

Geometric Semantic Genetic Programming

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Geometric Operators

Crossover and mutation
as operations on a metric space

Metric Spaces

Distance

$$d : S \times S \rightarrow \mathbb{R}^+$$

Identity

$$d(x, y) = 0 \text{ iff } x = y$$

Symmetry

$$d(x, y) = d(y, x)$$

Triangular inequality

$$d(x, y) \leq d(x, z) + d(z, y)$$

Segment

$$S(x, y) = \{z \mid d(x, z) + d(z, y) = d(x, y)\}$$

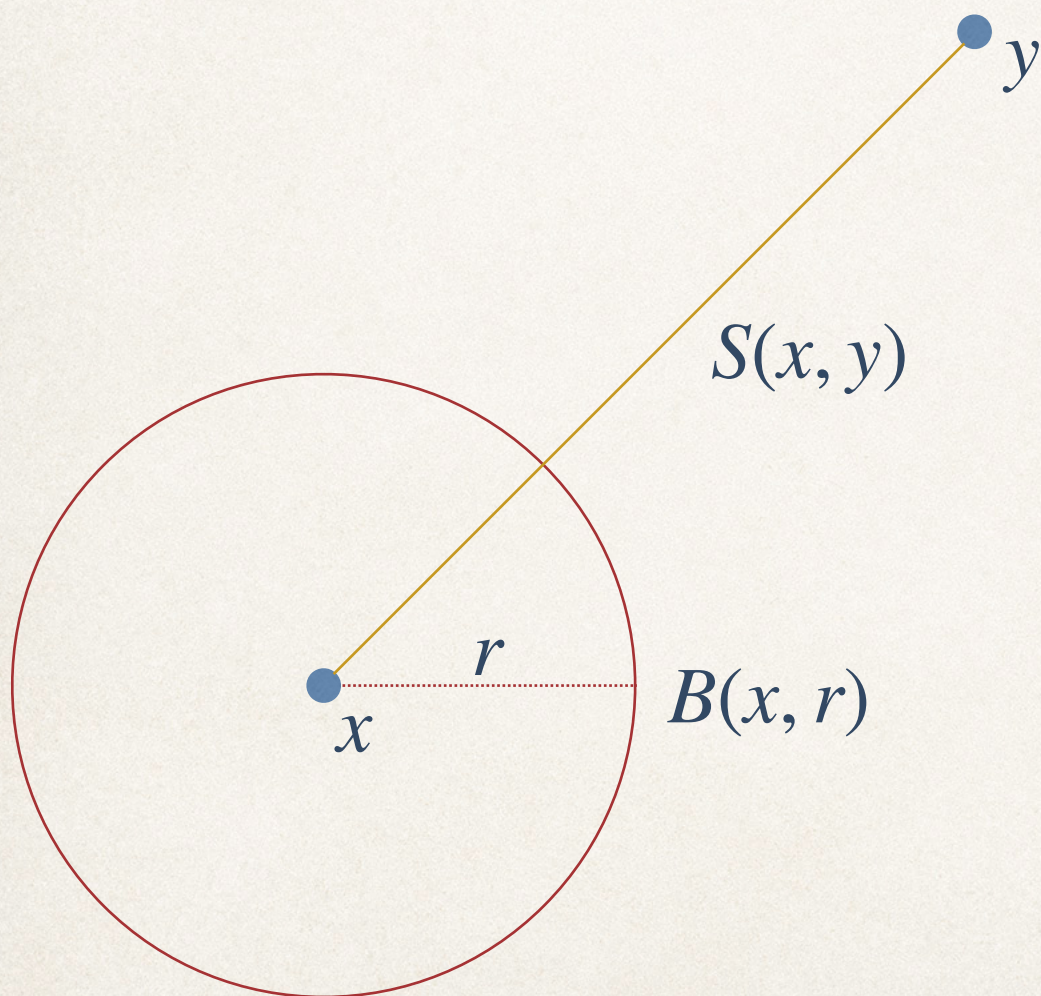
Ball of radius r

$$B(x, r) = \{y \mid d(x, y) \leq r\}$$

Segments and Balls

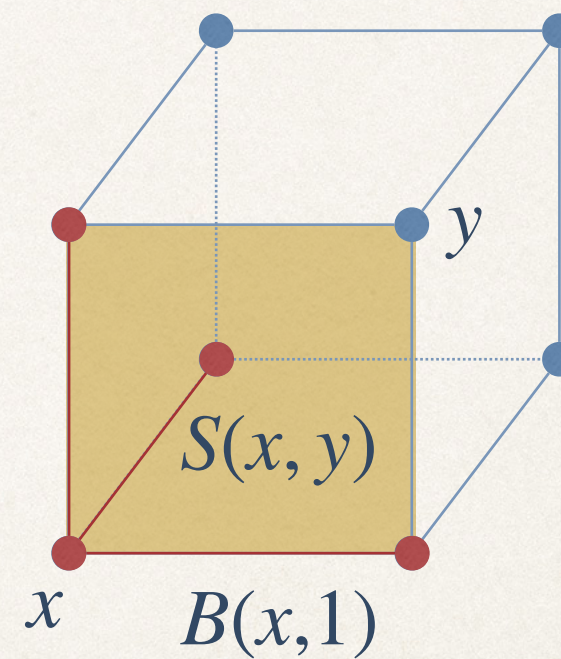
\mathbb{R}^2

