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A heuristic algorithm for solving hazardous materials distribution problems

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Abstract

A type of decision of major importance that directly affects the performance of a distribution system is the routing and scheduling of delivery trucks. The determination of hazardous materials distribution routes can be defined as a bi-objective vehicle routing problem with time windows since risk minimization accompanies the cost minimization in the objective function. The objective of this paper is to present a new heuristic algorithm for solving the bi-objective vehicle routing and scheduling problem. The proposed algorithm has been applied to several benchmark problems. The results of these applications seem to be quite encouraging. Furthermore, the proposed algorithm has been integrated within a GIS based decision support system for hazardous materials logistics operations providing valid preliminary results on a set of case studies.

Keywords: Vehicle routing and scheduling; Heuristic; Hazardous materials

1. Introduction

The hazardous materials transportation covers a large part of the economic activities of the industrialized countries. It has been estimated that four billion tons of hazardous materials are being transported annually at a worldwide level. Apart from its significant role in the economy of the industrialized countries, hazardous materials transportation raises concerns about the safety of the human and natural environment. Although hazardous material accidents are rare events, i.e., the accident probability is extremely low, their impacts can be severe for the human and natural environment [22]. This basic feature of hazardous materials transportation adds a special concern in the decision making process of the hazardous materials transportation since the minimization of risk should also be taken into account [12,20].

The risk imposed by the transportation of hazardous materials is expressed by the product of the probability of an accident times the measure of its consequences. Therefore, the risk minimization in hazardous materials transportation can be accomplished both by minimizing the hazardous materials accident probability and the reduction of

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the accident's impacts [21]. The consideration of risk as a criterion for selecting hazardous materials routes contributes substantially to the reduction of accident probability and the severity of the accident's impacts.

The determination of hazardous materials distribution routes can be defined as a bi-objective vehicle routing problem with time windows since risk minimization accompanies the cost minimization in the objective function. Vehicle routing is one of the most frequently faced logistical decisions in distribution management. Substantial research effort has been devoted for solving this broad category of problems [6,10]. The complexity of this problem causes a heavy computational burden for its solution especially in cases of large-scale distribution networks. Therefore, heuristic algorithms are proposed for the solution of the aforementioned problem.

The objective of this paper is to present a new heuristic algorithm for solving the bi-objective vehicle routing and scheduling problem. The proposed algorithm has been applied to several benchmark problems. The results of these applications seem to be quite encouraging.

The rest of this paper consists of four sections. The objective of Section 2 is to provide a self-contained description of the vehicle routing problem that emerges within the hazardous materials distribution process and the major research effort that has been devoted to its solution. Section 3 describes the new algorithmic approach in solving the vehicle routing problem for hazardous materials distribution while Section 4 presents the computational results of its application to the well-known Solomon benchmark problems and a set of case studies and presents the integration of the proposed heuristic algorithm into a GIS based decision support system for hazardous materials logistical decisions. Finally, Section 5 provides an overall assessment of the performance of the proposed algorithm focusing on its validity and applicability.

2. Hazardous materials distribution problem

A type of decision of major importance that directly affects the performance of a distribution system is the routing and scheduling of delivery trucks. It has been widely stated in the literature that this type of decision is modelled by difficult to solve combinatorial optimisation problem [1,4,6]. More specifically, vehicle routing and scheduling problem is NP-hard [19]. As a consequence exact solution methods cannot find the optimum solution even for moderate size problems [6]. Alternatively several heuristic approaches have been developed within the last two decades aiming at identifying near optimal solutions [3,8,9,11,13–18]. This section is focused on the definition and mathematical formulation of the routing and scheduling problem underlying the hazardous materials distribution process as a mixed integer programming model and present the relative literature review of the existing hazardous materials routing models and algorithms.

2.1. Definition of the problem

The hazardous materials transportation problem is defined by one or more pairs of origins and destinations of a specified quantity of hazardous materials. The objective of the problem is the transportation of the quantity from the origins to the destinations through the most economic and safe routes. Based on the transportation requirements several routing problems may emerge. The hazardous material distribution problem can be defined as the routing problem of a set of vehicles for servicing a set of customers' demand for a specific hazardous material under the following requirements:

(i) each vehicle starts and terminates its route at the depot,
(ii) each route could contain intermediate stops (deliveries or pick ups),
(iii) each point of demand must be serviced within a predeclared time window,
(iv) the objective of the identified routes is the achievement of minimum cost and the highest level of transportation safety (accident avoidance).

Based on the aforementioned requirements the distribution of hazardous goods could be defined as the determination of the safest and most economical routes assigned to a fleet of vehicles for the transportation of products from a depot to a set of demand points (customers). Furthermore, the basic characteristics of the vehicle routing and scheduling problem are: (a) the vehicle capacity constraint which implies that the total demand covered by each vehicle should not exceed the vehicle's capacity, (b) the route duration constraint which imposes a time limitation on the use of the vehicles and consequently on the duration of the routes, (c) the time windows constraint which refers to the limitation that is imposed on the earliest and latest delivery time at each point of demand, and (d) the objective function which defines the criterion for selecting the optimum routes [5]. The problem at hand possesses all of these main features of a typical vehicle routing and scheduling problem.

