Hybrid Metaheuristics for Multi-objective Combinatorial Optimization

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Summary. Many real-world optimization problems can be modelled as combinatorial optimization problems. Often, these problems are characterized by their large size and the presence of multiple, conflicting objectives. Despite progress in solving multi-objective combinatorial optimization problems exactly, the large size often means that heuristics are required for their solution in acceptable time. Since the middle of the nineties the trend is towards heuristics that "pick and choose" elements from several of the established metaheuristic schemes. Such hybrid approximation techniques may even combine exact and heuristic approaches. In this chapter we give an overview over approximation methods in multi-objective combinatorial optimization. We briefly summarize "classical" metaheuristics and focus on recent approaches, where metaheuristics are hybridized and/or combined with exact methods.

1 Introduction

The last two or three decades have seen the development and the improvement of approximate solution methods – usually called *heuristics* and *metaheuristics*. The success of metaheuristics, e.g., simulated annealing, tabu search, genetic algorithms, on hard single objective optimization problems is well recognized today. Although combinatorial optimization models have been successfully used in a vast number of applications, these models often neglect the fact that many real-life problems require taking into account several conflicting points of view corresponding to multiple objectives. Here are some examples.

- In portfolio optimization risk and return are the criteria that have generally been considered. Recently the classical Markowitz model has been criticized and other criteria have been mentioned: ratings by agencies, dividend, long-term performance, etc., see, e.g., Ehrgott et al. [32].
- In airline operations, scheduling technical and cabin crew has a major effect on cost and small percentage improvements may translate to multi-million

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Euro savings. However, cost is not the only concern in airline operations. Robust solutions are desired, which avoid the propagation of delays due to crew changing aircraft, see Ehrgott and Ryan [33].
- In railway transportation, the planning of railway network infrastructure capacity has the goals of maximizing the number of trains that can use the infrastructure element (e.g. a station) and to maximize robustness of the solution to disruptions in operation. This problem can be modelled as a large scale set packing problem with two objectives [20].
- In radiation therapy planning for cancer treatment conflicting goals are to achieve a high uniform dose in the tumour whereas the dose absorbed by healthy tissue is to be limited. For anatomical and physical reasons these objectives cannot be achieved simultaneously. A multi-criteria model is described in Hamacher and Küfer [62].
- In computer networks, internet traffic routing may be enhanced if based on a multi-objective routing procedure to prevent network congestion. Multi-objective shortest paths between one router and all the other routers of the network must be computed in real-time, by simultaneously optimizing linear objectives (cost, delay) and bottleneck ones (quality, bandwidth), see Randriamasy et al. [44, 107].

Multiple objective optimization differs from traditional single objective optimization in several ways.

- The usual meaning of the optimum makes no sense in the multiple objective case because a solution optimizing all objectives simultaneously does in general not exist. Instead, a search is launched for a feasible solution yielding the best compromise among objectives on a set of so-called efficient (Pareto optimal, non-dominated) solutions.
- The identification of a best compromise solution requires the preferences expressed by the decision maker to be taken into account.
- The multiple objectives encountered in real-life problems can often be expressed as mathematical functions of a variety of forms, i.e., not only do we deal with conflicting objectives, but with objectives of different structures.

1.1 Multi-objective Optimization

A multi-objective optimization problem is defined as

$$\min \{(z_1(x), \ldots, z_p(x)) : x \in X\}, \tag{MOP}$$

where $X \subset \mathbb{R}^n$ is a feasible set and $z : \mathbb{R}^n \to \mathbb{R}^p$ is a vector valued objective function. By $Y = z(X) := \{z(x) : x \in X\} \subset \mathbb{R}^p$ we denote the image of the feasible set in the objective space. We consider optimal solutions of (MOP) in the sense of efficiency, i.e., a feasible solution $x \in X$ is called efficient if there does not exist $x' \in X$ such that $z_k(x') \leqq z_k(x)$ for all $k = 1, \ldots, p$ and

$z_j(x') < z_j(x)$ for some j. In other words, no solution is at least as good as x for all objectives, and strictly better for at least one.

Efficiency refers to solutions x in decision space. In terms of the objective space, with objective vectors $z(x) \in \mathbb{R}^p$ we use the notion of non-dominance. If x is efficient then $z(x) = (z_1(x), \dots, z_p(x))$ is called non-dominated. The set of efficient solutions is X_E, the set of non-dominated points is Y_N. We may also refer to Y_N as the non-dominated frontier. For $y^1, y^2 \in \mathbb{R}^p$ we shall use the notation $y^1 \leqq y$ if $y^1_k \leqq y^2_k$ for all $k = 1, \dots, p$; $y^1 \leq y^2$ if $y^1 \leqq y^2$ and $y^1 \neq y^2$; and $y^1 < y^2$ if $y^1_k < y^2_k$ for all $k = 1, \dots, p$. $\mathbb{R}^p_{\geqq}$ denotes the non-negative orthant $\{y \in \mathbb{R}^p : y \geqq 0\}$, $\mathbb{R}^p_{\geq} 0$ and $\mathbb{R}^p_{>} 0$ are defined analogously.

To solve a multi-objective optimization problem means to find the set of efficient solutions or, in case of multiple x mapping to the same non-dominated point, for each $y \in Y_N$ to find an $x \in X_E$ with $z(x) = y$. This concept of a set of efficient solutions is the major challenge of multi-objective optimization. Most methods require the repeated solution of single objective problems which are in some sense related to the multi-objective problem, see e.g., Miettinen [91] or Ehrgott and Wiecek [35]. Many references on the state-of-the-art in multi-objective optimization are found in [29].

1.2 Multi-objective Combinatorial Optimization

In this chapter we focus on *multi-objective combinatorial optimization* problems formulated as

$$\min \{Cx : Ax \geqq b, x \in \mathbb{Z}^n\}. \tag{MOCO}$$

Here C is a $p \times n$ objective function matrix, where c^k denotes the k-th row of C. A is an $m \times n$ matrix of constraint coefficients and $b \in \mathbb{R}^m$. Usually the entries of C, A and b are integers. The feasible set $X = \{Ax \geqq b, x \in \mathbb{Z}^n\}$ may describe a combinatorial structure such as, e.g., spanning trees of a graph, paths, matchings etc. We shall assume that X is a finite set. By $Y = CX := \{Cx : x \in X\}$ we denote the image of X under C in $\mathbb{R}^p$, the feasible set in objective space.

MOCO has become a very active area of research since the 1990s as demonstrated in the bibliographies by Ehrgott and Gandibleux [27, 28].

The biggest additional challenge in solving MOCOs as compared to multi-objective linear programs (MOLPs) $\min\{Cx : Ax \geqq b, x \geqq 0\}$ results from the existence of efficient solutions which are *not* optimal for any scalarization using weighted sums

$$\min \left\{ \sum_{k=1}^{p} \lambda_k z_k(x) : x \in X \right\}, \tag{1}$$

called non-supported efficient solutions X_{NE}. Those that are optimal for some weighted sum problem (1) are called supported efficient solutions X_{SE}.

Based on this distinction between supported and non-supported efficient solutions, the so-called two-phase method has first been developed by Ulungu and Teghem [130, 131]. The method computes supported (in Phase 1) and non-supported (in Phase 2) efficient solutions. In Phase 2 information obtained in Phase 1 is exploited. Both phases rely on efficient algorithms for the solution of single objective problems. The method has been applied to a number of problems such as network flow [82], spanning tree [106], assignment [104, 131] and knapsack problems [135].

Notably, all these applications are for bi-objective problems and the first generalization to three objectives is very recent. Przybylski, Gandibleux and Ehrgott [102, 103] use a decomposition of the weight set $\left\{\lambda \in \mathbb{R}^p_> : \mathbf{1}^T\lambda = 1\right\}$ (where $\mathbf{1}$ is a vector of ones) in Phase 1 and upper bound sets (see Sect. 2.1) and ranking algorithms in Phase 2 in an application to the three-objective assignment problem.

Many methods generalizing single objective algorithms for the use with multiple objectives have been developed, including dynamic programming [11] and branch and bound [88]. For more details we refer again to the bibliographies by Ehrgott and Gandibleux [27, 28] and references therein.

From a methodological point of view non-supported efficient solution are the main reason why the computation of X_E, respectively Y_N is hard, from a more theoretical point of view the computational complexity is another. Most MOCO problems are NP-hard (finding an efficient solution is hard) as well as #P-hard (counting efficient solutions is hard), see Ehrgott [26]. The best illustration of this fact is perhaps the unconstrained problem

$$\min\left\{\left(\sum_{i=1}^{n} c_i^1 x_i, \sum_{i=1}^{n} c_i^2 x_i\right) : x \in \{0,1\}^n\right\},$$

with $c_i^1, c_i^2 \geqq 0$ for $i = 1, \ldots, n$ which is trivial with only one objective. Moreover, MOCO problems are often intractable, i.e., there may exist exponentially many non-dominated points and efficient solutions.

