= 0 + f[0,1](x-0) + f[0,1,2](x-0)(x-1) = 16x + 7x(x-1)$$
$$= 7x^2 + 9x$$

3. 设函数 $f(x) = \dfrac{1}{1+x^2}$，试写它在插值节点组 $\{-1,0,1\}$ 上的插值多项式

用它计算 $x = \pm\dfrac{1}{3}$ 处之值。

X	-1	0	1
y	1/2	1	1/2

$$L_2(x) = \sum_{j=0}^{2} f(x_j)l_j(x) = f(x_0)\frac{(x-x_1)(x-x_2)}{(x_0-x_1)(x_0-x_2)}$$

$$+ f(x_1)\frac{(x-x_0)(x-x_2)}{(x_1-x_0)(x_1-x_2)} + f(x_2)\frac{(x-x_0)(x-x_1)}{(x_2-x_0)(x_2-x_1)}$$

$$= \frac{1}{2}\frac{x(x-1)}{-1\cdot-2} + 1\cdot\frac{(x+1)(x-1)}{1\cdot-1} + \frac{1}{2}\frac{(x+1)x}{2\cdot1}$$

$$L_2(\pm\frac{1}{3}) = 1 - \frac{1}{2}\cdot\frac{1}{9} = \frac{17}{18}$$

$$= \frac{1}{4}(x^2 - x - 4x^2 + 4 + x^2 + x) = 1 - \frac{1}{2}x^2$$

对比：$f(\pm\frac{1}{3}) = \dfrac{9}{10}$

4. 令 $x_0 = 0$，$x_1 = 1$，写出 $y(x) = e^{-x}$ 的线性插值多项式 $L_1(x)$，并估计误差。

$$L_1(x) = \sum_{j=0}^{1} f(x_j) l_j(x) = f(x_0) \frac{(x - x_1)}{(x_0 - x_1)} + f(x_1) \frac{(x - x_0)}{(x_1 - x_0)}$$

$$= 1 \cdot \frac{x - 1}{-1} + \frac{1}{e} \cdot \frac{x - 0}{1 - 0} = 1 - x + \frac{x}{e}$$

$$E(x) = \frac{f^{(n+1)}(\xi)}{(n+1)!} p_{n+1}(x) = \frac{1}{2} e^{-x} (x - 0)(x - 1) = \frac{1}{2} e^{-x} (x^2 - x)$$

$$|E(x)| \leq \frac{1}{2} \cdot \frac{1}{4} = \frac{1}{8}$$

8. 如下列表函数：

x_i	0	1	2	3	4
$f(x_i)$	3	6	11	18	27

试计算此列表函数的差分表,并利用牛顿前插插值公式给出它的插值多项式。

f(x)	1	2	3	4
3	3	2	0	0
6	5	2	0	
11	7	2		
18	9			
27				

$$N_4(x+th) = \sum_{k=0}^{4} \frac{\Delta^k f_0}{k!} \prod_{j=0}^{k-1}(t-j) = 3 + 3t + \frac{2}{2!}(t-1)t = t^2 + 2t + 3$$

9. 设有实验数据如下：

x	0	1	2	3	4
f	1.1	1.9	3.1	3.9	4.9

要求按最小二乘法拟合上述数据。经验公式为 $S(x) = a_0 + a_1 x$。

$$(\varphi_0, \varphi_0) = \sum_{i=0}^{4} \omega_i = 5 \qquad (\varphi_0, \varphi_1) = (\varphi_1, \varphi_0) = \sum_{i=0}^{4} \omega_i x_i = 10$$

$$(\varphi_1, \varphi_1) = \sum_{i=0}^{4} \omega_i x_2^2 = 30 \qquad (f, \varphi_0) = \sum_{i=0}^{4} \omega_i y_i = 14.9$$

$$(f, \varphi_1) = \sum_{i=0}^{4} \omega_i x_i y_i = 1.9 + 6.2 + 11.7 = 19.6 = 39.4$$

$$\begin{pmatrix} 5 & 10 \\ 10 & 30 \end{pmatrix} \begin{pmatrix} a_0 \\ a_1 \end{pmatrix} = \begin{pmatrix} 14.9 \\ 39.4 \end{pmatrix} \qquad a_0 = 1.06, \ a_1 = 0.96$$

$$S(x) = 1.06 + 0.96x$$

x	0.125	0.250	0.375	0.500	0.625	0.750
$f(x)$	0.79618	0.77334	0.74371	0.70413	0.65632	0.60228

14. 用上题数据计算 $f(0.385)$：

(1) 取 $x_0 = 0.250$，用二次 Newton 前插公式；

(2) 取 $x_0 = 0.500$，用二次 Newton 后插公式；

二者计算结果是否相同？为什么？

f(x)	1	2
0.77334	-0.02963	-0.00995
0.74371	-0.03958	
0.70413		

h=0.125

$$N_2(x+th) = f_0 + \sum_{k=1}^{2} \frac{\Delta^k f_0}{k!} \prod_{j=0}^{k-1}(t-j) = f_0 + \frac{\Delta f_0}{1!}t + \frac{\Delta^2 f_0}{2!}t(t-1)$$

$$= 0.77334 - 0.02963t - 0.00995/2 \cdot t(t-1)$$

t=0.135/0.125=1.08
0.77334-0.02963*1.08-0.00995/2*1.08*(1.08-1)= 0.74091

$$N_2(x+th) = f_n + \sum_{k=1}^{2} \frac{\nabla^k f_n}{k!} \prod_{j=0}^{k-1}(t+j) = f_n + \frac{\nabla f_n}{1!}t + \frac{\nabla^2 f_n}{2!}t(t+1)$$

$$= 0.70413 - 0.03958t - 0.00995/2 \cdot t(t+1)$$

t=-0.115/0.125=-0.92

0.70413+0.03958*0.92+0.00995/2*0.92*(-0.92+1) = 0.74091

基本要求

- 拉格朗日插值法计算；
- 一次和二次分段拉格朗日插值法计算；
- 差商的计算，牛顿插值法计算；
- 最小二乘法数据拟合的法方程组方法。