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Review

An integer linear model for hospital bed planning

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ABSTRACT

In this paper we investigate the hospital bed management problem. Constrained by budgetary cuts, French hospitals must efficiently manage shared resources, especially beds, which are allocated to acute and elective patients. Nurses therefore have to manage beds and to plan their use according to the demand, including cases of acute patients from the emergency ward. Managing hospital bed planning is difficult due to the demand increase and restricted hospital resources. Our aim is to propose a decision support tool based on an integer linear program for hospital bed planning, considering two classes of patients: elective and acute cases. We take into account several constraints: incompatibility between pathologies, no mixed-sex rooms, continuity of care, etc. In order to solve the proposed model and to compare the results, we use three solver softwares: GLPK, LINGO of LINDO SYSTEMS and CPLEX of ILOG.

Contents

1. Introduction

Hospitals are complex systems which manage a large number of material and human resources. Due to budgetary cuts, the majority of French hospitals need to reduce costs and to be competitive, while ensuring the best quality of care to patients.

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In this context, they admit elective and acute (admitted from the emergency ward) patients who require hospitalization. The bed management between these two classes of patients is one of hospitals' main problems, consisting in handling a variable and unpredictable demand using a fixed resource. In fact, beds are limited and each delay or cancellation impacts on care quality and hospital profits, since hospital capacity is generally expressed in terms of numbers of beds. Hospitals therefore face a bed sizing problem at a strategic level and a bed planning problem at an operational level. Such problems have been identified by several authors in past years (Morgan, 1950; Manchester Regional Hospital Board, 1954; Croome et al., 1964; Orendi, 2008). In this paper, in order to identify the bed management problems, we analyze the bed planning organization in the booking service of the Saint Joseph/Saint Luc hospital located in Lyon. In the first part we present previous studies on hospital bed management. The second part is a description of the process of bed planning at the Saint Joseph/Saint Luc hospital. In the third part we propose a mathematical model using integer linear programming for the purpose of planning bed occupancy. We use three different solvers to solve the proposed model and the results obtained are then analyzed and commented on in the fourth part. Finally, we identify some directions for future research.

2. Previous works

Hospital emergency wards are overcrowded due to waiting time and levels of care which tend to worsen with the increase of patient admissions. Because of a shortage in the number of beds, some scheduled patient admissions are delayed or even transferred to other hospitals. Some patients are moreover hospitalized in inappropriate wards unsuited to their pathologies, with the risk of a lower quality care and more chance of infection (Orendi, 2008). The main previous investigations on the subject are the following.

Regarding bed resources, most studies reported in the literature deal with operating theater planning and refer to intensive care unit beds (Jebali et al., 2006; Kokangul, 2008), recovery room beds (Cardoen et al., 2009) or surgery unit beds (Guinet et al., 2003). Several papers on operating theater planning which do not take into account bed resources specify to integrate such characteristic in perspectives to make their work closer to the practical environment (Fei et al., 2008, 2009; Croome et al., 1964; Lamiri et al., 2009). Bed management can be easily linked to the operating room management, especially in terms of underutilization of care units and overutilization of units (Vissers et al., 2005). Beliën and Demeulemeester (2007) proposed several models for building surgery schedules while studying its impact on bed occupancy and minimizing the expected bed shortage. Few studies have been identified on hospital bed management, although several approaches are proposed to improve bed sizing and planning in hospitals (Mazier et al., 2010).

Some authors (Lapierre et al., 1999; Vissers 1994,1998) use mathematical modeling to develop time series models using hourly census data (number of patients in a care unit at a given time) in order to calculate the size of each unit. In this respect they use simulation and minimize the collected data and the modeling phase. By representing the time census as a time series, the model can produce a frequency distribution to illustrate the bed demand. Murray (2005) based his research on the study carried out by Lapierre et al. (1999). Rather than using simulation, Murray proposes a seasonality forecasting model to predict the future demand for beds. Walczak et al. (2002) uses neural networks to predict the length of stay for each patient arriving at the emergency ward, in order to improve bed requirement planning. Adan and Vissers (2002) analyzed the impact of the following question: "How can a hospital generate a patient admission profile for a specialty, given targets for patient throughput and utilization of the resources while satisfying given restrictions?". Therefore, they developed an integer linear programming model with an application on a orthopedics care unit.

Utley et al. (2003) studied the advantages of creating an intermediate unit of care between emergency and other specialized wards. In this approach, the sizing model allows the authors to determine the optimal number of beds in a care unit. This model takes into account the flow of patients, their waiting time, their length of stay and their transfer time from one service to another until they leave the care unit. Troy and Rosenberg (2009) used a Monte Carlo model in order to determine the number of intensive care unit beds for surgery patients. Kokangul (2008) used determinist and stochastic approaches to determine the optimal number of beds in an intensive pediatric care unit. Also with regard to bed sizing, Hguyen et al. (2005) propose a simple method to optimize hospital bed capacity, based on a score model with three parameters (number of beds, number of unscheduled admissions and number of unoccupied beds).

Previous research (Kim et al., 2000; Kim and Horowitz 2002; Ridge et al., 1998; Harris, 1986) has focused on simulation to further understand and examine in more detail the activities of a care unit in evaluating the various decisions which will be taken to plan the occupancy of beds. With the same philosophy, McGowan et al. (2007) focus on hospital unit organization to improve bed resource availability, with a view to increasing operating theater efficiency. Gallivan et al. (2002) also focused their research on patient admission and the booking system. To this end, they analyzed a cardiac surgery department in order to study the possible links between variability of length of stay, admissions already booked and capacity requirements. Bechar and Guinet (2006) relies on a method based on integer linear programming which allows for the insertion of emergency patients into an existing bed occupancy schedule. This method is based on two tools: an algorithm for the insertion of the patients according to their order of arrival, and a linear program for re-scheduling those patients for whom the hospitalization date was not confirmed and was therefore canceled to make room for emergency patients.

We have noted that the literature about hospital bed planning is scarce. The bed sizing of a care unit is the most frequently discussed problem. In this paper we propose a generic planning tool for bed capacity management, which takes into account elective and acute patients in relation to various constraints.

3. Description of bed management process in Saint Joseph/Saint Luc hospital

The Saint Joseph/Saint Luc hospital is a non-profit private hospital which is located in Lyon. It includes 350 beds and various specialized wards: maternity, reanimation, intensive care, emergency, etc.

