

Local Search in Two-Fold EMO Algorithm to Enhance Solution Similarity for Multi-objective Vehicle Routing Problems

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Abstract. In this paper, we propose a memetic EMO algorithm that enhances the similarity of two sets of non-dominated solutions. We employ our algorithm in vehicle routing problems (VRPs) where the demand of customers varies. We consider two periods of different demand in a problem that are Normal Demand Period (NDP) and High Demand Period (HDP). In each period, we can find a set of non-dominated solutions with respect to several objectives such as minimizing total cost for delivery, minimizing maximum cost, minimizing the number of vehicles, minimizing total delay to the date of delivery and so on. Although a set of non-dominated solutions can be searched independently in each period, drivers of vehicles prefer to have similar routes in NDP and HDP in order to reduce their fatigue to drive on a different route. In this paper, we propose a local search that enhance the similarity of routes in NDP and HDP. Simulation results show that the proposed memetic EMO algorithm can find a similar set of non-dominated solutions in HDP to the one in NDP.

Keywords: memetic algorithm, local search, solution similarity, vehicle routing problem.

1 Introduction

Although we have various approaches in EMO (Evolutionary Multi-criterion Optimization) community [1-3], there are few research works that investigate the similarity among obtained sets of non-dominated solutions. Deb considered topologies of several non-dominated solutions in Chapter 9 of his book [4]. He examined the topologies or structures of three-bar and ten-bar truss. He showed that neighboring non-dominated solutions on the obtained front are under the same topology, and NSGA-II can find the gap between the different topologies. While he considered the similarity of solutions in a single set of non-dominated solutions from a topological point of view, there is no research work relating to EMO that considers the similarity of solutions in different sets of non-dominated solutions. In this paper, we propose a memetic EMO algorithm that enhances the similarity of solutions in different sets of non-dominated solutions.

We employ Vehicle Routing Problems (VRPs) to consider the similarity in different sets of solutions. The VRP is a complex combinatorial optimization problem that can be seen as a merge of two well-known problems: Traveling Salesman Problems (TSP) and Bin Packing Problems (BRPs). This problem can be described as follows: Given a fleet of vehicles, a common depot, and several customers scattered geographically. Find the sets of routes for the fleet of vehicles. As for objective functions considered in VRPs, many research works [5-9] on the VRP try to minimize the total route cost that is calculated using the distance or the duration between customers. Among them the research works in [7-9] are related to multi-objective optimization. Tan *et al.* [7] and Saadah *et al.* [8] employed the travel distance and the number of vehicles to be minimized. Chitty and Hernandez [9] tried to minimize the total mean transit time and the total variance in transit time.

In this paper, we employ three objectives. One is to minimize maximum routing time and another is to minimize the number of vehicles in VRPs. It should be noted that we don't employ the total routing time of all the vehicles, but use the maximum routing time among the vehicles. We employed it in order to minimize the active duration of the central depot. Even if the total routing time is minimized, the central depot should be opened until the last vehicle comes back to the depot. In order to minimize the active duration of the central depot, the maximum routing time should be minimized.

As for the third objective, we consider the similarity of solutions. In this paper, we suppose two periods with different demands. One period has a normal demand of customers. The other has a higher demand. We refer the former period and the latter period as Normal Demand Period (NDP) and High Demand Period (HDP), respectively. We define the demand in the HDP as an extended demand of the NDP in this paper. For example, we assume that the demand in the HDP is a demand occurring in a high season such as Christmas season. In that season, the depot may have an extra demand as well as the demand in the normal season. In order to avoid big changes of each route from the depot, a solution (i.e., a set of route) in HDP should be similar to a solution in NDP. This situation requires us to consider the similarity of solutions on different non-dominated solutions in multi-objective VRPs.

In order to find a set of non-dominated solutions in the HDP that is similar to a set of non-dominated solutions in the NDP, we apply a two-fold EMO algorithm [10] to the problem. In a two-fold EMO algorithm, first we find a set of non-dominated solutions for the NDP by an EMO algorithm. In order to enhance the similarity between sets of non-dominated solutions in NDP and HDP, we showed the effectiveness of utilization of a solution set in NDP for population initialization in HDP [10]. In this paper, we propose a local search method in a memetic EMO algorithm to enhance the solution similarity in HDP to a solution set in NDP.

