A. Soundness for Typed Clojure

Assumption A.1 (JVM_{new}). If $\forall i. v_i = C_i \{ \overrightarrow{fld_j : v_j} \}$ or $v_i = \text{nil } and \ v_i \text{ is consistent } with \ \rho \text{ then } either$

- $\mathsf{JVM}_{\mathsf{new}}[C, [\overrightarrow{C_i}], [\overrightarrow{v_i}]] = C\{\overrightarrow{fld_k} : \overrightarrow{v_k}\}\ which is consistent with \ \rho,$
- $\bullet \ \ \mathsf{JVM}_{\mathsf{new}}[C, [\overrightarrow{C_i}], [\overrightarrow{v_i}]] = \mathsf{err}, \, \mathit{or} \,$
- $\mathsf{JVM}_{\mathsf{new}}[C, [\overrightarrow{C_i}], [\overrightarrow{v_i}]]$ is undefined.

Assumption A.2 (JVM_{getstatic}). If $v_1 = C_1 \{fld : v_f, \overline{fld_l : v_l}\}$, then either

- $\mathsf{JVM}_{\mathsf{getstatic}}[C_1, v_1, fld, C_2] = v_f$, and either
 - $\bullet v_f = C_2 \left\{ \overrightarrow{fld_m : v_m} \right\} or$
 - $v_f = \text{nil}, or$
- ullet JVM $_{
 m getstatic}[C_1,v_1,fld,C_2]={
 m err.}$

Assumption A.3 (JVM_{invokestatic}). If $v_1 = C_1$ { $\overrightarrow{fld_i : v_l}$ }, $\forall i. \ v_i = C_i$ { $\overrightarrow{fld_j : v_j}$ } or $v_i = \text{nil then either}$

- $\mathsf{JVM}_{\mathsf{invokestatic}}[C_1, v_m, mth, [\overrightarrow{C_i}], [\overrightarrow{v_i}], C_2] = v$ and either
 - $v = C_2 \{\overrightarrow{fld_m : v_m}\} \ or \ v = \mathsf{nil}, \ or$
- $\mathsf{JVM}_{\mathsf{invokestatic}}[C_1, v_m, mth, [\overrightarrow{C_i}], [\overrightarrow{v_i}], C_2] = \mathsf{err}, or$
- $\mathsf{JVM}_{\mathsf{invokestatic}}[C_1, v_m, mth, [\overrightarrow{C_i}], [\overrightarrow{v_i}], C_2]$ is undefined.

Lemma A.1. If ρ and ρ' agree on $\mathsf{fv}(\psi)$ and $\rho \models \psi$ then $\rho' \models \psi$.

Proof. Since the relevant parts of ρ and ρ' agree, the proof follows trivially.

Lemma A.2. If

- $\bullet \ \psi_1 = \psi_2[o/x],$
- $\rho_2 \models \psi_2$,
- $\forall v \in \mathsf{fv}(\psi_2) x. \ \rho_1(v) = \rho_2(v),$
- and $\rho_2(x) = \rho_1(o)$

then $\rho_1 \models \psi_1$.

Proof. By induction on the derivation of the model judgement.

Lemma A.3. If $\rho \models \Gamma$ and $\Gamma \vdash \psi$ then $\rho \models \psi$.

Proof. By structural induction on $\Gamma \vdash \psi$.

Lemma A.4. If $\Gamma \vdash \tau_{\pi(x)}$, $\rho \models \Gamma$ and $\rho(\pi(x)) = v$ then $\vdash v : \tau ; \psi'_{+} | \psi'_{-} ; o'$ for some ψ'_{+}, ψ'_{-} and o'.

Proof. Corollary of lemma A.3.

Lemma A.5 (Paths are independent). If $\rho(o) = \rho_1(o')$ then $\rho(\pi(o)) = \rho_1(\pi(o'))$

Proof. By induction on π .

Case
$$(\pi = \epsilon)$$
. $\rho(o) = \rho(o')$

As $\rho(\epsilon(o)) = \rho(o)$ and $\rho(\epsilon(o')) = \rho(o')$ we can conclude $\rho(\epsilon(o)) = \rho(\epsilon(o'))$.

Case $(\pi = pe :: \pi_1)$. $\rho(o) = \rho(o')$

By cases on pe.

Subcase $(pe = \mathbf{key}_k)$. By the induction hypothesis on π_1 we know $\rho(\pi_1(o)) = \rho_1(\pi_1(o'))$. By the definition of path translation $\rho(\pi_1(o)) = (\text{get } \rho(o) \ k)$ and $\rho(\pi_1(o')) = (\text{get } \rho(o') \ k)$

Lemma A.6 (class). If $\rho \vdash (\text{class } \rho(\pi(x))) \Downarrow C \text{ then } \rho \models C_{\pi(x)}$.

Proof. Induction on the definition of class.

Definition A.1. v is consistent with ρ iff $\forall [\rho_1, \lambda x^{\sigma}.e]_c$ in v, if $\vdash [\rho_1, \lambda x^{\sigma}.e]_c : \tau$; $\text{tt} \mid \text{ff}$; \emptyset and \forall o' in τ , either $o' = \emptyset$, or $o' = \pi'(x)$, or $\rho(o') = \rho_1(o')$.

Definition A.2. ρ is consistent iff

 $\forall v \in rng(\rho), v \text{ is consistent with } \rho.$

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Definition A.3. TrueVal(v) iff $v \neq$ false and $v \neq$ nil.

Definition A.4. FalseVal(v) iff v =false or v =nil.

Lemma A.7. If $\Gamma \vdash e : \tau ; \psi_{+} | \psi_{-} ; o, \rho \models \Gamma, \rho \text{ is consistent, and } \rho \vdash e \Downarrow \alpha \text{ then either}$

- $\rho \vdash e \Downarrow v$ and all of the following hold:
 - 1. either $o = \emptyset$ or $\rho(o) = v$,
 - 2. either TrueVal(v) and $\rho \models \psi_+$ or FalseVal(v) and $\rho \models \psi_-$,
 - 3. $\vdash v : \tau$; $\psi'_{+} | \psi'_{-}$; o' for some ψ'_{+} , ψ'_{-} and o', and
 - 4. v is consistent with ρ , or
- $\rho \vdash e \Downarrow \text{err.}$

Proof. By induction on the derivation of the typing judgement.

Case (T-True).
$$e = \text{true}, \ \tau = \text{true}, \ \psi_+ = \text{tt}, \ \psi_- = \text{ff}, \ o = \emptyset$$

Subcase (B-Val). v = true

Proving part 1 is trivial: o is \emptyset . To prove part 2, we note that v = true and ψ_+ = \mathfrak{tt} , so $\rho \models \psi_+$ by M-Top. Part 3 holds as e can only be reduced to itself via B-Val. Part 4 holds vacuously.

Case (T-HMap).
$$e = \{\overrightarrow{v_k \mapsto v_v}\}, \tau = (\mathbf{HMap}^{\mathcal{C}} \mathcal{M}), \psi_+ = \text{tt}, \psi_- = \text{ff}, o = \emptyset, \overrightarrow{\vdash v_k : (\mathbf{Val} \ k)}, \overrightarrow{\vdash v_v : \tau_v}, \mathcal{M} = \{\overrightarrow{k \mapsto \tau_v}\}$$

Subcase (B-Val). $v = \{\overrightarrow{v_k \mapsto v_v}\}$

Similar to T-True.

Case (T-Kw).
$$e = k$$
, $\tau = (Val k)$, $\psi_+ = tt$, $\psi_- = ff$, $o = \emptyset$

Subcase (B-Val). v = k

Similar to T-True.

Case (T-False).
$$e = \text{false}, \ \tau = \text{false}, \ \psi_+ = \text{ff}, \ \psi_- = \text{tt}, \ o = \emptyset$$

Subcase (B-Val). v = false

Proving part 1 is trivial: o is \emptyset . To prove part 2, we note that v = false and ψ_- = \mathfrak{tt} , so $\rho \models \psi_-$ by M-Top. Part 3 holds as e can only be reduced to itself via B-Val. Part 4 holds vacuously.

Case (T-Class).
$$e=C,\, \tau=$$
 (Val C), $\psi_+=$ tt , $\psi_-=\operatorname{ff}$, $o=\emptyset$

Subcase (B-Val). v = C

Similar to T-True.

$$\textit{Case} \text{ (T-Instance)}. \ \ e = C \ \{\overrightarrow{fld:v}\}, \ \tau = C, \ \psi_+ = \mathtt{tt}, \ \psi_- = \mathtt{ff}, \ o = \emptyset$$

Subcase (B-Val). $v = C \{ \overrightarrow{fld_i : v_i} \}$

Similar to T-True.

Part 4 holds by the induction hypothesis on $\overrightarrow{v_i}$.

Case (T-Nil).
$$e = \text{nil}, \tau = \text{nil}, \psi_+ = \text{ff}, \psi_- = \text{tt}, o = \emptyset$$
,

Subcase (B-Val). v = nil

Similar to T-False.

Case (T-Local).
$$e = x, \psi_+ = \overline{(\cup \text{ nil false})}_x, \psi_- = (\cup \text{ nil false})_x, o = x, \Gamma \vdash \tau_x,$$

Subcase (B-Local). $\rho(x) = v$, $\rho \vdash x \downarrow v$

Part 1 follows from $\rho(x) = v$ by B-Local.

Part 2 considers two cases: if v is not false or nil, then $\rho \models \overline{(\cup \text{ nil false})_x}$ holds by M-NotType; if v is false or nil, then $\rho \models \overline{(\cup \text{ nil false})_x}$ $(\cup$ **nil false** $)_{x}$ holds by M-Type.

We prove part 3 by observing $\Gamma \vdash \tau_x$ and $\rho \models \Gamma$, so $\rho(x) = v$ by B-Local gives us the desired result.

Part 4 holds vacuously.

Case (T-Do). $e = (\text{do } e_1 \ e_2), \Gamma \vdash e_1 : \tau_1 \ ; \ \psi_{1_+} | \psi_{1_-} \ ; \ o_1, \Gamma, \psi_{1_+} \lor \psi_{1_-} \vdash e_2 : \tau \ ; \ \psi_{+} | \psi_{-} \ ; \ o,$

Subcase (B-Do). $\rho \vdash e_1 \Downarrow v_1, \rho \vdash e_2 \Downarrow v$

For all parts we note since e_1 can be either a true or false value then $\rho \models \Gamma, \psi_{1_+} \lor \psi_{1_-}$ by M-Or, which together with $\Gamma, \psi_{1_+} \lor \psi_{1_-} \vdash$ $e_2:\tau$; $\psi_+|\psi_-$; o, and $\rho\vdash e_2\Downarrow v$ allows us to apply the induction hypothesis on e_2 .

To prove part 1 we use the induction hypothesis on e_2 to show either $o = \emptyset$ or $\rho(o) = v$, since e always evaluates to the result of e_2 . For part 2 we use the induction hypothesis on e_2 to show if $v \neq \mathsf{false}$ (or nil) then $\rho \models \psi_+$ or if $v = \mathsf{false}$ (or nil) then $\rho \models \psi_-$.

Parts 3 and 4 follow from the induction hypothesis on e_2 .

Subcase (BE-Do1). $\rho \vdash e_1 \Downarrow \mathsf{err}, \rho \vdash e \Downarrow \mathsf{err}$

Trivially reduces to an error.

Subcase (BE-Do2). $\rho \vdash e_1 \Downarrow v_1, \rho \vdash e_2 \Downarrow \mathsf{err}, \rho \vdash e \Downarrow \mathsf{err}$

As above.

