272SM: Introduction to Artificial Intelligence
Spring 2023
University of Trieste

Question	1	2	3	4	5	6	Total
Maximum Points	20	20	20	20	20	20	120
Achieved Points							

Instructors: Laura Nenzi, Tatjana Petrov June. 6th, 2023

Instructions:

Language: The questions of this examination are in English. Please answer in English.

Aids: The use of non-technical language dictionaries and non-programmable, scientific calculators is permitted.

Time allowed: **150 minutes**. Answer **all questions**. Write your name and student ID on the fields below. Use a **pen, not a pencil**, and **don't write in red or green ink** anywhere on the exam paper. Write legibly - you will not get a mark for answers that we cannot easily read. This examination paper is **14** pages long - check that you have all of them and let one of the proctors know if pages are missing!

The numbers in **squared brackets** after the question title denote the points assigned to that question. Write your answers on the question sheets in the designated fields and areas. If you need more space for your answer, you can ask for additional sheets of paper. Make sure to add your student ID on that page; note in the space below the question that you used an additional sheet and clearly state which is the final solution. If information appears to be missing from a question, **make a reasonable assumption**, state it, and proceed.

Individual discussions on the examination results can be arranged via email.

Good luck!

Name (please print): Bredariol Francesco

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Exam-ID: 272SM

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Question 1: State Spaces (12 points)

Pacman finds himself in an N by M grid. He starts in the top left corner of the grid and his goal is to reach the bottom right corner. However there are G ghosts in fixed locations around the grid that Pacman needs to kill before he reaches the goal. Pacman accomplishes this using his new car the Pacmobile. Every turn Pacman chooses a radius value, r, which can be any integer from 0 to R inclusive. This shocks and kills all ghosts within r grids of him (by Manhattan distance). However, the Pacmobile has a limited battery that contains E (a positive integer) units of charge. Whenever it produces an electric field of radius r, the Pacmobile loses r units of charge. The Pacmobile has 5 moving actions: left, right, up, down, and stop. All of those choices will cost 1 unit of charge as well. If the Pacmobile runs out of charge Pacman can no longer do anything and the game ends in a loss.

To further clarify, on each turn, Pacman chooses a movement action and radius value r. He then shocks all squares in a radius r around him, performs the movement, and loses $r + 1$ units of charge. Recall he starts with E units of charge and he has to kill G ghosts in fixed locations before reaching the bottom right square to win the game.

(a) [3 pts] Mark true or false for the following subparts

(1) [1 pt] Running Breadth First Search on this search problem will return the solution that takes the smallest amount of turns.

☒ (A) True ○ (B) False

(2) [1 pt]Running Breadth First Search on this search problem will return the solution that uses the smallest amount of charge.

○ (A) True ☒ (B) False

(3) [1 pt] Running Breadth First Search on this search problem will return the solution where Pacman travels the least amount of grid distance.

☒ (A) True ○ (B) False

(b) [4 pts] Write your answers to the following two subparts in terms of N,M,G,R,E and any scalars you need.

(1) [2 pts] What is the size of the state space, X, using a minimal state representation?

$X =$ NxMxEx2^(NxM)

(2) [2 pts] What is the maximum possible branching factor from any state?

Rx5

(c) [6 pts] Now we are going to investigate how the size of the state space changes with certain rule changes. Write your answers to the following two subparts in terms of X (the size of the original state space) and the terms N,M,G,R,E and any scalars. Each subpart is independent of the other.

(1) [3 pts] The ghosts are now more resilient so they have H health points. This means each ghost needs to spend H turns in the electric field before they die. What is the size of the state space with this adjustment?

$X \times$ NxMxExH^(NxM)

(2) [3 pts] The Pacmobile can no longer choose any radius value at every turn because it now takes time to increase and decrease the radius. If pacman is using a radius r on one turn, on the next turn he must choose radius values from $r-1$, r, or $r+1$

$X \times$ NxMxEx2^(NxM)

(d) [3 pts] Mark whether the following heuristics are admissible or not. Following the original problem description.

