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A multiobjective programming model for locating treatment sites and routing hazardous wastes

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Abstract

In this paper, a multiobjective model for locating disposal or treatment facilities and transporting hazardous waste along the links of a transportation network are presented. Some of the nodes of this network may be population centres generating hazardous waste which must be transported to the treatment facilities. Four objectives are considered: (1) minimisation of total operating cost, (2) minimisation of total perceived risk, (3) equitable distribution of risk among population centres and (4) equitable distribution of the disutility caused by the operation of the treatment facilities. A goal programming model to solve the problem is developed and a small hypothetical example is presented to illustrate how penalty functions can be used to obtain more satisfactory solutions in real life applications. © 1998 Elsevier Science B.V.

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1. Introduction

During the last two decades or so, modern societies have experienced an increasing public awareness of environmental issues and concern for the long lasting effects that human polluting activities may have on the environment. In particular, the risks associated with the transportation and treatment or disposal of hazardous materials and wastes have attracted considerable public attention. Several accidents related to nuclear and chemical industries have contributed significantly to this effect. In a recent incident in the UK (in January 1995), the removal of two drums containing uranium 238 from a farm where they had been dumped proved to be a much harder problem than was expected. The police had to supervise the transportation of the drums to ensure that they reached their destination safely.

Accidents involving such materials may occur during transportation as well as during their treatment and disposal. Accidents during transportation affect population centres along the routes used for transport whereas accidents during treatment affect population centres in the vicinity of the treatment facility where the accident has occurred. Hence, in order to minimise the effects of

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an accident during treatment, the treatment or disposal facilities can be located at sites where the minimum number of people would be put at risk. In addition, in order to reduce the transportation risk the safest possible routes from waste generation sites to disposal or treatment sites should be selected. However, the choice of destinations (treatment sites) will then affect the selection of the routes from the origins (waste generation sites) thus affecting the overall transportation risk. Hence, the problems of selecting the routes and locating the treatment facilities are clearly connected and any methodology used has to address them together.

Moreover, the transportation and treatment of hazardous materials and wastes involve decisions that may affect several interested parties in more than one way. The selection of the safest routes and the farthest locations may lead to an unacceptable increase in transportation and treatment cost. On the other hand, reducing the cost may increase the overall risk. Consequently, most models consider several objectives and present several alternative solutions from which the decision maker can choose.

Despite the fact that routing of hazardous wastes and location of treatment sites are clearly interdependent, most of the approaches in the literature seem to concentrate on only one aspect of the problem. List et al. (1991) present a review of models for routing of obnoxious vehicles, that is vehicles transporting undesirable materials. On the other hand Erkut and Neuman (1989) discuss models for locating obnoxious facilities such as chemical factories, nuclear power plants, etc.

Zografos and Samara (1989) introduce a combined location and routing model for hazardous waste transportation and disposal. They assume that waste is generated in several population centres and must then be transported to the disposal sites. They consider three objectives: (1) minimisation of transportation risk, (2) minimisation of travel time, and (3) minimisation of disposal risk. They also assume that each population centre is assigned to its nearest disposal facility, that the Euclidean distance between population centres and disposal sites is considered as the separation measure for the locational part of the model and that the number of treatment facilities to be located is predetermined. They then use pre-emptive goal programming to generate a number of solutions under alternative scenarios.

ReVelle et al. (1991) have developed a model that locates storage facilities and selects routes for shipments of spent nuclear fuel. They consider two criteria: minimisation of transportation burden and minimisation of perceived risk. Transportation burden is measured in ton-miles and perceived risk in tons-past-people. The authors combine methods of shortest paths, a 0-1 mathematical programming model for siting and the weighting method of multiobjective programming to solve the problem.

Zografos (1993) presents an extension of the earlier Zografos and Samara model that considers four objectives: (1) maximisation of the sum of the Euclidean distances from population centres to their nearest hazardous waste disposal or treatment facility, (2) maximisation of the minimum Euclidean distance from the disposal or treatment sites to any population centre, (3) minimisation of total risk associated with the transport of hazardous waste, and (4) minimisation of hazardous waste total travel time.