 $\textit{Case} \text{ (T-NewStatic)}. \quad e = (\text{new}_{[\overrightarrow{C_i}]} \ C \ \overrightarrow{e_i}), o = \emptyset, \psi_+ = \texttt{tt}, \psi_- = \texttt{ff}, \overrightarrow{\mathsf{Convert}(C_i)} = \overrightarrow{\tau_i}, \mathsf{Convert} \ (C) = \tau, \overrightarrow{\Gamma \vdash e_i} : \overrightarrow{\tau_i} = \overrightarrow{\tau_i} = \tau_i, \overrightarrow{\mathsf{Convert}(C_i)} = \overrightarrow{\tau_i} = \tau_i, \overrightarrow{\mathsf{Convert}(C_i)} = \tau_i, \overrightarrow{$

 $\textit{Subcase} \text{ (B-New). } \overrightarrow{\rho \vdash e_i \Downarrow v_i}, \mathsf{JVM}_{\mathsf{new}}[C_1, [\overrightarrow{C_i}], [\overrightarrow{v_i}]] = v$

Part 1 follows $o = \emptyset$.

Part 2 requires some explanation. The two false values in Typed Clojure cannot be constructed with new, so the only case is $v \neq f$ false (or nil) where $\psi_{+} = tt$ so $\rho \models \psi_{+}$.

Part 3 holds as B-New reduces to a *non-nilable* instance of C via JVM_{new} (by assumption A.1), and Convert $(C) = \tau$.

Subcase (BE-New1). $\rho \vdash e_{i-1} \Downarrow v_{i-1}, \rho \vdash e_i \Downarrow \mathsf{err}, \rho \vdash e \Downarrow \mathsf{err}$

Trivially reduces to an error.

Subcase (BE-New2). $\overrightarrow{\rho \vdash e_i \Downarrow v_i}$, $\mathsf{JVM}_{\mathsf{new}}[C_1, [\overrightarrow{C_i}], [\overrightarrow{v_i}]] = \mathsf{err}, \rho \vdash e \Downarrow \mathsf{err}$

Case (T-FieldStatic). $e = (.e_1 fld_{C_2}^{C_1})$, Convert $(C_1) = C$, Convert_{nil} $(C_2) = \tau$, $\Gamma \vdash e_1 : C$

Subcase (B-Field). $\rho \vdash e_1 \Downarrow C_1 \{fld : v\}$

Part 1 is trivial as o is always \emptyset . Part 2 holds trivially, v can be either a true or false value and both ψ_+ and ψ_- are \mathfrak{tt} . Part 3 relies on the semantics of $\mathsf{JVM}_{\mathsf{getstatic}}$ (assumption A.2) in B-Field, which returns a *nilable* instance of C_2 , and $\mathsf{Convert}_{\mathsf{nil}}(C_2) = \tau$.

Subcase (BE-Field). $\rho \vdash e_1 \Downarrow \mathsf{err}, \rho \vdash e \Downarrow \mathsf{err}$

Trivially reduces to an error.

 $\textit{Case} \; (\text{T-MethodStatic}). \;\; e = (. \; e_m \; (mth^{C_1}_{[[\overrightarrow{C_i}], C_2]} \; \overrightarrow{e_a})), \\ \psi_+ = \texttt{tt}, \\ \psi_- = \texttt{tt}, \\ o = \emptyset, \\ \overrightarrow{\mathsf{Convert}(C_i)} \; = \; \overrightarrow{\tau_i}, \\ \mathsf{Convert} \; (C_1) = C, \\ \mathsf{Convert}_{\mathsf{nil}}(C_2) =$

 $\textit{Subcase} \text{ (B-Method)}. \ \ \rho \vdash e_m \Downarrow v_m, \overrightarrow{\rho \vdash e_a \Downarrow v_a}, \\ \mathsf{JVM}_{\mathsf{invokestatic}}[C_1, v_m, mth, [\overrightarrow{C_a}], [\overrightarrow{v_a}], C_2] = v$

Part 1 is trivial as o is always \emptyset . Part 2 holds trivially, v can be either a true or false value and both ψ_+ and ψ_- are \mathfrak{tt} . Part 3 relies on the semantics of $\mathsf{JVM}_{\mathsf{invokestatic}}$ (assumption A.3) in B-Method, which returns a $\mathit{nilable}$ instance of C_2 , and $\mathsf{Convert}_{\mathsf{nil}}(C_2) = \tau$.

Subcase (BE-Method1). $\rho \vdash e_m \Downarrow \mathsf{err}, \rho \vdash e \Downarrow \mathsf{err}$ Trivially reduces to an error.

 $\textit{Subcase} \text{ (BE-Method 2)}. \ \ \rho \vdash e_m \Downarrow v_m, \overrightarrow{\rho \vdash e_{n-1} \Downarrow v_{n-1}}, \rho \vdash e_n \Downarrow \mathsf{err}, \rho \vdash e \Downarrow \mathsf{err}$

15 2015/2/28 Subcase (BE-Method3). $\rho \vdash e_m \Downarrow v_m, \overrightarrow{\rho \vdash e_a \Downarrow v_a}, \mathsf{JVM}_{\mathsf{invokestatic}}[C_1, v_m, mth, [\overrightarrow{C_a}], [\overrightarrow{v_a}], C_2] = \mathsf{err}, \rho \vdash e \Downarrow \mathsf{err}$

 $\textit{Case} \; (\text{T-DefMulti}). \; \; e = \left(\text{defmulti} \; \sigma \; e_d \right), \; \tau = \left(\textbf{Multi} \; \sigma \; \tau_d \right), \; \psi_+ = \text{\texttt{tt}} \; , \; \psi_- = \text{\texttt{ff}} \; , \; \sigma = x : \tau_1 \; \xrightarrow{\psi_1 + |\psi_1|} \tau_2, \; \tau_d = x : \tau_1 \; \xrightarrow{\psi_2 + |\psi_2|} \tau_3, \; \Gamma \vdash e_d : \tau_d \vdash \psi_1 = \psi_2 \vdash \psi_2 \vdash \psi_2 \vdash \psi_3 \vdash \psi_4 \vdash \psi_$

Subcase (B-DefMulti). $v = [v_d, \{\}]_m, \rho \vdash e_d \Downarrow v_d$

Part 1 and 2 hold for the same reasons as T-True. For part 3 we show $\vdash [v_d, \{\}]_m$: (**Multi** $\sigma \tau_d$) by T-Multi, since $\vdash v_d : \tau_d$ by the inductive hypothesis on e_d and $\{\}$ vacuously satisfies the other premises of T-Multi, so we are done.

 $\textit{Subcase} \ (\text{BE-DefMulti}). \ \ \rho \vdash e_d \Downarrow \mathsf{err}, \, \rho \vdash e \Downarrow \mathsf{err}$ Trivially reduces to an error.

Case (T-DefMethod).

- 1. $e = (\text{defmethod } e_m \ e_v \ e_f),$
- 2. $e_f = \lambda x^{\tau_1} . e_b$,
- 3. $\tau = ($ **Multi** $\tau_m \ \tau_d),$
- 4. $\psi_+ = tt$,
- 5. ψ_{-} = ff,
- 6. $o = \emptyset$,
- 7. $\tau_m = x : \tau_1 \xrightarrow[o_m]{\psi_{m+} \mid \psi_{m-}} \sigma$, 8. $\tau_d = x : \tau_1 \xrightarrow[o_d]{\psi_{d+} \mid \psi_{d-}} \sigma'$,
- 9. $\Gamma \vdash e_m : (\mathbf{Multi} \, \tau_m \, \tau_d),$
- 10. IsAProps $(o_d, \tau_v) = \psi_{i_+} | \psi_{i_-},$
- 11. $\Gamma \vdash e_v : \tau_v$,
- 12. $\Gamma, \tau_{1_x}, \psi_{i_+} \vdash e_b : \sigma ; \psi_{m_+} | \psi_{m_-} ; o_m$

Subcase (B-DefMethod).

13.
$$v = [v_d, t']_m$$
,

14.
$$\rho \vdash e_m \Downarrow [v_d, t]_m$$
,
15. $\rho \vdash e_v \Downarrow v_v$,

15.
$$\rho \vdash e_v \Downarrow v_v$$

16.
$$\rho \vdash e_f \Downarrow v_f$$
,

17.
$$t' = t[v_v \mapsto v_f]$$

Part 1 and 2 hold for the same reasons as T-True, noting that the propositions and object agree with T-Multi.

For part 3 we show $\vdash [v_d, t[v_v \mapsto v_f]]_m : (\mathbf{Multi} \ \tau_m \ \tau_d)$ by noting $\vdash v_d : \tau_d, \vdash v_v : \top$ and $\vdash v_f : \tau_m$, and since t is in the correct form by the inductive hypothesis on e_m we can satisfy all premises of T-Multi, so we are done.

Subcase (BE-DefMethod1). $\rho \vdash e_m \Downarrow \mathsf{err}, \rho \vdash e \Downarrow \mathsf{err}$ Trivially reduces to an error.

Subcase (BE-DefMethod2). $\rho \vdash e_m \Downarrow [v_d, t]_m, \rho \vdash e_v \Downarrow \mathsf{err}, \rho \vdash e \Downarrow \mathsf{err}$ As above.

Subcase (BE-DefMethod3). $\rho \vdash e_m \Downarrow [v_d, t]_m, \rho \vdash e_v \Downarrow v_v, \rho \vdash e_f \Downarrow \mathsf{err}, \rho \vdash e \Downarrow \mathsf{err}$ As above.

Case (T-App).

- $e = (e_1 \ e_2),$
- $\bullet \ \tau = \tau_f[o_2/x],$
- $\psi_+ = \psi_{f_+}[o_2/x],$
- $\psi_{-} = \psi_{f_{-}}[o_2/x],$
- $o = o_f[o_2/x],$ $\Gamma \vdash e_1 : x : \sigma \xrightarrow{\psi_{f_+} | \psi_{f_-}} \tau_f ; \psi_{1_+} | \psi_{1_-} ; o_1,$
- $\Gamma \vdash e_2 : \sigma \; ; \; \psi_{2_+} | \psi_{2_-} \; ; \; o_2,$

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Subcase (B-BetaClosure).

- $\bullet \rho \vdash e \Downarrow v$,
- $\rho \vdash e_1 \Downarrow [\rho_c, \lambda x^{\sigma}.e_b]_c$,
- $\bullet \rho \vdash e_2 \downarrow v_2,$
- $\bullet \rho_c[x \mapsto v_2] \vdash e_b \Downarrow v$

By inversion on e_1 from T-Clos there is some environment Γ_c such that

and also by inversion on e_1 from T-Abs

 $\blacksquare \Gamma_c, \sigma_x \vdash e_b : \tau_f ; \psi_{f_\perp} | \psi_{f_\perp} ; o_f.$

To prove part 1, we consider cases on the form of o_f :

- if $o_f = \emptyset$, then $o = \emptyset$ by substitution,
- if $o_f = \pi_f(x)$, we consider cases on the form of o_2 :
 - if $o_2 = \emptyset$, then $o = \emptyset$,
 - otherwise $o_2 = \pi_2(y)$, where, because $o = \pi_f(x)[o_2/x]$, by substitution $o = \pi_f(o_2)$. To prove $\rho(o) = v$ we note $\rho_c[x \mapsto c_1]$ $v_2(x) = v_2$ and $\rho(o_2) = v_2$, which implies $\rho_c(x) \mapsto v_2(x) = \rho(o_2)$, and by applying lemma A.5 we can derive $\rho(\pi_f(o_2)) = v_2(x) = \rho(o_2)$ $\rho_c[x \mapsto v_2](\pi_f(x))$. By the induction hypothesis on e_b we know $\rho_c[x \mapsto v_2](\pi_f(x)) = v$ so we can conclude $\rho(\pi_f(o_2)) = v$.
- otherwise $o_f = \pi_f(y)$ where $x \neq y$, and $o = o_f$ by substitution. We note $[\rho_c, \lambda x^{\sigma}.e_b]_c$ is consistent with ρ by the induction hypothesis on e_1 , so $\rho_c(y) = \rho(y)$, and because $x \neq y$ we also know $\rho_c[x \mapsto v_2](y) = \rho(y)$.

