Example Report using R markdown

V. Gioia

08/10/2024

Contents

Some notes on R Markdown

R Markdown is a powerful tool to create reports, by integrating text, formulas, R code, plots, links and so on.

Getting start:

- Create an R Markdown file (File, New File)
- Choose the title, the author/authors and the selected output
- The (default) header includes what you choose on the previous step
- We can add subtitle, fontsize and several output related options (not considered here)

The compilation is done by clicking on the knit button (there you can choose between html or pdf, despite your initial choice).

In moodle (Lab1 folder) you will find some materials on the basic use of R Markdown, but the web is a great mine of information.

The R code, in R Markdown term a chunk: each chunk is delimited by the symbols "‘ and must start with the syntax `{r chunk_name }`

You can mask the code by adding the argument `echo = FALSE` and you can avoid the evaluation by using `eval = FALSE`.

For a better rendering, it is better to introduce the option `fig.align = 'center'` in the options of knitr function otherwise you can use it as argument of `{r chunk_name }`.

In the Lab1 folder, you can also find an example on how generating slides using beamer in R Markdown (see the folder ExampleRMD_beamer).

Central Limit Theorem (CLT)

Let $X_1, X_2, \ldots, X_n$, be a sequence of independent and identically distributed (iid) random variables (rv) from a distribution with mean μ and finite variance σ^2.

For large n, the sample average

$$\bar{X}_n = \frac{1}{n}\sum_{i=1}^{n} X_i \dot{\sim} \mathcal{N}(\mu, \sigma^2/n)$$

where $\dot{\sim}$ indicates convergence in distribution. Equivalently

$$S_n = \sum_{i=1}^{n} X_i \dot{\sim} \mathcal{N}(n\mu, n\sigma^2)$$

The CLT supports the normal approximation to the distribution of a rv that can be viewed as the sum of other rv.

Approximation with CLT: application

The approximation above is useful in statistics for computing some quantities. For instance, let X and Y be two independent Binomial rv, such that $X \sim \text{Bin}(n, p)$ and $Y \sim \text{Bin}(m, q)$.

If we are interested in computing the probability $P(X > Y)$ the Normal approximation is the simplest way to do it. We can approximate:

$$X \approx \mathcal{N}(np, np(1-p)), \qquad Y \approx \mathcal{N}(mq, mq(1-q)).$$

Then, by using a well known probability result, the difference $W = X - Y$ of two independent normal distributions with means μ_X, μ_Y and variances σ_X^2, σ_Y^2, respectively, is **still a normal distribution** with mean $\mu_W = \mu_X - \mu_Y$ and variance $\sigma_W^2 = \sigma_X^2 + \sigma_Y^2$.

In such a case,

$$\mu_W = \mu_X - \mu_Y = np - mq, \qquad \sigma_W^2 = \sigma_X^2 + \sigma_Y^2 = np(1-p) + mq(1-q).$$

Approximation with CLT: real application with waterpolo goals

Tomorrow two professional Italian waterpolo teams, Posillipo and Pro Recco, compete against each other. Let X and Y be the random *goals scored* by Posillipo and Pro Recco, respectively.

We assume that X, Y follow two independent Binomial distributions. Thus, X and Y represent the number of shots converted in goal on the total number of shots n, m made by Posillipo and Pro Recco, with probabilities p and q, respectively.

Before the match, the number of shots is *unknown*. In what follows, we adopt a simplification, and we treat the quantities p, q, m, n as *known*, for instance fixing them upon historical experience: $p = 0.5, q = 0.7, n = 20, m = 20$.

We want to investigate the Posillipo probability of winning the next match against Pro Recco, that is

$$P(X > Y) = P(X - Y > 0) = ?$$

So, let W be the r.v., such that $W = X - Y$. We could compute the law of this rv but using the Normal approximation, $W \approx \mathcal{N}(\mu_W = \mu_X - \mu_Y, \sigma^2_W = \sigma^2_X + \sigma^2_Y)$, we can esily compute such a probability of interest.

```
p <- 0.5
q <- 0.7
n <- m <- 20
mW <-  p * n - q * m
sdW <- sqrt(n * p * (1 - p) + m * q * (1 - q))
# Probability that  W = X-Y > 0 (Posillipo win the match)
PWin_P <- pnorm(0, mean = mW, sd = sdW, lower.tail = FALSE)
PWin_P
```

```
## [1] 0.09362452
```

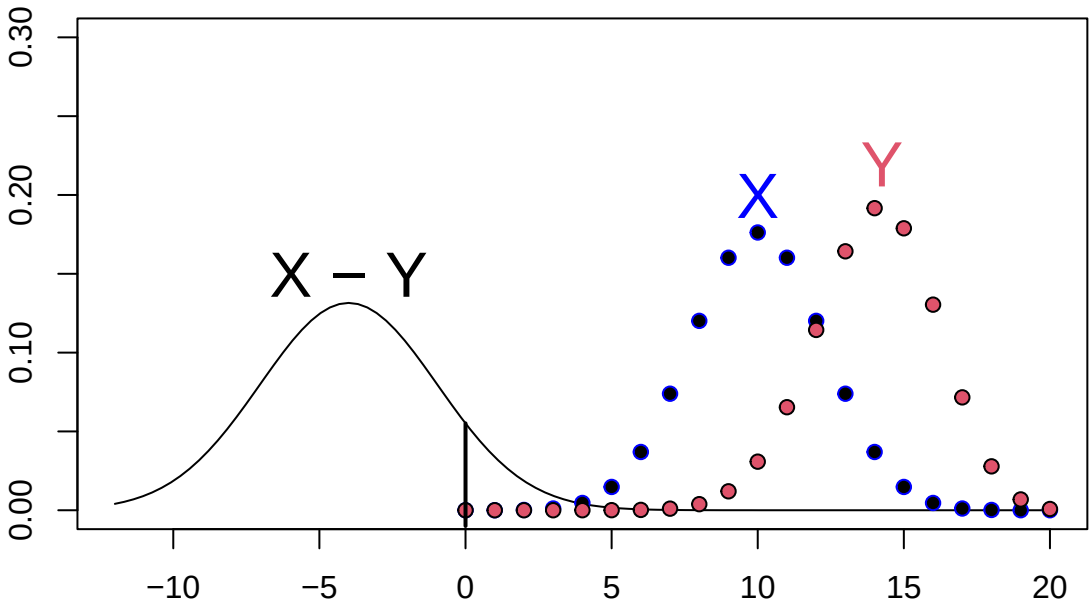
Here, we show the probability mass functions of X and Y and the probability density function of $W = X - Y$.

```
# pdf of W
curve(dnorm(x, mW, sdW), xlim = c(-12, 20), ylim = c(0, 0.3),
      xlab = "", ylab = "", cex.lab = 1.25)

# pmf of X and Y
points(0 : 20, dbinom(0 : n, n, p), pch = 21, bg = 1, col = "blue")
points(0 : 20, dbinom(0 : m, m, q), pch = 21, bg = 2)

segments(0, -0.01,              #(x_0, y_0)
         0, dnorm(0, mW, sdW),  #(x_1, y_1)
         lwd = 2)

text(14.25, 0.22, "Y", cex = 2, col = 2)
text(10, 0.2, "X", cex = 2, col = "blue")
text(-4, 0.15, "X - Y", cex = 2, col = 1)
```


0.30
0.20
0.10
0.00
X − Y
X
Y
−10
−5
0
5
10
15
20