In the following sections a weighted goal programming model is introduced for locating treatment facilities and selecting routes for hazardous waste shipments. Four objectives are considered: (1) minimisation of total operating cost, (2) minimisation of total perceived risk, (3) equitable distribution of risk among population centres and (4) equitable distribution of the disutility caused by the operation of the treatment facilities. A small hypothetical example is then presented and penalty functions are used to obtain more realistic solutions.

2. Model formulation

The disposal or treatment of a hazardous waste at certain sites, remote from its production, requires the existence of a directed transportation network $\{N, A\}$ along which the waste is transported. The nodes of this network are either population centres or candidate locations for the treatment facilities. Some of the population centres may be generating a certain amount of waste which must be treated or disposed of. The set of all sources is denoted by G whereas the set of the population centres that do not generate waste is denoted by P. Each population centre is associated with a positive weight reflecting its relative importance (e.g. the size of the population).

Other nodes of the network may be candidate locations for the treatment facilities. Let the set of candidate locations be denoted by L. We assume that population centres or sites generating waste are not candidates for treatment facilities i.e. $L \cap (G \cup P) = \emptyset$. Also, it will be assumed that these potential locations have been selected as a result of an initial screening process based on whatever criteria the decision maker considers important (distance, social and economic factors, etc.). In particular, Plastria (1992) suggests the use of continuous location models for generating these potential sites followed by the use of discrete models for selecting the best ones as an effective approach to real life location problems.

For each link (i,j) of the underlying network the unit cost of transporting waste from i to j is known. These unit costs may be based on the length of the link, the average travel time between i and j, the geography of the region, etc.

Each candidate location is associated with a fixed cost of opening a treatment facility there. This cost may depend on the size of the facility to be opened and may include land purchase cost, start up costs, annual operating costs depreciated over the expected life of a facility of that size, etc. The first objective of the combined routing and location model is to minimise total cost, that is, the sum of the transportation cost and the fixed cost of opening the treatment facilities.

The second objective is to minimise the total perceived risk due to the shipment of the hazardous waste. We assume that the amount of the hazardous waste transported through a population centre is a surrogate for the risk perceived by each individual. Multiplying this individual perceived risk by the weight of each population centre and then adding up for all population centres yields the total perceived risk.

In order to ensure that no population centre is unfairly treated we have to make sure that the perceived risk is distributed evenly among the population centres. Zografos and Samara (1989) try to achieve equity by imposing upper bounds on the amount of waste that can be transported along each link. Marsh and Shilling (1994) review several alternative equity measures (Gini coefficient, range, etc.). One of the approaches they assess is to focus on the worst off population centre. Equity is improved if maximum risk or disutility imposed on any population centre is minimised. Hence, the third objective of our model is to minimise the maximum individual perceived risk.

In addition to the risk imposed by the shipment of the hazardous waste, a certain disutility is also caused by the operation of the treatment facilities. We assume that the disutility caused by a treatment facility on a population centre i is an increasing function of the facility size and a decreasing function of the distance between them. Adding up for all treatment facilities we get the individual disutility at i. Hence, the fourth objective of the model is to minimise the maximum individual disutility.

Goal programming has been chosen to model this situation because it is a flexible tool which allows the investigation of alternative planning scenarios. The main idea of goal programming is to specify a target for each goal and then attempt to find a solution that comes as close as possible to these targets. Among the different goal programming approaches that appear in the literature we have chosen weighted goal programming (WGP) where each goal t is associated with a weight a_t reflecting its relative importance. The essence of WGP is to minimise the sum of the relevant deviations from the targets multiplied by the corresponding weights. For a detailed discussion of goal programming see Ignizio (1982) and Romero (1991). The goal programming formulation is given below.

