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# DCTD: Deep Conditional Target Densities for Accurate Regression

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Fredrik K. Gustafsson

Uppsala University

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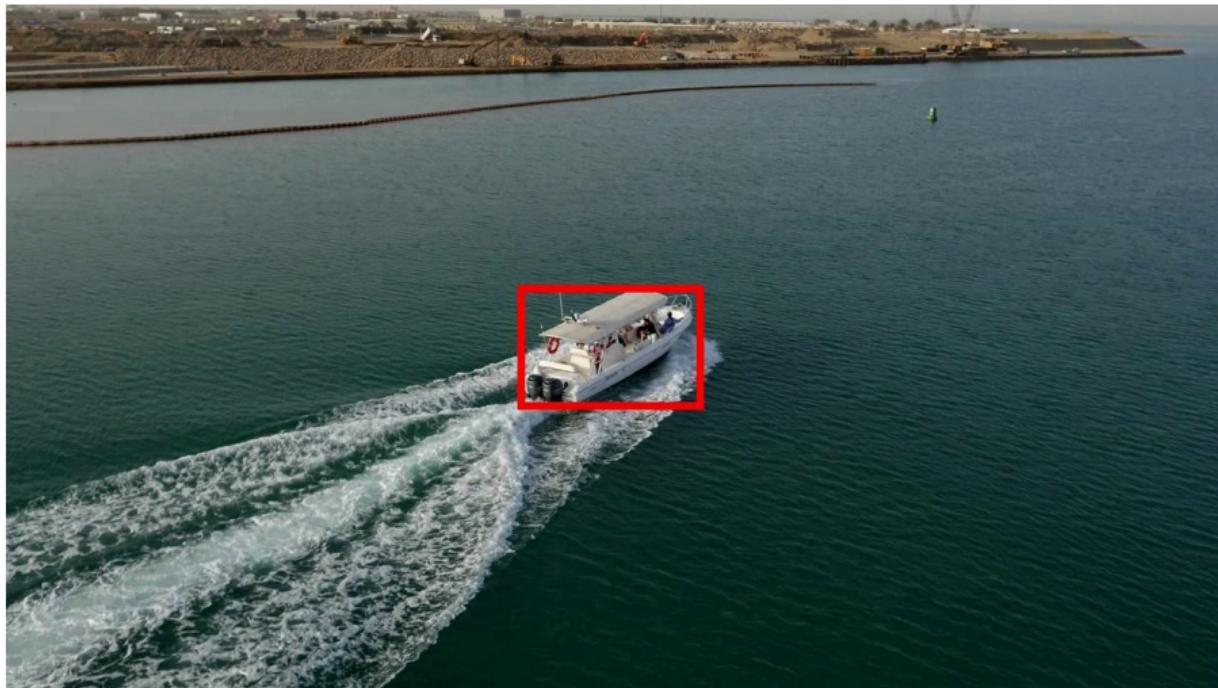
Fredrik K. Gustafsson\*, Martin Danelljan\* (ETH Zurich), Goutam Bhat (ETH Zurich), Thomas B. Schön

We propose DCTD, a general regression method with a clear probabilistic interpretation.

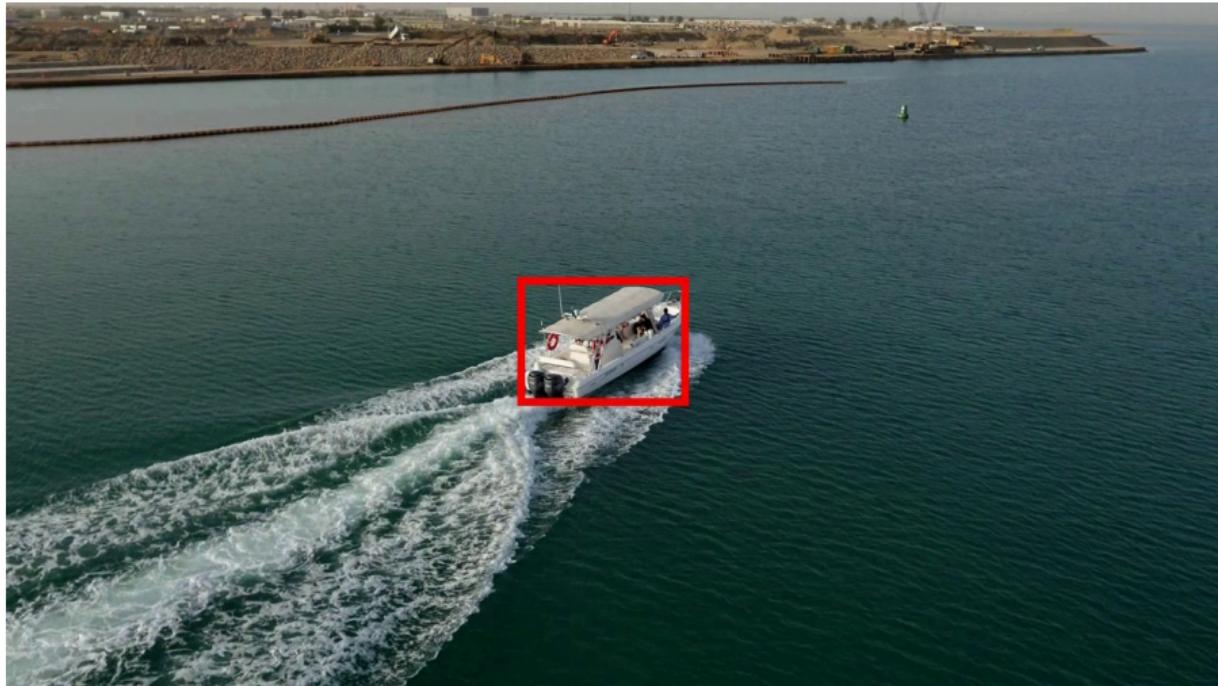
When applied for bounding box regression, DCTD sets a new state-of-the-art on the task of **generic visual object tracking**.



