Checking the equivalence of context-free session types

Andreia Mordido and Vasco T. Vasconcelos

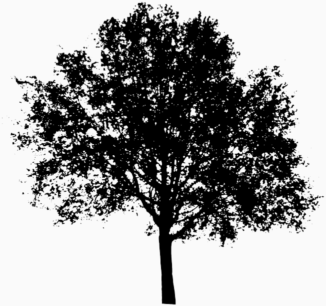
LASIGE, Faculdade de Ciências, Universidade de Lisboa

January 2019

Motivation

```
data Tree = Leaf | Node Int Tree Tree
```

```
sendTree Leaf c =
  select Leaf c
sendTree (Node x l r) c =
  let c1 = select Node c
      c2 = send x c1
      c3 = sendTree l c2
      c4 = sendTree r c3
   in c4
```



Plan

Definition (Type equivalence problem)

Given any context-free session types S and T, the type equivalence problem consists in deciding if types S and T are equivalent, i.e., $S \sim T$.

- Implement an algorithm that decides the *type equivalence problem*.
- Prove its soundness and completeness w.r.t. the metatheory of context-free session types proposed by Thiemann and Vasconcelos.
- Provide results on complexity.

Algorithm for checking the equivalence of CFST Main stages

Convert types to a grammar

Translates types into a (finite) set of productions

Prune unnormed productions

Streamlines the grammar by pruning unnormed productions

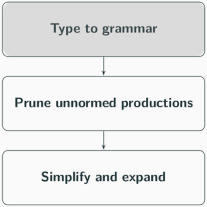
Simplify and expand

Alternates between simplification and expansion operations, until reaching a successful branch in the expansion tree or concluding that all branches are unsuccessful

Convert types to a grammar

We consider a finite set of productions $X \to a\vec{Y}$ where:

- X, Y represent *non-terminal symbols*
- a is a *terminal symbol* (a *label* in the labelled transition system)



Context-free session types are seen as *simple grammars*:

- context-free grammars in *Greibach normal form*
- s.t. for each X and a there is at most one production $X \to a\vec{Y}$

Convert types to a grammar

Example

$$S \triangleq (\mu x.\&\{n\colon x; x; ?\,\mathsf{int}, \ell\colon ?\,\mathsf{int}\}); (\mu z.!\,\mathsf{int}; z; z)$$

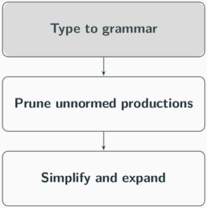
$$T \triangleq (\mu y.\&\{n\colon y; y, \ell\colon \mathsf{skip}\}; ?\,\mathsf{int}); (\mu z.!\,\mathsf{int}; z)$$

Type to grammar → **Prune unnormed productions** → **Simplify and expand**

Convert types to a grammar

Example

$$S \triangleq (\mu x.\&\{n\colon x; x; ?\,\mathsf{int}, \ell\colon\ ?\,\mathsf{int}\}); (\mu z.!\,\mathsf{int}; z; z)$$
$$T \triangleq (\mu y.\&\{n\colon y; y, \ell\colon \mathsf{skip}\}; ?\,\mathsf{int}); (\mu z.!\,\mathsf{int}; z)$$



Productions for S	Productions for T
$X_1 \rightarrow \&n\, X_1 X_1 X_2$	$Y_1 \rightarrow \&n\, Y_1\, Y_1\, Y_2$
$X_1 \rightarrow \&\ell\, X_3$	$Y_1 \rightarrow \&\ell\, Y_2$
$X_2 \rightarrow ?\mathsf{int}$	$Y_2 \rightarrow ?\mathsf{int}$
$X_3 \rightarrow ?\mathsf{int}$	$Y_3 \rightarrow !\mathsf{int}\, Y_3$
$X_4 \rightarrow !\mathsf{int}\, X_4 X_4$	

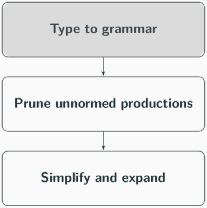
Convert types to a grammar

Example

$$S \triangleq (\mu x.\&\{n\colon x; x; ?\,\mathsf{int}, \ell\colon ?\,\mathsf{int}\}); (\mu z.!\,\mathsf{int}; z; z)$$

$$T \triangleq (\mu y.\&\{n\colon y; y, \ell\colon \mathsf{skip}\}; ?\,\mathsf{int}); (\mu z.!\,\mathsf{int}; z)$$

