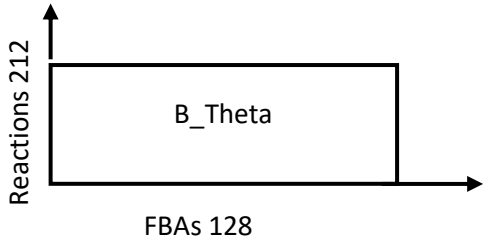
COMPLETED: Mutual Information Calculation of *B_theta and M_smithii* KBase Data Public Narrative Links at KBase Data:

https://narrative.kbase.us/narrative/ws.14662.obj.2
https://narrative.kbase.us/narrative/ws.14663.obj.1

INTRACELLULAR CALCULATIONS – STAGE I

MI_Calculation_Fig13_plotvalues.py file : only for 7 input compound calculation

B_theta_MI Calculation (Stage I – Intracellular) – When all 7 compounds present

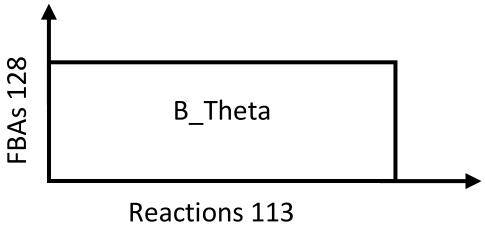


DetailsofDuplicateRows =

		Total Duplicates	Real Duplicates
1)	1,34,41,60,79,105,134,154,184,189,191,0,0,0,0,0,0	11	10
2)	2,178,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0	2	1
3)	4,57,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0	2	1
4)	5,15,17,32,74,78,95,108,111,112,130,135,164,167,171,195,198	17	16
5)	7,91,172,208,0,0,0,0,0,0,0,0,0,0,0,0,0	4	3
6)	10,56,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0	2	1
7)	11,52,55,131,175,0,0,0,0,0,0,0,0,0,0,0,0	5	4
8)	12,72,143,0,0,0,0,0,0,0,0,0,0,0,0,0,0	3	2
9)	18,144,152,0,0,0,0,0,0,0,0,0,0,0,0,0,0	3	2
10)	20,142,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0	2	1
11)	21,125,163,0,0,0,0,0,0,0,0,0,0,0,0,0,0	3	2
12)	23,201,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0	2	1
13)	25,179,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0	2	1
14)	26,36,102,149,0,0,0,0,0,0,0,0,0,0,0,0,0	4	3
15)	27,113,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0	2	1
16)	29,151,187,193,197,0,0,0,0,0,0,0,0,0,0,0,0	5	4
17)	35,51,210,0,0,0,0,0,0,0,0,0,0,0,0,0,0	3	2
18)	38,59,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0	2	1
19)	40,99,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0	2	1
20)	42,64,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0	2	1
21)	44,70,81,0,0,0,0,0,0,0,0,0,0,0,0,0,0	3	2
22)	47,71,109,136,0,0,0,0,0,0,0,0,0,0,0,0,0	4	3
23)	48,50,127,0,0,0,0,0,0,0,0,0,0,0,0,0,0	3	2
24)	58,94,119,138,169,0,0,0,0,0,0,0,0,0,0,0,0	5	4
25)	65,86,196,0,0,0,0,0,0,0,0,0,0,0,0,0,0	3	2
26)	66,96,157,0,0,0,0,0,0,0,0,0,0,0,0,0,0	3	2
27)	69,93,133,0,0,0,0,0,0,0,0,0,0,0,0,0,0	3	2
28)	73,140,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0	2	1
29)	84,110,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0	2	1
30)	88,106,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0	2	1
31)	98,100,126,147,158,182,0,0,0,0,0,0,0,0,0,0,0	6	5
32)	116,132,148,153,159,0,0,0,0,0,0,0,0,0,0,0,0	5	4
33)	121,212,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0	2	1
34)	122,162,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0	2	1
35)	150,173,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0	2	1
36)	156,209,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0	2	1
37)	180,181,185,192,0,0,0,0,0,0,0,0,0,0,0,0,0	4	3
38)	183,188,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0	2	1
39)	199,205,206,0,0,0,0,0,0,0,0,0,0,0,0,0,0	3	2
40)	202,204,207,0,0,0,0,0,0,0,0,0,0,0,0,0,0	3	2
	Sum	139	99

NumDuplicateRows = 99

Unique Reactions = 113 (212-99)



DetailsofDuplicateColumns =

		Total Duplicates	Real Duplicates
1)	1,4,17,42,55,94,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0	{6}	{5}
2)	2,5,7,18,20,22,24,38,43,45,52,56,58,62,74,76,78,80,91,93,95,96,98,111,113,115,126,128,0,0,0,0	{28}	{27}
3)	3,10,48,53,123,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0	{5}	{4}
4)	6,19,23,75,85,97,114,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0	{7}	{6}
5)	8,12,25,64,99,103,116,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0	{7}	{6}
6)	9,11,13,14,16,27,29,31,33,34,36,47,49,51,54,65,67,69,71,82,84,87,89,100,102,104,106,117,118,120,122,124	{32}	{31}
7)	15,28,32,35,66,70,83,88,101,105,119,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0	{11}	{10}
8)	21,59,108,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0	{3}	{2}
9)	26,30,46,50,68,81,86,107,121,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0	{9}	{8}
10)	37,77,90,112,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0	{4}	{3}
11)	39,44,61,79,110,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0	{5}	{4}
12)	40,60,73,109,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0	{4}	{3}
13)	41,57,92,127,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0	{4}	{3}
14)	63,72,125,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0	{3}	{2}
	Sum	128	114

NumDuplicateColumns = 114 (128-14)
Unique FBAs = 14 (128-114) **Hence 14 Groups in BT**

$$I(\{c_i\}_{i=1}^{N};\{r_i^*\}_{i=1}^{M}) = H(\{c_i\}_{i=1}^{N}) - H(\{c_i\}_{i=1}^{N}\,|\,\{r_i^*\}_{i=1}^{M}) \quad \textbf{(1)}$$

$$\boxed{\hat{H}(\{c_i\}_{i=1}^{N}) = \log_2(128) = 7 \text{ bits}}$$

$$H(\{c_{gl}, c_{lac}\}\,|\,\{r_i^*\}_{i=1}^{M}) = -\sum_{y=1}^{Y} P_Y \sum_{x=1}^{X_y} P(x|y)\log_2 P(x|y)$$

$$= -\sum_{\substack{i=1\\ No.Groups}}^{14} \frac{No.repetition\ i}{total\ combination} \times \sum_{\substack{No.repetition\\ each\ group\ i}} \frac{1}{No.repetition\ i} log_2\left(\frac{1}{No.repetition\ i}\right)$$