We organize this paper as follows: Section 2 gives the problem model for multi-objective VRPs. We define a measure of the similarity between solutions in Section 3. The outline of our two-fold EMO algorithm is described in Section 4. In section 5, we propose a local search algorithm that enhances the solution similarity introduced in the second phase of the two-fold memetic EMO algorithm. Section 6 describes simulation results that show the effectiveness of the proposed local search algorithm. Conclusions are drawn in Section 7.

2 Multi-objective Vehicle Routing Problems

The domain of VRPs has large variety of problems such as capacitated VRP, multiple depot VRP, periodic VRP, split delivery VRP, stochastic VRP, VRP with backhauls, VRP with pick-up and delivering, VRP with satellite facilities, VRP with time windows and so on. These problems have the basic architecture of the VRP except their own constraints. Those constraints are arisen in practical cases. Please see for the detail of the VRP problem in [11].

A solution of the VRPs is represented by a permutation of N customers, and we split it into M parts as shown in Figure 1. It shows eight customers that are served by three vehicles. The first vehicle denoted v_1 in the figure visits three customers in the order of Customers 1, 2, and 3. Each solution is divided by a closed triangle. Therefore the driving duration for v_1 is calculated by $c_{D,1} + c_{1,2} + c_{2,3} + c_{3,D}$. Figure 2 shows an example of three routes depicted on the map of eight customers and the depot. It should be noted that, we consider only problems with symmetric cost where $c_{1,2} = c_{2,1}$ in this paper.

The objective employed in many VRPs is to minimize a total cost is described as follows:

$$\text{Min. } \sum_{k=1}^M c_k, \quad (1)$$

where M is the number of vehicles that start from the depot and is routed by a sequence of customers, then return to the depot. The cost of k -th vehicle is denoted by c_k and described as follows:

$$c_k = c_{D,1} + \sum_{i=1}^{n_k-1} c_{i,i+1} + c_{n_k,D}, \quad (2)$$

where $c_{i,j}$ means the cost between Customers i and j . Let us denote D as the index for the depot in this paper. Equation (2) indicates the sum of the cost between the

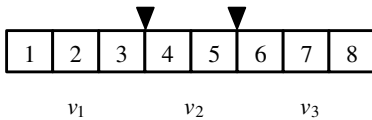


Fig. 1. An example of eight customers visited by three vehicles. Each triangle shows the split between the routes for vehicles.

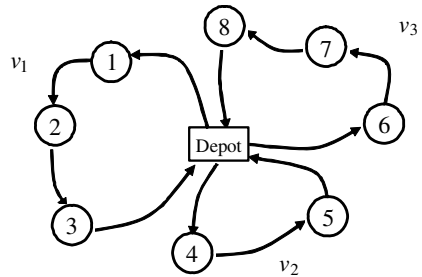


Fig. 2. An example of eight customers visited by three vehicles

depot and the first customer assigned to the k -th vehicle (i.e., $c_{D,1}$), the total cost from the 1st customer to the n_k -th customer (i.e., $\sum_{i=1}^{n_k-1} c_{i,i+1}$), and the cost between the final customer n_k and the depot. Each vehicle is assigned to visit n_k customers, thus we have $N = \sum_{k=1}^M n_k$ customers in total. The aim of this VRP is to find a set of sequences of customers that minimizes the total cost. Each customer should be visited exactly once by one vehicle.

While the total cost of all the vehicles is ordinarily employed in the VRP, we employ the maximum cost to be minimized in this paper. When the cost $c_{i,j}$ is related to the driving duration between Customers i and j in Equation (2), the total cost c_k for the k -th vehicle means the driving duration from the starting time from the depot to the returning time to the depot. In order to minimize the activity duration of the depot, the maximum duration of the vehicles should be minimized since the depot should wait until all the vehicles return to the depot. We also consider the minimization of the number of vehicles in our multi-objective VRP. The objectives in this paper can be described as follows:

$$\text{Min. } \max_k c_k, \quad (3)$$

$$\text{Min. } M. \quad (4)$$

When we have a solution with $M = 1$, our problem becomes the traveling salesman problem, the TSP. In that case, the other objective, to minimize the maximum driving duration in Equation (3), becomes just to minimize the total driving duration by one vehicle. On the other hand, the maximum driving duration becomes minimum when the number of vehicles equals to the number of customers (i.e., $M = N$). In that case, each vehicle visits only one customer. The driving duration for each vehicle in (2) can be described as follows:

$$c_k = c_{D,[1]_k} + c_{[1]_k,D}, \quad (5)$$

where $[1]_k$ denotes the index of the customer visited by the k -th vehicle. The maximum driving duration in (5) over M vehicles becomes the optimal value of that objective in the case of $M = N$. Therefore we face the trade off between these two objectives: the minimization of the maximum driving duration and the minimization of the number of vehicles.

We consider two periods with different demands: NDP and HDP. In NDP, a normal demand of customers should be satisfied. On the other hand, extra demands should also be satisfied in HDP. In this paper, we increase the number of customers in HDP. That is, $N_{NDP} < N_{HDP}$, where N_{NDP} and N_{HDP} are the number of customers in NDP and HDP, respectively. We can obtain a set of non-dominated solutions for each problem. We refer a set of non-dominated solutions for NDP as Ψ_{NDP} , and that for HDP as Ψ_{HDP} . These two sets of non-dominated solutions can be obtained by applying one of EMO algorithms such as NSGA-II [12]. But if we apply the

algorithm to each of NDP and HDP independently, we can not expect to obtain a set of solutions with similar routes in HDP to that obtained for NDP.

3 Similarity Between Sets of Non-dominated Solutions

We define a similarity measure between a non-dominated solution Ψ_{HDP} obtained for HDP and Ψ_{NDP} for NDP. Since the aim of measuring the similarity is to find a solution in HDP that is similar to one in NDP, we measure the similarity of a solution in HDP to the one in NDP. We measure it by a ratio of the number of the same edges to the number of all edges in a solution of NDP.

We define the similarity of solution x in HDP is as follows:

$$similarity(x) = \max_{y \in \Psi_{NDP}} (similarity(x, y)) = \max_{y \in \Psi_{NDP}} \left(\frac{sames(x, y)}{edges(y)} \right), x \in \Psi_{HDP}, (6)$$

where $similarity(x, y)$ is the similarity of the solution x to the solution y , that is calculated by $sames(x, y)$ (i.e., the number of the same edges) and $edges(y)$ (i.e., the number of edges in a solution y). Figure 3 shows an example to calculate the similarity of solution x (5, 1, 2, 3, 6, 4 with four vehicles) to three non-dominated solutions (2, 1, 3 with one vehicle, 1, 2, 3 with two, and 1, 3, 2 with three) obtained for NDP. The similarity of solutions x becomes the maximum similarity 0.8 through the calculation.

4 Two-Fold EMO Algorithm for Multi-objective VRPs

In this section, we show a Two-Fold EMO algorithm for our multi-objective VRPs [10]. Then we show how we apply a two-fold EMO algorithm to obtain a similar set of solutions in NDP and HDP.

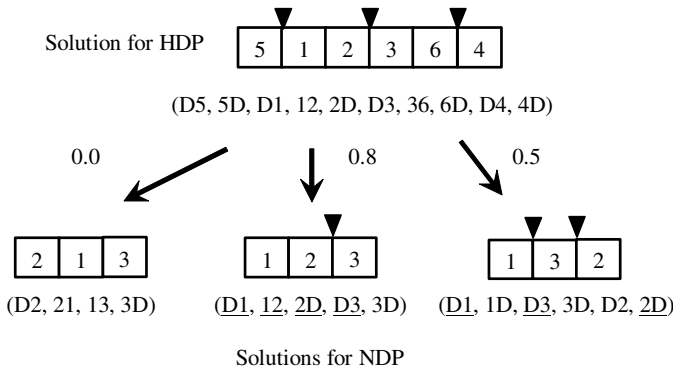


Fig. 3. The similarity of a solution for HDP that is calculated as the maximum similarity among three similarities

4.1 Genetic Operators

[Crossover]