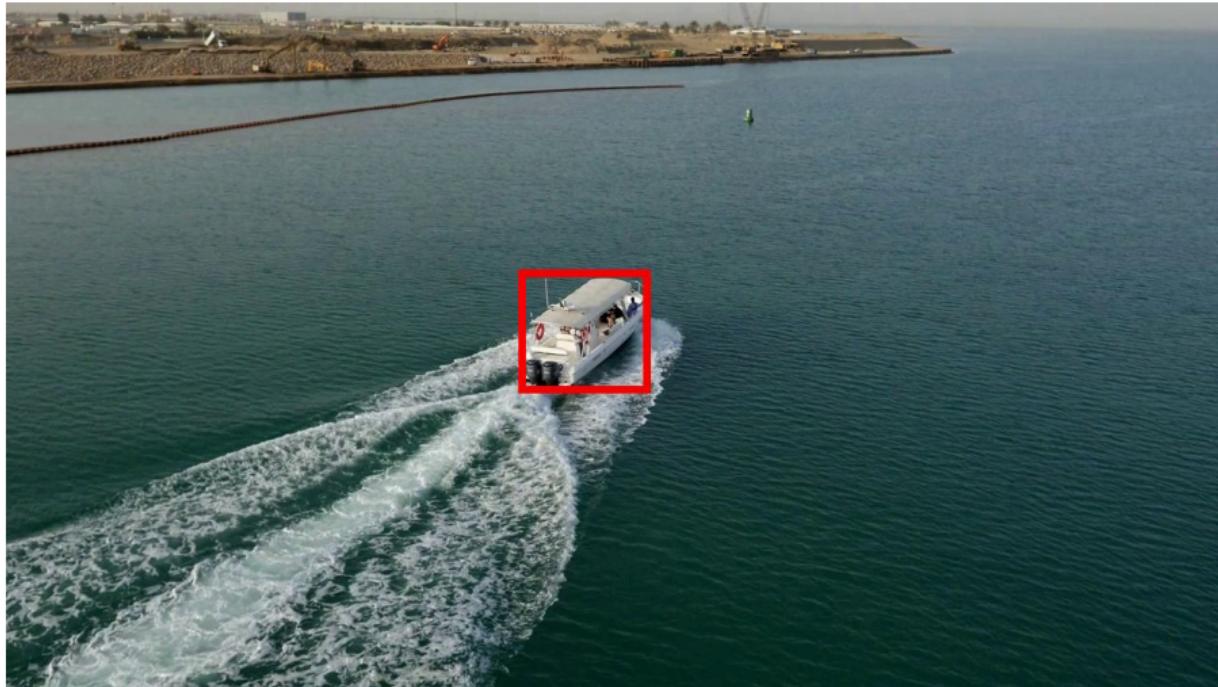
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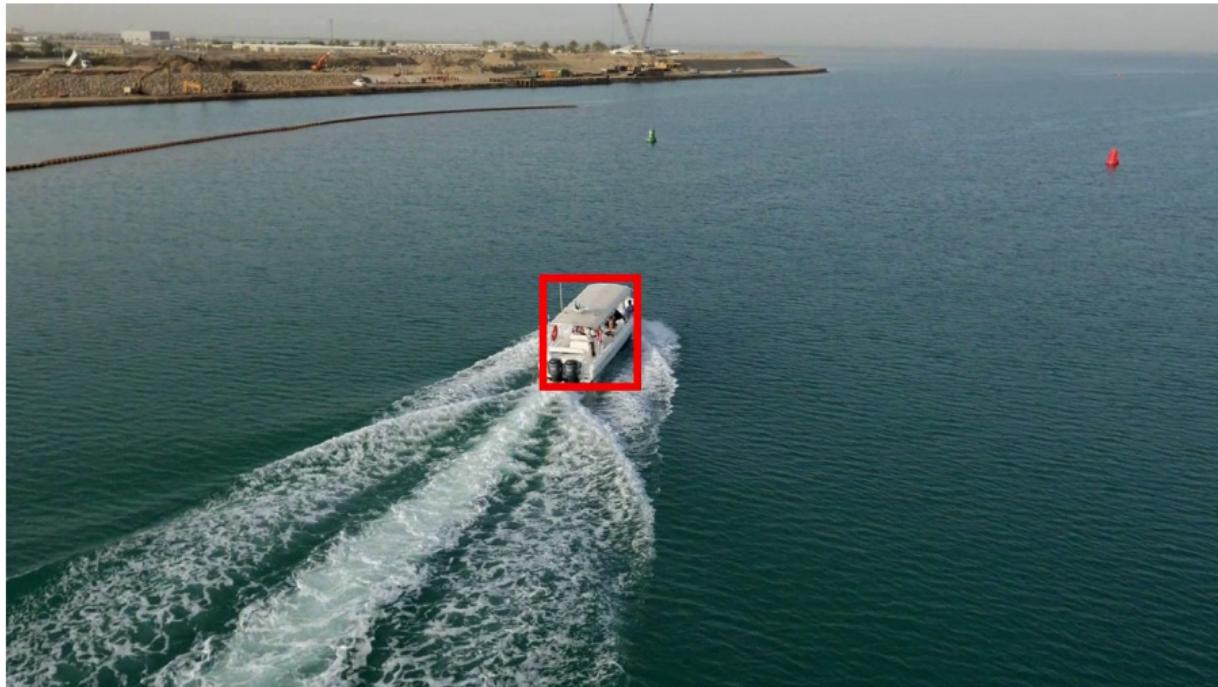
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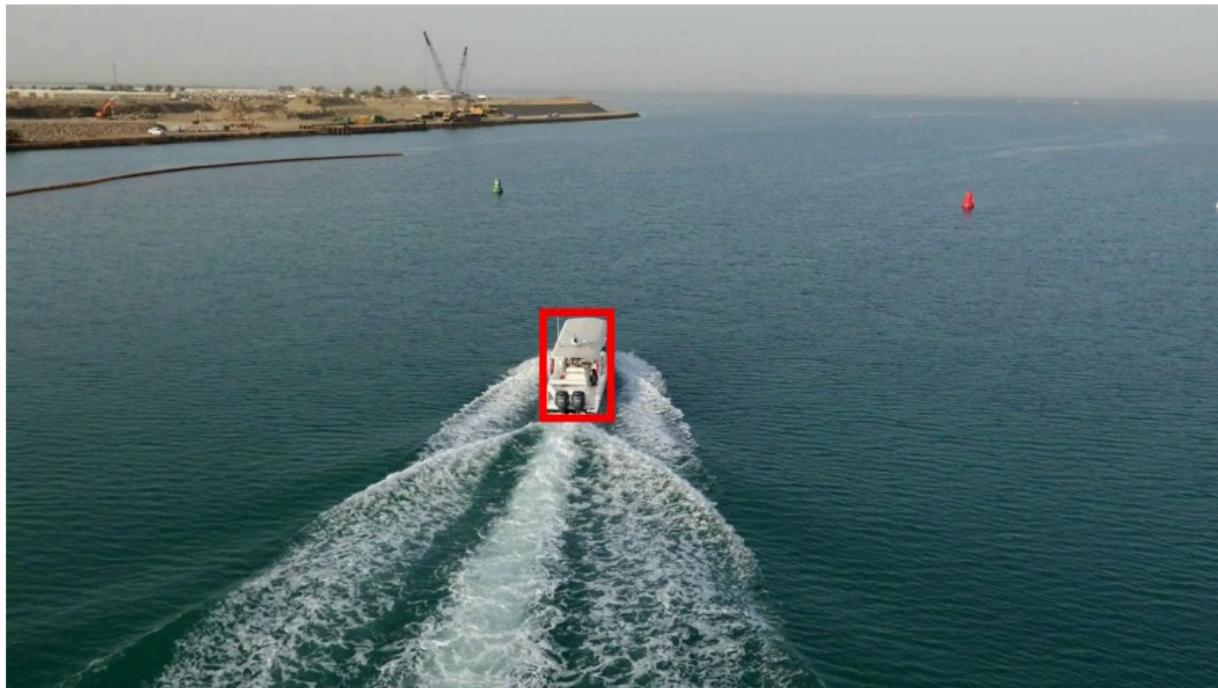
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