3.1. Hospitalization unit organization

There are two kinds of hospitalization units at Saint Joseph/Saint Luc:

- Daily Hospital Admission (DHA): this ward has two services. The first one includes an Anesthesia Service and an Ambulatory Surgery Ward (ASW). The second one consists of a part time hospitalization unit (PTHU). Day patients who have a surgical operation are admitted to the ambulatory surgery ward. The PTHU contains 10 beds and admits only daily medical hospitalized patients.

- Hospitalization Care Units (HCU): the hospitalization care units admit patients who are required to stay longer than one day. There are two kinds of hospitalization care units:
  - Medicine Hospitalization Care Unit (MHCU): there are 6 MHCU. Each one contains 25 beds. The 6 MHCU are divided up as follow: 3 weekly MHCU, 2 MHCU which open 5 days a week (from Monday to Friday) and one MHCU which opens 6 days a week (from Thursday to Tuesday).
  - Surgery Hospitalization Care Unit (SHCU): there are 4 SHCU. Each one comprises 25 beds. The 4 SHCU are distributed as follows: 2 weekly SHCU and 2 which open 5 days a week (one from Monday to Friday and the other from Thursday to Monday).
- Other beds are located in the Emergency Ward, the Serious Burns Ward, etc.

3.2. *Hospital bed planning*

The booking service and the emergency ward are the two main actors of bed management at the Saint Joseph/Saint Luc hospital (Guinet et al., 2003).

3.2.1. *Monthly bed planning*

When an admission request is received, the head nurse has to find an available bed according to the patient's length of stay, sex, pathology, etc. If a bed is available, the patient is inserted into the bed occupancy schedule. Otherwise, the head nurse consults the physician to shift the admission period and looks for an available bed with the new hospitalization period.

3.2.2. *Weekly pre-planning*

Every Thursday at midday, the next week's bed occupancy schedule is drawn up and transmitted to the hospitalization units.

3.2.3. *Daily bed planning*

Before the closing of a hospitalization unit (j-1 where j is the day the unit closes), the head nurse must find available beds for patients who are to remain in hospital after the unit has closed. If there are available beds in other units, the bed occupancy schedule is updated. Otherwise, the head nurse refers the matter to the hospital management in order to keep the hospitalization unit open, and a new team of staff is allocated to the unit. The hospital faces this situation two or three times per year.

The daily occupancy schedule is drawn up as follow: first, the head nurse has to plan the elective patient stays. The acute patients are then considered. If there are available beds, they are allocated to acute patients. Otherwise, these patients remain in the emergency ward or are moved to another hospital.

Fig. 1 describes the EPC (Sheer, 2001) process of transfer of acute patients and their insertion into a bed occupancy schedule.

In the following we propose a decision support tool in order to help nurses in bed occupancy planning. We provide a tool for finding available beds and updating the schedule for acute and elective patient's requirements, taking into account constraints such as no mixed-sex rooms, incompatibility between pathologies, etc.

3.3. *Bed planning organization*

The booking service wastes a lot of time in planning bed occupancy. In fact, several constraints (no mixed-sex rooms, contagious patients, incompatibility between pathologies, etc.) must be taken into account. Bed planning is moreover subject to repeated changes due to inaccuracy in the expected length of stay and to the insertion of acute patients. Due to the fact that the hospitalization units close regularly, patients sometimes have to be moved from one unit to another during their stay. It is difficult to find available beds for acute patients because they are fitted into the schedule after elective patients.

In the next section we describe the proposed decision support tool for bed capacity management. The purpose of our approach is to provide a model which takes into account both elective and the acute patients in bed planning, considering the constraints of single rooms, no mixed-sex double rooms, incompatibility between pathologies, and contagious patients.

4. Modeling

After the medical or surgical consultation, a diagnosis is established for the patient. According to this diagnosis, a bed is reserved and a hospitalization period is defined. Those patients are considered as elective. The acute patients arrive from the emergency ward and require a bed as soon as possible. They are inserted into the bed occupancy schedule, according to the bed availability in the hospitalization units.

4.1. *Hypothesis*

In our approach, we consider that:

- The length of stay is known.
- Each patient is assigned to the same bed throughout his/her hospitalization (no change from the hospitalization unit).
- For each patient, an availability time window is defined which corresponds to the earliest and latest hospitalization periods. The latest hospitalization period is calculated according to the patient's condition. For acute patients, the earliest and latest possible hospitalization periods are the same, in order to express urgency.
- The hospitalization unit includes double and single rooms.
- Mixed rooms are not allowed: men and women are not admitted together in the same room.
- Patients are assigned to double rooms with due consideration to compatibilities between pathologies.
- Contagious patients must stay in a single room or alone in a double room.

For each patient, the category (elective or acute), earliest and latest hospitalization period, sex (male or female), length of stay, pathology and contagiousness are known. A list of patients to be hospitalized is drawn up. The bed assignment must take into account the current schedule. The aim is to find an available bed for each patient, considering his/her expected length of stay and availability. If the patient is elective and there is no available bed, the case can be referred to the physician in order to change his/her period of stay and to find an available bed. If the patient is acute and there is no available bed, he/she is transferred to another hospital or kept in the emergency ward while waiting for a bed. The patient transfer depends on several criteria: age, number of available beds in the other hospital, physician's agreement, etc.

4.2. *Problem formulation*

We based our approach on an existing model (Ben Bachouch et al., 2007) used to insert acute patients into the bed occupancy