Another basic characteristic of the hazardous materials vehicle routing and scheduling problem that enhances its relative importance with regards to the other categories of routing problems is the introduction of the transportation risk into the criteria for route selection.

Transportation risk expresses the expected consequences of a hazardous materials accident. Although several types of consequences can be confronted in a hazardous materials accident such as injuries, fatalities, property damages, environmental impacts, it is the threat of human lives that has mainly attracted the interest of most researchers. Therefore, the common definition of risk faced by a population centre (i) and generated by the transportation of hazardous material m through a route p is given by the formula (1).

$$R_{ipm} = \mathrm{IR}_{ipm}\mathrm{POP}_i \tag{1}$$

where p is the hazardous material route, IR_{ipm} is the respective individual risk (i.e. by a person within population centre i) and POP_i is the number of persons within population center i. Individual risk is defined as the probability of a death expressed by the formula (2):

$$\mathrm{IR}_{ipm} = \sum_s P_s(A)P_{sm}(R/A)P_m(I/R)P_{ism}(D/I) \tag{2}$$

where $P_s(A)$ is the probability of a hazardous material accident in roadway segment s of route p, $P_{sm}(R/A)$ is the probability of release given a hazardous material accident, $P_m(I/R)$ is the probability of a consequence (e.g. fire, explosion) given a release, and $P_{ism}(D/I)$ is the probability of a death given a hazardous materials accident consequence. In most cases the estimation of $P_{ism}(D/I)$ is deemed as a hard task since it is very difficult to efficiently estimate the impacts of a consequence on a person. Alternatively, many researchers [7,12] have simplified the aforementioned risk expression by assuming that all persons lying within λ distance from the accident spot are bound to be exposed to the same consequences while the consequences outside distance λ are ignored. In other terms, $P_{ism}(D/I)$ is set equal to 1 if a person is within λ distance from roadway segment s and 0 otherwise. This simplification leads to the risk expression that is adopted within this paper and is defined by (3).

$$R_s = p_s C_s \tag{3}$$

where p_s is the hazardous materials accident probability and C_s is the population that is contained in a λ distance from roadway segment s. Furthermore the risk over the whole route is defined by (4) implying that the risk measure is additive.

$$R_p = \sum_{s \in p} p_s C_s. \tag{4}$$

A more formal definition and mathematical formulation of the hazardous materials vehicle routing and scheduling problem requires the definition of a mathematical graph $G(N, A)$ where N is the set of nodes and A is the set of links. Each node in the set $N \setminus \{0\}$ denotes a customer while $\{0\}$ denotes the depot. In this context every link (i, j) denotes the transportation link (i.e. the shortest path) from customer i to customer j. The main attributes of each link $(i, j) \in A$ are: (a) the travel time t_{ij} which is defined as the travel time of the shortest path from i to j, and (b) R_{ij} which denotes the transportation risk generated on the shortest path from i to j. Each point of demand $i \in N \setminus \{0\}$ is associated with two attributes: (a) demand (d_i), and (b) a time window of service $[a_i, b_i]$. The objective of the hazardous materials vehicle routing and scheduling problem is to identify a set of routes that service all the demand points within their time windows at the minimum cost and risk without violating the capacity constraint. The hazardous materials vehicle routing problem could be formulated as a bi-objective mixed integer programming model defined by constraints [(5)–(18)].

$$\text{Minimize } z_1 = \sum_{i=1}^{n}\sum_{j=1}^{n}\sum_{v=1}^{NV} x_{ij}^{v} c_{ij}, \tag{5}$$

$$z_2 = \sum_{i=1}^{n}\sum_{j=1}^{n}\sum_{v=1}^{NV} x_{ij}^{v} R_{ij}, \tag{6}$$

subject to

$$\sum_{i=0}^{n}\sum_{v=1}^{NV} x_{ij}^{v} = 1, \quad j = 1,\ldots,n, \tag{7}$$

$$\sum_{j=0}^{n}\sum_{v=1}^{NV} x_{ij}^{v} = 1, \quad i = 1,\ldots,n, \tag{8}$$

$$\sum_{i=1}^{n} x_{ip}^{v} - \sum_{j=1}^{n} x_{pj}^{v} = 0, \quad v = 1,\ldots,NV,\ p = 0,\ldots,n, \tag{9}$$

$$\sum_{i=1}^{n} d_i\left(\sum_{j=0}^{n} x_{ij}^{v}\right) \leqslant K_v, \quad v = 1,\ldots,NV, \tag{10}$$

$$x_{ij}^{v} = 1 \Rightarrow T_i + s_i + t_{ij} \leqslant T_j, \quad i = 1,\ldots,n,\ j = 1,\ldots,n,\ v = 1,\ldots,NV, \tag{11}$$

