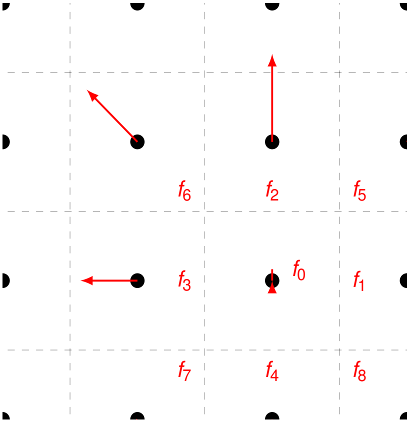
Stream Step

Particle distribution function f_i:
discrete in space and time,
continuous in probability values



One stream step for a D2Q9-stencil

The Lattice Boltzmann Equation

...is what we get when we apply both steps:

$$f_i(\boldsymbol{x} + \boldsymbol{c_i}\Delta t, t + \Delta t) = f_i(\boldsymbol{x}, t) + \Omega_i(\boldsymbol{x}, t)$$

With collision operator $\Omega_i(\boldsymbol{x}, t)$, e.g.:

- BGK: $\Omega(f) = \frac{1}{\tau}(f_i^{\mathrm{eq}} - f_i) = \omega(f_i^{\mathrm{eq}} - f_i)$
- TRT: is often nicer but needs more formulas $\rightarrow$ please look up

and the equilibrium particle distribution function $f_i^{\mathrm{eq}} = w_i \rho \left(1 + \frac{\vec{e}_i \vec{u}}{c_s^2} + \frac{(\vec{e}_i \vec{u})^2}{2c_s^4} - \frac{\vec{u}^2}{2c_s^2}\right)$

$w_0 = \frac{4}{9}, w_{1,2,3,4} = \frac{1}{9}, w_{5,6,7,8} = \frac{1}{36}$ for D2Q9

Moments - the Interesting Outputs in LBM

Density and velocity are given locally

$$\rho(\boldsymbol{x}, t) = \sum_i f_i(\boldsymbol{x}, t)$$

$$\rho\,\boldsymbol{u}(\boldsymbol{x}, t) = \sum_i \boldsymbol{c_i} f_i(\boldsymbol{x}, t)$$

Nondimensionalization – setting the step size

= Normalization in the three physical dimensions time, space, mass

Usual choice:

$$\Delta x^* = 1 = C_x \cdot \Delta x$$
$$\Delta t^* = 1 = C_t \cdot \Delta t$$
$$\Delta \rho^* = 1 = C_\rho \cdot \Delta \rho$$

The kinematic velocity in lattice units is directly linked to ω or τ:

$$\nu^* = (\tau - 0.5)c_s^2 = (\frac{1}{\omega} - 0.5)c_s^2, \ \ c_s^2 = \frac{1}{3}$$

$\Rightarrow 0.5 < \tau$ ... and not much larger than 1!