(1) [1 pt] $h_1(n)$ = The amount of charge remaining.

○ (A) Admissible ☒ (B) Not Admissible

(2) [1 pt] $h_2(n)$ = The number of ghosts still alive.

☒ (A) Admissible ○ (B) Not Admissible

(3) [1 pt] $h_3(n)$ = The Manhattan distance between the Pacmobile and the bottom right corner.

☒ (A) Admissible ○ (B) Not Admissible

Question 2: Sliding Puzzle (10 points)

Consider the sliding puzzle game as a search problem. The sliding puzzle game consists of a three-by-three board with eight numbered tiles and a blank space $\varnothing$. At each turn, only a tile adjacent to the blank space (left, right, above, below) can be moved into the blank space. The objective is to reconfigure any generic starting state to the goal state, $(\varnothing, 1, 2;\ 3, 4, 5;\ 6, 7, 8)$.

(a) [3 pts]

(1) [2 pts] What is the size of the state space?

○ (A) 9　　○ (B) $9 * 9$　　⊠ (C) $9!$

(2) [1 pt] Does every board have a unique sequence of moves to reach the goal state?

○ (A) Yes　　⊠ (B) No

(b) [12 pts] For the following heuristics, mark whether or not they are only admissible (admissible but not consistent), only consistent (consistent but not admissible), both, or neither.

(1) [2 pts] $h_1(n)$ = Total number of misplaced tiles.

○ (A) Only admissible　　○ (B) Only consistent

⊠ (C) Both　　○ (D) Neither

(2) [2 pts]

$$h_2(n) = \begin{cases} 0 & \text{if } h_1(n) < 3 \\ h_1(n) & \text{else} \end{cases}$$

⊠ (A) Only admissible　　○ (B) Only consistent

○ (C) Both　　○ (D) Neither

(3) [2 pts] $h_3(n)$ = Sum of Manhattan distances for each individual tile to their goal location.

- ○ (A) Only admissible
- ○ (B) Only consistent
- ⊠ (C) Both
- ○ (D) Neither

(4) [1 pt] $h_4(n) = max(h_1(n), h_3(n))$

- ○ (A) Only admissible
- ○ (B) Only consistent
- ⊠ (C) Both
- ○ (D) Neither

(5) [1 pt] $h_5(n) = min(h_1(n), h_3(n))$

- ⊠ (A) Only admissible
- ○ (B) Only consistent
- ○ (C) Both
- ○ (D) Neither

(6) [1 pt] $h_6(n) = h_1(n) + h_2(n)$

- ○ (A) Only admissible
- ○ (B) Only consistent
- ○ (C) Both
- ⊠ (D) Neither

(7) [3 pts] Comparing the heuristics above that you marked as admissible (or both), which one is best (dominates the others)? If multiple heuristics are tied mark them all.

- ☐ (A) h_1
- ☐ (B) h_2
- ☐ (C) h_3
- ☒ (D) h_4
- ☐ (E) h_5
- ☐ (F) h_6

Question 3 [18 pts]. Logic: Map Coloring

We'd like to express the map coloring problem in propositional logic. Consider a map with nodes X, Y, Z, U with two edges: between nodes X and Y, and between nodes X and Z.
We'd like to assign colors to nodes X,Y,Z,U such that no two adjacent nodes have the same color.

We have three colors available to use: R (Red), G (Green), and B (Blue).

Let the proposition symbol N_c represent node N being assigned color C . For example, X_r = true means that node X is colored Red.

(a) [2 pts] How many symbols are needed to express this problem in propositional logic? Your answer should be an integer.

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(b) [1 pt] For a general map coloring problem with n nodes and c colors, how many symbols are needed to express the problem in propositional logic?
Your answer should be an expression, possibly in terms of n and/or c.

n*c

Now we'll formulate several logical sentences that represent this map coloring problem.

