A Multiobjectivization Approach for Vehicle Routing Problems

Shinya Watanabe[1] and Kazutoshi Sakakibara[2]
[1]*Muroran Institute of Technology,*
[2]*Ritsumeikan Univ.,*
Japan

1. Introduction

In this chapter, we describe a new approach for vehicle routing problems (VRPs), which treats VRPs as multi-objective problems using the concept of multiobjectivization. The multiobjectivization approach has generated in the field of Evolutionary Multi-Criterion Optimization (EMO) (Knowles et al., 2001). This approach transforms a single-objective problem into a multi-objective problem. It is most important feature of this approach to provide more freedom to explore and to reduce the likelihood of becoming trapped in local optima by adding additional objectives.

There have been many studies using EMO algorithm to optimize multi-objective VRPs, with objectives including the number of routes and total travel distance or number of routes and duration of routes, etc (Jozefowiez et al., 2002). In these studies, EMO treats the original objective of VRPs directly as multi-objective.

On the other hand, our approach deals not only with the original objective of VRPs but also with newly defined objectives related to assignment of customers. Generally, VRPs seem to have two different determinations: the assignment of customers and the order of the route. The assignment of customers is known to have a stronger influence on the search than the order of the route in many studies (Doerner et al., 2005). Therefore, we expect that the proposed approach will get better solutions in minimization of the total travel distance than the approach using only the total travel distance as a single objective. In our approach, we add two new objective functions used by MOCK (Handl & Knowles, 2005) as the objective related to assignment of customers. Our multiobjectivization aims to accelerate the search for original objective by adding supplementary objective.

We investigated the characteristics and effectiveness of the proposed approach by comparing the performance of the conventional approach and multiobjectivization approach. In numerical experiments, we used Taillard's test functions as a benchmark problem. In addition, we used NSGA-II (Deb et al., 2002) in implementing our approach. Through numerical examples, we showed that the proposed multiobjectivization approach can obtain the solution with good quality and little variation in VRPs.

2. Vehicle routing problem

This paper deals with the most elementary version of VRPs, the capacitated VRPs (CVRPs), which can be described as follows (Braysy & Gendreau, 2005):

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- All vehicles start from the depot and visit the assigned customer points, then return to the depot.
- Here, a route is formed by the sequence of the depot and the customer points visited by a vehicle.
- Therefore the number of vehicles is same as the number of route. Moreover, each customer is visited only once by exactly one vehicle.
- Each customer asks for a weight $w_i (i = 1, \ldots, N)$ [1] of goods and a vehicle of capacity W is available to deliver the goods. In this paper, we used the same capacity W for all vehicles.
- A solution of the CVRP is a collection of routes where the total route demand is at most W.

VRPs have a number of objectives, such as minimization of the total travel distance, minimization of the number of routes, minimization of the duration of the routes, etc.
In this paper, we used minimization of the total travel distance (F_{sum}) as the objective of the VRPs. The formula of the objective is as follows:

$$\text{minimize } F_{\text{sum}} = \sum_{m=1}^{M} c^m \tag{1}$$

where M is the total number of routes and c^m indicates m th route distance. The formula for c^m is as follows:

$$c^m = c^m_{0,u_1^m} + \sum_{i=1}^{n_m - 1} c^m_{u_i^m, u_{i+1}^m} + c^m_{u_{n_m}^m, 0} \tag{2}$$

where $c^m_{i,j}$ indicates the distance from customer i to customer j. u_i^m represents the i th customer to be routed in the m th route and "0" is the depot. n_m indicates the total number of customers in the m th route. Here, the total number of customers is $N = \sum_{m=1}^{M} n_m$.
VRPs have a constraint on the vehicle capacity W. In this paper, we used the same capacity W for all vehicles.
The formula of the vehicle capacity W is as follows:

$$W \geq w^m = \sum_{i=1}^{n_m} w_{u_i^m} \qquad (m = 1, \ldots, M) \tag{3}$$

where w^m indicates the amount of customers' weight in the m th route and $w_{u_i^m}$ represents the weight of the goods for the i th customer to be routed in the m th route.
As noted above, in VRPs it is necessary to find a set of sequences of customers that will minimize the total travel distance. In addition, it is necessary to determine the following two points:

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[1] N is the number of customers.