No.of.Members in each group	3	4	5	6	7	9	11	28	32
	|	|	|	|	|	|	|	|	|
No. Groups	2	3	2	1	2	1	1	1	1

$$= -\Bigg[\left(\frac{6}{128}\times\left(\frac{1}{6}log_2\left(\frac{1}{6}\right)+\frac{1}{6}log_2\left(\frac{1}{6}\right)+\frac{1}{6}log_2\left(\frac{1}{6}\right)+\frac{1}{6}log_2\left(\frac{1}{6}\right)+\frac{1}{6}log_2\left(\frac{1}{6}\right)+\frac{1}{6}log_2\left(\frac{1}{6}\right)\right)\right) + \left(\frac{28}{128}\times\left(\frac{1}{28}log_2\left(\frac{1}{28}\right)+\frac{1}{28}log_2\left(\frac{1}{28}\right)+\frac{1}{28}log_2\left(\frac{1}{28}\right)+\cdots\times 28times\right)\right) +$$
$$\left(2\times\left(\frac{5}{128}\times\left(\frac{1}{5}log_2\left(\frac{1}{5}\right)+\frac{1}{5}log_2\left(\frac{1}{5}\right)+\frac{1}{5}log_2\left(\frac{1}{5}\right)+\cdots\times 5times\right)\right)\right) + \left(2\times\left(\frac{7}{128}\times\left(\frac{1}{7}log_2\left(\frac{1}{7}\right)+\frac{1}{7}log_2\left(\frac{1}{7}\right)+\frac{1}{7}log_2\left(\frac{1}{7}\right)+\cdots\times 7times\right)\right)\right) + \left(\frac{32}{128}\times\left(\frac{1}{32}log_2\left(\frac{1}{32}\right)+\right.\right.$$
$$\left.\left.\frac{1}{32}log_2\left(\frac{1}{32}\right)+\frac{1}{32}log_2\left(\frac{1}{32}\right)+\cdots\times 32times\right)\right) + \left(\frac{11}{128}\times\left(\frac{1}{11}log_2\left(\frac{1}{11}\right)+\frac{1}{11}log_2\left(\frac{1}{11}\right)+\frac{1}{11}log_2\left(\frac{1}{11}\right)+\cdots\times 11times\right)\right) + \left(2\times\left(\frac{3}{128}\times\left(\frac{1}{3}log_2\left(\frac{1}{3}\right)+\frac{1}{3}log_2\left(\frac{1}{3}\right)+\right.\right.\right.$$
$$\left.\left.\left.\frac{1}{3}log_2\left(\frac{1}{3}\right)\right)\right)\right) + \left(\frac{9}{128}\times\left(\frac{1}{9}log_2\left(\frac{1}{9}\right)+\frac{1}{9}log_2\left(\frac{1}{9}\right)+\frac{1}{9}log_2\left(\frac{1}{9}\right)+\cdots\times 9times\right)\right) + \left(3\times\left(\frac{4}{128}\times\left(\frac{1}{4}log_2\left(\frac{1}{4}\right)+\frac{1}{4}log_2\left(\frac{1}{4}\right)+\frac{1}{4}log_2\left(\frac{1}{4}\right)+\frac{1}{4}log_2\left(\frac{1}{4}\right)\right)\right)\right)\Bigg]$$

$$\left(\frac{6}{128}\times\left(\frac{1}{6}log_2\left(\frac{1}{6}\right)+\frac{1}{6}log_2\left(\frac{1}{6}\right)+\frac{1}{6}log_2\left(\frac{1}{6}\right)+\frac{1}{6}log_2\left(\frac{1}{6}\right)+\frac{1}{6}log_2\left(\frac{1}{6}\right)+\frac{1}{6}log_2\left(\frac{1}{6}\right)\right)\right) = -0.1212$$

$$\left(\frac{28}{128}\times\left(\frac{1}{28}log_2\left(\frac{1}{28}\right)+\frac{1}{28}log_2\left(\frac{1}{28}\right)+\frac{1}{28}log_2\left(\frac{1}{28}\right)+\cdots\times 28times\right)\right) = -1.0516$$

$$\left(2\times\left(\frac{5}{128}\times\left(\frac{1}{5}log_2\left(\frac{1}{5}\right)+\frac{1}{5}log_2\left(\frac{1}{5}\right)+\frac{1}{5}log_2\left(\frac{1}{5}\right)+\cdots\times 5times\right)\right)\right) = -0.1814$$

$$\left(2\times\left(\frac{7}{128}\times\left(\frac{1}{7}log_2\left(\frac{1}{7}\right)+\frac{1}{7}log_2\left(\frac{1}{7}\right)+\frac{1}{7}log_2\left(\frac{1}{7}\right)+\cdots\times 7times\right)\right)\right)=\ -0.3071$$

$$\left(\frac{32}{128}\times\left(\frac{1}{32}log_2\left(\frac{1}{32}\right)+\frac{1}{32}log_2\left(\frac{1}{32}\right)+\frac{1}{32}log_2\left(\frac{1}{32}\right)+\cdots\times 32times\right)\right)=\ -1.2500$$

$$\left(\frac{11}{128}\times\left(\frac{1}{11}log_2\left(\frac{1}{11}\right)+\frac{1}{11}log_2\left(\frac{1}{11}\right)+\frac{1}{11}log_2\left(\frac{1}{11}\right)+\cdots\times 11times\right)\right)=\ -0.2973$$

$$\left(2\times\left(\frac{3}{128}\times\left(\frac{1}{3}log_2\left(\frac{1}{3}\right)+\frac{1}{3}log_2\left(\frac{1}{3}\right)+\frac{1}{3}log_2\left(\frac{1}{3}\right)\right)\right)\right)=\ -0.0743$$

$$\left(\frac{9}{128}\times\left(\frac{1}{9}log_2\left(\frac{1}{9}\right)+\frac{1}{9}log_2\left(\frac{1}{9}\right)+\frac{1}{9}log_2\left(\frac{1}{9}\right)+\cdots\times 9times\right)\right)=\ -0.2229$$

$$\left(3\times\left(\frac{4}{128}\times\left(\frac{1}{4}log_2\left(\frac{1}{4}\right)+\frac{1}{4}log_2\left(\frac{1}{4}\right)+\frac{1}{4}log_2\left(\frac{1}{4}\right)+\frac{1}{4}log_2\left(\frac{1}{4}\right)\right)\right)\right)=\ -0.1875$$

$$H\left(\{c_{gl},c_{lac}\}|\{r_i^*\}_{i=1}^{M}\right)=-\sum_{y=1}^{Y}P_Y\sum_{x=1}^{X_y}P(x|y)\log_2 P(x|y)$$

$H(\{c_{gl},c_{lac}\}|\{r_i^*\}_{i=1}^{M}=\ -[-0.1212-1.0516-0.1814-0.3071\ -1.2500-0.2973\ -0.0743\ -0.2229\ -0.1875]$ = 3.6933 bits

Implement the values to equation (1),

$$I(\{c_i\}_{i=1}^{N};\{r_i^*\}_{i=1}^{M})=H(\{c_i\}_{i=1}^{N})-H(\{c_i\}_{i=1}^{N}\,|\,\{r_i^*\}_{i=1}^{M})$$