Productions for S	Productions for T
$X \to !(\,)\, X_1 X_4$	$Y \to !(\,)\, Y_1 Y_3$
$X_1 \to \&n\, X_1 X_1 X_2$	$Y_1 \to \&n\, Y_1 Y_1 Y_2$
$X_1 \to \&\ell\, X_3$	$Y_1 \to \&\ell\, Y_2$
$X_2 \to ?\mathsf{int}$	$Y_2 \to ?\mathsf{int}$
$X_3 \to ?\mathsf{int}$	$Y_3 \to !\mathsf{int}\, Y_3$
$X_4 \to !\mathsf{int}\, X_4 X_4$	



Convert types to a grammar

`LOHaskellCode` $\sim$ `100`

Soundness ✓

1. Conversion of types into BPAs is sound[a].
2. Conversion of BPAs to Greibach Normal Form without altering the solution is sound[b].

[a] Thiemann and Vasconcelos. Context-free session types. 2016.
[b] Baeten et al. Decidability of bisimulation equivalence for process generating context-free languages. 1993.

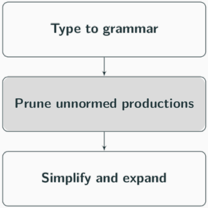
Prune unnormed productions

Simplify and expand

Prune unnormed productions

Definition ((Un)normed symbols[1])

A sequence of symbols α is *normed* if there is a path $\alpha \to \cdots \to \varepsilon$. Otherwise, α is said to be *unnormed*.

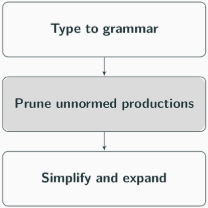


Christensen, Huttel, and Stirling[1] noted that:

$$\text{whenever } \alpha \text{ is unnormed, } \alpha \sim \alpha\beta.$$

[1] Christensen et al. Bisimulation equivalence is decidable for all CF processes. 1995

Prune unnormed productions



Definition ((Un)normed symbols[1])

A sequence of symbols α is *normed* if there is a path $\alpha \to \cdots \to \varepsilon$. Otherwise, α is said to be *unnormed*.

Christensen, Huttel, and Stirling[1] noted that:

$$\text{whenever } \alpha \text{ is unnormed, } \alpha \sim \alpha\beta.$$

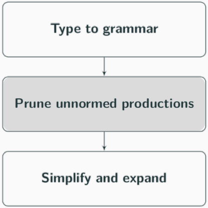
Example

$$S \triangleq (\mu x.\&\{n\colon x; x; ?\,\mathsf{int}, \ell\colon ?\,\mathsf{int}\}); (\mu z.!\,\mathsf{int}; z; z)$$

Productions for S
$X_1 \to \&n\, X_1 X_1 X_2$
$X_1 \to \&\ell\, X_3$
$X_2 \to ?\mathsf{int}$
$X_3 \to ?\mathsf{int}$
$X_4 \to !\mathsf{int}\, X_4 X_4$

[1] Christensen et al. Bisimulation equivalence is decidable for all CF processes. 1995

Prune unnormed productions



Definition ((Un)normed symbols[1])

A sequence of symbols α is *normed* if there is a path $\alpha \to \cdots \to \varepsilon$. Otherwise, α is said to be *unnormed*.

Christensen, Huttel, and Stirling[1] noted that:

whenever α is unnormed, $\alpha \sim \alpha\beta$.

Example

$$S \triangleq (\mu x.\&\{n\colon x; x; ?\,\mathsf{int}, \ell\colon ?\,\mathsf{int}\}); (\mu z.!\,\mathsf{int}; z; z)$$

Productions for S	Productions for *pruned* S
$X_1 \to \&n\, X_1 X_1 X_2$	$X_1 \to \&n\, X_1 X_1 X_2$
$X_1 \to \&\ell\, X_3$	$X_1 \to \&\ell\, X_3$
$X_2 \to ?\mathsf{int}$	$X_2 \to ?\mathsf{int}$
$X_3 \to ?\mathsf{int}$	$X_3 \to ?\mathsf{int}$
$X_4 \to !\mathsf{int}\, X_4 X_4$	$X_4 \to !\mathsf{int}\, X_4$

[1] Christensen et al. Bisimulation equivalence is decidable for all CF processes. 1995