We apply lemma A.5 with this fact to derive $\rho_c[x \mapsto v_2](\pi_f(y)) = \rho(\pi_f(y))$, and since $\rho_c[x \mapsto v_2](\pi_f(y)) = v$ by the induction hypothesis on e_b we can conclude $\rho(\pi_f(o_2)) = v$.

To prove part 2, we consider both cases:

- if $v \neq$ false (or nil) then $\rho \models \psi_+$.
- By the induction hypothesis on e_b we know $\rho_c[x \mapsto v_2] \models \psi_{f_+}$.

By the induction hypothesis on e_1 we know $[\rho_c, \lambda x^{\sigma}.e_b]_c$ is consistent with ρ , so we know $\forall v \in \mathsf{fv}(\psi_+) - x$, $\rho_c(v) = \rho(v)$.

By the induction hypothesis on e_2 we know $\rho(o_2) = v_2$ and since $\rho_c[x \mapsto v_2](x) = v_2$ then $\rho_c[x \mapsto v_2](x) = \rho(o_2)$.

Then with the premises

- 1. $\psi_+ = \psi_{f_+}[o_2/x]$,
- $2. \ \rho_c[x \mapsto v_2] \models \psi_{f_+},$
- 3. $\forall v \in \mathsf{fv}(\psi_+) x. \ \rho_c[x \mapsto v_2](v) = \rho(v),$
- 4. and $\rho_c[x \mapsto v_2](x) = \rho(o_2)$

we can apply lemma A.2, to derive $\rho \models \psi_+$ and we are done.

- if v = false (or nil) then $\rho \models \psi_-$.
 - Like above, but using ψ_{-} for ψ_{+} and $\psi_{f_{-}}$ for $\psi_{f_{+}}$.

For part 3

- $\bullet \rho \vdash e \Downarrow v$,
- $\bullet \rho_c[x \mapsto v_2] \vdash e_b \Downarrow v,$
- $\tau = \tau_f[o_2/x],$

Subcase (B-BetaMulti).

- 1. $\rho \vdash e_1 \Downarrow [v_d, m]_m$,
- $2. \rho \vdash e_2 \Downarrow v_2$
- 3. $\rho \vdash (v_d \ v_2) \Downarrow v_e$,
- 4. GM $(t, v_e) = v_a$,
- 5. $\rho \vdash (v_g \ v_2) \Downarrow v$,
- 6. $t = \{\overrightarrow{v_k} \mapsto \overrightarrow{v_v}\}$

By inversion on e_1 via T-Multi we know

- 7. $\Gamma \vdash e_1 : (\mathbf{Multi} \, \sigma_t \, \sigma_d) \; ; \; \psi_{1_+} | \psi_{1_-} \; ; \; o_1,$
- 8. $\sigma_t = x : \sigma \xrightarrow{\psi_{f_+} | \psi_{f_-} \rangle} \tau_f,$ 9. $\sigma_d = x : \sigma \xrightarrow{\psi_{d_+} | \psi_{d_-} \rangle} \tau_d,$
- 10. $\vdash v_d : \sigma_d ; \psi_{3_+} | \psi_{3_-} ; o_3,$
- 11. $\vdash v_k : \top$, and

By the inductive hypothesis on e_2 we know

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13. \Gamma \vdash v_2 : \sigma, and
    14. o_2 = \emptyset or \rho(o_2) = v_2.
     We know by the definition of GM that
    15. \{v_v|(v_k, v_v) \in t \text{ and } \mathsf{IsA}(v_k, v_e) = \mathsf{true}\} = (v_l, v_g), so
    16. IsA(v_l, v_e) = true
     and because v_g \in \{\overrightarrow{v_v}\} that
    17. \vdash v_g : \sigma_t.
    For part 1, we note
    18. o = o_f[o_2/x],
     Subcase (B-Delta). \rho \vdash e_1 \Downarrow c, \rho \vdash e_2 \Downarrow v_2, \delta(c, v_2) = v
     Prove by cases on c.
          \textit{Subcase} \ (c = \textit{class}). \ \ x : \sigma \xrightarrow[o_f]{\psi_{f_+} | \psi_{f_-}} \tau_f = x : \top \xrightarrow[\text{class}(x)]{\text{tilt}} (\bigcup \ \text{nil} \ \textbf{Class} \ )
          Prove by cases on v_2.
               Subcase (v_2 = C \ \{ \overrightarrow{fld_i : v_i} \}). v = C
To prove part 1, note o = o_f [o_2/x] and o_f = \mathbf{class}(x). There are two cases defined by substitution: if o_2 = \emptyset then o = \emptyset and we are
               done, or if o_2 = \pi(x') then o = \text{class}(o_2), by the induction hypothesis on e_2 \rho(o_2) = v_2 and by the definition of path translation
               \rho(\mathbf{class}(o_2)) = (class \ \rho(o_2)), which evaluates to v.
               Part 2 is trivial since both propositions are tt by substitution.
               Part 3 holds because \vdash v_2: Class.
               Subcase (v_2 = C). v = Class
               As above.
               Subcase (v_2 = \text{true}). v = \mathbf{B}
               As above.
               Subcase (v_2 = \text{false}). v = \mathbf{B}
               As above.
               Subcase (v_2 = [\rho, \lambda x^{\tau}.e]_c). v = \mathbf{Fn}
               As above.
               Subcase (v_2 = [v_d, t]_m). v = \mathbf{Map}
               As above.
               Subcase (v_2 = \{\overrightarrow{v_1 \mapsto v_2}\}). v = \mathbf{K}
               As above.
               Subcase (v_2 = \text{nil}). v = \text{nil}
               Parts 1 and 2 as above. Part 3 holds because \vdash v_2 : \mathbf{nil}.
     Subcase (BE-Beta1).
     Subcase (BE-Beta2).
     Subcase (BE-BetaClosure).
     Subcase (BE-BetaMulti1).
     Subcase (BE-BetaMulti2).
Case (T-IsA). e = (isa? e_1 e_2), \tau = \mathbf{B}, \Gamma \vdash e_1 : \tau_1 ; \psi_{1+} | \psi_{1-} ; o_1, \Gamma \vdash e_2 : \tau_2 ; \psi_{2+} | \psi_{2-} ; o_2, \mathsf{IsAProps}(o_1, \tau_2) = \psi_{+} | \psi_{-}, o = \emptyset
```

Subcase (B-IsA). $\rho \vdash e_1 \Downarrow v_1, \rho \vdash e_2 \Downarrow v_2, \mathsf{IsA}(v_1, v_2) = v$

Part 1 holds trivially with $o = \emptyset$.