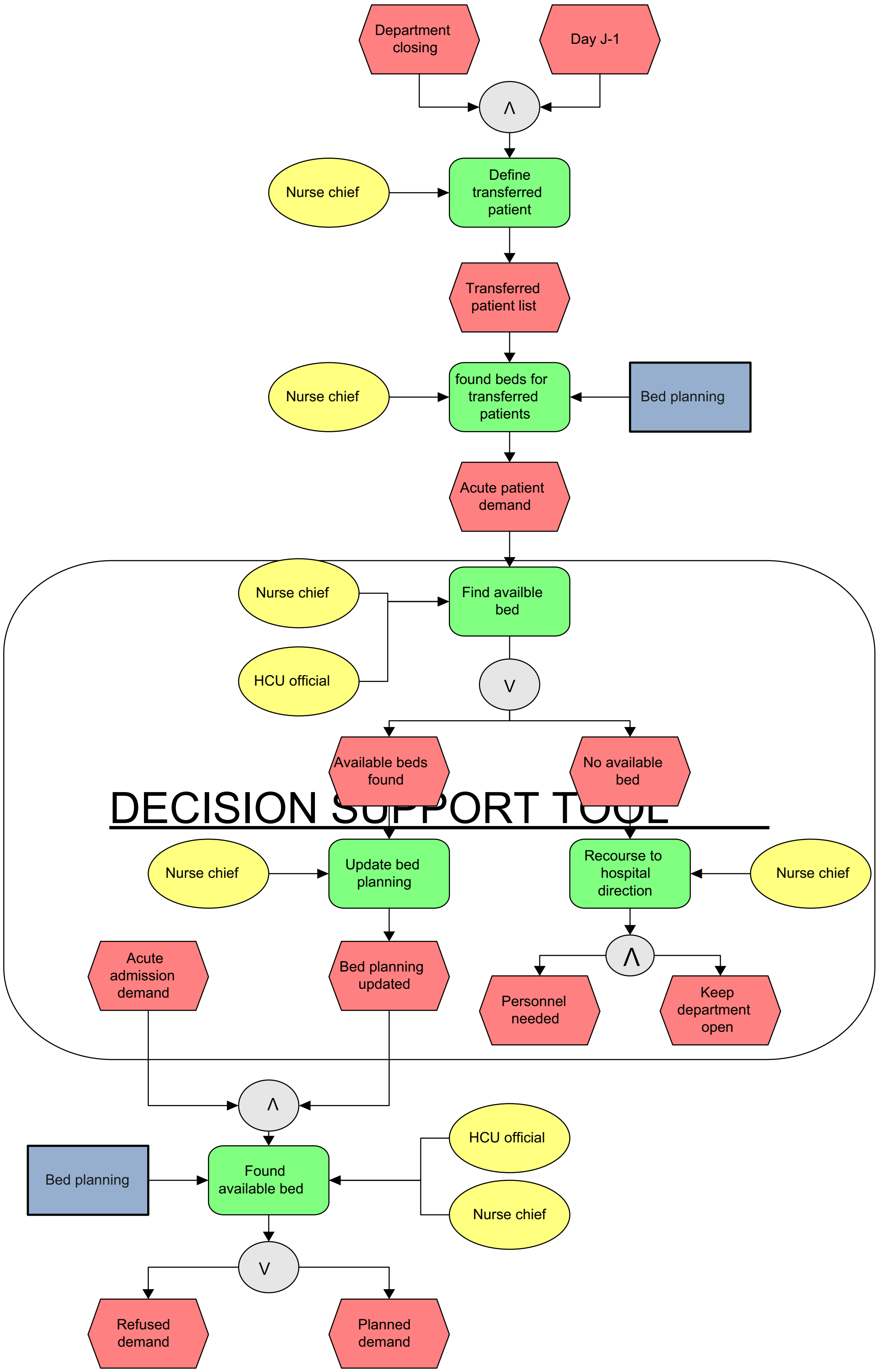

Fig. 1. Transfer and insertion of acute patients in bed planning.

schedule. We added constraints to take into account: single and double rooms, incompatibility between pathologies and contagious patients. The proposed model is described with the following notations.

Indexes

Set of patients indexed by $i \in P = 1, \ldots, N$
Set of days indexed by $t \in D = 1, \ldots, T$
Set of beds indexed by $l \in B = 1, \ldots, L$

Data

$debut_i$	earliest hospitalization period of patient i;
$tard_i$	latest hospitalization period of patient i;
S_i	sex of patient i;
P_i	pathology of patient i.
C_i	contagious state of patient i;
LoS_i	length of stay of patient i;
$T2A_i$	rates of activity for each patient;
H	cost of a lateness day regarding patient hospitalization;
HV	a high value;

Bed availability

$$B_{lt} = \begin{cases} -2 & \text{if bed } l \text{ is already assigned to a man during the period } t \\ 1 & \text{if bed } l \text{ is available during the period } t \\ 2 & \text{if bed } l \text{ is already assigned to a woman during the period } t \end{cases}$$

Bed location

$$M_{ll'} = \begin{cases} 1 & \text{if beds } l \text{ and } l' \text{ are in the same double room or} \\ & \text{if the bed } l \text{ is located in a single room } (l = l') \\ 0 & \text{otherwise} \end{cases}$$

Sex of patients

$$S_i = \begin{cases} -1 & \text{if patient } i \text{ is a man} \\ 1 & \text{if patient } i \text{ is a woman} \end{cases}$$

Bed availability by pathologies

$$PB_{lt} = \begin{cases} P_i & \text{if bed } l \text{ has been previously assigned to patient} \\ & i \text{ who has the pathology } P_i \text{ during period } t \\ 0 & \text{if bed } l \text{ is available during period } t \end{cases}$$

Binary variables

Assignment of patients to beds for each period

$$X_{ilt} = \begin{cases} 1 & \text{if patient } i \text{ is assigned to bed } l \text{ during period } t \\ 0 & \text{Otherwise} \end{cases}$$

Patient allocation to beds

$$A_{il} = \begin{cases} 1 & \text{if patient } i \text{ is assigned to bed } l \\ 0 & \text{otherwise} \end{cases}$$

Integer variables

J_i	hospitalization beginning period of patient i.
fin_i	hospitalization ending period of the patient i such as $fin_i = J_i + LoS_i - 1$.

Transfer variable

$$REFUS_i = \begin{cases} 0 & \text{if the patient } i \text{ is assigned to a bed} \\ LoS_i & \text{otherwise} \end{cases}$$

