Abstract

Categories and Subject Descriptors   CR-number [*subcategory*]:
third-level

Keywords   gradual types, Clojure
   Tobin-Hochstadt and Felleisen [1, 2]

References

[1] S. Tobin-Hochstadt and M. Felleisen.   The design and
    implementation of typed scheme.   *SIGPLAN Not.*, 43
    (1):395–406,   Jan.   2008.   ISSN   0362-1340.   URL
    http://doi.acm.org/10.1145/1328897.1328486.

[2] S. Tobin-Hochstadt and M. Felleisen. Logical types for untyped lan-
    guages. *SIGPLAN Not.*, 45(9):117–128, Sept. 2010. ISSN 0362-1340.
    URL http://doi.acm.org/10.1145/1932681.1863561.

$$
\begin{array}{lll}
v & ::= \#\mathsf{f} \mid \#\mathsf{t} & \text{Values}\\
e & ::= x \mid v \mid (e\ e) \mid \lambda x^\tau.e \mid (\textbf{if}\ e\ e\ e) \mid (\textbf{let}\ (x\ e)\ e) \mid (\textbf{begin}\ e\ e) \mid (cons\ e\ e) & \text{Expressions}\\[4pt]
\mathcal{R} & ::= \{\omega \mid \psi\} & \text{Refinement Types}\\[4pt]
\sigma,\tau & ::= \#\mathbf{t} \mid \#\mathbf{f} \mid \mathbf{N} \mid (\textstyle\bigcup \overrightarrow{\mathcal{R}}) \mid x{:}\mathcal{R} \to \mathcal{R} \mid \langle \mathcal{R}, \mathcal{R}\rangle & \text{Types}\\[4pt]
\kappa & ::= \omega \mid x & \text{Object Variables}\\[4pt]
\psi & ::= \tau_{\pi(\kappa)} \mid \overline{\tau}_{\pi(\kappa)} \mid \psi \supset \psi \mid \psi \wedge \psi \mid \psi \vee \psi \mid \mathbb{tt} \mid \mathbb{ff} \mid \pi(\kappa) \equiv \pi(\kappa) & \text{Propositions}\\
o & ::= \pi(\kappa) \mid \emptyset & \text{Objects}\\
\pi & ::= \overrightarrow{pe} & \text{Paths}\\
pe & ::= \textbf{car} \mid \textbf{cdr} & \text{Path Elements}\\[4pt]
\Gamma & ::= \overrightarrow{\psi} & \text{Proposition Environment}
\end{array}
$$

Figure 1. Syntax of Terms, Types, Propositions, and Objects

$$
\begin{array}{lcl}
\top & = & \{\omega \mid \mathbb{tt}\}\\
\bot & = & \{\omega \mid \mathbb{ff}\}\\
\mathbf{B} & = & (\textstyle\bigcup \{\omega \mid \#\mathbf{t}_\omega\}\ \{\omega \mid \#\mathbf{f}_\omega\})
\end{array}
$$

Figure 2. Refinement Type Abbreviations

T-IF
$$
\frac{\begin{array}{c}\omega_1 \notin \mathrm{fv}(\Gamma)\\ \Gamma \vdash e_1 : \{\omega_1 \mid \psi_1\}\\ \Gamma, \overline{\#\mathbf{f}}_{\omega_1}, \psi_1 \vdash e_2 : \mathcal{R}\\ \Gamma, \#\mathbf{f}_{\omega_1}, \psi_1 \vdash e_3 : \mathcal{R}\end{array}}{\Gamma \vdash (\textbf{if}\ e_1\ e_2\ e_3) : \mathcal{R}}
$$

T-VAR
$$
\frac{\begin{array}{c}\omega \notin \mathrm{fv}(\Gamma)\\ \Gamma \vdash \tau_x\end{array}}{\Gamma \vdash x : \{\omega \mid \tau_\omega \wedge \omega \equiv x\}}
$$