Euclidean distance



$\{0,1\}^3$

Hamming distance



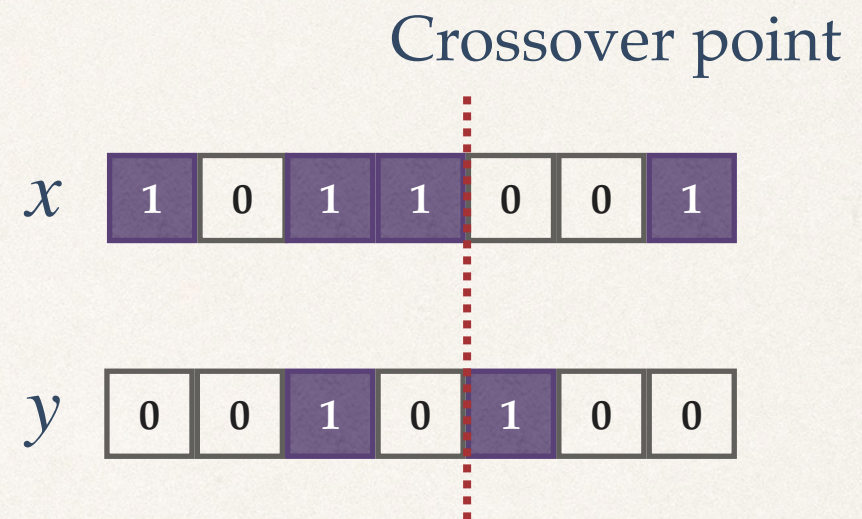
Geometric Crossover

Geometric Crossover

For all x, y

All possible results from crossover

Are inside $S(x, y)$



Geometric Crossover

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For all x, y

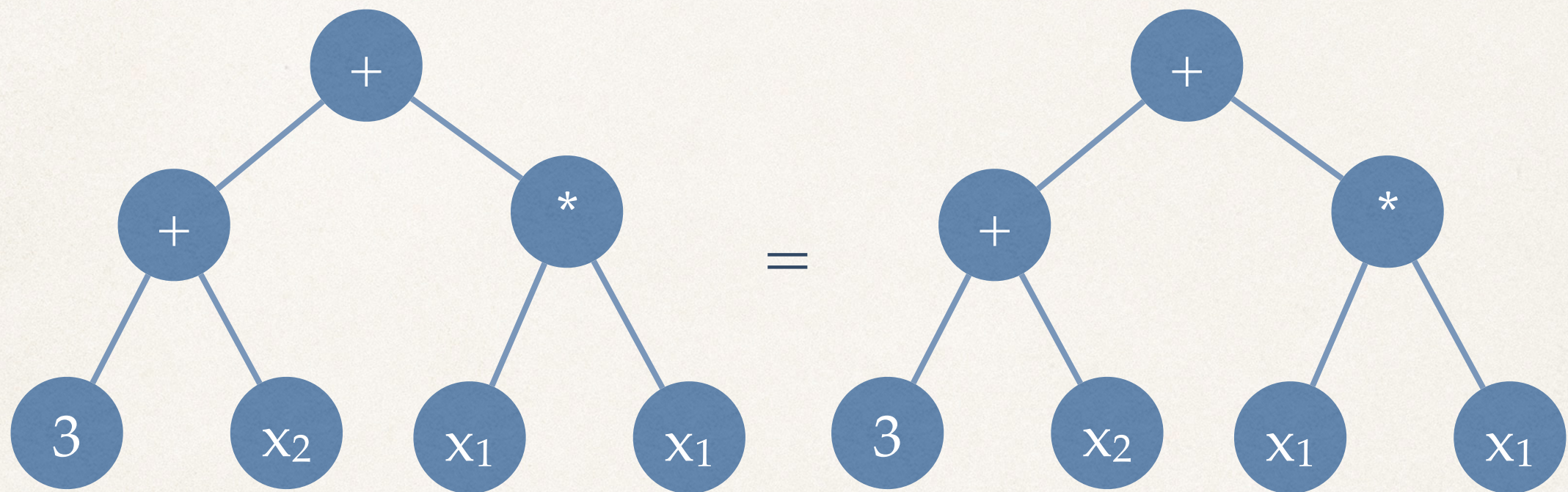
All possible results from crossover

Are inside $S(x, y)$

$$z_1 \begin{bmatrix} 1 & 0 & 1 & 1 & 1 & 0 & 0 \end{bmatrix} \in S(x, y)$$

$$z_2 \begin{bmatrix} 0 & 0 & 1 & 0 & 0 & 0 & 1 \end{bmatrix} \in S(x, y)$$

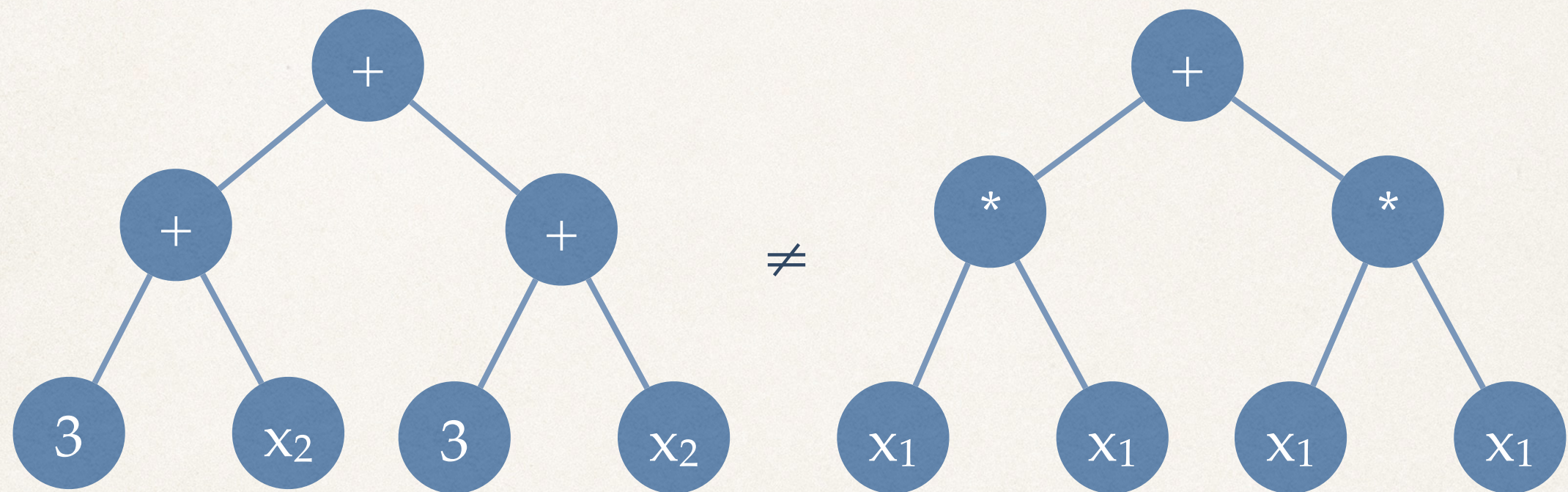
Are All Crossovers Geometric?



