Question 1.5.9

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Required to find points of contact, E_3 and F_3, of incircle with sides AC and AB respectively.

From previous questions we know the coordinates of the incircle are :

$$\mathrm{I} = \begin{bmatrix} \frac{-53-11\sqrt{37}+7\sqrt{61}+\sqrt{2257}}{12} \\ \\ \frac{5-\sqrt{37}+5\sqrt{61}-\sqrt{2257}}{12} \end{bmatrix}$$

Radius of incircle is :

$$r = \frac{185+41\sqrt{37}+-37\sqrt{61}-\sqrt{2257}}{6\sqrt{74}}$$

Equation of incircle is :

$$\|x - I\|^2 = r^2 \quad \text{——(1)}$$

points A, B and C are :

$$A = \begin{bmatrix} 1 \\ -1 \end{bmatrix}, B = \begin{bmatrix} -4 \\ 6 \end{bmatrix}, C = \begin{bmatrix} -3 \\ -5 \end{bmatrix}$$

Parametric equation of AC is :

$$x = A + k(A - B)$$

$$x = \begin{bmatrix} 1 \\ -1 \end{bmatrix} + k \begin{bmatrix} 4 \\ 4 \end{bmatrix}$$

On simplification :

$$x = \begin{bmatrix} 1 + k \\ -1 + k \end{bmatrix} \quad \text{——(2)}$$

On substituting Eq(2) in Eq(1) and solving, we find that k has only one solution :

$$k = \frac{-4-\sqrt{37}+\sqrt{61}}{2}$$

Substituting back into Eq(2), we get point of contact with AC,

$$E_3 = \begin{bmatrix} \frac{-2-\sqrt{37}+\sqrt{61}}{2} \\ \\ \frac{-6-\sqrt{37}+\sqrt{61}}{2} \end{bmatrix}$$

Parametric equation of AB is :

$$x = A + k(A - B)$$

$$x = \begin{bmatrix} 1 \\ -1 \end{bmatrix} + k \begin{bmatrix} 5 \\ -7 \end{bmatrix}$$

On simplification :

$$x = \begin{bmatrix} 1 + 5k \\ -1 - 7k \end{bmatrix}$$

Substituting back into Eq(2), we get point of contact with AB,

$$F_3 = \begin{bmatrix} \frac{-111-20\sqrt{37}+5\sqrt{2257}}{74} \\ \\ \frac{185+28\sqrt{37}-7\sqrt{2257}}{74} \end{bmatrix}$$

Diagram is shown below :

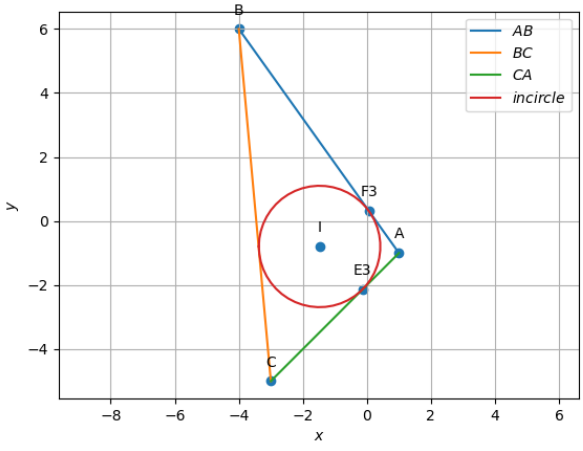

Fig. 0. Points of contact of incircle