$$x_{j0}^{v} = 1 \Rightarrow T_i + s_i + t_{i0} \leqslant T_{\mathrm{A}}^{v}, \quad i = 1,\ldots,n,\ v = 1,\ldots NV, \tag{12}$$

$$x_{0j}^{v} = 1 \Rightarrow T_{\mathrm{D}}^{v} + t_{0j} \leqslant T_j, \quad j = 1,\ldots,n,\ v = 1,\ldots,NV, \tag{13}$$

$$a_i \leqslant T_i \leqslant b_i, \quad i = 1,\ldots,n, \tag{14}$$

$$a_0 \leqslant T_{\mathrm{A}}^{v} \leqslant b_0, \quad v = 1,\ldots,NV, \tag{15}$$

$$a_0 \leqslant T_{\mathrm{D}}^{v} \leqslant b_0, \quad v = 1,\ldots,NV, \tag{16}$$

$$x_{ij}^{v} \in \{0,1\}, \tag{17}$$

$$T_i \in \Re, \tag{18}$$

where x_{ij}^{v} is a bi-valued variable that takes value 1 if vehicle v passes through node i and heads towards node j and 0 otherwise, T_i is the moment in time that service of node i begins, T_{A}^{v}, T_{D}^{v} are two variables that express the arrival (A) and departure (D) time of vehicle v respectively, and s_i is the duration of service at customer i.

The objective function z_1 expresses the total travel time of the distribution process while objective function z_2 expresses the respective total transportation risk. Constraints (7)–(9) secure that all nodes are serviced only once by a unique vehicle. Constraint (10) implies that the total demand serviced by each route should not exceed the capacity of vehicle. Constraints (14) and (15) impose upper and lower bounds on the times that the vehicles may arrive at each node. Constraint (16) implies that a vehicle may not depart from the depot sooner than a_0 nor later than b_0. Furthermore, constraints (11)–(13) assure that if a vehicle moves from node i to node j the time it needs to get at j is at least the service time at i plus the travel time from i to j.

Based on the above characteristics the hazardous materials distribution problem can be defined as follows: identify Pareto optimal cost/risk routes for a fleet of vehicles (not uniform necessarily) from one depot to a set of points of known demand and service time windows. In the next section there is an algorithmic approach for dealing with this problem.

2.2. Literature review

Hazardous materials routing is one of the most frequently faced logistical decisions in hazardous materials logistical operations. The public concern associated with the potential severity of the consequences of a hazardous material accident has motivated the researchers to develop several mathematical models for identifying safe and cost effective paths for the hazardous materials transportation [3,7]. Although the research on the origin to destination routing problem has been overwhelming during the last two decades, less focus has been placed on the hazardous materials vehicle routing and scheduling problem. The only relative work identified in the literature is on the routing and scheduling problem performed by Cox and Turnquist [3]. The focus of their work was the development of a dynamic programming algorithm for identifying the efficient frontier of an origin to destination bi-objective routing and scheduling problem on a graph. According to the knowledge of the authors there does not exist in the literature a research addressing the hazardous materials vehicle routing and scheduling problem.

3. The proposed heuristic algorithm

The analysis of the previous sections leads to the major conclusion that the hazardous materials distribution routes can be determined by solving a bi-objective vehicle routing and scheduling problem. The objective of any solution method for the bi-objective vehicle routing and scheduling problem should be the identification of the set of strictly non-dominated (or simply non-dominated) solutions S. However, no such efficient method exists for bi-objective problems in general while only some techniques can be found identifying subsets of the whole set of non-dominated solutions. The solution approach that has been used for the problem at hand was the weighting method. The weighting method specifies a set of non-dominated solutions $I(\subseteq S)$ by solving the integer-programming model which is defined by the constraint set of the original problem and an objective function defined by (19) and (20).

$$\text{Minimize}\left\{ w_1 \sum_{i=1}^{n} \sum_{j=1}^{n} \sum_{v=1}^{NV} x_{ij}^{v} c_{ij} + w_2 \sum_{i=1}^{n} \sum_{j=1}^{n} \sum_{v=1}^{NV} x_{ij}^{v} R_{ij} \right\}, \tag{19}$$

$$\sum_{i=1}^{2} w_i = 1 \quad \text{and} \quad w_i \in [0,1]. \tag{20}$$

The transformation of the bi-objective vehicle routing and scheduling problem into a set of single objective problems through the weighting method, justifies the use of a fast and efficient heuristic algorithm for approximating the efficient frontier of the problem at hand. However, residing at the stock of heuristic algorithms for the vehicle routing problem with time windows it can be verified that no such algorithm exists since the route building heuristics are fast but they do not produce near optimal solutions while on the other hand more sophisticated algorithms like Improvement heuristics, Tabu search and Genetic algorithms are time consuming in order to produce near optimal solutions [1]. Alternatively, a new route building algorithm for the single objective vehicle routing problem with time windows has been developed that outperforms the already existing common algorithms. In what follows there is a presentation of the new route building algorithm and a description of its use in solving the bi-objective vehicle routing problem with time windows.

