From Cellular Automata to the Lattice Boltzmann Method

BGCE Honours Project

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Outline

The Lattice Boltzmann Method

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Collide Step

Particle distribution function f_i:
discrete in space and time,
continuous in probability values

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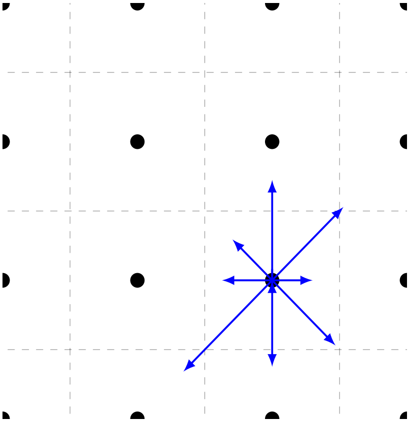

One collide step for a D2Q9-stencil

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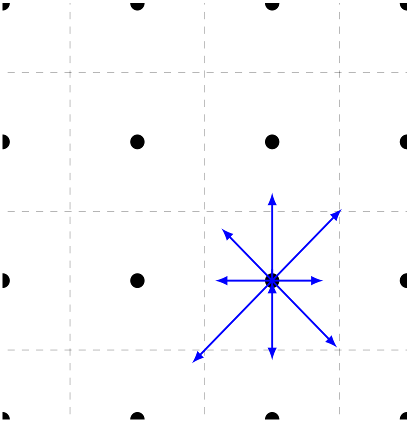

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Stream Step

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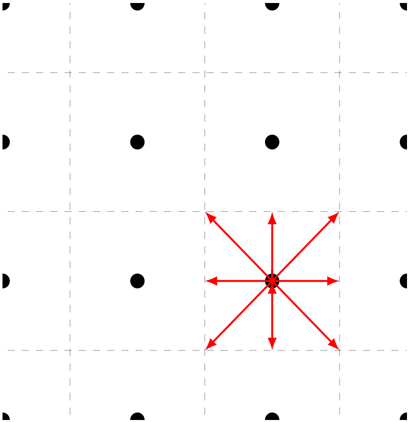

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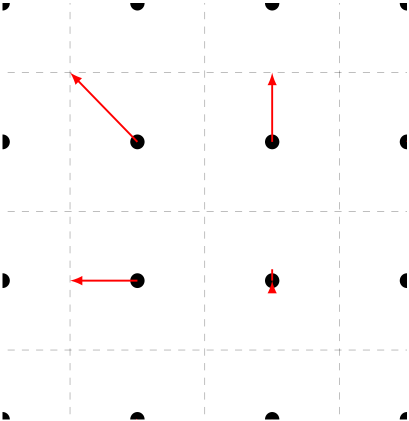

One stream step for a D2Q9-stencil

The Lattice Boltzmann Equation

...is what we get when we apply both steps:

$$f_i(\boldsymbol{x} + \boldsymbol{c_i}\Delta t, t + \Delta t) = f_i(\boldsymbol{x}, t) + \Omega_i(\boldsymbol{x}, t)$$

With collision operator $\Omega_i(\boldsymbol{x}, t)$, e.g.:

- BGK: $\Omega(f) = \frac{1}{\tau}(f_i^{eq} - f_i) = \omega(f_i^{eq} - f_i)$
- TRT:

and the equilibrium particle distribution function $f_i^{eq} =$

Cf. Boltzmann Equation

$$\frac{\partial f}{\partial t} + \frac{\boldsymbol{p}}{m} \cdot \nabla f = \left(\frac{\partial f}{\partial t}\right)_{coll}$$

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Moments - the Interesting Outputs in LBM

Density and velocity are given locally

$$\rho(\boldsymbol{x}, t) = \sum_i f_i(\boldsymbol{x}, t)$$

$$\rho\,\boldsymbol{u}(\boldsymbol{x}, t) = \sum_i \boldsymbol{c_i} f_i(\boldsymbol{x}, t)$$

Nondimensionalization – setting the step size

= Normalization in the three physical dimensions time, space, mass

Usual choice:

$$
\begin{aligned}
\Delta x^* = 1 = C_x \cdot \Delta x \\
\Delta t^* = 1 = C_t \cdot \Delta t \\
\Delta \rho^* = 1 = C_\rho \cdot \Delta \rho
\end{aligned}
$$

The kinematic velocity in lattice units is directly linked to ω/τ:

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Thanks for listening.

Any questions?

References

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