T-TRUE
$$
\Gamma \vdash \#\mathsf{t} : \{\omega \mid \#\mathbf{t}_\omega\}
$$

T-FALSE
$$
\Gamma \vdash \#\mathsf{f} : \{\omega \mid \#\mathbf{f}_\omega\}
$$

T-APP
$$
\frac{\begin{array}{c}\omega \notin \mathrm{fv}(\Gamma) \qquad \mathcal{R} = \{\omega \mid \psi\}\\ \Gamma \vdash e : \{\omega_0 \mid (x{:}\mathcal{R} \to \mathcal{R}')_{\omega_0}\}\\ \Gamma \vdash e' : \{\omega \mid \psi\}\end{array}}{\Gamma \vdash (e\ e') : \mathcal{R}[\omega/x]}
$$

T-SUBSUME
$$
\frac{\Gamma \vdash e : \mathcal{R} \qquad \Gamma \vdash \mathcal{R} <: \mathcal{R}'}{\Gamma \vdash e : \mathcal{R}'}
$$

T-LET
$$
\frac{\begin{array}{c}x, \omega_1 \notin \mathrm{fv}(\Gamma)\\ \Gamma \vdash e_1 : \{\omega_1 \mid \psi_1\}\\ \Gamma, \psi_1[\omega_1/x] \vdash e : \mathcal{R}\end{array}}{\Gamma \vdash (\textbf{let}\ (x\ e_1)\ e) : \mathcal{R}}
$$

T-CONS
$$
\frac{\Gamma \vdash e_1 : \mathcal{R}_1 \qquad \Gamma \vdash e_2 : \mathcal{R}_2}{\Gamma \vdash (cons\ e_1\ e_2) : \langle \mathcal{R}_1, \mathcal{R}_2\rangle}
$$

T-CAR
$$
\frac{\begin{array}{c}\omega, \omega_1 \notin \mathrm{fv}(\Gamma)\\ \Gamma \vdash e : \{\omega_1 \mid \psi_1\}\\ \psi_1 \vdash \langle \top, \top\rangle_{\omega_1}\end{array}}{\Gamma \vdash (car\ e) : \{\omega \mid \psi_1 \wedge \textbf{car}(\omega_1) \equiv \omega\}}
$$

T-CDR
$$
\frac{\begin{array}{c}\omega, \omega_1 \notin \mathrm{fv}(\Gamma)\\ \Gamma \vdash e : \{\omega_1 \mid \psi_1\}\\ \psi_1 \vdash \langle \top, \top\rangle_{\omega_1}\end{array}}{\Gamma \vdash (cdr\ e) : \{\omega \mid \psi_1 \wedge \textbf{cdr}(\omega_1) \equiv \omega\}}
$$

T-ABS
$$
\frac{\begin{array}{c}x \notin \mathrm{fv}(\Gamma) \qquad \mathcal{R} = \{\omega \mid \psi\}\\ \Gamma, \psi[x/\omega] \vdash e : \mathcal{R}'\end{array}}{\Gamma \vdash \lambda x^{\mathcal{R}}.e : x{:}\mathcal{R} \to \mathcal{R}'}
$$

Figure 3. Typing Rules

S-REFL
$$
\Gamma \vdash \tau <: \tau
$$

S-PAIR
$$
\frac{\Gamma \vdash \mathcal{R}_1 \sqsubseteq \mathcal{R}_1' \qquad \Gamma \vdash \mathcal{R}_2 \sqsubseteq \mathcal{R}_2'}{\Gamma \vdash \langle \mathcal{R}_1, \mathcal{R}_2\rangle <: \langle \mathcal{R}_1', \mathcal{R}_2'\rangle}
$$

S-FUN
$$
\frac{\begin{array}{c}x \notin \mathrm{fv}(\Gamma)\\ \Gamma \vdash \mathcal{R}_1' \sqsubseteq \mathcal{R}_1 \qquad \Gamma \vdash \mathcal{R}_2 \sqsubseteq \mathcal{R}_2'\end{array}}{\Gamma \vdash x{:}\mathcal{R}_1 \to \mathcal{R}_2 <: x{:}\mathcal{R}_1' \to \mathcal{R}_2'}
$$