```
For part 2, we must prove, where \mathsf{IsA}(v_1, v_2) = v, and \mathsf{IsAProps}(o_1, \tau_2) = \psi_+ | \psi_- \text{ that if TrueVal}(v) then \rho \models \psi_+ or if \mathsf{FalseVal}(v)
then \rho \models \psi.
Prove by cases on the definition of IsA:
    Subcase (IsA(v, v) = true, if v \neq C).
    v_1 = v_2, v_1 \neq C, v_2 \neq C, \text{TrueVal}(v)
    Since TrueVal(v) we prove \rho \models \psi_+ by cases on the definition of IsAProps:
         Subcase (IsAProps(class(\pi(x)), (Val C)) = C_{\pi(x)}|\overline{C}_{\pi(x)}).
         o_1 = \mathbf{class}(\pi(x)), \, \tau_2 = (\mathbf{Val} \, C), \, \psi_+ = C_{\pi(x)}
         Unreachable by inversion on the typing relation, since \tau_2 = (\text{Val } C), yet v_2 \neq C.
         Subcase (IsAProps(o, (\mathbf{Val}\, s)) = ((\mathbf{Val}\, s)_x | \overline{(\mathbf{Val}\, s)}_x)[o/x] if s \neq C).
         \tau_2 = (\text{Val } s), s \neq C, \psi_+ = (\text{Val } s)_x [o_1/x]
         Since \tau_2 = (Val \ s) where s \neq C, by inversion on the typing judgement v_2 is either true, false, nil or k by T-True, T-False, T-Nil or
         Since v_1 = v_2 then \tau_1 = \tau_2, and since \tau_2 = (\mathbf{Val} \, s) then \tau_1 = (\mathbf{Val} \, s), so \vdash v_1 : (\mathbf{Val} \, s)
         If o_1 = \emptyset then \psi_+ = \text{tt} and we derive \rho \models \text{tt} with M-Top.
         Otherwise o_1 = \pi(x) and \psi_+ = (\mathbf{Val}\,s)_{\pi(x)}, and since \vdash v_1 : (\mathbf{Val}\,s) then \vdash \rho(\pi(x)) : (\mathbf{Val}\,s), which we can use M-Type to derive
         \rho \models (\operatorname{Val} s)_{\pi(x)}.
         Subcase (IsAProps(o, \tau) = tt | tt).
         \psi_+=\mathrm{tt}
         \rho \models tt holds by M-Top.
    Subcase (IsA(C_1, C_2) = true, if \vdash C_1 <: C_2).
    v_1 = C_1, v_2 = C_2, \vdash C_1 <: C_2, \mathsf{TrueVal}(v)
    Since TrueVal(v) we prove \rho \models \psi_+ by cases on the definition of IsAProps:
         \begin{array}{l} \textit{Subcase} \; (\mathsf{IsAProps}(\mathbf{class}(\pi(x)), (\mathbf{Val}\, C)) = C_{\pi(x)} | \overline{C}_{\pi(x)}). \\ o_1 = \mathbf{class}(\pi(x)), \, \tau_2 = (\mathbf{Val}\, C_2), \, \psi_+ = C_{2\pi(x)} \end{array}
         By inversion on the typing relation, since class is the last path element of o_1 then \rho \vdash (class \ \rho(\pi(x))) \Downarrow v_1.
         Since \rho \vdash (class \ \rho(\pi(x))) \Downarrow C_1, as v_1 = C_1, we can derive from lemma A.6 \rho \models C_{1\pi(x)}.
         By the induction hypothesis we can derive \Gamma \vdash C_{1\pi(x)}, and with the fact \vdash C_1 <: C_2 we can use L-Sub to conclude \Gamma \vdash C_{2\pi(x)},
         and finally by lemma A.3 we derive \rho \models C_{2\pi(x)}.
         Subcase (IsAProps(o, (Val s)) = ((Val s)_x | \overline{(Val s)}_x)[o/x] if s \neq C).
         \tau_2 = (\text{Val } s), s \neq C, \psi_+ = (\text{Val } s)_x [o_1/x]
         Unreachable case since \tau_2 = (\text{Val } s) where s \neq C, but v_2 = C_2.
         Subcase (IsAProps(o, \tau) = \text{tt} | \text{tt}).
         \rho \models tt holds by M-Top.
    Subcase (IsA(v_1, v_2) = false, otherwise).
    v_1 \neq v_2, FalseVal(v)
    Since FalseVal(v) we prove \rho \models \psi_{-} by cases on the definition of IsAProps:
         Subcase (IsAProps(class(\pi(x)), (Val C)) = C_{\pi(x)}|\overline{C}_{\pi(x)}).
         o_1 = \mathbf{class}(\pi(x)), \, \tau_2 = (\mathbf{Val}\,C), \, \psi_- = \overline{C}_{\pi(x)}
         By inversion on the typing relation, since class is the last path element of o_1 then \rho \vdash (class \ \rho(\pi(x))) \ \psi \ v_1.
         By the definition of class either v_1 = C or v_1 = \text{nil}.
         If v_1 = \text{nil}, then we know from the definition of IsA that \rho(\pi(x)) = \text{nil}.
         Since \vdash \rho(\pi(x)): nil, and there is no v_1 such that both \vdash \rho(\pi(x)): C and \vdash \rho(\pi(x)): nil, we use M-NotType to derive
         \rho \models \overline{C}_{\pi(x)}.
         Similarly if v_1 = C_1, by the definition of IsAProps we know \vdash C_1 \nleq : C and \rho(\pi(x)) = C_1.
         Since \vdash \rho(\pi(x)) : C_1, and there is no v_1 such that both \vdash v_1 : C and \vdash v_1 : C_1, we use M-NotType to derive \rho \models \overline{C}_{\pi(x)}.
         Subcase (IsAProps(o, (\mathbf{Val}\, s)) = ((\mathbf{Val}\, s)_x | \overline{(\mathbf{Val}\, s)}_x)[o/x] if s \neq C).
         \tau_2 = (\mathbf{Val}\,s), s \neq C, \psi_- = \overline{(\mathbf{Val}\,s)}_x [o_1/x]
         Since \tau_2 = (\text{Val } s) where s \neq C, by inversion on the typing judgement v_2 is either true, false, nil or k by T-True, T-False, T-Nil or
         T-Kw.
```

If $o_1 = \emptyset$ then $\psi_- = \text{tt}$ and we derive $\rho \models \text{tt}$ with M-Top.

Otherwise $o_1 = \pi(x)$ and $\psi_- = \overline{(\mathbf{Val}\,s)}_{\pi(x)}$. Noting that $v_1 \neq v_2$, we know $\vdash \rho(\pi(x)) : \sigma$ where $\sigma \neq (\mathbf{Val}\,s)$, and there is no v_1 such that both $\vdash v_1 : (\mathbf{Val}\,s)$ and $\vdash v_1 : \sigma$ so we can use M-NotType to derive $\rho \models \overline{(\mathbf{Val}\,s)}_{\pi(x)}$.

Subcase (IsAProps $(o, \tau) = tt | tt$). $\psi_- = tt$

 $\rho \models \text{tt holds by M-Top.}$

Part 3 holds because by the definition of IsA v can only be true or false, and since $\Gamma \vdash \text{true} : \tau$ and $\Gamma \vdash \text{false} : \tau$ we are done.

Subcase (BE-IsA1). $\rho \vdash e_1 \Downarrow \mathsf{err}$

Trivially reduces to an error.

Subcase (BE-IsA2). $\rho \vdash e_1 \Downarrow v_1, \rho \vdash e_2 \Downarrow \mathsf{err}$

Trivially reduces to an error.

 $\begin{array}{c} \textit{Case} \ \ (\text{T-GetHMap}) \quad e = (\text{get} \ e_m \ e_k), \ \tau = (\bigcup \overrightarrow{\tau_i}) \ \psi_+ = \text{tt}, \ \psi_- = \text{tt}, \ o = \text{key}_k(x)[o_m/x], \ \Gamma \vdash e_k : (\text{Val} \ k), \ \Gamma \vdash e_m : (\bigcup \overline{(\text{HMap}^{\mathcal{E}} \ \mathcal{M} \ \mathcal{A})}) \ ; \ \psi_{m_+} | \psi_{m_-} \ ; \ o_m, \ \overline{\mathcal{M}[k] = \tau_i} \\ \end{array}$

Subcase (B-Get). $\rho \vdash e_m \Downarrow v_m, v_m = \{\overrightarrow{(v_a \ v_b)}\}, \rho \vdash e_k \Downarrow k, k \in dom(\{\overrightarrow{(v_a \ v_b)}\}), \{\overrightarrow{(v_a \ v_b)}\}[k] = v$ To prove part 1 we consider two cases on the form of o_m :

- if $o_m = \emptyset$ then $o = \emptyset$ by substitution, which gives the desired result;
- if $o_m = \pi_m(x_m)$ then $o = \mathbf{key}_k(o_m)$ by substitution. We note by the definition of path translation $\rho(\mathbf{key}_k(o_m)) = (\gcd \rho(o_m) \ k)$ and by the induction hypothesis on $e_m \ \rho(o_m) = \{(v_a \ v_b)\}$, which together imply $\rho(o) = (\gcd \{(v_a \ v_b)\} \ k)$. Since this is the same form as B-Get, we can apply the premise $\{(v_a \ v_b)\} \ [k] = v$ to derive $\rho(o) = v$.

Part 2 holds trivially as $\psi_{+} = tt$ and $\psi_{-} = tt$.

To prove part 3 we note that (by the induction hypothesis on e_m) $\vdash v_m : (\bigcup (\overline{\mathbf{HMap}^{\mathcal{E}} \mathcal{M} \mathcal{A}}))$, where $\overline{\mathcal{M}[k] = \tau_i}$, and both $k \in dom(\{(v_a \ v_b)\})$ and $\{(v_a \ v_b)\}[k] = v \text{ imply } \vdash v : (\bigcup \overrightarrow{\tau_i})$.

Subcase (B-GetMissing). $v = \operatorname{nil}, \rho \vdash e_m \underbrace{\Downarrow \{(\overrightarrow{v_a} \ v_b)\}}, \rho \vdash e_k \Downarrow k, k \not\in dom(\{(\overrightarrow{v_a} \ v_b)\})$ Unreachable subcase because $k \not\in dom(\{(\overrightarrow{v_a} \ v_b)\})$ contradicts $\mathcal{M}[k] = \tau$.

Subcase (BE-Get1).

Subcase (BE-Get2).

Case (T-GetHMapAbsent). $e = (\text{get } e_m \ e_k), \ \tau = \text{nil}, \ \psi_+ = \text{tt}, \ \psi_- = \text{tt}, \ o = \text{key}_k(x)[o_m/x], \ \Gamma \vdash e_k : (\text{Val}\,k), \ \Gamma \vdash e_m : (\text{HMap}^{\mathcal{E}}\,\mathcal{M}\,\mathcal{A}); \ \psi_{m+}|\psi_{m-}; \ o_m, k \in \mathcal{A}$

 $\textit{Subcase (B-Get).} \ \ \rho \vdash e_m \Downarrow \{\overrightarrow{(v_a \ v_b)}\} \ , \ \rho \vdash e_k \Downarrow k, \ k \in dom(\{\overrightarrow{(v_a \ v_b)}\}), \ \{\overrightarrow{(v_a \ v_b)}\} [k] = v$ Unreachable subcase because $k \in dom(\{\overrightarrow{(v_a \ v_b)}\}), \ contradicts \ k \in \mathcal{A}.$

Subcase (B-GetMissing). $v = \text{nil}, \rho \vdash e_m \Downarrow \{\overrightarrow{(v_a \ v_b)}\}, \rho \vdash e_k \Downarrow k, k \not\in dom(\{\overrightarrow{(v_a \ v_b)}\})$ To prove part 1 we consider two cases on the form of o_m :

- if $o_m = \emptyset$ then $o = \emptyset$ by substitution, which gives the desired result;
- if $o_m = \pi_m(x_m)$ then $o = \mathbf{key}_k(o_m)$ by substitution. We note by the definition of path translation $\rho(\mathbf{key}_k(o_m)) = (\gcd \rho(o_m) k)$ and by the induction hypothesis on $e_m \rho(o_m) = \{\overline{(v_a v_b)}\}$, which together imply $\rho(o) = (\gcd \{\overline{(v_a v_b)}\} k)$. Since this is the same form as B-GetMissing, we can apply the premise v = nil to derive $\rho(o) = v$.

Part 2 holds trivially as $\psi_+ = \text{$\pm t$}$ and $\psi_- = \text{$\pm t$}$. To prove part 3 we note that e_m has type (**HMap**^{\mathcal{E}} \mathcal{M} \mathcal{A}) where $k \in \mathcal{A}$, and the premises of B-GetMissing $k \notin dom(\{(v_a \ v_b)\})$ and v = nil tell us v must be of type τ .

Subcase (BE-Get1).

Subcase (BE-Get2).

Case (T-GetHMapPartialDefault). $e = (\text{get } e_m \ e_k), \ \tau = \top, \ \psi_+ = \text{tt}, \ \psi_- = \text{tt}, \ o = \text{key}_k(x)[o_m/x], \ \Gamma \vdash e_k : (\text{Val } k), \ \Gamma \vdash e_m : (\text{HMap}^{\mathcal{P}} \ \mathcal{M} \ \mathcal{A}); \ \psi_{m+}|\psi_{m-}; \ o_m, k \not\in dom(\mathcal{M}), k \not\in \mathcal{A}$

Subcase (B-Get). $\rho \vdash e_m \Downarrow \{(v_a v_b)\}\}$, $\rho \vdash e_k \Downarrow k$, $k \in dom(\{(v_a v_b)\})$, $\{(v_a v_b)\}\}[k] = v$ Parts 1 and 2 are the same as the B-Get subcase of T-GetHMap. Part 3 is trivial as $\tau = \top$.

Subcase (B-GetMissing). $v = \text{nil}, \rho \vdash e_m \Downarrow \{(v_a v_b)\}, \rho \vdash e_k \Downarrow k, k \not\in dom(\{(v_a v_b)\})$ Parts 1 and 2 are the same as the B-GetMissing subcase of T-GetHMapAbsent. Part 3 is trivial as $\tau = \top$.

Subcase (BE-Get1).

Subcase (BE-Get2).

Case (T-AssocHMap). $e = (assoc \ e_m \ e_k \ e_v), \ \tau = (\mathbf{HMap}^{\mathcal{E}} \ \mathcal{M}[k \mapsto \tau] \ \mathcal{A}), \ \psi_+ = \mathtt{tt}, \ \psi_- = \mathtt{ff}, \ o = \emptyset, \ \Gamma \vdash e_m : (\mathbf{HMap}^{\mathcal{E}} \ \mathcal{M} \ \mathcal{A}), \ \Gamma \vdash e_k : (\mathbf{Val} \ k), \ \Gamma \vdash e_v : \tau, k \not\in \mathcal{A}$

Subcase (B-Assoc). $v = \{ \overrightarrow{(v_a \ v_b)} \} [k \mapsto v_v], \rho \vdash e_m \Downarrow \{ \overrightarrow{(v_a \ v_b)} \}, \rho \vdash e_k \Downarrow k, \rho \vdash e_v \Downarrow v_v$ Parts 1 and 2 hold for the same reasons as T-True.

Subcase (BE-Assoc1).

Subcase (BE-Assoc2).

Subcase (BE-Assoc3).

 $\textit{Case} \; (\text{T-If}). \; \; e = (\text{if} \; e_1 \; e_2 \; e_3), \; \Gamma \vdash e_1 : \tau_1 \; ; \; \psi_{1_+} | \psi_{1_-} \; ; \; o_1, \; \Gamma, \psi_{1_+} \vdash e_2 : \tau \; ; \; \psi_{2_+} | \psi_{2_-} \; ; \; o, \; \Gamma, \psi_{1_-} \vdash e_3 : \tau \; ; \; \psi_{3_+} | \psi_{3_-} \; ; \; o, \; \psi_{+} = \psi_{2_+} \lor \psi_{3_+}, \; \psi_{-} = \psi_{2_-} \lor \psi_{3_-}$

Subcase (B-IfTrue). $\rho \vdash e_1 \Downarrow v_1, v_1 \neq \mathsf{false}, v_1 \neq \mathsf{nil}, \rho \vdash e_2 \Downarrow v$

For part 1, either $o = \emptyset$, or e evaluates to the result of e_2 .

To prove part 2, we consider two cases:

- if $v = \text{false (or nil) then } e_2$ evaluates to a false value so $\rho \models \psi_2$, and thus $\rho \models \psi_2 \lor \psi_3$ by M-Or,
- otherwise $v \neq \text{false}$ and $v \neq \text{nil}$, so e_2 evaluates to a true value so $\rho \models \psi_{2+}$, and thus $\rho \models \psi_{2+} \lor \psi_{3+}$ by M-Or.

Part 3 is trivial as $\rho \vdash e_2 \Downarrow v$ and $\vdash v : \tau$ by the induction hypothesis on e_2 .

Subcase (B-IfFalse). $\rho \vdash e_1 \Downarrow \mathsf{false}$ or $\rho \vdash e_1 \Downarrow \mathsf{nil}, \rho \vdash e_3 \Downarrow v$

For part 1, either $o = \emptyset$, or e evaluates to the result of e_3 .

To prove part 2, we consider two cases:

- if v = false (or nil) then e_3 evaluates to a false value so $\rho \models \psi_3$, and thus $\rho \models \psi_2 \lor \psi_3$ by M-Or,
- otherwise $v \neq \text{false}$ and $v \neq \text{nil}$, so e_3 evaluates to a true value so $\rho \models \psi_{3+}$, and thus $\rho \models \psi_{2+} \lor \psi_{3+}$ by M-Or.

Part 3 is trivial as $\rho \vdash e_3 \Downarrow v$ and $\vdash v : \tau$ by the induction hypothesis on e_3 .

Subcase (BE-If).

Subcase (B-Let). $\rho \vdash e_1 \Downarrow v_1, \rho[x \mapsto v_1] \vdash e_2 \Downarrow v$

For all the following cases (with a reminder that x is fresh) we apply the induction hypothesis on e_2 . We justify this by noting that occurrences of x inside e_2 have the same type as e_1 and simulate the propositions of e_1 because $\rho \vdash e_1 \Downarrow v_1$, and $\rho[x \mapsto v_1] \vdash e_2 \Downarrow v$, so $\rho \models \Gamma, \sigma_x, \psi', \psi''$, by M-And.

We prove parts 1, 2 and 3 by directly using the induction hypothesis on e_2 .

Subcase (BE-Let).

 $\textit{Case} \; (\text{T-Clos}). \;\; e = [\rho, \lambda x^{\sigma}.e_{1}]_{\mathsf{c}}, \; \psi_{+} \; \exists \Gamma'.\rho \models \Gamma' \; \; \text{and} \; \Gamma' \vdash \lambda x^{\sigma}.e_{1} : \tau \; ; \; \psi_{+} | \psi_{-} \; ; \; o \vdash \psi_{+} | \psi_{-} | \psi_{$

