

## Question 1.5.9

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Given triangle ABC with vertices,

$$\mathbf{A} = \begin{pmatrix} 1 \\ -1 \end{pmatrix}, \mathbf{B} = \begin{pmatrix} -4 \\ 6 \end{pmatrix}, \mathbf{C} = \begin{pmatrix} -3 \\ -5 \end{pmatrix} \quad (1)$$

find the points of contact,  $\mathbf{E}_3$  and  $\mathbf{F}_3$ , of the incircle with sides AC and AB respectively.

### Solution

Required to find points of contact,  $\mathbf{E}_3$  and  $\mathbf{F}_3$ , of incircle with sides AC and AB respectively.

From previous questions we know the coordinates of the incircle are :

$$\mathbf{I} = \begin{pmatrix} \frac{-53-11\sqrt{37}+7\sqrt{61}+\sqrt{2257}}{12} \\ \frac{5-\sqrt{37}+5\sqrt{61}-\sqrt{2257}}{12} \end{pmatrix} \quad (2)$$

Radius of incircle is :

$$r = \frac{185 + 41\sqrt{37} - 37\sqrt{61} - \sqrt{2257}}{6\sqrt{74}} \quad (3)$$

Equation of incircle is :

$$\|\mathbf{x} - \mathbf{I}\|^2 = r^2$$

points  $\mathbf{A}$ ,  $\mathbf{B}$  and  $\mathbf{C}$  are :

$$\mathbf{A} = \begin{pmatrix} 1 \\ -1 \end{pmatrix}, \mathbf{B} = \begin{pmatrix} -4 \\ 6 \end{pmatrix}, \mathbf{C} = \begin{pmatrix} -3 \\ -5 \end{pmatrix} \quad (5)$$

Parametric equation of AC is :

$$\mathbf{x} = \mathbf{A} + k(\mathbf{A} - \mathbf{C}) \quad (6)$$

Substituting (5) in (3) :

$$\|\mathbf{A} + k(\mathbf{A} - \mathbf{C}) - \mathbf{I}\|^2 = r^2 \quad (7)$$

$$(\mathbf{A} + k(\mathbf{A} - \mathbf{C}) - \mathbf{I}) \cdot (\mathbf{A} + k(\mathbf{A} - \mathbf{C}) - \mathbf{I}) = r^2 \quad (8)$$

$$k^2 \|\mathbf{A} - \mathbf{C}\|^2 + 2k(\|\mathbf{A}\|^2 - \mathbf{A}^T \mathbf{C} - \mathbf{I}^T \mathbf{A} + \mathbf{I}^T \mathbf{C}) + \|\mathbf{A} - \mathbf{I}\|^2 = r^2 \quad (9)$$

Since AC is tangent to the incircle, the discriminant of the obtained quadratic equation is zero and the value of k is given as :

$$k = -\frac{(\|\mathbf{A}\|^2 - \mathbf{A}^T \mathbf{C} - \mathbf{I}^T \mathbf{A} + \mathbf{I}^T \mathbf{C})}{\|\mathbf{A} - \mathbf{C}\|^2} \quad (10)$$

Upon substituting the values of  $\mathbf{A}$ ,  $\mathbf{C}$  and  $\mathbf{I}$  into (10) we get,

$$k = \frac{-4 - \sqrt{37} + \sqrt{61}}{2} \quad (11)$$

Substituting (11) back into (5), we get point of contact with AC,

$$\mathbf{E}_3 = \begin{pmatrix} \frac{-2 - \sqrt{37} + \sqrt{61}}{2} \\ \frac{-6 - \sqrt{37} + \sqrt{61}}{2} \end{pmatrix} \quad (12)$$

Now let us find the other point of contact, with AB.

Parametric equation of AB is :

$$\mathbf{x} = \mathbf{A} + k(\mathbf{A} - \mathbf{B}) \quad (13)$$

We can get the value of k by replacing  $\mathbf{C}$  with  $\mathbf{B}$  in Eq(10). Upon substituting the values, we get,

$$k = \frac{-37 - 4\sqrt{37} + \sqrt{2257}}{74} \quad (14)$$

Substituting (14) back into (13), we get point of contact with AB,

$$\mathbf{F}_3 = \begin{pmatrix} \frac{-111 - 20\sqrt{37} + 5\sqrt{2257}}{74} \\ \frac{185 + 28\sqrt{37} - \sqrt{2257}}{74} \end{pmatrix} \quad (15)$$

Diagram is shown on next page.

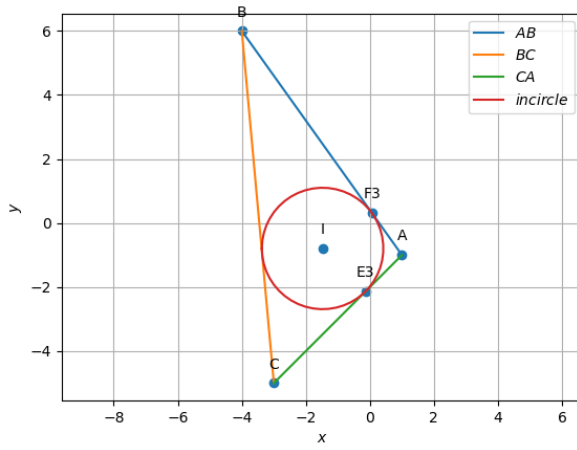


Fig. 0. Points of contact of incircle