MI = 7 bits - 3.6933 bits
MI = 3.3068 bits

MI_Calculation_Fig14_plotvalues.py file : only for 7 input compound calculation

M_smithii GROUP_MI Calculation (Stage I)– When all 7 compounds present

NumDuplicateRows = 420

Unique Reactions = 136 (556-420) P.S = Matrix information is excluded, because the size of the matrix is large to fit on a page

DetailsofDuplicateColumns =

		Total Duplicates	Real Duplicates
1)	1,108,0,0,0,0,0,0,0,0,0,0,0	2	1
2)	2,38,0,0,0,0,0,0,0,0,0,0,0	2	1
3)	4,21,59,63,77,112,0,0,0,0,0,0,0	6	5
4)	5,22,74,0,0,0,0,0,0,0,0,0,0	3	2
5)	6,44,79,85,97,114,0,0,0,0,0,0,0	6	5
6)	7,24,60,78,98,0,0,0,0,0,0,0,0	5	4
7)	8,26,46,81,99,107,116,0,0,0,0,0,0	7	6
8)	9,47,65,100,117,118,0,0,0,0,0,0,0	6	5
9)	10,28,48,66,83,101,119,0,0,0,0,0,0	7	6
10)	11,14,29,49,67,84,102,120,0,0,0,0,0	8	7
11)	12,25,50,68,86,103,121,0,0,0,0,0,0	7	6
12)	13,16,33,34,36,51,69,104,0,0,0,0,0	8	7
13)	15,32,35,53,70,88,105,123,0,0,0,0,0	8	7
14)	17,37,55,72,125,0,0,0,0,0,0,0,0	5	4
15)	18,20,40,52,56,58,73,76,91,93,109,111,126,128	14	13
16)	27,82,0,0,0,0,0,0,0,0,0,0,0	2	1
17)	31,54,71,87,89,106,122,124,0,0,0,0,0	8	7
18)	39,41,57,75,92,110,127,0,0,0,0,0,0	7	6
19)	42,94,0,0,0,0,0,0,0,0,0,0,0	2	1
20)	43,113,0,0,0,0,0,0,0,0,0,0,0	2	1
21)	45,62,80,0,0,0,0,0,0,0,0,0,0	3	2
	Sum	118	97

NumDuplicateColumns = 97 (118-21)
Unique FBAs = 31 (128-97) **Hence 31 Groups in MS**

$\hat{H}(\{c_i\}_{i=1}^{N})$ = $\log_2(128)$ = 7 bits

$$= -\sum_{\substack{i=1 \\ No.Groups}}^{31} \frac{No.repetition\ i}{total\ combination} \times \sum_{\substack{No.repetition \\ each\ group\ i}} \frac{1}{No.repetition\ i} log_2\left(\frac{1}{No.repetition\ i}\right)$$

No.of.Members in each group	1	2	3	5	6	7	8	14
No. Groups	10	5	2	2	3	4	4	1

$$= -\left[10 \times \left(\frac{1}{128} \times \left(\frac{1}{1} log_2\left(\frac{1}{1}\right)\right)\right) + 5 \times \left(\frac{2}{128} \times \left(\frac{1}{2} log_2\left(\frac{1}{2}\right) + \frac{1}{2} log_2\left(\frac{1}{2}\right)\right)\right) + \left(2 \times \left(\frac{3}{128} \times \left(\frac{1}{3} log_2\left(\frac{1}{3}\right) + \frac{1}{3} log_2\left(\frac{1}{3}\right) + \frac{1}{3} log_2\left(\frac{1}{3}\right)\right)\right)\right) + \left(2 \times \left(\frac{5}{128} \times \left(\frac{1}{5} log_2\left(\frac{1}{5}\right) + \frac{1}{5} log_2\left(\frac{1}{5}\right) + \frac{1}{5} log_2\left(\frac{1}{5}\right) + \cdots \times 5times\right)\right)\right) + 3 \times \left(\frac{6}{128} \times \left(\frac{1}{6} log_2\left(\frac{1}{6}\right) + \frac{1}{6} log_2\left(\frac{1}{6}\right) + \frac{1}{6} log_2\left(\frac{1}{6}\right) + \frac{1}{6} log_2\left(\frac{1}{6}\right) + \frac{1}{6} log_2\left(\frac{1}{6}\right) + \frac{1}{6} log_2\left(\frac{1}{6}\right)\right)\right) + \left(4 \times \left(\frac{7}{128} \times \left(\frac{1}{7} log_2\left(\frac{1}{7}\right) + \frac{1}{7} log_2\left(\frac{1}{7}\right) + \frac{1}{7} log_2\left(\frac{1}{7}\right) + \cdots \times 7times\right)\right)\right) + 4 \times \left(\frac{8}{128} \times \left(\frac{1}{8} log_2\left(\frac{1}{8}\right) + \frac{1}{8} log_2\left(\frac{1}{8}\right) + \frac{1}{8} log_2\left(\frac{1}{8}\right) + \cdots \times 8times\right)\right) + \frac{14}{128} \times \left(\frac{1}{14} log_2\left(\frac{1}{14}\right) + \frac{1}{14} log_2\left(\frac{1}{14}\right) + \frac{1}{14} log_2\left(\frac{1}{14}\right) + \cdots \times 14times\right)\right]$$

$$= 10 \times \left(\frac{1}{128} \times \left(\frac{1}{1} log_2\left(\frac{1}{1}\right)\right)\right) = 0$$

$$= -\left[5 \times \left(\frac{2}{128} \times \left(\frac{1}{2} log_2\left(\frac{1}{2}\right) + \frac{1}{2} log_2\left(\frac{1}{2}\right)\right)\right) + \left(2 \times \left(\frac{3}{128} \times \left(\frac{1}{3} log_2\left(\frac{1}{3}\right) + \frac{1}{3} log_2\left(\frac{1}{3}\right) + \frac{1}{3} log_2\left(\frac{1}{3}\right)\right)\right)\right) = -0.0781$$

$$= \left(2 \times \left(\frac{3}{128} \times \left(\frac{1}{3} log_2\left(\frac{1}{3}\right) + \frac{1}{3} log_2\left(\frac{1}{3}\right) + \frac{1}{3} log_2\left(\frac{1}{3}\right)\right)\right)\right) = -0.0743$$

$$= \left(2 \times \left(\frac{5}{128} \times \left(\frac{1}{5} log_2\left(\frac{1}{5}\right) + \frac{1}{5} log_2\left(\frac{1}{5}\right) + \frac{1}{5} log_2\left(\frac{1}{5}\right) + \cdots \times 5times\right)\right)\right) = -0.1814$$

$=3\times\left(\frac{6}{128}\times\left(\frac{1}{6}log_2\left(\frac{1}{6}\right)+\frac{1}{6}log_2\left(\frac{1}{6}\right)+\frac{1}{6}log_2\left(\frac{1}{6}\right)+\frac{1}{6}log_2\left(\frac{1}{6}\right)+\frac{1}{6}log_2\left(\frac{1}{6}\right)+\frac{1}{6}log_2\left(\frac{1}{6}\right)\right)\right)=\ -0.3635$