With increasing interest in multi-objective models for real world applications and the difficulty of solving multi-objective combinatorial optimization problems exactly, interest in approximate methods for solving MOP/MOCO problems arose. In this paper we present an overview of approximation methods for solving multi-objective combinatorial optimization problems, focussing on hybrid metaheuristics for multi-objective combinatorial optimization.

The chapter is organized as follows. Sect. 2 introduces approximation methods for MOCO problems. Bound sets and quality of approximation are discussed in some detail. In Sect. 3, we present classical evolutionary algorithms and neighbourhood search metaheuristics. New methods based on ant colony optimization and particle swarm optimization are also briefly presented. In Sect. 4 we develop the hybrid methods and organize the ideas in six classes.

2 Approximation Methods for MOCO

Approximation methods for multi-objective optimization include both approximation algorithms which have a guaranteed quality of approximation, i.e., polynomial time approximation algorithms, and *multiple objective (meta)heuristics*, *MO(M)H* for short. A MO(M)H is a method which finds either sets of locally potentially efficient solutions that are later merged to form a set of potentially efficient solutions – the approximation denoted by X_{PE} – or globally potentially efficient solutions according to the current approximation X_{PE}.

The interest in approximation methods for multi-objective optimization is relatively recent. The first polynomial time approximation algorithm with performance guarantee is due to Warburton for the shortest path problem [136]. In the last five years this field has been growing, and such algorithms for, e.g., knapsack [38], travelling salesman [3, 85] and scheduling problems [58] are now known. Papadimitriou and Yannakakis [96] give a general result on the existence of polynomial time approximation algorithms.

In this chapter, however, we concentrate on metaheuristics. The first MOMH algorithm is a genetic algorithm developed 1984 by Schaffer [109]. In 1992, the work of Serafini [111] started a stream of research on multiple objective extensions of local search based metaheuristics. While the first adaptation of metaheuristic techniques for the solution of multi-objective optimization problems has been introduced more than 20 years ago, the MO(M)H field has clearly mushroomed over the last ten years. The pioneer methods have three characteristics.

- They are inspired either by *evolutionary algorithms (EA)* or by *neighbourhood search algorithms (NSA)*.
- The early methods are direct derivations of single objective metaheuristics, incorporating small adaptations to integrate the concept of efficient solution for optimizing multiple objectives.
- Almost all methods were designed as a solution concept according to the principle of metaheuristics. In fact, problem specific heuristics that are so ubiquitous in combinatorial optimization are rare, even descriptions of local search procedures appeared only recently [30, 99]. Paquete and Stützle describe local search heuristics for the TSP and QAP in [97] and [98], respectively. [115] develops a local search for the optimization of mobile phone keymaps.

2.1 Bound Sets

The quality of a solution of a combinatorial optimization problem can be estimated by comparing lower and upper bounds on the optimal objective function value. In analogy to moving from the optimal value to a set of non-dominated points, the concept of bounds has to be extended to bound sets in multi-objective optimization. It is clear that the ideal and nadir point y^I and y^N defined by

$$\begin{aligned} y_k^I &= \min\{z_k(x) : x \in X\} \text{ for } k = 1, \ldots, p \text{ and} \\ y_k^N &= \max\{z_k(x) : x \in X_E\} \text{ for } k = 1, \ldots, p, \end{aligned}$$

respectively, are lower/upper bounds for Y_N. We sometimes refer to a utopian point $y^U = y^I - \varepsilon\mathbf{1}$, where $\mathbf{1}$ is a vector of all ones and ε is a small positive number. However, the ideal and nadir points are usually far away from non-dominated points and do not provide a good estimate of the non-dominated set. In addition, the nadir point is hard to compute for problems with more than two objectives, see [34]. Ehrgott and Gandibleux [31] present definitions, some general procedures and report results on lower and upper bound sets for the bi-objective assignment, knapsack, travelling salesman, set covering, and set packing problems.

Fernández and Puerto [39] use bound sets in their exact and heuristic methods to solve the multi-objective uncapacitated facility location problem. Spanjaard and Sourd [116] use bound sets in a branch and bound algorithm for the bi-objective spanning tree problem.

2.2 The Quality of Approximation

MO(M)H algorithms do compute a (usually feasible) set of solutions to a multi-objective optimization problem. According to the definition of Ehrgott and Gandibleux [31] these define an upper bound set. But what is a *good* approximation of the non-dominated set of a MOP? This is an ongoing discussion in the literature and there is no sign of a consensus at this point in time (see Figs. 1 and 2).

Kim et al. [10] propose the integrated preference functional (IFP), which relies on a weight density function provided by the decision maker, to compare the quality of algorithms for MOCO problems with two objectives. Sayın [108] proposes the criteria of coverage, uniformity, and cardinality to measure how well subsets of the non-dominated set represent the whole non-dominated set. Although developed for continuous problems the ideas may be interesting for MOCO problems. However, the methods proposed in [108] can be efficiently implemented for linear problems only.

Viana and Pinho de Sousa [134] propose distance based measures and visual comparisons of the generated approximations. The latter are restricted to bi-objective problems. Jaszkiewicz [70] also distinguishes between cardinal and geometric quality measures. He gives further references and suggests preference-based evaluation of approximations of the non-dominated set using outperformance relations. Collette and Siarry [15] mention the proportion of X_{PE} among all solutions generated in one iteration, the variance of the distance between points in objective space, and a metric to measure the speed of convergence. They also talk about the aesthetic of solution sets in the bi-objective case. Tenfelde-Podehl [126] proposes volume based measures. The

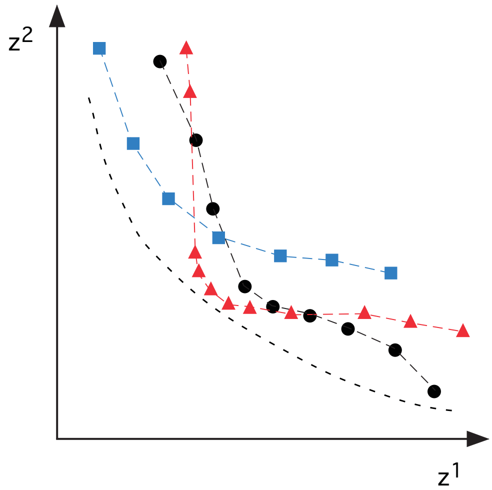

Fig. 1. Three approximations (bullet, triangle, square) and the non-dominated frontier (dotted line) are plotted. Square (better for z^1) and triangle (better for z^2) are complementary. Same conclusion for square and bullet. Triangle is better than bullet in the central part, and reversely. None of the three approximations is dominated. They are incomparable.

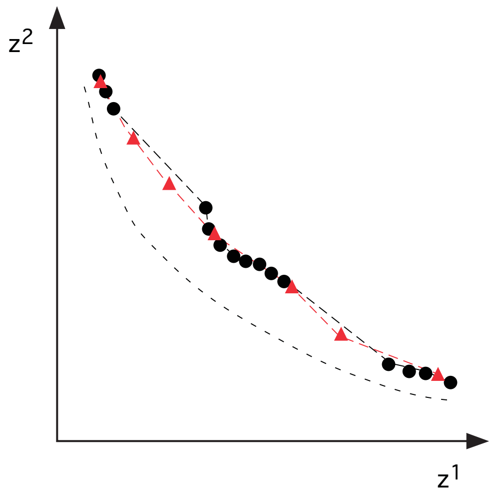

Fig. 2. Two approximations (bullet, triangle) and the non-dominated frontier (dotted line) are plotted. Density of points in the bullet approximation is higher than for the triangle one. However, points appear in clusters with the consequence of reporting gaps in the approximated frontier. Points in triangle approximation are better distributed along the frontier, but report a sparse approximated frontier. None of the two approximations dominates the other, they are incomparable.

hyper-volume measure introduced by Ziztler and Thiele [139] has become a popular means of comparing approximations because it has some important properties such as monotonicity with respect to set inclusion.

Zitzler et al. [140] present a review of performance measures, but none have been universally adopted in the multi-objective optimization literature, and further research is clearly needed.

3 Generation 1: Population Based Versus Neighbourhood Search MOMHs

3.1 Population Based Algorithms

Population based algorithms (see also Chap. 1 of this book) manage a "population" $\mathcal{P}$ of solutions rather than a single feasible solution. In general, they start with an initial population and combine principles of self adaptation, i.e., independent evolution (such as the mutation strategy in genetic algorithms), and cooperation, i.e., the exchange of information between individuals (such as the "pheromone" used in ant colony systems), to improve approximation quality. Because the whole population contributes to the evolutionary process, the generation mechanism is parallel along the non-dominated frontier, and thus these methods are also called *global convergence-based methods*. This characteristic makes population-based methods very attractive for solving multi-objective problems.