1. assignment of customers
2. routing (the order of customers)

3. The multiobjectivization of vehicle routing problem

In this section, we describe the purpose of multiobjectivization and the evaluation method related to assignment of customers.

3.1 Multiobjectivization approach

The term of multiobjectivization was born in the field of Evolutionary Multi-Criterion Optimization (EMO) (Knowles et al., 2001). The main concept of this approach is to translate single-objective optimization problems into multi-objective optimization problems and then apply EMO algorithm to the translated problem.
Previous studies of multiobjectivization can be divided roughly into two categories as follows(S. Watanabe & K. Sakakibara, 2005):

- Addition of new objectives to a problem.
- Decomposing a problem into sub-problems.

These multi-objectivizations have a number of effects, such as the reduction of the effect of local optima, making the problem easier, or increasing search paths to the global optimum.
Our multiobjectivization approach for VRPs is based on the addition of new objectives. In our approach, newly defined objective aims to accelerate the speed of the search for original objective. Fig.1 shows the concept of this approach.

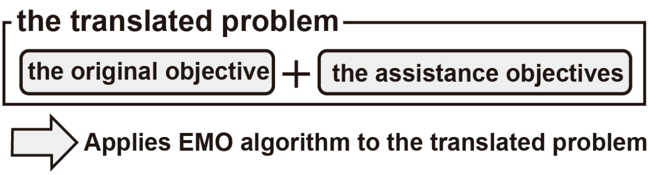

Fig. 1. The concept of multiobjectivization

3.2 The purpose of the proposed multiobjectivization approach

As described in section 2, two types of decision elements should be considered in VRPs. Among two decisions, the assignment of customers has a stronger influence on the search than the order of customers, because the order of customers can be determined under the fixed assignment of customers to the specific vehicle.If the assignment determination is not appropriate, good solutions cannot be obtained even if the best order is determined for all routes.
However, there have been no previous reports of VRPs explicitly taking into account evaluation of customer assignments. As a hierarchical search between the assignments and order determination, Bent et al. proposed a two-stage hybrid local search that first minimizes the number of vehicles using SA, and then minimizes the total travel distance using a large neighbourhood search (Bent & Hentenryck, 2004). In addition, Nanry et al. reported a hierarchical search using Reactive Tabu Search (RTS) to the customer assignments and the order determination (Nanry & Barnes, 2000). However, these approaches do not evaluate the customer assignments directly, and evaluate only the original objective of VRPs: the total travel distance and the number of vehicles.

The proposed approach treats two types of decision elements independently. But this approach uses the same solution strategy for these decisions, not using individual solution strategy as a hierarchical search. Treating VRP as multi-objective problem, we can handle two different decisions concurrently and independently.

3.3 The evaluation method related to assignment of customers

In the proposed approach, we capitalize on the objective functions using ''Multi-objective clustering with automatic determination of the number of clusters (MOCK) (Handl & Knowles, 2005)'' to evaluate customer assignments.

Clusters and data points in clustering problems can be assumed as routes and customers in VRPs, respectively. Therefore, the objective functions used by MOCK can be diverted to evaluation of the customer assignments in VRPs.

MOCK adopts the following two functions, which reflect two fundamentally different aspects of good clustering solutions.

1. The global concept of the compactness of clusters.
2. The local concept of the connectedness of data points.

The first of these clustering objectives evaluates the overall density of clusters as the compactness, and the latter evaluates the degree to which neighbouring data points have been placed in the same cluster as the connectedness.

The overall density of clustering solutions, which reflects the overall intra-cluster spread of the data, is computed as:

$$\mathrm{Dev}(C) = \sum_{C_k \in C} \sum_{i \in C_k} \delta(i, \mu_k) \tag{4}$$

where C is the set of all clusters, μ_k is the centroid of cluster C_k and $\delta(.,.)$ represents the distance function (Euclidean distance).

The second objective function, connectivity, evaluates the degree to which neighbourhood data points have been placed in the same cluster.

