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# Evaluating Scalable Bayesian Deep Learning Methods for Robust Computer Vision

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Fredrik K. Gustafsson

Uppsala University

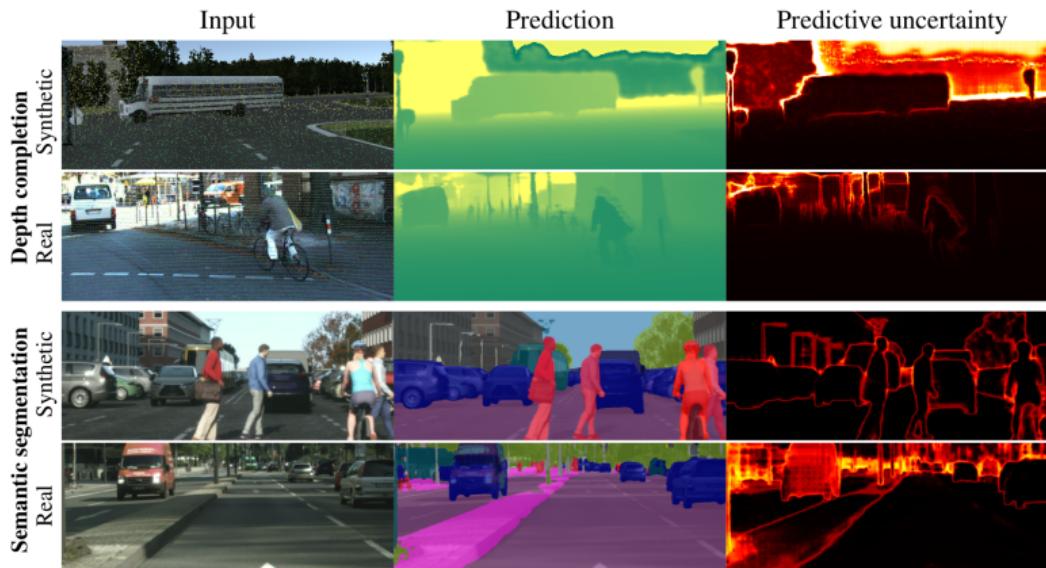
Zenuity

Göteborg, June 18, 2019

# Evaluating Scalable Bayesian Deep Learning Methods for Robust Computer Vision

Fredrik K. Gustafsson, Martin Danelljan (ETH Zurich), Thomas B. Schön (Uppsala University)

- arXiv: <https://arxiv.org/abs/1906.01620>
- Code: [https://github.com/fregu856/evaluating\\_bdl](https://github.com/fregu856/evaluating_bdl)
- These slides: <http://www.fregu856.com>



- Contributions:
  - We propose an **evaluation framework** for predictive uncertainty estimation that is specifically designed to test the robustness required in real-world vision applications.
  - We perform an **extensive comparison** of **ensembling** and **MC-dropout** on the tasks of *depth completion* and *street-scene semantic segmentation*.

1. Introduction
2. Predictive uncertainty estimation using Bayesian deep learning
3. Illustrative example
4. Ensembling as approximate Bayesian inference
5. Experiments
  - 5.1. Illustrative toy problems
  - 5.2. Depth completion
  - 5.3. Street-scene semantic segmentation
6. Discussion
7. Conclusion

# Outline

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# 1. Introduction

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- DNNs have become the go-to approach in computer vision, but generally fail to properly capture the **uncertainty** inherent in their predictions.
- Estimating this predictive uncertainty can be crucial, for instance in automotive and medical applications.

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- DNNs have become the go-to approach in computer vision, but generally fail to properly capture the **uncertainty** inherent in their predictions.
- Estimating this predictive uncertainty can be crucial, for instance in automotive and medical applications.
- **Bayesian deep learning** deals with predictive uncertainty by decomposing it into the distinct types of *aleatoric* and *epistemic* uncertainty.

# 1. Introduction - General setting

The task is to predict a target value  $y \in \mathcal{Y}$  given an input  $x \in \mathcal{X}$ . We are given a training set of i.i.d. sample pairs  $\mathcal{D} = \{X, Y\} = \{(x_i, y_i)\}_{i=1}^N$ ,  $(x_i, y_i) \sim p(x, y)$ .

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We view a DNN as a function  $f_\theta : \mathcal{X} \rightarrow \mathcal{U}$ , parameterized by  $\theta \in \mathbb{R}^P$ , that maps an input  $x \in \mathcal{X}$  to an output  $f_\theta(x) \in \mathcal{U}$ .

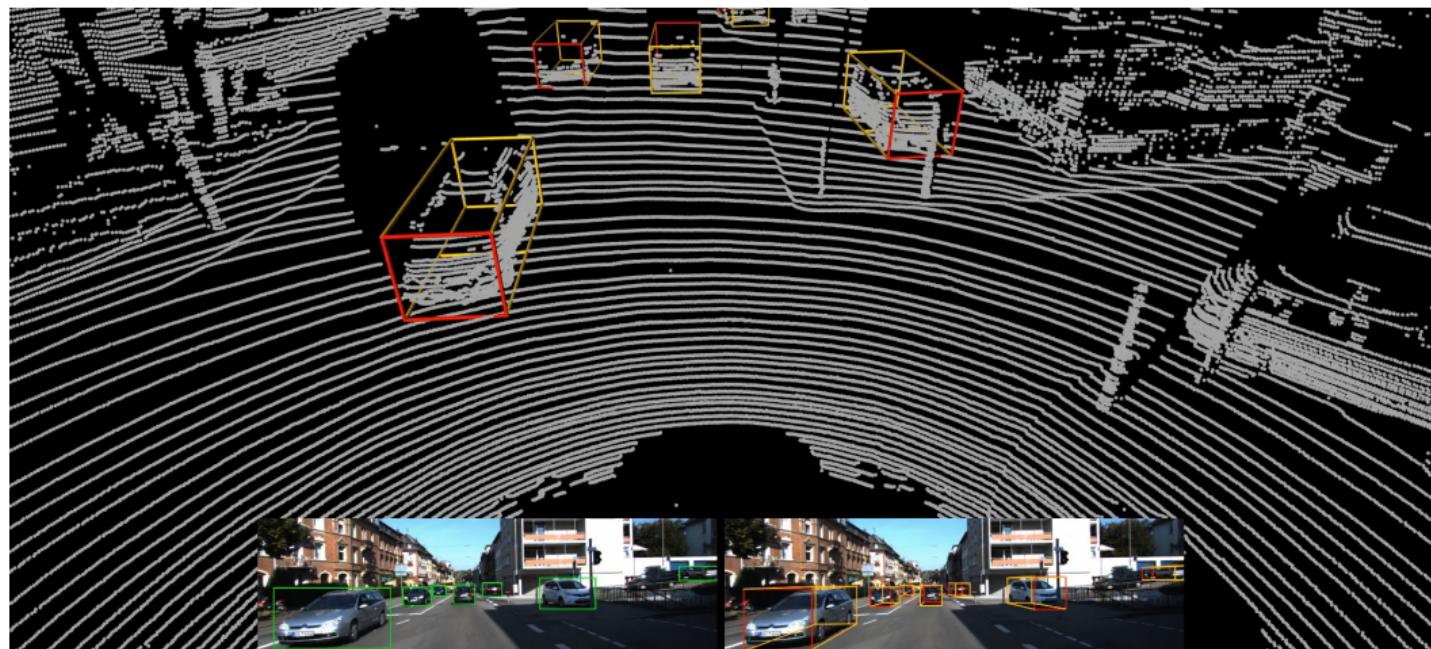
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- **Aleatoric** uncertainty captures this inherent and irreducible data uncertainty.
- *Input-dependent* aleatoric uncertainty is present whenever we expect the targets to be inherently more uncertain for some inputs.

# 1. Introduction - Aleatoric uncertainty

- This is true e.g. in 3D object detection, where the estimated location of distant objects generally is expected be more uncertain.



# 1. Introduction - Aleatoric uncertainty

- This is also true in semantic segmentation, where image pixels at object boundaries are inherently ambiguous.



# 1. Introduction - Epistemic uncertainty

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- **Epistemic** uncertainty accounts for uncertainty in the DNN model parameters.

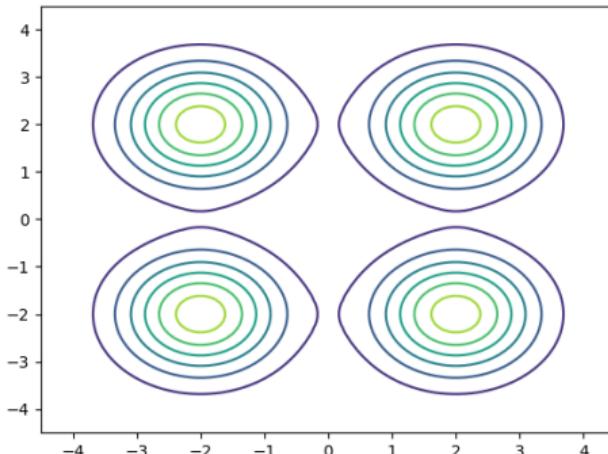
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- Given  $x^*$  at test time, the DNN then predicts the *distribution*  $p(y^*|x^*, \hat{\theta}_{\text{MLE}})$  over  $y^*$ , capturing aleatoric uncertainty.

## 2. Predictive uncertainty estimation using BDL - Aleatoric uncertainty

- In **classification**, a categorical model is commonly used:

$$p(y|x, \theta) = \text{Cat}(y; s_\theta(x)), \quad s_\theta(x) = \text{Softmax}(f_\theta(x)). \quad (1)$$

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- $-\log p(Y|X, \theta)$  corresponds to the following loss:

$$L(\theta) = \frac{1}{N} \sum_{i=1}^N \frac{(y_i - \mu_\theta(x_i))^2}{\sigma_\theta^2(x_i)} + \log \sigma_\theta^2(x_i).$$

## 2. Predictive uncertainty estimation using BDL - Epistemic uncertainty

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$$p(y^*|x^*, \mathcal{D}) = \int p(y^*|x^*, \theta)p(\theta|\mathcal{D})d\theta \approx \frac{1}{M} \sum_{i=1}^M p(y^*|x^*, \theta^{(i)}), \quad \theta^{(i)} \sim p(\theta|\mathcal{D}),$$

which captures both **aleatoric** and **epistemic** uncertainty.