(c) [2 pts] Below is a sentence that represents part of this map coloring problem. Fill in the blanks with $\wedge$ or $\vee$.

$$(X_R \_\_ X_G \_\_ X_B) \_\_ (Y_R \_\_ Y_G \_\_ Y_B) \_\_ (Z_R \_\_ Z_G \_\_ Z_B)$$

(d) [2 pts] What does the sentence in the previous subpart represent? You can answer in 10 words or fewer.

Everyone must be at least one color

(e) [2 pts] Write a logical sentence that encodes the constraint "X and Y cannot have the same color". Your sentence does not need to be in CNF.

((Xr and NotYr)or(Not Xr and Yr)) or .. (for every color)

(f) [2 pts] Suppose we've additionally added sentences that encode "X and Z cannot have the same color,".

This problem is still missing some sentence(s). Describe the missing sentence(s) (you can answer in 10 words or fewer):

The fact that every variable must have a single color and X must be different from Z

(g) [1 pt] How many symbols are assigned to *true* in an assignment that solves the problem? Your answer should be an integer.

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Pacman would like to prove the following statement: "If X is colored Red in this problem, then Y will not be colored Red," using only a SAT solver.

(h) [3 pts] In addition to the sentences defining the problem from subparts (c), (e), and (f), which other clauses should be part of the input to the SAT solver to ensure that Pacman gets the right answer? Select all that apply.

- ☑ (X_R)
- ☐ $(\neg X_R)$
- ☒ (Y_R)
- ☐ $(\neg Y_R)$
- ☐ $(\neg X_R \wedge \neg Y_R)$
- ☐ $(X_R \wedge Y_R)$
- ○ None of the above.

(i) [3 pts] Select all true statements:

- ☑ Every discrete, finite CSP can be represented as a SAT problem.
- ☑ Every SAT problem can be represented as a CSP.
- ☐ A correct SAT representation of a discrete, finite CSP has exactly the same number of satisfying assignments as the CSP has distinct solutions.
- ○ None of the above.

Question 4 [15 pts]. CSPs: The bike ride

Qui, Quo and Qua want to go Topolino's birthday, and your first task is to choose a vehicle for each of them.
Each can use one of the following: bicycle (B), electric bicycle (EB), scooter (S), car (C), where each vehicle can take up to three persons, and it is subject to the following constraints:

1. Quo and Qua cannot drive together,
2. Qua can only go with electric bike or scooter,
3. Quo cannot share the vehicle with anyone else.

(a) [2 pts] Consider the first constraint. Can this constraint be expressed using only binary constraints on the three variables ***I, O, A***?

- ● Yes, it can be expressed as 1 binary constraint.
- ○ Yes, it can be expressed as 2 different binary constraints.
- ○ Yes, it can be expressed as 4 different binary constraints.
- ○ No, this is necessarily a unary constraint.
- ○ No, this is necessarily a higher-order constraint.

(b) [3 pts] Suppose we enforce unary constraints, and then assign the Quo to use the scooter. The remaining values in each domain would be:

Qui:	B, EB, S, C
Quo:	S
Qua:	EB, S

In the table below, mark each value that would be **removed** by running forward-checking after this assignment.

	B	EB	S	C
Qui	☐	☐	☒	☐
Qua		☐	☒	

(c) [3 pts] Regardless of your answer to the previous subpart, suppose we've done some filtering and the following values

I:	EB, C
O:	S
A:	EB

- ● Yes, all arcs are consistent.
- ○ No, only $A \to I$ is not consistent.
- ○ No, only $I \to A$ is not consistent.
- ○ No, $A \to I$ and $I \to A$ are both not consistent.
- ○ No, only $I \to O$ is not consistent.
- ○ No, only $I \to O$ is not consistent.

- ○ No, $O \to I$ and $I \to O$ are both not consistent.

