50.004 Introduction to Algorithms, Fall 2015
Singapore University of Technology and Design 11 Oct., 2015
Simon Lui Problem Set - 1

Student Information

Name: _____ Student ID: _____
Due Date: 25-Oct. 11:59pm.
Submit written answer on paper in class or submit electronic version online through Dropbox. Submission without student information will **NOT** be marked!

Exercise 1 HeapSort

1. What are the minimum and maximum numbers of elements in a heap of height h?

2. What is the running time of HEAPSORT on an array A of length n that is already sorted in increasing order? What about decreasing order?

3. Where in a max-heap might the smallest element reside, assuming that all elements are distinct?

Exercise 2 Binary Search Tree

Suppose we have int values between 1 and 1000 in a BST and search for 363. Which of the following cannot be the sequence of keys examined, and why?

(a) $2 \quad 252 \quad 401 \quad 398 \quad 330 \quad 363$

(b) $399 \quad 387 \quad 219 \quad 266 \quad 382 \quad 381 \quad 278 \quad 363$

(c) $3 \quad 923 \quad 220 \quad 911 \quad 244 \quad 898 \quad 258 \quad 362 \quad 363$

(d) $4 \quad 924 \quad 278 \quad 347 \quad 621 \quad 299 \quad 392 \quad 358 \quad 363$

Exercise 3 AVL Tree

a. Insert the following sequence of elements into an AVL tree, starting with an empty tree: $10, 20, 15, 25, 30, 16, 18, 19$.

b. Delete 30 in the AVL tree that you got.

Note: Draw the tree step by step (This helps you master the main idea of AVL tree.).

Exercise 4 Radix Sorting

a. Here are five numbers: $259, 781, 636, 107, 548$. Sort the list of numbers (in increasing order) using radix sorting. (Write down the steps.)

b. Compare the sorting methods you have now learned (Merge Sort, Insertion Sort, Counting Sort, Heap Sort). (Hint: List running time of each sorting method and know the pros and cons.)

Exercise 5 Hashing

Suppose we use a hash function h to hash n distinct keys into an array T of length m. Assuming simple uniform hashing, what is the expected number of collisions? More precisely, what is the expected cardinality of $\{\{k, l\} : k \neq l, h(k) = h(l)\}$?