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- In practice, an approximate posterior  $q(\theta) \approx p(\theta|\mathcal{D})$  has to be used, resulting in:

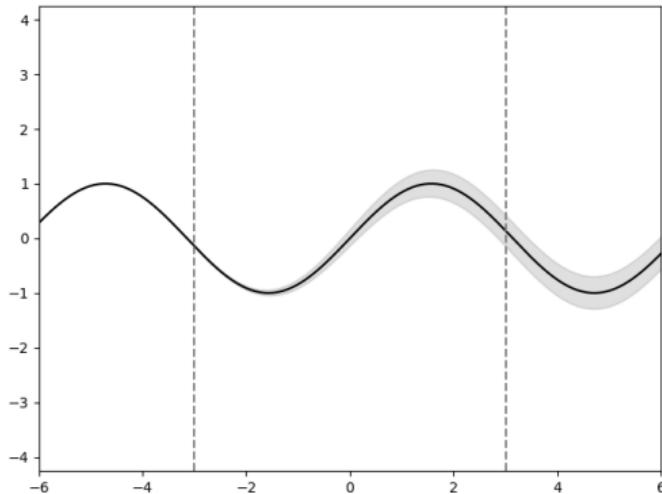
$$\hat{p}(y^*|x^*, \mathcal{D}) \triangleq \frac{1}{M} \sum_{i=1}^M p(y^*|x^*, \theta^{(i)}), \quad \theta^{(i)} \sim q(\theta). \quad (3)$$

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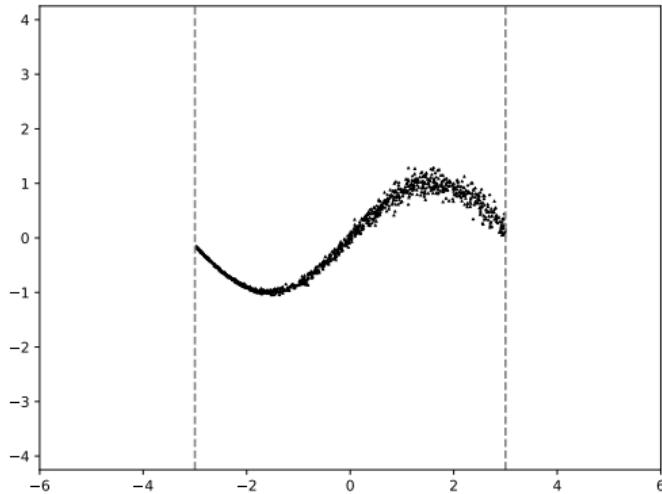
### 3. Illustrative example

We consider the following 1D regression problem:

$$y \sim \mathcal{N}(\mu(x), \sigma^2(x)), \quad \mu(x) = \sin(x), \quad \sigma(x) = \frac{0.15}{1 + e^{-x}}.$$



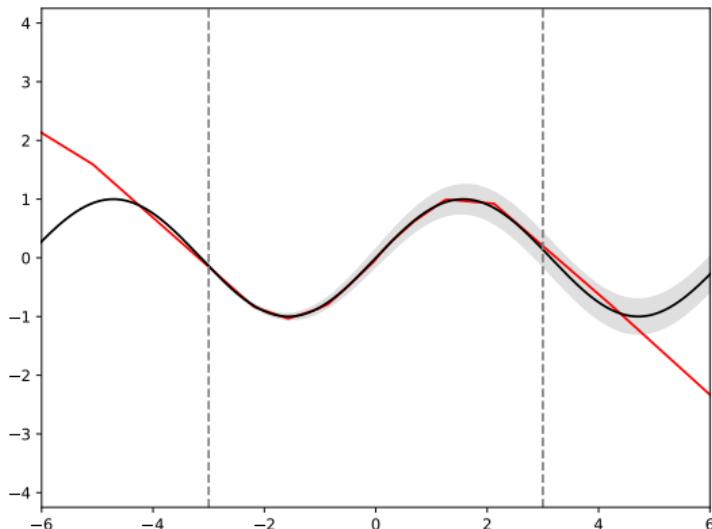
(a) True data generator.



(b) Training dataset.

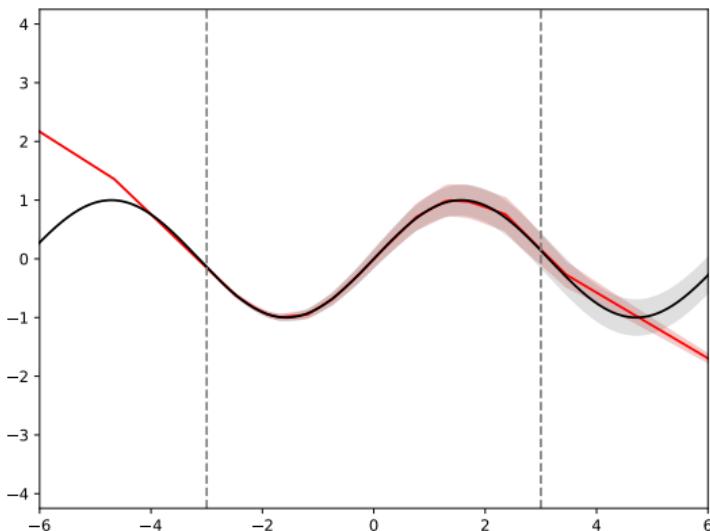
### 3. Illustrative example - Direct regression

- Following the conventional regression approach, a small DNN trained to directly predict targets,  $y^* = f_{\hat{\theta}}(x^*)$ , via the  $L^2$  loss is able to regress the mean for  $x^* \in [-3, 3]$ , but fails to capture any notion of uncertainty:



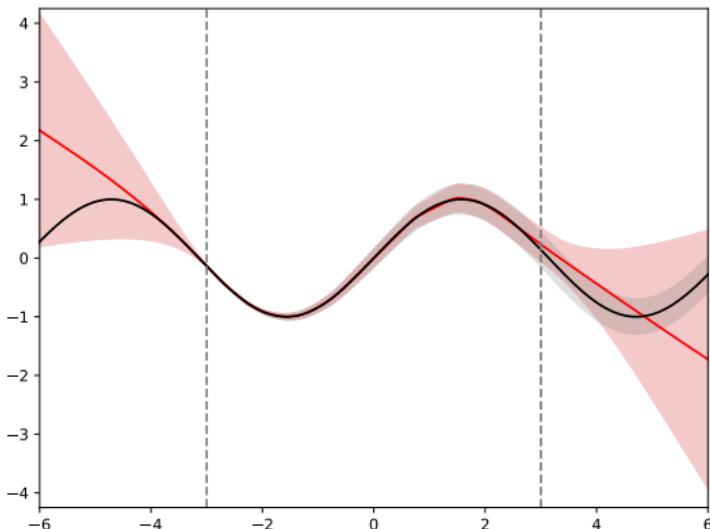
### 3. Illustrative example - Gaussian model, maximum-likelihood

- A corresponding Gaussian DNN model (2) trained via maximum-likelihood correctly accounts for aleatoric uncertainty, but generates overly confident predictions for inputs  $|x^*| > 3$  not seen during training:



### 3. Illustrative example - Gaussian model, approximate Bayesian inference

- A Gaussian DNN model trained via approximate Bayesian inference (3), with  $M = 1\,000$  samples obtained via HMC [11], is additionally able to predict more reasonable uncertainties in the region where no training data was available:



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## 4. Ensembling as approximate Bayesian inference

**Ensembling:** create a parametric model  $p(y|x, \theta)$  using a DNN  $f_\theta$ , learn  $M$  point estimates  $\{\hat{\theta}^{(m)}\}_{m=1}^M$  by repeatedly minimizing  $-\log p(Y|X, \theta)$  with *random initialization*, and average over the models to obtain the predictive distribution:

$$\hat{p}(y^*|x^*) \triangleq \frac{1}{M} \sum_{m=1}^M p(y^*|x^*, \hat{\theta}^{(m)}). \quad (4)$$

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- Since  $\{\hat{\theta}^{(m)}\}_{m=1}^M$  always can be seen as samples from *some* distribution  $\hat{q}(\theta)$ , we note that (4) and (5) are virtually identical.

## 4. Ensembling as approximate Bayesian inference

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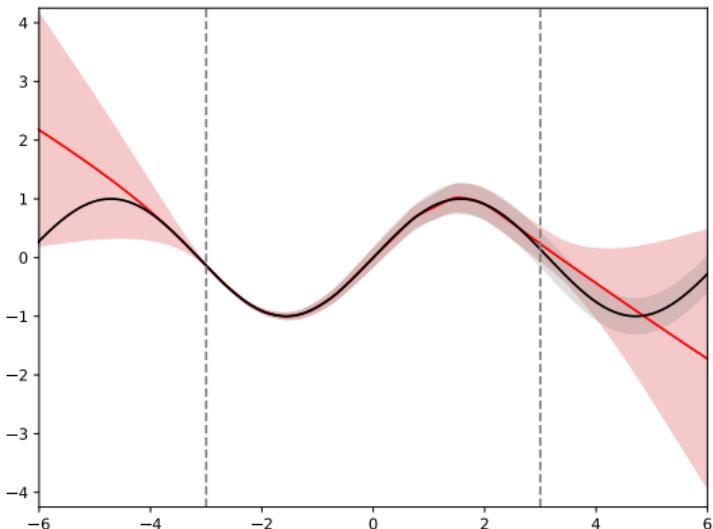
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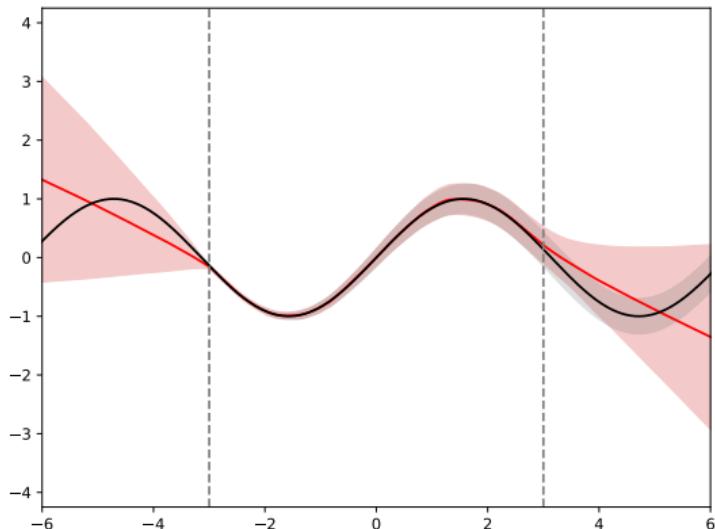
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- Also, by minimizing  $-\log p(Y|X, \theta)$  multiple times using SGD, starting from **randomly chosen** initial points, we are likely to find many different local optima.
- Ensembling can thus generate a compact set of samples  $\{\hat{\theta}^{(m)}\}_{m=1}^M$  that captures the important aspect of multi-modality in  $p(\theta|\mathcal{D})$ .

## 4. Ensembling as approximate Bayesian inference - Illustrative example

- On the 1D regression problem, we observe that ensembling provides reasonable approximations to HMC [11], even for relatively small values of  $M$ :



(a) HMC [11],  $M = 1\,000$ .



(b) Ensembling,  $M = 16$ .

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- Contributions:
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  - We perform an **extensive comparison** of **ensembling** and **MC-dropout** on the tasks of *depth completion* and *street-scene semantic segmentation*.

**MC-dropout:** simple and scalable method for epistemic uncertainty estimation. Entails using *dropout* also at test time and averaging  $M$  stochastic forward passes on the same input. Can be interpreted as performing variational inference [5, 8].

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- Since images captured in these different circumstances could all represent distinctly different regions of the vast input image space, it is infeasible to ensure that all encountered inputs will be well-represented by the training data. Thus, we argue that robustness to **out-of-domain inputs** is crucial.
- To simulate these challenging conditions and test the required robustness, we train models exclusively on **synthetic data** and evaluate the predictive uncertainty on **real-world data**.

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- To improve rigour of our evaluation, we repeat each experiment multiple times and report results together with the observed variation.

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- We evaluate the methods by quantitatively measuring how well the obtained predictive distributions approximate that of HMC [11] with  $M = 1\,000$  samples and prior  $p(\theta) = \mathcal{N}(0, I_P)$ .

## 5.1. Illustrative toy problems

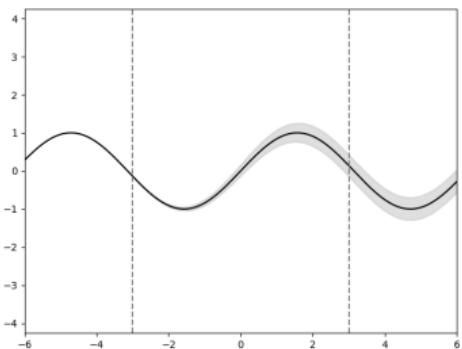
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- We thus take as our metric the **KL divergence**  $D_{\text{KL}}(p \parallel p_{\text{HMC}})$  with respect to this target predictive distribution  $p_{\text{HMC}}$ .