The second objective function is presented as the following formula:

$$\mathrm{Conn}(C) = \sum_{i=1}^{N} \left(\sum_{j=1}^{L} x_{i,nn_i(j)} \right), \qquad x_{i,nn_i(j)} = \begin{cases} 1/j & \text{if } \nexists C_k : i, nn_i(j) \in C_k \\ 0 & \text{otherwise,} \end{cases} \tag{5}$$

where $nn_i(j)$ is the j th nearest neighbour of datum i, and L is a parameter determining the number of neighbours that contribute to the connectivity measure. $x_{i,nn_i(j)}$ represents the penalty value related to whether data i and j th nearest neighbour of data i are placed in the same cluster or not.

In Eq.(5), if data i and j th nearest neighbour of data i are not placed in the same cluster , $1/j$ is added as the penalty value $x_{i,nn_i(j)}$. Therefore, greater values of Eq.(5) indicate the tendency of a clustering solution in which neighbouring data are not placed in the same cluster.

In these different objectives, it is very important that the value of $\mathrm{Dev}(C)$ is decreased with increasing number of clusters, while $\mathrm{Conn}(C)$ is increased by increasing the number of

clusters[2]. Therefore, $\mathrm{Dev}(C)$ and $\mathrm{Conn}(C)$ are in a trade-off relationship with the number of clusters.
Here, we examine the effectiveness of multiobjectivization to VRPs in which one or both of the above-mentioned objectives are added to the original objective.

4. Implementation of GA

In this section, we describe the implementation of GA to multi-objective VRPs based on multiobjectivization as described above.

4.1 Gene expression (String representation)

Various coding methods for VRPs have been proposed. In this numerical experiment, we used all routes directly as the genotype. In other words, the genotype in this numerical experiment is the same as the phenotype that represents the order of all routes.
Therefore, there is no need for translation between genotype and phenotype.

4.2 Population initialization

The average number of customers in one route can be calculated using the vehicle capacity W and a weight $w_i \quad (i = 1,\ldots,N)$ for each customer.

In this numerical experiment, the initial population was generated by random sampling that all routes for each individual must be equal to the average number of customers. The initialization process starts by inserting customers one by one into an empty route in random order until the number of customers in the route is equal to the average number of customers.
If an initial solution is not feasible, we used the repair method (stated in Section 4.6) to make it feasible. Therefore, all solutions of the initial population are feasible.

4.3 Crossover

As genotype has the same coding as phenotype, general crossover operators for VRPs, such as PMX, OX and EX, could not be adopted in this numerical experiment. Therefore, we implemented a new crossover operator, Partial Route Inheritance Crossover (PRIC), which aims to inherit as much as possible of the parents' route information.
In PRIC, the first child inherits half the routes of one parent directly, and then the remaining customers that are not in one parent are inherited using the route information of the other parent. The ratio of direct inheritance from parents to children should have strong effects on the search performance. Here, we designed the method to copy the half of routes in one parent directly.
Fig.2 shows the procedure of PRIC, the details of that are described as follows:
Step 1: Selecting two parents (Parent1 and Parent2) randomly.
Step 2: Copying the half of routes in Parent1 to child.

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[2] Because it becomes difficult to place near-neighbour data in the same cluster.

Step 3: The remaining customers that are not in Parent1 are inherited by the routes of Parent2. If the number of routes in the child is complete, the simulation goes to Step 4. If not, the simulation is terminated.

Step 4: Until the number of routes in the child becomes equal to the number of routes in Parent1, routes copied in Step 3 are integrated using the following procedure:

Step 4-1: The routes copied in Step 3 are sorted according to increasing number of customers included.

Step 4-2: Each of the routes according to the sorted order is integrated into the nearest route. The distance between routes is calculated using Euclidean distance between the centred coordinates of the route (including the depot). Therefore, the integration of routes is performed between the closest routes related to the centred coordinates. In this integration, customers are added to the bottom of the nearest route.

In Step 4 above, the routes with a small number of customers are integrated to decrease the total number of routes. Since the remaining customers that are not selected in Parent1 are picked out from Parent 2 routes, a lot of routes with small numbers of customers are produced (Step 3 in Fig.1).

PRIC includes not only the effect of the inheritance of parents' routes but also the effects of the integration of routes and the re-shuffling of customers between routes.

