Multi-Objective Vehicle Routing Problems Using Two-Fold EMO Algorithms to Enhance Solution Similarity on Non-dominated Solutions

Tadahiko Murata and Ryota Itai

Department of Informatics, Kansai University
2-1-1 Ryozenji, Takatsuki 569-1102, Osaka, Japan
murata@res.kutc.kansai-u.ac.jp
http://www.res.kutc.kansai-u.ac.jp/~murata/

Abstract. In this paper, we focus on the importance of examining characteristics of non-dominated solutions especially when a user should select only one solution from non-dominated solutions at a time, and select another solution due to the change of problem conditions. Although he can select any solution from non-dominated solutions, the similarity of selected solutions should be considered in practical cases. We show simulation results on vehicle routing problems that have two demands of customers: Normal Demand Problem (NDP) and High Demand Problem (HDP). In our definition the HDP is an extended problem of NDP. We examined two ways of applying an EMO algorithm. One is to apply it to each problem independently. The other is to apply it to the HDP with initial solutions generated from non-dominated solutions for the NDP. We show that the similarity of the obtained sets of non-dominated solutions is enhanced by the latter approach.

1 Introduction

Although we have many approaches in EMO (Evolutionary Multi-criterion Optimization) community [1, 2] recently, there are few research works that investigate the similarity of obtained non-dominated solutions. Deb considered topologies of several non-dominated solutions in Chapter 9 of his book [3]. He examined the topologies or structures of three-bar and ten-bar truss. He showed that neighboring non-dominated solutions on the obtained front are under the same topology, and NSGA-II can find the gap between the different topologies. While he considered the similarity of solutions in a set of non-dominated solutions from a topological point of view, there is no research work relating to EMO that considers the similarity of solutions in different sets of non-dominated solutions from that point of view.

We employ the Vehicle Routing Problem (VRP) to consider the similarity in different sets of solutions. The VRP is a complex combinatorial optimization problem, which can be seen as a merge of two well-known problems: the Traveling Salesman Problem (TSP) and the Bin Packing Problem (BRP). This problem can be described as follows: Given a fleet of vehicles, a common depot, and several customers scattered geographically. Find the sets of routes for the fleet of vehicles. Many research works [4, 5, 6, 7, 8] on the VRP try to minimize the total route cost that is calculated using the distance or the duration between customers. Several hybrid algorithms have been proposed to improve the search ability of genetic algorithms [4, 5]. The research works in [6, 7, 8] are related to multi-objective optimization. Tan *et al.* [6] and Saadah *et al.* [7] employed the travel distance and the number of vehicles to be minimized. Chitty and Hernandez [8] tried to minimize the total mean transit time and the total variance in transit time.

In this paper, we employ an EMO algorithm, NSGA-II [9], to our vehicle routing problems with minimizing the number of vehicles and the maximum routing time among the vehicles. It should be noted that we don't employ the total routing time of all the vehicles, but use the maximum routing time among the vehicles. We employed it in order to minimize the active duration of the central depot. We consider two problems with different demands. One problem has a normal demand of customers. The other has a high demand. We refer the former problem and the latter problem as NDP and HDP, respectively. We define the demand in the HDP as an extended demand of the NDP in this paper. For example, we assume that the demand in the HDP is a demand occurring in a high season such as Christmas season. In that season, the depot may have an extra demand as well as the demand in the normal season. In order to avoid a large change of each route from the depot, a solution (i.e., a set of route) in the HDP should be similar to a solution in the NDP. This situation requires us to consider the similarity of solutions on different non-dominated solutions in multi-objective VRPs.

In order to find a set of non-dominated solutions in the HDP that is similar to a set of non-dominated solutions in the NDP, we apply a two-fold EMO algorithm to the problem. In a two-fold EMO algorithm, first we find a set of non-dominated solutions for the NDP by an EMO algorithm. Then we generate a set of initial solutions for the HDP from the non-dominated solutions for the NDP. We apply an EMO algorithm to the HDP with initial solutions that are similar to those of the NDP problem.