2.1. Decision variables

x_{ij} amount of hazardous waste (HW) transported from i to j (link $(i,j) \in A$),

$$y_{mk} = \begin{cases} 1 & \text{if a treatment facility of size } k \text{ is open at } m, \\ 0 & \text{otherwise } (m \in L \text{ and } k = 1, \dots, K), \end{cases}$$

pd_t positive deviation from target of goal t ($t = 1, \ldots, 4$),
nd_t negative deviation from target of goal t ($t = 1, \ldots, 4$),
where K is the number of possible sizes (capacities) for the treatment facilities and L is the set of nodes which are candidate locations ($L \subset N$).

2.2. Objectives

(G1) *Total Cost goal* (minimise overachievement pd_1)

$$\sum_m \sum_k f_{mk} y_{mk} + \sum \sum_{(i,j) \in A} c_{ij} x_{ij} - \mathrm{pd}_1 + \mathrm{nd}_1 = \mathrm{ta}_1,$$

where f_{mk} is the fixed cost of opening a facility of size k at candidate location $m \in L$, c_{ij} is the unit transportation cost along (i,j) and ta_1 the target for the first goal. The first term in the above expression is the fixed cost of opening the treatment facilities and the second term the total transportation cost.

(G2) *Total Perceived Risk goal* (minimise overachievement pd_2)

$$\sum_{i \in P \cup G} w_i R_i - \mathrm{pd}_2 + \mathrm{nd}_2 = \mathrm{ta}_2,$$

where w_i is the weight of population centre i, R_i is the individual perceived risk at i due to the shipment of the hazardous waste and ta_2 is the target for the second goal.

(G3) *Individual Perceived Risk* goal (minimise overachievement pd_3)

$$R_{\max} - \mathrm{pd}_3 + \mathrm{nd}_3 = \mathrm{ta}_3,$$

where $R_{\max}$ is the maximum individual perceived risk and ta_3 the target for the third goal.

(G4) *Individual Disutility* goal (minimise overachievement pd_4)

$$E_{\max} - \mathrm{pd}_4 + \mathrm{nd}_4 = \mathrm{ta}_4,$$

where $E_{\max}$ is the maximum individual disutility caused by the operation of the treatment facilities and ta_4 is the target for the fourth goal.

2.3. Objective function

In order to obtain a meaningful objective function we must first normalise the four goals since they are expressed in different units. The easiest way to achieve this is to multiply each goal by 100 and then divide by the specified target, thus working in terms of percentages. In general, the goal t of our model can be written as:

$$\frac{100}{\mathrm{ta}_t} f_t(x,y) - p_t + n_t = 100 \quad \text{for } t = 1, \ldots, 4,$$

where $f_t(x,y)$ is the mathematical expression of the attribute of goal t (total cost, total risk, etc.) and p_t and n_t express the percentage deviations from the target. Note that $p_t = (100/\mathrm{ta}_t)\mathrm{pd}_t$ and $n_t = (100/\mathrm{ta}_t)\mathrm{nd}_t$ for $t = 1, \ldots, 4$. The objective function to be minimised is

$$\sum_{t=1}^{4} a_t p_t,$$

where a_t reflects the importance of goal t.

2.4. Constraints

$$\text{(C1)} \qquad \sum_{j \in \mathrm{ON}_g} x_{gj} - \sum_{k \in \mathrm{IN}_g} x_{kg} \geqslant D_g \quad \text{for } g \in G,$$

where D_g is the amount of the hazardous waste (HW) generated at population centre g, $\mathrm{ON}_g = \{j \in N \mid (g,j) \in A\}$, that is, the set of nodes connected to g by an edge starting from g, and $\mathrm{IN}_g = \{k \in N \mid (k,g) \in A\}$ the set of nodes reaching g, by an edge terminating at g.

$$\text{(C2)} \qquad \sum_{i \in \mathrm{IN}_m} x_{im} - \sum_{j \in \mathrm{ON}_m} x_{mj} \leqslant \sum_k C_k y_{mk} \quad \text{for } m \in L,$$

where C_k is the capacity of a facility of size k and IN_m and ON_m have been defined above.

$$\text{(C3)} \qquad \sum_{i \in \mathrm{IN}_p} x_{ip} - \sum_{j \in \mathrm{ON}_p} x_{pj} = 0 \quad \text{for } p \in P,$$

$$\text{(C4)} \qquad \sum_k y_{mk} \leqslant 1 \quad \text{for } m \in L,$$

(C5) $$R_i = \sum_{k \in \mathrm{IN}_i} x_{ki} \quad \text{for } i \in (P \cup G),$$

(C6) $$E_i = \sum_{m \in L} \frac{\sum_k (C_k y_{mk})^q}{(d_{im})^r} \quad \text{for } i \in (P \cup G),$$

where q and r are parameters for the individual disutility function and d_{im} is the distance between population centre i and facility location m in some distance metric.