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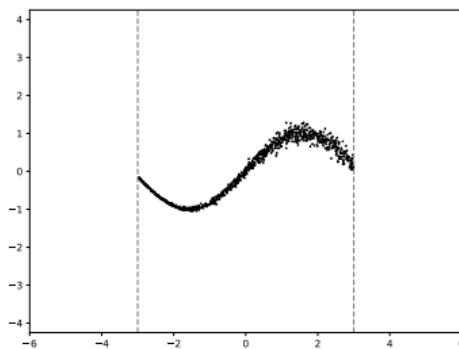
- First, we conduct experiments on **illustrative toy problems** to gain insights into how ensembling and MC-dropout fare against other approximate Bayesian inference methods.
- We compare with SGLD [13] and SGHMC [2], which are both Stochastic Gradient MCMC (SG-MCMC) [10] methods.
- We evaluate the methods by quantitatively measuring how well the obtained predictive distributions approximate that of HMC [11] with  $M = 1\,000$  samples and prior  $p(\theta) = \mathcal{N}(0, I_P)$ .
- We thus take as our metric the **KL divergence**  $D_{\text{KL}}(p \parallel p_{\text{HMC}})$  with respect to this target predictive distribution  $p_{\text{HMC}}$ .
- Note that HMC is considered a “gold standard” method for approximate Bayesian inference, but does not scale to large DNNs or large-scale datasets.

## 5.1. Illustrative toy problems - Regression

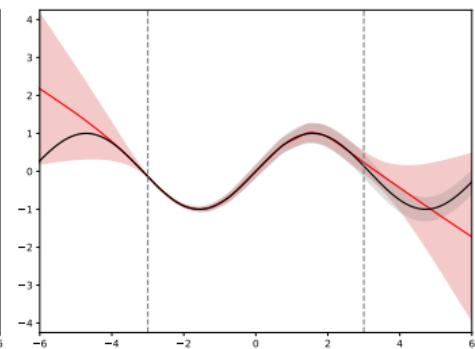
- For **regression**, we study the previously defined 1D problem:



(a) True data generator.



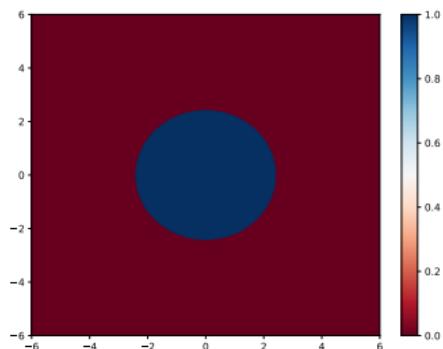
(b) Training dataset.



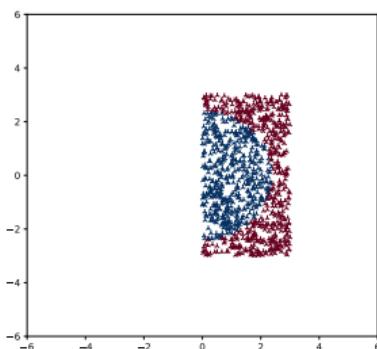
(c) HMC [11] “ground truth”.

## 5.1. Illustrative toy problems - Classification

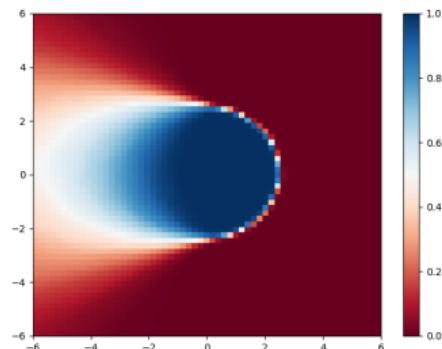
- For **classification**, we study the following binary classification problem:



(a) True data generator.

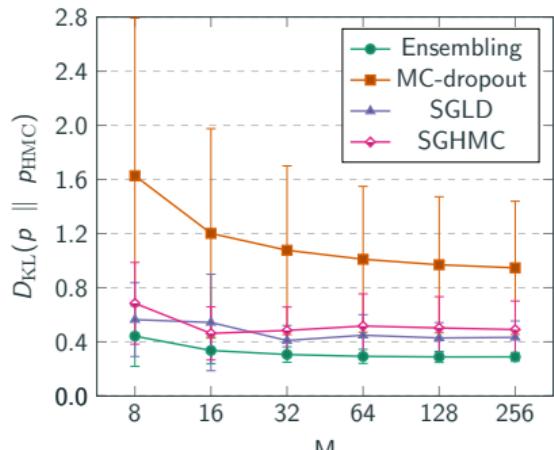


(b) Training dataset.

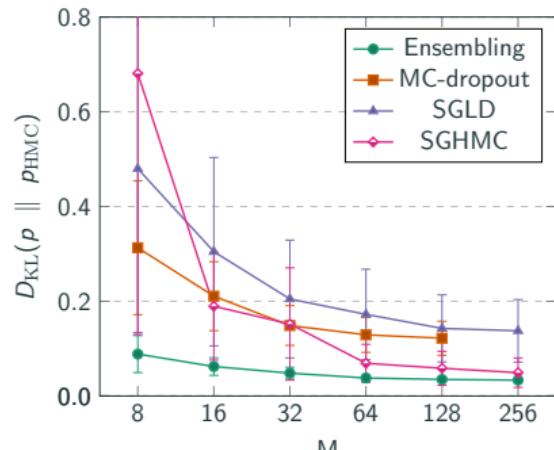


(c) HMC [11] “ground truth”.

## 5.1. Illustrative toy problems - Results



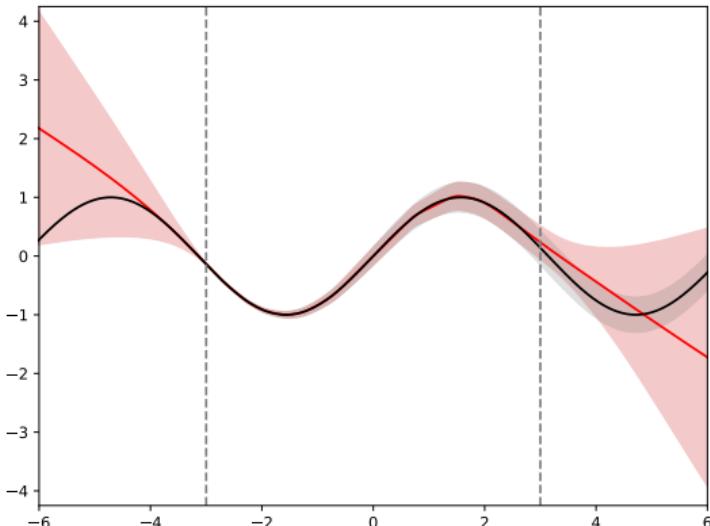
(a) Regression.



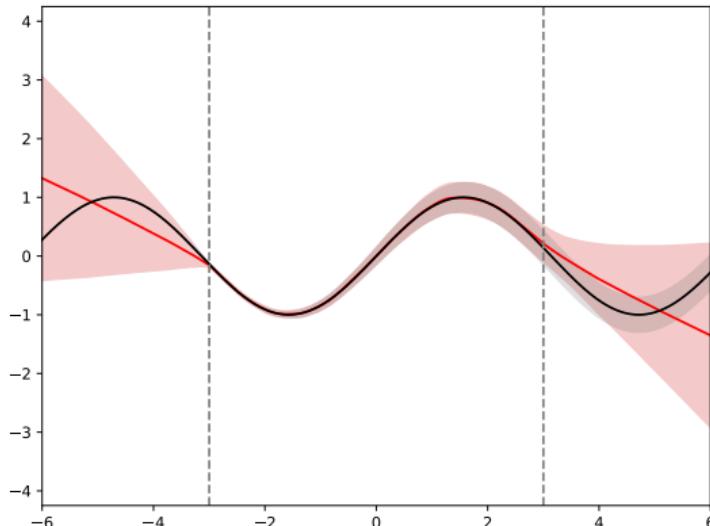
(b) Classification.

- We observe that **ensembling** consistently outperforms the compared methods, and MC-dropout in particular.

## 5.1. Illustrative toy problems - Qualitative results



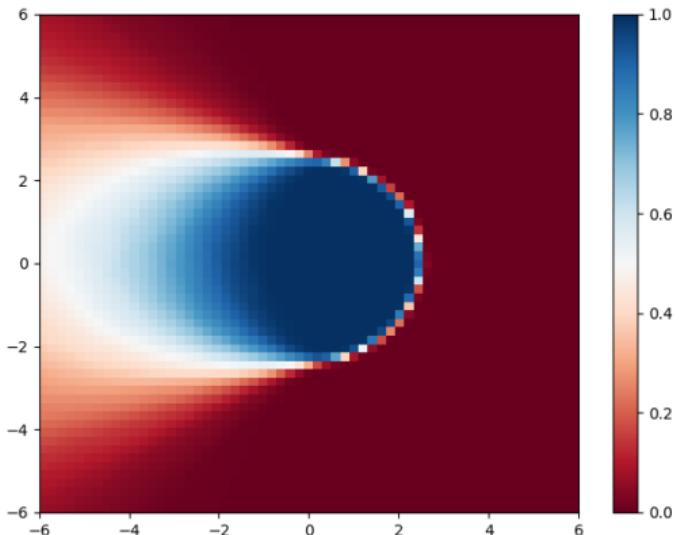
(a) HMC [11],  $M = 1\,000$ .



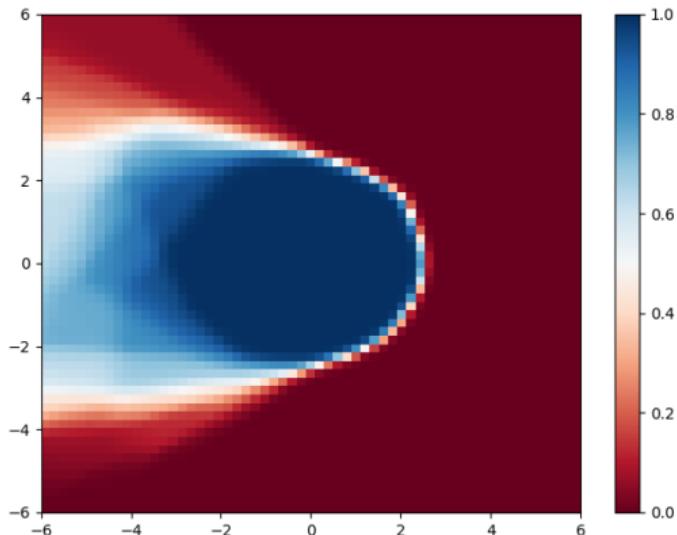
(b) Ensembling,  $M = 16$ .

- We observe that **ensembling** provides reasonable approximations to HMC [11], even for relatively small values of  $M$ .

## 5.1. Illustrative toy problems - Qualitative results



(a) HMC [11],  $M = 1\,000$ .

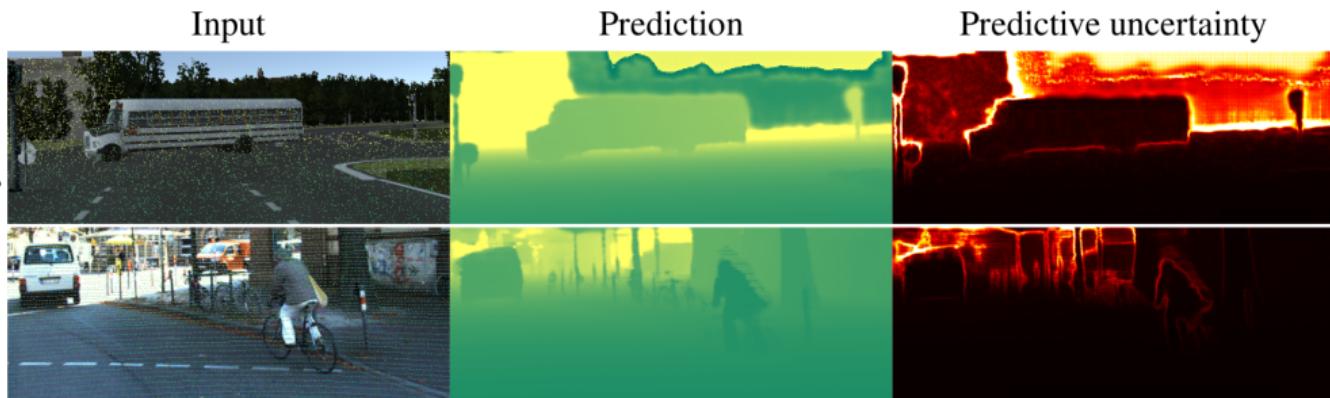


(b) Ensembling,  $M = 16$ .