Crossover of a Tree with itself

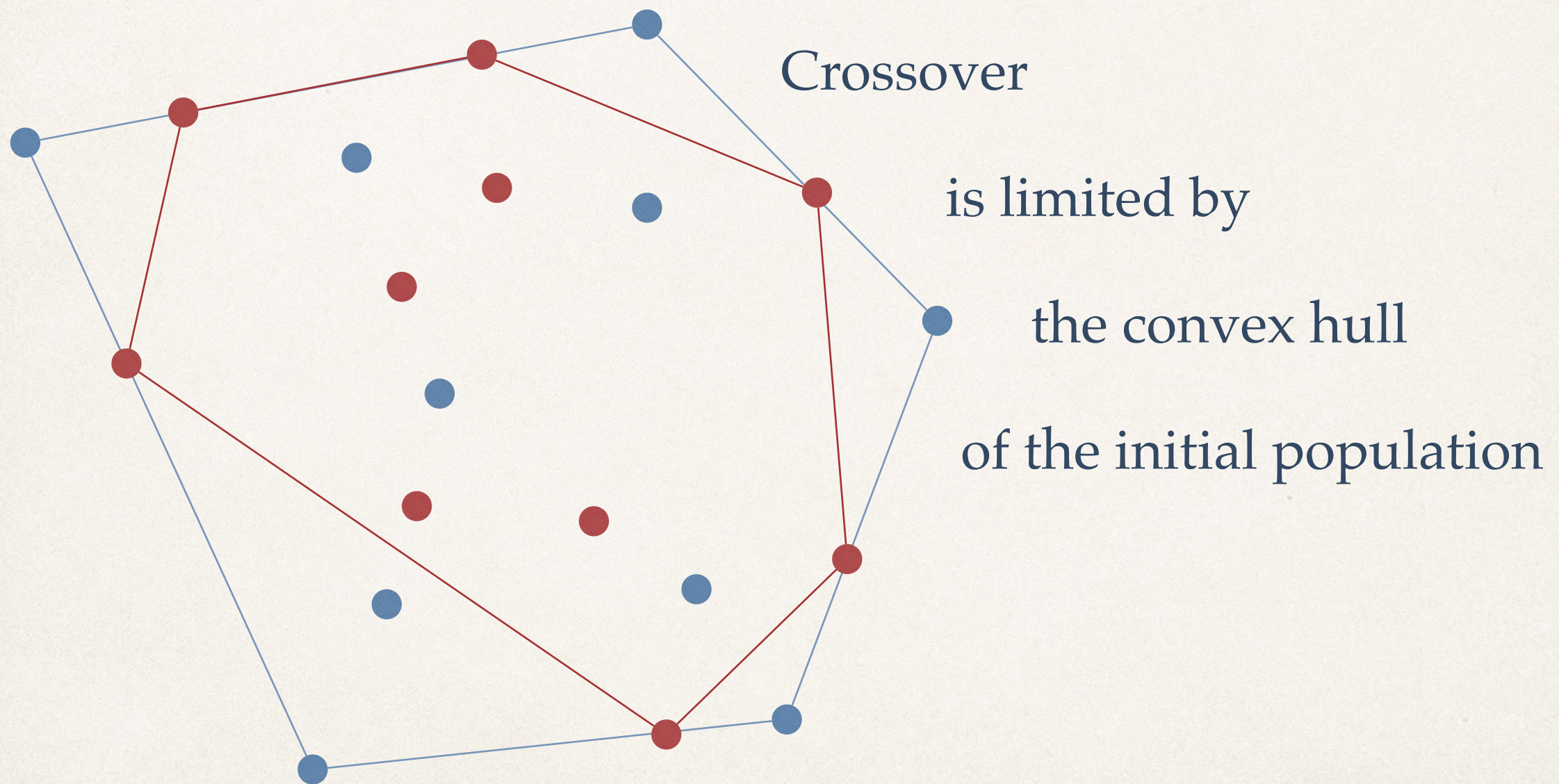
$$S(x, x) = \{x\}$$

Are All Crossovers Geometric?



Not a geometric crossover!

Consequences



Geometric Mutation

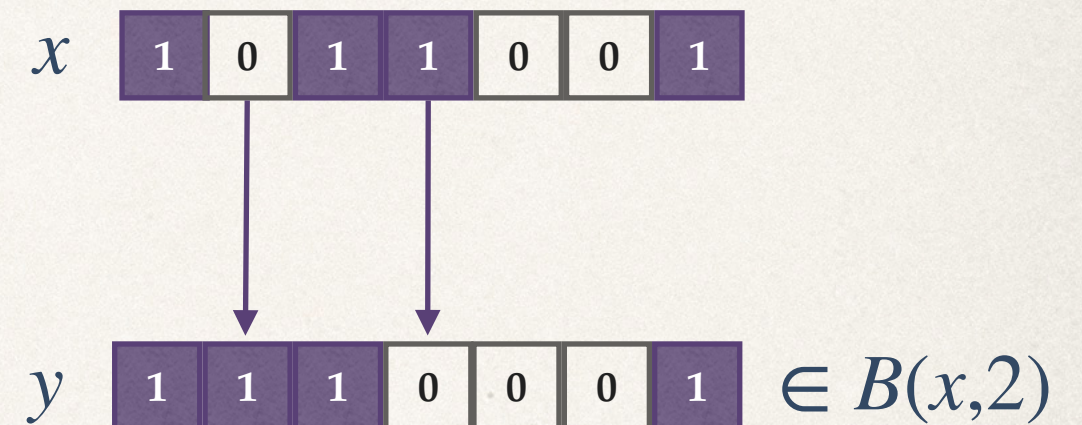
Geometric Mutation (of radius r)

