Optimized Keyword Persistent Hash Maps for Clojure

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Abstract

Clojure provides a suite of persistent data structures implemented by Hickey based on previous work by Bagwell. There are several implementations of persistent hash maps included in Clojure and provided by open source source libraries (like data.int-map), however there is no specific implementation for the most common case of hash map: relatively small hash maps (less than 32 entries) consisting of keyword keys. Keywords are effectively interned strings that are designed to be keys in maps. This paper presents how the current implementations of hash maps handle keyword usage, and present several experiments of specialized hash maps handling just keyword keys.

1. Introduction

Hash Array Mapped Tries (HAMT) have rocked the functional programming world with a fast, immutable and persistent alternative to a hash map. First described by Bagwell [2], they are featured in mainstream functional programming languages like Clojure and Scala, and have been ported to many others.

HAMT's compare very well to other similar data structures. They offer fast lookups by minimizing the depth and branching factors of their tree representation. They minimize memory usage by lazily creating subtries only when necessary.

Clojure is a dynamically typed programming language running on the JVM. It comes with a suite of persistent data structures that form the core of the language. These data structures are based on Array Mapped Tries [1], and include persistent vectors, hash maps, and hash sets.

To understand with a hash array mapped trie is, we first give some definitions. A *trie* is a way of formatting key/value pairs in a tree, where values are leaves and keys are spread across the paths to those nodes. Key prefixes occur on the shallow levels of the tree, and suffixes occur closer to the leaves. A *bit trie* assumes the mapping keys are strings of bits. Each level consumes one or more bits to index its elements. An *Array Mapped Trie* maps the bits of array indices as a bit trie.

In this paper, we explore Clojure's persistent hash map implementation (Section 3). It was implemented by Hickey [5], extending Bagwell's original formulation [2] to be persistent. Persistent data structures use *structural sharing* when extending themselves, so Clojure necessarily enforces hash maps to be *immutable*.

Contributions This paper offers several contributions.

1. We give an approachable walkthrough of mechanics behinds HAMTs.
2. We describe the internals of Clojure's persistent HAMT implementation, via our pure-Clojure reimplementation.
3. We present a port of Clojure's HAMT from Java to Clojure for pedagogical purposes, featuring trie visualization (Figure).
4. We evaluate a Clojure-specific optimization for Clojure's HAMT.

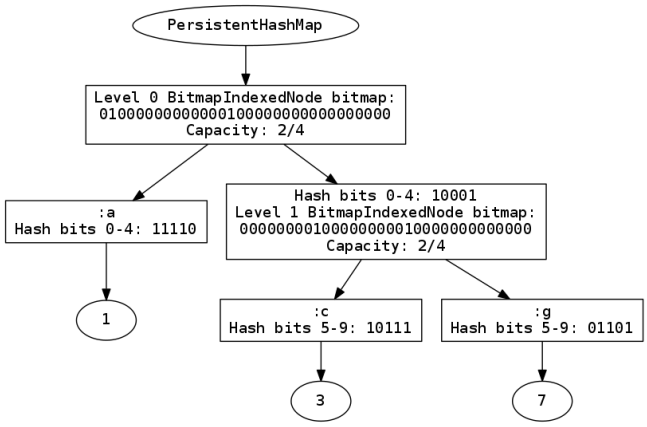


Figure 1. A hash array mapped trie {`:a 1, :c 3, :g 7`}. The first 5 bits of `:c`'s and `:g`'s hashes collide (`10001`), so a new level is used to disambiguate.

5. We speculate on future directions for HAMT's.

2. Walkthrough

Let's walk through some examples about how HAMTs work under different operations.

Firstly, a HAMT represents a search tree based on the *hash* of its keys. Each key is associated with a value. Figure 2 gives sample 32-bit hashes for six keys, which we will use only in this section.

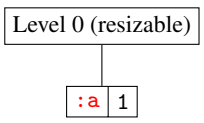
A HAMT starts as an empty tree with no root.

Insertion Let's insert a mapping into an empty tree. Since the tree is empty, we create the first (root) level, level 0. This corresponds to the first 5 bits of the hash. The maximum branching factor is $2^5 = 32$, but since we only need one entry, we create a *resizable* root node.

A resizable node of current capacity n entries, contains an array of length $2n$ and a 32-bit bitmap indicating the location of each entry. Initially, the entry array is of length zero, and the bitmap is zero.

To add a mapping from `:a` to `1`, we first retrieve the first 5 bits (since we are current at level 0) of `hash(:a)`—00001, or 1 in decimal. So, the second bit in the bitmap is set to 1. (Red text indicates the results of the current operation).

`0000 0000 0000 0000 0000 0000 0000 0010`



```
           6     5      4      3      2      1      0
hash(:a) = 00 00000 00000 00000 00000 00000 00001
hash(:b) = 00 00000 00000 00000 00000 00000 00011
hash(:c) = 00 00000 00000 00000 00000 00000 00101
hash(:d) = 00 00000 00000 00000 00000 00000 01010
hash(:e) = 00 00000 00000 00000 00000 00000 10100
hash(:f) = 00 00000 00000 00000 00000 00001 10100
```

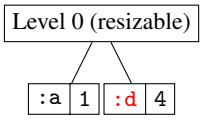
Figure 2. Example 32-bit binary hashes for six keys, partitioned into 6 groups.

Since there is only one entry, we allocate a length 2 array, and initialize it with the key and value.

