Parallel and hybrid models for multi-objective optimization: Application to the Vehicle Routing Problem

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Abstract. Solving a multi-objective problem means to find a set of solutions called the Pareto frontier. Since evolutionary algorithms work on a population of solutions, they are well-adapted to multi-objective problems. When they are designed, two purposes are taken into account: they have to reach the Pareto frontier but they also have to find solutions all along the frontier. It is the intensification task and the diversification task. Mechanisms dealing with these goals exist. But with very hard problems or benchmarks of great size, they may not be effective enough. In this paper, we investigate the utilization of parallel and hybrid models to improve the intensification task and the diversification task. First, a new technique inspired by the elitism is used to improve the diversification task. This new method must be implemented by a parallel model to be useful. Second, in order to amplify the diversification task and the intensification task, the parallel model is extended to a more general island model. To help the intensification task, a hybrid model is also used. In this model, a specially defined parallel tabu search is applied to the Pareto frontier reached by an evolutionary algorithm. Finally, those models are implemented and tested on a bi-objective vehicle routing problem.

1 What is to solve a multi-objective problem ?

The solution of a multi-objective problem (MOP) is not a unique optimal solution but a set of solutions called the Pareto frontier. These solutions, called Pareto optimal solutions, are the non-dominated solutions. A solution $\boldsymbol{y} = (y_1, y_2, \ldots, y_n)$ dominates [3] a solution $\boldsymbol{z} = (z_1, z_2, \ldots, z_n)$ if and only if $\forall\ i \in \{1 \ldots n\},\ y_i \leq z_i$ and $\exists\ i \in \{1 \ldots n\},\ y_i < z_i$.

But obtaining the complete set of Pareto optimal solutions for a multi-objective problem may be impossible to attain. That fact tends to discard exact methods. Instead, a *good* approximation to the Pareto set is sought. In this case, while solving a MOP, two purposes must be reached. On one hand, the algorithm

[3] In this paper, we assume that all the objectives must be minimized.

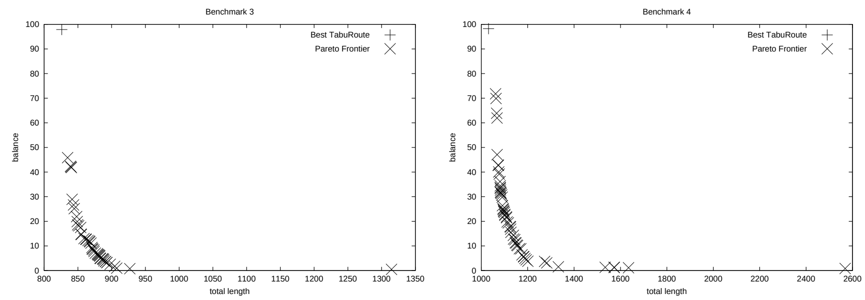

Fig. 1. Comparison between the Pareto frontier and the best solution of TabuRoute.

must converge to the optimal Pareto frontier. It is called the *intensification task*. On the other hand, a good approximation of the optimal Pareto frontier is required. The identified solutions should be well diversified along the frontier. It is called the *diversification task*.

Existing mechanisms are used to take those two goals into account. For example, one way to improve the intensification task is to arrange the solutions according to the Pareto dominance. It is what *ranking* methods like NSGA [16] do. The diversification task can be improved by ecological niche methods like the sharing [6]. However, with very hard instances or large scale benchmarks, these methods may not be sufficient. In this paper, we investigate the utilization of the parallelization and the hybridization to improve the intensification task and the diversification task. Section 2 describes the parallel multi-objective evolutionary algorithm (MOEA) we use as well as a new method to help the diversification task. Section 3 presents the hybrid model and a specially designed multi-start tabu search. Section 4 shows an implementation of those techniques for a bi-objective Vehicle Routing Problem (VRP). Finally, in section 5, the contribution of the different mechanisms is evaluated in order to show their interest.