We employ the edge exchange crossover (EXX) [13] as a crossover operator. This crossover produces offspring only by exchanging edges in parents chromosome, where an edge means a segment between two customers. Therefore offspring chromosomes preserve segments between customers well. The following is the algorithm of this crossover:

- Step 1) Select an edge randomly from one parent (Parent 1), and let i_1 be the position of the edge. Let i_2 be the position of the edge of the other parent (Parent 2) whose origin customer is the same as that of the i_1 -th edge in Parent 1.
- Step 2) Let j_2 be the position of the edge of Parent 2 whose origin customer is the same as the destination customer of the i_1 -th edge in Parent 1, and j_1 be the position of the edge of Parent 1 whose origin city is the same as the destination customer of the i_2 -th edge in Parent 2.
- Step 3) Exchange the i_1 -th edge of Parent 1 and the i_2 -th edge of Parent 2. If the destination customers of them are the same, terminate the algorithm.
- Step 4) Invert the order of the edges and their origin and destination customers of Parent 1 between the positions i_1 and j_1 , and those of Parent 2 between the positions i_2 and j_2 .
- Step 5) Let $i_1 = j_1$ and $i_2 = j_2$ and go to Step 2.

Figure 4 shows the above procedure between the following parents with one vehicle:

Parent 1: (1 2 3 4 5 6 7 8), and Parent 2: (2 5 4 1 6 7 3 8).

Their edges can be represented as follows:

Parent 1: (12 23 34 45 56 67 78 81), and Parent 2: (25 54 51 16 67 73 38 82).

As an example where the edge 23 of Parent 1 is taken as the starting edge in Step 1 of the above procedure. We have the following offspring after the crossover operation

Offspring 1: (1 2 5 4 3 8 7 6), and Offspring 2: (7 3 2 8 1 4 5 6).

In this paper, we consider any chromosome with multiple vehicles as that with one vehicle. Thus the following cases have the same result in the order of the customers while their positions of Depot do not change between parent and offspring.

Case A: Parent 1: (1 2 | 3 4 5 6 | 7 8), and Parent 2: (2 5 4 1 | 6 7 3 8).

Case B: Parent 1: (1 2 3 | 4 5 6 7 8), and Parent 2: (2 | 5 4 | 1 6 7 | 3 8).

Case A: Offspring 1: (1 2 | 5 4 3 8 | 7 6), and Offspring 2: (7 3 2 8 | 1 4 5 6).

Case B: Offspring 1: (1 2 5 | 4 3 8 7 6), and Offspring 2: (7 | 3 2 | 8 1 4 | 5 6).

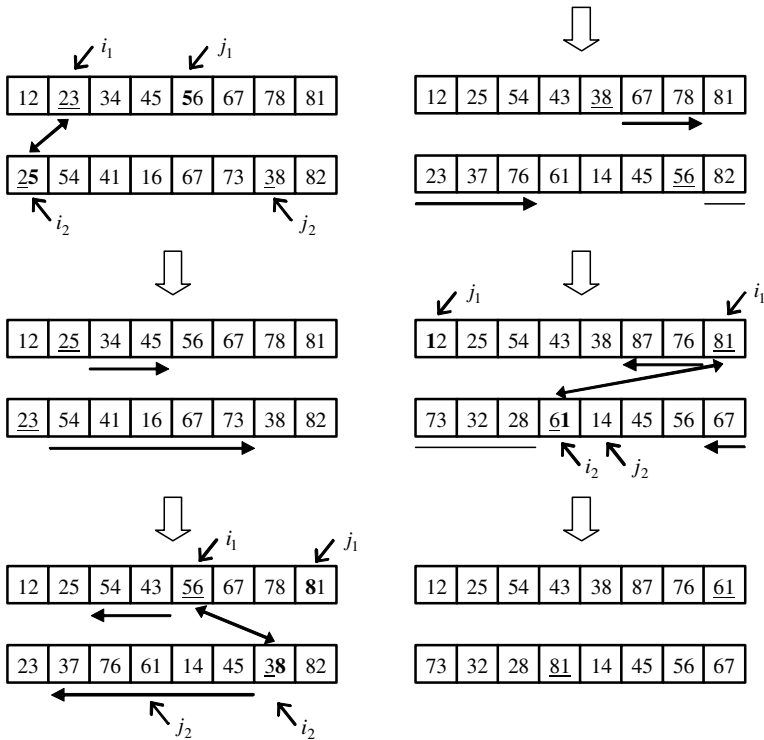


Fig. 4. Examples of Edge Exchange Crossover [13]

