# Question 10.13.3.1

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Two dice are thrown at the same time. Determine the probabiity that the difference of the numbers on the two dice is 2.

**Solution:** Let X, Y and Z be random variables with definition given as under:

| | | |
|---|---|---|
| $X$ | Number appearing on the first dice | |
| $Y$ | Number appearing on the second dice | |
| $Z$ | Difference of the numbers appearing on the dice | X - Y |

Table 1: Definition of Random variables.

$$p_X(k) = \frac{1}{6} \tag{1}$$

$$p_Y(k) = p_X(k) \tag{2}$$

PMF of $W$ using $z$-transform: applying the z-transform on both the sides

$$M_Z(z) = M_{X-Y}(z) \tag{3}$$

$$= M_X(z) \cdot M_Y(z^{-1}) \tag{4}$$

$$= \Sigma p_X(k) z^{-k} \cdot \Sigma p_Y(k) z^k \tag{5}$$

$$= \frac{1}{6}(z^{-1} + z^{-2} + z^{-3} + z^{-4} + z^{-5} + z^{-6}) \cdot \frac{1}{6}(z + z^2 + z^3 + z^4 + z^5 + z^6) \tag{6}$$

$$= \frac{1}{36}(z^{-5} + 2z^{-4} + 3z^{-3} + 4z^{-2} + 5z^{-1} + 6z^0 + 5z^1 + 4z^2 + 3z^3 + 2z^4 + z^5) \tag{7}$$

Now, we also know that,

$$M_Z(z) = \Sigma p_Z(k) z^{-k} \tag{8}$$

Therefore,

$$\Sigma p_Z(k) z^{-k} = \frac{1}{36}(z^{-5} + 2z^{-4} + 3z^{-3} + 4z^{-2} + 5z^{-1} + 6z^0 + 5z^1 + 4z^2 + 3z^3 + 2z^4 + z^5) \tag{9}$$

From Eq (9), we can extract the PMF of Z.

Now, let $E$ be the event that the difference of the numbers on the two dice is 2. Then,

$$p(E) = p_Z(2) + p_Z(-2) \tag{10}$$
$$\tag{11}$$

From Eq (9), we find that,

$$p_Z(2) = \frac{1}{9} \tag{12}$$
$$p_Z(-2) = \frac{1}{9} \tag{13}$$

Therefore,

$$p(E) = \frac{2}{9} \tag{14}$$

Shown below is a graph visualising a simulation of the given question, with the 2 dice being rolled 10,000 times and comparing it to the theoretical probability

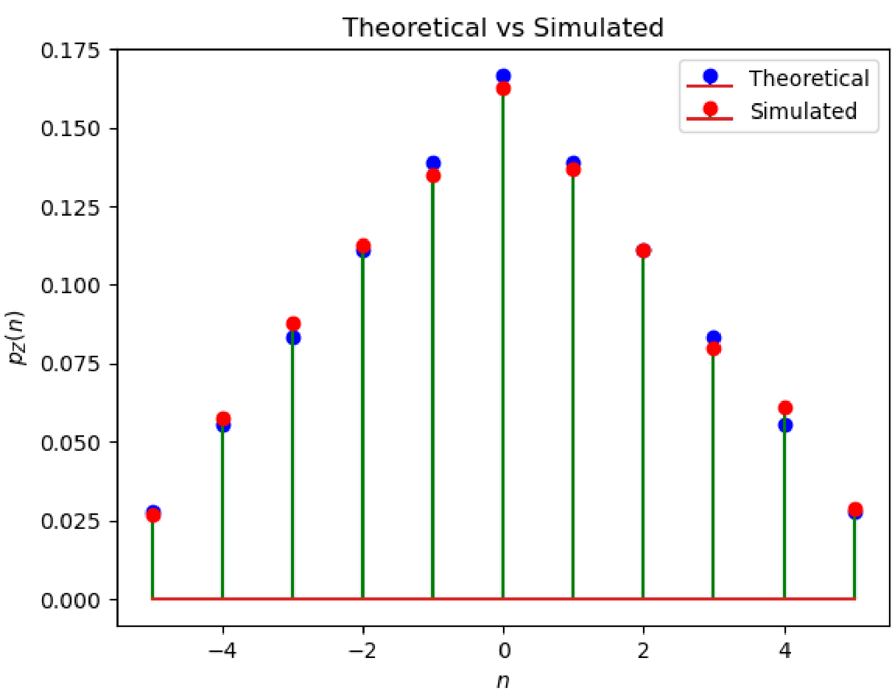


Figure 1: Stem plot for P(Z)