

Question 10.13.3.1

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Two dice are thrown at the same time. Determine the probability that the difference of the numbers on the two dice is 2.

Solution: Let X, Y and Z be random variables with definition given as under:

| | | |
|---|---|-------|
| X | Number appearing on the first dice | |
| Y | Number appearing on the second dice | |
| Z | Difference of the numbers appearing on the dice | X - Y |

Table 1: Definition of Random variables.

$$p_X(k) = \frac{1}{6} \quad (1)$$

$$p_Y(k) = p_X(k) \quad (2)$$

PMF of W using z -transform: applying the z -transform on both the sides

$$M_Z(z) = M_{X-Y}(z) \quad (3)$$

$$= M_X(z) \cdot M_Y(z^{-1}) \quad (4)$$

$$= \sum p_X(k) z^{-k} \cdot \sum p_Y(k) z^k \quad (5)$$

$$= \frac{1}{6} (z^{-1} + z^{-2} + z^{-3} + z^{-4} + z^{-5} + z^{-6}) \cdot \frac{1}{6} (z + z^2 + z^3 + z^4 + z^5 + z^6) \quad (6)$$

$$= \frac{1}{36} (z^{-5} + 2z^{-4} + 3z^{-3} + 4z^{-2} + 5z^{-1} + 6z^0 + 5z^1 + 4z^2 + 3z^3 + 2z^4 + z^5) \quad (7)$$

Now, we also know that,

$$M_Z(z) = \sum p_Z(k) z^{-k} \quad (8)$$

Therefore,

$$\sum p_Z(k) z^{-k} = \frac{1}{36} (z^{-5} + 2z^{-4} + 3z^{-3} + 4z^{-2} + 5z^{-1} + 6z^0 + 5z^1 + 4z^2 + 3z^3 + 2z^4 + z^5) \quad (9)$$

From (9), we can extract the PMF of Z .

Now, let E be the event that the difference of the numbers on the two dice is 2. Then,

$$p(E) = p_Z(2) + p_Z(-2) \quad (10)$$

$$(11)$$

From (9), we find that,

$$p_Z(2) = \frac{1}{9} \quad (12)$$

$$p_Z(-2) = \frac{1}{9} \quad (13)$$

Therefore,

$$p(E) = \frac{2}{9} \quad (14)$$

Shown below is a graph visualising a simulation of the given question, with the 2 dice being rolled 10,000 times and comparing it to the theoretical probability

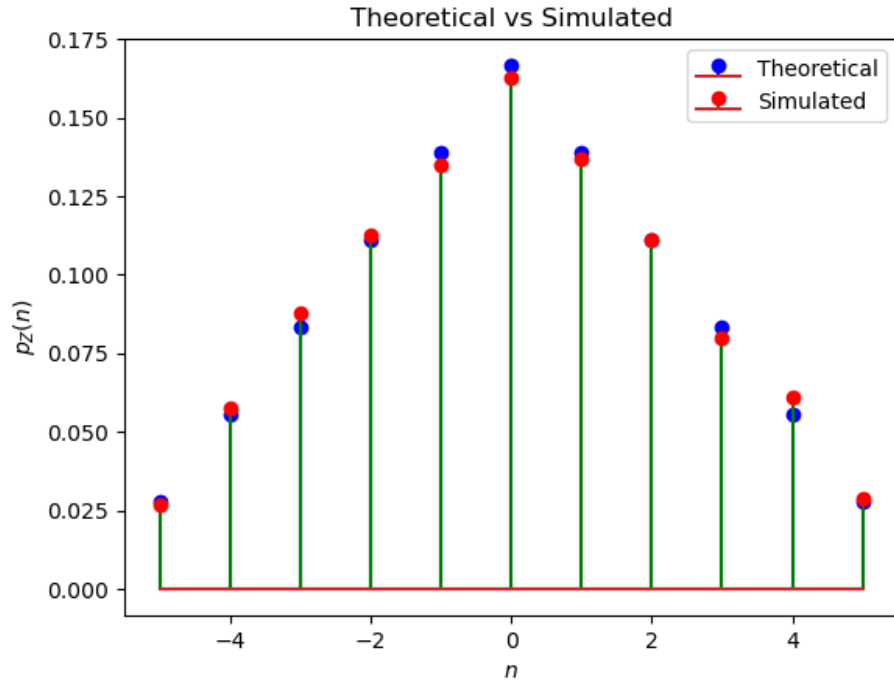


Figure 1: Stem plot for $P(Z)$