Algorithm for checking the equivalence of CFST Main stages

Convert types to a grammar

`LOHaskellCode` $\sim$ 100

Soundness ✓

Prune unnormed productions

`LOHaskellCode` $\sim$ 30

Soundness ✓

1. Pruning unnormed productions is sound[a]

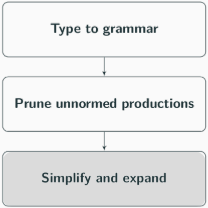
[a] Christensen, Huttel, and Stirling. Bisimulation equivalence is decidable for all context-free processes. 1995

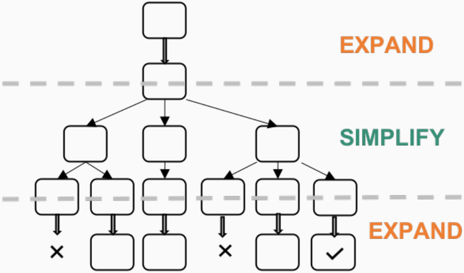
Simplify and expand

Simplify and expand

Following the ideas from Hirshfeld, Jančar, Moller, a bisimulation is seen as an **expansion tree**[2][3] that alternates between:

- expansion operations
- simplification operations





[2] Hirshfeld. Bisimulation trees and the decidability of weak bisimulations. 1997

[3] Jančar and Moller. Techniques for decidability and undecidability of bisimilarity. 1999

Simplify and expand

An expansion tree alternates between:

- **expansion operations** - a single derived node results from the expansion of the parent node.

[4] Jančar, Moller. Techniques for decidability and undecidability of bisimilarity. 1999
[5] Christensen et al. Bisimulation equivalence is decidable for all CF processes. 1995

Simplify and expand

An expansion tree alternates between:

- **expansion operations** - a single derived node results from the expansion of the parent node.
- **simplification operations**
  - **reflexive rule**: omit from a node N any reflexive pair.
  - **congruence rule**: omit from a node N any pair that belongs to the least congruence containing the ancestors of N.

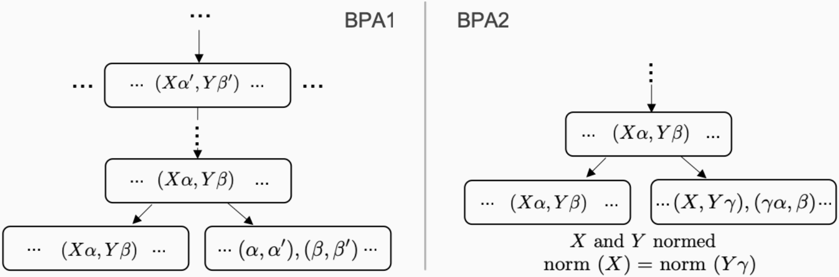
[4] Jančar, Moller. Techniques for decidability and undecidability of bisimilarity. 1999

[5] Christensen et al. Bisimulation equivalence is decidable for all CF processes. 1995

Simplify and expand

An expansion tree alternates between:

- **expansion operations** - a single derived node results from the expansion of the parent node.
- **simplification operations**
  - **reflexive rule**: omit from a node N any reflexive pair.
  - **congruence rule**: omit from a node N any pair that belongs to the least congruence containing the ancestors of N.
  - **basic process algebra rules**[4][5]



[4] Jančar, Moller. Techniques for decidability and undecidability of bisimilarity. 1999

[5] Christensen et al. Bisimulation equivalence is decidable for all CF processes. 1995

Simplify and Expand

All these transformation rules preserve the *safeness property*:

Safeness property[5]

$S \sim T$ iff the expansion tree rooted at $\{(S, T)\}$ has a successful branch.

The finite witness property holds[5]:

Finite witness property[5,6]

If $S \sim T$, then there exists a **finite successful branch** in the expansion tree.

[6] Christensen et al. Bisimulation equivalence is decidable for all CF processes. 1995

Example

$$S \triangleq (\mu x.\&\{n\colon x; x; ?\,\mathsf{int}, \ell\colon ?\,\mathsf{int}\}); (\mu z.!\,\mathsf{int}; z; z)$$
$$T \triangleq (\mu y.\&\{n\colon y; y, \ell\colon \mathsf{skip}\}; ?\,\mathsf{int}); (\mu z.!\,\mathsf{int}; z)$$

$\boxed{(X_1X_4, Y_1Y_3)}$

Productions for *pruned* S
$X_1 \to \&n\, X_1X_1X_2$
$X_1 \to \&\ell\, X_3$
$X_2 \to ?\mathsf{int}$
$X_3 \to ?\mathsf{int}$
$X_4 \to !\mathsf{int}\, X_4$