4.4 Mutation

In this paper, we used six kinds of operators as mutation:

1. 2-opt*(asterisk) (Braysy & Gendreau, 2005)
2. or-opt (Braysy & Gendreau, 2005)
3. Relocate Operator (Braysy & Gendreau, 2005)
4. Exchange Operator (Braysy & Gendreau, 2005)
5. Integration of different routes into one route
6. Division of a route into two routes

2-opt*(asterisk) swaps sub-routes between different two routes, and or-opt replaces the sub-route with L customers in a random chosen route. Also, Relocate Operator simply moves a customer from one route to another and Exchange Operator swaps two customers in different routes.

We randomly selected one out of the 6 operators as mutation operator at each generation.

4.5 The decision of start and end point in a route

In this paper, the start and end customers in a route are decided in the evaluation phase.

As the decision of start and end customers determines where to insert the depot in the sequence of customers, we used saving method (Braysy & Gendreau, 2005) to insert the depot with the minimum total travel distance. Therefore, the optimal insertion point of the depot in the sequence of customers can be decided.

4.6 Treatment of a solution with constraint violation

As VRPs have the constraint of the vehicle capacity, we should implement a repair method as a constraint handling technique. We used the repair method to divide an infeasible route into two routes. In this technique, a set of customer sequences satisfying the capacity constraint forms one route, and then the remaining customer sequences forms another route. Therefore, all solutions in this example are feasible.

5. Numerical examples

In this study, we investigated the characteristics and effectiveness of the proposed approach by comparing the performance of both the conventional approaches and multiobjectivization approaches.

To verify the effectiveness of multiobjectivization of VRPs, VRP instances provided by Taillard et al.[3] were used. In implementing our proposed approach, we used NSGA-II proposed by Deb et al. (Deb et al., 2002). Table 1 shows the GA parameters.

population size	200
crossover rate	1.0
mutation rate	1/bit length
number of trials	30

Table 1. GA Parameters

5.1 VRPs instances

We used six types of instances: tai75a, tai100d and newly defined four changing the variation coefficient (C) of the customer weight w_i $(i = 1,\dots,N)$ in tai75a and tai100d[4].

The characteristics of the instance are described in Table 2. Table 2 represents the number of customers(N), the vehicle capacity(W), the average of the customer weight($\overline{w}$), standard deviation($\sigma(w)$) of the customer weight w_i $(i = 1,\dots,N)$, and the variation coefficient ($C(w)$) of the customer weight.

tai75c(C=0)	75	1122	127	0.0
tai75c(C=0.6)	75	1122	163.2	0.6
tai75c(original)	75	1122	126.9	1.6
tai100d(C=0)	100	1297	136	0.0
tai100d(C=0.8)	100	1297	135.7	0.8
tai100d(original)	100	1297	135.7	1.6
tai75c(C=0)	75	1122	127	0.0

Table 2. Problem Instance

In Table 2, tai75c($C = 0$) and tai75c($C = 0.6$) are the problems of changing the amount of the customer weight w_i $(i = 1,\dots,N)$ in tai75c(original) so that the amounts of the variation coefficient of customer weights are about 0 and 0.6 respectively (the customer location and the vehicle capacity of these instances are the same as original instance). In the same way, tai100d($C = 0$) and tai100d($C = 0.8$) are modified so that the amounts of the variation coefficient of tai100d(original) are about 0 and 0.8 respectively.

The small value of the variation coefficient indicates that customer weights are homogenized. More homogenized customer weights make it easier to decide the assignment of customers, because simple heuristic approach, in which neighboring

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[3] These test problems are available at http://neo.lcc.uma.es/radi-aeb/WebVRP/.

[4] The variation coefficient (C) is the value that standard deviation was divided by the mean value. C is the index that represents the degree of variation.

customers merge into the same cluster, can be worked effectively in this case. In contradiction to this, the higher value of the variation coefficient make it more difficult to assign customer. In this case, the performance related to assignment of customers seems to influence the quality of the obtained solutions more strongly.