(C7) $$R_{\max} \geqslant R_i \quad \text{for } i \in (P \cup G),$$

(C8) $$E_{\max} \geqslant E_i \quad \text{for } i \in (P \cup G),$$

(C9) $$x_{ij} \geqslant 0 \quad \text{for } i, j \in N,$$

$$y_{mk} \in \{0, 1\} \quad \text{for } m \in L \text{ and } k = 1, \ldots, K,$$

where N is the set of all nodes of the network.

Constraints (C1) stipulate that the whole amount of hazardous waste generated at each source must be processed or disposed of. Constraints (C2) ensure that the capacity of each treatment facility is not exceeded whereas (C3) maintain the balance of flow in the population centres that do not generate waste. Constraints (C4) state that at most one facility size can be selected for each candidate location. Constraint set (C5) defines the individual perceived risk at a population centre i due to the shipment of waste whereas constraints (C6) define the individual disutility imposed at i by the operation of all treatment facilities. Finally, constraints (C7) and (C8) define the maximum individual perceived risk and the maximum individual disutility, respectively.

The individual disutility function is similar to the one used by Erkut and Neuman (1992) and Giannikos (1994). By the definition of E_i it can be seen that the disutility is an increasing function of facility size and a decreasing function of distance. In fact this form of disutility function can model a variety of situations; a large value of r, for instance, implies a rapidly decreasing function which is appropriate when the disutility is significant only in the immediate vicinity of the facility. A value of q greater than 1 implies that the population centres are sensitive to scale, i.e., if a facility A is twice as big as facility B then the disutility caused by A at any population centre is more than twice the disutility caused by B.

Note that the distance d_{im} in the definition of E_i need not be symmetric. Buchanan and Wesolowsky (1993), for instance, use the following asymmetric distance function:

$$d_{im} = \frac{\|S_{im}\|}{\rho(\theta(S_{im}))}$$

where S_{im} is the line segment joining population centre i and candidate location m, $\|S_{im}\|$ is the length of S_{im}, $\theta(S_{im})$ is the angle which this segment makes with the x-axis and $\rho(\theta) > 0$ for $0 \leqslant \theta \leqslant 2\pi$ is a function which defines the angular dependence of the distance function. Such asymmetric distance functions are particularly useful in situations where the disutility depends on the direction between treatment facility and population centre (e.g. in the case of prevalent winds which may diffuse emissions further in one direction than in others).

In Section 3 we present a small hypothetical application of the combined location-routing model.

3. Application of the model

In this section we present a small hypothetical problem with 13 population centres, three of which generate hazardous wastes, and five candidate locations for treatment facilities (see Fig. 1) to demonstrate the applicability of the model introduced in the previous section. In this particular example, population centres 11, 12 and 13 generate 60, 70 and 100 units of hazardous waste, respectively.

It will be assumed that there are three alternative sizes for the treatment facilities ($K = 3$). Table 1 shows the fixed cost of opening a facility of a certain size at a certain candidate location (nodes 14 to 18 in Fig. 1).

Table 2 shows the unit cost c_{ij} of transporting hazardous waste along each edge (i, j) in either direction.