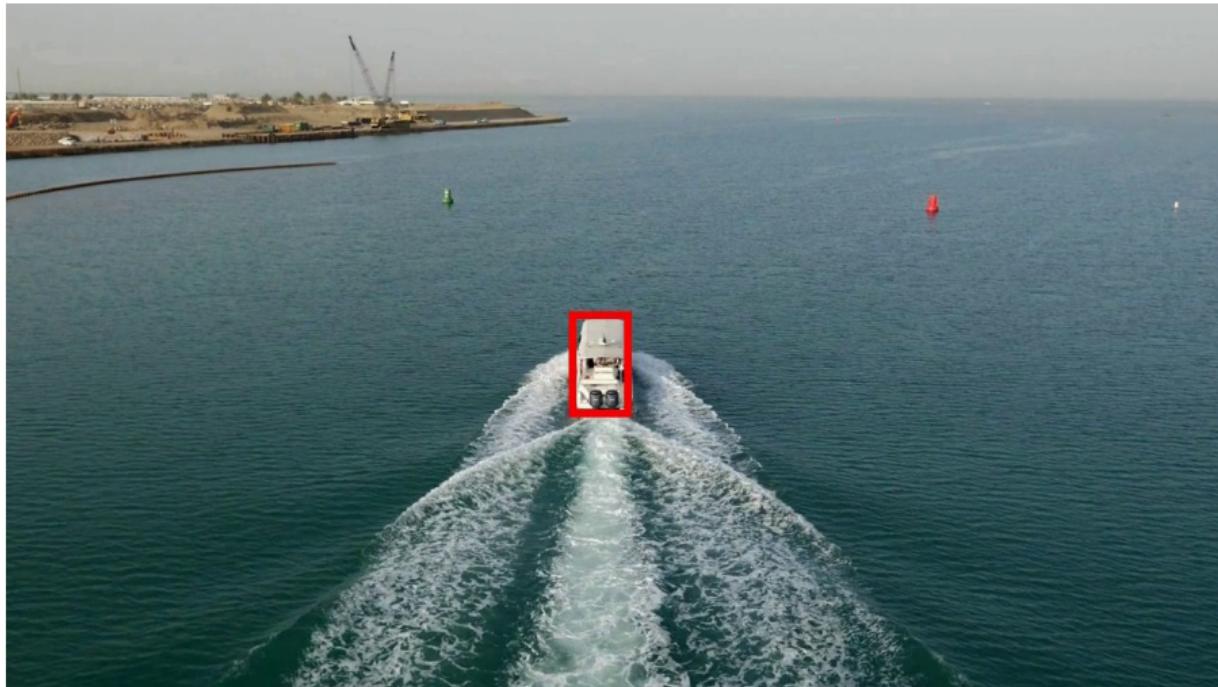
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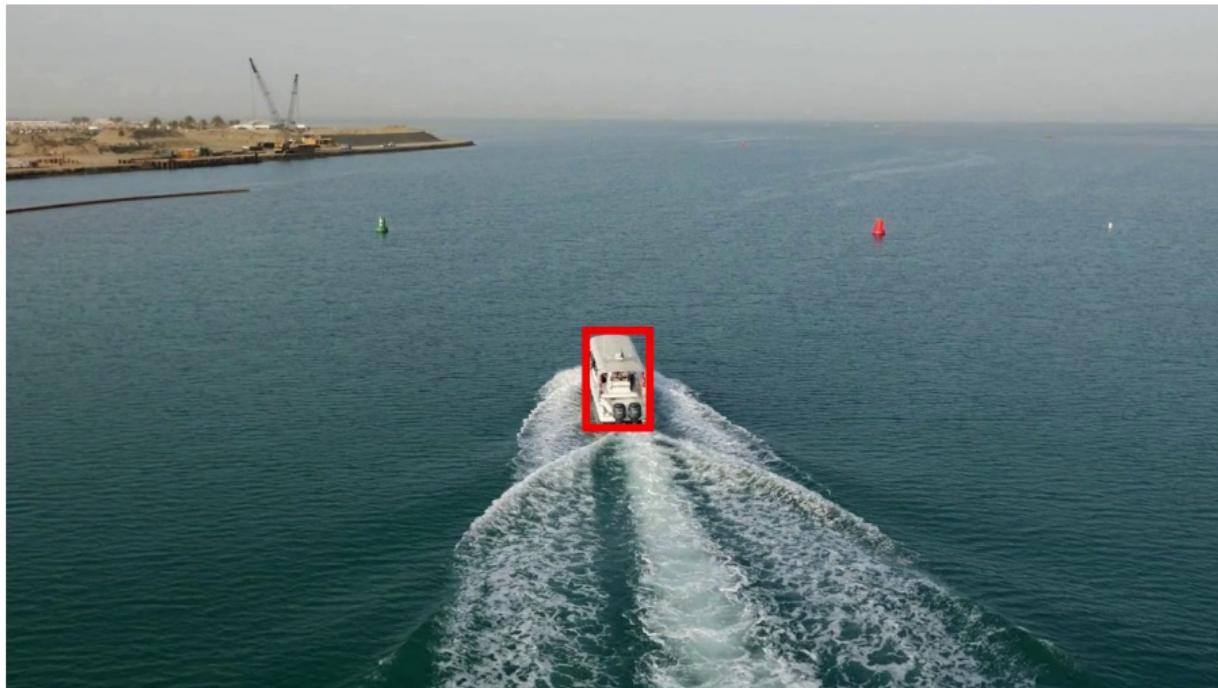
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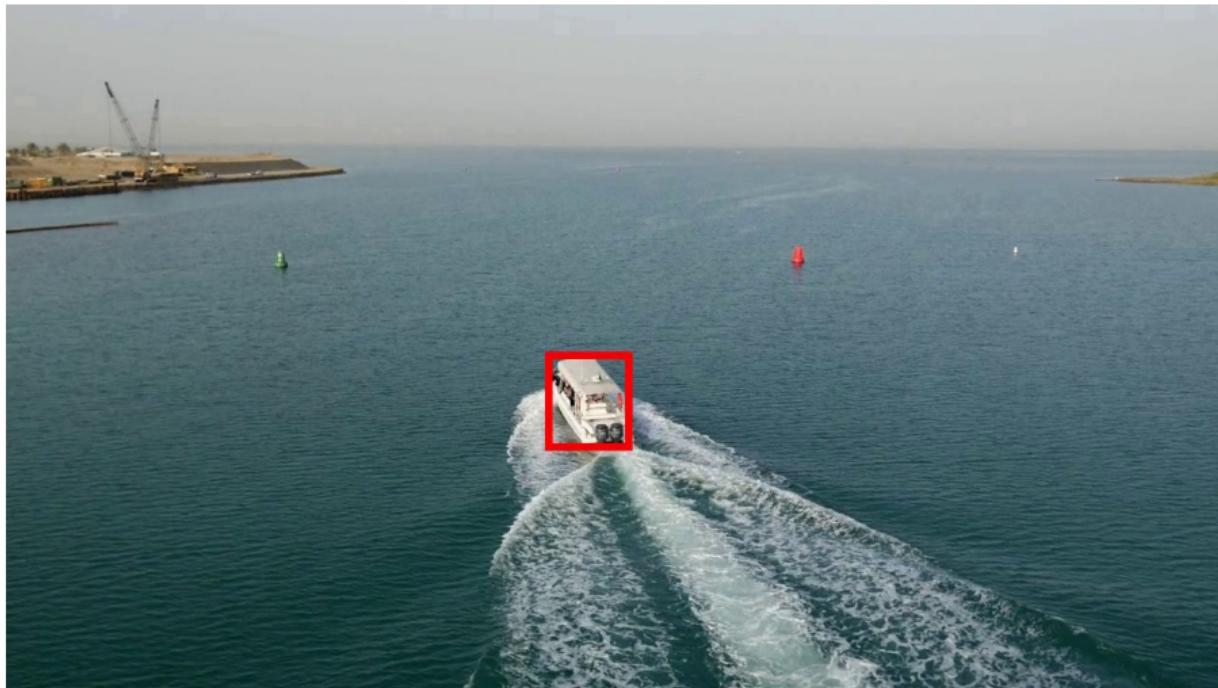