The heuristic algorithm proposed for solving the bi-objective vehicle routing problem is an insertion algorithm, i.e., it builds the routes step by step by inserting in the already existing routes a new demand point at each iteration. Although the proposed algorithm has many common features with the respective insertion algorithm developed by Solomon [19], it differs from it in the selection of the next demand point for insertion. More specifically, while the Solomon's algorithm allows the insertion of the unrouted demand points the proposed heuristic algorithm allows the insertion of both routed and unrouted demand points enabling in this way the reinsertion of the demand points at some more globally beneficial position. The basic steps of the proposed algorithmic approach are the following:

(1) Insertion of a new customer in a new route. In this step the construction of a new route is initiated. Three alternative ways are used for the initialisation of a route: (a) insertion of the unrouted customer that lies farthest from the depot, (b) insertion of the customer with the shortest lower time bound of its time window, or (c) insertion of the unrouted customer with shortest upper bound of its time window.
(2) If all customers are routed then the process terminates. Otherwise for every customer (both routed and unrouted) identify its best insertion position to the already existing routes. The best insertion position is defined on the basis of the minimization of the metric c_1:

$$c_i(i,u,j) = a_1(d_{i,u} + d_{u,j} - d_{ij}) + a_2(R_{i,u} + R_{u,j} - R_{ij}) + a_3(b_{j_u} - b_j), \quad a_1, a_2 \in [0,1], \quad (21)$$

where d_{ij} is the travel time from i to j, and b_i is the service start time at customer i, R_{ij} is the risk on link (i,j) and α_1, α_2, α_3 are weighting factors.

(3) Selection of the new customer to be inserted. The criterion for selecting the next customer to be routed is the minimization of metric c_2:

$$c_2(u) = a_1 t(u) + a_2 R(u) + a_3 S(u) \quad (22)$$

where $t(u)$, $R(u)$ are the total travel time and risk respectively after inserting u taking into account that each unrouted customer adds the quantity $a_1(t(0,u) + t(u,0)) + a_2(R(0,u) + R(u,0))$ to it. $S(u)$ denotes the total schedule time of the distribution process.

(4) Insertion of the selected customer, update the service start times of each routed customer and return to step 2.

The major objective of the proposed heuristic algorithm is to identify routes of minimum cost and risk. Intuitively cost minimization can be achieved by identifying short routes i.e. routes of minimum distance (or travel time) and waiting time (or schedule time). In this context, metric c_1 is defined as the weighted sum of the following quantities: (i) the savings in terms of distance gained by inserting customer u between the routed customers i and j, (ii) the savings in terms of risk gained in the same way and (iii) the shift in the schedule of customer j due to the insertion of customer u between i and j. Therefore the minimization of c_1 implies the maximization of the distance and risk savings and the minimization of the schedule time. Metric c_2 expresses the weighted sum of the travel time (distance), risk, and schedule time after inserting customer u in a route. It is necessary to emphasise at this point that this procedure is run for several values of $a_i \in [0,1]$ such that $\sum_i a_i = 1$. These runs lead to the proposed set of solutions. Furthermore it is vital to highlight some crucial technical details of the implementation of the algorithm. According to the state of the art and state of practice, the values of the hazardous materials accident probability are of order of magnitude ranging between 10^{-8}–10^{-6} per km [7,12]. As a direct consequence the respective risk values are of the same magnitude. However this fact raises an important impedance in capturing non-dominated solutions since weights $a_i \in [0,1]$ are taking values from 0 till 1 with step 0,1 it is inevitable that any of this values would favour the travel time objective. In order to overcome this technical obstacle in applying the algorithm, a scaling technique has been utilised. The objective of this technique is to transform the values of travel time and risk into a common range of values. The scaling has been accomplished by dividing both attributes of a link (i,j) by some specific scaling factors i.e. t_{sf}, R_{sf} respectively such that:

$$t_{sf} = \frac{\sum_{(i,j)} t_{ij}}{n(n-1)}, \quad (23)$$

$$R_{sf} = \frac{\sum_{(i,j)} R_{ij}}{n(n-1)}. \quad (24)$$

Based on this scaling technique, the attributes t_{ij}, R_{ij} appearing in the formulas of c_1, c_2 are replaced by the following attributes:

$$t'_{ij} = \frac{t_{ij}}{t_{sf}}, \quad (25)$$

$$R'_{ij} = \frac{R_{ij}}{R_{sf}}. \quad (26)$$

4. Application and computational results

The proposed algorithm can be applied on both single and bi-objective vehicle routing and scheduling problems. A common practice for testing the validity and efficiency of a heuristic algorithm is to apply it on a set of benchmark problems and compare the deviation of the produced solutions with the optimum solutions is attainable or the respective solutions produced by other competitive algorithms. Within the context of this practice, the proposed algorithm has been applied on the well-known single objective vehicle routing and scheduling problems of Solomon. Unfortunately, no benchmark problems for the bi-objective vehicle routing and scheduling problems were identified in the literature. However, the proposed algorithm was applied on a case study designed by the authors. The objective of the application of the proposed algorithm on the case study was to investigate the deviation of the produced solutions from the set of efficient (non-dominated) solutions under the following alternative hypotheses: (i) strict vs loose time windows, and (ii) large vs small variance of the values of risk. The rest of this section is devoted to the presentation of the application of the proposed algorithm on both single and bi-objective vehicle routing and scheduling problems, and the relevant computational results.

