Introduction to Financial Data Analysis

Week 10-Week 11: Factor Models and Risk-adjusted Performance Measures

Factor Models

- What is a Factor Model?
- A factor model is a financial model that employs multiple variables to explain variations of securities' returns.
- Factor models employs the regression methodology and use factors as explanatory variables to explain returns.
- Factor models can be used to:
- 1. Evaluate the performance of an actively managed portfolio.
- 2. Study the abnormal returns around an event that might affect asset returns.

Single Factor Model

- Single factor model: Capital Asset Pricing Model (CAPM).
- CAPM: returns of securities/portfolios only depend on one factor: market portfolio return.

Deriving the CAPM Model

- CAPM is trying price an asset in an efficient market.
- An asset is only efficiently priced if the compensation return for its risk is fair, that is, equal to the Sharpe Ratio given by the CML.
- Consider a portfolio with w portion invested in an asset i of expected return r_i
- The portfolio invest 1-w portion in the market portfolio of expected return r_m
- The return of this portfolio is (2)

$$r_w = wr_i + (1-w)r_m$$

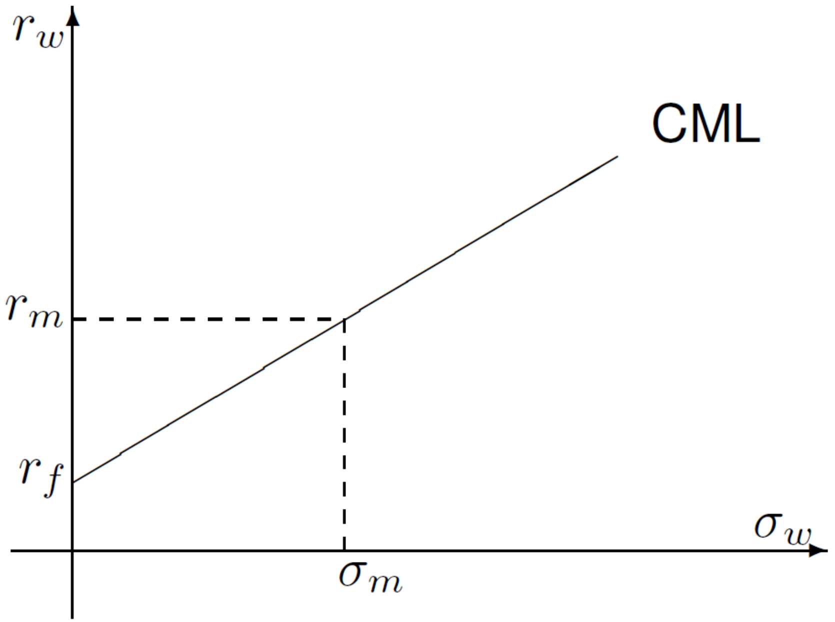
- The variance of the portfolio is ()

$$\sigma_w^2 = w^2\sigma_i^2 + 2w(1-w)\sigma_{im} + (1-w)^2\sigma_m^2$$

Deriving the CAPM Model

- The slope of the CML is the Sharpe ratio. At w=0, we have

$$\frac{r_m - r_f}{\sigma_m} = \left.\frac{dr_w}{d\sigma_w}\right|_{w=0}$$



Deriving the CAPM Model

- If the market is efficient, then the compensation return for additional risk should always equal to the Sharpe Ratio given by the CML, or say, the slope of the CML line.
- This means, the combination portfolio should also have the slope equal to the Sharpe Ratio of the complete portfolio.
- The slope of the portfolio is $\frac{dr_w}{d\sigma_w}$
- Use the chain rule:
- $$\frac{dr_w}{d\sigma_w} = \frac{\frac{dr_w}{dw}}{\frac{d\sigma_w}{dw}}$$

Deriving the CAPM Model

- From (2), we have $\frac{dr_w}{dw} = r_i - r_m$

- Form (3), we have

$$2\sigma_w \frac{d\sigma_w}{dw} = 2w\sigma_i^2 + 2(1-2w)\sigma_{im}, -2(1-w)\sigma_m^2$$

Equivalently,

$$\frac{d\sigma_w}{dw} = \frac{w\sigma_i^2 + (1-2w)\sigma_{im} - (1-w)\sigma_m^2}{\sigma_w}$$

Deriving the CAPM Model

- Putting everything together:

$$\frac{dr_w}{d\sigma_w} = \frac{\frac{dr_w}{dw}}{\frac{d\sigma_w}{dw}} = \frac{r_i - r_m}{\frac{w\sigma_i^2 + (1-2w)\sigma_{im} - (1-w)\sigma_m^2}{\sigma_w}}$$

