

# ECE539 Final Report

Victor Freire & Ian Ruh

2022

# MPC Problem in IPOPT Format

Let the MPC problem be:

$$\begin{aligned}
 & \text{minimize} && \frac{1}{2} \tilde{\mathbf{x}}_N^T Q_f \tilde{\mathbf{x}}_N + \frac{1}{2} \sum_{k=0}^{N-1} (\tilde{\mathbf{x}}_k^T Q \tilde{\mathbf{x}}_k + \mathbf{u}_k^T R \mathbf{u}_k) && (\text{NMPC}) \\
 & \text{subject to} && \mathbf{x}_{k+1} = f_d(\mathbf{x}_k, \mathbf{u}_k), \quad k = 0, \dots, N-1, \\
 & && \tilde{\mathbf{x}}_k = \mathbf{x}_k - \mathbf{x}_f, \quad k = 0, \dots, N, \\
 & && \text{and given } \mathbf{x}_0,
 \end{aligned}$$

with decision variables  $\mathbf{x}_i \in \mathbb{R}^{n_x}, i = 0, \dots, N$  and  $\mathbf{u}_j \in \mathbb{R}^{n_u}, j = 0, \dots, N-1$ .

The IPOPT problem format is:

$$\begin{aligned}
 & \text{minimize} && f(\mathbf{z}) && (\text{IPOPT}) \\
 & \text{subject to} && \mathbf{g}^L \leq g(\mathbf{z}) \leq \mathbf{g}^U, \\
 & && \mathbf{z}^L \leq \mathbf{z} \leq \mathbf{z}^U,
 \end{aligned}$$

with decision variable  $\mathbf{z} \in \mathbb{R}^n$ .

## Embedding

The vector  $\mathbf{z} \in \mathbb{R}^{6N+4}$  (i.e.,  $n = 6N + 4$ ) is formatted as follows:

$$\mathbf{z} = [\mathbf{x}_0^T \quad \cdots \quad \mathbf{x}_N^T \quad \mathbf{u}_0^T \quad \cdots \quad \mathbf{u}_{N-1}^T]^T. \quad (1)$$

We will also use zero indexing for elements of  $\mathbf{z}$  such that, for example:  $z_0 = x_0$ ,  $z_3 = \psi_0$ ,  $z_5 = y_1$ , etc. In general, we can observe the following indexing mappings:

$$\mathbf{x}_i = \mathbf{z}_{[4i, 4i+4]}, \quad (2a)$$

$$\mathbf{u}_i = \mathbf{z}_{[4N+2i+4, 4N+2i+6]}, \quad (2b)$$

where  $\mathbf{z}_{[i, j]} \triangleq (z_i, z_{i+1}, \dots, z_{j-1})^T$ .

## General Nonlinear Constraints

We require  $4N$  general nonlinear constraints (i.e.,  $m = 4N$ ). Begin by letting  $\mathbf{g}^L = \mathbf{g}^U = \mathbf{0}_{m \times 1}$ . Then, the function  $g : \mathbb{R}^n \rightarrow \mathbb{R}^m$  has the form:

$$g(\mathbf{z}) = \begin{bmatrix} f_d(\mathbf{x}_0, \mathbf{u}_0) - \mathbf{x}_1 \\ \vdots \\ f_d(\mathbf{x}_{N-1}, \mathbf{u}_{N-1}) - \mathbf{x}_N \end{bmatrix} \quad (3)$$

## Lower and Upper Bounds

The only required constraint of this type is to enforce  $\mathbf{x}_0 = \mathbf{x}_{init}$  where  $\mathbf{x}_{init}$  is a given parameter. Therefore, we can choose:

$$\mathbf{z}^L = [\mathbf{x}_{init}^T \quad -\infty \mathbf{1}_{6N \times 1}]^T, \quad (4a)$$

$$\mathbf{z}^U = [\mathbf{x}_{init}^T \quad \infty \mathbf{1}_{6N \times 1}]^T. \quad (4b)$$

## Objective Function

The objective function is already a function of  $\mathbf{z}$  by using the index mapping.