Finally, Table 3 shows the weight of each population centre and the distance between each popu-

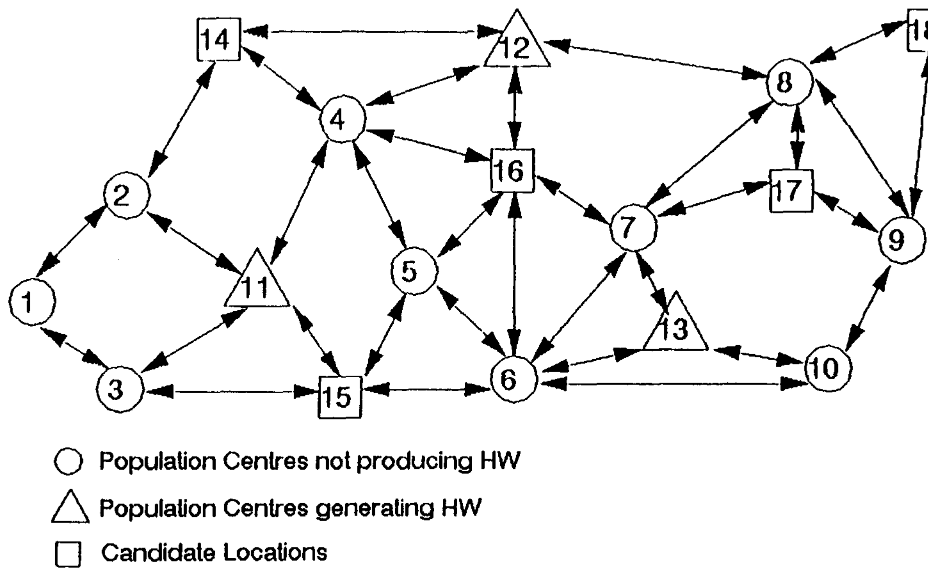

Fig. 1. The transportation network.

lation centre and each candidate location. For simplicity, we have used Euclidean distances and we have also set $p = q = 1$ in the disutility function.

Having presented the data for this hypothetical problem, the next step is to set the levels of targets ta_1 to ta_4. In order to set an acceptable target for the total cost we used the following formula:

$$ta_1 = 0.9\left[\frac{1}{4}\sum_m\sum_k f_{mk} + \left(\sum_g D_g\right)AP\right] = 963.0,$$

where AP is the average length (cost) of a path from the generation sites to the candidate locations. Hence, ta_1 is 90% of the sum of the average fixed cost and the average transportation cost.

Suppose that in this application a level of risk 25% higher than the "average" total perceived risk is considered acceptable. It can be seen from Fig. 1 that some generation sites are directly connected to candidate locations (e.g. generation site 12 to candidate location 14). Hence, we expect that "on average" half the amount of hazardous waste (HW) generated will be transported directly to the treatment centres (without affecting the population centres) whereas the other half will travel through the network thus imposing a certain perceived risk to the population centres it travels through. Hence, multiplying that "average" amount by the average weight and then by 1.25 gives us an acceptable target for the total perceived risk.

$$\begin{aligned} ta_2 &= (1.25)(\text{“Av.” amount of HW})(\text{Av. weight}) \\ &= (1.25)(\text{Total amount of HW}/2)(\text{Av. weight}) \\ &= 221.37. \end{aligned}$$

Our third objective is the minimisation of the maximum individual perceived risk. For this application we assume that an acceptable level for that maximum risk is 1/3 of the "average" amount of HW transported through the network. Hence,

$$ta_3 = \frac{230}{2}\frac{1}{3} = 38.33.$$

Finally, our fourth objective is to minimise the maximum disutility caused by the operation of

Table 1
Fixed facility costs

Facility sizes	Candidate locations				
	14	15	16	17	18
30	100	90	90	120	110
50	150	110	160	200	120
80	250	180	270	290	210