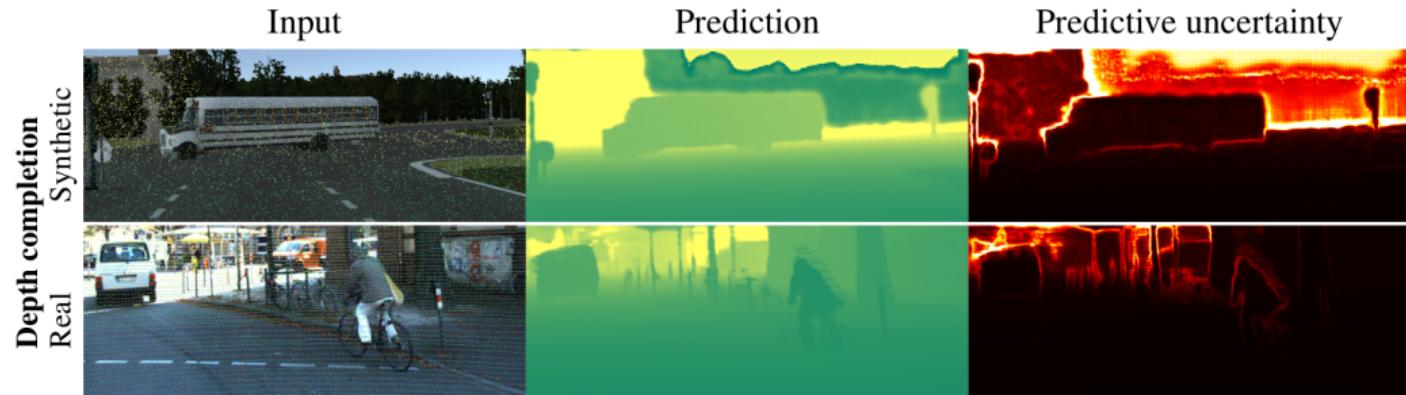
- We observe that **ensembling** provides reasonable approximations to HMC [11], even for relatively small values of  $M$ .

## 5.2. Depth completion

Depth completion  
Synthetic  
Real

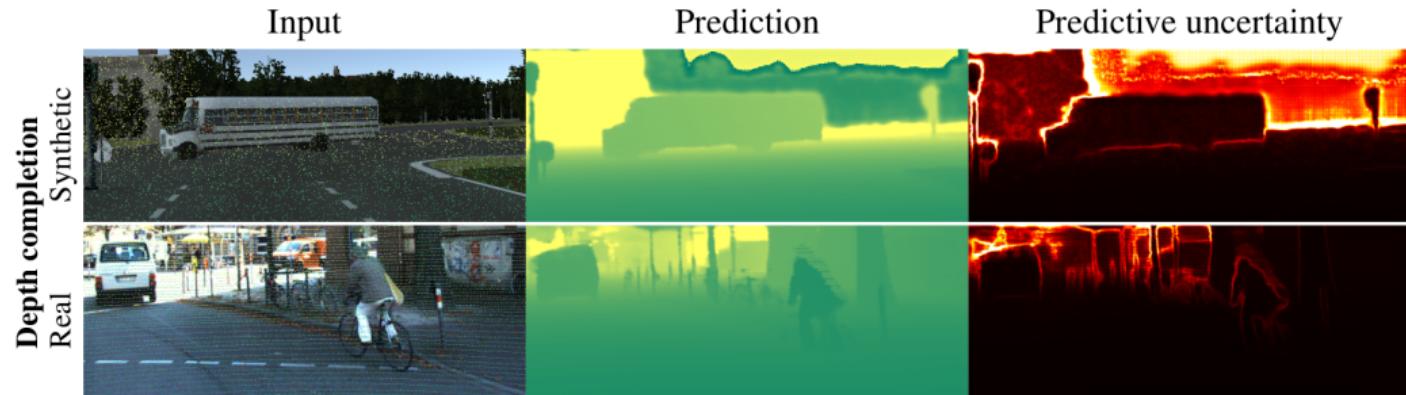


## 5.2. Depth completion



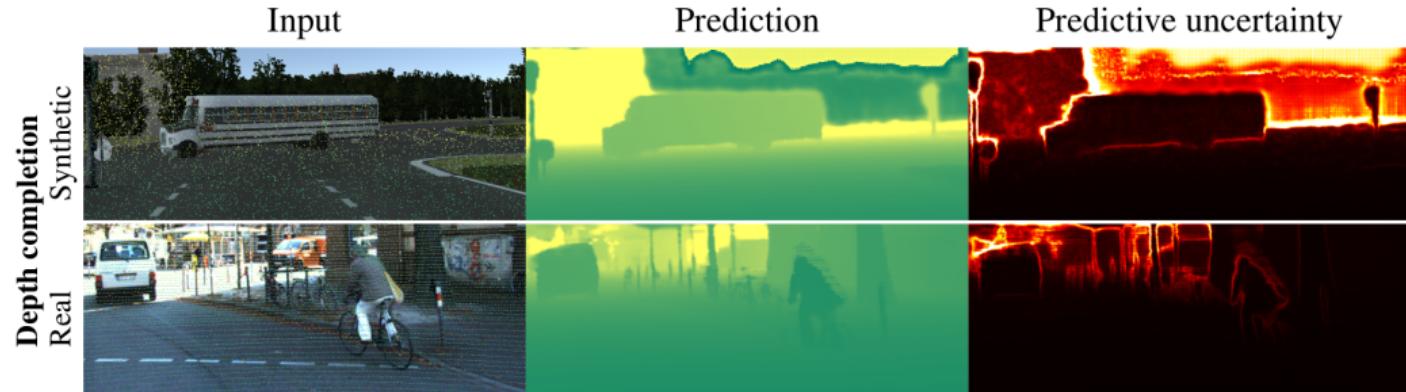
- In **depth completion**, we are given an image  $x_{\text{img}} \in \mathbb{R}^{h \times w \times 3}$  and an associated *sparse* depth map  $x_{\text{sparse}} \in \mathbb{R}^{h \times w}$ . Only non-zero pixels of  $x_{\text{sparse}}$  correspond to LiDAR depth measurements, projected onto the image plane.

## 5.2. Depth completion



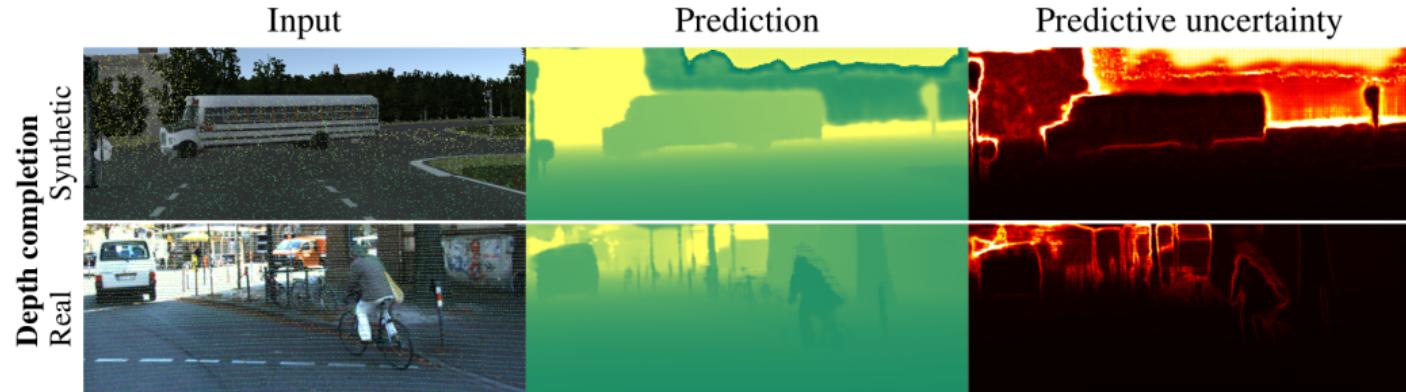
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- The goal is to predict a dense depth map  $y \in \mathbb{R}^{h \times w}$  of the scene.

## 5.2. Depth completion - Datasets



- We utilize the **KITTI depth completion** [6, 12] and **Virtual KITTI** [4] datasets.

## 5.2. Depth completion - Datasets



- We utilize the **KITTI depth completion** [6, 12] and **Virtual KITTI** [4] datasets.
- We train on Virtual KITTI (18 930 examples) and evaluate on KITTI depth completion (1 000 validation examples).

- We use the DNN model presented by Ma *et al.* [9].

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- The inputs  $x_{\text{img}}$ ,  $x_{\text{sparse}}$  are separately processed by initial convolutional layers, concatenated and fed to an encoder-decoder architecture based on ResNet34.

- We use the DNN model presented by Ma *et al.* [9].
- The inputs  $x_{\text{img}}$ ,  $x_{\text{sparse}}$  are separately processed by initial convolutional layers, concatenated and fed to an encoder-decoder architecture based on ResNet34.
- We employ the **Gaussian model (2)** by duplicating the final convolutional layer, outputting  $\mu \in \mathbb{R}^{h \times w}$  and  $\log \sigma^2 \in \mathbb{R}^{h \times w}$  instead of the plain depth  $\hat{y} \in \mathbb{R}^{h \times w}$ .

## 5.2. Depth completion - Evaluation metrics

- We evaluate the methods in terms of quality of the estimated predictive uncertainty, as measured by the *relative AUSE* metric [7] and the *absolute* measure of uncertainty **calibration**.

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- **Calibration**: our model outputs a Gaussian for each pixel, and we can thus construct prediction intervals of confidence level  $p \in ]0, 1[$  using the corresponding quantiles.

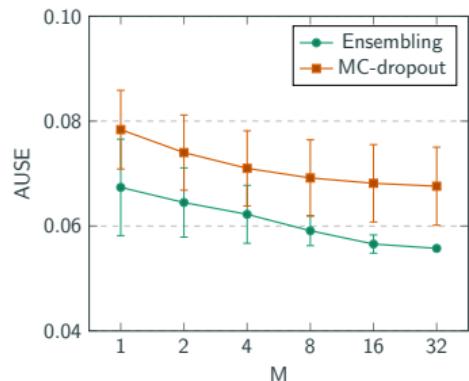
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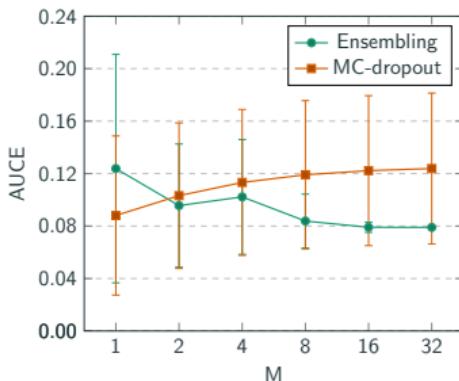
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- **Calibration**: our model outputs a Gaussian for each pixel, and we can thus construct prediction intervals of confidence level  $p \in ]0, 1[$  using the corresponding quantiles. The proportion of pixels for which the prediction interval covers the target is expected to equal  $p \in ]0, 1[$  for a perfectly calibrated model.
- We also evaluate in terms of the standard RMSE metric.