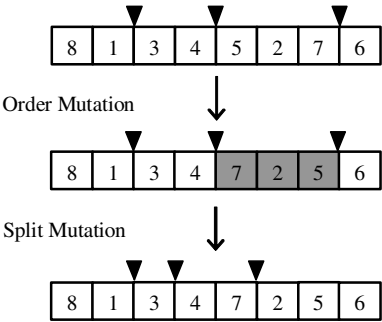


Fig. 5. Examples of two mutation operators. In the order mutation, a selected route is inverted its order of customers. In the split mutation, locations of splits are changed randomly.

[Mutation]

As for the mutation, we employ two kinds of operators in order to modify the order of customers and the locations of splits in a selected route. Figure 5 shows examples of these mutations. It should be noted that the order mutation itself does not affect the two objectives (i.e., the maximum driving duration and the number of vehicles). But it

can be useful to increase the variety of solutions when it is used with the crossover and the split mutation.

It should be noted that through crossover and mutation in this paper, the number of vehicles does not change. Therefore if there is no individual with a certain number of vehicles, no solution with that number of vehicles is generated through genetic search.

4.2 Two-Fold EMO Algorithm

In our multi-objective VRP, we have two periods, NDP and HDP. Since HDP has extra demands of customers with the demands of NDP, we have two approaches to search a set of non-dominated solutions for each of NDP and HDP. One approach is to apply an EMO algorithm individually to each of them. The other is to apply a two-fold EMO algorithm [10] to them. In the two-fold EMO algorithm, first we find a set of non-dominated solutions for the NDP by an EMO algorithm. Then we generate a set of initial solutions for the HDP from the non-dominated solutions for the NDP. We apply an EMO algorithm to the HDP with initial solutions that are similar to those of the NDP problem. In our former study [10], we showed that the two-fold EMO algorithm has the better performance than applying an EMO algorithm individually. The procedure of the two-fold EMO algorithm is described as follows:

[Two-Fold EMO Algorithm]

- Step 1: Initialize a set of solutions randomly for the NDP. The number of vehicles and the order of customers in each solution are defined randomly.
- Step 2: Apply an EMO algorithm to find a set of non-dominated solutions until the specified stopping condition is satisfied.
- Step 3: Obtain a set of non-dominated solutions for the NDP.
- Step 4: Initialize a set of solutions for the HDP using a set of non-dominated solutions of the NDP.
- Step 5: Apply an EMO algorithm to find a set of non-dominated solutions until the specified stopping condition is satisfied.
- Step 6: Obtain a set of non-dominated solutions for the HDP.

In Step 4, we initialize a set of solutions as follows:

- Step 4.1: Obtain a set of non-dominated solutions of the NDP.
- Step 4.2: Specify a solution of the set.
- Step 4.3: Insert new customers randomly into the solution.
- Step 4.4: Repeat Steps 4.2 and 4.3 until all solutions in the set of non-dominated solutions of the NDP are modified.

It should be noted that the number of vehicles of each solution is not changed by this initialization. The number of vehicles of each solution is changed by the crossover operation. Using this initialization method, we found that the similarity between non-dominated solutions for the NDP and those for the HDP can be increased [10].