5.2 Results and analysis

In this study, we used five types of NSGA-II experiment based on the implementation of objectives (f_1 and f_2). Table 3 shows the 5 experiments. In Table 3, the first objective, which is common to all experiments, is the total travel distance (Eq.(1)).
In this experiment, termination conditions of the instances with 75 customers (tai75c($C=0$), tai75c($C=0.6$) and tai75c(original)) was 5000 generations, and those of the instances with 100 customers (tai100d($C=0$), tai100d($C=0.8$) and tai100d(original)) was set to 7500 generations.
We performed 30 trials and all results are shown as averages of 30 trials. The results of the 6 instances are shown in Fig. 3. Fig. 3 shows the minimum, maximum and average values in which the objective value represents the total travel distance. Also, Fig. 4 shows the standard deviation of the solutions so as to evaluate the degree of variation of the solutions. In multiobjectivization approaches, the solution with the minimum total travel distance is treated as the final best solution in each trial. In concrete terms, the solutions with the minimum f_1 value of each experiment are used as the final results.

As shown in Fig. 3, the solutions of multiobjectivization (three proposed approaches) were better than those of the conventional approaches. Especially, the difference in quality of the obtained solutions between the conventional and the proposed approaches increased with larger customer size and variation coefficient (C) value.

method	f_1	f_2	f_3	method
Conventional 1	$f^{Eq.1}$	$f^{Eq.1}$	---	Conventional 1
Conventional 2	$f^{Eq.1}$	The number of routes	---	Conventional 2
Proposed 1	$f^{Eq.1}$	$f^{Eq.4}$	---	Proposed 1
Proposed 2	$f^{Eq.1}$	$f^{Eq.5}$	---	Proposed 2
Proposed 3	$f^{Eq.1}$	$f^{Eq.4}$	$f^{Eq.5}$	Proposed 3

Table 3. The four type experiments of NSGA-II

From the width between minimum and maximum in Fig. 3 and the degree of variation in Fig. 4, it was also clear that the proposed approaches can obtain the solution with good quality and little variation at each trial. These results confirmed that proposed multiobjectivization approaches are more effective for VRPs than both the conventional approaches.
The increase in the number of objectives usually degrades the convergence of solutions to the Pareto front. But the multiobjectivization in this paper doesn't show this tendency, since additional objectives don't yield the trade-off relationship between original and additional objectives. In this paper, additional objectives can help to accelerate the search for original objective. The results of Fig. 3 enhance the legitimacy of this inference.