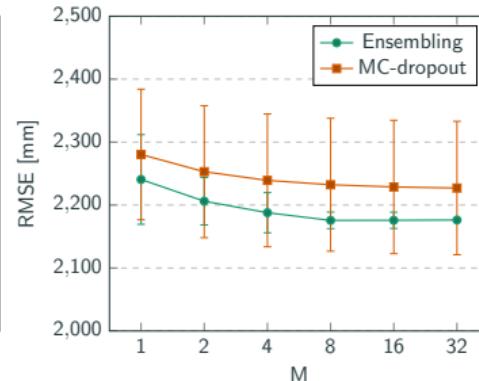
## 5.2. Depth completion - Results



(a) AUSE, lower is better.

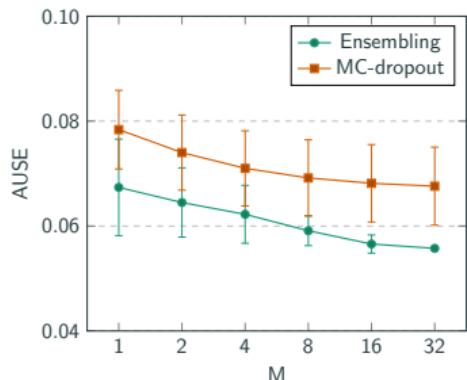


(b) Calibration (AUCE), lower is better.

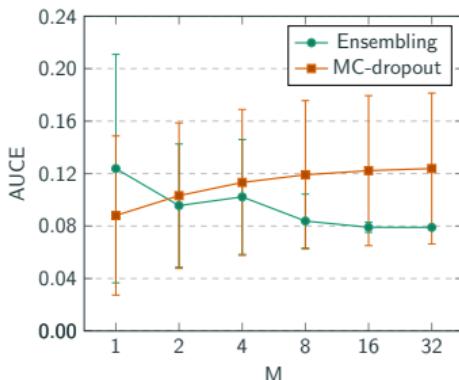


(c) RMSE, lower is better.

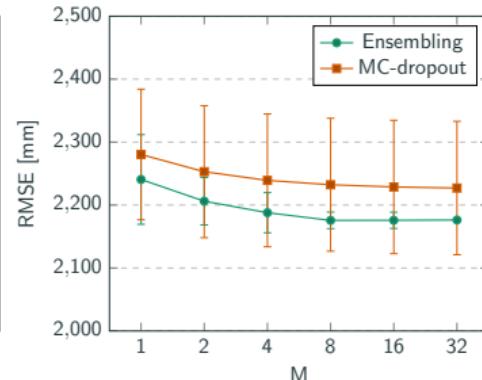
## 5.2. Depth completion - Results



(a) AUSE, lower is better.



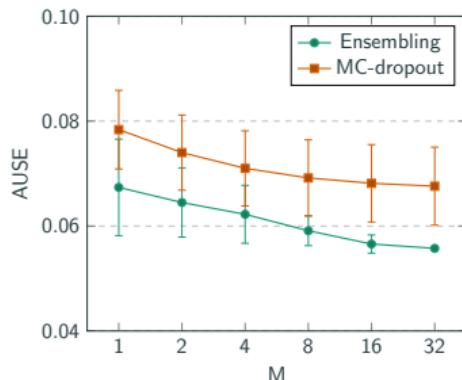
(b) Calibration (AUCE), lower is better.



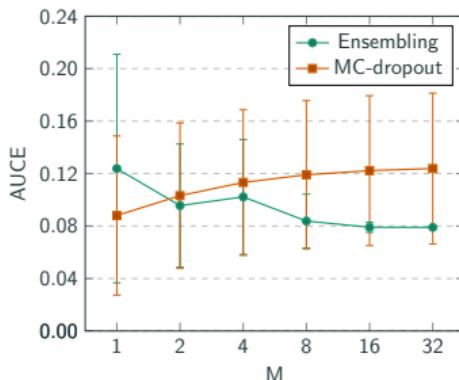
(c) RMSE, lower is better.

- We observe in (a) that ensembling consistently outperforms MC-dropout in terms of AUSE. However, the curves decrease as a function of  $M$  in a similar manner.

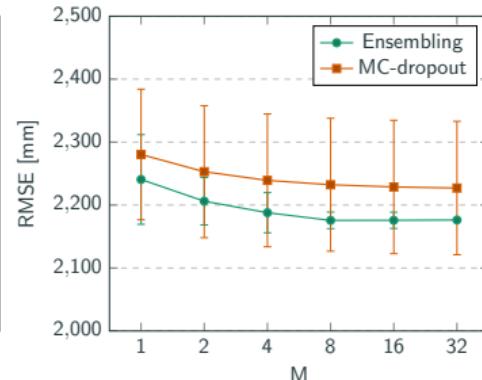
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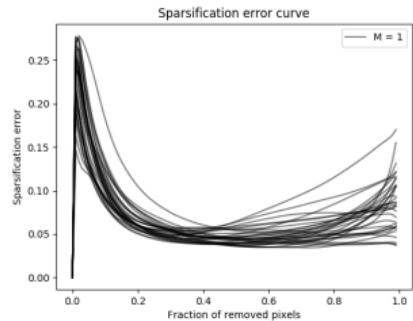
(b) Calibration (AUCE), lower is better.



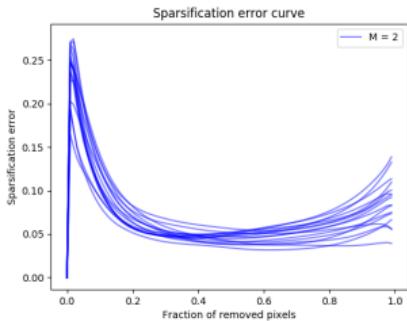
(c) RMSE, lower is better.

- We observe in (a) that ensembling consistently outperforms MC-dropout in terms of AUSE. However, the curves decrease as a function of  $M$  in a similar manner.
- A ranking can be more readily conducted based on (b), where we observe a clearly improving trend for **ensembling**, whereas MC-dropout gets progressively worse.

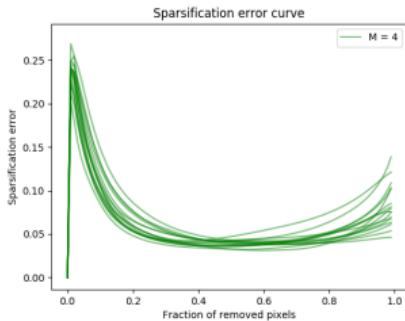
## 5.2. Depth completion - Results, sparsification, ensembling



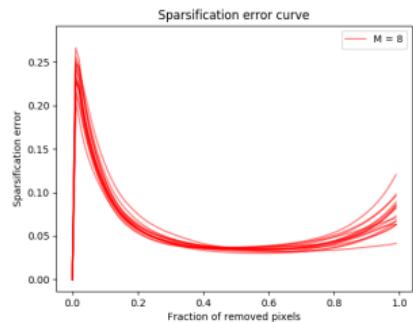
(a)  $M = 1$ .



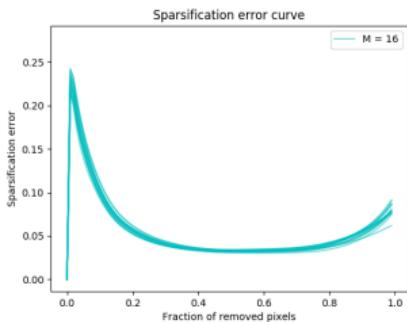
(b)  $M = 2$ .



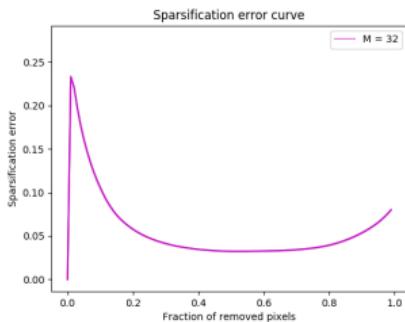
(c)  $M = 4$ .



(d)  $M = 8$ .

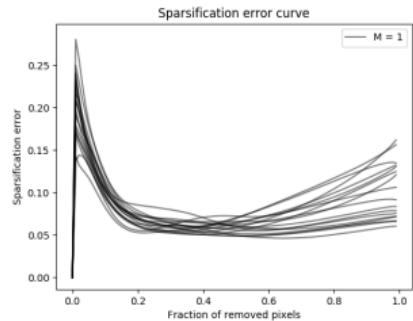


(e)  $M = 16$ .

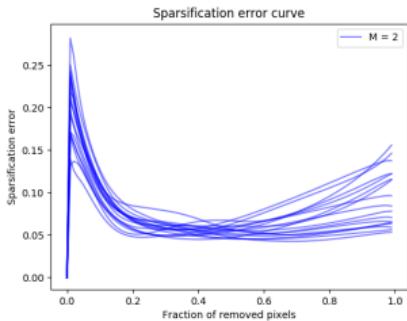


(f)  $M = 32$ .

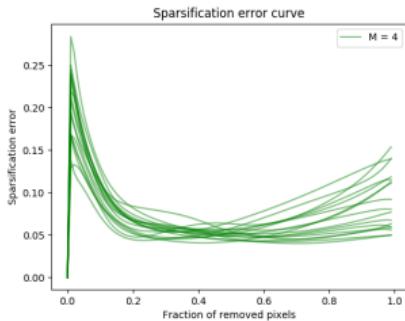
## 5.2. Depth completion - Results, sparsification, MC-dropout



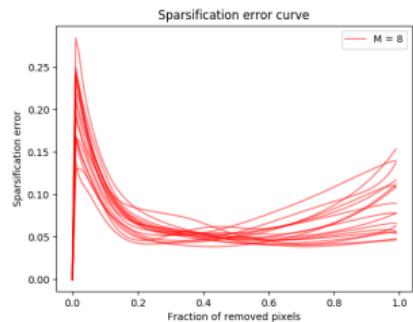
(a)  $M = 1$ .



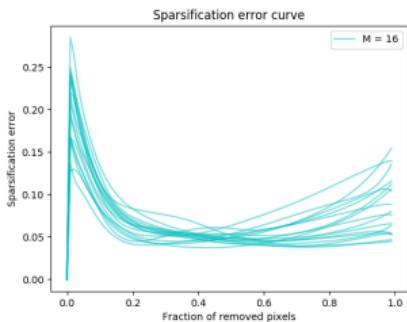
(b)  $M = 2$ .



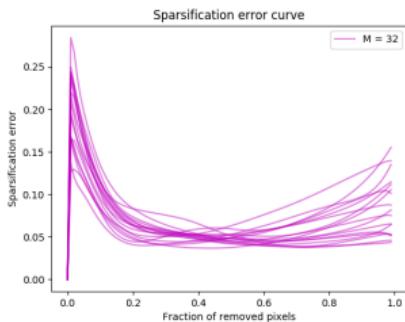
(c)  $M = 4$ .



(d)  $M = 8$ .

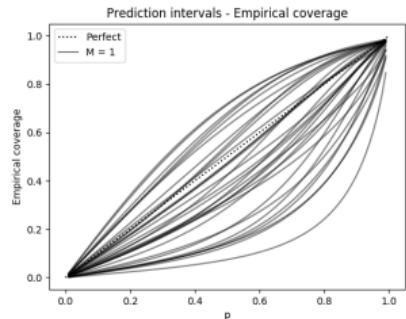


(e)  $M = 16$ .

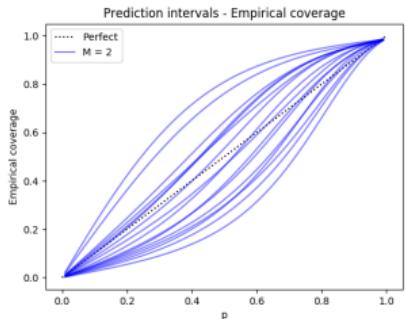


(f)  $M = 32$ .