$=\left(4\times\left(\frac{7}{128}\times\left(\frac{1}{7}log_2\left(\frac{1}{7}\right)+\frac{1}{7}log_2\left(\frac{1}{7}\right)+\frac{1}{7}log_2\left(\frac{1}{7}\right)+\cdots\times 7times\right)\right)\right)=\ -0.6141$

$=4\times\left(\frac{8}{128}\times\left(\frac{1}{8}log_2\left(\frac{1}{8}\right)+\frac{1}{8}log_2\left(\frac{1}{8}\right)+\frac{1}{8}log_2\left(\frac{1}{8}\right)+\cdots\times 8times\right)\right)=\ -0.7500$

$=\frac{14}{128}\times\left(\frac{1}{14}log_2\left(\frac{1}{14}\right)+\frac{1}{14}log_2\left(\frac{1}{14}\right)+\frac{1}{14}log_2\left(\frac{1}{14}\right)+\cdots\times 14times\right)=\ -0.4164$

$$H\left(\{c_{gl},c_{lac}\}|\{r_i^*\}_{i=1}^{M}\right)=-\sum_{y=1}^{Y}P_Y\sum_{x=1}^{X_y}P(x|y)\log_2 P(x|y)$$

$H(\{c_{gl},c_{lac}\}|\{r_i^*\}_{i=1}^{M}=\ -[-0.0781-0.0743-0.1814-0.3635-0.6141-0.7500\ -0.4164]=-[-2.4778]=\ 2.4778$

Implement the values to equation (1),

$$I(\{c_i\}_{i=1}^{N};\{r_i^*\}_{i=1}^{M})=H(\{c_i\}_{i=1}^{N})-H(\{c_i\}_{i=1}^{N}|\{r_i^*\}_{i=1}^{M})$$

MI = 7 bits - 2.4778 bits
MI = 4.5222 bits

STAGE II : Figure 17, Upper bounds of the steady-state mutual information for all the different combinations of seven compounds (extracellular) in E2E in B. theta with respect to Biomass only

B theta MI between 7 most influential factors and Biomass (Stage II) – zero values for all 14 groups for compound is eliminated

FBAGroups		Value	No.of.FBAs	Number of groups
1)	C,G	0.202862	C=4,G=5→9	2 (index: 4,8)
2)	L	0.203778	L=11→ 11	1 (index: 13)
3)	I	0.205694	I=7→ 7	1 (index: 10)
4)	F	0.206636	F=5→ 5	1 (index: 7)
5)	E,M	0.396393	E=4,M=28→32	2 (index: 6,14)
6)	N	0.398741	N=32→ 32	1 (index: 15)
7)	A,B,D,H	0.87827	A=3,B=3,D=4,H=6→16	4 (index: 2,3,5,9)
8)	J,K	0.883073	J=7,K=9→16	2 (index: 11,12)

$$5+7+9+11+2\times 16+2\times 32\ =128\ FBA$$

$\hat{H}(\{c_i\}_{i=1}^{N})$ = $\log_2(128)$ = 7 bits

$$H\left(\{c_{gl},c_{lac}\}|\{r_i^*\}_{i=1}^{M}\right)=-\sum_{y=1}^{Y}P_Y\sum_{x=1}^{X_y}P(x|y)\log_2 P(x|y)$$

No.of.Members in each group	1(11,7,5,32)	2(4,5,4,28,7,9)	4(3,3,4,6)
	|	|	|
No. Groups	4	3	1

$$\sum_{\substack{i=1\\ \text{No.Groups}}}^{8}\frac{\text{No. members i}}{\text{total combination}}\times\sum_{\substack{\text{No.members}\\ \text{eachgroup i}}}\frac{1}{\text{No. members i}}\log_2\left(\frac{1}{\text{No. members i}}\right)$$

$$\begin{aligned}=-\Bigg[&\left(\frac{5}{128}\times\left(\frac{1}{5}log_2\left(\frac{1}{5}\right)+\frac{1}{5}log_2\left(\frac{1}{5}\right)+\frac{1}{5}log_2\left(\frac{1}{5}\right)+\frac{1}{5}log_2\left(\frac{1}{5}\right)+\frac{1}{5}log_2\left(\frac{1}{5}\right)\right)\right)+\left(\frac{7}{128}\times\left(\frac{1}{7}log_2\left(\frac{1}{7}\right)+\frac{1}{7}log_2\left(\frac{1}{7}\right)+\frac{1}{7}log_2\left(\frac{1}{7}\right)+\frac{1}{7}log_2\left(\frac{1}{7}\right)+\frac{1}{7}log_2\left(\frac{1}{7}\right)+\frac{1}{7}log_2\left(\frac{1}{7}\right)+\frac{1}{7}log_2\left(\frac{1}{7}\right)\right)\right)\\&+\left(\frac{9}{128}\times\left(\frac{1}{9}log_2\left(\frac{1}{9}\right)+\frac{1}{9}log_2\left(\frac{1}{9}\right)+\frac{1}{9}log_2\left(\frac{1}{9}\right)+\cdots total\ 9\ times\right)+\left(\frac{11}{128}\times\left(\frac{1}{11}log_2\left(\frac{1}{11}\right)+\frac{1}{11}log_2\left(\frac{1}{11}\right)+\frac{1}{11}log_2\left(\frac{1}{11}\right)+\cdots total\ 11\ times\right)\right.\\&+2\times\left(\frac{16}{128}\times\left(\frac{1}{16}log_2\left(\frac{1}{16}\right)+\frac{1}{16}log_2\left(\frac{1}{16}\right)+\frac{1}{16}log_2\left(\frac{1}{16}\right)+\cdots total\ 16\ times\right)\right.\\&+\ 2\times\left(\frac{32}{128}\times\left(\frac{1}{32}log_2\left(\frac{1}{32}\right)+\frac{1}{32}log_2\left(\frac{1}{32}\right)+\frac{1}{32}log_2\left(\frac{1}{32}\right)+\cdots total\ 32\ times\right)\right)\Bigg]\end{aligned}$$

$=\left(\frac{5}{128}\times\left(\frac{1}{5}log_2\left(\frac{1}{5}\right)+\frac{1}{5}log_2\left(\frac{1}{5}\right)+\frac{1}{5}log_2\left(\frac{1}{5}\right)+\frac{1}{5}log_2\left(\frac{1}{5}\right)+\frac{1}{5}log_2\left(\frac{1}{5}\right)\right)\right)=\ -0.0907$

$=\left(\frac{7}{128}\times\left(\frac{1}{7}log_2\left(\frac{1}{7}\right)+\frac{1}{7}log_2\left(\frac{1}{7}\right)+\frac{1}{7}log_2\left(\frac{1}{7}\right)+\frac{1}{7}log_2\left(\frac{1}{7}\right)+\frac{1}{7}log_2\left(\frac{1}{7}\right)+\frac{1}{7}log_2\left(\frac{1}{7}\right)+\frac{1}{7}log_2\left(\frac{1}{7}\right)\right)\right)=\ -0.1535$