- At w = 0, $\sigma_w = \sigma_m$, moreover, given that the slope is the Sharpe Ratio,

$$\frac{r_m - r_f}{\sigma_m} = \frac{r_i - r_m}{\left(\frac{\sigma_{im} - \sigma_m^2}{\sigma_m}\right)} = \frac{r_i - r_m}{\left(\frac{\sigma_{im}}{\sigma_m^2} - 1\right)}$$

Deriving the CAPM Model

- For any asset that is not a market portfolio, $\frac{\sigma_{im}}{\sigma_m^2} - 1 \neq 0$
- So we multiply it to both sides to obtain,

$$(r_m - r_f)\left(\frac{\sigma_{im}}{\sigma_m^2} - 1\right) = r_i - r_m$$

$$\frac{\sigma_{im}}{\sigma_m^2}(r_m - r_f) - (r_m - r_f) = r_i - r_m$$

- Define $\frac{\sigma_{im}}{\sigma_m^2} = \beta_i$, we write, $\beta_i(r_m - r_f) = (r_m - r_f) + r_i - r_m = r_i - r_f$

- CAPM Formula：

$$r_i = r_f + \beta_i(r_m - r_f),$$

Single Factor Model

- Market portfolio?
  - Theoretically, market portfolio should be a very broad index including all stocks, bonds and other assets.
  - In practice, broad-based stock market index, such as the S&P 500 Index, MSCIWorld Index, etc. , are often used to represent the market portfolio return.
- CAPM model suggest the return of an asset should always equal to the

  beta * excess return of market portfolio.
- Anything different from this return suggested by CAPM should be purely random.

Single Factor Model

- Problems of CAPM: CAPM does not perform well in empirical testing.
- The regression residuals still have systematic patterns.
- For example:
  - Small market cap stocks tend to have positive residual.
  - Large market cap stocks tend to have negative residual.
  - Small stocks have better returns even after adjusted for risks measured by beta.

  - Value stocks defined as stocks with high Book Value to Market Value Ratio tend to have positive residual.
  - Growth stocks defined as stocks with low Book Value to Market Value Ratio tend to have negative residual.
  - Values stocks have better returns after adjusted risks by beta.

Multi-Factor Models

- Multi-factor model: Models include additional factors that help explain more of the variation in expected stock returns.
- Three Factor Model developed by Eugene Fama and Kenneth French (FF-Model)
- Fama and French were professors at the University of Chicago Booth School of Business. In 2013, Fama shared the Nobel Memorial Prize in Economics.
- FF model has 3 factors:
  - 1. The market portfolio return as in CAPM
  - 2. The difference in returns between small and large capitalization stocks
  - 3. The difference in returns of high and low book-to-market (i.e., value and growth stocks) stocks

Regression

- Factors models employs the regression method to explain the return using factors.
- Regression: For two variables y and x, regression is the method to study how y varies with changes in x, or say, explaining y in terms of x.
- To build a model in “explaining y in terms of x”.
    - Q1. What is the functional relationship between y and x? Linear/Nonlinear?
    - Q2. There is never an exact relationship between two variables, how do we allow for other factors to affect y?

Regression

- Q1. What is the functional relationship between y and x? Linear/Nonlinear?
- A1: A simple model assuming linear relationship between y and x

$$y = \beta_0 + \beta_1 x + u.$$

- The linear equation defines the simple linear regression model
  - y is called the dependent variable
  - x is called the independent variable
  - u is called the error term/noise, with an average equal to 0

Regression

- The functional relationship between y and x is linear:

$$\Delta y = \beta_1 \Delta x \quad \text{if} \;\; \Delta u = 0.$$

- If the change in u is zero, x has a linear effect on y.
- The change in y is simply beta1 multiplied by the change in x.
- This means that beta1 is the slope parameter in the relationship between y and x
- The intercept parameter beta0 measures the unconditional average of y and is not important in explaining y using x.

CAPM

- Capital Asset Pricing Model (CAPM): Use the market portfolio return (x) to explain a particular stock/portfolio’s return (y)
- The theoretical formula for the CAPM is:

$$r_i = r_f + \beta_i(r_m - r_f),$$

- where *ri* is the return on asset *i*
- *rf* is the return on the risk-free asset,
- *rm* is the return on the market portfolio proxy
- *rm-rf* is the excess market return or say market risk premium.
- *βi* is the sensitivity of asset *i* to the overall market risk premium .

CAPM Regression

- Empirical tests of the CAPM use the excess return form.

$$r_i - r_f = \alpha + \beta_i(r_m - r_f),$$

- The excess return is the return over the risk-free rate of both the subject security's return (y) and the market return (x).
- The α and βi in are estimated using an OLS regression.
  - For any stock, if the CAPM model holds, then α should be 0.
  - For an actively managed portfolio, if the fund manager indeed has a good skill in picking stocks, the α should be greater than 0.
  - For an event, if the information about the event is reflected into the stock returns quickly, then α after the event happened, should not be different from 0.