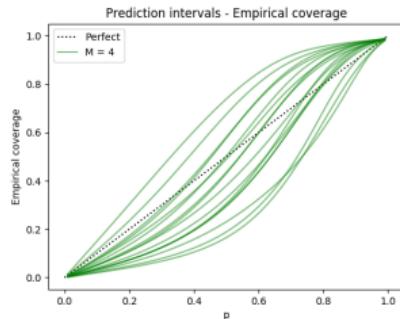
## 5.2. Depth completion - Results, calibration, ensembling



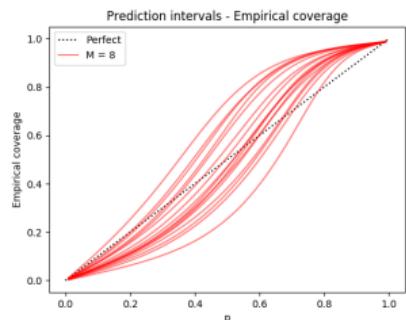
(a)  $M = 1.$



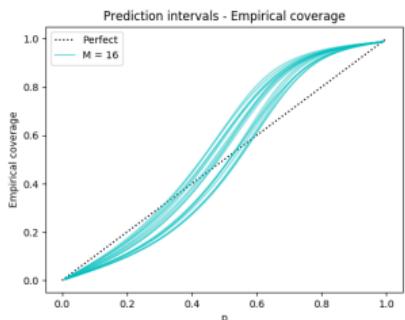
(b)  $M = 2.$



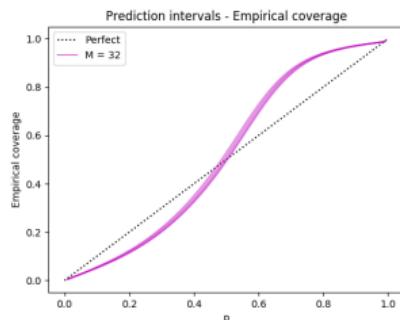
(c)  $M = 4.$



(d)  $M = 8.$

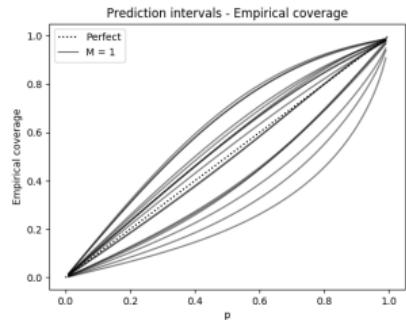


(e)  $M = 16.$

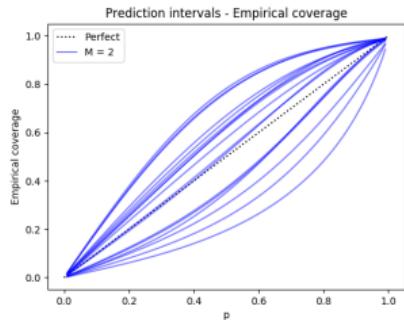


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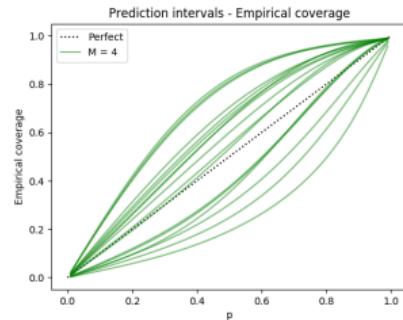
## 5.2. Depth completion - Results, calibration, MC-dropout



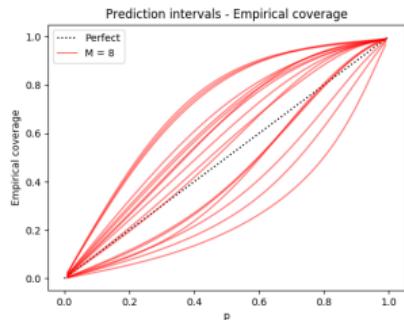
(a)  $M = 1$ .



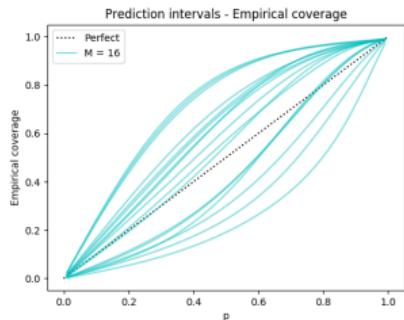
(b)  $M = 2$ .



(c)  $M = 4$ .

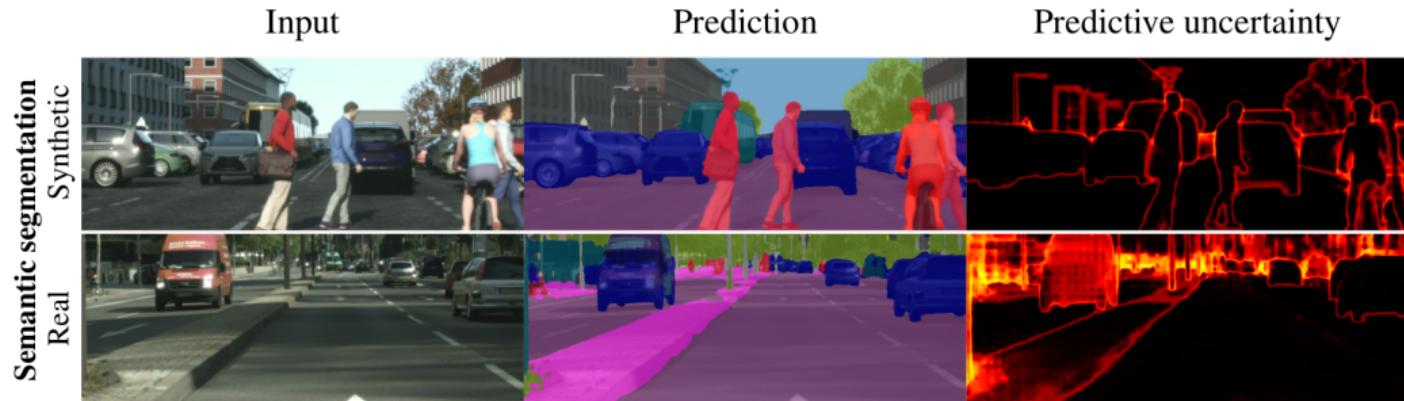


(d)  $M = 8$ .

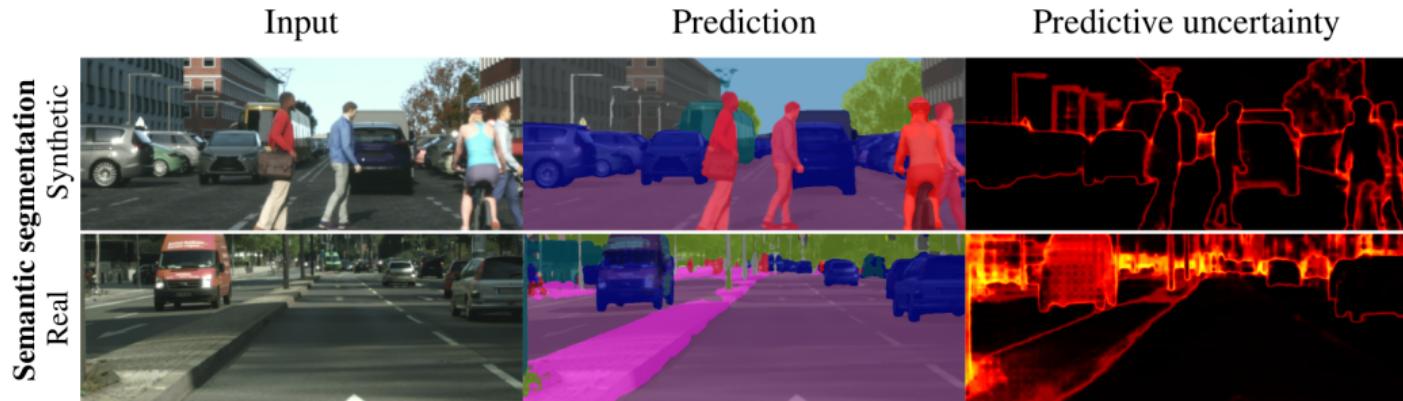


(e)  $M = 16$ .

## 5.3. Street-scene semantic segmentation

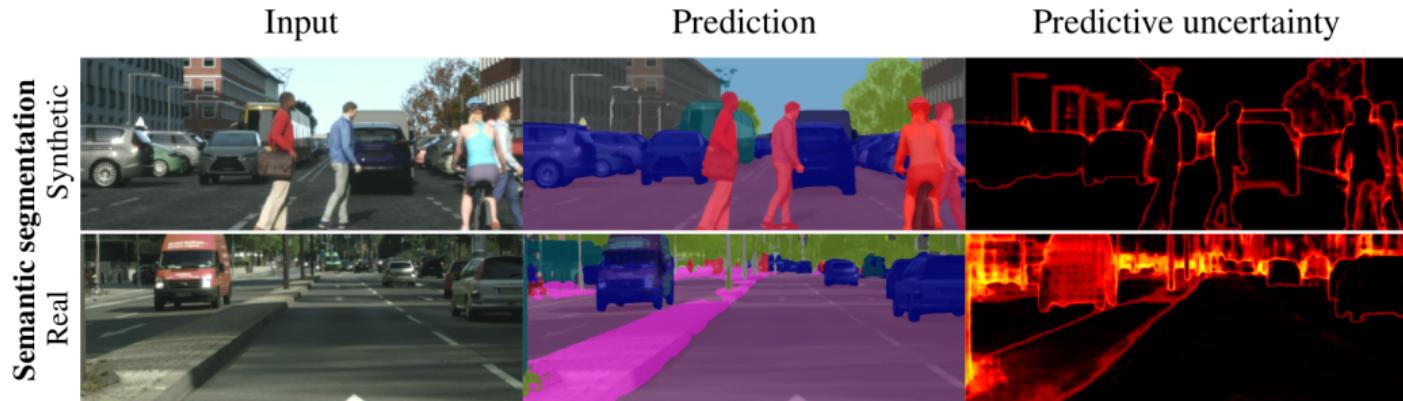


## 5.3. Street-scene semantic segmentation



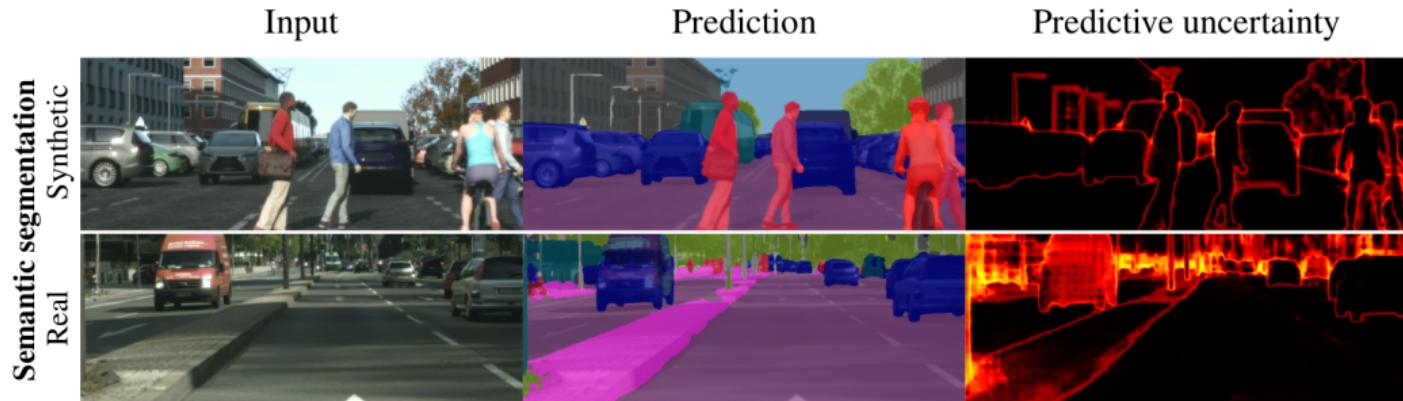
- In **street-scene semantic segmentation**, we are given an image  $x \in \mathbb{R}^{h \times w \times 3}$ .
- The goal is to predict  $y$  of size  $h \times w$ , in which each pixel is assigned to one of  $C$  different class labels (road, sidewalk, car, etc.).