2 A parallel MOEA

While developing a MOEA for a bi-objective VRP, the authors observe that the best-known solutions for one of the criteria were bad for the other objective. It seemed that the algorithm tended to converge prematurely to an area of the objective space. Then the used sharing method [6] was not able to fill the gap to the best-known solutions for one of the criteria (figure 1). In this article, we propose a technique, the *Elitist Diversification*, whose purpose is to maintain the population of the MOEA diversified. It is inspired by the elitism. The elitism is a way to speed up and improve the intensification task. It consists in maintaining an archive that will contain the Pareto solutions encountered during the search. Some solutions of this archive are included into the main population of the MOEA at each generation. In the elitist diversification, other archives are added. Those archives contain the solutions that are Pareto optimal when one of the

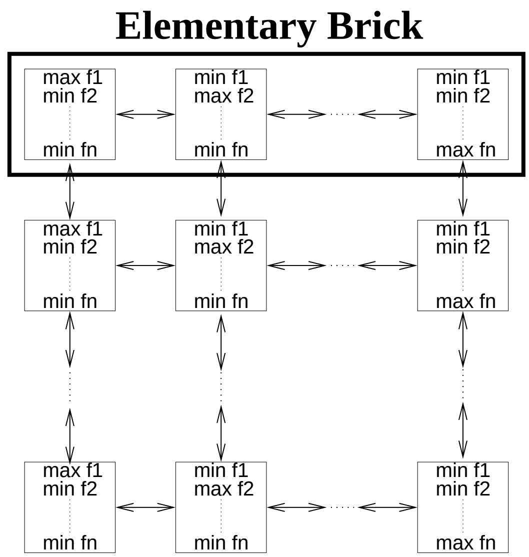

Fig. 2. The parallel model - the toric structure is not shown in order not to obfuscate the figure. The standard elitist archive is present in each island.

criteria is maximized instead of being minimized. For example, in the case of a bi-objective problem where f_1 and f_2 are the two objective functions, there are three archives. One is the standard elitist archive which contains the solutions that are Pareto optimal when both f_1 and f_2 are minimized. A second archive corresponds to the Pareto front when f_1 is minimized and f_2 is maximized. The last archive corresponds to the case where f_1 is maximized and f_2 is minimized. As in the elitism strategy, solutions from these new archives are included into the population of the MOEA at each generation. The role of the solutions of the new archives is to attract the population toward unexplored areas, and then to avoid the algorithm to converge prematurely to an area of the objective space. However, if all the archives are embedded in the same MOEA, the improvement of the diversification task is less important. This leads to the co-operative model that is the elementary brick in figure 2. In this model, an island has only one new archive. The standard elitist archive is present in each island. With a certain period in terms of the number of generations, the islands exchange their standard elitist archives. The communication topology is shown in figure 2.

To speed up the search and help the intensification task and the diversification task, a more general island model is defined. It consists in using more than one elementary brick. The connection between the islands is shown in figure 2. Therefore, an island has four neighbors. Two of them have the same kind of second archive. The communication between the islands is defined as follows: an island sends its standard elitist archive to all its neighbors. But it only sends the elitist diversification archive to the two neighbors that have the same kind of archive. It means the neighbors that are not in the same elementary brick.

3 A hybrid model

The principle of the hybridization is the following one: first, an approximation of the Pareto frontier is obtained using a MOEA; then, a local search [4] is applied in order to improve the approximation of the Pareto frontier. Therefore, the role of the hybridization is to help the intensification task. This model has already been studied in [4] and [17]. In [4], the local method was a simple local search, it cannot avoid local optima. Furthermore, it uses an aggregation method which needs to be correctly tuned and which is not able to find all the Pareto optimal solutions. Finally, for each solution of the frontier of the MOEA, only one new solution can be found. The method introduced in [17] has the main drawback to be very costly and it cannot be parallelized to reduce the computational time needed. Moreover, this method is not able to avoid the local optima.

The method introduced now, Π^2-TS (*Parallel Pareto Tabu Search*) deals with those difficulties. It is based on a tabu search approach. The starting points are the solutions of the Pareto set found by the MOEA. Thus, the meta-heuristic can escape from local optima. Furthermore, each tabu search correspond to a parallel process. No aggregation method is used. The selection of the next solution is based on the Pareto dominance. The neighborhood associated with the current solution can be partitioned into three subsets N_1, N_2 and N_3. N_1 contains the neighbors that dominate the current solution. N_2 includes the neighbors that do not dominate the current solution and are not dominated by the current solution either. N_3 is the subset of neighbors that are dominated by the current solution. Then, the next solution is the more dominating individual of N_1. If N_1 is empty, N_2 and N_3 are considered in sequence.