4.1. Application on single-objective problems

The proposed heuristic algorithm has been applied on single objective vehicle routing and scheduling problems by setting the weight a_2 to 0 without performing the scaling technique. More specifically, the proposed algorithm has been applied to the Solomon's benchmark problems that consist of several representative single objective vehicle routing and scheduling problems of 100 customers and a single depot. In particular Solomon's benchmark problems are divided to six categories namely R1, R2, C1, C2, RC1, and RC2. In categories R1, R2 the positions of the customers have been generated randomly from a uniform distribution. On the other hand the positions of the customers of problems within C1 and C2 have been generated in clusters while the respective positions in problems within RC1, RC2 are semi-clustered (6). The problems within R1, C1, and RC1 have been designed with short scheduling horizon resulting to routes with only a few customers. On the other hand problems within R2, C2, RC2 possess a long scheduling horizon enabling the identification of long routes of many customers. The assessment of the performance of the proposed algorithm on the Solomon's benchmark problems is based on the mean values over the problems of each category of the following criteria: (i) the number of routes, (ii) the travel time and (iii) the schedule time.

Within the context of testing the efficiency of the proposed algorithm the results of the application on Solomon's benchmark problems are compared to the respective results on the same problem set of the following route building heuristics: Sweep Algorithm, and Solomon's I_1, I_2 heuristics. The results of the application of the proposed algorithm on the Solomon's benchmark problems are presented in Table 1. The respective results of the Sweep, I_1, and I_2 algorithms were taken from [19] and are presented in Table 2. The routes produced by the aforementioned algorithms are compared with respect to the number of vehicles used (NV), the travel time (TT), and the schedule time (ST) i.e. the time elapsed between the moment of the departure of the first vehicle from the depot and the moment of the last vehicle returning at the depot. The comparison of the results of the alternative algorithms with the results of the proposed heuristic is quite encouraging for the performance of the new algorithm. The proposed algorithm outperforms the competitive route building algorithms i.e. Sweep, Solomon's I_1 and I_2 at problem sets R1, R2, RC1, and RC2.

Table 1
The results of the application of the heuristic algorithms Sweep, I_1, I_2 on the R1, R2, C1, C2, RC1, RC2 problems

	Sweep algorithm			Solomon's I_1			Solomon's I_2		
	NV	TT	ST	NV	TT	ST	NV	TT	ST
R1	14.6	1449.7	2817.4	13.6	1436.7	2695.5	14.5	1638.7	2888.1
R2	3.2	1448.6	2590.1	3.3	1402.4	2578.1	3.3	1470.7	2645.8
C1	10.0	940.8	10,133.8	10.0	951.9	10,104.2	10.1	1049.8	10,174.3
C2	3.0	711.9	9755.2	3.1	692.7	9921.4	3.4	921.5	10,151.4
RC1	14.9	1804.5	3094.5	13.5	1596.5	2775.0	14.2	1874.4	3029.5
RC2	4.0	1735.7	3007.9	3.9	1682.1	2955.4	4.1	1797.6	3128.4

Table 2
The results of the application of the heuristic algorithm New I_2 on the R1, R2, C1, C2, RC1, RC2 problems

	New-I_2		
	NV	TT	ST
R1	13.25	1367	2503
R2	3.09	1261	2407
C1	10.44	1150	10,299
C2	3.25	708	9756
RC1	13.25	1557	2673
RC2	3.6	1517	2673

The performance of the proposed algorithm on R1, R2, RC1, RC2 problem set outperforms the respective performance of the competitive algorithms in terms of all measures. This observation implies that the proposed algorithm is most suitable for problems that the positions of the customers (or points of demand in general) are either randomly scattered or semi-clustered. However, the performance of the algorithm on problems with clustered points of demand like those in C1, and C2 is outperformed by the competitive heuristics under all measures.

