



# Using the Vehicle Routing Problem for the Transportation of Hazardous Materials

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## Abstract

Transportation of hazardous materials (hazmats) is a decision problem that has been attracted much attention due to the risk factor involved. A considerable amount of models have been developed that employ single or multiple objective shortest path algorithms minimising the risks for a given origin-destination pair. However in many real life applications (i.e. transportation of gas cylinders), transportation of hazmats calls for the determination of a set of routes used by a fleet of trucks to serve a set of customers, rather than determination of a single optimal route as shortest path algorithms produce. In this paper, we focus on population exposure risk mitigation via production of truck-routes by solving a variant of the Vehicle Routing Problem. For this purpose we employ a single parameter metaheuristic algorithm. A case study of this approach is also demonstrated.

Keywords: Transportation, Vehicle Routing Problem, Metaheuristics, Risk

## 1. Introduction

**T**ransportation of hazardous materials (hazmats) is an important decision problem, which has received considerable attention [List et al. (1991), Karkazis and Boffey (1995), Verter and Erkut (1997), Erkut, and Verter (1998), Giannikos (1998), Erkut and Ingolfsson (2000), Frank et al. (2000), Leonelli et al. (2000)]. According to the Federal Hazardous Materials Transportation Law, material (including an explosive, radioactive material, etiologic agent, flammable or combustible liquid or solid, poison, oxidizing or corrosive material, and compressed gas) or a group or class of materials are designated as hazardous when transporting the material in commerce in a particular amount and form may pose an unreasonable risk to health and safety or property.

The main difference between the transportation of hazmats and transportation of other materials or goods is the risk associated with an accident release (incident) during the transportation. To understand the magnitude of the problem, we note that during the last 11 years, the transportation industry reported to United States Department of Transportation (DoT) that, on 800,000 shipments of hazmats that occur daily, about 22 deaths and 490 injuries each year occurred because of some 13,113 releases of hazmat in transportation. In addition, despite hazmat shippers implement measures to promote transportation safety, roughly 400 serious hazmat transportation incidents per year are reported to DoT. The problem becomes more and more serious since the number of hazmat inci-

dents reported to DoT has increased from 8,879 in 1990 to 16,881 in 1999 and 17,224 in 2000. Furthermore, hazmat incidents were responsible for \$57,530,562 property damage in 2000 [<http://hazmat.dot.gov>].

Though hazmats are transported by different modes (air, highway, railway, water, freight forwarder), the majority of the aforementioned incidents (over 80%) occur on highways. Thus, Hazardous Materials Transportation Uniform Safety Act of 1993 places considerably emphasis on route decision making as a means of reducing public risk from hazmat shipments.

A considerable amount of models have been developed that employ single or multiple objective shortest path algorithms that minimising the risks for a given origin-destination pair. However in many real life applications (i.e. transportation of gas cylinders), transportation of hazmats calls for the determination of a set of routes used by a fleet of trucks to serve a set of customers, rather than determination of a single optimal route as shortest path algorithms do. Such an algorithm that aims to mitigate risk via generation of routes, by solving a variant of the Vehicle Routing Problem, is presented in this paper.

## ***2. The Vehicle Routing Problem***

The Vehicle Routing Problem (VRP) lies in the heart of distribution management, which deals with the determination of the optimal sequence of deliveries conducted by a fleet of vehicles (trucks in our case) based at one or more depots to serve a set of customers. In real-life applications, several operational constraints are imposed on the route construction; the service may involve both pickups and deliveries, the capacity of a vehicle must not be exceeded, the vehicle routing plan (tour) has to be completed within a pre-specified time window, the service of the customer may occur within given time windows, the deliveries may be conducted by heterogeneous vehicles, the customer demands may not be completely known in advance, the demands or the travel times may vary dynamically, the service of customer may be split among different vehicles, vehicles may not need to return to depot (gas distribution centre).

