

Question 1.5.9

EE22BTECH11054 - Umair Parwez

Question 1.5.9

Given triangle ABC with vertices,

$$A = \begin{pmatrix} 1 \\ -1 \end{pmatrix}, B = \begin{pmatrix} -4 \\ 6 \end{pmatrix}, C = \begin{pmatrix} -3 \\ -5 \end{pmatrix} \quad (1)$$

find the points of contact, E_3 and F_3 , of the incircle with sides AC and AB respectively.

Solution

Required to find points of contact, (E_3) and (F_3) , of incircle with sides AC and AB respectively.

From previous questions we know the coordinates of the incircle are :

$$(I) = \begin{pmatrix} \frac{-53-11\sqrt{37}+7\sqrt{61}+\sqrt{2257}}{12} \\ \frac{5-\sqrt{37}+5\sqrt{61}-\sqrt{2257}}{12} \end{pmatrix} \quad (2)$$

Radius of incircle is :

$$r = \frac{185 + 41\sqrt{37} - 37\sqrt{61} - \sqrt{2257}}{6\sqrt{74}} \quad (3)$$

Equation of incircle is :

$$\|x - I\|^2 = r^2$$

points A, B and C are :

$$(A) = \begin{pmatrix} 1 \\ -1 \end{pmatrix}, (B) = \begin{pmatrix} -4 \\ 6 \end{pmatrix}, (C) = \begin{pmatrix} -3 \\ -5 \end{pmatrix} \quad (5)$$

Parametric equation of AC is :

$$x = A + k(A - C) \quad (6)$$

Substituting (5) in (3) :

$$\|A + k(A - C) - I\|^2 = r^2 \quad (7)$$

$$(A + k(A - C) - I) \cdot (A + k(A - C) - I) = r^2 \quad (8)$$

$$k^2 \|A - C\|^2 + 2k(\|A\|^2 - A^T C - I^T A + I^T C) + \|A - I\|^2 = r^2 \quad (9)$$

Since AC is tangent to the incircle, the discriminant of the obtained quadratic equation is zero and the value of k is given as :

$$k = -\frac{(\|A\|^2 - A^T C - I^T A + I^T C)}{\|A - C\|^2} \quad (10)$$

Upon substituting the values of A, C and I into (10) we get,

$$k = \frac{-4 - \sqrt{37} + \sqrt{61}}{2} \quad (11)$$

Substituting (11) back into (5), we get point of contact with AC,

$$(E_3) = \begin{pmatrix} \frac{-2 - \sqrt{37} + \sqrt{61}}{2} \\ \frac{-6 - \sqrt{37} + \sqrt{61}}{2} \end{pmatrix} \quad (12)$$

Now let us find the other point of contact, with AB.

Parametric equation of AB is :

$$x = A + k(A - B) \quad (13)$$

(4) We can get the value of k by replacing C with B in Eq(10). Upon substituting the values, we get,

$$k = \frac{-37 - 4\sqrt{37} + \sqrt{2257}}{74} \quad (14)$$

Substituting (14) back into (13), we get point of contact with AB,

$$(F_3) = \begin{pmatrix} \frac{-111 - 20\sqrt{37} + 5\sqrt{2257}}{74} \\ \frac{185 + 28\sqrt{37} - \sqrt{2257}}{74} \end{pmatrix} \quad (15)$$

Diagram is shown on next page.

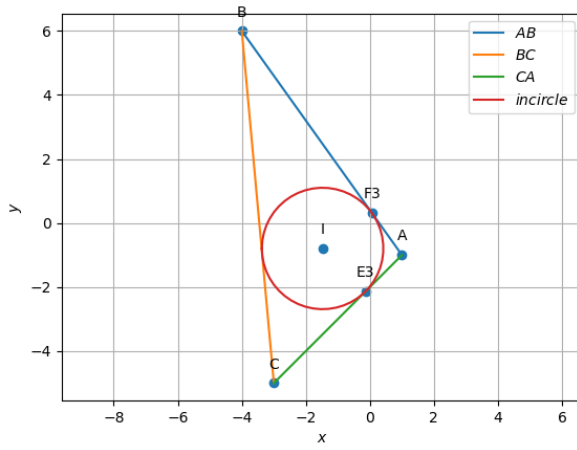


Fig. 0. Points of contact of incircle