We organize this paper as follows: Section 2 gives the problem model for multi-objective VRPs. The outline of our two-fold EMO algorithm is described in Section 3. We define a measure of the similarity between solutions in Section 4. A small example of our multi-objective VRP is also shown in Section 4. Section 5 presents the extensive simulations and compares results of the two-fold EMO algorithm and those obtained individually for the HDP and NDP. Conclusions are drawn in Section 6.

2 Multi-objective Vehicle Routing Problems

The domain of VRPs has large variety of problems such as capacitated VRP, multiple depot VRP, periodic VRP, split delivery VRP, stochastic VRP, VPR with backhauls, VRP with pick-up and deliverring, VRP with satellite facilities, and VRP with time windows. These problems have the basic architecture of the VRP except their own constraints. Their constraints are arisen in practical cases. Please see for the detail of the VRP problem in [10].

The objective of the basic problem is to minimize a total cost is described as follows:

$$\text{Min. } \sum_{m=1}^{M} c_m \ , \tag{1}$$

where M is the number of vehicles each of them starts from the depot and is routed by a sequence of customers, then return to the depot. The cost of each vehicle is denoted by c_m and described as follows:

$$c_m = c_{0,1} + \sum_{i=1}^{n_m - 1} c_{i,i+1} + c_{n_m,0} \ , \tag{2}$$

where $c_{i,j}$ means the cost between Customers i and j. Let us denote 0 as the index for the depot in this paper. Equation (2) indicates the sum of the cost between the depot and the first customer assigned to the m-th vehicle (i.e., $c_{0,1}$), the total cost from the 1st customer to the n_m-th customer (i.e., $\sum_{i=1}^{n_m-1} c_{i,i+1}$), and the cost between the final customer n_m and the depot. Each vehicle is assigned to visit n_m customers, thus we have $N = \sum_{m=1}^{M} n_m$ customers in total. The aim of the VRP is to find a set of sequences of customers that minimizes the total cost. Each customer should be visited exactly once by one vehicle.

While the total cost of all the vehicles is ordinarily employed in the VRP, we employ the maximum cost to be minimized in this paper. When the cost $c_{i,j}$ is related to the driving duration between Customers i and j in Equation (2), the total cost c_m for the m-th vehicle means the driving duration from the starting time from the depot to the returning time to the depot. In order to minimize the activity duration of the depot, the maximum duration of the vehicles should be minimized since the depot should wait until all the vehicles return to the depot. We also consider the minimization of the number of vehicles in our multi-objective VRP. The objectives in this paper can be described as follows:

$$\text{Min. } \max_{m} c_m \ , \tag{3}$$

$$\text{Min. } M \ . \tag{4}$$

When we have a solution with $M = 1$, our problem becomes the traveling salesman problem, the TSP. In that case, however, the other objective to minimize the maximum driving duration in Equation (3) can be reduced to at most the optimal value of the TSP with only one vehicle. That is, the maximum driving duration can not be reduced as in the case of using multiple vehicles. On the other hand, the maximum driving duration becomes minimum when the number of vehicles equals to the number of customers (i.e., $M = N$). In that case, each vehicle needs to visit only one customer. The driving duration for each vehicle in (2) can be described as follows:

$$c_m = c_{0,[1]_m} + c_{[1]_m,0} \,, \tag{5}$$

where $[k]_m$ denotes the index of the customer who is the k-th customer visited by the m-th vehicle. The maximum driving duration in (5) over M vehicles becomes the optimal value of that objective in the case of $M = N$. We have the trade off between these two objectives: the minimization of the maximum driving duration and the minimization of the number of vehicles.