S-UNIONSUPER
$$
\frac{\exists i.\ \Gamma \vdash \tau <: \sigma_i}{\Gamma \vdash \tau <: (\textstyle\bigcup \overrightarrow{\sigma}^i)}
$$

S-UNIONSUB
$$
\frac{\overrightarrow{\Gamma \vdash \tau_i <: \sigma}^i}{\Gamma \vdash (\textstyle\bigcup \overrightarrow{\tau}^i) <: \sigma}
$$

SR-REFINE
$$
\frac{\begin{array}{c}\omega \notin \mathrm{fv}(\Gamma)\\ \Gamma \vdash \psi_1 \supset \psi_2\end{array}}{\Gamma \vdash \{\omega \mid \psi_1\} \sqsubseteq \{\omega \mid \psi_2\}}
$$

Figure 4. Subtyping rules

$$
\begin{array}{lll}
\text{L-ATOM} & & \text{L-ANDI} \\
\dfrac{\psi \in \Gamma}{\Gamma \vdash \psi} \quad \text{L-TRUE} \quad \dfrac{\text{L-FALSE}}{\Gamma \vdash \mathbb{ff}} \quad \dfrac{\Gamma \vdash \psi_1 \quad \Gamma \vdash \psi_2}{\Gamma \vdash \psi_1 \wedge \psi_2} \quad \text{L-ANDE} \quad \dfrac{\Gamma, \psi_1 \vdash \psi \ \text{or} \ \Gamma, \psi_2 \vdash \psi}{\Gamma, \psi_1 \wedge \psi_2 \vdash \psi} \quad \text{L-IMPLI} \quad \dfrac{\Gamma, \psi_1 \vdash \psi_2}{\Gamma \vdash \psi_1 \supset \psi_2} \quad \dfrac{\text{L-IMPLE}}{\Gamma \vdash \psi_1 \quad \Gamma \vdash \psi_1 \supset \psi_2}{\Gamma \vdash \psi_2}
\end{array}
$$

Figure 5 — Proof System

L-ATOM
$$\dfrac{\psi \in \Gamma}{\Gamma \vdash \psi}$$

L-TRUE
$$\Gamma \vdash \mathbb{tt}$$

L-FALSE
$$\dfrac{\Gamma \vdash \mathbb{ff}}{\Gamma \vdash \psi}$$

L-ANDI
$$\dfrac{\Gamma \vdash \psi_1 \quad \Gamma \vdash \psi_2}{\Gamma \vdash \psi_1 \wedge \psi_2}$$

L-ANDE
$$\dfrac{\Gamma, \psi_1 \vdash \psi \ \text{or} \ \Gamma, \psi_2 \vdash \psi}{\Gamma, \psi_1 \wedge \psi_2 \vdash \psi}$$

L-IMPLI
$$\dfrac{\Gamma, \psi_1 \vdash \psi_2}{\Gamma \vdash \psi_1 \supset \psi_2}$$

L-IMPLE
$$\dfrac{\Gamma \vdash \psi_1 \quad \Gamma \vdash \psi_1 \supset \psi_2}{\Gamma \vdash \psi_2}$$

L-ORI
$$\dfrac{\Gamma \vdash \psi_1 \ \text{or} \ \Gamma \vdash \psi_2}{\Gamma \vdash \psi_1 \vee \psi_2}$$

L-ORE
$$\dfrac{\Gamma, \psi_1 \vdash \psi \quad \Gamma, \psi_2 \vdash \psi}{\Gamma, \psi_1 \vee \psi_2 \vdash \psi}$$

L-SUB
$$\dfrac{\Gamma \vdash \tau_{\pi(\kappa)} \quad \vdash \tau <: \sigma}{\Gamma \vdash \sigma_{\pi(\kappa)}}$$

L-SUBNOT
$$\dfrac{\Gamma \vdash \overline{\sigma}_{\pi(\kappa)} \quad \vdash \tau <: \sigma}{\Gamma \vdash \overline{\tau}_{\pi(\kappa)}}$$

