An Evolutionary Algorithm for the Multiobjective Risk-Equity Constrained Routing Problem

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Abstract

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1. Introduction

The transportation of hazardous materials (*hazmat* from now on) has received a large interest in recent years, which results from the increase in public awareness of the dangers of hazmats and the enormous amount of hazmats being transported [4]. The main target of this problem is to select routes from a given origin-destination pair of nodes such that the risk for the surrounding population and the environment is minimized—without producing excessive economic costs. When solving such a problem by minimizing both cost and the total risk, typically several vehicles share the same (short) routes which results in high risks associated to regions surrounding these paths whereas other regions are not affected. In this case, one may wish to distribute the risk in an equitable way over the population and the environment. Several studies consider this additional minimization of the equity risk, but most of them consist of a *single* origin-destination hazmat routing for a specific hazmat, transport mode and vehicle type (see for example [1, 4]). A more realistic *multi*-commodity flow model was proposed in [4] where each commodity is considered as one hazmat type. The objective function is formulated as the sum of the economical cost and the cost related to the consequences of an incident for each material. To deal with risk equity, the costs are defined as functions of the flow traversing the arcs which imposes an increase of the arc's cost and risk when the number of vehicles transporting a given material increases on the arc.

The majority of all hazmat routing studies deal with a single-objective scenario although the problem itself is multiobjective in nature and it is important to study the trade-offs among the objectives. Evolutionary Multiobjective Optimization (EMO) algorithms are able to compute a set of solutions showing these trade-offs within a single algorithm run which is the reason why we propose to use them for the problem of hazmat routing in this study (Sec. 3). Before, we formalize the routing of hazmat problem with three objectives (minimize total routing cost, total routing risk *and* risk equity) as a multicommodity flow model as in [4] since this model is the most realistic one permitting to manage several hazmat types simultaneously (Sec. 2).

2. The multiobjective risk-equity constrained routing problem

Let the transportation network be represented as a directed graph $G = (N, A)$, with N being the set of nodes and A the set of arcs. Let C be the set of commodities, given as a set of point-to-point demands to transport a certain amount of hazmats. For any commodity $c \in C$, let s^c and t^c be the source node and the destination node respectively, and let d^c be the amount of hazmat to be shipped from s^c to t^c. We assume that the risk is computed on each arc of the network and is proportional to the arc length and the population density around this arc, and we define r_{ij}^c as the risk imposed on the population when the arc $(i, j) \in A$ is used for the transportation of one unit of hazardous materials of type c. With each arc $(i, j) \in A$ a cost c_{ij} is associated, equivalent to the length of arc (i, j).

2.1. Multiple objective functions

The problem of transporting hazmat is multiobjective in nature: one usually wants to minimize the *total cost* of transportation, the *total risk* of transportation and the *distributed risk*, which can be defined as a measure of risk that is shared among different arcs or different paths.

2.2. An optimization model

We introduce a flow variable f_{ij}^c defining the portion of commodity c being transported on arc (i, j). These variables are subject to flow conservation constraints:

$$\sum_{j\in\delta^+(i)} f_{ij}^c - \sum_{j\in\delta^-(i)} f_{ji}^c = b_i^c, \forall i \in N, c \in C$$

where $\delta^-(i)$ and $\delta^+(i)$ are the forward and backward star of i, i.e., $\delta^-(i) = \{j \in N : (j,i) \in A\}$ and $\delta^+(i) = \{j \in N : (i,j) \in A\}$, and $b_i^c = \begin{cases} 1 & \text{if } i = s^c \\ -1 & \text{if } i = t^c \\ 0 & \text{otherwise.} \end{cases}$

The first objective is a cost function given as follows:

$$\min \sum_{c\in C} \sum_{(i,j)\in A} c_{ij} f_{ij}^c$$

The second objective is the minimization of the total risk

$$\min \sum_{c\in C} \sum_{(i,j)\in A} r_{ij}^c f_{ij}^c$$

We define the risk w_{ij} imposed on the population around the arc (i,j) as a linear combination of the flow variables:

$$w_{ij} = \sum_{c\in C} r_{ij}^c f_{ij}^c$$

and add a new variable $z = \max_{(i,j)\in A} w_{ij}$, which therefore is subject to the constraints

$$z \geq \sum_{c\in C} r_{ij}^c f_{ij}^c, \forall (i,j) \in A$$

The proposed model is defined as follows:

$$\begin{array}{llll}
\min & \sum_{c\in C} \sum_{(i,j)\in A} c_{ij} f_{ij}^c & & (1) \\
\min & \sum_{c\in C} \sum_{(i,j)\in A} r_{ij}^c f_{ij}^c & & (2) \\
\min & z & & (3) \\
s.t. & \sum_{j\in\delta^+(i)} f_{ij}^c - \sum_{j\in\delta^-(i)} f_{ji}^c = d_i^c & \forall i \in N, c \in C & (4) \\
 & z \geq \sum_{c\in C} r_{ij}^c f_{ij}^c & \forall (i,j) \in A & (5) \\
 & & & (6)
\end{array}$$