For all x

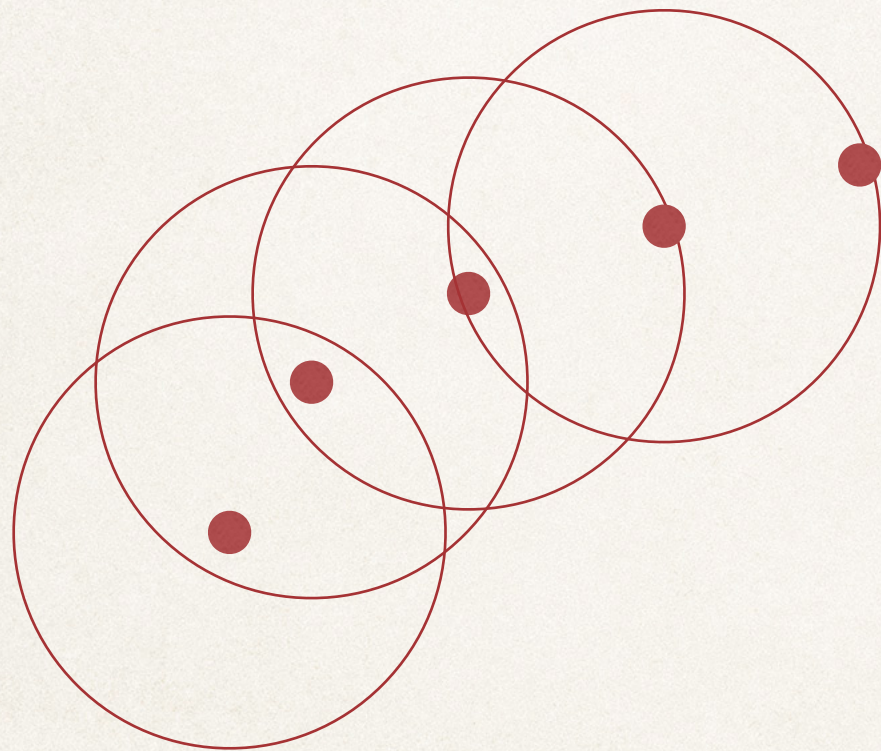
All possible results from the mutation

Are inside $B(x, r)$

$$r = 2$$



Mutation Can Take Us Everywhere*



Exploration

Mutation enlarges the search space

Crossover restricts the search space

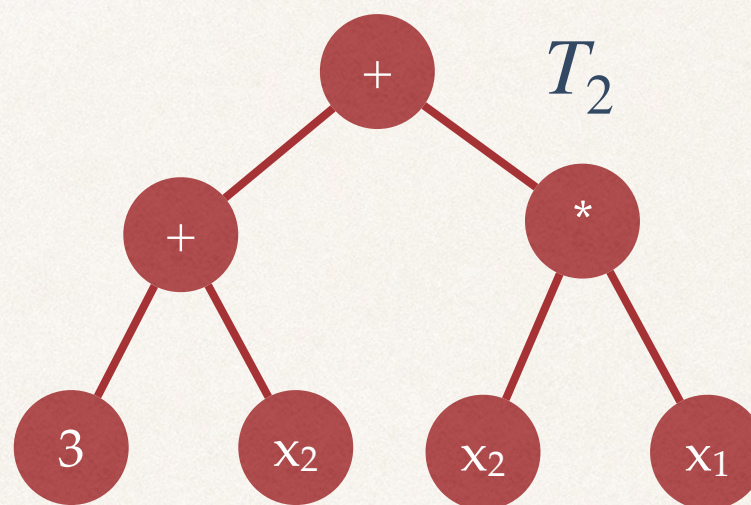
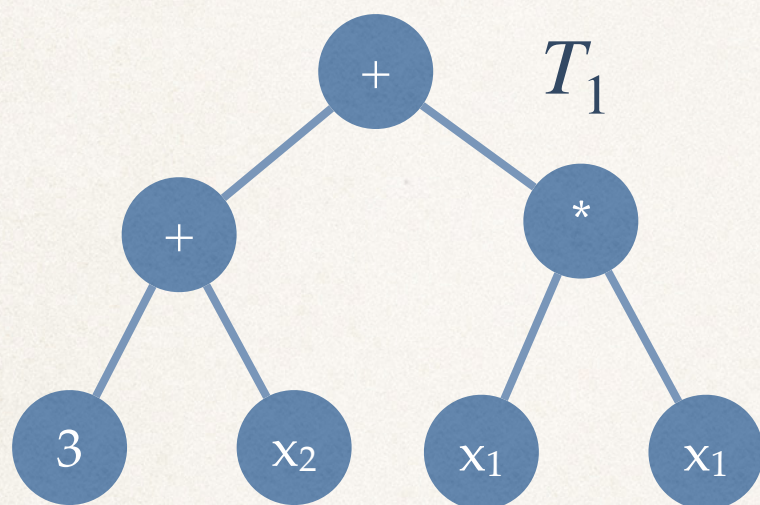
Exploitation

* Restrictions may apply for particular spaces.

Semantic Operators

Controlling crossover and mutation effects
on the *outputs* of an individual

Syntax vs Semantics



Syntax

x_1	x_2	T_1	T_2
2	3	10	12
4	1	20	8
1	2	6	7

Semantic vectors

(10, 20, 6)