$=\left(\frac{9}{128}\times\left(\frac{1}{9}log_2\left(\frac{1}{9}\right)+\frac{1}{9}log_2\left(\frac{1}{9}\right)+\frac{1}{9}log_2\left(\frac{1}{9}\right)+\cdots total\ 9\ times\right)=\ -0.2229$

$= \left(\frac{11}{128} \times \left(\frac{1}{11} log_2 \left(\frac{1}{11}\right) + \frac{1}{11} log_2 \left(\frac{1}{11}\right) + \frac{1}{11} log_2 \left(\frac{1}{11}\right) + \cdots total\ 11\ times\right) = -0.2973$

$= 2 \times \left(\frac{16}{128} \times \left(\frac{1}{16} log_2 \left(\frac{1}{16}\right) + \frac{1}{16} log_2 \left(\frac{1}{16}\right) + \frac{1}{16} log_2 \left(\frac{1}{16}\right) + \cdots total\ 16\ times\right) = -1$

$= 2 \times \left(\frac{32}{128} \times \left(\frac{1}{32} log_2 \left(\frac{1}{32}\right) + \frac{1}{32} log_2 \left(\frac{1}{32}\right) + \frac{1}{32} log_2 \left(\frac{1}{32}\right) + \cdots total\ 32\ times\right) = -2.5000$

$= -[-0.0907 - 0.1535 - 0.2229 - 0.2973 - 1 - 2.5000]$
$= 4.2644$

$$I(\{c_i\}_{i=1}^{N}; \{r_i^*\}_{i=1}^{M}) = H(\{c_i\}_{i=1}^{N}) - H(\{c_i\}_{i=1}^{N} \mid \{r_i^*\}_{i=1}^{M})$$

MI = 7 bits – 4.2644 bits
MI = 2.7356 bits

B_theta_ONLY BIOMASS (Stage II) – with respect to intracellular metabolism as inputs and only biomass out put14 groups

FBAGroups		Value	Number of groups
1)	C,G	0.202862	2 (index: 4,8)
2)	L	0.203778	1 (index: 13)
3)	I	0.205694	1 (index: 10)
4)	F	0.206636	1 (index: 7)
5)	E,M	0.396393	2 (index: 6,14)
6)	N	0.398741	1 (index: 15)
7)	A,B,D,H	0.87827	4 (index: 2,3,5,9)
8)	J,K	0.883073	2 (index: 11,12)

$\hat{H}(\{c_i\}_{i=1}^{N})$ = $\log_2(14)$ = 3.8074 bits

$$H(\{c_{gl}, c_{lac}\}|\{r_i^*\}_{i=1}^{M}) = -\sum_{y=1}^{Y} P_Y \sum_{x=1}^{X_y} P(x|y)\log_2 P(x|y)$$

$$= -\sum_{No.Groups}^{8}{}_{i=1} \frac{No.members\ i}{total\ combination} \times \sum_{\substack{No.members \\ eachgroup\ i}} \frac{1}{No.members\ i} log_2\left(\frac{1}{No.\ members\ i}\right)$$

No.of.Members in each group	1	2	4
	|	|	|
No. Groups	4	3	1

$$1 \times 4 + 2 \times 3 + 1 \times 4 = 14\ FBAgroups$$

$$= -\left[4 \times \left(\frac{1}{14} \times \left(\frac{1}{1} log_2\left(\frac{1}{1}\right)\right)\right) + 3 \times \left(\frac{2}{14} \times \left(\frac{1}{2} log_2\left(\frac{1}{2}\right) + \frac{1}{2} log_2\left(\frac{1}{2}\right)\right)\right) + 1 \times \left(\frac{4}{14} \times \left(\frac{1}{4} log_2\left(\frac{1}{4}\right) + \frac{1}{4} log_2\left(\frac{1}{4}\right) + \frac{1}{4} log_2\left(\frac{1}{4}\right) + \frac{1}{4} log_2\left(\frac{1}{4}\right)\right)\right)\right]$$

$$= 4 \times \left(\frac{1}{14} \times \left(\frac{1}{1} \log_2\left(\frac{1}{1}\right)\right)\right) = 0$$

$$=3 \times \left(\frac{2}{14} \times \left(\frac{1}{2} log_2\left(\frac{1}{2}\right) + \frac{1}{2} log_2\left(\frac{1}{2}\right)\right)\right) = -0.4286$$

$$=1 \times \left(\frac{4}{14} \times \left(\frac{1}{4} log_2\left(\frac{1}{4}\right) + \frac{1}{4} log_2\left(\frac{1}{4}\right) + \frac{1}{4} log_2\left(\frac{1}{4}\right) + \frac{1}{4} log_2\left(\frac{1}{4}\right)\right)\right) = -0.5714$$

$$= -[0 - 0.4286 - 0.5714]$$
$$= 1.0000$$

$$I(\{c_i\}_{i=1}^{N}; \{r_i^*\}_{i=1}^{M}) = H(\{c_i\}_{i=1}^{N}) - H(\{c_i\}_{i=1}^{N} | \{r_i^*\}_{i=1}^{M})$$

MI = 3.8074 bits - 1.0000 bits
MI = 2.8074 bits

BT MI of all excreted and up-taken, secreted compounds including Biomass (Stage II) – 14 Groups (with respect to intracellular metabolism as inputs and excreted and up-taken, secreted, and biomass

FBAGroups	Value	Number of groups
1) C,G	0.202862	2 (index: 4,8)
2) L	0.203778	1 (index: 13)
3) I	0.205694	1 (index: 10)
4) F	0.206636	1 (index: 7)
5) E	0.396393	1 (index: 6)
6) M	0.396393	1 (index: 14)
7) N	0.398741	1 (index: 15)
8) A	0.87827	1 (index: 2)
9) B	0.87827	1 (index: 3)
10) D	0.87827	1 (index: 5)
11) H	0.87827	1 (index: 2,3,5,9)
12) J	0.883073	1 (index: 11)
13) K	0.883073	1 (index: 12)

$\hat{H}(\{c_i\}_{i=1}^{N})$ = $\log_2(14)$ = 3.8074 bits

$$H(\{c_{gl}, c_{lac}\} | \{r_i^*\}_{i=1}^{M}) = -\sum_{y=1}^{Y} P_Y \sum_{x=1}^{X_y} P(x|y) \log_2 P(x|y)$$

$$= -\sum_{No.Groups}^{13}{}_{i=1} \frac{No.members\ i}{total\ combination} \times \sum_{\substack{No.members \\ eachgroup\ i}} \frac{1}{No.members\ i} log_2 \left(\frac{1}{No.\ members\ i}\right)$$