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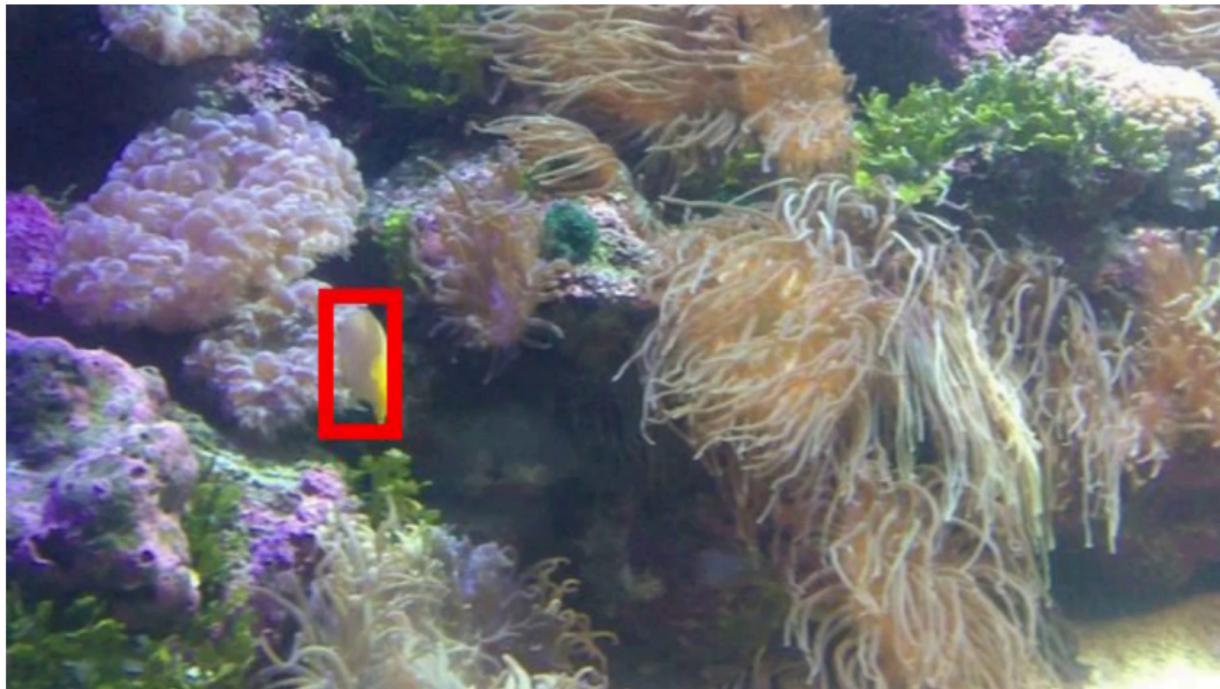
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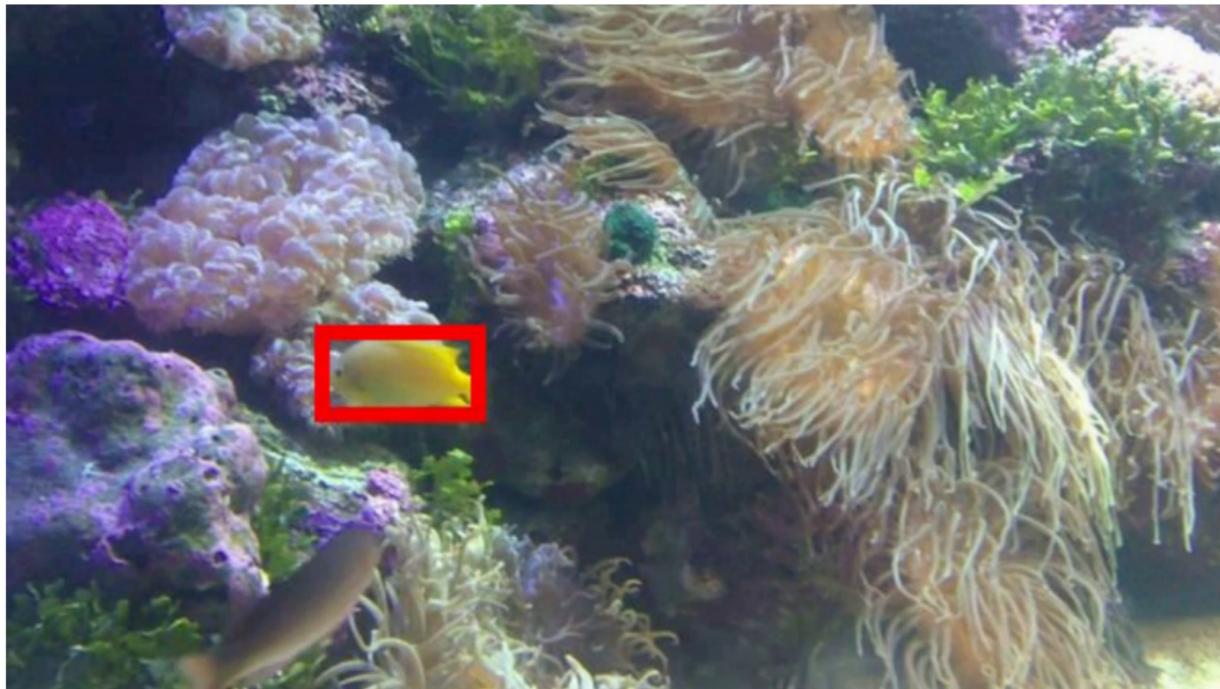
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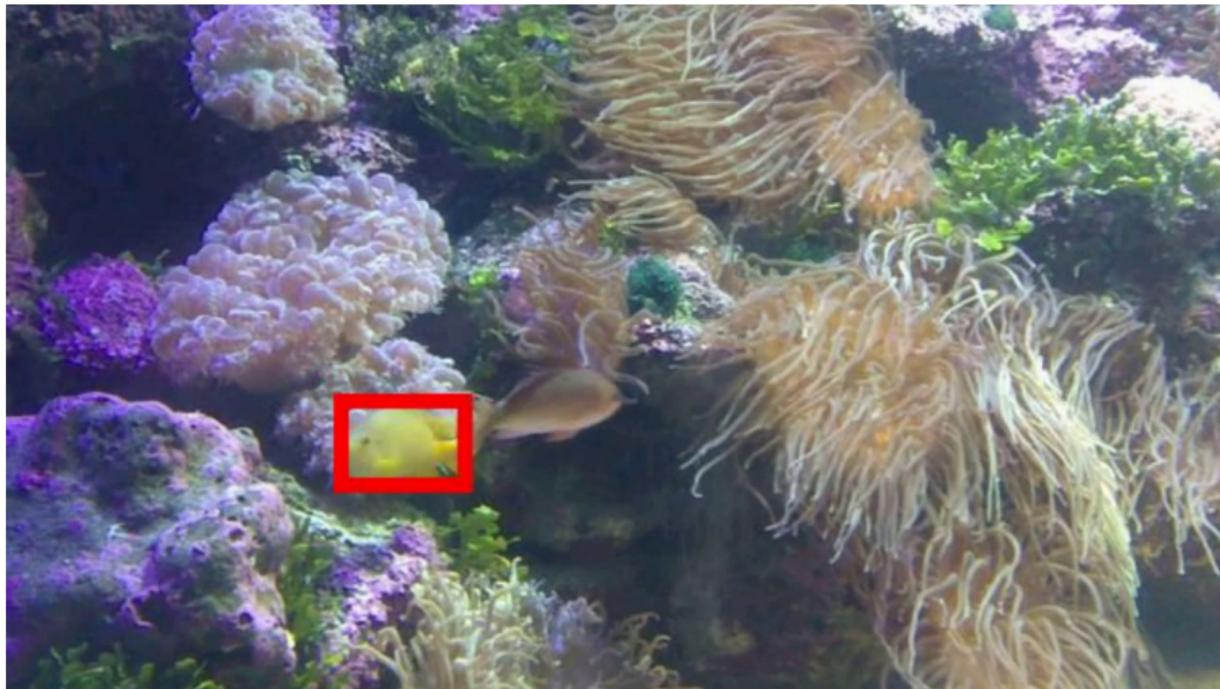
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  - 1.2 Probabilistic regression
  - 1.3 Confidence-based regression
2. Deep Conditional Target Densities (DCTD) for accurate regression
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# 1. Background: regression using deep neural networks