We consider two problems with different demands: the NDP and the HDP. In the NDP, a normal demand of customers should be satisfied. On the other hand, extra demands should also be satisfied in the HDP. In this paper, we increase the number of customers in the HDP. That is, $N_{NDP} < N_{HDP}$, where N_{NDP} and N_{HDP} are the number of customers in the NDP and the HDP, respectively. We can obtain a set of non-dominated solutions for each problem. We refer a set of non-dominated solutions for the NDP as Ψ_{NDP}, and that for the HDP as Ψ_{HDP}. These two sets of non-dominated solutions can be obtained by applying one of multi-objective algorithms such as EMO algorithms. But if we apply the algorithm to each of the NDP and the HDP independently, we can not expect to obtain a set of solutions with similar routes in the HDP to that obtained for the NDP. Before introducing a measure of similarity of a solution set in Section 4, we describe a two-fold EMO algorithm for our multi-objective VRP in the next section.

3 Two-Fold EMO Algorithm for Multi-objective VRPs

In this section, we show the employed coding scheme to find a set of non-dominated solutions using the genetic operations to apply an EMO algorithm to our multi-objective VRPs. Then we show how we apply a two-fold EMO algorithm to obtain a similar set of solutions in the NDP and the HDP.

3.1 Coding Scheme to Describe Solutions

We code a solution of the VRPs by a permutation of N customers, and we split it into M parts as shown in Fig. 1. There are eight customers in Fig. 1, and they are served by one of three vehicles. The first vehicle denoted v_1 in the figure visits three customers in the order of Customers 1, 2, and 3. It is noted that the depot is not appeared in the coding. Each route is divided by a closed triangle. Therefore the driving duration for v_1 is calculated by $c_{0,1} + c_{1,2} + c_{2,3} + c_{3,0}$. Fig. 2 shows an example of three routes depicted on the map of eight customers and the depot.

3.2 Genetic Operations

We employ the cycle crossover [11] as a crossover operator in this paper. Fig. 3 shows an example of generating an offspring from selected two parents by the crossover. From this figure we can find that the number and the location of splits are changed by this crossover. In this figure, the generation of the offspring is started from Customer 3 in Parent 1. Another offspring is also generated by starting from Parent 2.

As for the mutation, we employ two kinds of operators in order to modify locations of splits and the order of customers in a selected route. Fig. 4 shows examples of these mutations. It should be noted that the order mutation itself does not affect the two objectives (i.e., the maximum driving duration and the number of vehicles). But it can be useful to increase the variety of solutions when it is used with the cycle crossover and the split mutation.

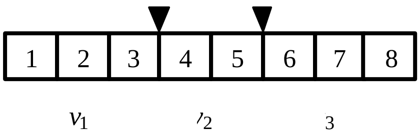

Fig. 1. An example of eight customers visited by three vehicles. Each triangle shows the split between the routes for vehicles.

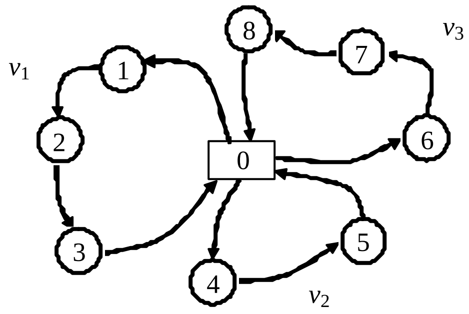

Fig. 2. An example of eight customers visited by three vehicles. Each triangle shows the split between the routes for vehicles.

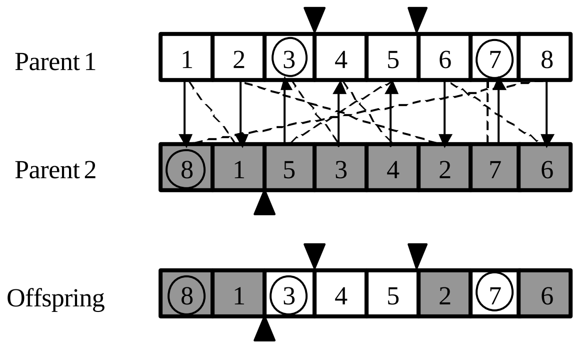

Fig. 3. An example of the cycle crossover. Customer 3 in Parent 1 is randomly chosen and inherited to an offspring. Then Customer 3 in the other parent is found (the dotted line). The customer in Parent 1 locating in the same position of Customer 3 in Parent 2 is inherited to the offspring (i.e., Customer 4). This operation is repeated until returning to Customer 3 in Parent 1. One of remaining customers is chosen from Parent 2 (Customer 8 in this figure). Repeat these operations until all customers are inherited to the offspring. It is noted that a split denoted by a closed triangle is also inherited to the offspring when a customer that is the final one in a route is inherited (Customers 3 and 5 from Parent 1, and Customer 1 from Parent 1 in this figure).