No.of.Members in each group	1	2
	|	|
No. Groups	12	1

$$1 \times 12 + 2 \times 2 = 14\ FBAgroups$$

$$= -\left[12 \times \left(\frac{1}{14} \times \left(\frac{1}{1} log_2 \left(\frac{1}{1}\right)\right) + 1 \times \left(\frac{2}{14} \times \left(\frac{1}{2} log_2 \left(\frac{1}{2}\right) + \frac{1}{2} log_2 \left(\frac{1}{2}\right)\right)\right)\right]$$

$$= 12 \times \left(\frac{1}{14} \times \left(\frac{1}{1} log_2 \left(\frac{1}{1}\right)\right)\right) = 0$$

$$= 1 \times \left(\frac{2}{14} \times \left(\frac{1}{2} log_2 \left(\frac{1}{2}\right) + \frac{1}{2} log_2 \left(\frac{1}{2}\right)\right)\right) = -0.1429$$

$$= -[0 - 0.1429]$$
$$= 0.1429$$

$$I(\{c_i\}_{i=1}^{N}; \{r_i^*\}_{i=1}^{M}) = H(\{c_i\}_{i=1}^{N}) - H(\{c_i\}_{i=1}^{N} | \{r_i^*\}_{i=1}^{M})$$

MI = 3.8074 bits – 0.1429 bits
MI = 3.6645 bit
P.S.: Exactly same result is obtained as above for MI of all excreted and up-taken compounds excluding Biomass

STAGE II : Figur 18, Upper bounds of the steady-state mutual information for all the different combinations of seven compounds (extracellular) in E2E in M. smithii with respect to Biomass only

M_Smitti_MI between 7 most influential factors and Biomass (Stage II) – zero values for all 31 groups for compound is eliminated

FBAGroups	BiomassValue	No.of.FBAs	Number of groups	
1) S	0	S=5→5	1 (index: 20)	- Data start from row index2, row 1 is title of the compounds)
2) J,Z	0.102111	J=1,Z=7→8	2 (index: 11,27)	
3) W	0.102548	W=8→8	1 (index: 24)	
4) E,K,R	0.129823	E=1,K=2,R=5→8	3 (index: 6,12,19)	
5) D,Y	0.154711	D=1,Y=7→8	2 (index:5,26)	
6) C	0.162135	C=1→1	1 (index:4)	
7) M,T	0.170735	M=2, T=6→8	2 (index:14,21)	
8) I,U	0.182998	I=1, U=6→7	2 (index:10,22)	
9) O,V,AB	0.23574	O=2,V=6,AB=8→16	3 (index:6,23,29)	
10) L,AE	0.259537	L=2,AE=14→16	2(index:13,32)	
11) B, AA	0.289634	B=1, AA=7→8	2 (index:3,28)	
12) H,X	0.306145	H=1,X=7→8	2 (index:9,25)	
13) A,N,P,Q	0.417114	A=1,N=2,P=3,Q=3→9	4 (index: 2,15,17,18)	- matlab index(1,14, 16, 17)
14) AC,AD	0.422201	AC =8, AD=8→16	2 (index: 30,31)	
15) F,G	0.433068	F=1,G=1→2	2 (index: 7,8)	

$$1 + 2 + 5 + 7 + 7 \times 8 + 9 + 16 \times 3 = 128\ FBA$$

$H(\{c_i\}_{i=1}^{N})$ = $\log_2(128)$ = 7 bits

$$H\left(\{c_{gl}, c_{lac}\} \mid \{r_i^*\}_{i=1}^{M}\right) = -\sum_{y=1}^{Y} P_Y \sum_{x=1}^{X_y} P(x|y) \log_2 P(x|y)$$

$$= -\sum_{No.Groups}^{15}{}_{i=1} \frac{No.members\ i}{total\ combination} \times \sum_{eachgroup\ i}^{No.members} \frac{1}{No.members\ i} log_2\left(\frac{1}{No.\ members\ i}\right)$$

No.of.Members in each group	1(5,8,1)	2(8,8,8,7,16,8,816,2)	3(8,16)	4(9)
	|	|	|	|
No. Groups	3	9	2	1

$$= -\left[\left(\frac{1}{128} \times \left(\frac{1}{1} log_2\left(\frac{1}{1}\right)\right)\right) + \left(\frac{2}{128} \times \left(\frac{1}{2} log_2\left(\frac{1}{2}\right) + \frac{1}{2}\ log_2\left(\frac{1}{2}\right)\right)\right) + \left(\frac{5}{128} \times \left(\frac{1}{5} log_2\left(\frac{1}{5}\right) + \frac{1}{5} log_2\left(\frac{1}{5}\right) + \frac{1}{5} log_2\left(\frac{1}{5}\right) + \frac{1}{5} log_2\left(\frac{1}{5}\right) + \frac{1}{5} log_2\left(\frac{1}{5}\right)\right)\right) + \left(\frac{7}{128} \times \left(\frac{1}{7} log_2\left(\frac{1}{7}\right) + \frac{1}{7} log_2\left(\frac{1}{7}\right) + \frac{1}{7} log_2\left(\frac{1}{7}\right) + \frac{1}{7} log_2\left(\frac{1}{7}\right) + \frac{1}{7} log_2\left(\frac{1}{7}\right) + \frac{1}{7} log_2\left(\frac{1}{7}\right) + \frac{1}{7} log_2\left(\frac{1}{7}\right)\right)\right) + \left(7 \times \left(\frac{8}{128} \times \left(\frac{1}{8} log_2\left(\frac{1}{8}\right) + \frac{1}{8} log_2\left(\frac{1}{8}\right) + \frac{1}{8} log_2\left(\frac{1}{8}\right) + \frac{1}{8} log_2\left(\frac{1}{8}\right) + \frac{1}{8} log_2\left(\frac{1}{8}\right) + \frac{1}{8} log_2\left(\frac{1}{8}\right) + \frac{1}{8} log_2\left(\frac{1}{8}\right) + \frac{1}{8} log_2\left(\frac{1}{8}\right)\right)\right)\right) + \left(\frac{9}{128} \times \left(\frac{1}{9} log_2\left(\frac{1}{9}\right) + \frac{1}{9} log_2\left(\frac{1}{9}\right) + \frac{1}{9} log_2\left(\frac{1}{9}\right) + \frac{1}{9} log_2\left(\frac{1}{9}\right) + \frac{1}{9} log_2\left(\frac{1}{9}\right) + \frac{1}{9} log_2\left(\frac{1}{9}\right) + \frac{1}{9} log_2\left(\frac{1}{9}\right) + \frac{1}{9} log_2\left(\frac{1}{9}\right) + \frac{1}{9} log_2\left(\frac{1}{9}\right)\right)\right) + \left(3 \times \left(\frac{16}{128} \times \left(\frac{1}{16} log_2\left(\frac{1}{16}\right) + \frac{1}{16} log_2\left(\frac{1}{16}\right) + \frac{1}{16} log_2\left(\frac{1}{16}\right) + \frac{1}{16} log_2\left(\frac{1}{16}\right) + \frac{1}{16} log_2\left(\frac{1}{16}\right) + \frac{1}{16} log_2\left(\frac{1}{16}\right) + \frac{1}{16} log_2\left(\frac{1}{16}\right) + \frac{1}{16} log_2\left(\frac{1}{16}\right) + \frac{1}{16} log_2\left(\frac{1}{16}\right) + \frac{1}{16} log_2\left(\frac{1}{16}\right) + \frac{1}{16} log_2\left(\frac{1}{16}\right) + \frac{1}{16} log_2\left(\frac{1}{16}\right) + \frac{1}{16} log_2\left(\frac{1}{16}\right) + \frac{1}{16} log_2\left(\frac{1}{16}\right) + \frac{1}{16} log_2\left(\frac{1}{16}\right) + \frac{1}{16} log_2\left(\frac{1}{16}\right)\right)\right)\right)$$