**Supervised regression:** learn to predict a continuous target value  $y^* \in \mathcal{Y} = \mathbb{R}^K$  from a corresponding input  $x^* \in \mathcal{X}$ , given a training set  $\mathcal{D}$  of i.i.d. input-target examples,  $\mathcal{D} = \{(x_i, y_i)\}_{i=1}^N$ ,  $(x_i, y_i) \sim p(x, y)$ .

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**Deep neural network (DNN):** a function  $f_\theta : \mathcal{U} \rightarrow \mathcal{O}$ , parameterized by  $\theta \in \mathbb{R}^P$ , that maps an input  $u \in \mathcal{U}$  to an output  $f_\theta(u) \in \mathcal{O}$ .

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For example, the  $L^2$  loss corresponds to a fixed-variance Gaussian model (1D case):

$$p(y|x; \theta) = \mathcal{N}(y; f_\theta(x), \sigma^2).$$

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## 1.3 Confidence-based regression

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Commonly employed for image-coordinate regression, e.g. human pose estimation [11], where the DNN predicts a 2D confidence heatmap over image-coordinates  $y$ . Recently, the approach was also employed by IoU-Net [4] for bounding box regression in object detection, which in turn was utilized by the ATOM [3] visual tracker.

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$$\begin{aligned} -\log p(y_i|x_i; \theta) &= \log \left( \int e^{f_\theta(x_i,y)} dy \right) - f_\theta(x_i, y_i) \\ &= \log \left( \int \frac{e^{f_\theta(x_i,y)}}{q(y)} q(y) dy \right) - f_\theta(x_i, y_i) \\ &\approx \log \left( \frac{1}{M} \sum_{k=1}^M \frac{e^{f_\theta(x_i,y^{(k)})}}{q(y^{(k)})} \right) - f_\theta(x_i, y_i), \quad y^{(k)} \sim q(y). \end{aligned}$$

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We use a proposal distribution  $q(y) = q(y|y_i) = \frac{1}{L} \sum_{l=1}^L \mathcal{N}(y; y_i, \sigma_l^2)$  that depends on the ground truth target  $y_i$ .

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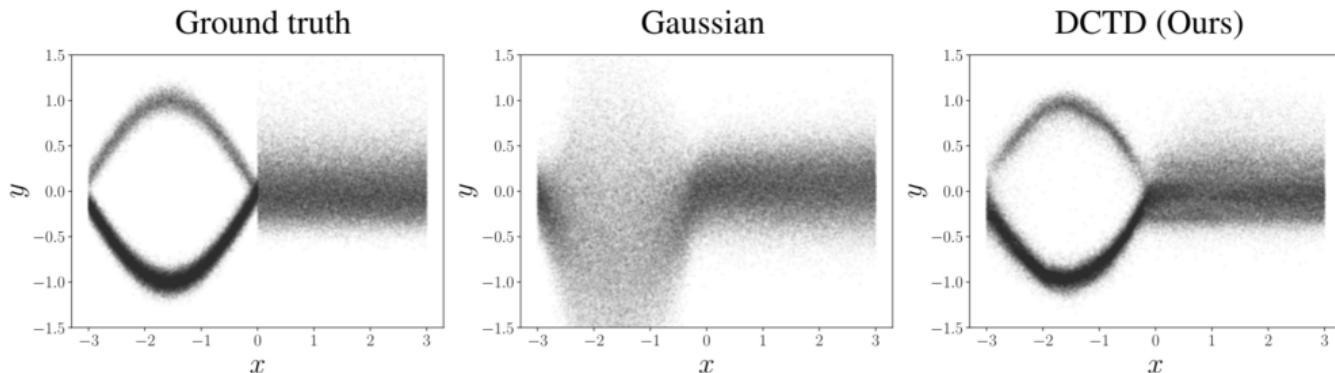
$$-\log p(y_i|x_i; \theta) \approx \log \left( \frac{1}{M} \sum_{k=1}^M \frac{e^{f_\theta(x_i, y^{(k)})}}{q(y^{(k)})} \right) - f_\theta(x_i, y_i), \quad y^{(k)} \sim q(y).$$