## 5.3. Street-scene semantic segmentation - Datasets



- We utilize the **Cityscapes** [3] and **Synscapes** [14] datasets.

## 5.3. Street-scene semantic segmentation - Datasets



- We utilize the **Cityscapes** [3] and **Synscapes** [14] datasets.
- We train on Synscapes (2 975 examples) and evaluate on Cityscapes (500 validation examples).

## 5.3. Street-scene semantic segmentation - Model

- We use the DeepLabv3 DNN model presented by Chen *et al.* [1].

## 5.3. Street-scene semantic segmentation - Model

- We use the DeepLabv3 DNN model presented by Chen *et al.* [1].
- The input image  $x$  is processed by a ResNet101 and then fed to an ASPP module, outputting logits at  $1/8$  of the original resolution. These are then upsampled to image resolution using bilinear interpolation.

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- The input image  $x$  is processed by a ResNet101 and then fed to an ASPP module, outputting logits at  $1/8$  of the original resolution. These are then upsampled to image resolution using bilinear interpolation.
- The conventional **Categorical model (1)** is thus used for each pixel.

## 5.3. Street-scene semantic segmentation - Evaluation metrics

- We evaluate the methods in terms of quality of the estimated predictive uncertainty, as measured by the *relative AUSE* metric [7] and the *absolute* measure of uncertainty **calibration**.

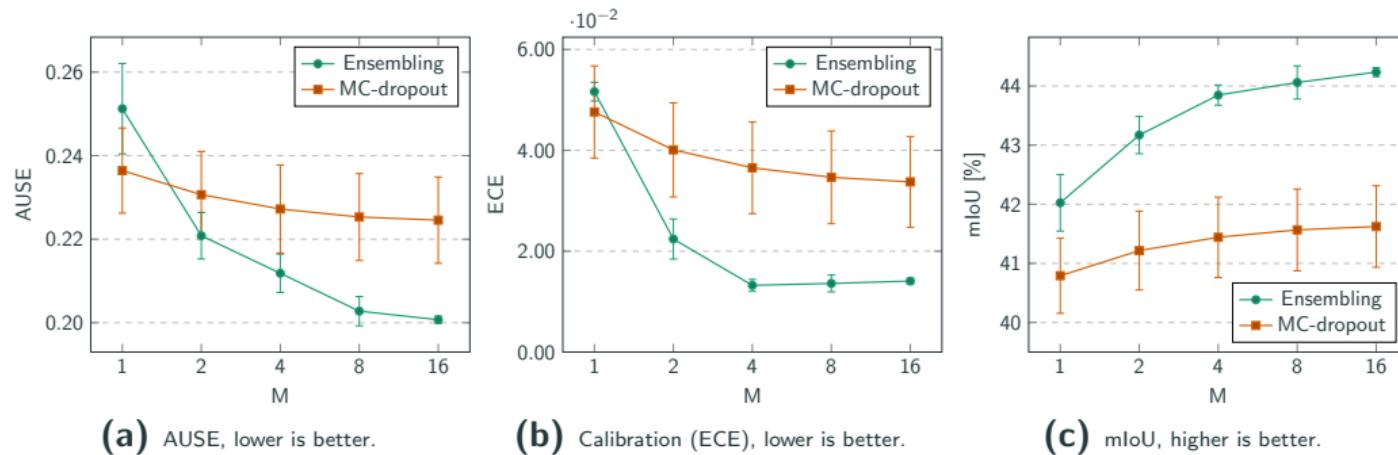
## 5.3. Street-scene semantic segmentation - Evaluation metrics

- We evaluate the methods in terms of quality of the estimated predictive uncertainty, as measured by the *relative AUSE* metric [7] and the *absolute* measure of uncertainty **calibration**.
- **Calibration:** all predictions are partitioned into  $L$  bins based on the maximum assigned confidence. For each bin, the difference between the average predicted confidence and the actual accuracy is then computed, and ECE (*Expected Calibration Error*) is obtained as the weighted average of these differences.

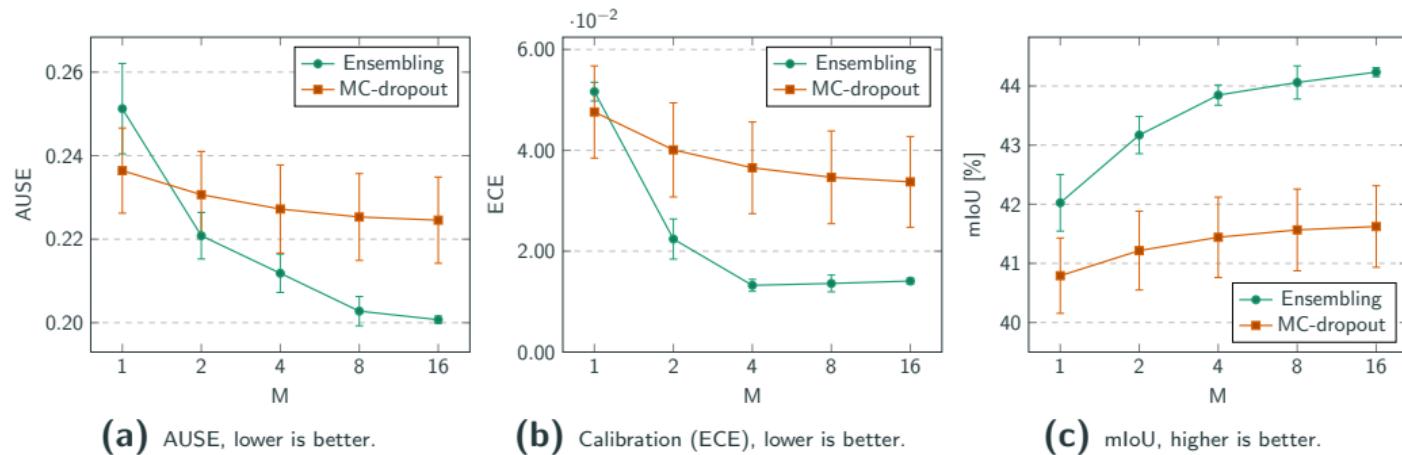
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- We also evaluate in terms of the standard mIoU metric.

## 5.3. Street-scene semantic segmentation - Results

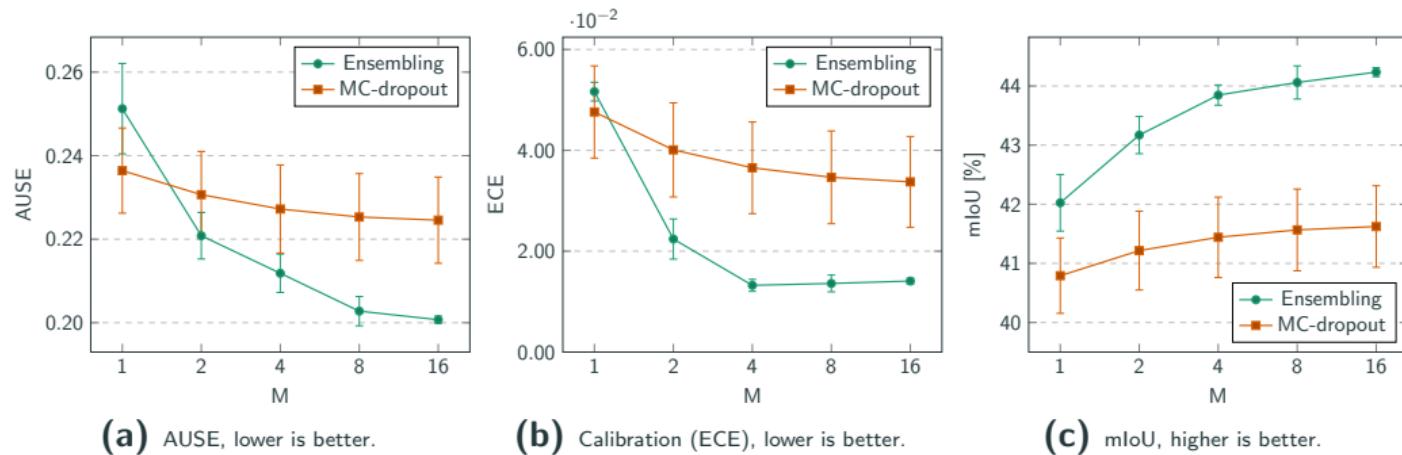


## 5.3. Street-scene semantic segmentation - Results



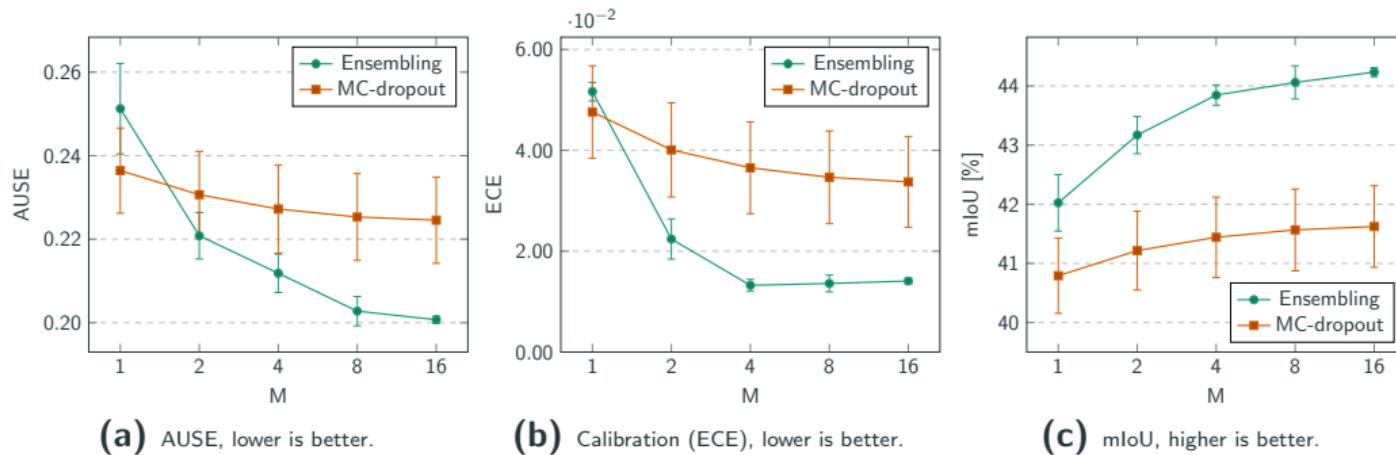
- Note that  $M = 1$  corresponds to the baseline of only estimating aleatoric uncertainty.

## 5.3. Street-scene semantic segmentation - Results



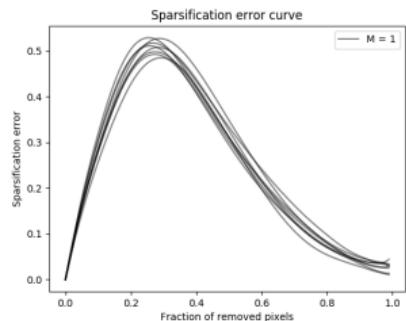
- Note that  $M = 1$  corresponds to the baseline of only estimating aleatoric uncertainty. The metrics clearly improve as functions of  $M$  for both methods, demonstrating the importance of epistemic uncertainty estimation.