$$= \left(\frac{1}{128} \times \left(\frac{1}{1} log_2\left(\frac{1}{1}\right)\right)\right) = 0$$

$$= \left(\frac{2}{128} \times \left(\frac{1}{2} log_2\left(\frac{1}{2}\right) + \frac{1}{2}\ log_2\left(\frac{1}{2}\right)\right)\right) = -0.0156$$

$$= \left(\frac{5}{128} \times \left(\frac{1}{5} \log_2\left(\frac{1}{5}\right) + \frac{1}{5} \log_2\left(\frac{1}{5}\right) + \frac{1}{5} \log_2\left(\frac{1}{5}\right) + \frac{1}{5} \log_2\left(\frac{1}{5}\right) + \frac{1}{5} \log_2\left(\frac{1}{5}\right)\right)\right) = -0.0907$$

$$= \left(\frac{7}{128} \times \left(\frac{1}{7} log_2\left(\frac{1}{7}\right) + \frac{1}{7} log_2\left(\frac{1}{7}\right) + \frac{1}{7} log_2\left(\frac{1}{7}\right) + \frac{1}{7} log_2\left(\frac{1}{7}\right) + \frac{1}{7} log_2\left(\frac{1}{7}\right) + \frac{1}{7} log_2\left(\frac{1}{7}\right) + \frac{1}{7} log_2\left(\frac{1}{7}\right)\right)\right) = -0.1535$$

$$= \left(7 \times \left(\frac{8}{128} \times \left(\frac{1}{8} log_2\left(\frac{1}{8}\right) + \frac{1}{8} log_2\left(\frac{1}{8}\right) + \frac{1}{8} log_2\left(\frac{1}{8}\right) + \frac{1}{8} log_2\left(\frac{1}{8}\right) + \frac{1}{8} log_2\left(\frac{1}{8}\right) + \frac{1}{8} log_2\left(\frac{1}{8}\right) + \frac{1}{8} log_2\left(\frac{1}{8}\right) + \frac{1}{8} log_2\left(\frac{1}{8}\right)\right)\right)\right) = -1.3125$$

$$= \left(\frac{9}{128} \times \left(\frac{1}{9} log_2\left(\frac{1}{9}\right) + \frac{1}{9} log_2\left(\frac{1}{9}\right) + \frac{1}{9} log_2\left(\frac{1}{9}\right) + \frac{1}{9} log_2\left(\frac{1}{9}\right) + \frac{1}{9} log_2\left(\frac{1}{9}\right) + \frac{1}{9} log_2\left(\frac{1}{9}\right) + \frac{1}{9} log_2\left(\frac{1}{9}\right) + \frac{1}{9} log_2\left(\frac{1}{9}\right) + \frac{1}{9} log_2\left(\frac{1}{9}\right)\right)\right) = -0.2229$$

$$= 3 \times \left(\frac{16}{128} \times \left(\frac{1}{16} log_2\left(\frac{1}{16}\right) + \frac{1}{16} log_2\left(\frac{1}{16}\right) + \frac{1}{16} log_2\left(\frac{1}{16}\right) + \frac{1}{16} log_2\left(\frac{1}{16}\right) + \frac{1}{16} log_2\left(\frac{1}{16}\right) + \frac{1}{16} log_2\left(\frac{1}{16}\right) + \frac{1}{16} log_2\left(\frac{1}{16}\right) + \frac{1}{16} log_2\left(\frac{1}{16}\right) + \frac{1}{16} log_2\left(\frac{1}{16}\right) + \frac{1}{16} log_2\left(\frac{1}{16}\right) + \frac{1}{16} log_2\left(\frac{1}{16}\right) + \frac{1}{16} log_2\left(\frac{1}{16}\right) + \frac{1}{16} log_2\left(\frac{1}{16}\right) + \frac{1}{16} log_2\left(\frac{1}{16}\right) + \frac{1}{16} log_2\left(\frac{1}{16}\right) + \frac{1}{16} log_2\left(\frac{1}{16}\right)\right)\right) = -2.5850$$

$$= -[0 - 0.0156 - 0.0907 - 0.1535 - 1.3125 - 0.2229 - 1.5]$$

$$= 3.2952$$

$$I(\{c_i\}_{i=1}^{N}; \{r_i^*\}_{i=1}^{M}) = H(\{c_i\}_{i=1}^{N}) - H(\{c_i\}_{i=1}^{N} \mid \{r_i^*\}_{i=1}^{M})$$

MI = 7 bits- 3.2952bits

MI = 3.7048 bits

M_Smitti_ONLY BIOMASS (Stage II) – 31 Groups

FBAGroups	Value	Number of groups	
1) S	0	1 (index: 20)	- Data start from row index2, row 1 is title of the compounds)
2) J,Z	0.102111	2 (index: 11,27)	
3) W	0.102548	1 (index: 24)	
4) E,K,R	0.129823	3 (index: 6,12,19)	
5) D,Y	0.154711	2 (index:5,26)	
6) C	0.162135	1 (index:4)	
7) M,T	0.170735	2 (index:14,21)	
8) I,U	0.182998	2 (index:10,22)	
9) O,V,AB	0.23574	3 (index:6,23,29)	
10) L,AE	0.259537	2(index:13,32)	
11) B, AA	0.289634	2 (index:3,28)	
12) H,X	0.306145	2 (index:9,25)	
13) A,N,P,Q	0.417114	4 (index: 2,15,17,18)	- matlab index(1,14, 16, 17)
14) AC,AD	0.422201	2 (index: 30,31)	
15) F,G	0.433068	2 (index: 7,8)	

$\hat{H}(\{c_i\}_{i=1}^{N})$ = $\log_2(31)$ = 4.9542 bits

$$H(\{c_{gl}, c_{lac}\} | \{r_i^*\}_{i=1}^{M}) = -\sum_{y=1}^{Y} P_Y \sum_{x=1}^{X_y} P(x|y) \log_2 P(x|y)$$

$$= -\sum_{\substack{i=1 \\ No.Groups}}^{15} \frac{No.members\ i}{total\ combination} \times \sum_{\substack{No.members \\ eachgroup\ i}} \frac{1}{No.members\ i} log_2\left(\frac{1}{No.\ members\ i}\right)$$