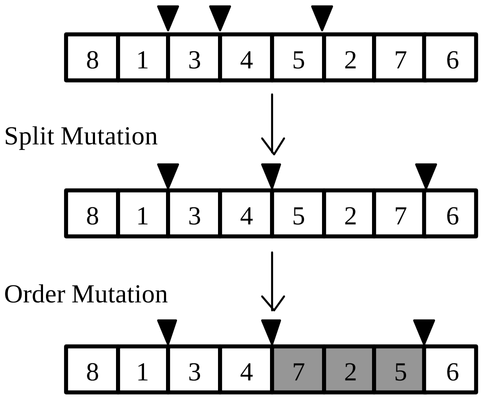

Fig. 4. Examples of two mutation operators. In the split mutation, locations of splits are changed randomly. In the order mutation, a selected route is inversed its order of customers.

3.3 Two-Fold EMO Algorithm

In our multi-objective VRP, we have two problems, the NDP and the HDP. Since the HDP has extra demands of customers with the demands of the NDP, we have two approaches to search a set of non-dominated solutions for each of the NDP and the HDP. One approach is to apply an EMO algorithm individually to each of them. The other is to apply a two-fold EMO algorithm to them. In the two-fold EMO algorithm, first we find a set of non-dominated solutions for the NDP by an EMO algorithm. Then we generate a set of initial solutions for the HDP from the non-dominated solutions for the NDP. We apply an EMO algorithm to the HDP with initial solutions that are similar to those of the NDP problem. The procedure of the two-fold EMO algorithm is described as follows:

[Two-Fold EMO Algorithm]

Step 1: Initialize a set of solutions randomly for the NDP.
Step 2: Apply an EMO algorithm to find a set of non-dominated solutions until the specified stopping condition is satisfied.
Step 3: Obtain a set of non-dominated solutions for the NDP.
Step 4: Initialize a set of solutions for the HDP using a set of non-dominated solutions of the NDP.
Step 5: Apply an EMO algorithm to find a set of non-dominated solutions until the specified stopping condition is satisfied.
Step 6: Obtain a set of non-dominated solutions for the HDP.

In Step 4, we initialize a set of solutions as follows:

Step 4.1: Obtain a set of non-dominated solutions of the NDP.

Step 4.2: Specify a solution of the set.
Step 4.3: Insert new customers randomly into the solution.
Step 4.4: Repeat Steps 4.2 and 4.3 until all solutions in the set of non-dominated solutions of the NDP are modified.

It should be noted that the number of vehicles of each solution is not changed by this initialization. The number of vehicles of each solution is changed by the crossover operation. Using this initialization method, we show that the similarity between non-dominated solutions for the NDP and those for the HDP can be increased. Before showing the simulation results we explain a measure of the similarity of solutions in the next section

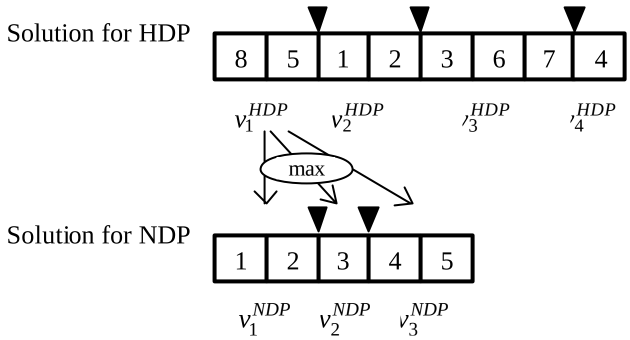

Fig. 5. A set of example solutions for the NDP and the HDP.