OLS Regression

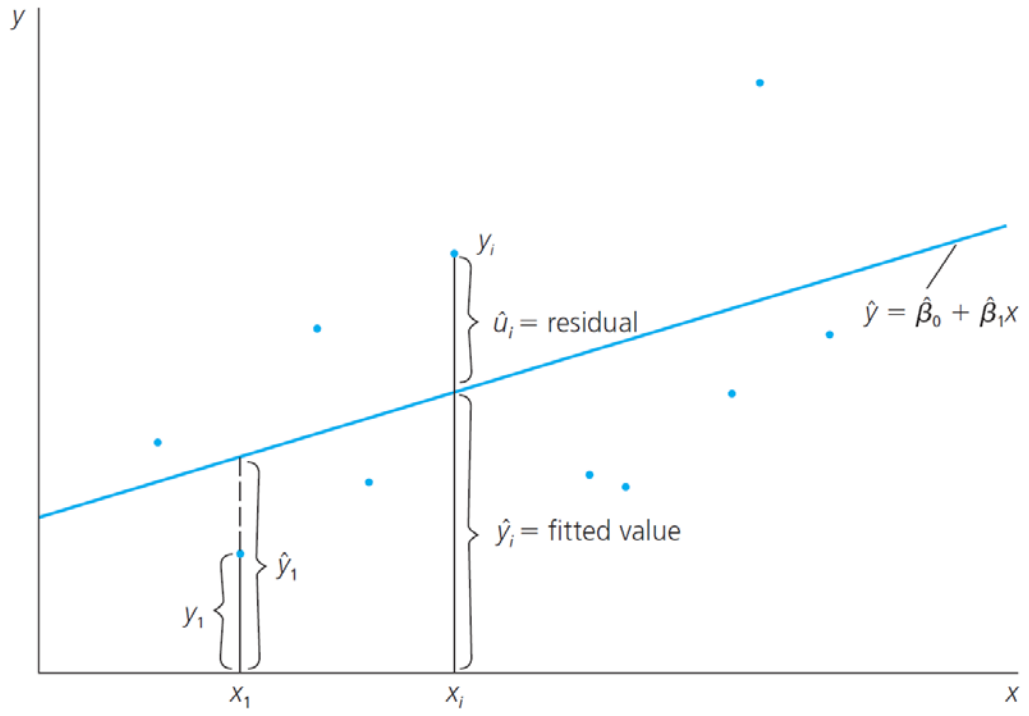
- Q: How to estimate the α and βi from data?
- Q: How to estimate beta1 and beta0 from

$$y = \beta_0 + \beta_1 x + u.$$

- For a given fitted relationship:

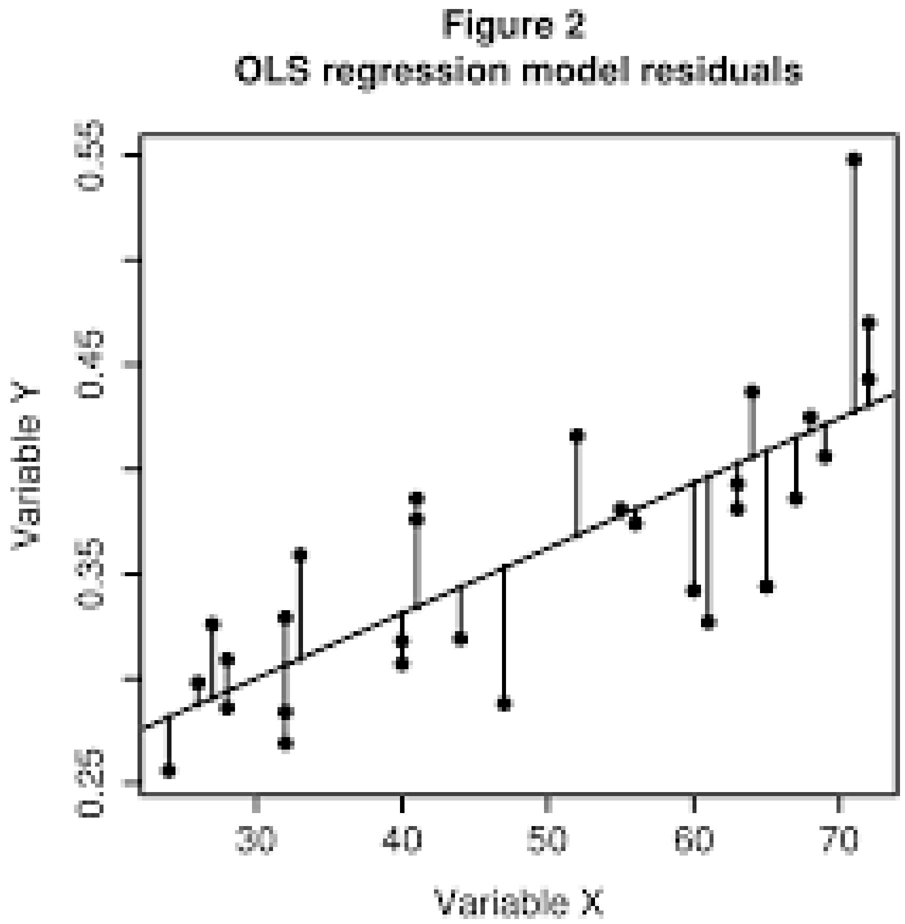
$$\hat{y}_i = \hat{\beta}_0 + \hat{\beta}_1 x_i.$$