4.2. Application on bi-objective problems

The assessment of the efficiency and validity of the algorithm involves the evaluation of its performance on a wide range of bi-objective vehicle routing and scheduling problems. The lack of bi-objective benchmark problems was handled by designing a set of test scenarios aiming at investigating. The test scenarios were build on a basic bi-objective vehicle routing and scheduling problem which is defined by one depot, a set of eight customers with demands that range from 1 to 5 tons and an available homogeneous fleet of vehicles with capacity equal to 20 tons. The positions of the customers on the plane were randomly selected within the square defined by the points $(0,0)$, $(0,100)$, $(100,100)$, and $(100,0)$. The service times at the customers range from 15 to 25 minutes. The travel time between each pair of customers equals the Euclidian distance. On the other hand the risk between each pair of points in the plane (customer or depot) is measured by using formula (27).

$$R_{ij} := n_{ij} \; * \; d_{ij} \; * \; \mathrm{POP}_{ij} \tag{27}$$

where n_{ij} is the frequency of hazardous materials accident per km, d_{ij} is the distance between i and j, and POP_{ij} is the population within 2 km from the spot of a hazardous materials accident.

The proposed algorithm has been tested under the four alternative scenarios: (1) tight time windows and non-uniform risk values, (2) loose time windows and non uniform risk values, (3) tight time windows and uniform risk values, and (4) loose time windows and uniform risk values. The duration of a tight time window does not exceed 10% of the planning horizon of the problem while the respective value for the loose time window is 25%. In addition, the terms uniform and non-uniform risk values relate to the variance of the risk values between each pair of customers.

The scope of the evaluation of the algorithm is to test its capability to identify non-dominated solutions to all four scenarios. A subset of the set of non-dominated solutions was determined by solving 100 single objective problems for each scenario by using the Mathematical Programming Platform AIMMS [2]. The objective of the algorithms evaluation is to secure that its solutions are not dominated by the efficient solutions identified by the AIMMS application.

Tables 3 and 4 summarize the results of both solution methods (Algorithm vs AIMMS application respectively) under the four routing and scheduling scenarios. The efficient solutions produced by the AIMMS applications are denoted by S_{ij} where i denotes the identification number of the specific scenario and j denotes the identification number of the solutions within the set of solutions identified for scenario i. An analogous symbolism is used for the solutions produced by the proposed algorithm. Each of the two tables consists of five columns. Looking from left to right the first column contains the identification symbol of each solution, the second column contains the scenario identification number for each solution, the third column contains the travel time and the fifth column contains the total risk for each solution. As it can be verified the solutions that are generated by the algorithm are either equal to the efficient so-

Table 3
Features of the non-dominated solutions produced by the AIMMS application to Scenarios 1–4

Solution	Scenario	Number of routes	Route time (min)	Total risk ($\times 10^{-6}$)
S_{11}	1	3	476	93,228
S_{12}	1	3	482	83,623
S_{13}	1	3	565	65,150
S_{14}	1	3	651	54,771.4
S_{21}	2	2	426	36,567.1
S_{22}	2	2	512	26,188.4
S_{23}	2	2	597	22,727.2
S_{31}	3	2	405	10,116.3
S_{41}	4	3	476	18,307.8
S_{42}	4	3	488	14,812.6
S_{43}	4	3	588	14,069.2

Table 4
Features of the solutions produced by the application of the proposed algorithm to Scenarios 1–4

Solution	Scenario	Number of routes	Route time (min)	Total risk ($\times 10^{-6}$)
H_{11}	1	3	482	83,616
H_{12}	1	3	540	79,668
H_{13}	1	3	565	65,145
H_{14}	1	3	620	65,099
H_{15}	1	3	649	58,680
H_{16}	1	3	651	54,766
H_{17}	1	4	689	50,924
H_{21}	2	2	405	61,970
H_{22}	2	2	426.1	36,562
H_{23}	2	2	512	26,183
H_{31}	3	2	405	10,110
H_{41}	4	3	482	18,749
H_{42}	4	3	495	15,253

lutions defined by the AIMMS application or at least not dominated by them.

Scenario 1 is defined by the basic bi-objective vehicle routing and scheduling problem enhanced with the following hypotheses:

- The planning horizon of the distribution process endures six hours starting at 08:00 and finishing at 14:00 while the depot is open between 07:00 and 14:00. For the convenience of the reader the time will be expressed from now on in minutes taking as 0 minutes the 00:00 hours, e.g., the planning horizon is defined as 480–800 minutes.
- The width of time windows equals 10% of the width of the planning horizon. Therefore, the time window of each point of demand is determined by: (i) selecting a randomly a point of time within the planning horizon, and (ii) add and subtract 5% of the planning horizon i.e. 18 min, from this point in time in order to identify the upper and lower time limits respectively.
- The values of risk per link (i, j), are defined in Table 5.