In this paper, we consider the static and deterministic version of the problem, known as the Capacitated VRP (CVRP). The CVRP calls for the determination of the optimal sequence of deliveries conducted by a fleet of homogeneous vehicles, based at one depot, to serve a set of customers such that

- the total distance travelled by the fleet is minimized
- the fixed capacity of a vehicle cannot be exceeded
- each customer has known demand that must be satisfied
- the demand of each customer is satisfied by exactly one visit of a single vehicle
- each vehicle must leave and return to the depot
- the proposed tour has to be completed within a pre-specified time interval.

The CVRP is one of the most difficult problems (NP-hard) in the field of combinatorial optimisation. Thus, in practical applications that involve more than 75 customers, a large number of approaches focus on using heuristics [Laporte et al. 2000]. The reason is that the solution space that has to be swept in such problems is enormous and most exact algorithms fail to come up with reliable solutions in polynomial time computation terms. Heuristics produce solutions within modest computing times but due to the relatively limited exploration of the solution space they perform, their solutions are not of high quality. Therefore, new modern heuristics algorithms were developed, termed as meta-heuristics [Taillard (1993), Osman (1993), Rochat and Taillard (1995), Barbarosoglu and Ozgur (1999), Campos and Mota (2000), Ichoua et al. (2000), Van Breedam, A. (2001), Rego (2001)], which focused on performing a deep exploration of the most promising regions of the solution space by using sophisticated improvement procedures, and produced much higher quality solutions than that obtained by heuristics. Such a meta-heuristic is presented in this paper.

### 3. Link risk

In this paper, we focus on risk mitigation via selection of routes by solving a variant of the VRP. Our goal is to select routes not close to aggregate population points in order to reduce the number of people placed at risk (population exposure risk). Hence, routing decisions is made in a new space, the so-called risk space, where risk of a point is defined as the product of population ( $N_i$ ) of an aggregate population point (city, town, ward)  $i$  and distance-length ( $d_i$ ) between this point and the aggregate population point  $i$ :

$$\text{risk} = \rho(x, y) = \sum_i \frac{N_i}{\sqrt{[(x - x_i)^2 + (y - y_i)^2]}} \quad (1)$$

The risk space is a Riemannian manifold because the metric is different from point to point in this space. Moreover, the length of road in the risk space is different from the length of the same road in the real space (due to the transformation of real space because of the risk function). Hence, contours of different risk level (i.e. low, medium, and high risk) are determined around the geographic population points.

Since the length of the road in the risk space is different from the length of the same road in the real space, the length between the two nodes that define a projected road in the risk space has to be computed (a road is considered as link which is defined by the up- and down-node respectively). In addition, because of the risk of a point is expressed by the analytical function (1), the length of the projected road in the risk space is a curve that is computed by the following integral (since routing of the trucks takes place on a *real* (not straight lines) road network

$$\text{Length} = \int_C dl = \int_{x_1}^{x_2} \left\| \frac{dr}{dx}(x) \right\| dx = \int_{x_1}^{x_2} \left\| \frac{\partial \rho}{\partial x} + \frac{\partial \rho}{\partial y} \left( \frac{y_2 - y_1}{x_2 - x_1} \right) \right\| dx \quad (2)$$

where

$$r(x) = \rho \left[ x, y_1 + \frac{y_2 - y_1}{x_2 - x_1} (x - x_1) \right] \quad l \text{ is the projection of a road in the risk space, } l \text{ is a}$$

small segment of the curve,  $(x_1, y_1)$  and  $(x_2, y_2)$  are the coordinates of the down-node and the up-node that define a link (road).

In order to be able to employ our metaheuristic algorithm, we have to construct the Origin-Destination matrix, which consists of the shortest paths for a given pair of customers or given pair of depot and customer. The computation of the shortest path tree associated with each and every one of the network's nodes requires the employment of Dijkstra algorithm that computes the shortest paths from a "source" node to all other nodes of the real road network. However, the repeated application of Dijkstra algorithm finds the shortest path tree associated with each and every one of the network's nodes.

Provided that the shortest paths are known, our metaheuristic algorithm is employed in order to determine the best sequence of customer to be served such that the set of composed routes will minimise risk of population exposure.

