ECE539 Final Report

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MPC Problem in IPOPT Format

Let the MPC problem be:

$$\begin{aligned}
\text{minimize} \quad & \frac{1}{2}\tilde{\mathbf{x}}_N^T Q_f \tilde{\mathbf{x}}_N + \frac{1}{2}\sum_{k=0}^{N-1}\left(\tilde{\mathbf{x}}_k^T Q \tilde{\mathbf{x}}_k + \mathbf{u}_k^T R \mathbf{u}_k\right) \\
\text{subject to} \quad & \mathbf{x}_{k+1} = f_d(\mathbf{x}_k, \mathbf{u}_k), \quad k = 0, \ldots, N-1, \\
& \tilde{\mathbf{x}}_k = \mathbf{x}_k - \mathbf{x}_f, \quad k = 0, \ldots, N, \\
& \text{and given} \quad \mathbf{x}_0,
\end{aligned} \tag{NMPC}$$

with decision variables $\mathbf{x}_i \in \mathbb{R}^{n_x}, i = 0, \ldots, N$ and $\mathbf{u}_j \in \mathbb{R}^{n_u}, j = 0, \ldots, N-1$.

The IPOPT problem format is:

$$\begin{aligned}
\text{minimize} \quad & f(\mathbf{z}) \\
\text{subject to} \quad & \mathbf{g}^L \leq g(\mathbf{z}) \leq \mathbf{g}^U, \\
& \mathbf{z}^L \leq \mathbf{z} \leq \mathbf{z}^U,
\end{aligned} \tag{IPOPT}$$

with decision variable $\mathbf{z} \in \mathbb{R}^n$.

Embedding

The vector $\mathbf{z} \in \mathbb{R}^{6N+4}$ (i.e., $n = 6N+4$) is formatted as follows:

$$\mathbf{z} = \begin{bmatrix} \mathbf{x}_0^T & \cdots & \mathbf{x}_N^T & \mathbf{u}_0^T & \cdots & \mathbf{u}_{N-1}^T \end{bmatrix}^T. \tag{1}$$

We will also use zero indexing for elements of $\mathbf{z}$ such that, for example: $z_0 = x_0$, $z_3 = \psi_0$, $z_5 = y_1$, etc. In general, we can observe the following indexing mappings:

$$\mathbf{x}_i = \mathbf{z}_{[4i,\ 4i+4]}, \tag{2a}$$

$$\mathbf{u}_i = \mathbf{z}_{[4N+2i+4,\ 4N+2i+6]}, \tag{2b}$$

where $\mathbf{z}_{[i,\ j]} \triangleq (z_i, z_{i+1}, \ldots, z_{j-1})^T$. It follows that:

$$x_i = z_{4i}, \tag{3a}$$

$$y_i = z_{4i+1}, \tag{3b}$$

$$v_i = z_{4i+2}, \tag{3c}$$

$$\psi_i = z_{4i+3}, \tag{3d}$$

$$\dot{v}_i = z_{4N+2i+4}, \tag{3e}$$

$$\dot{\psi}_i = z_{4N+2i+5}. \tag{3f}$$

General Nonlinear Constraints

We require $4N$ general nonlinear constraints (i.e., $m = 4N$). Begin by letting $\mathbf{g}^L = \mathbf{g}^U = \mathbf{0}_{m\times 1}$. Then, the function $g : \mathbb{R}^n \to \mathbb{R}^m$ has the form:

$$g(\mathbf{z}) = \begin{bmatrix} f_d(\mathbf{x}_0, \mathbf{u}_0) - \mathbf{x}_1 \\ \vdots \\ f_d(\mathbf{x}_{N-1}, \mathbf{u}_{N-1}) - \mathbf{x}_N \end{bmatrix} \tag{4}$$

Lower and Upper Bounds

The only required constraint of this type is to enforce $\mathbf{x}_0 = \mathbf{x}_{init}$ where $\mathbf{x}_{init}$ is a given parameter. Therefore, we can choose:

$$\mathbf{z}^L = \begin{bmatrix} \mathbf{x}_{init}^T & -\infty \mathbf{1}_{6N\times 1} \end{bmatrix}^T, \tag{5a}$$

$$\mathbf{z}^U = \begin{bmatrix} \mathbf{x}_{init}^T & \infty \mathbf{1}_{6N\times 1} \end{bmatrix}^T. \tag{5b}$$

Objective Function

The objective function is already a function of $\mathbf{z}$ by using the index mapping.

Alternative Objective Function

This is a different objective function to try:

$$J = \tilde{\mathbf{x}}_N^T Q_f \tilde{\mathbf{x}}_N + \sum_{k=0}^{N-1} (\dot{v}_k^2 + v_k^2 \dot{\psi}_k^2) \tag{6}$$