Question 10.13.3.1

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EE22BTECH11009

Two dice are thrown at the same time. Determine the probabiity that the difference of the numbers on the two dice is 2.

Solution: Let X, Y and Z be random variables with definition given as under:

X	Number appearing on the first dice	
Y	Number appearing on the second dice	
Z	Difference of the numbers appearing on the dice	X - Y

Table 1: Definition of Random variables.

$$p_X(k) = \frac{1}{6} \tag{1}$$

$$p_Y(k) = p_X(k) \tag{2}$$

PMF of W using z-transform: applying the z-transform on both the sides

$$M_Z(z) = M_{X-Y}(z) \tag{3}$$

Using the expectation operator:

$$E\left[z^{-Z}\right] = E\left[z^{-X+Y}\right] \tag{4}$$

$$= M_X(z) \cdot M_Y(z^{-1}) \tag{5}$$

$$= \Sigma p_X(k) z^k \cdot \Sigma p_Y(k) z^{-k} \tag{6}$$

$$= \frac{1}{6}(z + z^2 + z^3 + z^4 + z^5 + z^6) \cdot \frac{1}{6}(z^{-1} + z^{-2} + z^{-3} + z^{-4} + z^{-5} + z^{-6}) \tag{7}$$

$$= \frac{1}{36}(z^{-5} + 2z^{-4} + 3z^{-3} + 4z^{-2} + 5z^{-1} + 6z^0 + 5z^1 + 4z^2 + 3z^3 + 2z^4 + z^5) \tag{8}$$

Now, we also know that,

$$M_Z(z) = \Sigma p_Z(k) z^k \tag{9}$$

Therefore,

$$\Sigma p_Z(k)z^k = \frac{1}{36}(z^{-5} + 2z^{-4} + 3z^{-3} + 4z^{-2} + 5z^{-1} + 6z^0 + 5z^1 + 4z^2 + 3z^3 + 2z^4 + z^5) \tag{10}$$

From Eq (10), we can extract the PMF of Z.

Now, let E be the event that the difference of the numbers on the two dice is 2. Then,

$$p(E) = p_Z(2) + p_Z(-2) \tag{11}$$

$$\tag{12}$$

From Eq (10), we find that,

$$p_Z(2) = \frac{1}{9} \tag{13}$$

$$p_Z(-2) = \frac{1}{9} \tag{14}$$

Therefore,

$$p(E) = \frac{2}{9} \tag{15}$$

Shown below is a graph visualising a simulation of the given question, with the 2 dice being rolled 1,000,000 times.

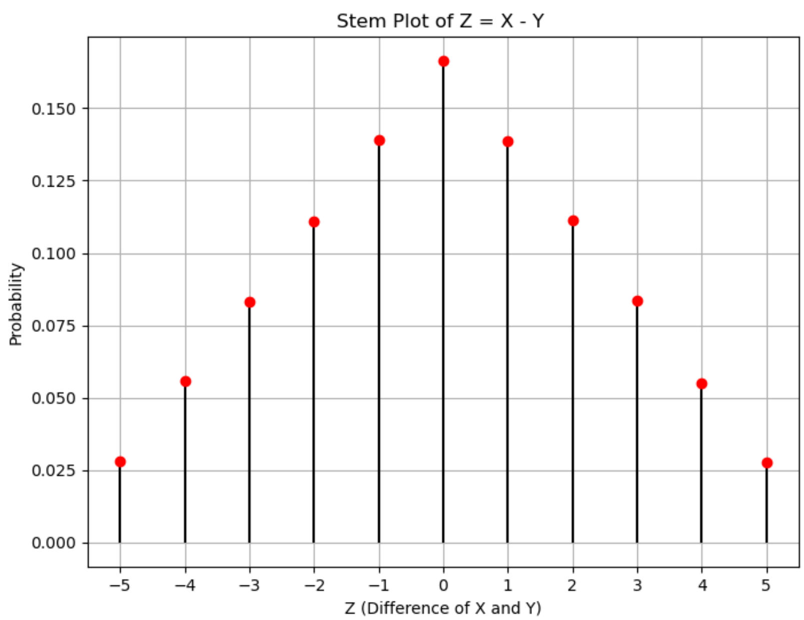

Figure 1: Stem plot for P(Z)