#### 4. The List Based Threshold Accepting (LBTA) Algorithm

List Based Threshold Accepting is a metaheuristic algorithm that belongs to the class of threshold accepting based algorithms [Dueck and Scheuer (1990)]. LBTA has simple structure, as it is described in Figure 1, since the inner and outer loops have been merged into a single iteration loop, opposite to a typical threshold accepting algorithm and gener-

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Initial system configuration  $s$ , initial threshold list ;
Loop: while outer loop criterion .NOT. satisfied do
    Derive configuration  $s'$  from  $s$  ;

    If  $T_{new} = \frac{c(s') - c(s)}{c(s)} \leq T_{max}$  then
        {set  $s = s'$  ; check if  $c(s) < c(s_{best})$  then  $s_{best} = s$  ;}

    If ( $s$  has changed then insert  $T_{new}$  in list)
        { Insert:  $T_{new} \rightarrow List$ ; Pop:  $List \rightarrow T_{max}$  }

Repeat loop;
Report best solution found;

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Figure 1. The LBTA metaheuristic algorithm

al stochastic methods based on Simulated Annealing [Kirkpatrick et al.(1983)]. In addition, opposite to the majority of metaheuristics, LBTA is a single parameter (list size) stochastic search method, hence it is very easy to tune. Acceptances throughout the LBTA method occur for any maximum threshold value held in the list, without requiring back-tracking [Tarantilis and Kiranoudis (2001)] of the threshold value since they were inserted by acceptance at that point.

LBTA iteratively searches the solution space guided by a deterministic control parameter, in the same units as the cost function, to reveal promising regions for better configurations. This parameter is called threshold and reduced (cooling) throughout the method. The proposed solution is derived by employing local search moves [Waters (1987)]. These moves improve an initial solution  $s$  (set of truck routes) by taking a customer from its position and moving it to another in the same or different truck route or by exchanging road segments (links) in the same way.

LBTA method is described as follows:

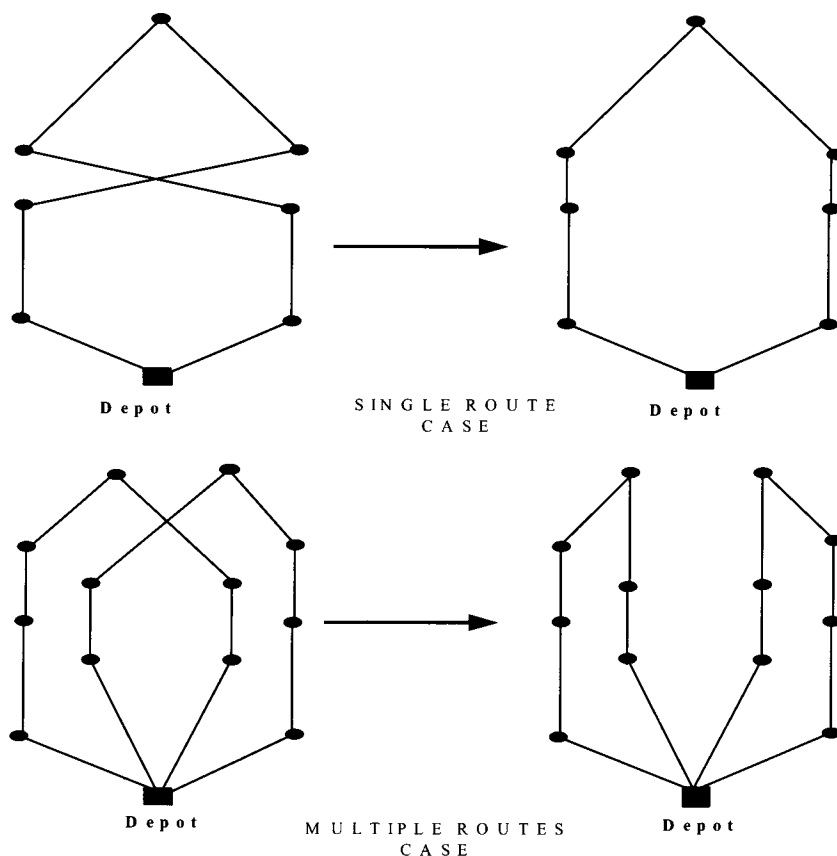


Figure 2. The 2-opt move

### Step 1 (Initialisation phase)

An initial positive value of threshold is selected ( $T > 0$ ), and LBTA algorithm starts with an initial solution (set of routes) where each truck serves only one customer.