For a long time, the problems investigated with these methods were often unconstrained bi-objective problems with continuous variables and non-linear functions. Population based algorithms are appreciated by the engineering community, which could explain the large number of multi-objective evolutionary algorithm (MOEA) applications to real world problems (in mechanical design or electronics, for example). Many of these applications are characterized by long computation times for the evaluation of a single solutions. Surprisingly, MOCO problems have hardly been attacked by population based methods until recently.

3.2 The Pioneer: VEGA by Schaffer, 1984

In 1984 Schaffer [109, 110] introduced the *Vector Evaluated Genetic Algorithm (VEGA)*, which is an extension of Grefenstette's GENESIS program [60] to include multiple objective functions. The vector extension concerns only the selection procedure.

For each generation in VEGA, three stages are performed (Algorithm 17). The selection procedure is performed independently for each objective. In the first stage, the population is divided into p subpopulations S^k according to their performance in objective k (routine `pickIndividuals`). Each subpopulation is entrusted with the optimization of a single objective. In the next stage, subpopulations are shuffled to create a mixed population (routine `shuffle`). In the final stage, genetic operators, such as mutation and crossover, are applied to produce new potentially efficient individuals (routine `evolution`). This process is repeated for N_{gen} iterations.

Algorithm 17 VEGA, Vector Evaluated Genetic Algorithm

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input:	pop,	the population size
	N_{gen},	the generation limit
	$parameters$,	the crossover probability and mutation rate
output:	X_{PE},	the set of potentially efficient solutions

begin `VEGA`

`--|` Generate an initial population of *pop* individuals

$\mathcal{P}_0 \leftarrow$ `initialization`(pop)

`--|` Generation process

for n **in** $1, \ldots, N_{gen}$ **loop**

`--|` 1. Elaborate p subpopulations of size pop/p using each objective k
`--|` in turn

$S^k \leftarrow$ `pickIndividuals`$(\ pop/p\ ,\ k\ ,\ \mathcal{P}_{n-1}),\ \forall k = 1, \ldots, p$

`--|` 2. Set population of size *pop* shuffling together
`--|` the p subpopulations S^k

$S \ \leftarrow$ `shuffle`$(\cup_{k=1,\ldots,p} S^k)$

`--|` 3. Apply genetic operators

$\mathcal{P}_n \leftarrow$ `evolution`$(S, parameters)$

endLoop

$X_{PE} \leftarrow \mathcal{P}_{N_{gen}}$

end `VEGA`

As VEGA selects individuals who excel in one performance dimension without looking at the other dimensions, the speciation problem can arise with that method. This implies that individuals with a balanced performance on all objectives will not survive under this selection mechanism. Speciation is undesirable because it is opposed to the generation of compromise solutions. Due to this characteristic VEGA is termed a non-Pareto approach [13]. Additional heuristics were developed (like crossbreeding among the species) and studied to overcome this tendency.

3.3 Other Evolutionary Algorithms

Since VEGA many MOEAs have been developed. Significant progress concerns corrections of shortcomings observed in the first algorithms introduced and propositions of new algorithmic primitives to generate a better approximation of X_E. Modern MOEAs are characterized according to the way they handle population structure, archiving, selection/elitism mechanisms, and fitness functions.

Two central questions motivate the research about MOEAs: (1) how to accomplish both fitness assignment and selection in order to guide the search toward the non-dominated frontier and (2) how to maintain a diversified population in order to avoid premature convergence and find a uniform distribution of points along the non-dominated frontier?

For the first question MOEAs are distinguished by the way the performance of individuals is evaluated in the selection. If the objectives are considered separately, the selection of individuals is performed by considering each objective independently (Schaffer [110]), or the selection is based on a comparison procedure according to a predefined (or random) order on the objectives (Fourman [42]), or the selection takes into account probabilities assigned to each objective in order to determine a predominant objective (Kursawe [78]). If the objectives are aggregated into a single parameterized objective function, the parameters of the function are systematically updated during the same runs (at random or using a particular weight combination) taking advantage of information collected on the population of individuals (Hajela and Lin [61] and Murata and Ishibuchi [94]). Each aggregation defines a search direction in the objective space and the idea is to optimize in multiple directions simultaneously. If, on the other hand, the concept of efficiency is directly used, (non-domination ranking) the fitness of an individual (i.e. a solution) is calculated on the basis of the dominance definition. The idea is to take advantage of information carried by the population of solutions using the notion of domination for selection. This is the most common approach and has led to several Pareto-based fitness assignment schemes, see [40, 55, 67, 117, 139], etc.

The majority of the other components of a MOEA deal with the second question. Fitness sharing based on a principle of niches is the most frequently used technique and most MOEAs are implementing it, e.g., [40, 67, 117, 138]. Niches are solution neighbourhoods with a radius σ_{sh} in objective space, centered on candidate points. Based on the number of solutions in these niches, the selection of individuals can be influenced to generate more in areas where niches are sparsely populated, with the goal of greater distribution uniformity along the non-dominated frontier. A *sharing function*, which measures the distance $d(i,j)$ between a candidate point i and a neighbour j, is defined by

$$\phi(d(i,j)) = 1 - \left(\frac{d(i,j)}{\sigma_{sh}}\right)^{\alpha}$$

if $d(i,j) < \sigma_{sh}$ and 0 otherwise. The parameter α amplifies ($\alpha > 1$) or attenuates ($\alpha < 1$) the sharing value computed. Thus the shared fitness f_{s_i} of candidate i is

$$f_{s_i} = \frac{f_i}{\sum_{j=1}^{N} \phi(d(i,j))}$$

such that the shared fitness of candidates increases the fitness f_i if ϕ values are small, i.e. the distance of neighbours from i is close to σ_{sh}.

A number of important implementations of MOEAs have been published in recent years. There are even a number of surveys on the topic (see [13, 14, 41, 72]). The most outstanding among the pioneer MOEAs are mentioned briefly below:

- Vector Evaluated Genetic Algorithm (VEGA) by Schaffer, 1984 [109].
- Multiple Objective Genetic Algorithm (MOGA93) by Fonseca and Fleming, 1993 [40]. MOGA93 uses a ranking procedure in which the rank of an individual is equal to the number of solutions which dominate this individual.
- Non-dominated Sorting Genetic Algorithm (NSGA) by Srinivas and Deb, 1994 [117]. NSGA implements Goldberg's ranking idea in which the rank of an individual is equal to its domination layer, computed by ranking the population on the basis of domination.
- Niched Pareto Genetic Algorithm (NPGA) by Horn, Nafpliotis and Goldberg, 1994 [67]. NPGA combines the Pareto dominance principle and a Pareto tournament selection where two competing individuals and a set of individuals are compared to determine the winner of the tournament.
- Strength Pareto Evolutionary Algorithm (SPEA) by Zitzler and Thiele, 1998 [138]. SPEA takes the best features of previous MOEAs and combines them to create a single algorithm. The multi-objective multi-constraint knapsack problem has been used as a benchmark to evaluate the method [139].
- Pareto Archived Evolution Strategy (PAES) by Knowles and Corne, 1999 [77]. PAES is an evolutionary strategy that employs local search to generate new candidate solutions and a reference archive to compute solution quality.

3.4 Other Population Based Methods

Ant colony optimization (ACO) based heuristics are population based methods imitating the foraging behaviour of ants. The principle is based on the positive reinforcement of elementary decisions via the use of pheromone trails, which are left by the ants. The cooperation between ants is accomplished by pheromone trails which act as a common memory of the colony.

Roughly speaking, in the case of a combinatorial optimisation problem, the pheromone trails induce a probability distribution over the search space. An ACO algorithm consists in an iterative process where, at each iteration the ants build solutions by exploiting the pheromone trails. Hence, they have the ability of learning "good and bad" decisions when building solutions. Once a solution is completed, pheromone trails are updated according to the quality of some of the generated solutions. Recently, ACO algorithms have been proposed for MOPs, introducing a novel stream of population based algorithms. We summarize some of the published papers.

- The MOAQ algorithm of Mariano and Morales [86, 87] uses one ant colony for each objective function. All colonies have the same number of ants. (Partial) solutions of each colony are used in the next colony. The algorithm is applied to two literature problems (and compared with VEGA [109]). The research is motivated by the real world problem of designing a water distribution network for irrigation to minimise network cost and maximise profit.
- McMullen [89] addresses a just-in-time sequencing problem with the objective of minimizing the number of setups and minimizing the usage rates of raw materials. The problem is reformulated as a TSP by spatialising the data and applying a standard single objective ant colony algorithm.
- Iredi and Middendorf [68] propose a number of ant colony optimization algorithms to solve bi-criteria combinatorial problems, including ones with single and multiple colonies. Various methods for pheromone update and weight assignment (in order to browse the whole non-dominated frontier) are proposed and tested on a single machine scheduling problem to minimise total tardiness and changeover cost. Numerical tests are presented.
- Doerner et al. [21, 22, 23, 24] work on a multi-objective portfolio selection problem. They consider a rather large number of objectives $p = (B+R)T$, where B is the number of benefit categories, R is the number of resources, and T is the planning period. They use one colony and a random selection of weights of objectives for each ant. The global update considers the best and second best solutions for each objective found in the current iteration. The results on test problems are compared with NSGA [117], PSA [16], and the true efficient set (for small problems).
- Doerner et al. [25] solve a special case of the pickup and delivery problem with the linearly combined objectives of total number of vehicles and empty vehicle movements by a multi-colony approach. The colonies use different heuristic information, and their sizes change during the algorithm.
- Gravel et al. [59] consider the problem of sequencing orders for the casting of aluminium. Four objectives are considered in a lexicographic sense. A distance function based on penalties for bad performance is used to translate the problem into a TSP setting. Global pheromone update considers only the primary objective.