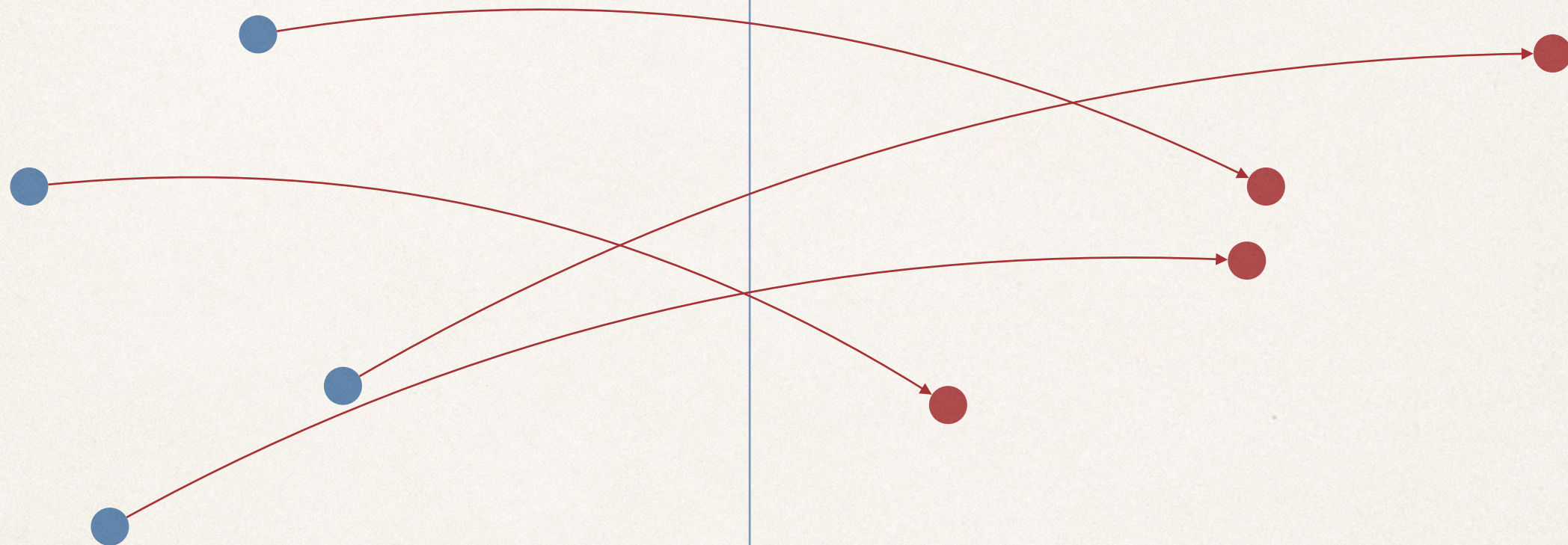
(12, 8, 7)