We formulate the bed capacity planning problem as an integer linear program:

$$Min \sum_{i \in P} (J_i - debut_i) \cdot H + \sum_{i \in P} \left(REFUS_i / LoS_i\right) \cdot T2A_i \tag{1}$$

subject to

$$\forall i \in P, \forall t \in D : \sum_{l \in B} X_{ilt} \leq 1 \tag{2}$$

$$\forall l \in B, \forall t \in D : \sum_{i \in P} X_{ilt} \leq 1 \tag{3}$$

$$\forall i \in P : \sum_{l \in B} \sum_{t = debut_i}^{T} X_{ilt} + REFUS_i = LoS_i \tag{4}$$

$$\forall i \in P : \sum_{l \in B} \sum_{t = debut_i}^{T} (X_{ilt} \cdot (B_{lt} - 2) \cdot (B_{lt} + 2))/(-3) = LoS_i \cdot A_{il} \tag{5}$$

$$\forall i \in P, \forall t \in D : J_i \leq \sum_{l \in B} t \cdot X_{ilt} + (-X_{ilt} + 1) \cdot HV \tag{6}$$

$$\forall i \in P, \forall t \in D : fin_i \geq \sum_{l \in B} t \cdot X_{ilt} \tag{7}$$

$$\forall i \in P : fin_i = J_i + LoS_i - 1 \tag{8}$$

$$\forall i \in P : J_i \geq debut_i \tag{9}$$

$$\forall i \in P : J_i \leq tard_i \tag{10}$$

$$\forall l, l' \in B, l \neq l', \forall t \in D, \forall M_{ll'} = 1 : \sum_{i \in P/C_i = 2} X_{ilt} \cdot S_i - \sum_{i \in P/C_i = 2} X_{il't} \cdot S_i \leq 1 \tag{11}$$

$$\forall l, l' \in B, l \neq l', \forall t \in D, \forall M_{ll'} = 1 : \sum_{i \in P/C_i = 2} X_{ilt} \cdot S_i - \sum_{i \in P/C_i = 2} X_{il't} \cdot S_i \geq -1 \tag{12}$$

$$\forall l, l' \in B, l \neq l', \forall t \in D, \forall M_{ll'} = 1 : B_{lt} \cdot \sum_{i \in P/C_i = 2} S_i \cdot X_{il't} \leq 2 \tag{13}$$

$$\forall l, l' \in B, l \neq l', \forall t \in D, \forall M_{ll'} = 1 : B_{lt} \cdot \sum_{i \in P/C_i = 2} S_i \cdot X_{il't} \geq -1 \tag{14}$$

$$\forall l, l' \in B, l \neq l', \forall t \in D, \forall M_{ll'} = 1 :$$
$$\sum_{i \in P/C_i = 2} X_{ilt} \cdot P_i - \sum_{i \in P/C_i = 2} X_{il't} \cdot P_i \leq 1 + \left(1 - \sum_{i \in P/C_i = 2} X_{il't}\right) \cdot HV \tag{15}$$

$$\forall l, l' \in B, l \neq l', \forall t \in D, \forall M_{ll'} = 1 :$$
$$\sum_{i \in P/C_i = 2} X_{ilt} \cdot P_i - \sum_{i \in P/C_i = 2} X_{il't} \cdot P_i \geq -1 - \left(1 - \sum_{i \in P/C_i = 2} X_{ilt}\right) \cdot HV \tag{16}$$

$$\forall l, l' \in B, l \neq l', \forall t \in D, \forall M_{ll'} = 1 :$$
$$PB_{lt} - \sum_{i \in P/C_i = 2} P_i \cdot X_{il't} \leq 1 + \left(1 - \sum_{i \in P/C_i = 2} X_{il't}\right) \cdot HV \tag{17}$$

$$\forall l, l' \in B, l \neq l', \forall t \in D, \forall M_{ll'} = 1 :$$
$$PB_{lt} - \sum_{i \in P/C_i = 2} P_i \cdot X_{il't} \geq -1 - \left(1 - \sum_{i \in P/C_i = 2} X_{il't}\right) \cdot HV \tag{18}$$

$$\forall l, l' \in B, l \neq l', \forall t \in D, \forall M_{ll'} = 1 : B_{l't} \cdot \left(\sum_{i \in P/C_i = 1} X_{ilt} + \sum_{i \in P} X_{il't}\right) \leq 1 \tag{19}$$

$$\forall l,l' \in B, l \neq l', \forall t \in D, \forall M_{ll'} = 1 : B_{l't} \cdot \left(\sum_{i \in P/C_i = 1} X_{ilt} + \sum_{i \in P} X_{il't} \right) \geq 0 \tag{20}$$

$$\forall i \in P, \forall l \in B, \forall t \in D : X_{ilt} \in \{0,1\} \tag{21}$$

$$\forall i \in P, \forall l \in B : A_{il} \in \{0,1\} \tag{22}$$

$$\forall i \in P : REFUS_i \geq 0 \tag{23}$$

$$\forall i \in P : J_i \geq 0 \tag{24}$$

$$\forall i \in P : fin_i \geq 0 \tag{25}$$

The objective function (1) minimizes two costs: the cost generated by the lateness of the patient's hospitalization (difference between the beginning of hospitalization and the earliest hospitalization period, multiplied by a fixed cost); and the cost of a refused admission due to bed unavailability. Every hospitalization is associated with a defined activity-based payment *T2A*: Tarification à l'activité which induces a loss if the patient's stay is not scheduled. We use the same cost for each day of hospitalization lateness because we admit that beds remain available irrespective of the pathology associated with the hospitalization lateness. This cost is equal to the average lost income of one extra or regular day regarding to *T2A*.

Constraints (2) and (3) ensure that each patient is assigned to at most one bed during a period. Constraints (4) and (5) guarantee that each patient is assigned to an available bed for a period equal to his/her length of stay. If no bed is found, the patient i is not scheduled and the variable $REFUS_i$ takes a value equal to his/her length of stay. The constraints (5) ensure that the patient i is assigned to an available bed. Thus, the patient i can be assigned to the bed l only when the value of $B_{l,t}$ is equal to "1" i.e. when the bed is available. Constraints (6) and (7) ensure that J_i and fin_i take respectively the smallest and the greatest period values of the stay. The constraints (8) require that the difference between J_i and fin_i is equal to the length of stay of patient i. Constraints (9) and (10) ensure that the beginning period of the hospitalization for each patient is included in the time window $[debut_i,fin_i]$. For acute patients, J_i must be equal to $debut_i$. These patients cannot stay at the emergency ward more than a few hours and if there are no available beds for them, they are transferred to another hospital. Constraints (11) and (12) guarantee that the rooms of the hospital are not mixed. Thus, if a room is empty, it will be allocated to two patients of the same sex (two women or two men) or to only one patient (man or woman). Table 1 describes the different possibilities of allocation of patients to double rooms (S_l and $S_{l'}$ represent the sex of the patient assigned respectively to beds j and k which are in the same room).

Constraints (11) and (12) do not guarantee that the requirement of segregated rooms is respected when the room is double and one bed is already assigned to a patient. Thus, we defined constraints (13) and (14). Table 2 illustrates the results of these constraints according to all possible situations (B_{lt} indicates the bed state and S_i designates the sex of the patient assigned to the bed l).