Table 2
Unit transportation cost for each edge

Edge	Unit cost
1–2	0.7
1–3	1.2
2–11	0.5
2–14	1.0
3–11	0.8
3–15	1.3
4–5	1.2
4–11	0.5
4–12	1.3
4–14	0.6
4–16	0.9
5–6	0.4
5–15	0.4
5–16	0.9
6–7	1.1
6–10	1.5
6–13	1.8
6–15	0.7
6–16	1.9
7–8	0.7
7–13	0.3
7–16	1.4
7–17	0.5
8–9	1.3
8–12	1.3
8–17	1.7
8–18	0.5
9–10	1.1
9–17	0.9
9–18	1.3
10–13	1.1
11–15	1.2
12–14	0.6
12–16	1.7

the treatment centres. Recall that the individual disutility caused at a certain population centre by a treatment facility is an increasing function of the facility size and a decreasing function of the distance between them. If we divide the total supply of waste by the average distance between population centres and candidate locations (12.53), we get an indication of the disutility imposed on the average population centre. We assume that an acceptable target for the fourth goal is four times this number:

$$\mathrm{ta}_4 = \frac{230}{12.53}\,4 = 73.42.$$

Table 3
Distance from each population centre to each candidate location

Population centre	Candidate locations				
	14	15	16	17	18
1 ($w_1 = 1$)	7	7	13	25	35
2 ($w_2 = 1.3$)	2	8	11	23	33
3 ($w_3 = 1.7$)	10	2	12	22	34
4 ($w_4 = 1.2$)	1	6	4	19	26
5 ($w_5 = 1$)	8	2	2	15	28
6 ($w_6 = 2.5$)	13	3	3	8	26
7 ($w_7 = 1.8$)	18	8	1	3	18
8 ($w_8 = 1.4$)	23	20	5	1	9
9 ($w_9 = 1.8$)	32	25	8	2	8
10 ($w_{10} = 1.6$)	30	15	10	3	14
11 ($w_{11} = 1.5$)	2	2	6	19	28
12 ($w_{12} = 2$)	15	10	1	6	10
13 ($w_{13} = 1.2$)	20	8	3	3	11

It can be argued that these targets have been selected somewhat arbitrarily. However, we had to make these assumptions since this is a hypothetical example. In a real life application the decision maker (government, local authority, etc.) would set these targets, possibly based on any international standard(s) relevant for such situations.

Using these target levels we construct several alternative scenarios to investigate the effect on the optimal solution of different priority weights of the goals. Table 4 shows the priority weight of each goal and the optimal solution for each scenario. The first observation is that the final solution is affected by changes in the weights attached to the four goals, as shown by scenarios 1–4. Furthermore, the solution may also be affected by increases in the level of any target. In scenarios 5 and 6 the target level of the total cost goal (ta_1) is increased by 20% and 30%, respectively. It can be seen that a 20% increase in ta_1 results in a new configuration of treatment facilities. However, increases in ta_1 beyond 1092.84 do not affect the other objectives at all. Consequently, it does not make sense to attempt to reduce perceived risk and disutility by accepting a higher cost. A similar analysis can be performed by varying the target levels of the other goals as well.

The results of Table 4 have been obtained based on an implicit assumption regarding the

Table 4
Results for alternative scenarios

	Scenarios					
	1	2	3	4	5	6
Priority weights	$a_1=1$ $a_2=1$ $a_3=1$ $a_4=1$	$a_1=1$ $a_2=5$ $a_3=2$ $a_4=2$	$a_1=1$ $a_2=2$ $a_3=5$ $a_4=5$	$a_1=5$ $a_2=1$ $a_3=1$ $a_4=1$	$a_1=1$ $a_2=1$ $a_3=1$ $a_4=1$	$a_1=1$ $a_2=1$ $a_3=1$ $a_4=1$
ta_1	963.00	963.00	963.00	963.00	1155.60	1251.90
ta_2	221.37	221.37	221.37	221.37	221.37	221.37
ta_3	38.33	38.33	38.33	38.33	38.33	38.33
ta_4	73.42	73.42	73.42	73.42	73.42	73.42
Treatment facilities open	14 cap = 50 15 cap = 80 16 cap = 30 17 cap = 30 18 cap = 50	14 cap = 50 15 cap = 80 16 cap = 50 17 cap = 50	14 cap = 50 15 cap = 80 16 cap = 30 17 cap = 50 18 cap = 30	14 cap = 50 15 cap = 80 16 cap = 50 17 cap = 50	14 cap = 50 15 cap = 80 16 cap = 30 17 cap = 50 18 cap = 30	14 cap = 50 15 cap = 80 16 cap = 30 17 cap = 50 18 cap = 30
Total cost	1030.34	1033.62	1089.84	1005.34	1093.84	1093.84
Total perc. risk	260.00	224.10	251.67	274.00	251.67	251.67
Maximum perc. risk	38.33	47.45	38.33	38.33	38.33	38.33
Maximum disutility	74.34	79.44	74.62	77.76	74.62	74.62