L-ALIAS
$$\dfrac{\Gamma \vdash \psi \quad \Gamma \vdash \pi(\kappa_1) \equiv \kappa_2 \ \text{or} \ \Gamma \vdash \kappa_2 \equiv \pi(\kappa_1)}{\Gamma \vdash \psi[\pi(\kappa_1)/\kappa_2]}$$

L-BOT
$$\dfrac{\Gamma \vdash \perp_{\pi(\kappa)}}{\Gamma \vdash \psi}$$

L-UPDATE
$$\dfrac{\Gamma \vdash \tau_{\pi'(\kappa)} \quad \Gamma \vdash \nu_{\pi(\pi'(\kappa))}}{\Gamma \vdash \mathsf{update}(\Gamma, \tau, \nu, \pi)_{\pi'(\kappa)}}$$

Figure 5. Proof System

$$
\begin{aligned}
\Gamma[o/\kappa] &= \overrightarrow{\psi[o/\kappa]} & \Gamma = \overrightarrow{\psi} \\
\psi_+ | \psi_-[o/\kappa] &= \psi_+[o/\kappa] | \psi_-[o/\kappa] & \\[4pt]
\nu_{\pi(\kappa)}[\pi'(\kappa')/\kappa] &= (\nu[\pi(\kappa')/\kappa])_{\pi(\pi'(\kappa'))} & \\
\nu_{\pi(\kappa)}[\emptyset/\kappa]_+ &= \mathbb{tt} & \\
\nu_{\pi(\kappa)}[\emptyset/\kappa]_- &= \mathbb{ff} & \\
\nu_{\pi(\kappa)}[o/\kappa'''] &= \nu_{\pi(\kappa)} & \kappa \neq \kappa''' \text{ and } \kappa''' \notin \mathrm{fv}(\nu) \\
\nu_{\pi(\kappa)}[o/\kappa''']_+ &= \mathbb{tt} & \kappa \neq \kappa''' \text{ and } \kappa''' \in \mathrm{fv}(\nu) \\
\nu_{\pi(\kappa)}[o/\kappa''']_- &= \mathbb{ff} & \kappa \neq \kappa''' \text{ and } \kappa''' \in \mathrm{fv}(\nu) \\
\mathbb{tt}[o/\kappa] &= \mathbb{tt} & \\
\mathbb{ff}[o/\kappa] &= \mathbb{ff} & \\
(\psi_1 \supset \psi_2)[o/\kappa]_+ &= \psi_1[o/\kappa]_- \supset \psi_2[o/\kappa]_+ & \\
(\psi_1 \supset \psi_2)[o/\kappa]_- &= \psi_1[o/\kappa]_+ \supset \psi_2[o/\kappa]_- & \\
(\psi_1 \vee \psi_2)[o/\kappa] &= \psi_1[o/\kappa] \vee \psi_2[o/\kappa] & \\
(\psi_1 \wedge \psi_2)[o/\kappa] &= \psi_1[o/\kappa] \wedge \psi_2[o/\kappa] & \\
(\pi'(\kappa') \equiv \pi''(y'))[\pi(\kappa''')/\kappa] &= \pi'(\kappa')[\pi(\kappa''')/\kappa] \equiv \pi''(y')[\pi(\kappa''')/\kappa] & \\
(\pi(\kappa'') \equiv \pi'(\kappa'''))[\emptyset/\kappa] &= \pi(\kappa'') \equiv \pi'(\kappa''') & \kappa \neq \kappa'' \text{ and } \kappa \neq \kappa''' \\
(\pi(\kappa) \equiv \pi'(\kappa''))[\emptyset/\kappa'''] &= \mathbb{tt} & \\[4pt]
\pi(\kappa)[\pi'(\kappa'')/\kappa] &= \pi(\pi'(\kappa'')) & \\
\pi(\kappa)[\emptyset/\kappa] &= \emptyset & \\
\pi(\kappa)[o/\kappa'''] &= \pi(\kappa) & \kappa \neq \kappa''' \\
\emptyset[o/\kappa] &= \emptyset & \\[4pt]
\{\omega \mid \psi\}[o/\kappa] &= \{\omega \mid \psi[o/\kappa]\} & \kappa \neq \omega \\
\{\omega \mid \psi\}[o/\kappa] &= \{\omega \mid \psi\} & \\
(x{:}\sigma \rightarrow \tau)[o/\kappa] &= x{:}\sigma[o/\kappa] \rightarrow \tau[o/\kappa] & x \neq \kappa \\
(x{:}\sigma \rightarrow \tau)[o/\kappa] &= x{:}\sigma[o/\kappa] \rightarrow \tau &
\end{aligned}
$$

