

## Question 10.13.3.26

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**Question:** Two dice are thrown at the same time. Determine the probability that the difference of the numbers on the two dice is 2.

**Solution:** Let  $X$  and  $Y$  represent the two dice. Now, the PMF of  $X$  and  $Y$  are,

$$P_X(m) = \frac{1}{6}, (0 < m < 7) \quad (1)$$

$$= 0, \text{ otherwise} \quad (2)$$

$$P_Y(m) = \frac{1}{6}, (0 < m < 7) \quad (3)$$

$$= 0, \text{ otherwise} \quad (4)$$

Let  $Z = X - Y$ . The PMF of  $Z$  is,

$$P_Z(k) = P(Z = k) \quad (5)$$

$$= P(X - Y = k) \quad (6)$$

$$= P(X = k + Y) \quad (7)$$

$$= P_X(k + Y) \quad (8)$$

$$= \sum_{m=1}^6 P_X(k + m) P_Y(m) \quad (9)$$

$$= \frac{1}{6} \sum_{m=1}^6 P_X(k + m) \quad (10)$$

Now, let  $E$  be the event that the difference of the two dice is 2. Then,

$$P(E) = P_Z(-2) + P_Z(2) \quad (11)$$

Substituting (10),

$$P(E) = \frac{1}{6} \sum_{m=1}^6 P_X(m - 2) + \frac{1}{6} \sum_{m=1}^6 P_X(m + 2) \quad (12)$$

$$= \frac{1}{6} \cdot \frac{2}{3} + \frac{1}{6} \cdot \frac{2}{3} \quad (13)$$

$$= \frac{2}{9} \quad (14)$$