Particle swarm optimization (PSO) is a population based metaheuristic inspired by the social behaviour of bird flocks and fish schools (swarms) searching for food. Each individual determines its velocity based on its experience and information obtained from interacting with other members. In the optimization context, the individual members of the swarm (particles) "travel" through solution space. Each particle is characterized by three vectors x, v and p, where x is the current position (solution), v is the velocity, and p is the best position occupied so far. In each iteration k, the position and velocity of the particles are updated: $v^{k+1} := f(v^k, p^k, x^k, \hat{p}^k)$ and $x^{k+1} = x^k + v^{k+1}$. The update function f depends not only on the particles own experience but also on $\hat{p}$, the best position of any particle so far, thus modelling the interaction in the swarms behaviour.

The PSO method has been introduced by Kennedy and Eberhart [76] in 1995 and applications in multi-objective optimization date back to [92]. Implementations usually use an archive of potentially efficient solutions. Very few applications in the MOCO area exist.

In [105] PSO is applied to a permutation flowshop problem to minimize weighted mean completion time and weighted mean tardiness. Yapicioglu et al. [137] use it to solve a bi-objective formulation of the semi-desirable facility location problem. The algorithm uses local search to improve solutions obtained by the basic PSO mechanism.

The *scatter search* principle has been proposed by Glover, Laguna, and Martí [54]. It uses a population of solutions called the reference set. The method forms combinations of solutions which are subsequently improved, before updating the reference set. A scatter search algorithm consists of

1. a diversification method to generate a diverse set of solutions,
2. an improvement method to be applied to solutions,
3. a reference set update method,
4. a subset generation method to select solutions from the reference set that is used for creating combinations,
5. and a solution combination method.

An adaptation to multi-objective optimization appeared shortly after the original method [8]. The only MOCO papers using scatter search we are aware of are [56] and [57], who apply it to the bi-objective knapsack problem. The former paper uses exact solutions of the LP relaxation as diversification, simple heuristics to obtain feasible and improved solutions, a clustering method for reference set update, selects pairs of consecutive solutions from the reference set, and uses path relinking for combining solutions. The latter is a hybrid method that improves some of the methods used in [56] and includes the exact solution of small residual problems in the improvement method.

3.5 Neighbourhood Search Algorithms

In *neighbourhood search algorithms (NSA)* the generation of solutions relies upon one individual, a current solution x^n, and its neighbours $x \in \mathcal{N}(x^n)$. Using a local aggregation mechanism for the objectives (often based on a weighted sum), a weight vector $\lambda \in \Lambda$, and an initial solution x^0, the procedure iteratively projects the neighbours into the objective space in a search direction λ by optimizing the corresponding parametric single objective problem. A local approximation of the non-dominated frontier is obtained using archives of the successive potentially efficient solutions detected. This generation mechanism is sequential along the frontier, producing a local convergence to the non-dominated frontier, and so such methods are called *local* *convergence-based methods*. The principle is repeated for diverse search directions to completely approximate the non-dominated frontier. NSAs are well-known for their ability to locate the non-dominated frontier, but they require more effort in diversification than EAs in order to cover the non-dominated frontier completely.

The first approximation methods proposed for MOCO problems were "pure" NSA strategies and were straightforward extensions of well-known metaheuristics for dealing with the notion of non-dominated points. Simulated annealing (the MOSA method [132]), tabu search (the MOTS method [46], the method of Sun [119]), or GRASP (the VO-GRASP method [52]) are examples.

3.6 The Pioneer: MOSA by Ulungu, 1992

In 1992 (EURO XII conference, Helsinki), Ulungu introduced Multi-objective Simulated Annealing, MOSA [130], a direct derivation of the simulated annealing principle for handling multiple objectives (see Algorithm 18). Starting from an initial, randomly generated solution x^0 and a neighbourhood structure $\mathcal{N}(x^n)$, MOSA computes a neighbour $x \in \mathcal{N}(\cdot)$ using a set of weights Λ that define search directions $\lambda \in \Lambda$. The comparison of x with x^n according to p objectives $z_k(x)$, $k = 1, \dots, p$ gives rise to three possible cases. If $\Delta z_k = z_k(x) - z_k(x^n)$ is the difference between solution x and x^n in the objective k:

(a) $\Delta z_k \leqq 0$ for all k: x improves all objectives. x (weakly) dominates x^n.
(b) $\Delta z_k < 0$ and $\Delta z_{k'} > 0$ for some k and k': Improvement and deterioration occur simultaneously for different objectives. Both solutions x and x^n are potentially efficient.
(c) $\Delta z_k \geqq 0$ for all k: All objectives deteriorate with at least one strict inequality. Solution x is dominated by x^n.

A neighbour x is always accepted if it dominates x^n (a). When x is dominated (c), it can be accepted with decreasing probability, depending on the current "temperature" of the cooling schedule (Routine `isAccepted`). In the initial version of MOSA, a neighbour in situation (b) was also always accepted (Routine `isBetter`). This acceptance principle has been revised in a later version of the method to include the search direction in the decision.

To measure the degradation in the routine `isAccepted`, the values are aggregated using a scalarizing function $S(z(x), \lambda)$. Such a function makes a "local aggregation" of the objectives which allows the computation of the "weighted distance" $\Delta s = S(z(x), \lambda) - S(z(x^n), \lambda)$ between $z(x)$ and $z(x^n)$.

If a neighbour is accepted, the set of potentially efficient solutions $X_{PE\lambda}$ in direction λ is updated. The search stops after a certain number of iterations, or when a predetermined temperature is reached (Routine `isFinished`). At the end, MOSA combines the sequential processes in the objective space Y in a set X_{PE} by merging the sets PE_λ (Routine `merge`). The outline of MOSA for maximizing objectives is given in Algorithm 18.

Algorithm 18 MOSA, multi-objective Simulated Annealing

input:	Λ,	set of weights
	$T, \alpha, N_{step}, T_{stop}, N_{stop}$,	SA parameters
output:	X_{PE},	set of potentially efficient solutions

begin `MOSA`
  $X_{PE} \leftarrow \emptyset$
  for all $\lambda \in \Lambda$ **loop**
    $T_0 \leftarrow T$; $N_{count} \leftarrow 0$; $n \leftarrow 0$
    randomly draw $x^n \in X$; $X_{PE\lambda} \leftarrow \{x^n\}$
    repeat   randomly draw $x \in \mathcal{N}(x^n)$
      if `isBetter`(x, x^n) **or else** `isAccepted`$(x, x^n, n, T_n, \lambda)$ **then**
        $X_{PE\lambda} \leftarrow$ `archive`$(X_{PE\lambda}, x)$; $x^{n+1} \leftarrow x$; $N_{count} \leftarrow 0$
      else
        $x^{n+1} \leftarrow x^n$; $N_{count} + +$
      endIf
      $n + +$;   `updateParameters`(α, n, T_n)
    until `isFinished`(N_{count}, T_n)
  endLoop
  $X_{PE} \leftarrow$ `merge`$(X_{PE\lambda})$
end `MOSA`

3.7 Other Neighbourhood Search Methods

In 1996 (MOPGP 96 conference, Torremolinos), Gandibleux et al. introduced the first TS-based method called MOTS for multi-objective tabu search [46], designed to compute a set of potentially efficient solutions. Using a scalarizing function and a reference point, the method performs a series of tabu processes guided automatically in the objective space by the current approximation of the non-dominated frontier. Intensification, diversification and tabu daemon (usually called aspiration criteria) are designed for the multi-objective case. Two tabu lists are used, one on the decision space $TmemX$, the second on the objective space $TmemY$. The former is an attribute-based tabu list preventing a return to already visited solutions during a tabu process. The latter is related to the objectives and based on an improvement measure of each objective. It is used for updating weights between two consecutive tabu processes.