- It is a line on the right chart

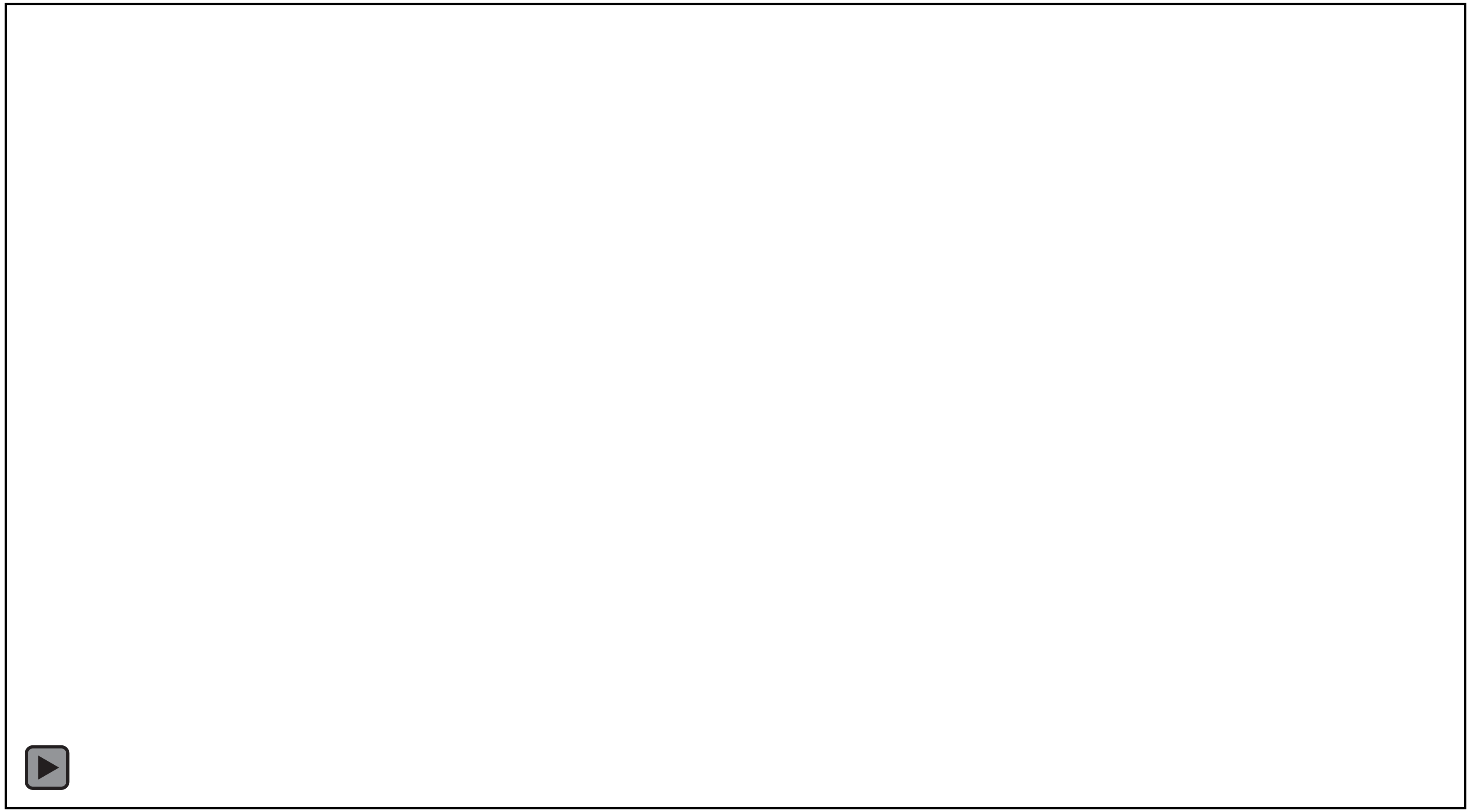


OLS Regression

- OLS Regression find the fitted relationship by making the overall distance between observations and the estimations smallest.
- The distance are calculated using the squares of the real distance.
- The overall is simply summing all the distance together.