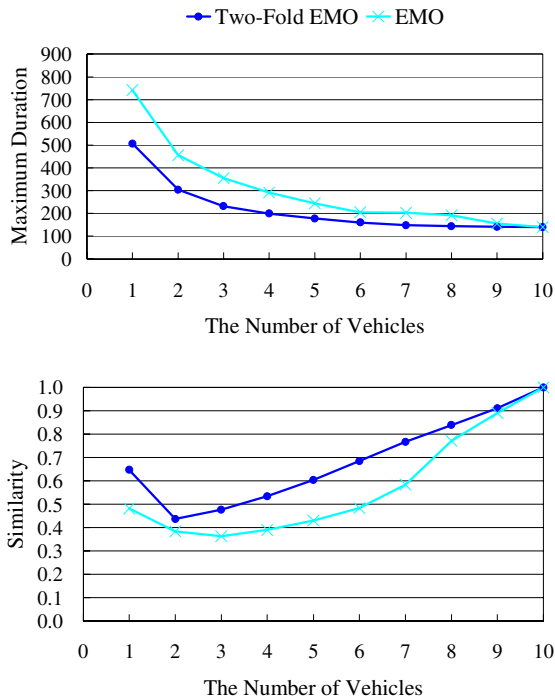


Fig. 6. The obtained non-dominated solutions in the HDP with ten customers. The similarity is to be maximized, the maximum duration to be minimized, and the number of vehicles to be minimized.

We applied the two-fold EMO algorithm to a VRP that has five customers in NDP and ten customers in HDP. As for an EMO algorithm, we employed NSGA-II [12]. Figure 6 shows the results of the two-fold EMO, and the EMO applied the HDP with a population initialized randomly. In this problem, we consider only two objectives: the maximum duration and the number of vehicles. We obtained the average maximum duration of a set of non-dominated solutions over 100 trials. We calculate the average similarity after obtaining a set of non-dominated solutions for HDP. In the first figure of Figure 6, we can find that the two-fold EMO can find better non-dominated solutions with respect to the minimization of the maximum duration and the number of vehicles. From the second figure, we can find that the similarity of non-dominated solutions obtained by the two-fold EMO algorithm is better than that obtained by the EMO algorithm. In this experiment, we can see that improving solutions with respect to the maximum duration does not lead to deterioration of the similarity of non-dominated solutions to those in NDP. Therefore we can say that the two-fold EMO could find the better solutions compared to the EMO for HDP without initial solutions from NDP.

5 Two-Fold Memetic EMO Algorithm

In this paper, we propose a local search that enhances the similarity of non-dominated solutions for HDP. In order to increase the similarity of a solution for HDP, we incorporate segments between customers from a solution of NDP to a solution of HDP. Therefore we introduce this procedure in an EMO search for HDP not for NDP. The algorithm of the proposed local search can be described as follows:

[Local Search Algorithm]

- Step 1: Select a solution x from the current Ψ_{HDP} .
- Step 2: Select a non-dominated solution y from Ψ_{NDP} that is used for the calculation of the maximum similarity of a solution x .
- Step 3: Select an edge between two customers in y . Therefore the edge should be selected within a vehicle.
- Step 4: Find a start customer of the selected edge in x .
- Step 5: Incorporate the edge to x at the position of the first customer in x . Since the following customer to the first customer in x is replaced by the second customer in the edge, a repairing process should be followed. Find the second customer in x , and replace that with the following customer.
- Step 6: Return to Step 3 until all edges in y is incorporated in x .

Figure 7 shows an example of this local search. We apply this local search to each solution of the current set of non-dominated solutions. Since this local search process is introduced to an EMO search in HDP, the two-fold memetic EMO algorithm can be depicted as Figure 8.

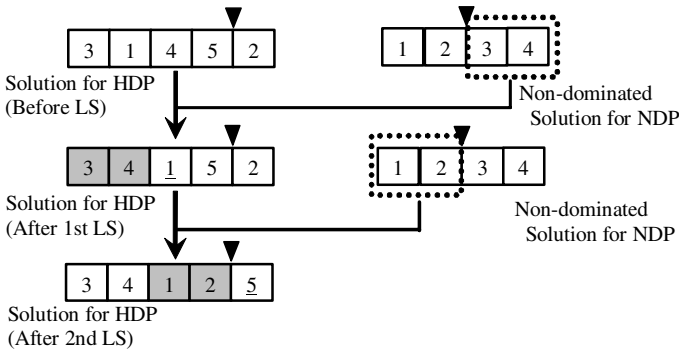


Fig. 7. Local search applied to a solution for HDP. An edge (3,4) in a non-dominated solution for NDP is incorporated to a solution for HDP. Since (3,1) in the solution for HDP is replaced with (3,4) the customers 1 and 4 in the solutions for HDP should be exchanged in a repairing process. Since the solution for NDP has two edges in its string, the local search process terminates at the second time.