Subcase (B-Abs). $v = [\rho, \lambda x^{\sigma}.e_1]_c$ We assume some Γ' , such that

$$\bullet \rho \models \Gamma'$$

$$\bullet \Gamma' \vdash \lambda x^{\sigma}.e_1 : \tau \; ; \; \psi_+ | \psi_- \; ; \; o.$$

Note the last rule in the derivation of $\Gamma' \vdash \lambda x^{\sigma} \cdot e_1 : \tau \; ; \; \psi_+ | \psi_- \; ; \; o \; \text{must be T-Abs, so} \; \psi_+ = \text{tt} \; , \; \psi_- = \text{ff} \; \text{and} \; o = \emptyset$. Thus parts 1 and 2 hold for the same reasons as T-True. Part 3 holds as v has the same type as λx^{σ} . e_1 under Γ' .

Case (T-Multi).
$$e = [v_1, \{\overrightarrow{v_k \mapsto v_v}\}]_m, \tau = (\textbf{Multi } \sigma \tau_1), \psi_+ = \texttt{tt}, \psi_- = \texttt{ff}, o = \emptyset, \vdash v_1 : \tau_1, \overrightarrow{\vdash v_k : \top}, \overrightarrow{\vdash v_v : \sigma}$$

Subcase (B-Val). $v = [v_1, \{\overrightarrow{v_k \mapsto v_v}\}]_m$ Similar to T-True.

$$\textit{Case} \; (\text{T-Abs}). \; \; e = \lambda x^{\sigma}.e_{1}, \; \tau = x : \sigma \xrightarrow[o_{1}]{\psi_{1} - |\psi_{1}|} \tau_{1}, \; \psi_{+} = \text{ tt}, \; \psi_{-} = \text{ ff}, \; o = \emptyset, \; \Gamma, \sigma_{x} \vdash e_{1} : \tau \; ; \; \psi_{1} + |\psi_{1}| \; ; \; o_{1}, \; \Gamma \vdash \lambda x^{\sigma}.e_{1} : \tau \; ; \; \psi_{+} | \psi_{-} \; ; \; o = \emptyset, \; \Gamma, \sigma_{x} \vdash e_{1} : \tau \; ; \; \psi_{1} + |\psi_{1}| \; ; \; o_{1}, \; \Gamma \vdash \lambda x^{\sigma}.e_{1} : \tau \; ; \; \psi_{+} | \psi_{-} \; ; \; o = \emptyset, \; \Gamma, \sigma_{x} \vdash e_{1} : \tau \; ; \; \psi_{1} + |\psi_{1}| \; ; \; o_{1}, \; \Gamma \vdash \lambda x^{\sigma}.e_{1} : \tau \; ; \; \psi_{+} | \psi_{-} \; ; \; o = \emptyset, \; \Gamma, \sigma_{x} \vdash e_{1} : \tau \; ; \; \psi_{1} + |\psi_{1}| \; ; \; o_{1}, \; \Gamma \vdash \lambda x^{\sigma}.e_{1} : \tau \; ; \; \psi_{+} | \psi_{-} \; ; \; o = \emptyset, \; \Gamma, \sigma_{x} \vdash e_{1} : \tau \; ; \; \psi_{1} + |\psi_{1}| \; ; \; o_{1}, \; \Gamma \vdash \lambda x^{\sigma}.e_{1} : \tau \; ; \; \psi_{+} | \psi_{-} \; ; \; o = \emptyset, \; \Gamma, \sigma_{x} \vdash e_{1} : \tau \; ; \; \psi_{1} + |\psi_{1}| \; ; \; o_{1}, \; \Gamma \vdash \lambda x^{\sigma}.e_{1} : \tau \; ; \; \psi_{+} | \psi_{-} \; ; \; o = \emptyset, \; \Gamma, \sigma_{x} \vdash e_{1} : \tau \; ; \; \psi_{1} + |\psi_{1}| \; ; \; o = \emptyset, \; \Gamma, \sigma_{x} \vdash e_{1} : \tau \; ; \; \psi_{1} + |\psi_{1}| \; ; \; o = \emptyset, \; \Gamma, \sigma_{x} \vdash e_{1} : \tau \; ; \; \psi_{1} + |\psi_{1}| \; ; \; o = \emptyset, \; \Gamma, \sigma_{x} \vdash e_{1} : \tau \; ; \; \psi_{1} \vdash \psi_{1} \; ; \; \sigma_{x} \vdash e_{1} : \tau \; ; \; \psi_{1} \vdash \psi_{1} \; ; \; \sigma_{x} \vdash e_{1} : \tau \; ; \; \psi_{1} \vdash \psi_{1} \; ; \; \sigma_{x} \vdash e_{1} : \tau \; ; \; \psi_{1} \vdash \psi_{1} \; ; \; \sigma_{x} \vdash \varphi_{x} \vdash \psi_{x} \vdash$$

Subcase (B-Abs). $v = [\rho, \lambda x^{\sigma}.e_1]_{c}, \ \rho \vdash \lambda x^{\tau}.e_1 \Downarrow [\rho, \lambda x^{\sigma}.e_1]_{c}$

Parts 1 and 2 hold for the same reasons as T-True. Part 3 holds directly via T-Clos, since v must be a closure.

Case (T-Error).
$$e = \text{err}, \, \tau = \bot, \, \psi_+ = \text{ff}, \, \psi_- = \text{ff}, \, o = \emptyset$$

Subcase (BE-Error). $\rho \vdash e \Downarrow err$

Trivially reduces to an error.

Case (T-Subsume).
$$\Gamma \vdash e : \tau' ; \ \psi'_{+} | \psi'_{-} ; \ o', \Gamma, \psi'_{+} \vdash \psi_{+}, \Gamma, \psi'_{-} \vdash \psi_{-}, \vdash \tau' <: \tau, \vdash o' <: o' \vdash \psi_{-}, \vdash \tau' =: \tau, \vdash v' := v' \vdash v' \vdash \psi_{-}, \vdash v' \vdash \psi_{-}, \vdash v' \vdash \psi_{-}, \vdash v' \vdash v' \vdash \psi_{-}, \vdash v' \vdash \psi_{-}, \vdash v' \vdash v' \vdash \psi_{-}, \vdash v' \vdash v' \vdash \psi_{-}, \vdash v'$$

Case (T-Subsume). $\Gamma \vdash e : \tau' \; ; \; \psi'_+ | \psi'_- \; ; \; o', \Gamma, \psi'_+ \vdash \psi_+, \Gamma, \psi'_- \vdash \psi_-, \vdash \tau' <: \tau, \vdash o' <: o$ Part 1 holds because o' is the object of e and $\vdash o' <: o$. Part 2 holds because the then and else propositions of e are ψ'_+ and $\psi'_$ respectively, so ψ_+ and ψ_- are also respectively as $\Gamma, \psi'_+ \vdash \psi_+$ and $\Gamma, \psi'_- \vdash \psi_-$. Part 3 holds because e is of type τ' and $\vdash \tau' <: \tau$. Case (T-Const). $e=c,\, \tau=\delta_{\tau}(c),\, \psi_{+}=\mathrm{tt},\, \psi_{-}=\mathrm{ff}$, $o=\emptyset$

Subcase (B-Val). Parts 1, 2 and 3 hold for the same reasons as T-True.

Theorem A.1 (Well-typed programs don't go wrong). If $\vdash e : \tau \; ; \; \psi_+ | \psi_- \; ; \; o \; then \; \forall \; e \; \Downarrow \; wrong.$

Proof. Corollary of lemma A.7, since by lemma A.7 when $\vdash e : \tau$; $\psi_+ | \psi_-$; o, either $\vdash e \Downarrow v$ or $\vdash e \Downarrow$ err, therefore $\not\vdash e \Downarrow wrong$.

Theorem A.2 (Type soundness). If $\Gamma \vdash e : \tau$; $\psi_+ | \psi_- |$; o and $\rho \vdash e \Downarrow v$ then $\vdash v : \tau$; $\psi'_+ | \psi'_- |$; o' for some ψ'_+ , ψ'_- and o'

Proof. Corollary of lemma A.7.

Theorem A.3 (nil cannot be accidentally introduced). If $\Gamma \vdash v : \tau$; $\psi_+ | \psi_- |$; o and \vdash nil $\not<: \tau$ then $v \neq \text{nil}$.

Proof. Corollary of lemma A.7.

Lemma A.8 (Field lookup never fails). *Proof.* Corollary of lemma A.7.

Theorem A.4 (Field lookup on nil is disallowed). *Proof.* Corollary of lemma A.8.

Theorem A.5 (Method invocation on nil is disallowed). *Proof.* Corollary of lemma A.7.

Theorem A.6 (nil invocation is disallowed in typed code). If $\Gamma \vdash e : \tau \; ; \; \psi_+ | \psi_- \; ; \; o \; then \; e \; is \; not \; (nil \; e'), \; (. \; nil \; fld), \; or \; (. \; nil \; (mth \; \overrightarrow{e'})).$

Proof. Corollary of lemma A.7.

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d,e ::= x \mid v \mid (e \mid e) \mid \lambda x^{\tau}.e \mid (\text{if } e \mid e \mid e) \mid (\text{do } e \mid e) \mid (\text{let} \mid x \mid e \mid e) \mid \beta \mid R \mid E \mid M \mid H \mid G
                                                                                                                                                                                Expressions
Values
                                                                                                                                                                                Map Values
c ::= class \mid inc \mid number?
                                                                                                                                                                                Constants
H ::= \widehat{\ } C x \mid (\operatorname{let} [\widehat{\ } C x e] e)
                                                                                                                                                                                Type Hinted Expressions
G ::= (\text{get } e \ e) \mid (\text{assoc } e \ e)
E ::= (.e \ fld_C^C) \mid (.e \ (mth_{[[\overrightarrow{C}],C]}^C \overrightarrow{e})) \mid (\text{new}_{[\overrightarrow{C}]} C \overrightarrow{e})
R ::= (.e \ fld) \mid (.e \ (mth \overrightarrow{e})) \mid (\text{new } C \overrightarrow{e})
                                                                                                                                                                                Hash Maps
                                                                                                                                                                                Non-Reflective Java Interop
                                                                                                                                                                                Reflective Java Interop
M ::= (\operatorname{defmulti} \tau e) \mid (\operatorname{defmethod} e e e) \mid (\operatorname{isa}? e e)
                                                                                                                                                                                Immutable First-Class Multimethods
\sigma, \tau ::= \top \mid C \mid (\mathbf{Val}\, s) \mid (\bigcup \overrightarrow{\tau}) \mid x : \tau \xrightarrow{\psi \mid \psi} \tau \mid (\mathbf{HMap}^{\mathcal{E}}\, \mathcal{M}\, \mathcal{A}) \mid (\mathbf{Multi}\, \tau\, \tau)
                                                                                                                                                                                Typed Clojure Types
\mathcal{M} ::= \{\overrightarrow{k} \mapsto \overrightarrow{\tau}\}
                                                                                                                                                                                HMap mandatory entries
 \mathcal{A} ::= \{\overrightarrow{k}\}
                                                                                                                                                                                HMap absent entries
    \mathcal{E}
                                                                                                                                                                                HMap completeness tags
                                                                                                                                                                                 Value types
s
       ::= true | false
                                                                                                                                                                                Boolean values
    Value environments
                                                                                                                                                                                Type Hints
      ::= \tau_{\pi(x)} \mid \overline{\tau}_{\pi(x)} \mid \psi \supset \psi
                                                                                                                                                                                Propositions
        \mid \psi \wedge \psi \mid \psi \vee \psi \mid \mathsf{tt} \mid \mathsf{ff}
      :=\pi(x)\mid\emptyset
                                                                                                                                                                                Objects
\pi ::= \overrightarrow{pe}
                                                                                                                                                                                Paths
                                                                                                                                                                                Path Elements
pe ::= \mathbf{class} \mid \mathbf{key}_k
       := \overrightarrow{\psi}
Γ
                                                                                                                                                                                Proposition Environment
       ::=\{\overrightarrow{v\mapsto v}\}
                                                                                                                                                                                Multimethod dispatch table
      ::=\{\overrightarrow{x}:\overrightarrow{\gamma}\}
                                                                                                                                                                                Type Hint Environment
ce ::= \{\mathsf{mths} \mapsto \{[mth, [\overrightarrow{C}], C]\},
                                                                                                                                                                                Class descriptors
               \mathsf{flds} \mapsto \{ \overline{[fld,C]} \},\
               \underline{\mathsf{ctors}} \mapsto \{[\overrightarrow{\overline{C}}]\}\}
\mathcal{CT} \, ::= \{\overrightarrow{C \mapsto ce}\}
                                                                                                                                                                                Class Table
     ::= Object | K | Class | B | Fn | Multi | Map | Void
                                                                                                                                                                                Class literals
       := C \{ \overrightarrow{fld} : \overrightarrow{v} \}
                                                                                                                                                                                Class Values
β
      ::= wrong \mid err
                                                                                                                                                                                Wrong or error
\alpha ::= v \mid \beta
                                                                                                                                                                                Defined reductions
pol ::= pos \mid neg
                                                                                                                                                                                Substitution Polarity
```