OLS Regression



OLS Regression

- OLS: the method estimates the relationship between y and x by minimizing the sum of the squares in the difference between the observed and predicted values of the dependent variable configured as a straight line.
- That is find the betas that minimize the following formula

$$\sum_{i=1}^{n} \hat{u}_i^2 = \sum_{i=1}^{n} (y_i - \hat{\beta}_0 - \hat{\beta}_1 x_i)^2,$$

- The solution are the following:

$$\hat{\beta}_0 = \bar{y} - \hat{\beta}_1 \bar{x}.$$

$$\hat{\beta}_1 = \frac{\sum_{i=1}^{n} (x_i - \bar{x})(y_i - \bar{y})}{\sum_{i=1}^{n} (x_i - \bar{x})^2}.$$

- Where $\bar{y} = n^{-1} \sum_{i=1}^{n} y_i$ and is the sample average and likewise for *x.*

Measures of Fit

- Q: How well the model describes the data?
- A: Visually, this amounts to assessing whether the observations are tightly clustered around the regression line, this is measured by R^2.
- R^2 is the fraction of the sample variance of y_i that is explained by x_i.
- Mathematically, the R^2 can be written as the ratio of the explained sum of squares to the total sum of squares.
- $R^2 = \frac{ESS}{TSS}$
  - The explained sum of squares (ESS) is the sum of squared deviations of the predicted values $\widehat{y}_i$ from the average of the y^i , $\Sigma_{i=1}^{n}(\hat{y}_i - \overline{y})^2$
  - The total sum of squares (TSS) is the sum of squared deviations of the y_i from their average.
- R^2 lies between 0 and 1.
  - It is easy to see that a perfect fit, i.e., no errors made when fitting the regression line, implies $R^2 = 1$,since then we have ESS = TSS
  - On the contrary, if our estimated regression line does not explain any variation in the y_i ,we have $ESS = 0$ and $R^2 = 0$

The Sampling Distribution of the OLS Estimator

- $\widehat{\beta_0}$ and $\widehat{\beta_1}$ are computed from a sample, the estimators themselves are random variables with a probability distribution
- The distribution is the so-called sampling distribution of the estimators - which describes the values they could take on over different samples.
- When the sample size n is sufficiently large, $\widehat{\beta_0}$ and $\widehat{\beta_1}$ are unbiased estimators of β_0 and β_1, the true parameters.
- In large samples, we can assume $\widehat{\beta_0}$ and $\widehat{\beta_1}$ are normally distributed
  - $\hat{\beta}_0 \sim N(\beta_0, \sigma^2_{\hat{\beta}_0})$, $\hat{\beta}_1 \sim N(\beta_1, \sigma^2_{\hat{\beta}_1})$