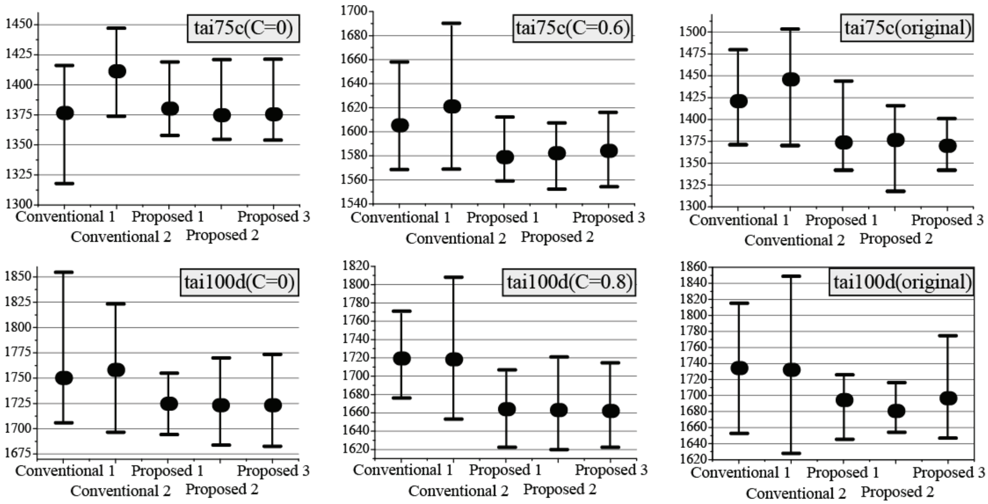

Fig. 3. The results of the total travel distance

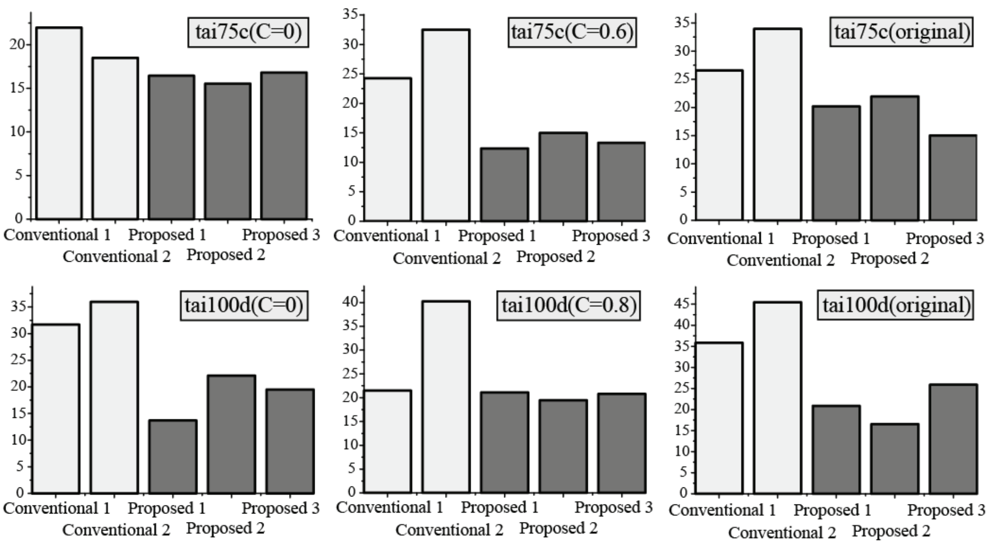

Fig. 4. The standard deviation of the solutions

The transition of the objective values

Here, we describe the transition of the objective values in each approach. The transitions of the objective values in tai100d(original) are shown in Fig. 5. The four objective values in Fig. 5 are the total travel distance(Eq.(1)), the number of routes, compactness ($\mathrm{Dev}(C)$) and connectivity ($\mathrm{Conn}(C)$)). As these objective values of Fig. 5, we used the objective value of the individuals with minimum value of the total travel distance in the generation[5]. In these figures, the horizontal axes indicate generation and the vertical axes indicate each objective value and the values of generation are described on a log scale (common logarithm). Also, all results are shown as averages of 30 trials.

From Fig. 5, there seem to be some sort of relationships between each objective values, because all objective values were decreased by a large generation number. But the values of compactness and connectivity in three proposed approaches were increased with more than 200 generations. Therefore the correlations of compactness and connectivity with near the minimum value of the total travel distance are guessed to be low or slight trade-off relation.

The transitions of the total travel distance had similar tendencies in all approaches. But the transition of Conventional2 with the total travel distance and the number of routes as objectives were a more slower slope for less than 150 generations as compared to the other approaches. On the other hand, when considering the transition of the number of route, Conventional2 got the smaller value in earlier generations as compared to the other approaches. This was because Conventional2 explicitly evaluate the number of route.

And in terms of the objective value of compactness and connectivity, three proposed approaches yield better results. It is interesting that Proposed3 with both objective functions of MOCK was better than Proposed1 and Proposed2 with one or other of two MOCK functions. Therefore it is apparent from Fig. 5 that Proposed3 is better in terms of assignment of customers than other multiobjectivization approaches with one or other of MOCK functions.

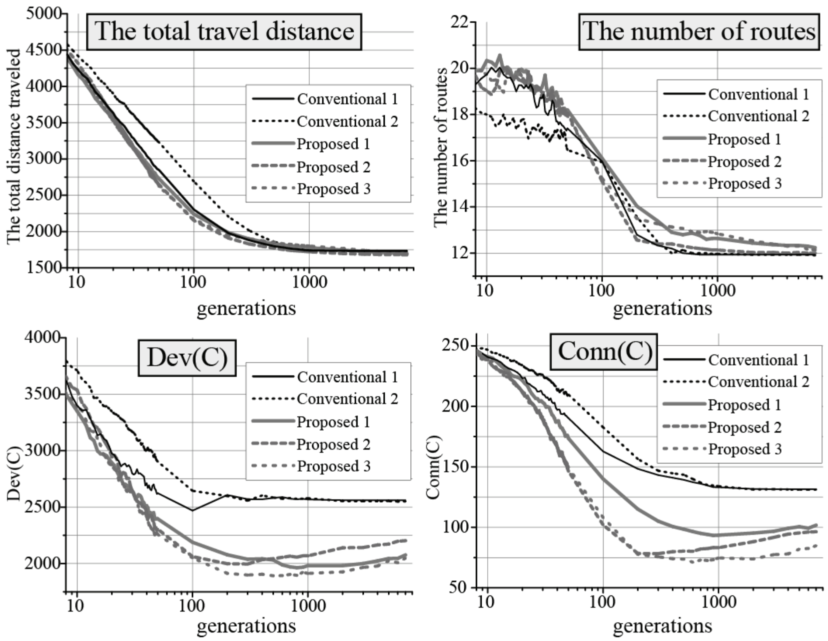

Fig. 5. The transition of the objective values

[5] Therefore, all objective values except the total travel distance may make a change for the worse.