(d) [2 pts] Regardless of your answer to the previous subpart, suppose we start over and just enforce the third constraint. Then the remaining values in each domain are:

I:	B, EB, S, C
O:	B, EB, S, C
A:	EB, S

What does the minimum remaining values (MRV) heuristic suggest doing next?

- ○ Assign B or C to a variable next.
- ○ Assign E or S to a variable next.
- ● Assign a value to A next.
- ○ Assign a value to I or O next.

(e) [2 pts] Again, consider the CSP after just enforcing the third constraint:

I:	B, EB, S, C
O:	B, EB, S, C
A:	EB, S

Which assignment would the least constraining value (LCV) heuristic prefer?

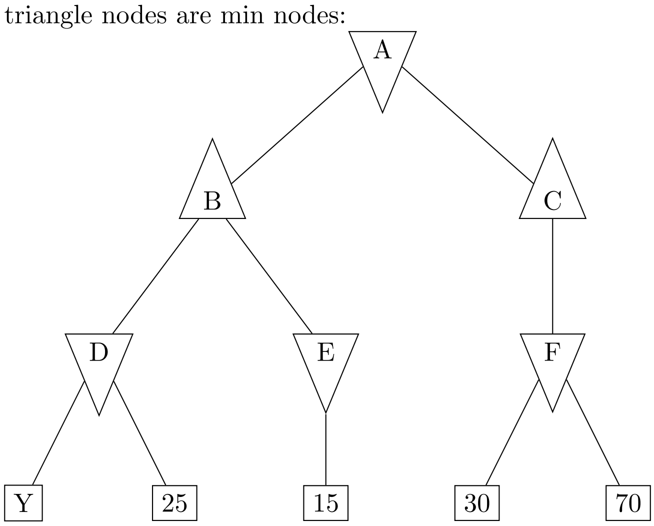
- ● Assign B to O.
- ○ Assign EB to O.
- ○ LCV is indifferent between these two assignments.

(f) [3 pts] Suppose we add another constraint:
"The car has place for only one person."
Can this constraint be expressed using only binary constraints on the three variables ***I, O, A***?

- ○ Yes, it can be expressed as 1 binary constraint.
- ○ Yes, it can be expressed as 2 different binary constraints.
- ○ Yes, it can be expressed as 3 different binary constraints.
- ● No, this is necessarily a higher-order constraint.

Question 5 [16 pts]. Games

Consider the following game tree where upward triangle nodes are max nodes, and downward triangle nodes are min nodes:



(a) [1 pt] If $Y = 20$, what is the minimax value at the root node A
Your answer should be an integer.

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(b) [3 pts] What range of values for Y would cause the minimax value at the root A to be equal to Y?
Your answer should be an inequality using $\leq$, such as $Y \leq 940$, or $940 \leq Y \leq 1440$, or $1940 \leq Y \leq 1940$, or "impossible" (if there are no such values for Y).

15 <= y <= 25

(c) [4 pts] Assume that $Y = 20$. If we run minimax with alpha-beta pruning, visiting nodes from left to right, which terminal nodes will we **not** visit because of pruning? Select all that apply.

- ☐ Y
- ☐ 25
- ☐ 15
- ☐ 30
- ☐ 70
- ● None of the above.

(d) [2 pts] When is the statement $A \leq max(Y, 70)$ true?

- ● Always true.
- ○ Sometimes true.
- ○ Never true.

(e) [2 pts] The root of this tree is a min node. You would like to represent this same game with a new tree that has a max node at the root.

Select all changes that you would need to make in the new tree.

- ☒ Convert all min nodes to max nodes.
- ☒ Convert all max nodes to min nodes.
- ☐ Add a layer of chance nodes to the new tree.
- ☒ Multiply each of the terminal node values by -1.
- ○ None of the above -it is impossible to represent this game with a new tree with a max node at the root.

(f) [4 pts] Suppose the max nodes B and C are replaced with chance nodes that select actions uniformly at random. What range of values for Y would cause the value at the root A to be equal to Y?