Distribution of Beta

Testing Two-Sided Hypotheses Concerning the Slope Coefficient

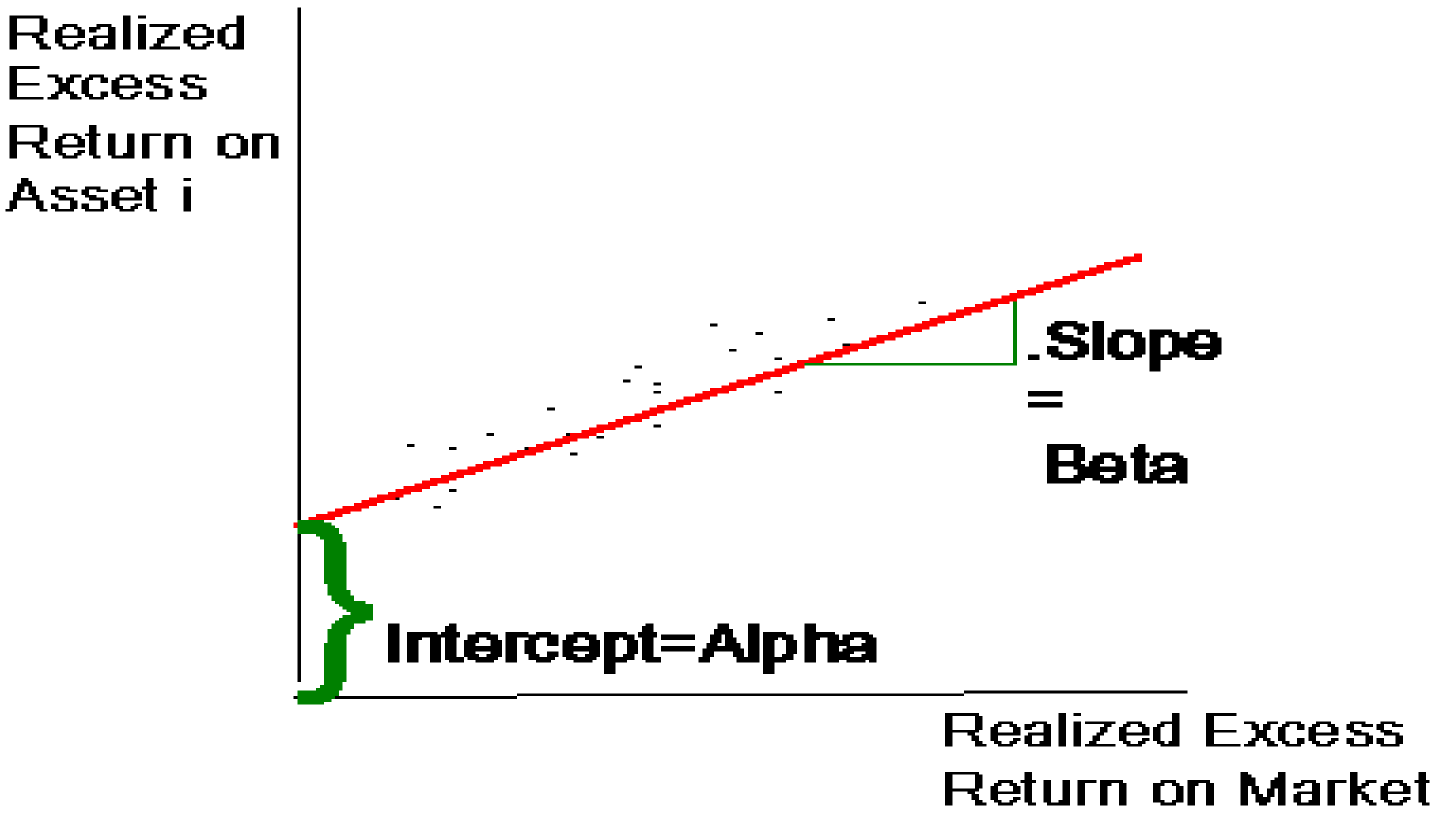
- Q: How confident we are in that our estimated beta is not zero so that there is some relationship between our x and y?
- If we get a relative large/small beta, it is unlikely that the real beta is 0 because given the true beta is 0, the probability of getting a large/small beta is small.
- The probability that our true beta is zero is given by the p-value.
- The smaller the p-value, the less likely that our beta is 0.

OLS Regression in R

- The function in R to perform a OLS regression is
- lm(y~x)
- y~x is the formula of estimation: $y = \beta_0 + \beta_1 x + u.$
- If you have a dataframe df with variable y and x:
- You could either call the regression in two ways
  - lm(df$y~df$x)
  - lm(y~x, data=df)

CAPM Regression

$$R_{it} - R_{ft} = \alpha_i + \beta_i\,(R_{mt} - R_{ft}) + \epsilon_{it};$$



Interpreting the CAPM Regression

$$R_{it} - R_{ft} = \alpha_i + \beta_i \left(R_{mt} - R_{ft} \right) + \epsilon_{it},$$

- Investing context:
  - The intercept from the CAPM model is called alpha.
  - The slope from the CAPM model is called beta.
- Beta: measures how an asset (i.e. a stock, an ETF, or portfolio) moves versus a benchmark (i.e. an index). Beta is a historical measure of volatility.
  - Beta reflects the compensation an investor earns by taking the additional risk as reflected in beta.
- Alpha: measures an asset's return on investment compared to the risk adjusted expected return.
  - Alpha reflects the extra returns/abnormal returns an investor earnings beyond taking the risk reflected in beta.

Alpha and Beta Example

- Let's assume company XYZ's stock has a return on investment of 12% for the year and a beta of 1.5. Our benchmark is the S&P500 which was up 10% during the period. Is this a good investment?
  - A beta of 1.5 implies volatility 50% greater than the benchmark therefore the stock should have had a return of 15% to compensate for the additional volatility risk taken by owning a higher volatility investment.
  - The stock only had a return of 12%; three percent lower than the rate of return needed to compensate for the additional risk. The alpha for this stock was -3% and tells us it was not a good investment even though the return was higher than the benchmark.

Example of CAPM Regression

- Example: Performance of the minimum variance portfolio
- Some empirical evidence shows that active funds/assets that has low volatility tend to have positive alpha.
- We use 10 stocks randomly selected from the CRSP data base and build up a portfolio that achieved the global minimum variance among all possible allocations using these 10 stocks.
- Take this portfolio as given, we examine the performance against CAPM model.