Figure A.1. Syntax of Terms, Types, Propositions, and Objects

```
egin{array}{lll} \mbox{nil} &\equiv & (\mbox{Val nil}) \ \mbox{true} &\equiv & (\mbox{Val true}) \ \mbox{false} &\equiv & (\mbox{Val false}) \ \end{array}
```

Figure A.2. Type abbreviations

```
\begin{array}{lll} \Gamma \vdash e : \tau & \equiv & \Gamma \vdash e : \tau \; ; \; \psi_+ | \psi_- \; ; \; o & \text{for some } \psi_+, \psi_- \text{and } o \\ \tau[o/x] & \equiv & \tau[o/x]^{\mathsf{pos}} \\ \psi[o/x] & \equiv & \psi[o/x]^{\mathsf{pos}} \\ \psi|\psi[o/x] & \equiv & \psi|\psi[o/x]^{\mathsf{pos}} \\ o[o/x] & \equiv & o[o/x]^{\mathsf{pos}} \end{array}
```

Figure A.3. Judgement abbreviations

Figure A.4. Standard Typing Rules

$$\begin{array}{c|c} \frac{\text{T-NewStatic}}{\text{Convert}(C_i) = \tau_i} & \text{Convert}(C) = \tau & \overline{\Gamma \vdash e_i : \tau_i} \\ \hline \Gamma \vdash (\text{new}_{[\overrightarrow{C_i}]} \ C \ \overrightarrow{e_i}) : \tau \ ; \ \text{tt} \mid \text{ff} \ ; \ \emptyset \\ \hline \end{array} \qquad \begin{array}{c|c} \frac{\text{T-FieldDStatic}}{\text{Convert}(C_1) = \sigma} & \text{Convert}_{\text{nil}}(C_2) = \tau & \Gamma \vdash e : \sigma \\ \hline \Gamma \vdash (. \ e \ fld_{C_2}^{C_1}) : \tau \ ; \ \text{tt} \mid \text{tt} \ ; \ \emptyset \\ \hline \end{array}$$

Figure A.5. Java Interop Typing Rules

Т-DefMulti
$$\sigma = x:\tau \xrightarrow{\psi_{+}|\psi_{-}} \sigma' \qquad \sigma' = x:\tau \xrightarrow{\psi'_{+}|\psi'_{-}} \tau'' \qquad \Gamma \vdash e:\sigma'$$

$$\Gamma \vdash (\text{defmulti } \sigma \ e) : (\textbf{Multi } \sigma \ \sigma') \ ; \ \text{tt | fff } ; \ \emptyset$$

$$\Gamma \vdash (\text{defmethod } e_{m} \ e_{v} \ \lambda x^{\tau}.e_{b}) : (\textbf{Multi } \tau_{m} \ \tau_{d}) \ ; \ \text{tt | fff } ; \ \emptyset$$

$$\Gamma \vdash (\text{defmethod } e_{m} \ e_{v} \ \lambda x^{\tau}.e_{b}) : (\textbf{Multi } \tau_{m} \ \tau_{d}) \ ; \ \text{tt | fff } ; \ \emptyset$$

$$\Gamma \vdash (\text{defmethod } e_{m} \ e_{v} \ \lambda x^{\tau}.e_{b}) : (\textbf{Multi } \tau_{m} \ \tau_{d}) \ ; \ \text{tt | fff } ; \ \emptyset$$

$$\Gamma \vdash (\text{defmethod } e_{m} \ e_{v} \ \lambda x^{\tau}.e_{b}) : (\textbf{Multi } \tau_{m} \ \tau_{d}) \ ; \ \text{tt | fff } ; \ \emptyset$$

$$\Gamma \vdash (\text{defmethod } e_{m} \ e_{v} \ \lambda x^{\tau}.e_{b}) : (\textbf{Multi } \tau_{m} \ \tau_{d}) \ ; \ \text{tt | fff } ; \ \emptyset$$

$$\Gamma \vdash (\text{defmethod } e_{m} \ e_{v} \ \lambda x^{\tau}.e_{b}) : (\textbf{Multi } \tau_{m} \ \tau_{d}) \ ; \ \text{tt | fff } ; \ \emptyset$$

Figure A.6. Multimethod Typing Rules

$$\frac{\Gamma\text{-HMAP}}{\vdash v_k: (\mathbf{Val}\,k)} \xrightarrow{\vdash v_v: \tau_v} \mathcal{M} = \{\overrightarrow{k} \mapsto \overrightarrow{\tau_v}\} \qquad \Gamma\text{-KW} \\ \Gamma \vdash k: (\mathbf{Val}\,k) ; \text{ tt} \mid \text{fff} ; \emptyset$$

$$\frac{\Gamma\text{-GETHMAP}}{\Gamma \vdash e: (\bigcup} \xrightarrow{(\mathbf{HMap}^{\mathcal{E}} \mathcal{M} \mathcal{A})^i} ; \psi_{1+} \mid \psi_{1-} ; o \qquad \Gamma \vdash e_k: (\mathbf{Val}\,k) \qquad \overrightarrow{\mathcal{M}[k] = \tau}$$

$$\frac{\Gamma\text{-GETHMAPABSENT}}{\Gamma \vdash e: (\mathbf{HMap}^{\mathcal{E}} \mathcal{M} \mathcal{A}) ; \psi_{1+} \mid \psi_{1-} ; o \qquad \Gamma \vdash e_k: (\mathbf{Val}\,k) \qquad k \in \mathcal{A}}{\Gamma \vdash (\text{get } e \cdot e_k) : \mathbf{nil} ; \text{ tt} \mid \text{tt} ; \mathbf{key}_k(x)[o/x]}$$

$$\frac{\Gamma\text{-GETHMAPABSENT}}{\Gamma \vdash e: (\mathbf{HMap}^{\mathcal{E}} \mathcal{M} \mathcal{A}) ; \psi_{1+} \mid \psi_{1-} ; o \qquad \Gamma \vdash e_k: (\mathbf{Val}\,k) \qquad k \in \mathcal{A}}{\Gamma \vdash (\text{get } e \cdot e_k) : \mathbf{nil} ; \text{ tt} \mid \text{tt} ; \mathbf{key}_k(x)[o/x]}$$

$$\frac{\Gamma\text{-GETHMAPPARTIALDEFAULT}}{\Gamma \vdash e: (\mathbf{HMap}^{\mathcal{P}} \mathcal{M} \mathcal{A}) ; \psi_{1+} \mid \psi_{1-} ; o \qquad \Gamma \vdash e_k: (\mathbf{Val}\,k) \qquad k \notin dom(\mathcal{M}) \qquad k \notin \mathcal{A}}{\Gamma \vdash (\text{get } e \cdot e_k) : \Gamma ; \text{ tt} \mid \text{tt} ; \mathbf{key}_k(x)[o/x]}$$

$$\frac{\Gamma\text{-Assochmap}}{\Gamma \vdash e: (\mathbf{HMap}^{\mathcal{E}} \mathcal{M} \mathcal{A}) \qquad \Gamma \vdash e_k: (\mathbf{Val}\,k) \qquad \Gamma \vdash e_v: \tau \qquad k \notin \mathcal{A}}{\Gamma \vdash (\text{assoc } e \cdot e_k \cdot e_v) : (\mathbf{HMap}^{\mathcal{E}} \mathcal{M}[k \mapsto \tau] \mathcal{A}) ; \text{ tt} \mid \text{fff} ; \emptyset}$$