4 Measure of Similarity Between Sets of Non-dominated Solutions

4.1 Similarity Measure

We define a similarity measure to compare non-dominated solutions obtained for the NDP and the HDP. Figure 5 shows a set of example solutions to be compared. Suppose that we have five customers in the NDP and eight in the HDP. The five customers in the NDP are denoted by 1, 2, ..., 5. The other three customers inserted in the HDP are denoted by 6, 7 and 8. When we obtain a solution with three vehicles for the NDP and one with four for the HDP, the similarity of a solution in the HDP to one in the NDP can be calculated as follows:

$$s(v_i^{HDP}) = \max_j rsr(v_i^{HDP}, v_j^{NDP}), \; i = 1, \ldots, M_{v_i^{HDP}}, \tag{6}$$

where $s(v_i^{HDP})$ is the similarity of the i-th vehicle v_i^{HDP} of a solution for the HDP to vehicles v_j^{NDP} ($j = 1, ..., M_{v_j^{NDP}}$) of a solution for the NDP, and $rsr(v_i^{HDP}, v_j^{NDP})$ is the ratio of the same route in v_i^{HDP} and v_j^{NDP}. The number of vehicles in these solutions is denoted by $M_{v_i^{HDP}}$ and $M_{v_j^{NDP}}$, respectively. In the example of Fig. 5, $M_{v_i^{HDP}} = 4$ and $M_{v_j^{NDP}} = 3$. We calculate $rsr(v_i^{HDP}, v_j^{NDP})$ as follows:

$$rsr(v_i^{HDP}, v_j^{NDP}) = | R_{v_i^{HDP}} \cap R_{v_j^{NDP}} | / | R_{v_j^{NDP}} |, \tag{7}$$

where R_{v_i} shows a set of routes for Vehicle v_i, and $| R_{v_i} |$ indicates that the number of routes of Vehicle v_i. For example, $R_{v_1^{HDP}}$ consists of routes {0 to 8, 8 to 5, 5 to 0}. The number of routes of v_1^{HDP} is three. When we compare the routes of v_1^{HDP} to the routes of vehicles in the solution obtained for the NDP, we can obtain the similarity of v_1^{HDP} using (6) as follows:

$$\begin{aligned} s(v_1^{HDP}) &= \max_j rsr(v_1^{HDP}, v_j^{NDP}), \\ &= \max\{rsr(v_1^{HDP}, v_1^{NDP}), rsr(v_1^{HDP}, v_2^{NDP}), rsr(v_1^{HDP}, v_3^{NDP})\}, \\ &= \max\{0/3, 0/2, 1/3\} = 1/3. \end{aligned} \tag{8}$$

As shown in the above equations, v_1^{HDP} has no common routes with v_1^{NDP} and v_2^{NDP}, and 1/3 as its similarity to the route of v_3^{NDP}.

4.2 Example

We show a small example of our multi-objective vehicle routing problem. Fig. 6 shows a map of a depot and customers. The rectangle in the map shows a central depot from which every vehicle starts and to which it returns. Open circles show customers in the NDP, and closed circles show those added in the HDP. Fig. 7 shows an example solution with three vehicles for the NDP with five customers. Figs. 8 and 9 show solutions with a high maximum similarity and a low maximum similarity. From these figures, we can see that the solution for the HDP in Fig. 8 has the same routes (thick routes) in Fig. 7 more than the one in Fig. 9. If we can obtain a solution with a high similarity, we can reduce the effort of drivers of vehicles to be informed of routes among customers.

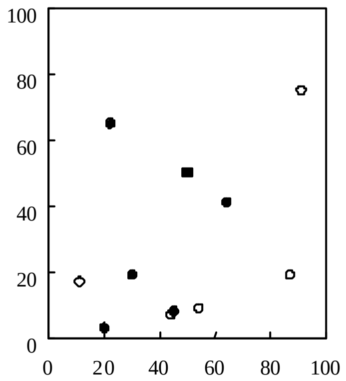

Fig. 6. The locations of customers in a vehicle routing problem with five customers in the NDP, and ten in the HDP.