The major feature of this test scenario is the narrow time windows and the wide variance of the resulting values of the risk measure. The solution of the emerging problem was achieved by using the AIMMS application. It resulted in four efficient solutions, namely S_{11}, S_{12}, S_{13}, S_{14}, that are presented in the first four rows of Table 3. The application of the proposed algorithm on the same test scenario lead to seven alternative solutions which are presented in the seven first rows of Table

Table 5
The risk values used in Scenario 1 ($\times 10^{-6}$)

Nodes	1	2	3	4	5	6	7	8	9
1	0	6907	163	28,196	24,739	2175	276	1754	4593
2	6907	0	9420	29,416	1393	2415	1546	5121	1392
3	163	9420	0	33,916	6428	2556	3363	1585	330
4	26,196	29,416	33,916	0	6720	1502	444	70	1952
5	24,739	1383	6428	6720	0	23,846	233	823	1271
6	2175	2415	2556	1502	23,846	0	5460	5990	42,440
7	276	1546	3363	444	233	5460	0	8097	5247
8	1754	5121	1585	70	823	8990	8097	0	6364
9	4593	1392	330	1952	1271	42,440	5247	5247	0

4. Comparing one by one the solutions produced by both methods it can be verified that the heuristic solutions are either identical or non-dominated by the exact solutions. Besides the comparison of the solutions produced by the exact and heuristic method it would be stimulating to investigate the sensitivity of the resulting routes under several pairs of priorities assigned to the two objectives of the problem i.e. travel time and risk. Fig. 1 presents the routes of H_{11} solution, which has been derived by assigning minor weight to risk while travel time has received enormous weight. The depot is denoted by the square box on the bottom left corner while the points of demand are represented by the numbered circles. Each route is presented by a sequence of arcs. The style of the line of the arcs is different for each route while the number in parenthesis beside each node denotes the service start time at the respective customer.

However in Fig. 2 which presents the routes of solution H_{12} it can be seen that by increasing the priority of the risk, several low risk links are entering the routes even though they are lengthy e.g. link $(4, 8)$ that has a risk value of just 70, while links of high risk are avoided e.g. link $(2, 8)$.

This observation can be more clearly verified by comparing Fig. 2 with Fig. 3, which presents the routes of solution H_{17}. It is apparent from the comparison of these two figures that an additional route is constructed in order to determine the safest possible distribution.

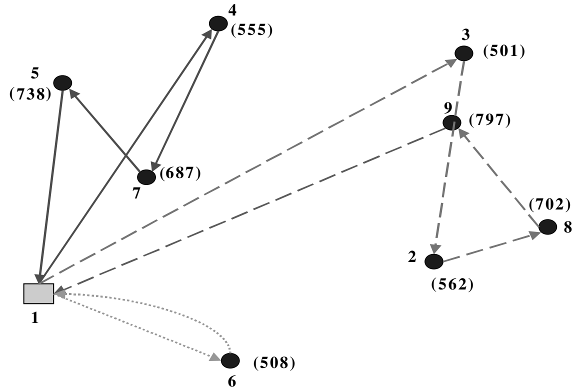

Fig. 1. The vehicle routes presentation of heuristic solution H_{11} of Scenario 1.

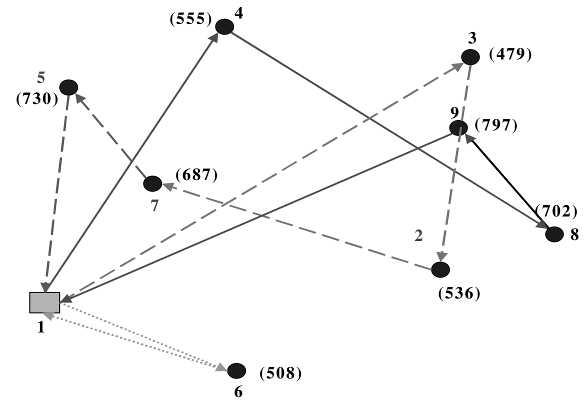

Fig. 2. The vehicle routes presentation of heuristic solution H_{12} of Scenario 1.

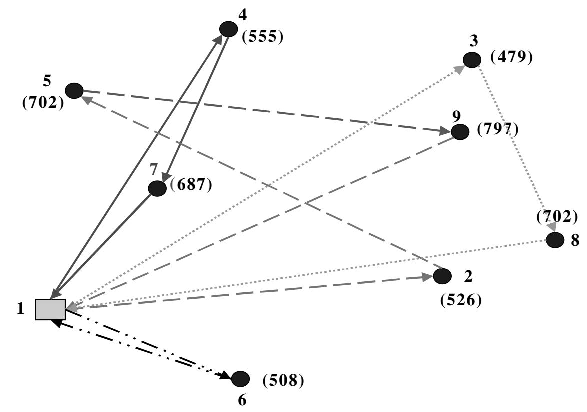

Fig. 3. The vehicle routes presentation of heuristic solution H_{17} of Scenario 1.

Scenario 2 is different from Scenario 1 only at the width of the time windows. More specifically, in Scenario 2 the time windows are loose i.e. their width covers 25% of the planning horizon or 90 min. The solutions produced by the AIMMS application on Scenario 2 are denoted by S_{21}, S_{22}, S_{23}, S_{24} and are presented in Table 3 while the respective solutions determined by the proposed algorithm are presented in Table 4. The comparison of both sets of solution leads to the conclusions that both sets of solutions are non-dominated.