On the other hand, the results concerning compactness and connectivity of Conventional2 with the minimization of the number of routes as second objective function and that of Conventional1 are not so different and worse than that of Proposed3. Therefore, the minimization of the number of routes doesn't have a strong effect on the assignment of customers.

6. Conclusions

In this paper, we proposed a new approach based on multiobjectivization for vehicle routing problems (VRPs) with a single objective. This approach treats VRPs as multi-objective problems in which a newly defined objective related to assignment of customers is added. As the objective related to assignment of customers, we used two objective functions used by MOCK, i.e., the compactness of clusters and the connectedness of data points.

This multiobjectivization aims to accelerate the search for original objective by adding supplementary objective. This approach assumes that the adding objective is empirically-predicted to be useful in improving the objective value. We think that our approach is one of effective means for using the empirical knowledge of the problem.

We investigated the effectiveness of the proposed multiobjectivization approaches by comparison of its performance with that of the conventional approaches.

Numerical experiments clarified the following points:

- Multiobjectivization approaches can obtain the solution with better quality and less variation at each trial. Also, the experimental results indicate that multiobjectivization using both additional objectives is more effective than using either alone.
- From the results of the transition of the objective values in the course of the search process, it was confirmed that the multiobjectivization approach using MOCK functions is very effective for the assignment of customers, while the minimization of the number of routes have little effect on the assignment of customers. Also, the three multiobjectivization approach using both objective functions of MOCK can derive better solutions than other approaches with only one MOCK function.

7. References

R. Bent and P. Van Hentenryck. A two stage hybrid local search for the vehicle routing problem with time windows. In Transportation Science, volume 38, pages 515–530, 2004.

O. Braysy and M. Gendreau. Vehicle routing problem with time windows, part i:Route construction and local search algorithms. Transportation Science, 39(1):104–118, 2005.

K. Deb, A. Pratap, S. Agarwal, and T. Meyarivan. A Fast and Elitist Multiobjective Genetic Algorithm: NSGA–II. IEEE Transactions on Evolutionary Computation,6(2):182–197, April 2002.

J.Handl and J.Knowles. Exploiting the Trade-Off—The Benefits of Multiple Objectives in Data Clustering. In Evolutionary Multi-Criterion Optimization. Third International Conference, EMO 2005, pages 547–560, 2005.

K.Deb. Multi-Objective Optimization using Evolutionary Algorithms. Chichester, UK:Wiley, 2001.

K.F.Doerner, R.F. Hartl, and M. Lucka. A parallel version of the d-ant algorithm for the vehicle routing problem. In Theory an Applications, pages 109–118, 2005.

D. Knowles, A. Watson, and W. Corne. Reducing local optima in single-objective problems by multi-objectivization. In Evolutionary Multi-Criterion Optimization. First International Conference, EMO 2001, pages 268–282, 2001.

W. P. Nanry and J. W. Barnes. Solving the pickup and delivery problem with time windows using reactive tabu search. Transportation Research Part B, 34:107–121,2000.

N.Jozefowiez, F.Semet, and E.Talbi. Parallel and Hybrid Models for Multiobjective Optimization: Application to the Vehicle Routing Problem. In Parallel Problem Solving from Nature—PPSN VII, pages 271–280, 2002.

J.Y. Potvin and S. Bengio. The vehicle routing problem with time windows - partII: Genetic search. In INFORMS Journal on Computing, volume 8, pages 165–172,1996.

S. Watanabe and K. Sakakibara. Multi-objective approaches in a single-objective optimization environment, 2005 IEEE Congress on Evolutionary Computation (CEC2005),pp.1714-1721, 2005