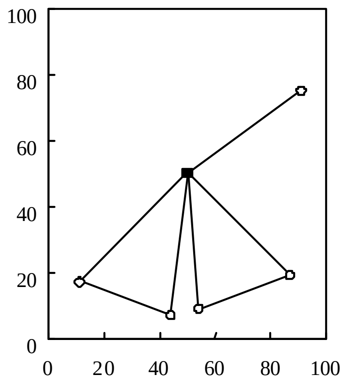

Fig. 7. A solution with three vehicles for the NDP.

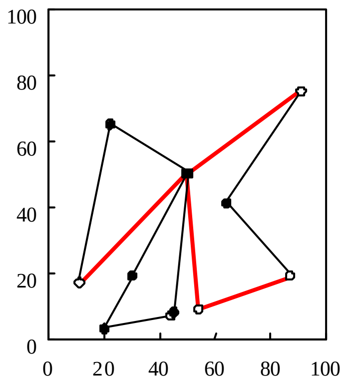

Fig. 8. A solution with three vehicles with a high similarity.

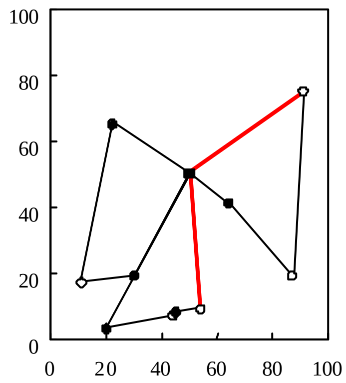

Fig. 9. A solution with three vehicles with a low similarity.

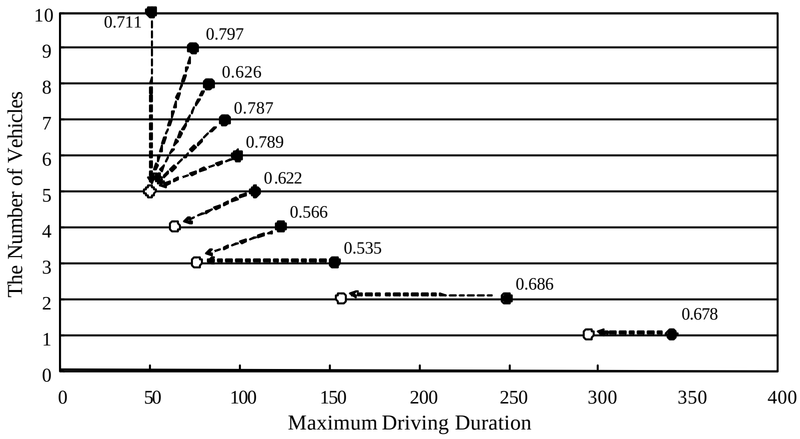

Fig. 10. Sets of Pareto-optimal solutions for the NDP and the HDP in Fig. 6.

When we apply an exact search algorithm to find optimal Pareto set for the NDP and the HDP, we obtained the Pareto front depicted in Fig. 10 (In this paper, we employed an exhaust method to examine all possible solutions in the problem. We could employ the exhaust method since the problem is small). The open circles in Fig. 10 show the Pareto solutions for the NDP with the five customers in Fig. 6 with respect to the maximum driving duration and the number of vehicles. The closed circles show the Pareto solutions for the HDP. Each dotted arrow shows the corresponding solution in the NDP, that has the maximum similarity to each solution in the HDP. For example, the solution with four vehicles in the HDP has the routes with the maximum similarity with the solution with three vehicles in the HDP. This figure shows that not all solutions in the NDP are similar to solutions obtained in the HDP. We can increase the number of vehicles according to the increase of demands of customers in the HDP with a small effort of drivers to be informed of new routes between customers.