Once more the sensitivity of the routes with regards to different objectives priorities can be traced. For instance, one could compare the routes

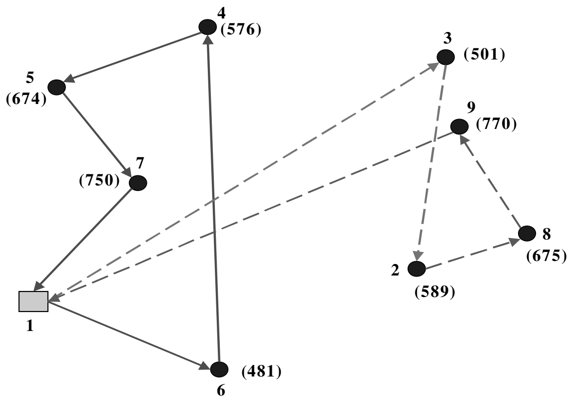

Fig. 4. Presentation of vehicle routes for Scenario 2–heuristic solution H_{21}.

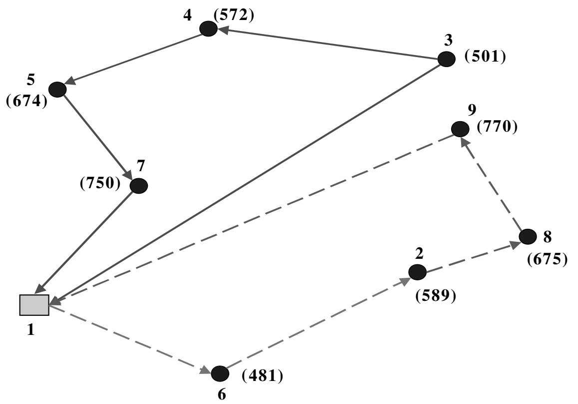

Fig. 5. Presentation of vehicle routes for Scenario 2–heuristic solution H_{23}.

of a low risk priority solution H_{21} presented in Fig. 4 as compared to the respective routes of a high risk priority solution H_{23} (Fig. 5). It is apparent that high-risk links (e.g. (4, 3)) are excluded from the low risk priority solution.

In Scenario 3 the time windows are kept the same as in Scenario 2 while the risk values are modified to decrease their variance leading to the values presented in Table 6.

Furthermore Scenario 4 contains the updated values of risk introduced in Scenario 3 while the time windows of the demand points are tight. The solutions both exact and heuristic are presented in Tables 3 and 4 respectively. It should be emphasized that both the exact and the heuristic method identify only one solution in Scenario 3 implying that loose time windows in combination with minimum risk values variance extinguish the trade off between travel time and risk. Fortunately this result is within our expectation since the assumption of minimum risk values variance implies that no significant difference with regards to the risk objective exists between alternative routes. Therefore it is be expected that the minimum travel time set of routes would be the unique solution of the problem. The role of the loose time windows in forming this intuition is highlighted in Scenario 4 where for the same values of risk, the time windows are tightened. Not surprisingly more than one non-dominated solution are produced by the proposed algorithm since the tight time windows exclude the routes identified in Scenario 3 leaving several equivalent alternative sets of routes in terms of travel time and risk producing once more a trade off between these two objectives.

Table 6
The risk values used in Scenario 3 ($\times 10^{-6}$)

Nodes	1	2	3	4	5	6	7	8	9
1	0	3452	1629	7049	825	2175	276	677	1531
2	3453	0	1256	5229	5571	805	1546	591	348
3	1629	1256	0	1357	1928	2556	1121	396	330
4	7049	5229	1357	0	672	1502	444	700	1952
5	825	5571	1928	672	0	2385	233	823	1271
6	2175	805	2556	1502	2385	0	1456	1199	1415
7	276	1546	1121	444	233	1456	0	4049	525
8	877	591	396	700	823	1199	4049	0	849
9	1531	348	330	1952	1271	1415	525	849	0

5. Conclusions

The overall assessment of the proposed heuristic algorithm for solving the hazardous materials vehicle routing and scheduling problem indicates that it provides valid solutions to all alternative scenarios identified in a hazardous materials distribution network. The algorithm has been applied on both single and bi-objective vehicle routing and scheduling problems providing encouraging results.

The application of the algorithm on the Solomon's benchmark problems proved that its performance on vehicle routing problems with randomly scattered and semi-clustered customers outperforms the respective performance of the most successful competitive route building heuristic algorithms. However, the results of the algorithm on problems with clustered customers were inferior to those achieved by the other route building heuristics. It can be argued though that the proposed heuristic algorithm is appropriate for the vast majority of the vehicle routing problems emerging from the hazardous materials distribution process.

The lack of benchmark bi-objective vehicle routing and scheduling problems inspired the authors to construct a set of cases studies in order to test its validity and efficiency under different times windows width (loose vs strict) and variance of risk values (large vs minor). It should be emphasized that the solutions produced by the application of the algorithm on these case studies were non-dominated in comparison with the respective solutions produced by the an exact method. Furthermore the results of the algorithm did manage to capture the trade off between the risk and travel time at all scenarios.

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