The tabu search algorithm does not provide a unique solution, but a set of solutions that are not dominated. Therefore, a small archive is associated with each tabu search. It contains all the non-dominated solutions found during the search.

To intensify the search, each tabu search focuses on a limited area of the objective space. An example of space restriction in a bi-objective case is shown in figure 5.

4 Application to a bi-objective VRP

4.1 A bi-objective VRP

The Vehicle Routing Problem (VRP) is a well-known problem often studied since the end of the 50's. Many practical applications exist for various industrial areas (eg. transport, logistic, workshop problem ...). The VRP has been proved NP-hard [10] and applied solution methods range from exact methods [7] to specific heuristics [8], and meta-heuristics [8][14][13].

[4] Here, the term *local search* is used in a general way. As a matter of fact, it can be a local search, a tabu search, a simulated annealing ...

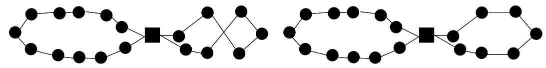

Fig. 3. (a) is better-balanced than (b), but (b) does not artificially improve the balance.

The most elementary version of the vehicle routing problem is the *Capacitated Vehicle Routing Problem* (CVRP). The CVRP is a graph problem that can be described as follows : n customers must be served from a unique depot a quantity q_i of goods ($i = 1, \ldots, n$). To deliver those goods, a fleet of vehicles with a capacity Q is available. A solution of the CVRP is a collection of tours where each customer is visited only once and the total tour demand is at most Q.

Existing studies of the VRP are almost all concerned with the minimization of the total distance only. The model studied here introduces a second objective whose purpose is to balance the length of the tours. This new criterion is expressed as the minimization of the difference between the length of the longest tour and the length of the shortest tour. As far as we know, the balancing of the tours as a criterion has been studied in two other cases [9][15]. However, the balance of a solution was not expressed in the same way.

4.2 A Parallel Pareto Genetic Algorithm

The implemented MOEA is based on a generational Genetic Algorithm (GA). The GAs have widely been used to solve multi-objective problems, as they are working on a population of solutions [3]. Two implementations of the GA were implemented. They differ by the crossovers they use. In a first version, the RBX [13] and the split crossover, which is based on the GA defined by C. Prins [14], are used. The split crossover does not work well on benchmarks with clustered customers. With those benchmarks, the split crossover is replaced by the SBX crossover [13]. The mutation operator is the Or-opt. It consists in moving 1 to 3 consecutive customers from a tour to another position in the same tour or to another tour. A 2-opt local search is applied to each tour of each solution. It has three purposes: it allows the solution to be less chaotic, it can improve the total length, and it does not allow the second criterion to be distorted (as shown in figure 3).

In order to favor the intensification task and the diversification task, multi-objective mechanisms are used. The ranking function NSGA [16] is used. To maintain diversity along the Pareto frontier, we use a sharing technique that aims to spread the population along the Pareto frontier by penalizing individuals that are strongly represented in the population. The used elitism is the one defined in section 2.

Preliminary experiments have shown that the second criterion is easier to solve than the first one. Therefore, to save computational resources, only the part of the parallel model corresponding to the minimization of the total length

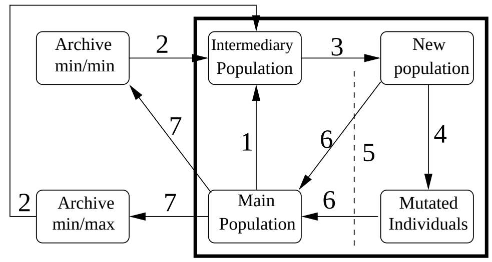

Fig. 4. The genetic algorithm.

and the maximization of the second criterion was implemented. Moreover, maintaining the archives is costly. To avoid that and a too strong pressure, we use a small archive, as shown by Zitzler and Thiele [19]. To reduce the size of the archive, a clustering algorithm, *average linking method* [12], is used. It has been proved to work well with data such as the Pareto frontier [19].

The structure of the GA is summarized in figure 4. The different steps are:

1. Selection: SUS.
2. Elitism.
3. Recombination.
4. Mutation.
5. 2-opt local search.
6. Replacement.
7. Updating of the archives.