Example of CAPM

- To perform the CAPM OLS regression, we need to choose
  - 1. the length of the estimation period
  - 2. the frequency of the returns data
  - 3. the risk-free rate used in the calculation.
- For 1. 2. , the example use 3 years of monthly returns or 36 monthly return observations for both the subject portfolio's return and market return. This setting is used a lot to evaluate the ability of fund manager.
- We use market return and risk-free return from FF website.
- We check the alpha and beta of this minvariance portfolio using 3 years monthly data from 2008-01-01 to 2010-12-31.
- See "L4_CAPM.html"

Example: The performance of Mean-Variance Efficient Portfolio

- Step 1. Load the port.sr.RData which contains 10 stocks returns, restrict the data to 3 years from 2008-01-01 to 2010-12-31
- Step 2. Get the risky portfolio weights with the minimum variance from mean-variance efficient frontier
  - Use the function **minvariancePortfolio()** from the fportfolio package to get the minvariance portfolio
- Step 3. Get the min variance portfolio monthly returns
- Step 4. Load the FF Data from lecture 2 and use market excess return as benchmark
- Step 5. Combine FF Data and our minvariance portfolio return data and calculate the excess return of the portfolio
- Step 6: Run regression on excess return of the portfolio on excess return of the market return which gives us the alpha and beta of the minvariance portfolio performance relative to the benchmark

Event Study using Factor Model

- Event study is import in corporate finance research.
- An **event study** tests the impact of new information on a firm's stock price.
- An event study estimates the impact of a specific event on a company or a group of companies by looking at the firm's stock price performance.
- The event study uses the historical relationship between the firm's stock return and one or more independent variables (For example: market return of CAPM or FF 3 factors).
- Based on this historical relationship, an event study predicts a return for the company on the event date.
- An abnormal return is then calculated as the difference between the firm's actual return on the event date or after the event date and the predicted return on the event date or after the event date.

Event

- Event: An “event” could be something that affects a particular firm (e.g., corporate earnings announcements) or something that affects an entire industry (e.g., passing of new legislation). Moreover, some events are not reported in isolation (i.e., events are announced together with other information that could also affect the firm’s stock price).

Periods around the Event

- Two time periods on the event study: 1. Event Window 2. Estimation Period
- 1. Event window : the day or days around which the effects of an event has occurred. The length of the event window depends on the type of event being studied.
    - In the US, empirical evidence exists to show that stock prices react very quickly, usually within minutes, of material news announcements. A one-day event window is likely sufficient when the timing of the event.
    - In some instances, news may be leaked days or weeks prior to the official announcement. For example, mergers. In this case, the event window can be extended to longer periods to fully capture the effect of the event

Periods around the Event

- 2. Estimation Period: From which we estimate the parameters under "normal" conditions unaffected by the event. The estimation procedure uses a market model of the CAPM form or FF 3 factor form.
- For example: Using CAPM assumption:

$$r_{i,t} = \alpha + \beta r_{m,t} + \epsilon_{i,t},$$

- It is typical to calculate the returns in an event study using log returns:

$$r_{i,t} = \ln(P_{i,t}/P_{1,t-1}) \qquad r_{m,t} = \ln(P_{m,t}/P_{m,t-1})$$

- Arithmetic returns is also used in practice.

Normal Estimation Periods

- Q: When is the normal periods for the event study?
- A: Depend on the event.
  - For example, if we are studying earnings announcements, then placing the estimation period prior to the event window may be appropriate.
  - For example, if we are studying the event of merger. After the merger, it is normal periods since ahead of the merger, leak of information lead to abnormal returns.
- Non-event factors might matters:
  - For example, we likely will find many significant days if we analyze an event during the 2008/2009 financial crisis when we choose an estimation period before or after the crisis since the volatility is likely to be high.

Length of Estimation Periods

- Length of the estimation period:
  - When daily data is used, a 6-month or 1-year period is sufficient.
  - Alternatives could be weekly or monthly data over a longer date range.
- Trade-off in choosing longer period:
  - More data
  - The risk of including less relevant data.

What is Abnormal Return?

- How abnormal is abnormal? Or if the realized return is different from what is predicted by factor model, is it an abnormal return?
- A: No. Define the abnormal return based on the stocks' volatility.
  - For example, a highly volatile stock may move up or down 5–10% in any given day, which means that a 10 abnormal return for that stock cannot be discerned from the normal volatility of the stock. Hence, we have to test whether the abnormal return is sufficiently large that it can be considered statistically significant.
- Use t-test to perform this task

$$t_{i,t} = \frac{\varepsilon_{i,t}}{\sigma},$$

- sigma is the Standard Error of the Regression (SER) of the market model regression
- The corresponding *p*-value of the t-statistic tells us whether the t-statistic is significant, that is abnormal or not.

Example: Earnings Announcement

- Case Study on Netflix Earnings Announcement
- After markets closed on July 22, 2013, Netflix announced earnings that more than quadrupled from a year ago but its new subscriber numbers for the second quarter of 630,000 failed to meet investors' expectations of 800,000 new subscribers.
- Netflix stock dropped more than 4% on July 23, 2013, the first full trading day on which investors could have reacted to Netflix's information. News reports attributed the drop to the miss in new subscriber numbers.
- Examine whether the price reaction on July 23, 2013 can be distinguished from normal volatility in Netflix stock.