Semantics

Syntax vs Semantics

Syntax Space

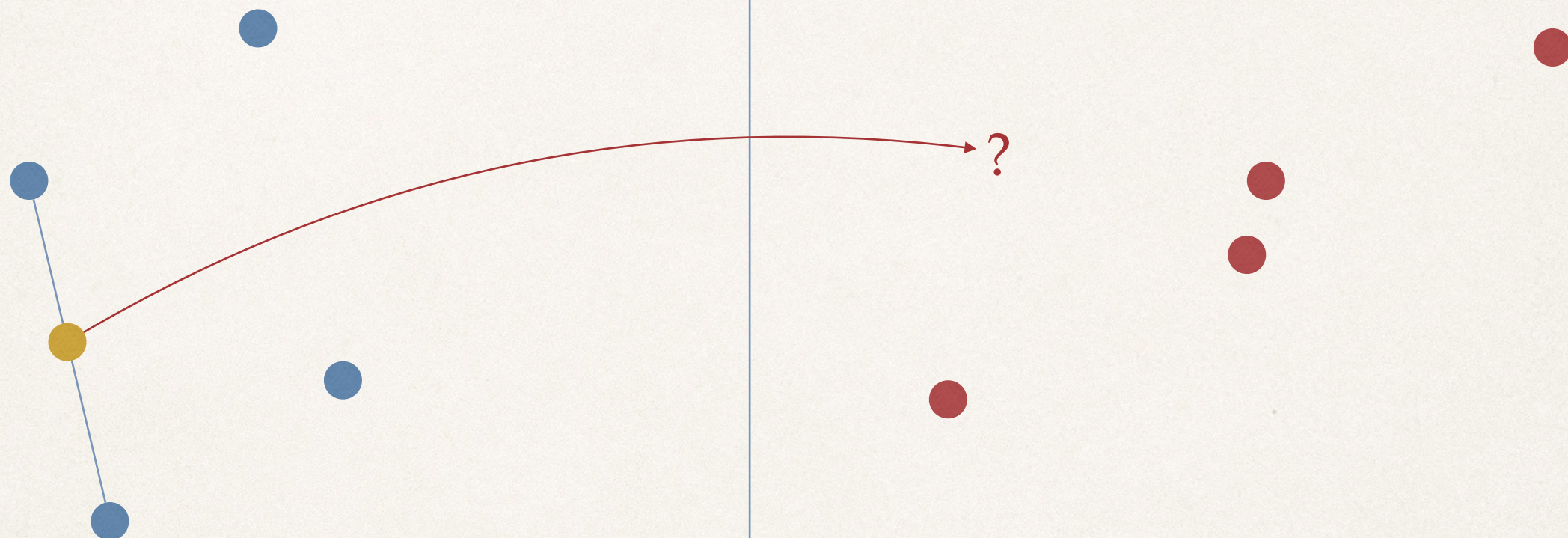
Semantic Space



Syntax vs Semantics

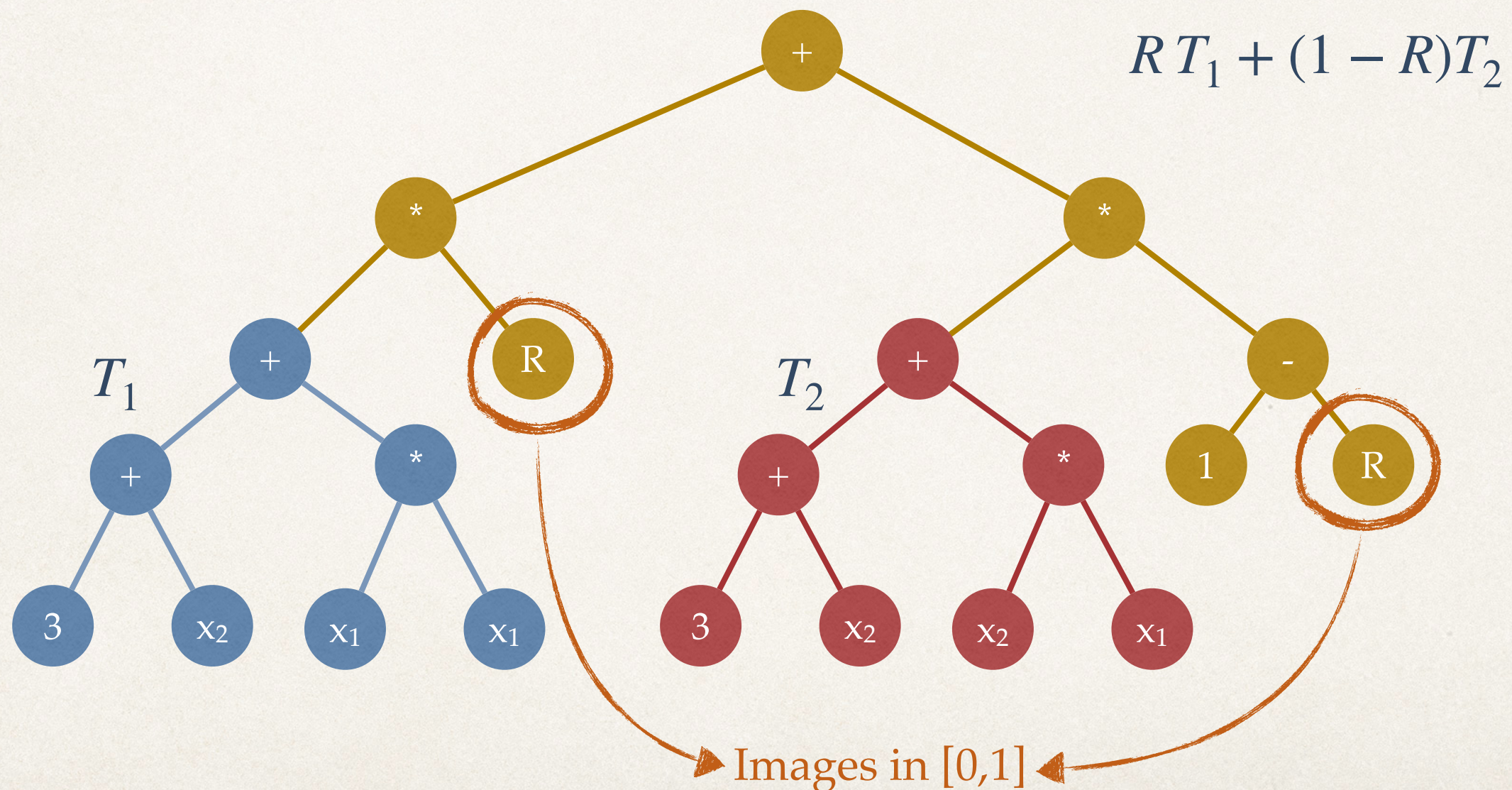
Syntax Space

Semantic Space

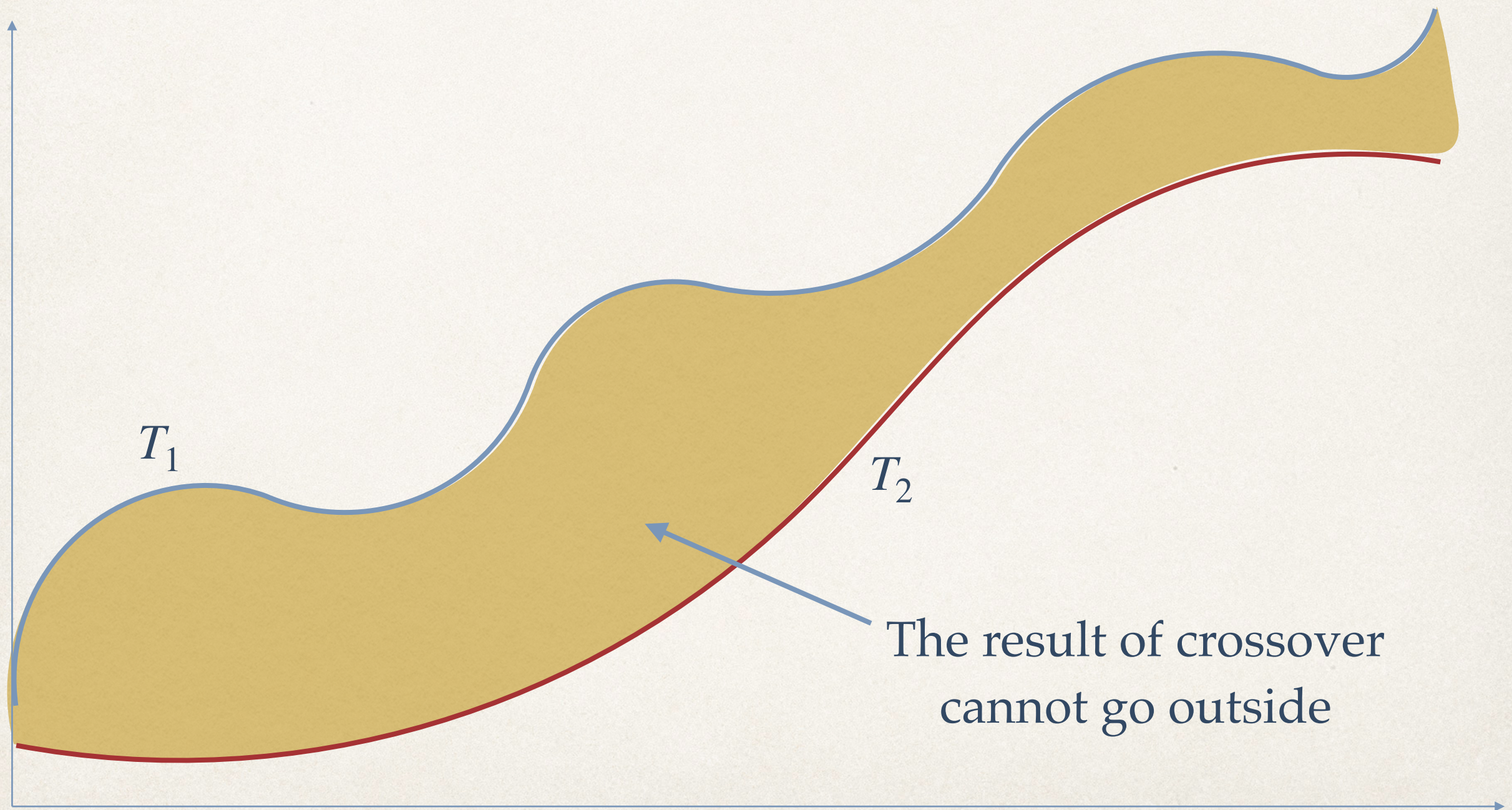


Can we make operators that are
geometric in the **semantic** space?