5 Simulation Results by Two-Fold EMO Algorithm

We show the simulation result on the NDP and HDP problems. We apply our two-fold EMO algorithm to the problem with 100 different initial solution sets. That is, we obtain 100 results for the problem in Fig. 6. We also apply an EMO algorithm to the HDP individually from the solution obtained for the NDP. In this paper, we refer a two-fold EMO algorithm and an individually applied EMO as 2F-EMO and I-EMO, respectively. As an EMO algorithm, we employed the NSGA-II [9], that is known as one of high performance algorithm among EMO algorithms. It should be noted that any EMO algorithm can be used in a 2F-EMO and an I-EMO.

Figs. 11 and 12 show examples of non-dominated solutions obtained by a 2F-EMO and by an I-EMO. In these figures, we compare each set of non-dominated solutions obtained by a 2F-EMO or an I-EMO for a HDP to a set of non-dominated solutions for a NDP. A value attached to each solution of non-dominated solutions for the HDP means a maximum similarity of that solution calculated by (8). By comparing these figures, we can see that the obtained sets of non-dominated solutions are not so different each other. However, when we measure the maximum similarity of each solution for the HDP to a set of solutions for the NDP, we can see the difference between those sets of solutions. That is, while the average value of the maximum similarity for each solution obtained by a 2F-EMO in Fig. 11 is 0.5717, that obtained by an I-EMO in Fig. 12 is 0.3858. These values show that the set of non-dominated solutions obtained by a 2F-EMO has higher similarity to the set obtained for the NDP.

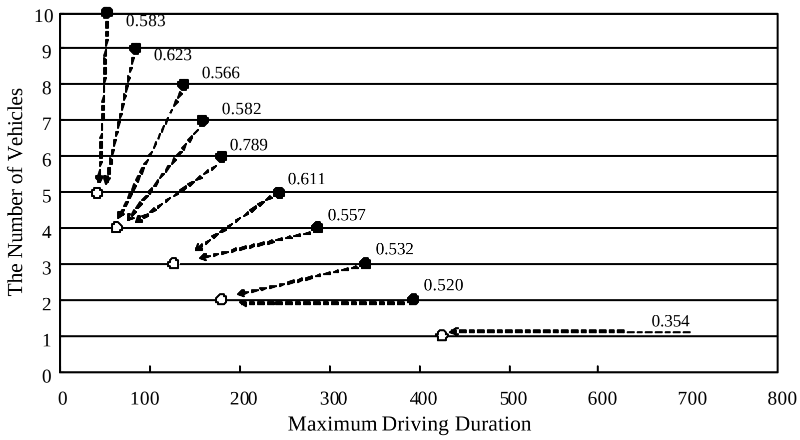

Fig. 11. Similarities of solutions obtained by a two-fold EMO algorithm (2F-EMO). Solutions of the HDP are obtained by an EMO algorithm with initial solutions generated from the solutions obtained in the NDP.

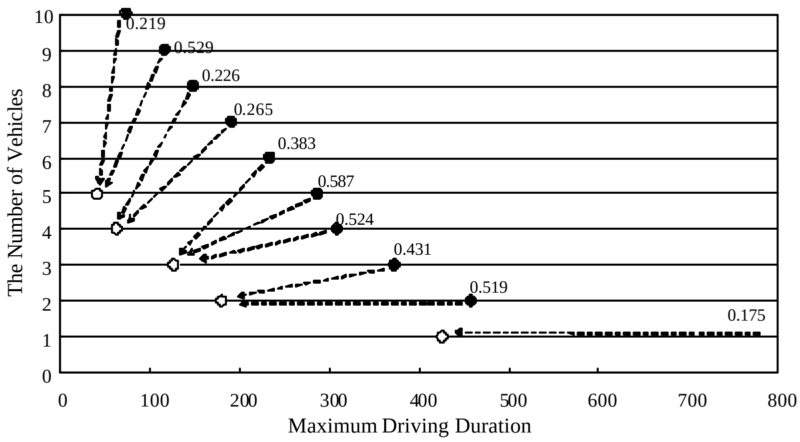

Fig. 12. Similarities of solutions obtained by an individually applied EMO algorithm (I-EMO). Solutions of the HDP are obtained by an EMO algorithm applied individually.