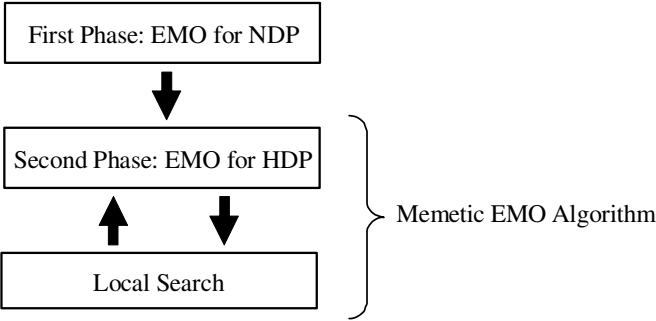


Fig. 8. Two-Fold Memetic EMO Algorithm. A local search is introduced in the second phase of EMO search for HDP.

Table 1. The parameter specifications in EMO algorithms

# of population	30
Crossover rate	1.0
Order mutation rate	0.04
Split mutation rate	0.02
Terminal Generation	2000

6 Simulation Results by Two-Fold Memetic EMO Algorithm

We show the simulation result on a multi-objective VRP with NDP and HDP. In that problem, there are five customers in NDP, and ten customers in HDP. Table 1 shows the parameter specifications in our two-fold memetic EMO algorithm. We apply our two-fold memetic EMO algorithm to the problem with 100 different initial solution sets. That is, we obtain average results over 100 trials in a problem. In this section, first we examine the effect of introducing the similarity as third objective. Then we show the effectiveness of the proposed local search to enhance the similarity.

6.1 Effect of Similarity

We apply two EMO algorithms to a problem in HDP. One is the two-fold EMO algorithm with three objectives (2F-EMO-3). The other is the two-fold EMO algorithm with two objectives (2F-EMO-2). We don't employ the proposed local search in this section. Therefore both algorithms have an initial population generated by a non-dominated solutions for NDP. The same result is obtained by Two-Fold EMO in Figure 6 by 2F-EMO-2. We calculate the similarity of non-dominated solutions obtained by 2F-EMO-2 after the search. Figure 9 shows the simulation results obtained by these algorithms. Since the 2F-EMO-3 finds non-dominated solutions on the surface of three objectives, we project them onto the two objective space in Figure 9. Therefore they are projected between two lines. We depicted two lines of extreme cases. That is, the lowest and the highest similarity on the space with the maximum duration and the number of vehicles. On the other hand, the shortest

and the longest maximum duration on the space with the similarity and the number of vehicles. From Figure 9, we can see that slightly better solutions are obtained by the 2F-EMO-2 with respect to the minimization of the maximum duration. On the other hand, the 2F-EMO-2 finds worse solutions with respect to the maximization of the similarity. As for the 2F-EMO-3, it produces slightly better non-dominated solutions in the similarity when their maximum duration becomes near to those of the 2F-EMO-2. On the other hand, when the 2F-EMO-3 sacrifices the minimization of the maximum duration, the similarity of non-dominated solutions becomes much better than the 2F-EMO-2. Through this figure, we can find that the similarity of non-dominated solutions has the trade-off relationship with the maximum duration. Therefore the introduction of the similarity as third objective is needed for those who wants to have similar routes in HDP to NDP.

6.2 Effect of Local Search to Enhance the Similarity in HDP

In this section, we examine the effectiveness of the proposed local search to enhance the similarity of non-dominated solutions for HDP. We compare the 2F-EMO-3 and the two-fold memetic EMO algorithm (2F-mEMO). From Figure 10, We can see that the 2F-EMO-3 could find better solutions with respect to the maximum duration when