The MOTS search strategy is encapsulated in a *tabu process*, which is composed of a series of iterations. Let us consider, at the n^{th} iteration, the current solution x^n and its (sub)neighbourhood $\mathcal{N}(x^n)$ obtained according to a suitable move $x^n \to x$ defined according to the structure of the feasible set X (routine `exploreNeighbourhood`). The successor $\bar{x}$ of x^n for the next iteration is selected from the list of neighbour solutions $\mathcal{L} = \{x \in \mathcal{N}(x^n)\}$ as the best according to a weighted scalarizing function $S(z(x), y^U, \lambda)$

$$S(z(x), y^U, \lambda) = \max_{1 \leq k \leq p} \left\{\lambda_k \left(y_k^U - z_k(x)\right)\right\} + \rho \sum_{k=1}^{p} \lambda_k \left(y_k^U - z_k(x)\right),$$

with $\rho > 0$. The number of candidates in list $\mathcal{L}$ is limited to K solutions. The value of this parameter depends on the neighbourhood size $(1 \leq K \leq |\mathcal{N}(x^n)|)$. The reference point y^U in the scalarizing function is the locally determined utopian point $y^U = (y_1^U, \ldots, y_p^U)$ over $\mathcal{L}$, where $y_k^U < \inf\{z_k(x) : x \in \mathcal{L}\}$. This point dominates the ideal point given by the lowest objective function value on each objective among the solutions in the neighbourhood of the current solution. The tabu list $TmemX$ is used to avoid cycling. The selected solution $\bar{x} \in \mathcal{L}$, which minimizes $S(z(x), y^U, \lambda)$ over $\mathcal{L}$ such that the move $x^n \to \bar{x}$ is not tabu, becomes the new current solution x^{n+1}. The *tabu daemon* overrides the tabu status of a solution $x' \in \mathcal{N}(x^n)$ if $s(z(x'), \lambda) \leq s(z(\bar{x}), \lambda) - \Delta$, with Δ being a static or dynamic threshold value. As $\mathcal{L}$ is generally a finite subset of X, the successor solution x^{n+1} can be found easily. However, the time complexity depends on the size of the neighbourhood $\mathcal{N}(x^n)$. Each iteration ends with the identification of the potentially efficient solutions in $\mathcal{L}$, which represents a local approximation of the non-dominated frontier (routine `archive`). More details about MOTS are available in [30, 46]. This is a generic method, rather than a ready-to-use technique. All of its primitives need to be stated in a suitable manner, according to the MOCO problem to be solved.

Few implementations have been developed in this stream. It can be explained by the fact that the advantage of NSA algorithms hybridized with other techniques has been recognized early, giving birth to the hybrid MOMH wave. Among the "pure NSA" algorithms discussed, we find:

- Multi-objective tabu search (MOTS) by Gandibleux, Mezdaoui and Fréville, 1996 [46].
- Sun's Method, 1997 [119]. An interactive procedure using tabu search for general multiple objective combinatorial optimization problems, the procedure is similar to the combined Tchebycheff/aspiration criterion vector method [118]. The tabu search is used to solve subproblems in order to find potentially efficient solutions. The principles used for designing the TS search strategy are similar to those defined for MOTS. This method has been used for facility location planning [1].
- Adaptations of metaheuristics, such as the greedy randomized adaptive search procedure GRASP [52].
- Other simulated annealing-based methods. Nam and Park's method, 2000 [95] is another simulated annealing-based method. The authors report

good results in comparison with MOEA. Applications include aircrew rostering problems [84], assembly line balancing problems with parallel workstations [90], and analogue filter tuning [127].

4 Generation 2: Hybrid MOMHs

The methods that followed the pioneer ones, designed to be more efficient algorithms in the MOCO context, have been influenced by two important observations.

The first observation is that on the one hand, NSAs focus on convergence to efficient solutions, but must be guided along the non-dominated frontier. On the other hand, EAs are very well able to maintain a population of solutions along the non-dominated frontier (in terms of diversity, coverage, etc.), but often converge too slowly to the non-dominated frontier. Naturally, methods have been proposed that try to take advantage of both EA and NSA features by combining components of both approaches, introducing *hybrid algorithms* for MOPs.

The second observation is that MOCO problems contain information deriving from their specific combinatorial structure, which can be advantageously exploited by the approximation process. Single objective combinatorial optimization is a very active field of research. Many combinatorial structures are very well understood. Thus combinatorial optimization represents a useful source of knowledge to be used in multi-objective optimization. This knowledge (e.g., cuts for reducing the search space) are more and more taken into account when designing a very efficient approximation method for a particular MOCO. It is not surprising to see an evolutionary algorithm – for global convergence – coupled with a tabu search algorithm – for the exploitation of the combinatorial structure – within one approximation method.

Modern MOMH's for MOCO problems appear more and more as problem-oriented techniques, i.e., selections of components that are advantageously combined to create an algorithm which can tackle the problem in the most efficient way. By nature the algorithm is hybrid, including evolutionary components, neighbourhood search components, and problem specific components.

This section gives an overview of the literature on hybrid MOMH's classified in three main categories: metaheuristic with metaheuristic (Sects. 4.1, 4.2 and 4.3), metaheuristic with other techniques (Sect. 4.4) and metaheuristic with preferences provided by a decision-maker (Sect. 4.5). An overview of the trends in the design of hybrid multi-objective metaheuristics closes this section (Sect. 4.6).

4.1 Hybridization to Make a Method More Aggressive

To enhance the aggressiveness of the approximation procedure, the first step of hybridization has been the inclusion of a local search inside a MOEA. In that scheme, EA drives the search procedure, and activates an NSA, with the aim to improve as far as possible the promising solutions resulting from the evolutionary operators. The NSA can simply be a depth first search method, or a more sophisticated method like a (truncated) tabu search exploiting advantageously the combinatorial structure of the optimization problem.

Figs. 3 and 4 report the results obtained for a bi-objective knapsack problem with two constraints and 250 items. The exact frontier has been computed with CPLEX. Approximations are the output of a single run of five MOMHs. The approximations obtained with three MOEAs without NSA (VEGA, NSGA, SPEA) are concentrated in the middle part of the non-dominated frontier (Fig. 3). Notice the gap of efficiency between VEGA, the pioneer method, and the two well-known methods, NSGA and SPEA, which is well visible on this example. The two MOEAs with a NSA (MOGLS and MOGTS) compute comparable approximations, both are well spread along the non-dominated frontier (Fig. 4).

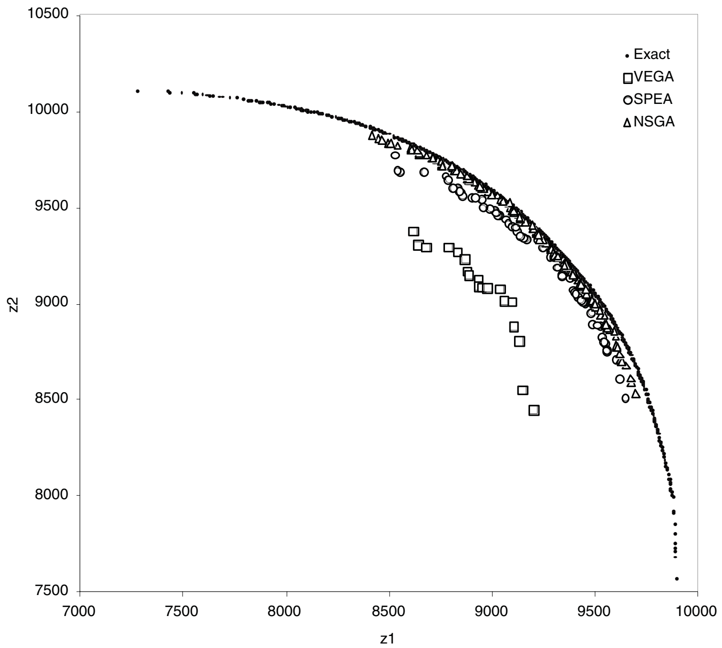

Fig. 3. Multi-objective multi-constrained knapsack problem (250 items, 2 objectives, and 2 constraints), solved with VEGA, NSGA, and SPEA.

The following references give an overview of methods falling in this category.

- Multiple Objective Genetic Algorithm (MOGA) by Murata and Ishibuchi, 1995 [94]. This method is not based on the Pareto ranking principle but

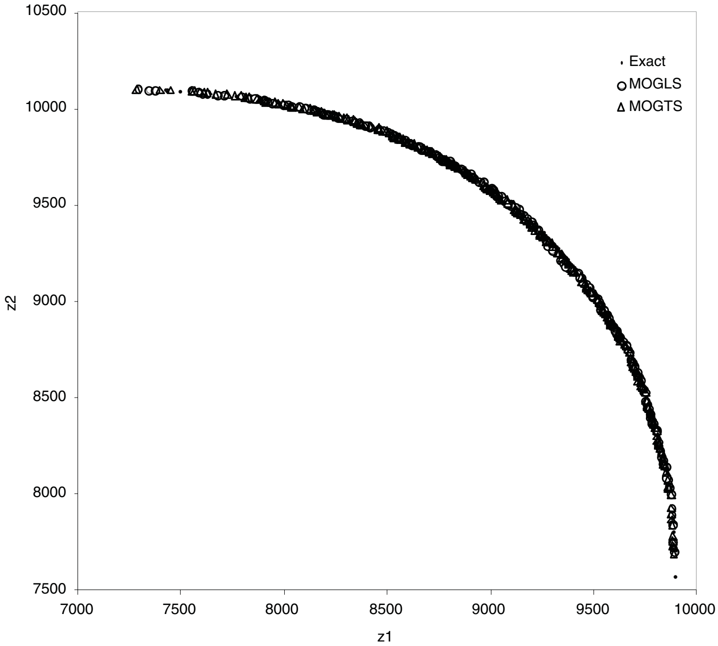

Fig. 4. Multi-objective multi-constrained knapsack problem (250 items, 2 objectives, and 2 constraints), solved with MOGLS and MOGTS.

on a weighted sum of objective functions, combining them into a scalar fitness function that uses randomly generated weight values in each iteration. Later, the authors coupled a local search with a genetic algorithm, introducing the memetic algorithm principle for MOPs.