We use a proposal distribution  $q(y) = q(y|y_i) = \frac{1}{L} \sum_{l=1}^L \mathcal{N}(y; y_i, \sigma_l^2)$  that depends on the ground truth target  $y_i$ . The final minimization objective  $J(\theta)$  is thus given by:

$$J(\theta) = \frac{1}{N} \sum_{i=1}^N \log \left( \frac{1}{M} \sum_{m=1}^M \frac{e^{f_\theta(x_i, y^{(i, m)})}}{q(y^{(i, m)}|y_i)} \right) - f_\theta(x_i, y_i), \quad \{y^{(i, m)}\}_{m=1}^M \sim q(y|y_i).$$

## 2.1 Training - Illustrative toy example

The DCTD model  $p(y|x; \theta) = e^{f_\theta(x,y)} / Z(x, \theta)$  is highly flexible and can learn complex target densities directly from data, including multi-modal and asymmetric densities.



**Figure 4:** An illustrative 1D regression problem. The training data  $\{(x_i, y_i)\}_{i=1}^{2000}$  is generated by the ground truth conditional target density  $p(y|x)$ .

**Deep Conditional Target Densities (DCTD):** train a DNN  $f_\theta : \mathcal{X} \times \mathcal{Y} \rightarrow \mathbb{R}$  to predict a scalar value  $f_\theta(x, y)$ , then model  $p(y|x)$  with:

$$p(y|x; \theta) = \frac{e^{f_\theta(x, y)}}{Z(x, \theta)}, \quad Z(x, \theta) = \int e^{f_\theta(x, y)} dy.$$

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Given an input  $x^*$  at test time, we predict the target  $y^*$  by maximizing  $p(y|x^*; \theta)$ :

$$y^* = \operatorname{argmax}_y p(y|x^*; \theta) = \operatorname{argmax}_y f_\theta(x^*, y).$$

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By designing the DNN  $f_\theta$  to be differentiable w.r.t. targets  $y$ , the gradient  $\nabla_y f_\theta(x^*, y)$  can be efficiently evaluated using auto-differentiation. We can thus perform *gradient ascent* to find a local maximum of  $f_\theta(x^*, y)$ , starting from an initial estimate  $\hat{y}$ .

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#### Algorithm 1 Prediction via optimization-based refinement

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**Input:**  $x^*$ ,  $\hat{y}$ ,  $T$ ,  $\lambda$ ,  $\Omega_1$ ,  $\Omega_2$ .

- 1:  $y \leftarrow \hat{y}$ .
- 2: **for**  $t = 1, \dots, T$  **do**
- 3:   PrevValue  $\leftarrow f_\theta(x^*, y)$ .
- 4:    $y \leftarrow y + \lambda \nabla_y f_\theta(x^*, y)$ .
- 5:   NewValue  $\leftarrow f_\theta(x^*, y)$ .
- 6:   **if**  $|\text{PrevValue} - \text{NewValue}| < \Omega_1$  **or**  $(\text{NewValue} - \text{PrevValue}) < \Omega_2$  **then**
- 7:     **Return**  $y$ .
- 8: **Return**  $y$ .

1. Background: regression using deep neural networks
  - 1.1 Direct regression
  - 1.2 Probabilistic regression
  - 1.3 Confidence-based regression
2. Deep Conditional Target Densities (DCTD) for accurate regression
  - 2.1 Training
  - 2.2 Prediction
3. Experiments
  - 3.1 Age estimation, head-pose estimation, object detection
  - 3.2 Generic visual object tracking

### 3. Experiments

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We evaluate DCTD on four diverse computer vision regression tasks: **age estimation**, **head-pose estimation**, **object detection** and **generic visual object tracking**.

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(IoU-Net trains a DNN  $f_\theta : \mathcal{X} \times \mathcal{Y} \rightarrow \mathbb{R}$  to predict the IoU overlap between a bounding box  $y$  and the corresponding ground truth  $y_i$ . For training, boxes are sampled around  $y_i$  and the difference between predicted and true IoU is minimized. For prediction, an initial estimate  $\hat{y}$  is refined using gradient-based maximization of the predicted IoU.)

### 3.1 Age estimation

**Age estimation:** refinement using DCTD consistently improves MAE (lower is better) for the age predictions outputted by a number of baselines.

+DCTD	Cao et al. [2]	Direct	Gaussian	Laplace	Softmax (CE, $L^2$ )	Softmax (CE, $L^2$ , Var)
	$5.47 \pm 0.01$	$4.81 \pm 0.02$	$4.79 \pm 0.06$	$4.85 \pm 0.04$	$4.78 \pm 0.05$	$4.81 \pm 0.03$
✓	-	<b>4.65</b> $\pm 0.02$	$4.66 \pm 0.04$	$4.81 \pm 0.04$	<b>4.65</b> $\pm 0.04$	$4.69 \pm 0.03$

### 3.1 Head-pose estimation

**Head-pose estimation:** refinement using DCTD consistently improves the average MAE for Yaw, Pitch and Roll for the predicted pose outputted by our baselines.