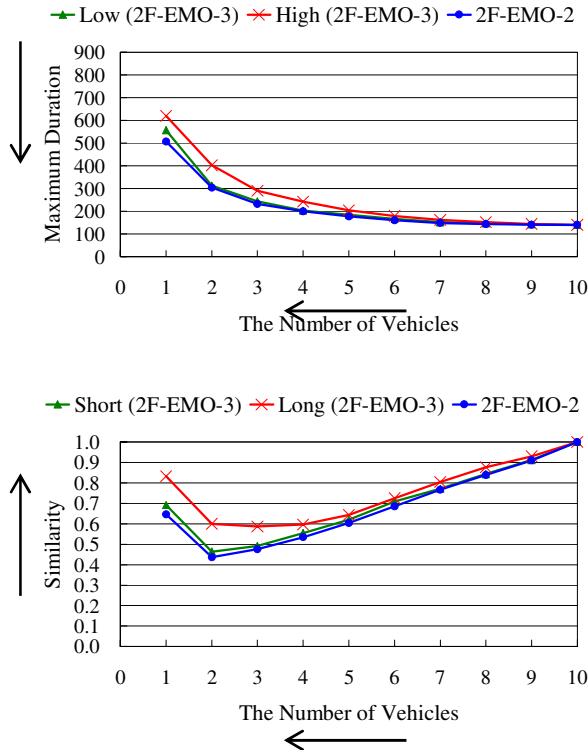


Fig. 9. The effect of the similarity

it sacrifices the similarity. On the other hand, almost similar maximum durations are obtained by both the algorithms when they seek to maximize the similarity. Although both the algorithms have similar maximum durations in the case of high similarity, the degree of the similarity of these algorithms is quite different in the latter figure of Figure 10. Using the proposed local search, the 2F-mEMO could enhance the similarity especially in non-dominated solutions with two through six vehicles. As for the solutions with more than seven vehicles the similarity is not improved well. This is because each vehicle should not visit so many customers when the number of vehicles is similar to the number of customers. Similar routes are required when each vehicle has several customers to visit. From Figure 10, we can see that the proposed local search is very much effective in enhancing the similarity with a slight deterioration in the maximum duration.

7 Conclusion

In this paper, we proposed a local search that can be used in a two-fold memetic EMO algorithm for multiple-objective VRPs with different demands. The simulation results show that the proposed method have the fine effectiveness to enhance the similarity of

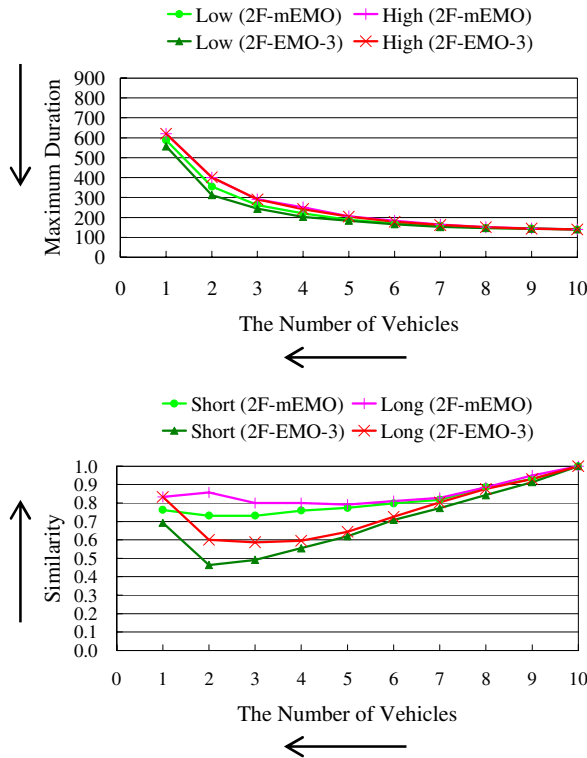


Fig. 10. The effect of the local search in HDP

obtained routes for vehicles. Although the local search slightly deteriorates the maximum duration, we can see the effectiveness of the similarity of the routes because the possibility of getting lost the way of drivers may be decreased. If drivers get lost their ways during their delivery, the cost of his routes may increase. The enhancing the similarity of set of non-dominated solutions seems important when we apply EMO algorithms to practical problems.

Since the algorithm of the proposed local search to enhance the similarity depends on the problem specifications, we should make further research on the similarity of a set of non-dominated solutions with different problems. We may define similarity on the genotype, and it on the phenotype. Since the similarity on the phenotype may depend on problems, we should research further on the similarity on the genotype of various problems.

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