I. Subset, superset

When all members of set A are present in another set B, then A is a **subset** of B. Let's say set B is the set of all movies ever produced. Then A (movies I've watched) is clearly a subset of B. This notion is expressed like so:

$$A \subseteq B \tag{1}$$

To look at things from the other end, B is a **superset** of A:

$$B \supseteq A \tag{2}$$

You know what else is a subset of B? An empty set!

$$\varnothing \subseteq B \tag{3}$$

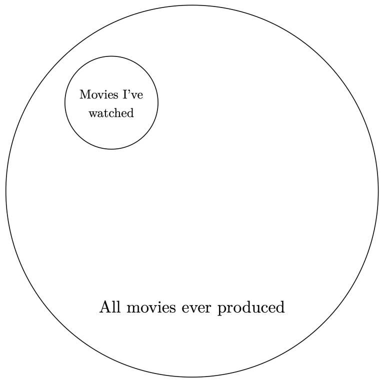

Figure 1: A set and its subset.

This either sounds absolutely natural to you or extremely weird. It makes perfect sense to a mathematician, because it's easy to argue: *all* members of $\varnothing$ are present in B, all zero of them.

It gets weirder. As per our definition, if all members of a set are also present in another set, then the one is a subset of the other. This means any set is a subset of itself.

$$A \subseteq A \qquad B \subseteq B \qquad Z \subseteq Z$$

(4)

By extension, if two sets are the same, then either of them is a subset of the other.

$$\text{if } A \subseteq B \text{ and } B \subseteq A \text{ then } A = B \tag{5}$$

When we look at a statement $A \subseteq B$, we often need to know whether $A = B$ or not. To distinguish between the two cases, mathematicians use a special notion of a **proper subset**.

If $A \in B$, but $A \neq B$, then A is a proper subset of B.

$$A \subset B \tag{6}$$

Since I haven't watched all the movies ever produced, I can say that A is a proper subset of B. So, a set is a subset of itself, but is never a proper subset of itself.