Example: Earnings Announcement

- See “L4_EventStudy.html”
- Step 1. Connect to WRDS and get Netflix daily returns from July 23, 2012 to July 22, 2013
- Step 2. Get SP500 Index returns from COMPUSTAT
- Step 3. Get Risk-free return from Fama French website
- Step 4. Merge the Netflix, SP500 and Fama French data by date into one table
- Step 5. Calculate excess returns of NFLX and SP500

Example: Earnings Announcement

- Step 6. Perform Market Model Regression During the Estimation Period
- Step 7. Calculate Abnormal Return, t-Statistic, and p-Value for Dates in Event Window
- Step 8. Check the performance of Netflix ahead of the event
- Step 9. Choose another estimation period after the price jump by removing some outlier returns.
- Step 10. Calculate the market model parameters of this alternative estimation period

Multi-factor Model

- The CAPM is very simple: only one source of risk (market risk) affects expected returns.
- Advantage
  - Simplicity and provides a good benchmark (e.g. performance evaluation).
- Disadvantage
  - It is likely that other sources of risk exist (omitted variable bias in the estimates of beta)
- CAPM suggests that if alpha is 0, then stocks with high beta should have higher returns.
- Empirical evidence is against this CAPM implication: high beta stocks earns lower returns in general.
- This suggests other factors may need to be added to help explain the remaining variation in asset returns unexplained by the market.
- One such model is the Fama-French Three Factor (FF) Model
- The FF Model has taken the place of the CAPM for routine risk adjustment in empirical work.

Multi-factor Model

- FF Model

$$r_i = r_f + \beta_i(r_m - r_F) + hHML + sSMB$$

- where *HML* is the difference in the returns of portfolios with high B/M ratios and low B/M ratios
- *SMB* is the difference in returns of portfolios of small company stocks and big company stocks.
- The data needed to implement the FF Model can be downloaded from Professor Kenneth French’s Data Library (U.S. Research Returns Data)

Rolling Window Regressions

- In the CAPM estimation, we use a fixed 3 year window in estimating the alpha and beta.
- The estimates of alpha and betas are sensitive to the time period over which we select to calculate them.
- For example, if you are using 1 year of data, did you use data from January to December 2011, July 2008 to June 2009, January 2008 to December 2008, and so. We can see that there are many possible variations of the assumption we can choose from.
- Practitioners usually run regressions over a rolling window, so that we can calculate the alphas and betas for the portfolio over multiple periods to analyze the variation of the alpha and beta through time.

Rolling Window Regressions

- Example: Calculate the alphas and betas of multiple stocks from FF model using regressions on 252 days daily returns from 2008 to 2010 and rolling 1 day ahead in each regression.
- A minimum number of data points are usually required to ensure the quality of the values estimated by the regression.
- In the case of daily data over a one-year period, a reasonable requirement may be that the regression be fit using at least 200 data points.

Rolling Window Regression

- “L4_Multifactor.html”
- Step 1. Get 5 stocks daily returns from WRDS, the 5 stocks are the same as in L2. Date is from '2008-01-01' to '2010-12-31’
- Step 2. Get Fama French daily factor data and merge it with stock returns ‘2008-01-01' to '2010-12-31’
  - Merge the Fama French daily factor data and stock returns by date
  - Calculate the excess returns of each stock

Rolling Window Regression

- Step 3. Rolling window regression example for one stock
  - Introduce the following functions to make us able to keep many different fields from regressions.
  - tibbletime::rollify(): rollify function from library(tibbletime) to write a rolling function using **multiple** vectors as input variables. Specifically, we use rollify to write a rolling linear regression function of excess returns on all 3 factors (Roll.Lm)
  - dplyr::slice(): slice(df,n) will keep the row indexed by n. slice(df,-n) will delete rows indexed by n. Use slices to remove first 251 regression results as they are empty.
  - broom::tidy(): tidy() function from library(broom) summarizes information about the components of a linear regression model. In general, tidy() can turn an structured object into a dataframe of tibble format.
  - purrr::map(): Similar to apply with input as a vector or a list. The map() function from library(purrr) transform their input(a list or a vector) by applying a function to each element and returning a vector the same length as the input. Use map() together with tidy() to make the result from regressions which is originally a complicated list into a dataframe.
  - tidyr::unnest() : unnest() function from tidyr can flattens a list back out into regular columns. We flatten the dataframe column back to multiple regular columns containing results from regression.

Rolling Window Regression

- Step 4. Group for each stock and redo the process in Step 3 to get betas for all stocks over all rolling periods
- Step 5. Visualize each stocks' alpha and beta over time
- Step 6. Monthly persistence of beta of each stock by checking the correlations of beta and lag.beta