- Method of Morita et al. (MGK) by Morita, Gandibleux and Katoh, 1998 [47]. Seeding solutions, either greedy or supported efficient, are put in the initial population in order to initialise the algorithm with good genetic information. The bi-objective knapsack problem is used to validate the principle. This method becomes a memetic algorithm when a local search has been performed on each new potentially efficient solution [48].
- Pareto Archived Evolution Strategy (PAES) by Knowles and Corne, 1999 [77]. PAES is an evolutionary strategy that employs local search to generate new candidate solutions and a reference archive to compute solution quality.
- Multiple Objective Genetic Local Search (MOGLS) by Jaszkiewicz, 2001 [69]. This method hybridizes recombination operators with local improvement heuristics. A scalarizing function is drawn at random for selecting solutions, which are then recombined, and the offspring of the recombination are improved using heuristics.

- Multiple Objective Genetic Tabu Search (MOGTS) by Barichard and Hao, 2002 [5]. This is another hybrid method in which a genetic algorithm is coupled with a tabu search. MOGTS has been evaluated on the multi-constraint knapsack problem.
- Multi-colony Ant System (MACS) by Gambardella, Taillard and Agazzi, 1999 [43]. A bi-criteria vehicle routing problem with time windows with the lexicographically sorted objectives of minimizing the number of vehicles and minimizing total travel distance is solved. MACS uses one colony for each objective and local as well as global pheromone update. The colonies cooperate through the use of the global best solution for the global pheromone update. Local search is applied to improve the quality of each solution found.
- T'Kindt et al., 2002 [128]. A (single) ant colony optimization approach is proposed for a two machine bi-criteria flowshop problem to minimize makespan and total flowtime in a lexicographic sense. The solutions produced by the ants are improved by local search.

Other references in this most popular category include [123, 124, 122], all of which apply an evolutionary algorithm combined with local search heuristics to variants of the vehicle routing problem. In [71] this idea is applied to the bi-objective set covering problem and [79] deals with the bi-objective arc routing problem following this strategy and [74] apply it to the TSP with profits.

4.2 Hybridization to Drive a Method

One fundamental question for "pure" NSA based MOMHs, like MOSA or MOTS, is the guidance mechanism along the non-dominated frontier. Influenced by the success of MOGA, authors introduced the principle of deriving search directions from a population of individuals. Here, global information about the current approximation is deduced and *drives local search processes* in order to "guarantee" a good coverage of the non-dominated frontier.

Using, for example, mechanisms based on notions of repulsion between non-dominated points, the search is guided toward subareas of the frontier containing (i) a high density of solutions or (ii) areas not yet explored.

This is the principle of the PSA [17] and the TAMOCO [63] methods. Dealing with the same question, Engrand's revised method [36, 100], and Shelokar et al. [114] use the non-domination definition to avoid the management of search directions.

- Pareto Simulated Annealing (PSA) by Czyzak and Jaszkiewicz, 1995 [16, 17, 18]. This method combines simulated annealing and genetic algorithm principles. The main ideas concern the management of weights and the consideration of a set of current solutions. A sample $S \subset X$ of $\#S$ solutions is determined and used as initial solutions. Each solution in this set is "optimized" iteratively, i.e., by generating neighbouring solutions that may be accepted according to a probabilistic strategy. For a given solution $\bar{x} \in S$ the weights are changed in order to increase the probability of moving it away from its closest neighbour in S denoted by $\bar{x}'$. Solutions in S play the role of agents working almost independently but exchanging information about their positions in the objective space. Thus, the interaction between solutions guides the generation process through the values of λ.

- Multi-objective Tabu Search (TAMOCO) by Hansen, 1997 [64]. This method uses a set of "generation solutions", each with its own tabu list. These solutions are dispersed throughout the objective space in order to allow searches in different areas of the non-dominated frontier (see Fig. 5). Weights are defined for each solution with the aim of forcing the search into a certain direction of the non-dominated frontier and away from other current solutions that are efficient with respect to it. Diversification is ensured by a set of generation solutions and a drift criterion. Results for the knapsack problem and for a resource constrained project scheduling problem are available in [134].

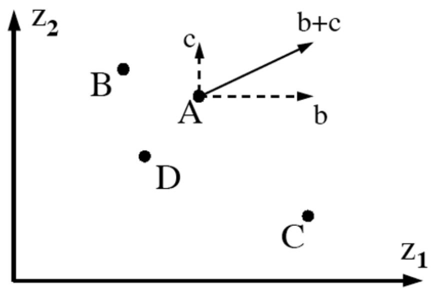

Fig. 5. The positions of four solutions A, B, C and D in objective space are shown (for a maximization problem). Solution A should be improved to move towards the non-dominated frontier but at the same time it should move away from other current solutions, which are non-dominated with respect to A (solutions B and C). Solution B pushes solution A away and this is shown by an optimization influence in the direction of vector b. Likewise does solution C influence solution A to move away from it in direction c. The final optimization direction for solution A is found by adding these weighted influence vectors. (The figure is reproduced from [63].)

- Engrand's method, 1997 [36, 37]. This is a hybridization of simulated annealing principles with genetic algorithms originally applied to a nuclear fuel management problem. Engrand's method has been revised later by Park and Suppapitnarm [100, 120, 121] and applied to the pressurized water reactor reload core design optimization problem. The main characteristic of this revised version is its ability to work without search directions, using a population of individuals to ensure the exploration of the complete trade-off surface. Each objective is considered separately. The method uses only the non-domination definition to select potentially efficient solutions, thus avoiding the management of search directions and aggregation mechanisms. Advanced strategies use the population of potentially efficient solutions to drive the approximation mechanism, thus ensuring the detection of the whole non-dominated frontier.

- The method of Shelokar et al., 2000 [112, 113, 114]. An ant algorithm for multi-objective continuous optimization is proposed. An interesting feature is that it combines the ant system methodology with the strength Pareto fitness assignment of [138] and clustering methods. The algorithm is applied to reliability engineering problems in [112] and to the optimization of reactor regenerator systems in [113].
- Armentano and Arroyo's method [4] is based on the tabu search principle. However, they work on a set of solutions following several paths, each having its own tabu-list. To diversify the search they apply a clustering technique to the set of potentially efficient solutions in any given iteration and use the centroids of clusters to define search directions. They report experiments on a bi-objective flowshop scheduling problem, obtaining good results compared to exact methods, local search, and tabu search.

4.3 Hybridization for Exploiting Complementary Strengths

The idea of using the complementary forces of metaheuristics was a natural way for the emergence of hybrid methods. A MOMH is often designed as a two step method, switching from method A to method B, with a communication process of solutions from A to B. The couple EA+NSA aims to take the advantage of, for example, a GA-based algorithm for building a set of good solutions in a first step followed by an aggressive search method, e.g., a TS-based algorithm, in a second step. The couple NSA+GA makes sense for example when an efficient algorithm is known for the single objective version of the optimization problem. The first step aims to poke around the non-dominated frontier, providing a sample of very good solutions covering very well the whole non-dominated frontier, while the second step has to fill out the approximation in terms of distribution. The following references illustrate three MOMHs falling in this category.

- Ben Abdelaziz et al.'s hybrid method, 1999 [9]. The authors present a hybrid algorithm using both EA and NSA independently. The goal of the EA (a genetic algorithm) is to produce a first diversified approximation, which is then improved by the NSA (a tabu search algorithm). Results have been reported on the multi-objective knapsack problem.
- Delorme et al. [19] design a scheme based on an NSA interfaced with an EA for solving the bi-objective set packing problem. The idea is to take advantage of an efficient heuristic known for the single objective problem in order to compute an initial set of very good solutions $\mathcal{P}_0$ in a first phase. The heuristic (a GRASP algorithm) is encapsulated in a basic generation

procedure, for example using a convex combination of the objectives: λ-GRASP. The second phase works on a population of individuals $\mathcal{P}$ derived from the initial set $\mathcal{P}_0$, and performs an EA (a modified version of SPEA dealing with all potential efficient solutions and integrating a local search: A-SPEA) in order to consolidate the approximation of the non-dominated frontier (see Fig. 6).