Figure A.7. Map Typing Rules

$$\frac{\text{TA-Local}}{\sum(x) = \gamma} \quad \frac{\text{TA-NIL}}{\sum \vdash_{\mathsf{h}} \text{ nil}} :? \quad \frac{\text{TA-True}}{\sum \vdash_{\mathsf{h}} \text{ true}} : \mathbf{Boolean} \quad \frac{\text{TA-False}}{\sum \vdash_{\mathsf{h}} \text{ false}} : \mathbf{Boolean} \quad \frac{\text{TA-Kw}}{\sum \vdash_{\mathsf{h}} k : \mathbf{K}} \quad \frac{\text{TA-Class}}{\sum \vdash_{\mathsf{h}} k : \mathbf{K}}$$

$$\frac{\text{TA-Lethint}}{\sum \vdash_{\mathsf{h}} k : \mathbf{K}} \quad \frac{\text{TA-Lethint}}{\sum \vdash_{\mathsf{h}} k : \mathbf{K}} \quad \frac{\text$$

Figure A.8. Type Hint Inference

$$\begin{array}{c} \text{R-Local } \\ \text{R-Local } \\ \text{E-F}_r^{CT} x \Rightarrow x \end{array} \qquad \begin{array}{c} \text{R-Local Hint} \\ \text{E-F}_r^{CT} \cap C x \Rightarrow \cap C x \end{array} \qquad \begin{array}{c} \text{R-New Refl} \\ \text{E-F}_r^{CT} v \Rightarrow v \end{array} \qquad \begin{array}{c} \text{R-New Refl} \\ \text{E-F}_r^{CT} e_i \Rightarrow e_j \\ \text{E-F}_r^{CT} e_i \Rightarrow e_j \end{array} \qquad \begin{array}{c} \text{E-R-New Static} \\ \text{E-F}_r^{CT} e_i \Rightarrow e_j \\ \text{E-F}_r^{CT} (\text{new } C \overrightarrow{e_i}) \Rightarrow (\text{new } [\overrightarrow{c_k}] \cap C \overrightarrow{e_j}) \end{array} \qquad \begin{array}{c} \text{R-New Static} \\ \text{E-F}_r^{CT} (\text{new } C \overrightarrow{e_i}) \Rightarrow (\text{new } [\overrightarrow{c_k}] \cap C \overrightarrow{e_j}) \end{array} \qquad \begin{array}{c} \text{R-R-Res Static} \\ \text{E-F}_r^{CT} e_i \Rightarrow e_j \\ \text{E-F}_r^{CT} (\text{new } [\overrightarrow{c_k}] \cap C \overrightarrow{e_j}) \Rightarrow (\text{new } [\overrightarrow{c_k}] \cap C \overrightarrow{e_j}) \end{array} \qquad \begin{array}{c} \text{R-ABS} \\ \text{E-ET} \\ \text{E-F}_r^{CT} (\text{new } [\overrightarrow{c_k}] \cap C \overrightarrow{e_j}) \Rightarrow (\text{new } [\overrightarrow{c_k}] \cap C \overrightarrow{e_j}) \end{array} \qquad \begin{array}{c} \text{R-LETHINT} \\ \text{E-F}_r^{CT} e_i \Rightarrow e_i \\ \text{E-F}_r^{CT} (\text{let } [x e_1] e) \Rightarrow (\text{let } [x e_1' e) \Rightarrow (\text{le$$

Figure A.9. Java Reflection Resolution

Figure A.10. Subtyping rules

```
\begin{array}{lll} \mathsf{FId}(\mathcal{CT},C,fld) & = & [C,C_f] & \text{if } [fld,C_f] \in \mathcal{CT}[C][\mathsf{fIds}] \\ \mathsf{Ctor}(\mathcal{CT},C,[\overrightarrow{C_p}]) & = & [\overrightarrow{C_p}] & \text{if } [\overrightarrow{C_p}] \in \mathcal{CT}[C][\mathsf{ctors}] \\ \mathsf{Mth}(\mathcal{CT},C,mth,[\overrightarrow{C_p}]) & = & [C,[\overrightarrow{C_p}],C_r] & \text{if } [mth,[\overrightarrow{C_p}],C_r] \in \mathcal{CT}[C][\mathsf{mths}] \end{array}
```

Figure A.11. Class table lookup

```
\begin{array}{lll} \mathsf{Convert}(\mathbf{Void}\,) &=& \mathbf{nil} \\ \mathsf{Convert}(C) &=& C \\ \mathsf{Convert}_{\mathsf{nil}}(\mathbf{Void}\,) &=& \mathbf{nil} \\ \mathsf{Convert}_{\mathsf{nil}}(C) &=& (\bigcup \ \mathbf{nil} \ C) \end{array}
```

Figure A.12. Java Type Conversion

$$\delta_{\tau}(class) = x : \top \xrightarrow{\text{tt} \mid \text{tt}} (\bigcup \text{ nil } \mathbf{Class})$$

Figure A.13. Constant Typing

```
\delta(class, C\{\overrightarrow{fld}:\overrightarrow{v}\})
                                               = C
                                                                           \delta(class, true)
                                                                                                            = B
\begin{array}{l} \delta(class,C)\\ \delta(class,C)\\ \delta(class,[\rho,\lambda x^{\tau}.e]_{\text{c}})\\ \delta(class,[v_{d},t]_{\text{m}}) \end{array}
                                                      Class
                                                                           \delta(class, false)
                                                                                                            = B
                                                = Fn
                                                                           \delta(class, k)
                                                                                                            = K
                                                = Multi
                                                                           \delta(class, nil)
                                                                                                            = nil
\delta(class, m)
                                                        Map
```

Figure A.14. Primitives

```
\mathsf{IsAProps}(\mathbf{class}(\pi(x)), (\mathbf{Val}\,C)) \quad = \quad C_{\pi(x)}|\overline{C}_{\pi(x)}|
                                                                     \frac{((\operatorname{Val} s)_x|(\operatorname{Val} s)_x)[o/x]}{\text{if } s \neq C} 
\mathsf{IsAProps}(o, (\mathbf{Val}\,s))
                                                                  tt|tt
\mathsf{IsAProps}(o, \tau)
                                                                                        otherwise
                                                    v \neq C
\mathsf{IsA}(v,v)
                               true
\mathsf{IsA}(C,C')
                                            \vdash C < : C'
                       =
                              true
\mathsf{IsA}(v,v')
                                              otherwise
                        = false
```

Figure A.15. Definition of isa?

```
\begin{array}{l} \mathsf{GM}(t,v_e) = v_f \quad \text{if } \overrightarrow{v_{fs}} = \{v_f\} \\ \quad \text{where } \overrightarrow{v_{fs}} = \{v_f | (v_v,v_f) \in t \text{ and } \mathsf{IsA}(v_v,v_e) = \mathsf{true}\} \\ \mathsf{GM}(t,v_e) = \mathsf{err} \quad \text{otherwise} \end{array}
```

Figure A.16. Definition of get-method

$$\begin{array}{c} \text{B-LOCAL} \\ \frac{\rho(x) = v}{\rho \vdash x \Downarrow v} & \frac{\rho \vdash e_1 \Downarrow v_1}{\rho \vdash e \Downarrow v} & \frac{\rho \vdash e_a \Downarrow v_a}{\rho \vdash (\log e e) \Downarrow v} & \frac{\rho \vdash e_a \Downarrow v_a}{\rho \vdash (\log e e) \Downarrow v} & \frac{\rho \vdash e_a \Downarrow v_a}{\rho \vdash (\log e e) \Downarrow v} & \frac{\rho \vdash e_a \Downarrow v_a}{\rho \vdash (\log e e) \Downarrow v} & \frac{\rho \vdash e_a \Downarrow v_a}{\rho \vdash (\log e e) \Downarrow v} & \frac{\rho \vdash e_a \Downarrow v_a}{\rho \vdash (\log e e) \Downarrow v} & \frac{\rho \vdash e_a \Downarrow v_a}{\rho \vdash (\log e e) \Downarrow v} & \frac{\rho \vdash e_a \Downarrow v_a}{\rho \vdash (\log e e) \Downarrow v} & \frac{\rho \vdash e_a \Downarrow v_a}{\rho \vdash (\log e e) \Downarrow v} & \frac{\rho \vdash e_a \Downarrow v_a}{\rho \vdash (\log e e) \Downarrow v} & \frac{\rho \vdash e_a \Downarrow v_a}{\rho \vdash (\log e e) \Downarrow v} & \frac{\rho \vdash e_a \Downarrow v_a}{\rho \vdash (\log e) \Downarrow v} & \frac{\rho \vdash e_a \Downarrow v_a}{\rho \vdash (\log e) \Downarrow v} & \frac{\rho \vdash e_a \Downarrow v_a}{\rho \vdash (\log e) \Downarrow v} & \frac{\rho \vdash e_a \Downarrow v_a}{\rho \vdash (\log e) \Downarrow v} & \frac{\rho \vdash e_a \Downarrow v_a}{\rho \vdash (\log e) \Downarrow v} & \frac{\rho \vdash e_a \Downarrow v_a}{\rho \vdash (\log e) \Downarrow v} & \frac{\rho \vdash e_a \Downarrow v_a}{\rho \vdash (\log e) \Downarrow v} & \frac{\rho \vdash e_a \Downarrow v_a}{\rho \vdash (\log e) \Downarrow v} & \frac{\rho \vdash e_a \Downarrow v_a}{\rho \vdash (\log e) \Downarrow v} & \frac{\rho \vdash e_a \Downarrow v_a}{\rho \vdash (\log e) \Downarrow v} & \frac{\rho \vdash e_a \Downarrow v_a}{\rho \vdash (\log e) \Downarrow v} & \frac{\rho \vdash e_a \Downarrow v_a}{\rho \vdash (\log e) \Downarrow v} & \frac{\rho \vdash e_a \Downarrow v_a}{\rho \vdash (\log e) \Downarrow v} & \frac{\rho \vdash e_a \Downarrow v_a}{\rho \vdash (\log e) \Downarrow v} & \frac{\rho \vdash e_a \Downarrow v_a}{\rho \vdash (\log e) \Downarrow v} & \frac{\rho \vdash e_a \Downarrow v_a}{\rho \vdash (\log e) \Downarrow v} & \frac{\rho \vdash e_a \Downarrow v_a}{\rho \vdash (\log e) \Downarrow v} & \frac{\rho \vdash e_a \Downarrow v_a}{\rho \vdash (\log e) \Downarrow v} & \frac{\rho \vdash e_a \Downarrow v_a}{\rho \vdash (\log e) \Downarrow v} & \frac{\rho \vdash e_a \Downarrow v_a}{\rho \vdash (\log e) \Downarrow v} & \frac{\rho \vdash e_a \Downarrow v_a}{\rho \vdash (\log e) \Downarrow v} & \frac{\rho \vdash e_a \Downarrow v_a}{\rho \vdash (\log e) \Downarrow v} & \frac{\rho \vdash e_a \Downarrow v_a}{\rho \vdash (\log e) \Downarrow v} & \frac{\rho \vdash e_a \Downarrow v_a}{\rho \vdash (\log e) \Downarrow v} & \frac{\rho \vdash e_a \Downarrow v_a}{\rho \vdash (\log e) \Downarrow v} & \frac{\rho \vdash e_a \Downarrow v_a}{\rho \vdash (\log e) \Downarrow v} & \frac{\rho \vdash e_a \Downarrow v_a}{\rho \vdash (\log e) \Downarrow v} & \frac{\rho \vdash e_a \Downarrow v_a}{\rho \vdash (\log e) \Downarrow v} & \frac{\rho \vdash e_a \Downarrow v_a}{\rho \vdash (\log e) \Downarrow v} & \frac{\rho \vdash e_a \Downarrow v_a}{\rho \vdash (\log e) \Downarrow v} & \frac{\rho \vdash e_a \Downarrow v_a}{\rho \vdash (\log e) \Downarrow v} & \frac{\rho \vdash e_a \Downarrow v_a}{\rho \vdash (\log e) \Downarrow v} & \frac{\rho \vdash e_a \Downarrow v_a}{\rho \vdash (\log e) \Downarrow v} & \frac{\rho \vdash e_a \Downarrow v_a}{\rho \vdash (\log e) \Downarrow v} & \frac{\rho \vdash e_a \Downarrow v_a}{\rho \vdash (\log e) \Downarrow v} & \frac{\rho \vdash e_a \Downarrow v_a}{\rho \vdash (\log e) \Downarrow v} & \frac{\rho \vdash e_a \Downarrow v_a}{\rho \vdash (\log e) \Downarrow v} & \frac{\rho \vdash e_a \Downarrow v_a}{\rho \vdash (\log e) \Downarrow v} & \frac{\rho \vdash e_a \Downarrow v_a}{\rho \vdash (\log e) \Downarrow v} & \frac{\rho \vdash e_a \Downarrow v_a}{\rho \vdash (\log e) \Downarrow v} & \frac{\rho \vdash e_a \Downarrow v_a}{\rho \vdash (\log e) \Downarrow v} & \frac{\rho \vdash e_a \Downarrow v_a}{\rho \vdash (\log e) \Downarrow v} & \frac{\rho \vdash e_a \Downarrow v_a}{\rho \vdash (\log e) \Downarrow v} & \frac{\rho \vdash e_a \Downarrow v_a}{\rho \vdash (\log e) \Downarrow v} & \frac{\rho \vdash e_a \Downarrow v_a}{\rho \vdash (\log e) \Downarrow v} & \frac{\rho \vdash e_a \Downarrow v_a}{\rho \vdash (\log e) \Downarrow v} & \frac{\rho \vdash e_a \Downarrow$$