3. Evolutionary multiobjective optimization

Evolutionary algorithms (EAs) and Evolutionary Multiobjective Optimization (EMO) algorithms in particular are general-purpose randomized search heuristics and as such well suited for problems where the objective function(s) can be highly non-linear, noisy, or even given only implicitly, e.g., by expensive simulations [6, 5]. Since the third objective in the above problem formulation is nonlinear, we propose to use an EMO algorithm for the multiobjective risk-equity constrained routing problem here. Our EMO algorithm follows the standard iterative/generational cycle of an EA of mating selection, variation, objective function evaluation, and environmental selection and is build upon the state-of-the-art selection scheme in HypE [2] as implemented in the PISA platform [3]. The variation operators as well as the representation of the solutions, however, have to be adapted to the problem at hand in the following way in order to fulfill the problem's constraints at all times.

3.1. Representation

We choose a variable length representation as it has been theoretically shown to be a good choice for multiobjective shortest paths problems [7]: A solution is thereby represented by a list of paths of variable lengths with one path per truck. For the moment, we consider a fixed amount of trucks for each commodity and therefore a fixed number of paths through the network. In order to have every variable length path represent an uninterrupted path from source to destination at any time (see the constraints in (4)), we ensure all paths to always start with the source s^c for the corresponding commodity c, ensure with the variation operator that all neighbored vertices in the path are connected by an arc, and complete each path by the shortest path between the path's actual end node and the commodity's destination node t^c.

3.2. Initialization

Initially, we start with a single solution where the paths p for all trucks are empty ($p = (s^c)$). This corresponds to the situation where all trucks choose the shortest s^c–t^c path for their assigned commodity—implying the smallest possible overall cost but a high risk along the used route(s). Nevertheless, the initial solution is already Pareto-optimal and is expected to be a good starting point for the algorithm.

3.3. Variation

As mutation operator, we suggest to shorten or lengthen the path of one or several trucks. In order to generate a new solution s' from s, for each truck path, we draw a binary value $b \in \{0, 1\}$ uniformly at random and create the new path p' from the old one $p = (v_1 = s^c, v_2, \ldots, v_l)$ as in [7]:

- if $b = 0$ and $l =: \text{length}(p) \geq 2$, set $p' = (s^c, \ldots, v_{l-1})$
- if $b = 1$ and $|V_{\text{rem}} = \{v \in V \mid (v_l, v) \in A\}| \neq \emptyset$, choose v_{l+1} from V_{rem} uniformly at random and set $p' = (v_0, \ldots, v_l, v_{l+1})$.
- otherwise, use the same path p also in the new solution s'.

4. Experimental analysis

We concentrated our analysis on a real-world case study [8]. We considered the road network of the Lazio region (located in the middle of Italy), and, in particular, its main transport roads. The network is composed of 311 nodes and 441 links. The risk $r_{ij}^c, \forall (i, j) \in A, c \in C$ is evaluated as the societal risk computed as the number of people living inside the exposure zone around link (i, j) (whose size depends on the hazmat type c times the accident probability involving the vehicle. We considered from 2 to 10 origin-destination pairs on the network, each one associated with a number of commodities ranging from 1 to 3, for an overall number of shipments (commodities) between 2 and 30 (we recall that in our model each commodity is associated with one origin-destination pair). With each one of the latter scenarios we associated 10 instances; each instance has been generated by assigning uniformly at random a demand from 100 to 1000 tons to each shipment.

5. Conclusions

The transportation of hazmats is an important optimization problem in the field of sustainable development and in particular the equitable distribution of risks is of high interest. Within this study, we formalize this transportation problem as the minimization of three objectives and propose to use an evolutionary algorithm to cope with the non-linear equity risk objective.

The third objective function of our problem can be rewritten by minimizing the additional variable z as third objective and adding the constraints

$\forall q \in Q : z \geq \sum_{c\in C} \sum_{(i,j)\in A} r_{ij}^{cq} f_{ij}^{c}$. Although this equivalent formulation makes the problem linear (with additional linear constraints), classical algorithms are expected to have difficulties with this formulation as well and our algorithm is supposed to be more efficient in the current formulation due to the fewer number of constraints. Note that, for the moment, the proposed EMO algorithm exists on paper only and an actual implementation has to prove in the future which additional algorithm components (such as problem-specific initialization, recombination operators, or other exact optimization (sub-)procedures) are necessary to generate solutions of sufficient quality and whether adaptively changing the number and capacity of trucks is beneficial.

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