No.of.Members in each group	1	2	3	4
	|	|	|	|
No. Groups	3	9	2	1

$$1 \times 3 + 2 \times 9 + 3 \times 2 + 1 \times 4 = 31\ FBAgroups$$

$$-\left[3 \times \left(\frac{1}{31} \times \left(\frac{1}{1} log_2\left(\frac{1}{1}\right)\right)\right) + 9 \times \left(\frac{2}{31} \times \left(\frac{1}{2} log_2\left(\frac{1}{2}\right) + \frac{1}{2} log_2\left(\frac{1}{2}\right)\right)\right) + \left(2 \times \left(\frac{3}{31} \times \left(\frac{1}{3} log_2\left(\frac{1}{3}\right) + \frac{1}{3} log_2\left(\frac{1}{3}\right) + \frac{1}{3} log_2\left(\frac{1}{3}\right)\right)\right)\right) + \left(1 \times \left(\frac{4}{31} \times \left(\frac{1}{4} log_2\left(\frac{1}{4}\right) + \frac{1}{4} log_2\left(\frac{1}{4}\right) + \frac{1}{4} log_2\left(\frac{1}{4}\right) + \frac{1}{4} log_2\left(\frac{1}{4}\right)\right)\right)\right)\right.$$

$=3 \times \left(\frac{1}{31} \times \left(\frac{1}{1} log_2\left(\frac{1}{1}\right)\right)\right) = 0$

$=9 \times \left(\frac{2}{31} \times \left(\frac{1}{2} log_2\left(\frac{1}{2}\right) + \frac{1}{2} log_2\left(\frac{1}{2}\right)\right)\right) = -0.5806$

$=2 \times \left(\frac{3}{31} \times \left(\frac{1}{3} log_2\left(\frac{1}{3}\right) + \frac{1}{3} log_2\left(\frac{1}{3}\right) + \frac{1}{3} log_2\left(\frac{1}{3}\right)\right)\right)$ =-0.3068

$=\left(1 \times \left(\frac{4}{31} \times \left(\frac{1}{4} log_2\left(\frac{1}{4}\right) + \frac{1}{4} log_2\left(\frac{1}{4}\right) + \frac{1}{4} log_2\left(\frac{1}{4}\right) + \frac{1}{4} log_2\left(\frac{1}{4}\right)\right)\right)\right)$ = -0.2581

$= -[0 - 0.5806 - 0.3068 - 0.2581]$

= 1.1455

$$I(\{c_i\}_{i=1}^{N}; \{r_i^*\}_{i=1}^{M}) = H(\{c_i\}_{i=1}^{N}) - H(\{c_i\}_{i=1}^{N} | \{r_i^*\}_{i=1}^{M})$$

MI = 4.9542 bits - 1.1455 bits
MI = 3.8087 bit

M_Smitti_MI of all excreted and up-taken compounds including Biomass (Stage II) – 31 Groups

FBAGroups	Indexes	Number of groups	
1) A,N,P,Q	1 14 16 17	4	-matlab index
2) B,AA	2 27	2	
3) D,Y	4 25	2	
4) E,K,R	5 11 18	3	
5) F,G	6 7	2	
6) H,X	8 24	2	
7) I, U	9 21	2	
8) J,Z	10 26	2	
9) L,AE	12 31	2	
10) M,T	13 20	2	
11) O,V,AB	15 22 28	3	
12) C	3	1	
13) S	19	1	
14) W	23	1	
15) AC	29	1	
16) AD	30	1	

$\hat{H}(\{c_i\}_{i=1}^{N})$ = $\log_2(31)$ = 4.9542 bits

$$H(\{c_{gl}, c_{lac}\}|\{r_i^*\}_{i=1}^{M}) = -\sum_{y=1}^{Y} P_Y \sum_{x=1}^{X_y} P(x|y)\log_2 P(x|y)$$

$$= -\sum_{i=1}^{16}{}_{No.Groups} \frac{No.members\ i}{total\ combination} \times \sum_{\substack{No.members \\ eachgroup\ i}} \frac{1}{No.members\ i} log_2\left(\frac{1}{No.\ members\ i}\right)$$

No.of.Members in each group	1	2	3	4
	|	|	|	|
No.of groups	5	8	2	1

$$1 \times 5 + 2 \times 8 + 3 \times 2 + 1 \times 4 = 31\ FBAgroups$$

$$-\left[5 \times \left(\frac{1}{31} \times \left(\frac{1}{1} log_2\left(\frac{1}{1}\right)\right) + 8 \times \left(\frac{2}{31} \times \left(\frac{1}{2} log_2\left(\frac{1}{2}\right) + \frac{1}{2} log_2\left(\frac{1}{2}\right)\right) + \left(2 \times \left(\frac{3}{31} \times \left(\frac{1}{3} log_2\left(\frac{1}{3}\right) + \frac{1}{3} log_2\left(\frac{1}{3}\right) + \frac{1}{3} log_2\left(\frac{1}{3}\right)\right)\right)\right) + \left(1 \times \left(\frac{4}{31} \times \left(\frac{1}{4} log_2\left(\frac{1}{4}\right) + \frac{1}{4} log_2\left(\frac{1}{4}\right) + \frac{1}{4} log_2\left(\frac{1}{4}\right) + \frac{1}{4} log_2\left(\frac{1}{4}\right)\right)\right)\right)\right)\right)$$

$=5 \times \left(\frac{1}{31} \times \left(\frac{1}{1} log_2\left(\frac{1}{1}\right)\right)\right) = 0$

$=8 \times \left(\frac{2}{31} \times \left(\frac{1}{2} log_2\left(\frac{1}{2}\right) + \frac{1}{2} log_2\left(\frac{1}{2}\right)\right)\right) = -0.5161$

$=2 \times \left(\frac{3}{31} \times \left(\frac{1}{3} log_2\left(\frac{1}{3}\right) + \frac{1}{3} log_2\left(\frac{1}{3}\right) + \frac{1}{3} log_2\left(\frac{1}{3}\right)\right)\right)$ =-0.3068

$=\left(1 \times \left(\frac{4}{31} \times \left(\frac{1}{4} log_2\left(\frac{1}{4}\right) + \frac{1}{4} log_2\left(\frac{1}{4}\right) + \frac{1}{4} log_2\left(\frac{1}{4}\right) + \frac{1}{4} log_2\left(\frac{1}{4}\right)\right)\right)\right)$ = -0.2581

$= -[0 - 0.5161 - 0.3068 - 0.2581]$

= 1.0810

$$I(\{c_i\}_{i=1}^{N}; \{r_i^*\}_{i=1}^{M}) = H(\{c_i\}_{i=1}^{N}) - H(\{c_i\}_{i=1}^{N} | \{r_i^*\}_{i=1}^{M})$$

MI = 4.9542 bits - 1.0810 bits

MI = 3.8732 bit

P.S.: Exactly same result is obtained as above for MI of all excreted and up-taken compounds excluding Biomass