Figure A.17. Operational Semantics

Figure A.18. Stuck programs

Figure A.19. Error and stuck propagation

$$\begin{array}{lll} \rho(x) & = & v & (x,v) \in \rho \\ \rho(\mathbf{key}_k(o)) & = & (\det \rho(o) \ k) \\ \rho(\mathbf{class}(o)) & = & (\mathit{class} \ \rho(o)) \end{array}$$

Figure A.20. Path translation

```
if \mathcal{M}[k] = \tau
update((\mathbf{HMap}^{\mathcal{E}} \mathcal{M} \mathcal{A}), \nu, \pi :: \mathbf{key}_{k})
                                                                                                                                      (\mathbf{HMap}^{\mathcal{E}} \mathcal{M}[k \mapsto \mathsf{update}(\tau, \nu, \pi)] \mathcal{A})
update((\mathbf{HMap}^{\mathcal{E}} \mathcal{M} \mathcal{A}), \tau, \pi :: \mathbf{key}_k)
                                                                                                                                                                                                                                                                 if \vdash nil \not<: \tau and k \in \mathcal{A}
                                                                                                                          =
\begin{array}{l} \mathsf{update}((\mathbf{HMap}^{\mathcal{E}}\,\mathcal{M}\,\mathcal{A}),\tau,\pi::\mathbf{key}_k) \\ \mathsf{update}((\mathbf{HMap}^{\mathcal{E}}\,\mathcal{M}\,\mathcal{A}),\tau,\pi::\mathbf{key}_k) \\ \mathsf{update}((\mathbf{HMap}^{\mathcal{E}}\,\mathcal{M}\,\mathcal{A}),\nu,\pi::\mathbf{key}_k) \\ \mathsf{update}((\mathbf{HMap}^{\mathcal{P}}\,\mathcal{M}\,\mathcal{A}),\tau,\pi::\mathbf{key}_k) \end{array}
                                                                                                                                     \perp
                                                                                                                                                                                                                                                                 if \vdash nil <: \tau and k \in \mathcal{A}
                                                                                                                          =
                                                                                                                                   (\mathbf{HMap}^{\mathcal{E}} \mathcal{M} \mathcal{A})
                                                                                                                                                                                                                                                                 if k \in \mathcal{A}
                                                                                                                                      (\cup \ (\mathbf{HMap}^{\mathcal{P}} \ \mathcal{M}[k \mapsto \tau] \ \mathcal{A}) \\ (\mathbf{HMap}^{\mathcal{P}} \ \mathcal{M} \ (\mathcal{A} \cup \{k\})))
                                                                                                                                                                                                                                                                 if \vdash nil <: \tau,
                                                                                                                                                                                                                                                                 k \not\in dom(\mathcal{M}) and k \not\in \mathcal{A}
                                                                                                                                      \begin{array}{l} (\mathbf{HMap}^{\mathcal{P}}\,\mathcal{M}[k\mapsto \mathsf{update}(\top,\nu,\pi)]\,\mathcal{A})\\ (\mathbf{HMap}^{\mathcal{P}}\,\mathcal{M}[k\mapsto \mathsf{update}(\top,\nu,\pi)]\,\mathcal{A}) \end{array}
\mathsf{update}((\mathbf{HMap}^{\mathcal{P}} \, \mathcal{M} \, \mathcal{A}), \nu, \pi :: \mathbf{key}_k)
                                                                                                                                                                                                                                                                 if k \not\in dom(\mathcal{M}) and k \not\in \mathcal{A}
update((\mathbf{HMap}^{\mathcal{P}} \mathcal{M} \mathcal{A}), \nu, \pi :: \mathbf{key}_{k})
\begin{array}{l} \operatorname{update}((\bigcup \ \overrightarrow{(\mathbf{HMap}^{\mathcal{E}} \ \mathcal{M} \ \mathcal{A})}^{i}), \nu, \pi :: \mathbf{key}_{k}) \\ \operatorname{update}(\tau, (\mathbf{Val} \ C), \pi :: \mathbf{class}) \end{array}
                                                                                                                                      (\bigcup \mathsf{update}((\mathsf{HMap}^{\mathcal{E}} \mathcal{M} \mathcal{A}), \nu, \pi :: \mathsf{key}_{k})
                                                                                                                                      update(\tau, C, \pi)
update(\tau, \overline{(Val C)}, \pi :: class)
                                                                                                                                      update(\tau, \overline{C}, \pi)
                                                                                                                                                                                                                                                                 if \not\exists C'. \vdash C' <: C \text{ and } C' \neq C
                                                                                                                                      update(\tau, \mathbf{Object}, \pi)
                                                                                                                                                                                                                                                                 if \vdash \sigma <: \mathbf{Object}
update(\tau, \sigma, \pi :: class)
update(\tau, \overline{\sigma}, \pi :: class)
                                                                                                                                                                                                                                                                 if \vdash Object \stackrel{\circ}{<}: \sigma
                                                                                                                                      update(\tau, \mathbf{nil}, \pi)
                                                                                                                                                                                                                                                                 if \vdash \sigma \lessdot: nil
update(\tau, \sigma, \pi :: class)
                                                                                                                                      update(\tau, nil, \pi)
update(\tau, \overline{\sigma}, \pi :: class)
                                                                                                                                      update(\tau, \mathbf{Object}, \pi)
                                                                                                                                                                                                                                                                 if \vdash nil <: \sigma
update(\tau, \nu, \pi :: class)
                                                                                                                           =
update(\tau, \sigma, \epsilon)
                                                                                                                                      restrict(\tau, \sigma)
                                                                                                                           =
update(\tau, \overline{\sigma}, \epsilon)
                                                                                                                                      remove(\tau, \sigma)
restrict(\tau, \sigma)
                                                                                                                                      if \not\exists v. \vdash v : \tau ; \psi_1 ; o_1
                                                                                                                                      and \vdash v : \sigma ; \psi_2 ; o_2
\operatorname{restrict}((\bigcup \overrightarrow{\tau}), \sigma)
                                                                                                                                      (\bigcup \mathsf{restrict}(\tau, \sigma))
restrict(\tau, \sigma)
                                                                                                                                                                                                                                                                 if \vdash \tau <: \sigma
                                                                                                                           =
restrict(\tau, \sigma)
                                                                                                                                                                                                                                                                 otherwise
                                                                                                                           =
                                                                                                                                      \sigma
\mathsf{remove}(\tau, \sigma)
                                                                                                                                                                                                                                                                 \text{if} \vdash \tau \mathop{<:} \sigma
                                                                                                                                      remove((\bigcup \overrightarrow{\tau}), \sigma)
                                                                                                                                      (\bigcup \text{remove}(\tau, \sigma))
remove(\tau, \sigma)
                                                                                                                                                                                                                                                                 otherwise
```

Figure A.21. Type Update

$$\frac{\text{M-OR}}{\Gamma \models \psi_{1} \text{ or } \Gamma \models \psi_{2}}{\Gamma \models \psi_{1} \vee \psi_{2}} \qquad \frac{\prod_{i}^{\text{M-IMP}} \Gamma \models \psi_{1} \text{ implies } \Gamma \models \psi_{2}}{\Gamma \models \psi_{1} \supset \psi_{2}} \qquad \frac{\prod_{i}^{\text{M-AND}} \Gamma \models \psi_{1}}{\Gamma \models \psi_{1} \wedge \psi_{2}} \qquad \frac{\prod_{i}^{\text{M-TOP}} \Gamma \models \text{tt}}{\Gamma \models \text{tt}}$$

$$\frac{\prod_{i}^{\text{M-TYPE}} \Gamma \mid \psi_{1} \mid$$

Figure A.22. Satisfaction Relation

Figure A.23. Proof System

$$\begin{array}{llll} \psi_{+}|\psi_{-}[o/x]^{pol} & = & \psi_{+}[o/x]^{pol}|\psi_{-}[o/x]^{pol} \\ \nu_{\pi(x)}[\pi'(y)/x]^{pol} & = & (\nu[\pi'(y)/x]^{pol})_{\pi(\pi'(y))} \\ \nu_{\pi(x)}[\emptyset/x]^{\rm pos} & = & {\rm tt} \\ \nu_{\pi(x)}[\emptyset/x]^{\rm neg} & = & {\rm ff} \\ \nu_{\pi(x)}[o/z]^{pol} & = & \nu_{\pi(x)} & x \neq z \ {\rm and} \ z \not\in {\rm fv}(\nu) \\ \nu_{\pi(x)}[o/z]^{\rm pos} & = & {\rm tt} & x \neq z \ {\rm and} \ z \in {\rm fv}(\nu) \\ \nu_{\pi(x)}[o/z]^{\rm neg} & = & {\rm ff} \\ \nu_{\pi(x)}[o/z]^{\rm neg} & = & {\rm ff} \\ \nu_{\pi(x)}[o/z]^{\rm pol} & = & {\rm tt} \\ {\rm ff} \ [o/x]^{pol} & = & {\rm ff} \\ (\psi_1 \supset \psi_2)[o/x]^{\rm pos} & = & \psi_1[o/x]^{\rm neg} \supset \psi_2[o/x]^{\rm pos} \\ (\psi_1 \supset \psi_2)[o/x]^{\rm neg} & = & \psi_1[o/x]^{\rm pos} \supset \psi_2[o/x]^{\rm neg} \\ (\psi_1 \lor \psi_2)[o/x]^{\rm pol} & = & \psi_1[o/x]^{\rm pol} \lor \psi_2[o/x]^{\rm pol} \\ (\psi_1 \land \psi_2)[o/x]^{\rm pol} & = & \psi_1[o/x]^{\rm pol} \land \psi_2[o/x]^{\rm pol} \\ (\psi_1 \land \psi_2)[o/x]^{\rm pol} & = & \psi_1[o/x]^{\rm pol} \land \psi_2[o/x]^{\rm pol} \\ \pi(x)[\pi'(y)/x]^{\rm pol} & = & \emptyset \\ \pi(x)[o/z]^{\rm pol} & = & \emptyset \\ \pi(x)[o/z]^{\rm pol} & = & \emptyset \\ \end{array}$$

Substitution on types is capture-avoiding structural recursion.

Figure A.24. Substitution