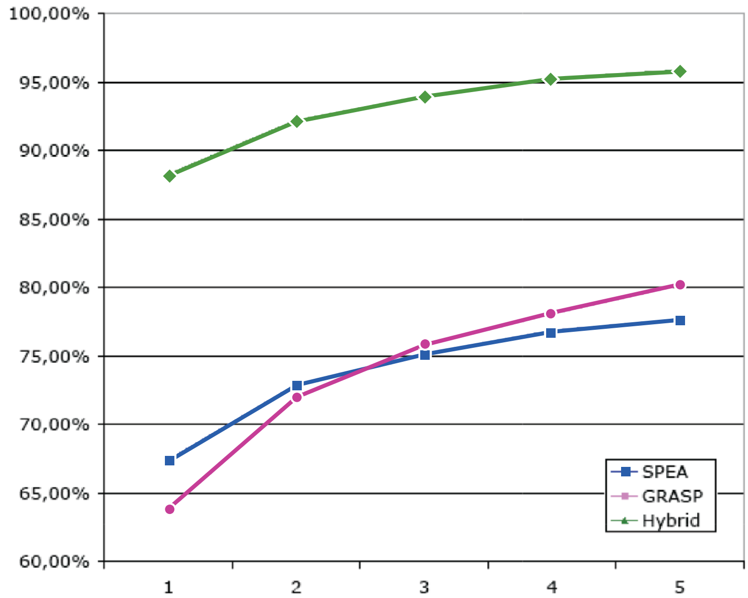

Fig. 6. The figure illustrates the average percentage of exact solutions found using λ-GRASP, A-SPEA, and the hybrid for the set packing problem with two objectives, when all three methods are allowed the same computational effort. (The figure is reproduced from [19].)

- López-Ibáñez, Paquete and Stützle [83] compare hybridizations of a multi-objective ant colony optimization algorithm and the evolutionary algorithm SPEA with short runs of a tabu search or a simple iterative improvement algorithm. The goal of the hybridization is to achieve an acceptable trade-off between solutions speed and solution quality. Their experimental study on the bi-objective quadratic assignment problem shows that characteristics of the instance have a strong influence on the performance of the variants.

4.4 Hybridization with Other Techniques

Embedding (meta)heuristics in an exact solution method or reversely is common in single objective optimization. More broadly speaking, the integration of techniques from several fields as operations research, artificial intelligence, and constraint programming has lead to interesting results on large and complex single objective problems. But it is astonishing to observe that this way to proceed is marginal in the multi-objective context. Four successful cases of that kind of hybridization are reported.

- Gandibleux and Fréville [45] propose a procedure for the bi-objective knapsack problem combining an exact procedure for reducing the search space with a tabu search process for identifying the potentially efficient solutions. The reduction principle is based on cuts which eliminate parts of the decision space where (provably) no exact efficient solution exists (see Fig. 7). It uses an additional constraint on the cardinality of an optimal solution for computing a utopian reference point and an approximation set for verifying if the reference point is dominated. The tabu search is triggered on the reduced space and dynamically updates the bounds in order to guarantee the tightest value at any time. Here the cuts help the metaheuristic.

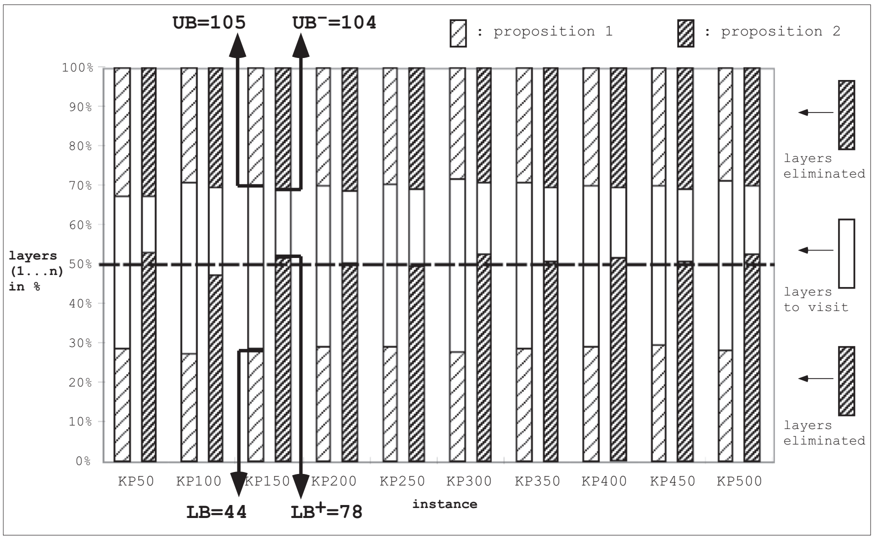

Fig. 7. The figure shows how two pre-processing procedures reduce the search area in decsision space by proving that solutions with small or large cardinality can not be efficient. E.g., for KP 150, the first procedure establishes that all efficient solutions must have between 44 and 105 items in the knapsack, whereas the second procedure shows they must have between 78 and 104 items. The vertical axis is relative because the results are for instances of size 50 to 500. (The figure is reproduced from [45].)

- Przybylski et al. [104] introduced the *seek and cut method* for solving the bi-objective assignment problem. The "seek" computes a local approximation of the non-dominated frontier (i.e., bounds are computed by a population-based algorithm coupled with path relinking) which is then used for “cutting” the search space of an implicit enumeration scheme (see Fig. 8). Here the metaheuristic helps the enumeration scheme in providing bounds of good quality for generating the exact non-dominated points.

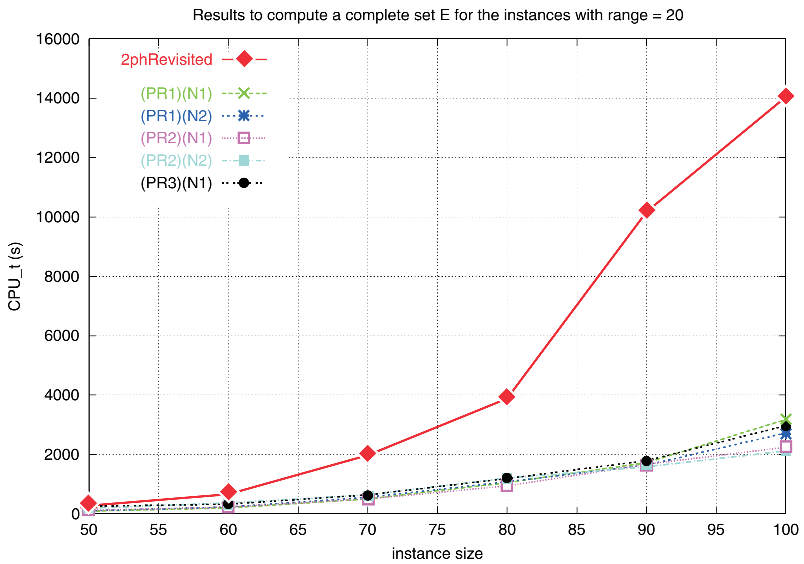

Fig. 8. CPU time used by an exact method for solving the assignment problem with two objectives without (the upper curve) and with (the lower curves) the use of approximate solutions for the pruning test inside the method of [104].

- Barichard introduced constraint programming techniques in the solution procedure of MOP problems with the PICPA method [6, 7]. The main goal of CP in PICPA is to build a sharp bound around the non-dominated frontier in the decision space using value propagation mechanisms over variables.
- Jozefowiez proposed Target Aiming Pareto Search (TAPaS) [73], a MOMH where the search directions of the procedure are given by the current set X_{PE}, similar to the principle of almost all the tabu search adaptations for MOP [46, 64, 119]. A series of goals is deduced from X_{PE} and a scalarizing function is used for guiding an NSA, defining a two phase strategy. For the covering tour problem, the method is coupled with an EA plus a branch and cut algorithm specifically designed for the single objective version of the problem.

We mention a few more papers with combinations of metaheuristics with exact methods for MOCO. In [57] scatter search is combined with the exact solution of small residual problems. [75] combine an evolutionary algorithm with a branch and cut algorithm to solve subproblems. The approach by [80] adaptively constructs right hand side values for the ε-constraint scalarization and solves the single objective subproblems, e.g., by an evolutionary algorithm.

Chen et al. [12] integrate simulation to model uncertainty and a genetic algorithm to solve a bi-level multi-objective stochastic program arising in built-operate-transfer network design problems.

4.5 Hybridization with Preferences Provided by a Decision-Maker

A decision-maker can be invited to play a role in a resolution process of an MOP. Information provided by the decision maker often concerns his preferences. Here, it is usual to distinguish three modes following the role of the decision maker in the resolution process. In "a priori mode", all the preferences are known at the beginning of the decision making process. The techniques used seek for a solution on the basis of these parameters. In "a posteriori mode" the set of all efficient solutions is generated for the considered problem. At the end, this set is analyzed according to the decision maker's preferences. Many MOMHs are designed following this solution mode. In the "interactive mode", the preferences are introduced by the decision maker during the solution process. The methods involve a series of computing steps alternating with dialogue steps and can be viewed as the interactive determination of a satisfying compromise for the decision maker. Thus they require a high participation level on the part of the decision maker. Practical problems are often solved according to the interactive mode.

