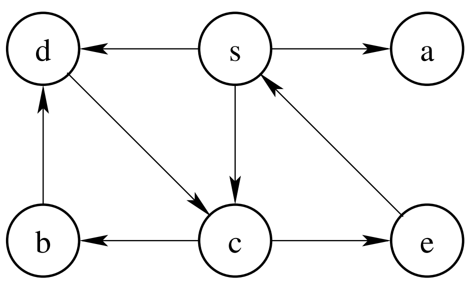
50.004 Introduction to Algorithms, Fall 2015
Singapore University of Technology and Design — 13 Nov. 2015
Simon Lui — Problem Set- 2

Student Information

Name: ________________________ Student ID: ________________________
Due Date: 10am Wednesday 2 Dec. 2015.

Submit written answer on paper in class or submit electronic version online through Dropbox.
Note: Please name your file correctly, as: ps2_1001234_JohnSimth.doc.

Exercise 1 BFS/DFS



$$\begin{aligned} adj(s) &= [a, c, d], \\ adj(a) &= [\,], \\ adj(c) &= [e, b], \\ adj(b) &= [d], \\ adj(d) &= [c], \\ adj(e) &= [s]. \end{aligned}$$

Figure 1: Graph for Exercise 1. **(a).**

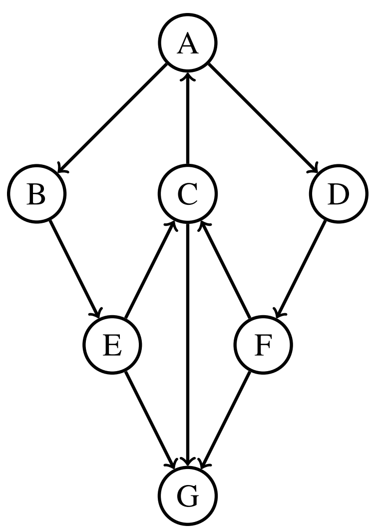


Figure 2: Graph for Exercise 1. **(b).**

(a). Given the graph and corresponding adjacency lists in Fig. 2, write down the visited nodes order for each type of graph search (BFS,DFS), starting with s.

(b). Perform a DFS on the graph in Fig. 2, starting at A. Label every edge in the graph with T if it is a tree edge, B if it is a back edge, F if it is a forward edge, and C if it is a cross edge. Assume that neighbors of the same node are visited in alphabetical order.

Exercise 2 Bellman Ford Algorithm

Given a weighted directed graph $G = (V, E)$ with no negative-weight cycles, let $l(s, v)$ be the minimum number of edges in a shortest path from source s to v, for $v \in V$. (Here, the shortest path is by weight, not the number of edges.) Define m be the maximum of $l(s, v)$ over all vertices $v \in V$. Suggest a simple change to the Bellman-Ford algorithm that allows it to terminate in $m+1$ passes, assume that m is not known in advance.

Exercise 3 Dijkstra's Algorithm

Run Dijkstra's algorithm on graph in Fig. 3, starting from vertex S. What is the order in which vertices get removed from the priority queue? What is the resulting shortest-path tree?

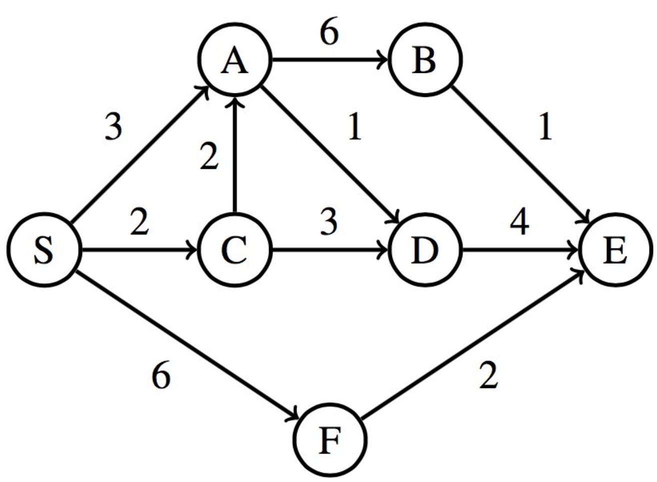


Figure 3: Graph for Exercise 3

Exercise 4

Given a directed graph $G = (V, E)$ on which each edge $(u, v) \in E$ has an associated value $r(u, v)$, which is a real number in the range $0 \le r(u, v) \le 1$ that represents the reliability of a communication channel from vertex u to vertex v. We interpret $r(u, v)$ as the probability that the channel from u to v will not fail, and assume that these probabilities are independent. Give an efficient algorithm to find the most reliable path between two given vertices.

Exercise 5 System of Difference Constraints

You have 6 variables $x_1, x_2, x_3, x_4, x_5, x_6$, find a feasible solution or determine that no feasible solution exists for the following system of difference constraints:

$$
\begin{aligned}
x_1 - x_2 &\leq 1\,, \\
x_1 - x_4 &\leq -4\,, \\
x_2 - x_3 &\leq 2\,, \\
x_2 - x_5 &\leq 7\,, \\
x_2 - x_6 &\leq 5\,, \\
x_3 - x_6 &\leq 10\,, \\
x_4 - x_2 &\leq 2\,, \\
x_5 - x_1 &\leq -1\,, \\
x_5 - x_4 &\leq 3\,, \\
x_6 - x_3 &\leq -8\,.
\end{aligned}
$$

Hint: Draw the constraint graph corresponding to the system of difference constraints, solve the problem by finding the shortest path weights in this graph.
(More detail in textbook: An Introduction to Algorithms, By Thomas H. Cormen, p664-p668)