Productions for T
$Y_1 \to \&n\, Y_1Y_1Y_2$
$Y_1 \to \&\ell\, Y_2$
$Y_2 \to ?\mathsf{int}$
$Y_3 \to !\mathsf{int}\, Y_3$

Example

$$S \triangleq (\mu x.\&\{n\colon x; x; ?\,\mathsf{int}, \ell\colon ?\,\mathsf{int}\}); (\mu z.!\,\mathsf{int}; z; z)$$

$$T \triangleq (\mu y.\&\{n\colon y; y, \ell\colon \mathsf{skip}\}; ?\,\mathsf{int}); (\mu z.!\,\mathsf{int}; z)$$

$$\begin{tikzcd}
 & \boxed{(X_1X_4, Y_1Y_3)} \arrow[dl, "\text{bpa2}"'] \arrow[dr, "\text{self}"] & \\
\boxed{(X_1, Y_1), (X_4, Y_3)} & & \boxed{(X_1X_4, Y_1Y_3)}
\end{tikzcd}$$

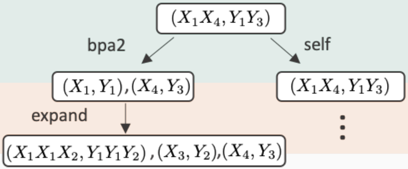
Productions for *pruned* S
$X_1 \rightarrow \&n\, X_1 X_1 X_2$
$X_1 \rightarrow \&\ell\, X_3$
$X_2 \rightarrow ?\mathsf{int}$
$X_3 \rightarrow ?\mathsf{int}$
$X_4 \rightarrow !\mathsf{int}\, X_4$

Productions for T
$Y_1 \rightarrow \&n\, Y_1 Y_1 Y_2$
$Y_1 \rightarrow \&\ell\, Y_2$
$Y_2 \rightarrow ?\mathsf{int}$
$Y_3 \rightarrow !\mathsf{int}\, Y_3$

Example

$$S \triangleq (\mu x.\&\{n\colon x; x; ?\,\mathsf{int}, \ell\colon\ ?\,\mathsf{int}\}); (\mu z.!\,\mathsf{int}; z; z)$$

$$T \triangleq (\mu y.\&\{n\colon y; y, \ell\colon \mathsf{skip}\}; ?\,\mathsf{int}); (\mu z.!\,\mathsf{int}; z)$$



Productions for *pruned* S

$X_1 \to \&n\, X_1X_1X_2$
$X_1 \to \&\ell\, X_3$
$X_2 \to ?\mathsf{int}$
$X_3 \to ?\mathsf{int}$
$X_4 \to !\mathsf{int}\, X_4$

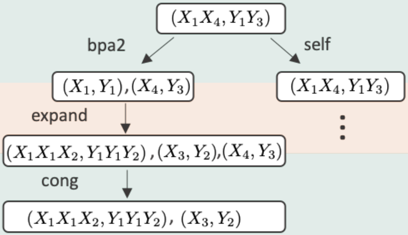
Productions for T

$Y_1 \to \&n\, Y_1Y_1Y_2$
$Y_1 \to \&\ell\, Y_2$
$Y_2 \to ?\mathsf{int}$
$Y_3 \to !\mathsf{int}\, Y_3$

Example

$$S \triangleq (\mu x.\&\{n\colon x; x; ?\,\mathsf{int}, \ell\colon ?\,\mathsf{int}\}); (\mu z.!\,\mathsf{int}; z; z)$$

$$T \triangleq (\mu y.\&\{n\colon y; y, \ell\colon \mathsf{skip}\}; ?\,\mathsf{int}); (\mu z.!\,\mathsf{int}; z)$$



Productions for *pruned* S

$X_1 \to \&n\, X_1X_1X_2$
$X_1 \to \&\ell\, X_3$
$X_2 \to ?\mathsf{int}$
$X_3 \to ?\mathsf{int}$
$X_4 \to !\mathsf{int}\, X_4$