4.3 A parallel Pareto tabu search for the VRP

The model proposed in section 3 is used. The following implementation of Π^2-TS is made. The starting points are the Pareto frontier found by the GA. The neighborhood operator is the or-opt. The tabu list is defined as follows: when a customer is moved from a tour, it cannot be put back into that tour for N iterations. As suggested in [5], the solutions that violate the capacity constraint can be accepted. The zone from the objective space associated to a tabu search is defined as follows: they are the solutions so that the distance between a solution and the line of slope $\frac{1}{2}$ which goes through the starting point is smaller than a value Γ. It is illustrated in figure 5.

5 Evaluation

The evaluation[5] was done on the standard benchmarks of Christofides [2]. More precisely on the benchmarks numbers 1, 2, 3, 4, 5, 11 and 12. They correspond to CVRP instances. The first five benchmarks correspond to maps where the customers are uniformly distributed on the map, while benchmarks 11 and 12 correspond to clustered maps.

To evaluate the contribution of the elementary brick parallel model, we use the *entropy* measure [11][1]. The entropy indicator gives an idea about the diversity of a Pareto front in comparison with another Pareto front. It is defined

[5] The results can be found at the url http://www.lifl.fr/~jozef.

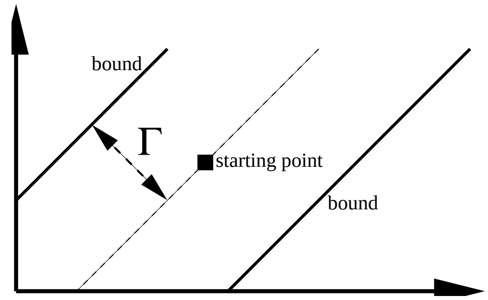

Fig. 5. Restriction of the objective space.

Problem	E(W, Wo)	E(Wo, W)
1 (50)	0.84	0.94
2 (75)	0.73	0.67
3 (100)	0.83	0.74
4 (150)	0.85	0.74
5 (199)	0.86	0.79
11 (120)	0.92	0.91
12 (100)	0.93	0.82

Table 1. Contribution to the diversification task.

as follows: Let PO_1 and PO_2 be two sets of solutions. Let PO^* be the set of optimal Pareto solutions of $PO_1 \cup PO_2$. Let N_i be the cardinality of solutions of $PO_1 \cup PO^*$ which are in the niche of the i^{th} solution of $PO_1 \cup PO^*$. Let n_i be the cardinality of solutions PO_1 which are in the niche of the i^{th} solution of $PO_1 \cup PO^*$. Let C be the cardinality of the solutions of $PO_1 \cup PO^*$. Let $\gamma = \sum_{i=1}^{C} \frac{1}{N_i}$ be the sum of the coefficients affected to each solution. The more concentrated a region of the solution space, the lower the coefficient of its solutions. Then the following formula is applied to evaluate the entropy E of PO_1 relatively to the space occupied by PO^*:

$$E(PO_1, PO_2) = \frac{-1}{\lg \gamma} \sum_{i=1}^{C} \left(\frac{1}{N_i} \frac{n_i}{C} \lg \frac{n_i}{C}\right) \tag{1}$$

The results are shown in table 1. In this table, E(W, Wo) is the entropy of the algorithm with the elitist diversification compared to the algorithm without. E(Wo, W) is the reverse. Except for the first benchmark, the new mechanism improves the diversity. For the first benchmark, the GA was able to reach the best solution for the first objective without the new mechanism which is therefore useless. Moreover, the new mechanism has the effect to slow down the convergence for the second objective (figure 6). However, this consequence is compensated

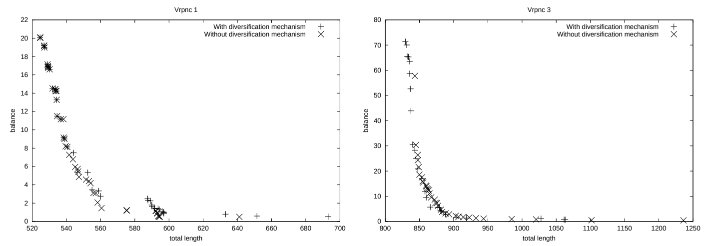

Fig. 6. Two examples of Pareto frontiers for the instances 1 and 3.