Table 1
Various possible cases of patients' assignment to rooms.

	S_l	$S_{l'}$	Result of the constraint
A woman and a man	1	−1	2*
Only a woman	1	0	1
Two women	1	1	0
Two men	−1	−1	0
Only a man	−1	0	−1
A man and a woman	−1	1	−2*

* Undesirable cases.

Table 2
Different patient assignments to beds in double rooms already used.

	B_{lt}	S_l	Result of the constraint
Bed already assigned to a man	−2	−1	2
Bed already assigned to a man	−2	1	−2*
Bed already assigned to a woman	2	−1	−2*
Bed already assigned to a woman	2	1	2
Available bed	1	−1	−1
Available bed	1	1	1

* Undesirable cases.

Table 3
Symmetric incompatibility between pathologies.

P_l	$P_{l'}$
1	3
1	4
2	4

Table 4
Different pathology assignments to double rooms.

P_l	$P_{l'}$	Result of the constraint
1	1	0
1	2	−1
1	3	−2*
1	4	−3*
2	1	1
2	2	0
2	3	−1
2	4	−2*
3	1	2*
3	2	1
3	3	0
3	4	−1
4	1	3*
4	2	2*
4	3	1
4	4	0

* Undesirable cases.

Constraints (15) and (16) guarantee that patients are assigned to double rooms according to compatibility between pathologies. In this case, we defined four pathologies $P=\{1,2,3,4\}$. Table 3 describes the pathologies which are incompatible. These constraints are available irrespective of the pathology number insofar they are pair wise compatible in the increasing order, given that the incompatibility between pathologies is intransitive.

Table 4 describes the different possibilities of pathology assignments to double rooms (P_l and $P_{l'}$ represent the patient pathology assigned respectively to bed l and bed l').

If a double room is already occupied by only one patient, we must ensure that the second patient assigned to the available bed in this room had a compatible pathology. This is expressed with constraints (17) and (18). Table 5 describes the different situation of pathology assignments to double rooms.

Constraints (11)–(18) concern only non-contagious patients ($C_i=2$).

Constraints (19) and (20) assign contagious patients to single rooms or alone to double rooms. The different cases of contagious patient assignments to rooms are shown in Table 6.

Constraints (21)–(25) are integrity constraints.

Table 5
Different cases of pathology assignments to double rooms.

PB_{lt}	Pl'	Result of the constraint
1	1	0
1	2	−1
1	3	−2*
1	4	−3*
2	1	1
2	2	0
2	3	−1
2	4	−2*
3	1	2*
3	2	1
3	3	0
3	4	−1
4	1	3*
4	2	2*
4	3	1
4	4	0

* Undesirable cases

Table 6
Different cases of contagious patient assignments to rooms.

C_l	$C_{l'}$	Result of the constraint
1	0	1
1	1	2*
1	2	3*

5. Resolution

In order to solve the integer linear program defined above, we use three solver softwares that enable us to compare their efficiency:

- LINGO: academic solver from LINDO SYSTEMS INC.
- ILOG OLP-CPLEX Development Studio: solver from ILOG.
- GLPK (GNU Linear Programming Kit): free solver software.

All our experiments are performed on a Pentium M with 1500 MHz CPU, using LINGO 8, ILOG OPL Development Studio 4.2 and GLPK 4.8.

We take into account:

- A planning horizon of 14 days.
- A weekly hospitalization care unit of 10 double rooms and 5 single rooms i.e. 25 beds, knowing that we intend to use the model for each care unit.
- A cost of €365 per day late for the patient's hospitalization (Cohen et al., 1980). We considered this cost, knowing that the average cost of a hospitalization day is about €500 (De Pouvourville 2009). However, it is not so certain that the bed will be occupied, and a day beyond the length of stay defined by the activity-based payment cost is about €250. Thus, we considered an average of all these costs, which is €365.
- Four pathologies. For each one, the corresponding rates on activity *T2A* is defined below in Table 7.
- Four groups of pathologies have been defined with $P=\{1,2,3,4\}$

5.1. Bed planning

We plan for 20 patients. We aim to illustrate how our model can be used to generate a T-period bed occupancy schedule. Table 8 describes the availability of beds before planning.

Table 7
Summary of rates on activity (*T2A*) associated to each pathology.

Pathology	Length of stay	*T2A* (€)
P_1	≤ 2 *days*	1650
P_2	≤ 2 *days*	1127
P_3	≤ 18 *days*	3064
P_4	≤ 15 *days*	1740
Contagious_1	≤ 21 *days*	2468
Contagious_2	≤ 20 *days*	2069
Contagious_3	≤ 2 *days*	1745

Table 9 illustrates the results obtained after solving the model by Lindo System (1999). We admit that for acute patients, the earliest and the latest periods are the same and $debut_i$ take the value 2, meaning that an acute patient is already in the emergency room waiting for a bed. We obtained results with LINGO within 6mn31s. We studied the example of 20 patients, of which 3 were contagious and 5 were acute (2, 5, 10, 15 and 16). We obtained only 3 refusals.

Table 10 illustrates the bed occupancy schedule obtained after solving the model with LINGO. We note that only 3 patients are not scheduled. Patients are assigned to rooms, considering continuity of patient stay, segregated rooms, incompatibility between pathologies and contagiousness.

5.2. Experimental results

In this section we make a performance comparison between the solvers used. We consider four experiments to schedule 15, 20, 30 and 40 patients. The results obtained are illustrated in Table 11.

We note that the results obtained with LINGO and CPLEX for the case of 15 patients are almost identical. The objective function value is optimal and the number of unscheduled patients is the same for the two solvers. The only difference is that LINGO is a little bit faster (about 13 s) than CPLEX for finding the optimal solution.

For the case of 20 patients, we note that all the results are the same except for computation times which are longer with LINGO. The difference varies from 3 min to 2 h for the same number of unscheduled patients and an identical optimal performance for the objective function.

We noticed that LINGO could not find the best solution for scheduling 30 patients. We therefore decided to stop the resolution after a time limit because we found that the best solutions obtained did not change after 8 h (case of 30 contagious patients) and 14 h (case of 30 patients of whom 10 are contagious). Moreover, CPLEX reaches the optimal solution within a few minutes.