theoretical structure of our goal programming model. More specifically, the model we introduced in the previous section assumes that any deviation from the target level of a goal is penalised according to a constant marginal penalty, i.e., any marginal deviation is of equal importance no matter how distant it is from the target. The total perceived risk under scenario 4, for instance, is 274 (23.8% away from the target) and is penalised at the same rate as the total risk under scenario 2 which is merely 1.2% away from the target.

Clearly, in the context of hazardous waste transportation and obnoxious facility location, this is not satisfactory. In Section 4 we demonstrate how monotonically increasing penalty functions can be used to overcome this slight inadequacy.

4. Solving the problem using penalty functions

In the previous section we highlighted one of the weaknesses of conventional goal programming where any deviational variable with respect to its target value is penalised according to a constant marginal penalty. This slight drawback can be overcome easily by introducing monotonically increasing penalties.

Recall that in our model all goals were normalised by multiplying the corresponding expressions by $100/\mathrm{ta}_t$ where ta_t is the target level of goal t. Now, assume that the decision maker in our hypothetical example considers that for the total cost goal deviations between 10% and 40% with respect to the target are twice as important as deviations between 0% and 10% of the target and also that deviations above 40% of the target are unacceptable. Suppose also that for the other three goals deviations between 10% and 40% are three times as important as deviations between 0% and 10% of the target and that deviations above 40% are not acceptable.

This function can be incorporated into our model using a method proposed by Can and Houck (1984). Following this method, goal t of our model is written:

$$\frac{100}{\mathrm{ta}_t} f_t(x,y) - \mathrm{pd}_{t1} - \mathrm{pd}_{t2} + \mathrm{nd}_{t1} = 100,$$

$$0 \leqslant \mathrm{pd}_{t1} \leqslant 10, \quad 0 \leqslant \mathrm{pd}_{t2} \leqslant 30, \quad \mathrm{nd}_{t1} \geqslant 0,$$

Table 5
Results for alternative scenarios using penalty functions

Priority weights	Scenarios					
	1	2	3	4	5	6
	$a_1=1$ $a_2=1$ $a_3=1$ $a_4=1$	$a_1=1$ $a_2=5$ $a_3=2$ $a_4=2$	$a_1=1$ $a_2=2$ $a_3=5$ $a_4=5$	$a_1=5$ $a_2=1$ $a_3=1$ $a_4=1$	$a_1=1$ $a_2=1$ $a_3=1$ $a_4=1$	$a_1=1$ $a_2=1$ $a_3=1$ $a_4=1$
ta_1	963.00	963.00	963.00	963.00	1155.60	1251.90
ta_2	221.37	221.37	221.37	221.37	221.37	221.37
ta_3	38.33	38.33	38.33	38.33	38.33	38.33
ta_4	72.66	72.66	72.66	72.66	72.66	72.66
Treatment facilities open	14 cap = 50 15 cap = 80 16 cap = 50 17 cap = 50	14 cap = 50 15 cap = 80 16 cap = 50 17 cap = 50	14 cap = 50 15 cap = 80 16 cap = 30 17 cap = 50 18 cap = 30	14 cap = 50 15 cap = 80 16 cap = 50 17 cap = 50	14 cap = 50 15 cap = 80 16 cap = 50 17 cap = 30 18 cap = 30	14 cap = 50 15 cap = 80 16 cap = 50 17 cap = 30 18 cap = 30
Total cost	1056.65	1047.38	1089.84	1056.65	1095.77	1095.77
Total perc. risk	243.51	234.59	251.67	243.51	243.51	243.51
Maximum perc. risk	38.60	42.16	38.33	38.60	38.60	38.60
Maximum disutility	79.44	79.44	74.62	79.44	78.57	78.57

where $f_t(x,y)$ is the mathematical expression of the attribute of goal t (given in G1–G4 earlier). The deviational variables nd_{t1}, pd_{t1} and pd_{t2} measure the percentage achievement of the goal in the ranges $(-\infty, 100]$, $(100, 110]$ and $(110, 140]$, respectively.