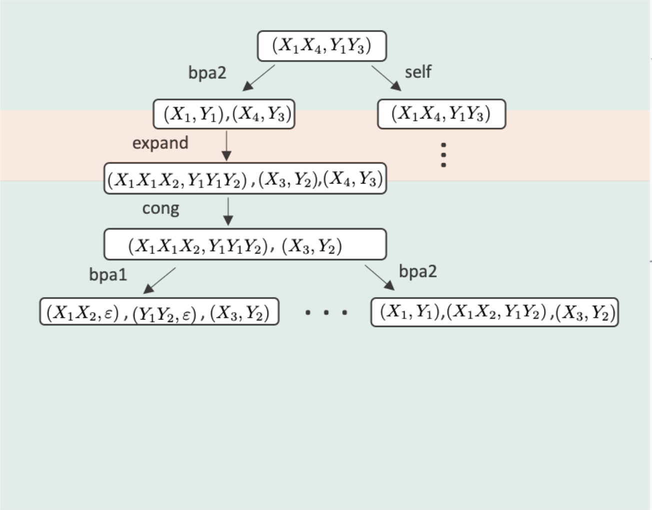
Productions for T

$Y_1 \to \&n\, Y_1Y_1Y_2$
$Y_1 \to \&\ell\, Y_2$
$Y_2 \to ?\mathsf{int}$
$Y_3 \to !\mathsf{int}\, Y_3$

Example

$$S \triangleq (\mu x.\&\{n\colon x; x; ?\,\mathsf{int}, \ell\colon ?\,\mathsf{int}\}); (\mu z.!\,\mathsf{int}; z; z)$$

$$T \triangleq (\mu y.\&\{n\colon y; y, \ell\colon \mathsf{skip}\}; ?\,\mathsf{int}); (\mu z.!\,\mathsf{int}; z)$$



Productions for *pruned* S

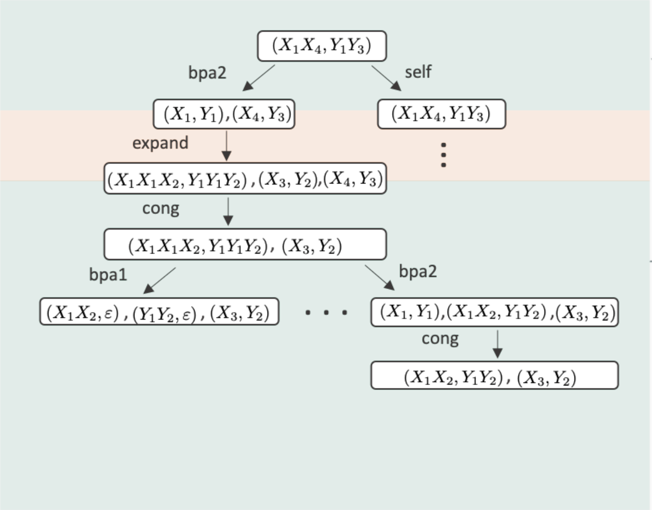
$X_1 \to \&n\, X_1X_1X_2$
$X_1 \to \&\ell\, X_3$
$X_2 \to ?\mathsf{int}$
$X_3 \to ?\mathsf{int}$
$X_4 \to !\mathsf{int}\, X_4$

Productions for T

$Y_1 \to \&n\, Y_1Y_1Y_2$
$Y_1 \to \&\ell\, Y_2$
$Y_2 \to ?\mathsf{int}$
$Y_3 \to !\mathsf{int}\, Y_3$

Example

$$S \triangleq (\mu x.\&\{n\colon x; x; ?\,\mathsf{int}, \ell\colon\ ?\,\mathsf{int}\}); (\mu z.!\,\mathsf{int}; z; z)$$
$$T \triangleq (\mu y.\&\{n\colon y; y, \ell\colon \mathsf{skip}\}; ?\,\mathsf{int}); (\mu z.!\,\mathsf{int}; z)$$



Productions for *pruned* S

$X_1 \to \&n\, X_1X_1X_2$
$X_1 \to \&\ell\, X_3$
$X_2 \to ?\mathsf{int}$
$X_3 \to ?\mathsf{int}$
$X_4 \to !\mathsf{int}\, X_4$