The interactive mode defines a kind of hybrid MOMH. The following references give an overview of some procedures proposed in that context.

- Alves and Clímaco [2] propose a general interactive method for solving 0-1 multi-objective problems where simulated annealing and tabu search work as two alternative and complementary computing procedures. It is a progressive and selective search of potentially efficient solutions by focusing the search on a subregion delimited by information for the objective function values specified by the decision-maker. Computational results for multiple-constraint knapsack problems with two objectives are reported.
- Pamuk and Köksalan proposed an evolutionary metaheuristic that interacts with the decision maker to guide the search effort towards his or her preferred solutions.
- PSA (see Sect. 4.2) has been coupled with an interactive procedure (light beam search) in order to organize an interactive search in [65].
- An interactive version of MOSA (see Sect. 3.6) has been introduced. In [125], a simulation with a fictitious decision maker is reported for the knapsack problem with four objectives and the assignment problem with three objectives. In [133] the interactive version is used for solving a real situation, the problem of homogeneous grouping of nuclear fuel.

4.6 Conclusions and Discussion

Literature overviews published these last 20 years about progress in the field of MOMH, show clearly that the methods have quickly transcended the perimeter of principles endogenous of metaheuristics. Various ideas and techniques have been progressively integrated for making the method more aggressive, for better tackling the specificities and difficulties of multi-objective optimization problems, for reusing well-established results in the single objective case, etc. MOMHs became a blend of heterogeneous components, hybrid methods by nature. Another important change relates to the abandonment of the idea to design a single universal method (solver). Methods are more and more specific, strongly related to the optimization problem to be solved. This double observation is particularly true when the problem to be solved is a MOCO problem.

The research field now has a significant background which allows to measure the strengths and weaknesses of ideas which were introduced. Postulating that an approximation method in multi-objective optimization is specialized for the problem to be solved, a MOMH appears today as collection of various techniques available in a library, which have to be combined together and instantiated on a given problem. Without being exhaustive, among the relevant techniques who can be embedded in components available for a MOMH designer, we find: a population of solutions, evolutionary operators, strategies of ranking/guiding/clustering, a neighbourhood structure, an exploration strategy (partial, exhaustive), a scalarizing function, etc. Three components have been recently underlined as important for the efficiency of a MOMH designed for solving MOCO problems:

- **The initial solutions.** In [81] Zitzler has underlined the significant role played by elite (potentially efficient) solutions in a MOEA. However, the initial population is often composed of feasible solutions built randomly. For MOCO problems, subsets of exact solutions or approximate solutions computed with a greedy or an ad-hoc algorithm can be advantageously exploited. The role of elite solutions in the generation of the non-dominated frontier has been investigated by Gandibleux et al. [48, 93] for the knapsack problem. Using greedy solutions, or efficient supported solutions, in the initial population makes the algorithm more apt to generate quickly efficient solutions. Haubelt et al. [66] obtained the same conclusion for a MOCO problem in the field of embedded system synthesis. Gandibleux et al. [51] compute a complete set of exact supported solutions for the bi-objective assignment problem. Pasia et al. [101] use a single objective ACO algorithm for generating individuals objective by objective for a flowshop problem.
- **Lower and upper bounds on the non-dominated frontier.** A local search is usually an expensive procedure in terms of computing time. Triggering a local search from a solution, while it has little chance of generating new potentially efficient solutions is a waste of time. A solution

can be a candidate for performing a local search basically if it close to the non-dominated frontier. Knowing a lower bound on the non-dominated frontier can help to implement such a strategy. Bounds and bound sets are discussed by Ehrgott and Gandibleux [31]. The principle has been implemented successfully by Gandibleux et al. [51] and Paisa et al. [101].

- **A path relinking operator.** In Gandibleux et al. [50], similarities in solutions and subsets of exact solutions are used advantageously by the components of an evolutionary method. Here, interesting performance results are measured with a *path relinking* operator [53], given a subset of optimal solutions (or approximations) in the initial population. Path relinking generates new solutions by exploring the trajectories that connect good solutions. A path relinking operation starts by randomly selecting I_A (the initiating solution) and I_B (the guiding solution), two individuals from the current population (Fig. 9). The path relinking operation generates a path $I_A(= I_0), I_1, \ldots, I_B$ through the neighbourhood space, such that the distance between I_i and I_B decreases monotonically in i, where the distance is defined as the number of positions for which different values are assigned in I_i and I_B.
  Although many such paths may possibly exist, one path is chosen using, for example, random moves based on a swap operator. Such randomness introduces a form of diversity to the solutions generated along the path. For every intermediate solution I_i, a single solution is generated in the neighbourhood (Fig. 9). Introduced for the first time in 2003 in multi-objective optimization [49], path relinking has shown clearly its impact for the approximation of efficient solutions. This principle has been successfully implemented for computing the approximation of the complete non-dominated frontier of assignment and knapsack problems with two objectives [51], and recently on a bi-objective flowshop problem [101]. Fig. 10 provides a sample of these results. Using the same numerical instances, this population-based method based on specific operators outperforms MOSA [129].

Despite all the work done, many hot topics are open in the field of hybrid metaheuristics for multi-objective combinatorial optimization. Among them, we mention the scalability problem, the reduction of (decision and objective) spaces, the impact of objective functions, or the solution of problems with more than two objectives.

Answers may come from effectively reusing 50 years of knowledge in (single objective) optimization, the coupling with others techniques like constraint programming, or again the design of new efficient components for combinatorial problems, like the path relinking. These challenges promise many future papers.

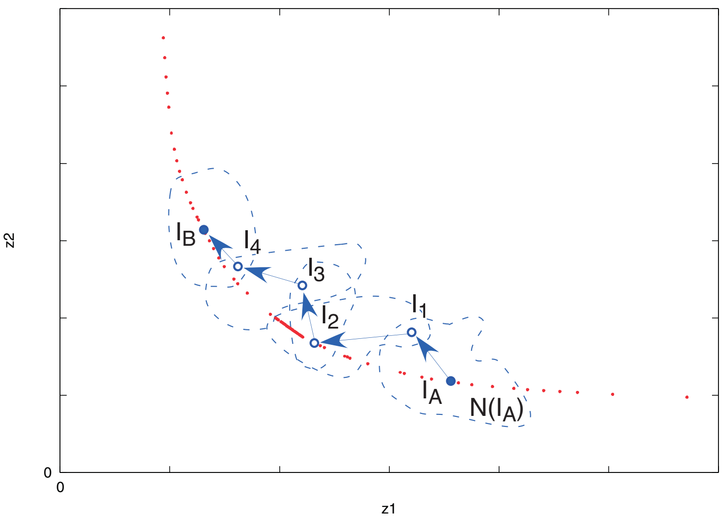

Fig. 9. Illustration of a possible path construction (see [50]). I_A and I_B are two individuals randomly selected from the current elite population (small bullets). I_A is the initiating solution, and I_B is the guiding solution. $\mathcal{N}(I_A)$ is the feasible neighbourhood according to the move defined. $I_A - I_1 - I_2 - I_3 - I_4 - I_B$ is the path that is built.

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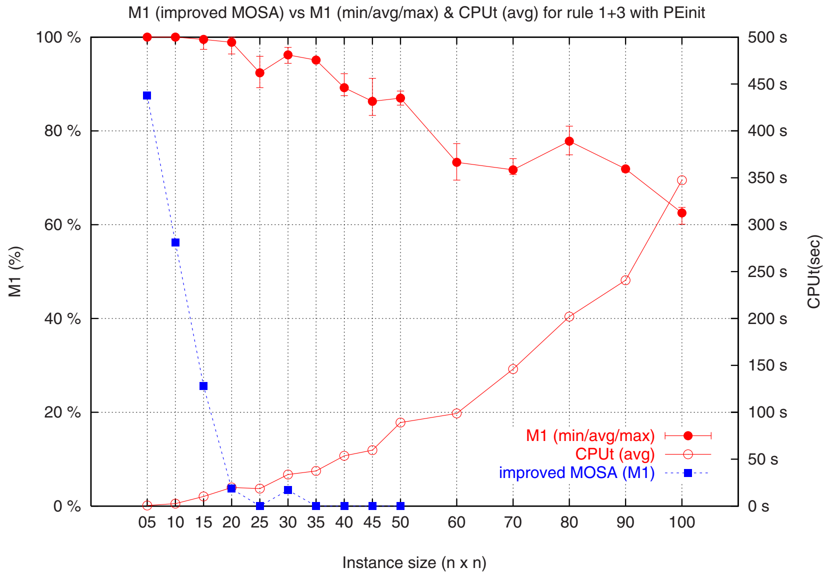

Fig. 10. Approximations obtained with the MOSA method and a population-based method for the assignment problem with two objectives. The comparison is based on the performance measure M_1 [130], which measures the percentage of exact solutions included in the final approximation set.

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