## 5.3. Street-scene semantic segmentation - Results

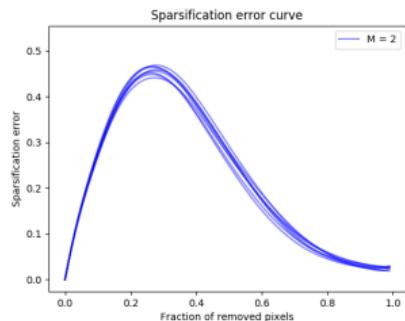


- Note that  $M = 1$  corresponds to the baseline of only estimating aleatoric uncertainty. The metrics clearly improve as functions of  $M$  for both methods, demonstrating the importance of epistemic uncertainty estimation.
- We observe that the rate of improvement is generally greater for **ensembling**.

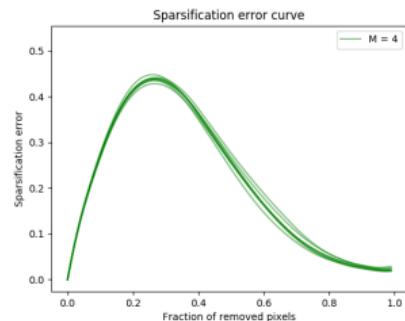
## 5.3. Semantic segmentation - Results, sparsification, ensembling



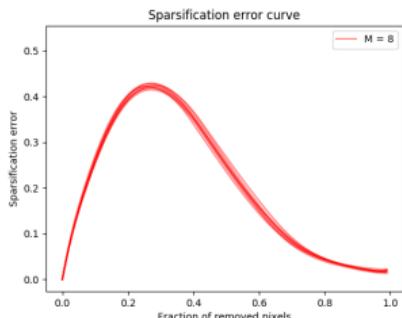
(a)  $M = 1$ .



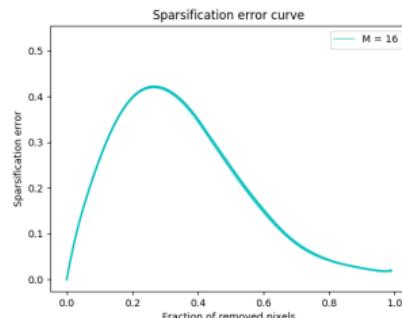
(b)  $M = 2$ .



(c)  $M = 4$ .

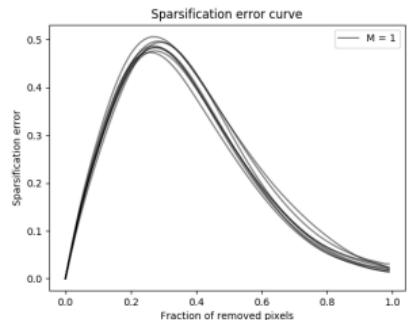


(d)  $M = 8$ .

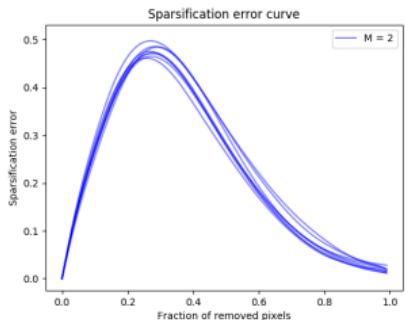


(e)  $M = 16$ .

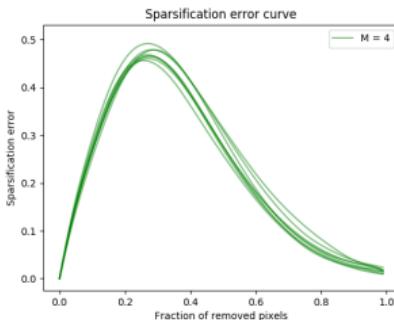
## 5.3. Semantic segmentation - Results, sparsification, MC-dropout



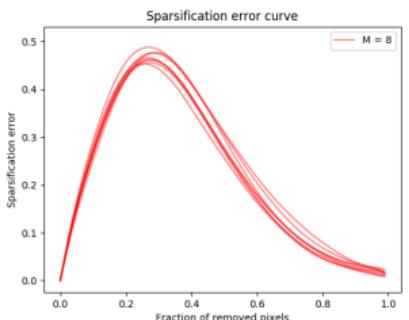
(a)  $M = 1$ .



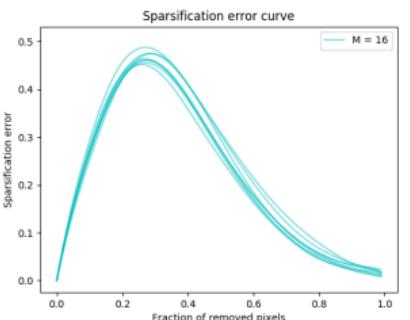
(b)  $M = 2$ .



(c)  $M = 4$ .

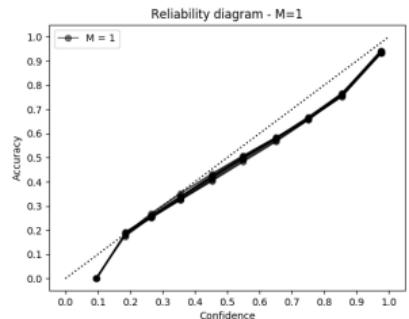


(d)  $M = 8$ .

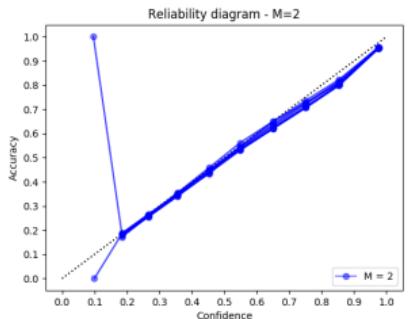


(e)  $M = 16$ .

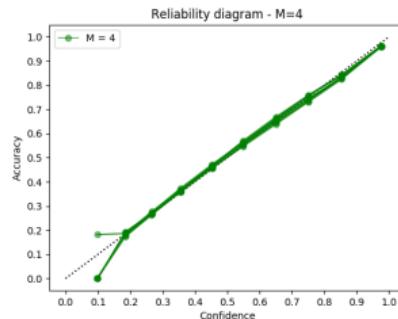
## 5.3. Semantic segmentation - Results, calibration, ensembling



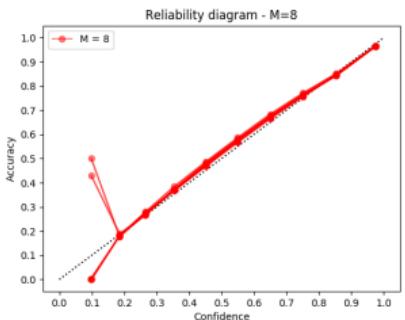
(a)  $M = 1$ .



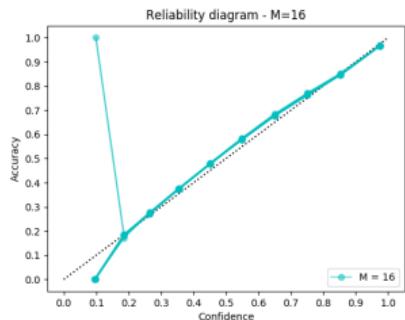
(b)  $M = 2$ .



(c)  $M = 4$ .

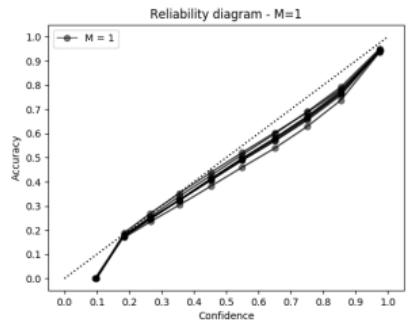


(d)  $M = 8$ .

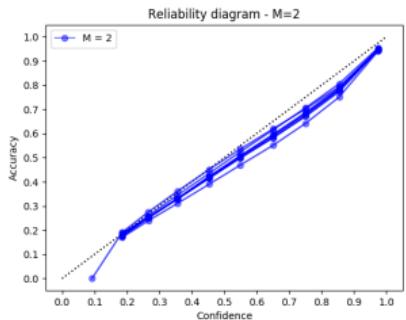


(e)  $M = 16$ .

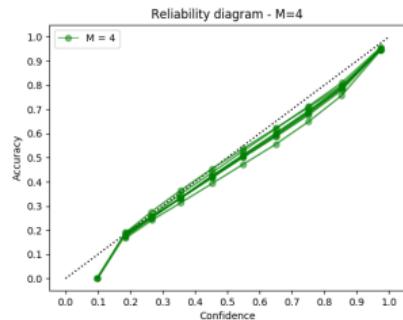
## 5.3. Semantic segmentation - Results, calibration, MC-dropout



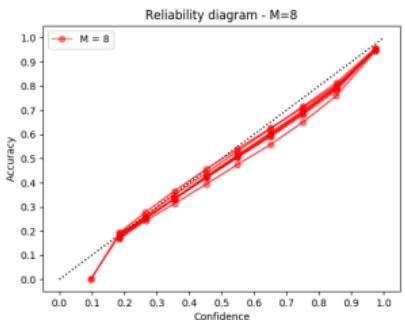
(a)  $M = 1$ .



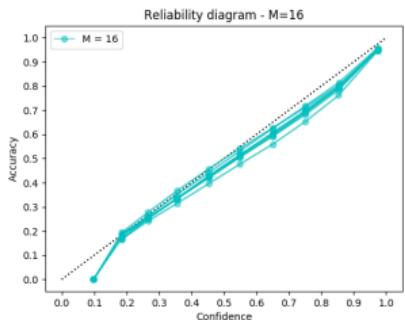
(b)  $M = 2$ .



(c)  $M = 4$ .



(d)  $M = 8$ .



(e)  $M = 16$ .

- **Video:** <https://youtu.be/CabPVqtzs0I>.
- All shown results are for **ensembling** with  $M = 8$ .

## 5.4. Qualitative results - Depth completion

- **Video:** <https://youtu.be/CabPVqtzs0I>.
- **Depth completion:** 8:22 - 14:18.
  - Trained on Virtual KITTI, evaluated on Virtual KITTI (synthetic to synthetic): 8:22.
  - Trained on Virtual KITTI, evaluated on KITTI (synthetic to real): 9:26.
- The input image, input sparse depth map, ground truth depth map, prediction, predictive uncertainty, aleatoric uncertainty and epistemic uncertainty are visualized.
- Black: minimum uncertainty, white: maximum uncertainty.

## 5.4. Qualitative results - Street-scene semantic segmentation

- **Video:** <https://youtu.be/CabPVqtzs0I>.
- **Street-scene semantic segmentation:** 0:00 - 8:22.
  - Trained on Cityscapes, evaluated on Cityscapes (real to real): 0:00.
  - Trained on Synscapes, evaluated on Cityscapes (synthetic to real): 2:30.
  - Trained on Synscapes, evaluated on Synscapes (synthetic to synthetic): 5:00.
  - Trained on Cityscapes, evaluated on Synscapes (real to synthetic): 6:41
- On Cityscapes, the input image, prediction and predictive entropy are visualized.
- On Synscapes, the input image, ground truth, prediction and predictive entropy are visualized.
- Black: minimum uncertainty, white: maximum uncertainty.

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  - Here, **future work** includes exploring the effect of model pruning techniques on predictive uncertainty quality. For ensembling, sharing early stages of the DNN among ensemble members is also an interesting future direction.
- A **weakness** of ensembling is the additional training required, which also scales linearly with  $M$ . The training of different ensemble members can however be performed in parallel, making it less of an issue in practice given appropriate computing infrastructure.

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- We proposed an **evaluation framework** for predictive uncertainty estimation that is specifically designed to test the robustness required in real-world computer vision applications.
- We performed an **extensive comparison** of ensembling and MC-dropout on the tasks of depth completion and street-scene semantic segmentation, the results of which suggest that **ensembling** consistently provides more reliable and useful predictive uncertainty estimates.

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