For the case of 40 patients, we observed that CPLEX found the optimal solution within a few seconds whereas LINGO spent hours without obtaining the best solution. The number of unscheduled patients was moreover lower when the solution was calculated with CPLEX.

GLPK is not suited to our model because the constraint number is too high. The solver LINGO spends too much time in solving the model and this time increases with the number of patients to schedule. It probably uses only a Branch and Bound method which consists in exploring all the solution possibilities. Even if the best solution is found, LINGO keeps on searching as long as all the solutions are not swept. Moreover, LINGO does not allow us to know the computation time to find the best solution. The solver CPLEX is the most powerful regarding computation times. Indeed, irrespective of the number of patients, CPLEX calculates the best solution in a few seconds and provides the bed occupancy

Table 8
Bed planning in progress.

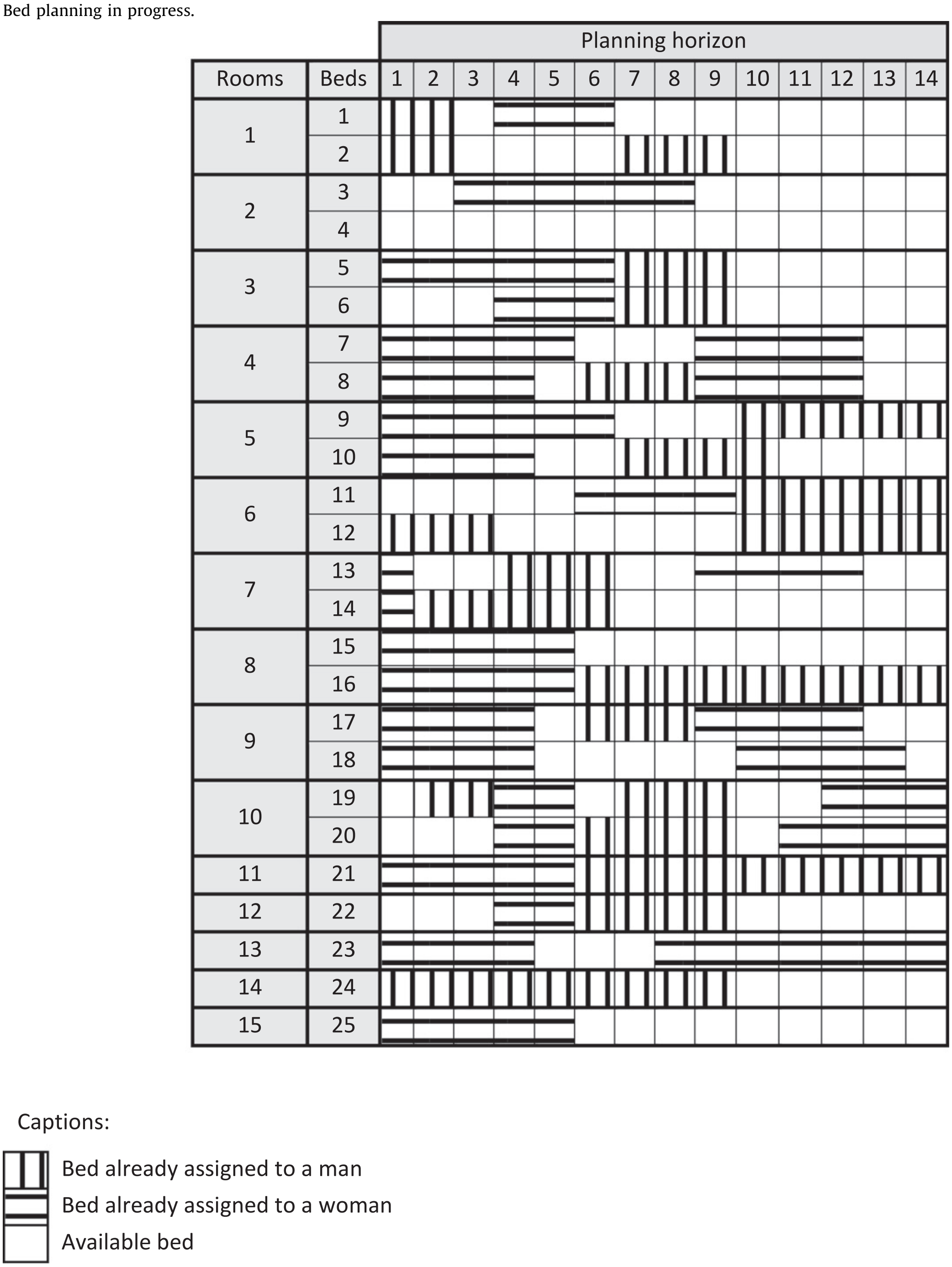

Table 9
Results obtained by solving the program with Lingo solver.

i	$DEBUT_i$	$TARDi$	S_i	LoS_i	P_i	C_i	$T2A_i$	J_i	FIN_i	Bed_i	$Room_i$	$REFUS_i$
1	10	11	1	4	1	2	3064	10	13	22	12	0
2	**2**	**2**	**1**	**3**	**1**	**2**	**3064**	**2**	**4**	**0**	**0**	**3**
3	5	8	1	2	−1	1	1127	5	6	2	1	0
4	6	8	−1	3	2	2	1740	6	8	7	4	0
5	2	2	−1	2	1	2	3064	2	3	20	10	0
6	9	13	−1	2	2	2	1740	10	11	6	3	0
7	4	6	1	4	−1	1	2069	4	7	4	2	0
8	9	10	1	3	2	2	1740	9	11	25	15	0
9	5	7	1	2	−1	2	1127	5	6	23	13	0
10	**2**	**2**	**−1**	**4**	**2**	**2**	**1740**	**2**	**5**	**0**	**0**	**4**
11	9	12	−1	3	1	2	3064	10	12	5	3	0
12	9	10	1	2	1	1	3064	9	10	14	7	0

Table 9 (*continued*)

i	$DEBUT_i$	$TARDi$	S_i	LoS_i	P_i	C_i	$T2A_i$	J_i	FIN_i	Bed_i	$Room_i$	$REFUS_i$
13	8	10	−1	3	2	2	1740	9	11	4	2	0
14	9	12	−1	2	−2	2	1650	9	10	1	1	0
15	2	2	−1	1	1	2	3064	2	2	3	2	0
16	**2**	**2**	**1**	**3**	**2**	**2**	**1740**	**2**	**4**	**0**	**0**	**3**
17	4	8	1	3	1	2	2069	4	6	12	6	0
18	6	9	−1	3	2	2	1740	6	8	25	15	0
19	8	9	−1	3	2	2	1740	9	11	3	2	0
20	9	11	−1	2	2	2	1745	10	11	24	14	0

Table 10
Bed planning after solving.