The objective function is:

$$\text{Minimise } 1\sum_{t=1}^{4} a_t \mathrm{pd}_{t1} + 2a_1 \mathrm{pd}_{12} + 3\sum_{t=2}^{4} a_t \mathrm{pd}_{t2}$$

Table 5 shows the optimal solution for each of the six scenarios of the previous section under this penalty scheme. It can be seen that the introduction of monotonically increasing penalties can have a significant effect on the optimal solution especially in scenarios where the total perceived risk is high. In scenario 1, for instance, the penalty scheme has resulted in a new solution with lower total risk but slightly higher total cost and higher maximum disutility. In scenario 4, although the set of facilities that are open remains the same, the routing pattern of the hazardous waste is different thus reducing total perceived risk. Similar changes on a smaller scale are observed in scenarios 2, 5 and 6.

The code used to solve the problem was the FORTMP optimiser combined with the MPL modelling language. Computational experiments with different sizes of the problem have shown that the computation time is affected by the number of candidate locations as well as the number of alternative facility sizes due to the combinatorial nature of the problem. However, we were able to solve problems with 40 population centres, 20 candidate locations and five alternative facility sizes in less than three minutes on a 486 machine running at 66 MHz. In addition, an initial screening process to select the candidate locations usually takes place in real life applications. Hence, it is reasonable to assume that the number of candidate sites does not exceed 30 in real world problems.

5. Discussion and conclusions

In this paper a goal programming model for locating disposal or treatment centres and routing hazardous wastes through an underlying transportation network has been presented. It was assumed that the amount of hazardous waste transported through a population centre is a surrogate of the risk perceived by the inhabitants of the centre. Also assumed was that a treatment or disposal facility imposes a certain disutility on nearby population centres; this disutility depends on the capacity of the facility and its distance from the population centres. Consequently, we considered four objectives: (1) minimisation of total operating cost, (2) minimisation of total perceived risk, (3) minimisation of maximum individual risk, and (4) minimisation of maximum individual disutility. Goal programming was used to solve the problem and it was shown how monotonically increasing penalty functions can be used to obtain more satisfactory solutions in the sense that large deviations from the targets are penalised accordingly.

Clearly, there were several issues not included in our analysis. Firstly, the selection of the priority weight expressing the relative importance of each goal. For the purposes of our model we assumed that these weights were given. In real life problems it is more appropriate to consider a whole range of values for these weights expressing the decision maker's preferences thus obtaining several alternative solutions. In fact, the use of systematic methods such as the Analytical Hierarchy Process (AHP) to obtain these weights is the subject of ongoing research in operations research.

Secondly, the selection of the increasing marginal penalties must also be analysed thoroughly. The decision maker(s) can experiment with alternative penalty functions and assess the effect they may have on the final solution.

Determining the target levels for the goals is also important since they may affect the final outcome significantly. Setting some of the targets too optimistically makes it extremely difficult to achieve the other goals, thus rendering them redundant. However, in most real world problems these target levels are implied by the problem specification or obtained following discussions with the decision makers.

Although our goal programming model is based on some rather simplifying assumptions, it is relatively easy to understand and offers a significant amount of flexibility to the decision makers who can experiment with alternative weighting schemes and penalty functions. In addition, the model can offer invaluable insights into the trade-offs involved in a situation where conflicting objectives must be achieved.

Hence, this model must be regarded as a decision aid tool that can help decision makers understand a situation better, rather than as a black box aiming to find *the* optimum. The model can merely offer a number of solutions that may be considered satisfactory; however, the ultimate decision (and responsibility) always lies with the decision maker(s).

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