EXAM – Algebra

Exercise 1. Solve the equation $(2+3i)\bar{z}+3z=7+i$.

Exercise 2. The number $z = -1 + 2i$ is a root of the polynomial

$$W(z) = z^4 + 2z^3 + 13z^2 + 16z + 40.$$

Find all the other roots.

Exercise 3. Find the relationship between the number of solutions of the following system and the parameter $a \in R$. Justify your answer.

$$\begin{cases} 3x + ay + 3z = a \\ x + y + z = 1 \\ ax + ay - z = 2 \end{cases}$$

Exercise 4. Solve the following matrix equation $A(X^T - I) = B$, where

$$A = \begin{bmatrix} 1 & 2 & 0 \\ 0 & -1 & 1 \\ 0 & 1 & -2 \end{bmatrix} \qquad B = \begin{bmatrix} 0 & -1 & 2 \\ 0 & 2 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

Exercise 5. Find the eigenvalues and eigenvectors for the matrix $\begin{bmatrix} 3 & -1 & 0 \\ 0 & 3 & 0 \\ 1 & -3 & 0 \end{bmatrix}$

Exercise 6. Let A_L be the standard matrix of a linear transformation L,

$$A_L = \begin{bmatrix} 1 & 2 & 3 \\ 1 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 2 & 3 \end{bmatrix}$$

a) Identify the domain and codomain of L
b) Find the basis of Ker L and the dimension dim Ker L,