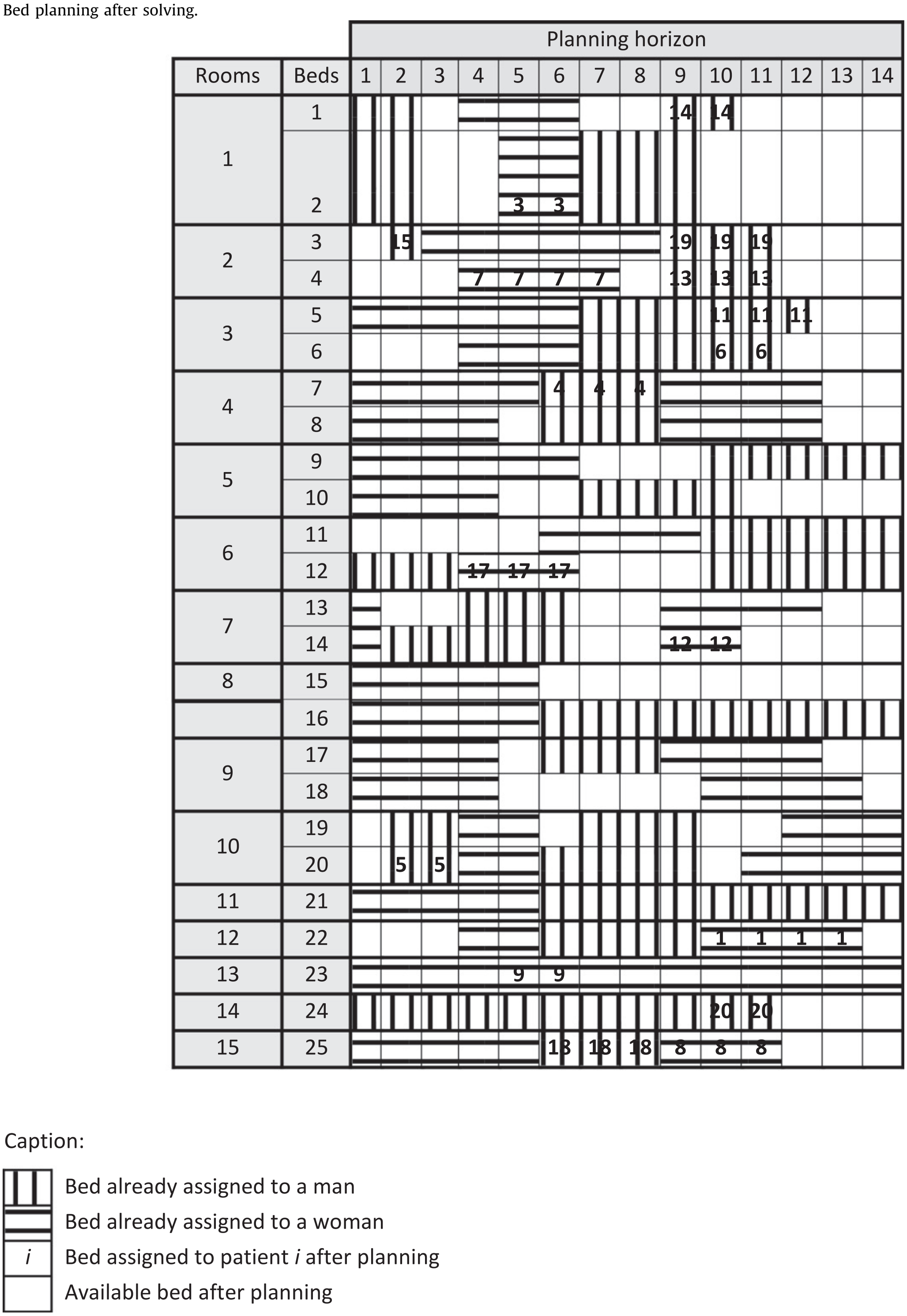

Table 11
Results of the resolution.

Patient number	Solver	Computation time (in s)	Optimum
15 patients (2 acute patients)	LINGO	5s	Yes
	CPLEX	21	Yes
15 patients (3 contagious and 2 acute patients)	LINGO	2s	Yes
	CPLEX	17	Yes
20 patients (3 acute patients)	LINGO	1 h04 mn	Yes
	CPLEX	34	Yes
20 patients (4 contagious and 5 acute patients)	LINGO	19 mn20 s	Yes
	CPLEX	10	Yes
30 contagious patients	LINGO	1 h*	No
	CPLEX	115 (1 mn55 s)	Yes
30 patients of whom 10 are contagious	LINGO	1h*	No
	CPLEX	139 (2 mn19 s)	Yes
40 contagious patients	LINGO	1 h*	No
	CPLEX	56	Yes

schedule with a better assignment of patients to beds (for example it assigns contagious patient in single rooms, and gives preference to rooms already occupied, when possible, rather than assigning patients to free double rooms).

We tested the efficiency of the proposed model by considering a schedule of 21 and 28 days. We obtained results in all cases. These results are optimal for the cases of scheduling 15 and 20 patients but for 30 and 40 patients with an horizon of 28 days, we interrupted the resolution after one hour and also obtained a feasible schedule. Other tests were performed where we tried to identify the model limits. In the case where $H \ll T2Ai$ or where $H \gg T2Ai$, we obtained results which were not optimal but we produced a feasible occupancy schedule. Moreover, where we used more incompatibility pathologies, it became harder to obtain a feasible solution.

6. Conclusion

In this paper we focus on the problem of planning bed occupancy by elective patients and acute patients in the Saint Joseph/Saint Luc hospital. The acute patient's stays need to be scheduled as soon as possible to relieve the emergency ward's resources. Our aim is to provide a tool which allows the head nurse to draw up a bed occupancy schedule that takes into account both acute and elective patients, as well as the constraints of segregated rooms, contagious patients, incompatibility between pathologies and continuity of patient's stays. Moreover, the choice of a solver to solve our model depends on many criteria. It is obvious that if we need to for a large number of patients, CPLEX is the most appropriate, but for reduced numbers of patients, LINGO and CPLEX are both powerful.