### Step 2

A local search procedure is conducted in order to compute the threshold values that represent the list of the method. Local search uses a blend of the moves, which convert one truck tour into another:

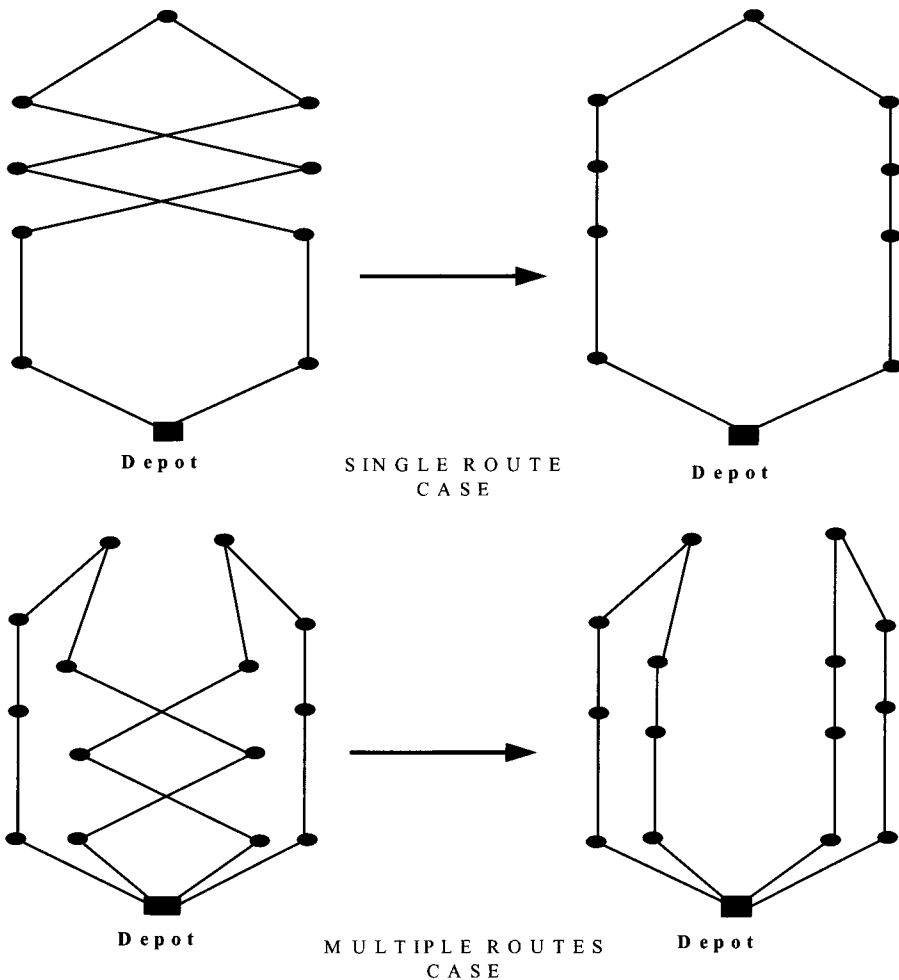


Figure 3. The Exchange move

- **2-opt move**

In the case of a single route, 2-opt move is employed as follows: Suppose a single route consists of the following set of customers (nodes) in the given order  $(u_o, u_1, u_2, \dots, u_n, u_o)$ , and let  $\{(u_i, u_{i+1}); (u_j, u_{j+1})\}$  be a set of two edges belong to this route that form a criss-cross. 2-opt move eliminates any criss-cross and reverses a section of the route by deleting the edges  $(u_i, u_{i+1}), (u_j, u_{j+1})$  and replacing them with  $(u_i, u_j), (u_{i+1}, u_{j+1})$  to reconstruct the route. 2-opt move is applied exactly in the same way when the edges  $(u_i, u_{i+1})$  and  $(u_j, u_{j+1})$  belong to different routes but they again form a criss-cross. This is demonstrated in Figure 2.