Geometric Semantic Crossover

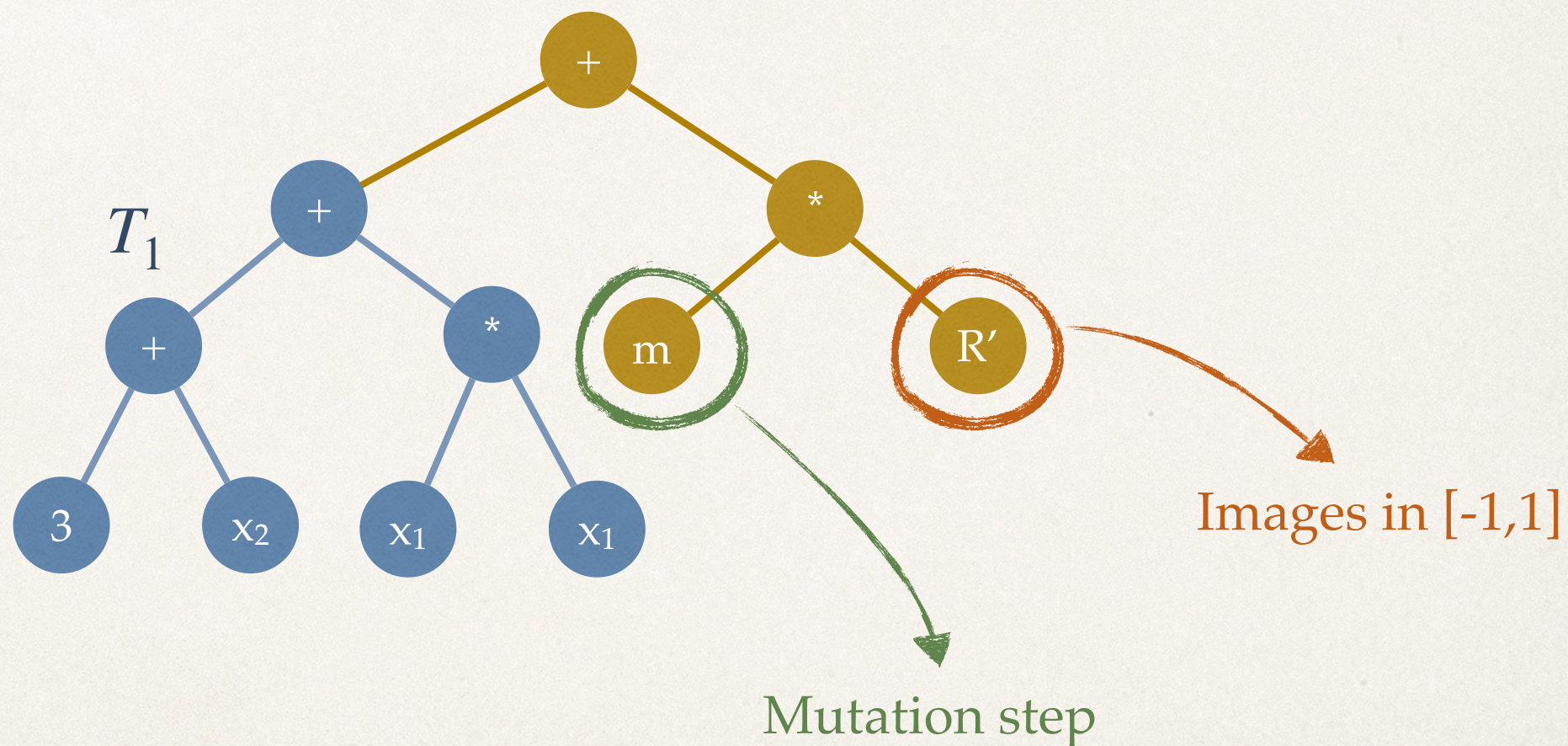


Effect on the Semantics

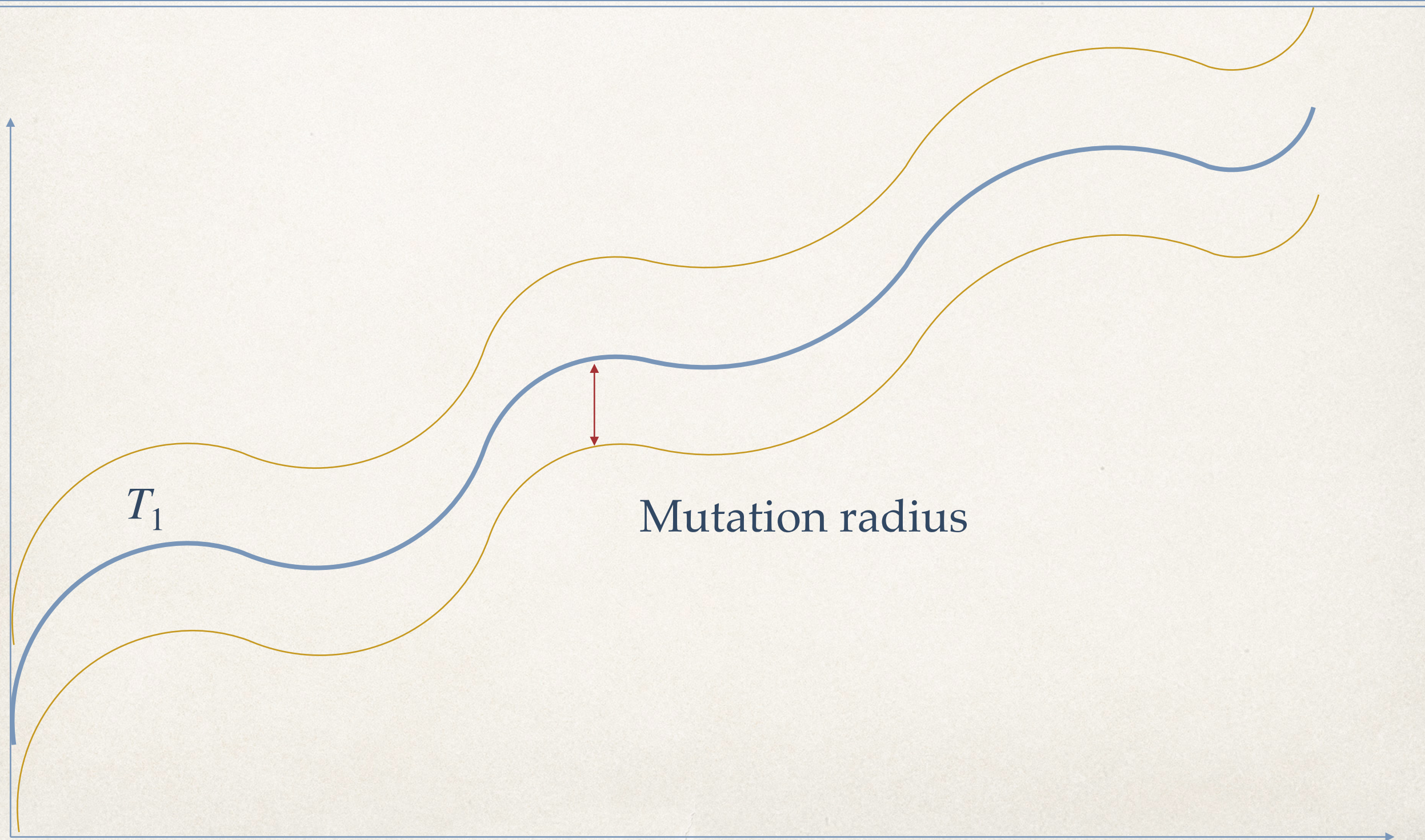


Geometric Semantic Mutation

$$T_1 + m R'$$



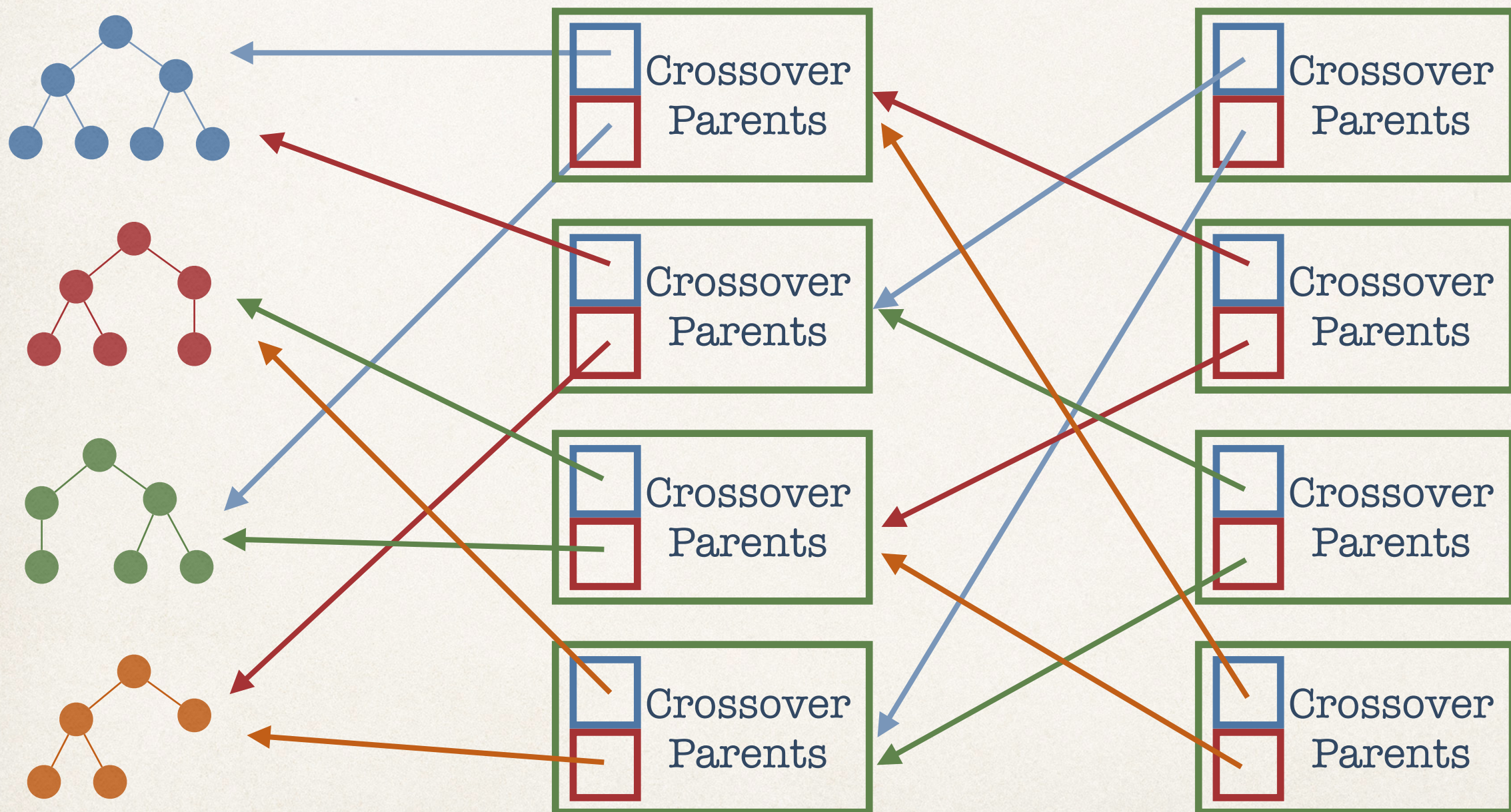
Effect on the Semantics



Is this everything?

- ❖ Different operators for different domains:
Boolean, programs, etc.
- ❖ The tree-based representation is not the best one
 - ◆ Exponential growth wrt number of generations
 - ◆ Linear growth with subtree sharing.

Fast GSGP



An Example