Regarding our comparative study, we conclude that the solver GLPK is not suited to our model (Ben Bachouch et al., 2007) even if it was proved in other studies to be a powerful solver (Bechar and Guinet, 2006). The choice is therefore limited to two solvers: CPLEX and LINGO. For LINGO, the computation time is exponential to the growth of patient numbers. Hence, the choice of the most appropriate solver for the model resolution depends on the number of patients that have to be scheduled.

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References

Adan, I.J., et Vissers, J.M., 2002. Patient mix optimisation in hospital admission planning: a case study. International Journal of Operations and Production Management 22, 445–461.
Bechar, S., et Guinet, A., 2006. Planification des lits d'un établissement de soins, Conference GISEH 2006, Luxembourg ville, 14–16 Septembre, pp. 645–653.
Beliën, J., et Demeulemeester, E., 2007. Building cyclic master surgery schedules with levelled resulting bed occupancy. European Journal of Operational Research 176, 1185–1204.
Ben Bachouch, R., Guinet, A., Hajri-Gabouj, S., 2007. Planification des lits mutualisés d'un établissement de soins. In: Proceedings of the 7th International Congress of Industrial engineering CIGI'2007. Trois-Rivières Québec, Canada, 5–8 Juin 2007, CDROM, 2007 p.
Cardoen, B., Demeulemeester, E., Belien, J., 2009. Optimizing a multiple objective surgical case sequencing problem. International Journal of Production Economics 119, 354–366.
Cohen, M.A., Heshey, J.C., Weiss, E.N., 1980. Analysis of capacity decisions for progressive patient care hospital facilities. Health Services Research 15, 145–160.
Croome, E.M., Ingram, J., Qvist, G., 1964. Hospital bed occupancy. The Lancet 284, 1122–1123.
De Pouvourville, G., 2009. Exploitation seconde de la base de l'Étude nationale de coûts (ENC). Dossier solidarité et santé 9, 1–53.
Fei, H., Meskens, N., Combes, C., Chu, C., 2009. The endoscopy scheduling problem: a case study with two specialised operating rooms. International Journal of Production Economics 120, 452–462.
Fei, H., Chu, C., Meskens, N., Artiba, A., 2008. Solving surgical case assignment problem by a branch-and-price approach. International Journal of Production Economics 112, 96–108.
Gallivan, S., Utley, M., Treasure, T., Valencia, O.S., 2002. Booked inpatient admissions and hospital capacity: Mathematical modeling study. British Medical Journal 324, 280–282.
Guinet, A., Chaabane, S., 2003. Operating theater planning. International Journal of Production Economics 85, 69–81.
Harris, R.A., 1986. Hospital bed requirement planning. European Journal of Operational Research 25, 121–126.
Hguyen, J.M., Six, P., Antonioli, D., Glemain, P., Potel, G., Lombrail, P., Le Beux, P., 2005. A simple method to optimize hospital bed capacity. International Journal of Medical Informatics 74, 39–49.
I.D.S. Sheer Academy, 2001. ARIS6 Collaborative Suite version 6. Method ARIS.
Jebali, A., Atidel, B.H.A., Ladet, P., 2006. Operating room scheduling. International Journal of Production Economics 99, 52–62.
Kim, S.C., Horowitz, I., Young, K.K., Buckley, T.A., 2000. Flexible bed allocation and performance in the intensive care unit. Journal of Operation Management, 427–443.
Kim, S.C., Horowitz, L., 2002. Scheduling hospital services: the efficacy of elective-surgery quotas. Omega-International Journal of Management 18, 427–443.
Kokangul, A., 2008. A combination of deterministic and stochastic approaches to optimize bed capacity in a hospital unit. Computer Methods and Programs in Biomedicine 90 (1), 56–65.
Lapierre, S.D., Goldsman, D., Cochran, R., DuBow, J., 1999. Bed allocation techniques based on census data. Socio-Economic Planning Sciences 33, 25–38.
Lamiri, M., Grimaud, F., Xiaolan, X., 2009. Optimization methods for a stochastic surgery planning problem. International Journal of Production Economics 120, 400–410.
Lindo System INC., 1999. , Lingo: The Modeling Language and Optimizer User's Guide, October 1999.
Manchester Regional Hospital Board, 1954. Use of Hospital Beds. The Lancet 4, 488–489.

Mazier, A., Xie, X., Sarazin, M., 2010. Real-Time Patients Assignment: a Method for Improving Emergency Department Flow. In: Proceedings of the IEEE workshop on Health Care Management. Venetia (Italia).

McGowan, J.E., Truwit, J.D., Cipriano, P., Howell, R.E., VanBree, M., Garson, A., Hanks, J.B., 2007. Operating room efficiency and hospital capacity: factors affecting operating room use during maximum hospital census. American College of Surgeons 204, 865–871.

Morgan, D.G., 1950. Hospital beds. The Lancet 11 281 p.

Murray, C.J., 2005. A note on bed allocation technique based on census data. Socio Economic Planning Sciences 39, 183–192.

Orendi, J., 2008. Health-care organisation, hospital-bed occupancy, and MRSA. The Lancet 371, 1401–1402.

Ridge, J.C., Jones, S.K., Nielsen, M.S., Shahani, A.K., 1998. Capacity planning for intensive care units. European Journal of Operational Research 105, 346–355.

Troy, P.M., Rosenberg, L., 2009. Using simulation to determine the need for ICU beds for surgery patients. Surgery 146, 608–620.

Utley, M., Gallivan, S., Davis, K., Daniel, P., Reeves, P., Worrall, J., 2003. Estimating bed requirements for an intermediate care facility. European Journal of Operational Research, 92–100.

Vissers, J.M.H., 1994. Patients Flow-Based Allocation of Hospital Resource. Graduate School of Industrial Engineering and Management Science, Eindhoven University of Technology.

Vissers, J.M.H., 1998. Patients flow-based allocation of inpatient resources: a case study. European Journal of Operational Research 105, 356–370.

Vissers, J.M.H., Adan, I.J.B.F., Bekkers, J.A., 2005. Patient mix optimization in tactical cardiothoracic surgery planning: a case study. IMA Journal of Management Mathematics 16, 281–304.

Walczak, S., Pofahl, W.E., Scorpio, R.J., 2002. A decision support tool for allocating hospital resources and determining required acuity of care. Decision Support System, 445–446.