- **Exchange move**

The Exchange move swaps two customers from the same route. Consequently, if the initial tour consists of the following set of customers  $(u_o, \dots, u_{i-1}, u_i, u_{i+1}, \dots, u_{j-1}, u_j, u_{j+1}, \dots, u_o)$ , the improved one is constructed as  $(u_o, \dots, u_{i-1}, u_j, u_{i+1}, \dots, u_{j-1}, u_i, u_{j+1}, \dots, u_o)$ . The same procedure is conducted in the case of multiple routes but the swapping of customers takes place between different routes. This is demonstrated in Figure 3.

- **Relocate move**

The relocate move transfers a node from its position in one route to another in either the same or a different route. Consequently, while the initial tour was  $(u_o, \dots, u_i, u_{i+1}, \dots, u_{j-1}, u_j, u_{j+1}, \dots, u_o)$ , the improved one is constructed as  $(u_o, \dots, u_i, u_j, u_{i+1}, \dots, u_{j-1}, u_{j+1}, \dots, u_o)$ . This is demonstrated in Figure 4.

The customers involved in the implementation of the above moves and the type of the move are selected stochastically. The list serves as a memory of the variability of local function values, stored in the form of value changes from each old configuration. The storage scheme employs up to  $M$  (user defined parameter) values in a binary tree list. This is a special list structure suitable for inserting and retrieving information on shorted elements in extremely fast computation time ( $O(n \log n)$  complexity). Local search starts as follows:

- A neighbour  $s'$  is derived from a current solution  $s$  by employing one of the local search moves
- The new threshold value is computed,  $T_{new} = \frac{c(s') - c(s)}{c(s)}$ . The normalization of the threshold values stored in the list helps the LBTA method accept easy cost-increasing neighbouring solutions of a current solution. If  $T_{new}$  is positive and lower than the maximum element of the list,  $T_{max}$ ,  $T_{new}$  is inserted in the list. This iterative procedure is repeated for each one of the moves, until the list is filled up with  $M$  "sufficiently large" threshold values.

### Step 3

In this step, the maximum threshold value stored in the list,  $T_{max}$ , is selected to decide on new acceptances.