+DCTD	Yang et al. [12]	Direct	Gaussian	Laplace	Softmax (CE, $L^2$ )	Softmax (CE, $L^2$ , Var)
	3.60	$3.09 \pm 0.07$	$3.12 \pm 0.08$	$3.21 \pm 0.06$	$3.04 \pm 0.08$	$3.15 \pm 0.07$
✓	-	$3.07 \pm 0.07$	$3.11 \pm 0.07$	$3.19 \pm 0.06$	<b><math>3.01 \pm 0.07</math></b>	$3.11 \pm 0.06$

### 3.1 Object detection

**Object detection:** when applied to refine the Faster-RCNN detections on COCO [6], DCTD both improves the original detections and outperforms the IoU-Net refinement.

Formulation Approach	Direct Faster-RCNN [10]	Gaussian	Laplace	Confidence IoU-Net [4]	Confidence IoU-Net <sup>†</sup>	DCTD
AP (%)	37.2	36.7	37.1	38.3	38.2	<b>39.1</b>
AP <sub>50</sub> (%)	<b>59.2</b>	58.7	59.1	58.3	58.4	58.5
AP <sub>75</sub> (%)	40.3	39.6	40.2	41.4	41.4	<b>41.8</b>

**Generic visual object tracking:** given *any* target object defined by a bounding box in the first frame of a video, estimate its bounding box in all subsequent video frames.

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ATOM [3] trains a classifier *online* to first roughly localize the target object in a new frame. Its bounding box is then estimated by using an IoU-Net, trained *offline*, to refine this initial estimate.

**Video:** [https://youtu.be/UP\\_eLvwsKzU](https://youtu.be/UP_eLvwsKzU)

### 3.2 Generic visual object tracking - Results

**Results:** when applied to refine the initial estimate provided by the classifier in ATOM, DCTD outperforms the original method (which uses IoU-Net for refinement). DCTD also outperforms other state-of-the-art trackers.

Dataset	Metric	SiamFC [1]	MDNet [9]	DaSiamRPN [13]	SiamRPN++ [5]	ATOM [3]	ATOM <sup>†</sup>	DCTD
TrackingNet [7]	Precision (%)	53.3	56.5	59.1	<b>69.4</b>	64.8	66.7	68.9
	Norm. Prec. (%)	66.6	70.5	73.3	<b>80.0</b>	77.1	78.3	79.5
	Success (%)	57.1	60.6	63.8	73.3	70.3	72.1	<b>73.7</b>
UAV123 [8]	OP <sub>0.50</sub> (%)	-	-	73.6	75*	78.9	79.6	<b>80.1</b>
	OP <sub>0.75</sub> (%)	-	-	41.1	56*	55.7	56.0	<b>59.8</b>
	AUC (%)	-	52.8	58.4	61.3	65.0	65.0	<b>66.5</b>

### Qualitative results for DCTD:

<https://youtu.be/AAnr0g38UeA>

<https://youtu.be/JyhgUYpwQ5c>

# References

- [1] L. Bertinetto, J. Valmadre, J. F. Henriques, A. Vedaldi, and P. H. Torr. Fully-convolutional siamese networks for object tracking. In *ECCV workshop*, 2016.
- [2] W. Cao, V. Mirjalili, and S. Raschka. Rank-consistent ordinal regression for neural networks. *arXiv preprint arXiv:1901.07884*, 2019.
- [3] M. Danelljan, G. Bhat, F. S. Khan, and M. Felsberg. ATOM: Accurate tracking by overlap maximization. In *Proceedings of the IEEE Conference on Computer Vision and Pattern Recognition (CVPR)*, pages 4660–4669, 2019.
- [4] B. Jiang, R. Luo, J. Mao, T. Xiao, and Y. Jiang. Acquisition of localization confidence for accurate object detection. In *Proceedings of the European Conference on Computer Vision (ECCV)*, pages 784–799, 2018.
- [5] B. Li, W. Wu, Q. Wang, F. Zhang, J. Xing, and J. Yan. Siamrpn++: Evolution of siamese visual tracking with very deep networks. In *CVPR*, 2019.
- [6] T.-Y. Lin, M. Maire, S. Belongie, J. Hays, P. Perona, D. Ramanan, P. Dollár, and C. L. Zitnick. Microsoft coco: Common objects in context. In *Proceedings of the European Conference on Computer Vision (ECCV)*, pages 740–755, 2014.
- [7] M. Müller, A. Bibi, S. Giancola, S. Al-Subaihi, and B. Ghanem. Trackingnet: A large-scale dataset and benchmark for object tracking in the wild. In *ECCV*, 2018.
- [8] M. Müller, N. Smith, and B. Ghanem. A benchmark and simulator for uav tracking. In *ECCV*, 2016.
- [9] H. Nam and B. Han. Learning multi-domain convolutional neural networks for visual tracking. In *CVPR*, 2016.
- [10] S. Ren, K. He, R. B. Girshick, and J. Sun. Faster r-cnn: Towards real-time object detection with region proposal networks. *IEEE Transactions on Pattern Analysis and Machine Intelligence*, 39:1137–1149, 2015.
- [11] B. Xiao, H. Wu, and Y. Wei. Simple baselines for human pose estimation and tracking. In *Proceedings of the European Conference on Computer Vision (ECCV)*, pages 466–481, 2018.
- [12] T.-Y. Yang, Y.-T. Chen, Y.-Y. Lin, and Y.-Y. Chuang. FSA-Net: Learning fine-grained structure aggregation for head pose estimation from a single image. In *Proceedings of the IEEE Conference on Computer Vision and Pattern Recognition (CVPR)*, pages 1087–1096, 2019.
- [13] Z. Zhu, Q. Wang, L. Bo, W. Wu, J. Yan, and W. Hu. Distractor-aware siamese networks for visual object tracking. In *ECCV*, 2018.

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