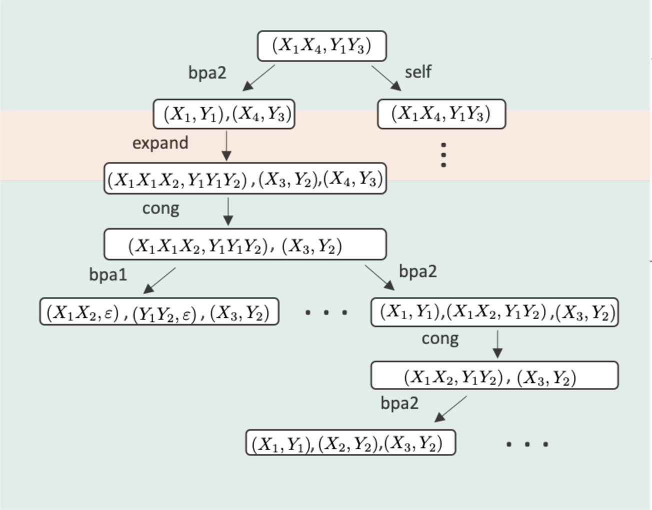
Productions for T

$Y_1 \to \&n\, Y_1Y_1Y_2$
$Y_1 \to \&\ell\, Y_2$
$Y_2 \to ?\mathsf{int}$
$Y_3 \to !\mathsf{int}\, Y_3$

Example

$$S \triangleq (\mu x.\&\{n\colon x; x; ?\,\mathsf{int}, \ell\colon ?\,\mathsf{int}\}); (\mu z.!\,\mathsf{int}; z; z)$$

$$T \triangleq (\mu y.\&\{n\colon y; y, \ell\colon \mathsf{skip}\}; ?\,\mathsf{int}); (\mu z.!\,\mathsf{int}; z)$$



Productions for *pruned* S

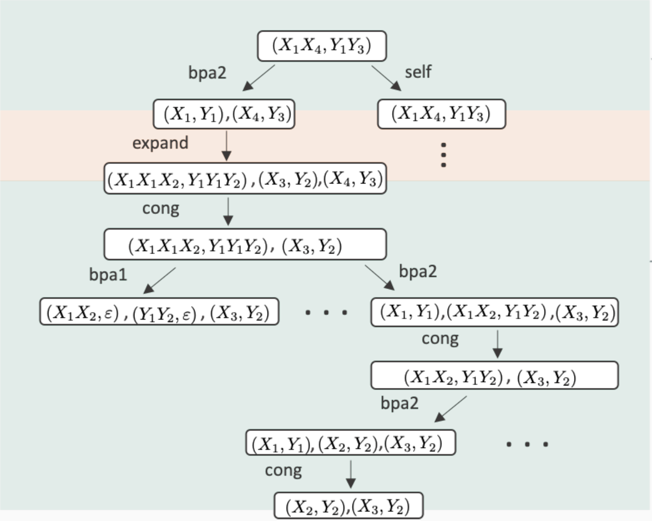
$X_1 \to \&n\, X_1X_1X_2$
$X_1 \to \&\ell\, X_3$
$X_2 \to ?\mathsf{int}$
$X_3 \to ?\mathsf{int}$
$X_4 \to !\mathsf{int}\, X_4$

Productions for T

$Y_1 \to \&n\, Y_1Y_1Y_2$
$Y_1 \to \&\ell\, Y_2$
$Y_2 \to ?\mathsf{int}$
$Y_3 \to !\mathsf{int}\, Y_3$

Example

$$S \triangleq (\mu x.\&\{n\colon x; x; ?\,\mathsf{int}, \ell\colon ?\,\mathsf{int}\}); (\mu z.!\,\mathsf{int}; z; z)$$
$$T \triangleq (\mu y.\&\{n\colon y; y, \ell\colon \mathsf{skip}\}; ?\,\mathsf{int}); (\mu z.!\,\mathsf{int}; z)$$



Productions for *pruned* S

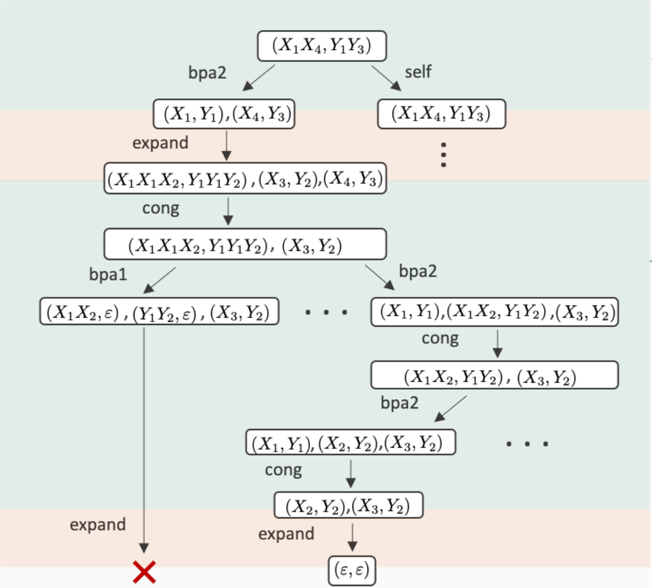
$X_1 \to \&n\, X_1X_1X_2$
$X_1 \to \&\ell\, X_3$
$X_2 \to ?\mathsf{int}$
$X_3 \to ?\mathsf{int}$
$X_4 \to !\mathsf{int}\, X_4$