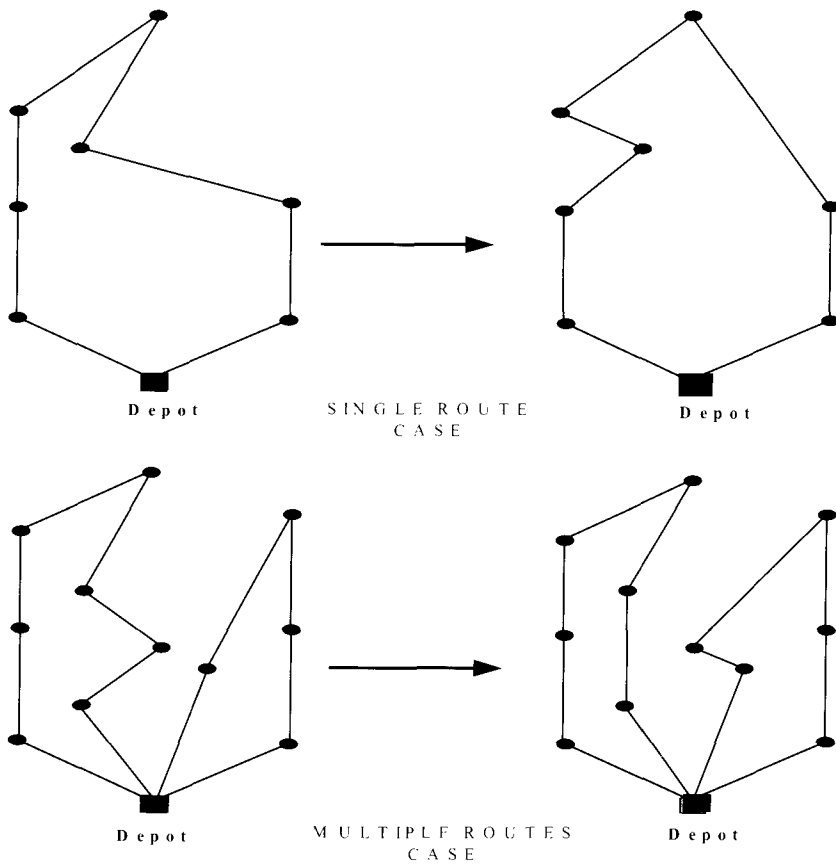


Figure 4. The Relocate move

- If the proposed solution  $s'$  is accepted, i.e.  $T_{new} = \frac{c(s') - c(s)}{c(s)} \leq T_{max}$ ,  $T_{new}$  is appropriately inserted while the old  $T_{max}$  is potted from the list.
- If  $T_{new}$  is not accepted, the list is not modified and a new proposed solution is generated by using a local search move.
- The same procedure is repeated until no proposed solution is found that satisfies the relation  $T_{new} \leq T_{max}$ , for a number of feasible moves (stopping criterion)

It's worth mentioning that the proposed LBTA method has already tested for the basic version of the CVRP (minimisation of total distance in non-risk space)[Tarantilis and Kiranoudis (2001)] on a set of well-known benchmark instances [Christofides et al. (1979)], producing high quality solutions.



## 5. Case study

In this section, we demonstrate an example of determination of hazmat routes with respect to minimisation of population exposure risk, by solving a variant of the VRP.

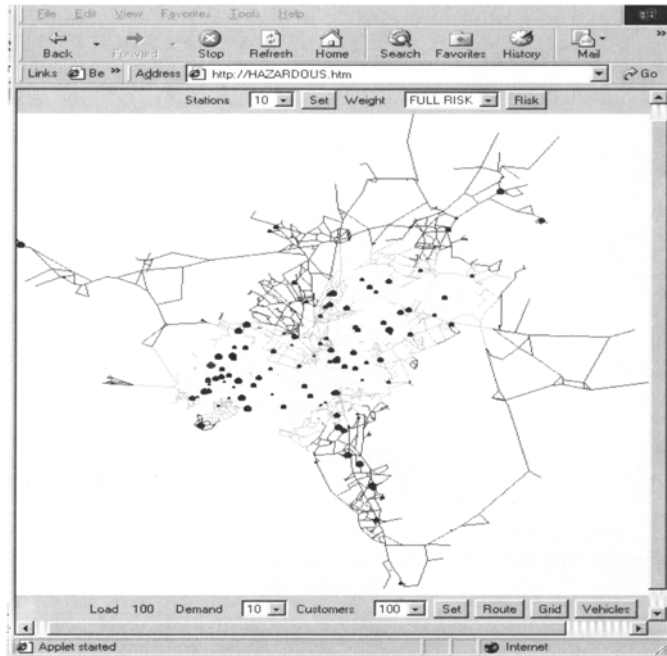
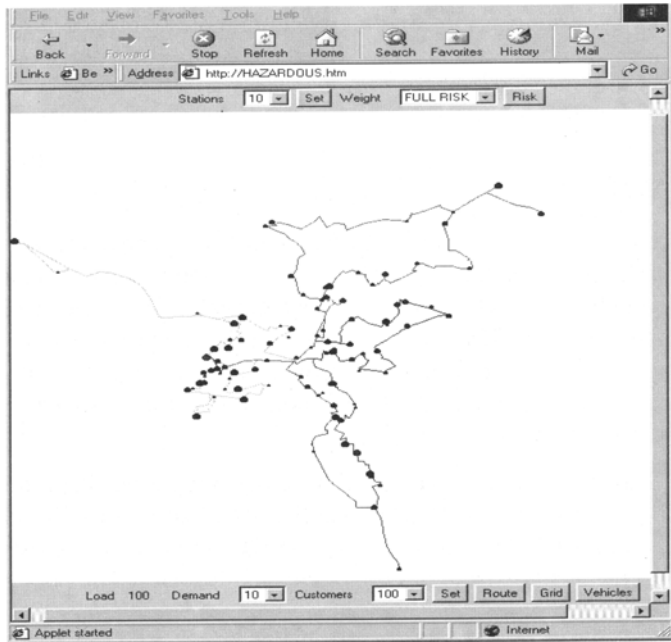