Problem	E(P, NP)	E(NP, P)	C(P, NP)	C(NP, P)
1 (50)	0.86	0.78	0.75	0.25
2 (75)	0.85	0.69	0.91	0.08
3 (100)	0.90	0.73	1.00	0.00
4 (150)	0.90	0.78	0.85	0.15
5 (199)	0.91	0.71	0.88	0.12
11 (120)	0.94	0.70	0.73	0.27
12 (100)	0.92	0.83	0.70	0.30

Table 2. Contribution of the parallelization.

by the tabu search of the hybridization. It can also be avoided by using the full parallel model described in section 2. To save computational resources for the convergence to the total distance objective which is more difficult, only half of the model was implemented. Furthermore, it can be dealt by the hybridization.

In order to show the interest of the general parallel model, we have compared with the results obtained when only one island is used. The measures used are the *entropy* measure E, which has already been used before, and the *contribution* measure C. It quantifies the domination between two sets of non-dominated solutions. The contribution of a set of solutions PO_1 relatively to a set of solutions PO_2 is the ratio of non-dominated solutions produced by PO_1. Let D be the set of solutions in $PO_1 \cap PO_2$. Let W_1 (resp. W_2) be the set of solutions in PO_1 (resp. PO_2) that dominate some solutions of PO_2 (resp. PO_1). Let L_1 (resp. L_2) be the set of solutions in PO_1 (resp. PO_2) that are dominated by some solutions of PO_2 (resp. PO_1). Let N_1 (resp. N_2) be the other solutions of PO_1 (resp. PO_2): $N_i = PO_i \backslash (C \cup W_i \cup L_i)$. Let PO^* be the set of Pareto solutions of $PO_1 \cup PO_2$. So, $\|PO^*\| = \|D\| + \|W_1\| + \|N_1\| + \|W_2\| + \|N_2\|$. The *contribution* of a set PO_1 relatively to PO_2 is stated as follows:

$$C(PO_1, PO_2) = \frac{\frac{\|D\|}{2} + \|W_1\| + \|N_1\|}{\|PO^*\|} \qquad (2)$$

Problem	Run 1	Run 2	Run 3	Run 4	Run 5
1 (50)	0.37	0.48	0.29	0.53	3.39
2 (75)	1.89	0.60	2.33	1.46	1.15
3 (100)	0.41	0.81	1.13	0.62	0.85
4 (150)	24.16	1.73	3.67	3.71	1.82
5 (199)	6.11	20.40	5.13	2.89	4.54
11 (120)	12.44	3.44	1.10	2.18	4.50
12 (100)	1.31	0.99	0.41	2.01	0.62

Table 3. Contribution of the local search.

Problem	Best-known	Best found	%	Avg. found	%
1 (50)	524.61	524.61	0.00	525.89	0.24
2 (75)	835.26	840.00	0.56	846.19	1.38
3 (100)	826.14	829.43	0.39	832.62	0.78
4 (150)	1028.42	1059.09	2.98	1069.32	3.97
5 (199)	1291.45	1353.52	4.80	1365.56	5.73
11 (120)	1042.11	1042.11	0.00	1047.49	0.51
12 (100)	819.56	819.56	0.00	819.56	0.00

Table 4. Comparison with the best-known total length.

The results are given in table 2, where P is for *Parallel algorithm* and NP for *Non-Parallel algorithm*.

To quantify the contribution of the local search, the generational distance [18] is used. Normally it is used between a front and the optimal one. But regarding the GA front, the front given by the local search can be considered as an optimal one. The results are reported in table 3 for five different runs for each problem.

One of the main problems in multi-objective optimization is that optimal Pareto frontiers are not known. We can only compare the value obtained for the total length criterion with the best-known solutions. The results are shown in table 4.

6 Conclusion

In this paper, we have investigated the utilization of parallel and hybrid metaheuristics to improve the intensification task and the diversification task. First, a new mechanism, the elitist diversification, was proposed to favor the diversification. Its utilization leads us to the design of a parallel model that improves the intensification and the diversification. Second, a tabu search, Π^2-TS, was specially designed for the hybridization with a MOEA. These methods were implemented and tested on a bi-objective Vehicle Routing Problem. The measures show that the proposed techniques ensure a better convergence to the Pareto frontier and help the diversification of the found set. Furthermore, the implemented algorithm leads to good results for the VRP studied.

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