Figure 6. Substitution

$$
\begin{aligned}
\delta_\tau(number?) &= x{:}\top \rightarrow \{\omega \mid \mathbf{B}_\omega \wedge \overline{\#\mathbf{f}}_\omega \supset \mathbf{N}_x \wedge \#\mathbf{f}_\omega \supset \overline{\mathbf{N}}_x\} \\[6pt]
\delta_\tau(boolean?) &= x{:}\top \rightarrow \{\omega \mid \mathbf{B}_\omega \wedge \overline{\#\mathbf{f}}_\omega \supset \mathbf{B}_x \wedge \#\mathbf{f}_\omega \supset \overline{\mathbf{B}}_x\} \\[6pt]
\delta_\tau(cons?) &= x{:}\top \rightarrow \{\omega \mid \mathbf{B}_\omega \wedge \overline{\#\mathbf{f}}_\omega \supset \langle \top, \top \rangle_x \wedge \#\mathbf{f}_\omega \supset \overline{\langle \top, \top \rangle}_x\} \\[6pt]
\delta_\tau(add1) &= x{:}\mathbf{N} \rightarrow \mathbf{N} \\[6pt]
\delta_\tau(zero?) &= x{:}\mathbf{N} \rightarrow \mathbf{B}
\end{aligned}
$$

Figure 7. Constant Typing

$$
\begin{array}{lcll}
\text{update}(\Gamma, \langle \{\omega \mid \psi\}, \mathcal{R} \rangle, \nu, \pi :: \mathbf{car}) & = & \langle \{\omega \mid \psi \wedge \nu_{\pi(\omega)}\}, \mathcal{R} \rangle \\
\text{update}(\Gamma, \langle \mathcal{R}, \{\omega \mid \psi\} \rangle, \nu, \pi :: \mathbf{cdr}) & = & \langle \mathcal{R}, \{\omega \mid \psi \wedge \nu_{\pi(\omega)}\} \rangle \\
\text{update}(\Gamma, \tau, \sigma, \epsilon) & = & \text{restrict}(\Gamma, \tau, \sigma) \\
\text{update}(\Gamma, \tau, \overline{\sigma}, \epsilon) & = & \text{remove}(\Gamma, \tau, \overline{\sigma}) \\[2ex]
\text{restrict}(\Gamma, \tau, \sigma) & = & \bot & \text{if } \not\exists v.\, \Gamma \vdash v : \{\omega \mid \sigma_\omega\} \text{ and } \Gamma \vdash v : \{\omega \mid \tau_\omega\} \\
\text{restrict}(\Gamma, (\bigcup \overrightarrow{\tau}), \sigma) & = & (\bigcup \overrightarrow{\text{restrict}(\Gamma, \tau, \sigma)}) \\
\text{restrict}(\Gamma, \tau, \sigma) & = & \tau & \text{if } \Gamma \vdash \tau <: \sigma \\
\text{restrict}(\Gamma, \tau, \sigma) & = & \sigma & \text{otherwise} \\[2ex]
\text{remove}(\Gamma, \tau, \sigma) & = & \bot & \text{if } \Gamma \vdash \tau <: \sigma \\
\text{remove}(\Gamma, (\bigcup \overrightarrow{\tau}), \sigma) & = & (\bigcup \overrightarrow{\text{remove}(\Gamma, \tau, \sigma)}) \\
\text{remove}(\Gamma, \tau, \sigma) & = & \tau & \text{otherwise}
\end{array}
$$

Figure 8. Type Update