Figure 5. Customers (blue points) located on the road network of Greater Athens area

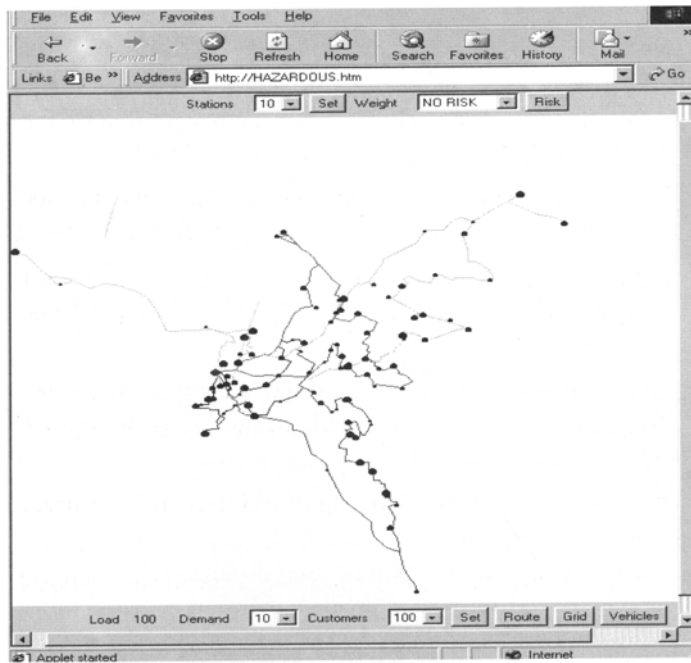
The problem deals with the transportation of gas cylinders that takes place on the road network of Greater Athens area that and it involves more or less 10,000 roads, as it is given in Figure 5.

For demonstration reasons, three hundred customers are located upon this network represented by the blue points. The size of points that represent customers is proportional to the required demand. The red point represents the gas distribution centre. Using ten aggregate population points (black points), the network is divided into regions of variable risk. Three contours areas are presented: the areas of high-risk (yellow), medium-risk (green), low-risk (red). Each truck leave and after satisfying the demand of each customer, returns to the depot.

The tour, which presents the best sequence of deliveries with respect to minimisation of population exposure risk (minimisation of total distance in the risk space) is demonstrated in Figure 6. On the contrary the optimal sequence of deliveries with respect to minimisation of the total distance travelled by the trucks in the non-risk space is demonstrated in Figure 7. The different sequences of deliveries between these routing plans are obvious.



*Figure 6. The best sequence of deliveries with respect to minimisation of population exposure risk (minimisation of total distance travelled by trucks in the risk space)*



*Figure 7. The best sequence of deliveries with respect to minimisation of total distance travelled by trucks in the real (non-risk) space*

For this particular problem, the computational time of the proposed metaheuristic algorithm is equal to 6 seconds (Pentium II - 400 MHz). An instance of the problem was coded in Java 2.0 and run on Microsoft Internet Explorer 6.0.

## 6. Conclusions

Transportation of hazmats is a decision problem that has been attracted much attention due to the risks involved. A considerable amount of models have been developed that employ single or multiple objective shortest path algorithms that minimised the risks for a given origin-destination pair. However, in many real life applications (e.g. transportation of gas cylinders), transportation of hazmats calls for the determination of a set of routes, used by a fleet of trucks to serve a set of customers, rather than a single optimal route as in the case of shortest path algorithms.

In this paper, we focus on population exposure risk mitigation via production of truck-routes by solving a variant of the Vehicle Routing Problem (VRP). We employ a metaheuristic, called List Based Threshold Accepting (LBTA) that minimize risk by minimising the total distance travelled by trucks in the so-called risk space. The population exposure risk of point in the risk space is defined as the product of population ( $N_i$ ) of a geographic population object (city, town, ward)  $i$  and distance-length ( $d_i$ ) between the point and the geographic population point  $i$ .

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