Your answer should be an inequality using $\leq$, such as $Y \leq 940$, or $940 \leq Y \leq 1440$, or $1940 \leq Y \leq 1940$, or "impossible" (if there are no such values for Y).

y<=25 y = 15

Question 6 [17 pts]. MDPs: Flying Pacman

Pacman is in a 1-dimensional grid with squares labeled 0 through n, inclusive, as shown below:

0	1	2	3	4	5		$n-1$	n

Pacman's goal is to reach square n as cheaply as possible. From state n, there are no more actions or rewards available.

At any given state, if Pacman is not in n, Pacman has two actions to choose from:

- **Run**: Pacman deterministically advances to the next state (i.e., from state i to state $i+1$). This action costs Pacman 1.
- **Fly**: With probability p, Pacman directly reaches state n. With probability $1-p$, Pacman is stuck in the same state. This action costs Pacman 2.

(a) [3 pts] Fill in the blank boxes below to define the MDP. i represents an arbitrary state in the range $\{0, \ldots, n-1\}$.

s	a	s'	$T(s,a,s')$	$R(s,a,s')$
i	Run	$i+1$	1	-1
i	Fly	i	1-p	-2
i	Fly	n	p	-2

For the next three subparts, assume that $\gamma = 1$.

Let π_R denote the policy of always selecting Run, and π_F denote the policy of always selecting Fly.

Compute the values of these two policies. Your answer should be an expression, possibly in terms of n, p, and/or i.

(b) [2 pts] What is $V^{\pi_R}(i)$?

n-i

(c) [2 pts] What is $V^{\pi_F}(i)$?
Hint: Recall that the mean of a geometric distribution with success probability p is $\sum_{k=1}^{\infty} k(1-p)^{k-1}p = \frac{1}{p}$.

-2/p

(d) [4 pts] Given the results of the two previous subparts, we can now find the optimal policy for the MDP.
Which of the following are true? Select all that apply. (Hint: consider what value of i makes $V^{\pi_R}(i)$ and $V^{\pi_F}(i)$ equal.) Note: $\lceil x \rceil$ is the smallest integer greater than or equal to x.

- ☐ If $p < 2/n$, Fly is optimal for all states.
- ☑ If $p < 2/n$, Run is optimal for all states.
- ☐ If $p \geq 2/n$, Fly is optimal for all $i \geq \lceil n{-}2/p \rceil$ and Run is optimal for all $i < \lceil n{-}2/p \rceil$.
- ☑ If $p \geq 2/n$, Run is optimal for all $i \geq \lceil n{-}2/p \rceil$ and Fly is optimal for all $i < \lceil n{-}2/p \rceil$.
- ○ None of the above.

Regardless of your answers to the previous parts, consider the following modified transition and reward functions (which may not correspond to the original problem). As before, once Pacman reaches state n, no further actions or rewards are available.

For each modified MDP and discount factor, select whether value iteration will converge to a finite set of values.

(e) [2 pts] $\gamma = 1$

s	a	s'	$T(s,a,s')$	$R(s,a,s')$
i	Run	$i+1$	1.0	+5
i	Fly	$i+1$	1.0	+5

- ● Value iteration converges
- ○ Value iteration does not converge
- ○ Not enough information to decide

(f) [2 pts] $\gamma = 1$

s	a	s'	$T(s,a,s')$	$R(s,a,s')$
i	Run	$i+1$	1.0	+5
i	Fly	$i-1$	1.0	+5

- ○ Value iteration converges
- ● Value iteration does not converge
- ○ Not enough information to decide

(g) [2 pts] $\gamma < 1$

s	a	s'	$T(s,a,s')$	$R(s,a,s')$
i	Run	$i+1$	1.0	+5
i	Fly	$i-1$	1.0	+5

- ● Value iteration converges
- ○ Value iteration does not converge
- ○ Not enough information to decide

End of exam.