Productions for T

$Y_1 \to \&n\, Y_1Y_1Y_2$
$Y_1 \to \&\ell\, Y_2$
$Y_2 \to ?\mathsf{int}$
$Y_3 \to !\mathsf{int}\, Y_3$

Example

$$S \triangleq (\mu x.\&\{n\colon x; x; ?\,\mathrm{int}, \ell\colon ?\,\mathrm{int}\}); (\mu z.!\,\mathrm{int}; z; z)$$
$$T \triangleq (\mu y.\&\{n\colon y; y, \ell\colon \mathrm{skip}\}; ?\,\mathrm{int}); (\mu z.!\,\mathrm{int}; z)$$



Productions for *pruned* S

$X_1 \to \&n\, X_1X_1X_2$
$X_1 \to \&\ell\, X_3$
$X_2 \to ?\mathrm{int}$
$X_3 \to ?\mathrm{int}$
$X_4 \to !\mathrm{int}\, X_4$

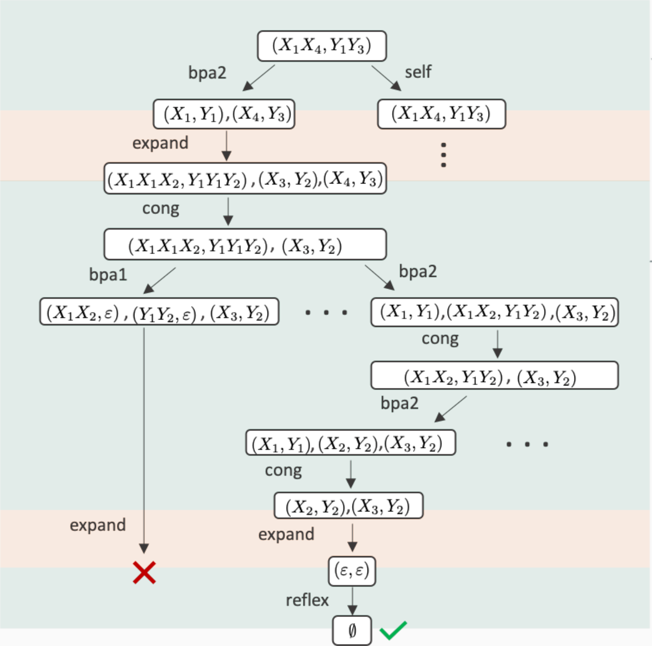
Productions for T

$Y_1 \to \&n\, Y_1Y_1Y_2$
$Y_1 \to \&\ell\, Y_2$
$Y_2 \to ?\mathrm{int}$
$Y_3 \to !\mathrm{int}\, Y_3$

Example

$$S \triangleq (\mu x.\&\{n\colon x; x; ?\,\mathsf{int}, \ell\colon ?\,\mathsf{int}\}); (\mu z.!\,\mathsf{int}; z; z)$$

$$T \triangleq (\mu y.\&\{n\colon y; y, \ell\colon \mathsf{skip}\}; ?\,\mathsf{int}); (\mu z.!\,\mathsf{int}; z)$$



Productions for *pruned* S

$X_1 \to \&n\, X_1X_1X_2$
$X_1 \to \&\ell\, X_3$
$X_2 \to ?\mathsf{int}$
$X_3 \to ?\mathsf{int}$
$X_4 \to !\mathsf{int}\, X_4$

Productions for T

$Y_1 \to \&n\, Y_1Y_1Y_2$
$Y_1 \to \&\ell\, Y_2$
$Y_2 \to ?\mathsf{int}$
$Y_3 \to !\mathsf{int}\, Y_3$

Algorithm for checking the equivalence of CFST **Main stages**

Convert types to a grammar

`LOHaskellCode` $\sim$ 100

Soundness ✓

↓

Prune unnormed productions

`LOHaskellCode` $\sim$ 30

Soundness ✓

↓

Simplify and expand

`LOHaskellCode` $\sim$ 170

Soundness ✓

1. The *safeness property ensures soundness*[a]

[a] Jančar, Moller. Techniques for decidability and undecidability of bisimilarity. 1999

Implementation strategies

Implementation choice:

- Breadth-first search on the tree

Implementation strategies

Implementation choice:

- Breadth-first search on the tree

Strategic options that can enhance performance:

- Instead of looking for a fixed point, iterate the simplification phase
- Apply BPA rules wrapped with blocks of reflexive and congruence rules

Implementation strategies

Implementation choice:

- Breadth-first search on the tree

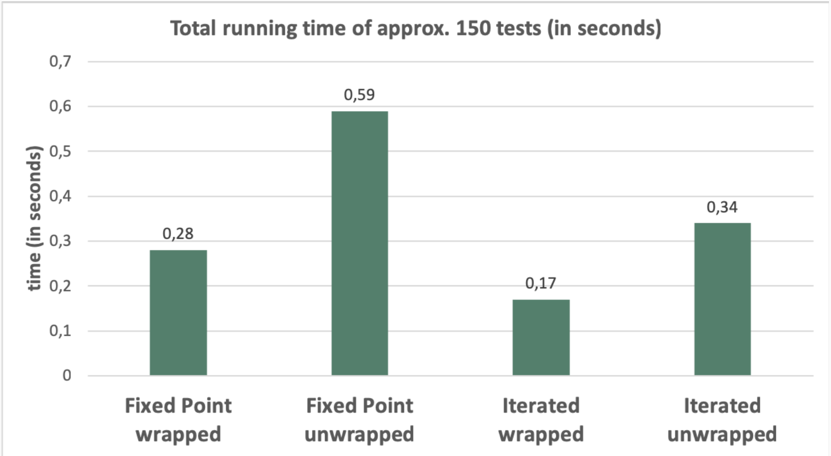
Strategic options that can enhance performance:

- Instead of looking for a fixed point, iterate the simplification phase
- Apply BPA rules wrapped with blocks of reflexive and congruence rules

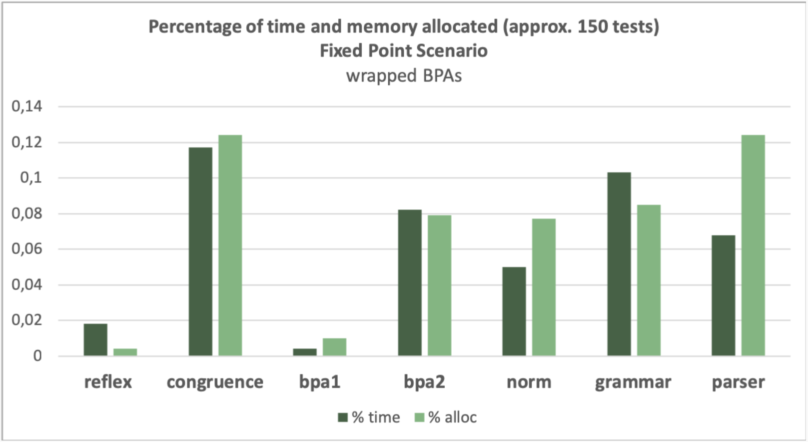
We showcase runtimes for four scenarios:

	Fixed Point Scenario		**Iterated Scenario**	
	wrapped	unwrapped	wrapped	unwrapped
Simplification	find fixed point		iterate	
BPAs wrapped	✓	×	✓	×

Running times ($\sim$ 150 tests)



Specs:	MacBook Air
	1.8 GHz Intel Core i5
	8GB of memory


Percentage of time and memory allocated (approx. 150 tests)
Fixed Point Scenario
wrapped BPAs
0,14
0,12
0,1
0,08
0,06
0,04
0,02
0
reflex
congruence
bpa1
bpa2
norm
grammar
parser
% time
% alloc

Towards completeness...

Finite witness property[7,8]

If $S \sim T$, then there exists a **finite successful branch** in the expansion tree.

Implementation choices aiming to achieve completeness:

- Instead of looking for a fixed point, iterate the simplification phase (there may not be a fixed point)
- Use double ended enqueue, prepending *promising* nodes, as opposed to queuing all new nodes

[7] Christensen et al. Bisimulation equivalence is decidable for all CF processes. 1995
[8] Jančar, Moller. Techniques for decidability and undecidability of bisimilarity. 1999

Ongoing Work

- Soundness ✓
- Completeness ? ... on our way to achieve it.
- Complexity ? ... in practice it does not seem to take much longer than parsing.
- Lines of Haskell code: approx. 300

Coming soon:

FreeST, a compiler for context-free session types!

(demos on demand)

Thank you!