Table 1. The average maximum similarity over 100 trials.

# of vehicles	10	9	8	7	6	5	4	3	2	1	Ave.
2F-EMO	0.41	0.39	0.43	0.41	0.38	0.33	0.31	0.20	0.13	0.05	0.304
I-EMO	0.37	0.44	0.38	0.35	0.36	0.28	0.25	0.19	0.13	0.04	0.277

Table 1 shows that the average simulation results over 100 trials on the NDP and the HDP shown in Fig. 6. It summarizes the average maximum similarity for the solutions of each number of vehicles. We can see that the maximum similarity tends to become small for the solutions with the small number of vehicles. In a solution with a small number of vehicles, it has a small similarity since each vehicle should visit many customers.

5 Conclusion

In this paper, we consider the similarity of sets of non-dominated solutions that are obtained for a vehicle routing problem and its variant. When the number of customers are increased in a vehicle routing problem, it is better to reduce the effort of drivers to be informed of routes among customers when the new vehicle routing problem is solved. In this paper, we have just considered the influence of using initial solutions generated by a solution obtained for the former problem. Simulation results show that it is better to use initial solutions for an EMO algorithm to solve the increased problem. We are tackling to develop an algorithm to increase the similarity using genetic operations such as crossover and mutation.

References

1. Zitzler, E., Deb, K., Thiele, L., Coello Coello, C. A. and Corne, D. (eds.), *Proc. of First International Conference on Evolutionary Multi-Criterion Optimization* (EMO 2001).
2. Fonseca, C. M., Fleming, P. J., Zitzler, E., Deb, K., and Thiele, L. (eds.), *Proc. of Second International Conference on Evolutionary Multi-Criterion Optimization* (EMO 2003).
3. Deb, K., "Applications of multi-objective evolutionary algorithms," In: *Multi-objective Optimization Using Evolutionary Algorithms* (John Wiley & Sons, England), pp.447-479 (2001).
4. Tavares, J., Pereira, F. B., Machado, P., and Costa, E., "Crossover and diversity: A study about GVR," *Proc. of AdoRo (Workshop held at GECCO 2003)*, 7 pages.
5. Berger, J., and Barkaoui, M., "A hybrid genetic algorithm for the capacitated vehicle routing problem," *Proc. of GECCO 2003*, pp.646-656 (2003).
6. Tan, K. C., Lee, T. H., Chew, Y. H., and Lee, L. H., "A hybrid multiobjective evolutionary algorithms for solving vehicle routing problem with time windows," *Proc. of IEEE*

International Conf. on Systems, Man, and Cybernetics 2003 (Washington D.C., U.S.A., Oct. 5-8, 2003), pp.361-366 (2003).

7. Saadah, S., Ross, P., and Paechter, B., "Improving vehicle routing using a customer waiting time colony," *Proc. of 4th European Conf. on Evolutionary Computation in Combinatorial Optimization*, pp.188-198 (2004).
8. Chitty, D. M., and Hernandez, M. L., "A hybrid ant colony optimisation technique for dynamic vehicle routing," *Proc. of GECCO 2004*, pp.48-59 (2004).
9. Deb, K., Pratap, A., Agarwal, S., and Meyarivan, T., "A fast and elitist multiobjective genetic algorithm: NSGA-II," *IEEE Trans. on Evolutionary Computation*, **6** (2), pp.182-197, 2002.
10. Lenstra, J. K., and Rinnooy Kan, A.H.G., "Complexity of vehicle routing and scheduling problems", *Networks*, 11, pp. 221-227 (1981).
11. Oliver, I. M., Smith, D. J., and Holland, J. R. C., "A study of permutation crossover operators on the traveling salesman problem," *Proc. of 2nd International Conference on Genetic Algorithm and Their Application*, pp.224-230 (1987).