Adding another entry reveals a crucial invariant of resizable nodes: the array index of an entry corresponding to bit i in the bitmap, is the sum of all the 1 bits below bit i, multiplied by 2.

We add a mapping from `:d` to 4. The first 5 bits of its hash is 01010, 10 in decimal. so we set the 11th bit of the bitmap to 1.

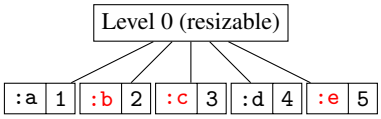
```
0000 0000 0000 0000 0000 0100 0000 0010
```



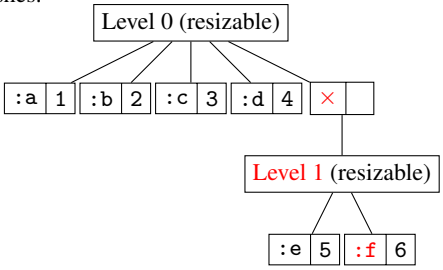
Does the new node go before or after `:a`? It is decided by counting the number of 1's below the 11th bit—assigning the result to i—and inserting the entry into the array after skipping i nodes. Here, the sum is 1, so we insert `:d` after visiting 1 node. The actual index to insert the new key and value is $2i$ and $2i + 1$ respectively.

We repeat this with a few more keys—we leave the intermediate stages as an exercise for the reader.

```
0000 0000 0001 0000 0000 0100 0010 1010
```



Inserting new levels We must deal with the case of hashes clashing at a particular level. Notice `hash(:e)` and `hash(:f)` both have 10100 for their level 0 hash. If the hashed values were actually identical, we would simply swap out the old mapped value with the new mapped value. Since they are actually different keys, we must create a new level and compare them on the next 5 bits of their hashes.



The new level 1 node has its own bitmap based on the 6th-10th bits of hashes. Currently, the bitmap's 1st and 2nd bits are set to 1, since the level 1 hashes of `:e` and `:f` in decimal are 0 and 1 respectively. Since the 2nd bit has a single 1 bit below it in the bitmap, `:f` goes after `:e`.

We can also see how resizable nodes avoid some indirection—if an entry is unambiguous at a level, it is inserted directly into the array; otherwise, a × indicates the next array member is pointer to a subtrie to continue searching.

Search To perform a lookup on a given key, we use each level of its hash to traverse the tree, until the tree ends, or we find the entry.

For example, let's lookup the entry for `:e` in the previous trie.

At level 0, its hash is 10100, or 20 in decimal. We lookup the 21st bit in the root bitmap (in red) and it is 1—this indicates the entry exists. We count four 1 bits below it (in blue).

```
0000 0000 0001 0000 0000 0100 0010 1010
```

We lookup entry $2 \cdot 4 = 8$ in the root array, and find an ×. We follow the pointer to level 1, and repeat for the next 5 hash bits (00000; 0 in decimal) with the next node's bitmap:

```
0000 0000 0000 0000 0000 0000 0000 0011
```

We find a 1 in the 1st bit position, and zero 1's below it, so we lookup entry $2 \cdot 0 = 0$ in the level 1 array. Since it is not ×, we test to see if the entry we have travelled to `:e` is equals to our query—it is, so the search has succeeded and the mapped value is at index $2 \cdot 0 + 1 = 1$; otherwise the search fails.

Deletion Node deletion follows the same procedure as search to discover the location of an entry. Instead of returning the mapped value, it removes the entry from the array, and sets the bitmap for that entry to 0.

Full nodes Once a resizable array reaches a certain threshold size, it is no longer efficient to allocate an array of length $2n$ to hold n nodes. For example, if we have over 16 entries, which would require copying arrays over length 32, we could instead once-and-for-all allocate a length 32 array where each member is a subtrie (without a × flag)—we call this a *full* node.

This removes the need to bitmap bits—the 32 bitmap bits now map one-to-one to the subtries.

Hash collision nodes If two different keys hash to the same value, we use a *hash collision node* to differentiate them. One approach is to default to a linear search—with the assumption that the hash function has a low probabilities of collisions, this should rarely be an issue.

3. Clojure's HAMT, in detail

Now, we will give a detailed description of Hickey's HAMT implementation for Clojure, based on Bagwell's original HAMT.

3.1 Understanding the bit operations

The core of the HAMT implementation has 3 important bit operations, which we will cover in detail.

Bit masking The `mask` function isolates a multiple of 5 bits from a given hash, interpreting them as a 32-bit integer.

We can view hashes as 5 partitions by of a 32-bit integer from right-to-left.

```
6     5      4      3      2      1      0
00 00000 00000 00000 00000 00000 00000
```

The source for the mask function (below), selects partitions by shifting the hash's bits right until the first 5 bits is the parition needed, then the first 5 bits of the result interpretted as an integer (Appendix A explains `>>>`).

```
// returns integer between 0 and 31
int mask(int hash, int shift) {
  return (hash >>> shift) & 0x01f;
}
```

The return value of `mask` is an integer value from 0 to 31 which indicates which bit in the bitmap is relevant (in `ArrayNode`, it is the

actual array index, `BitmapIndexedNode` requires you to count the number of 1's below this index in the bitmap, as described below).

For example, to isolate partition 0 from a hash, we call `mask(hash, 0)`, which simply isolates the first 5 bits (`0x01f` is `11111` in binary). To isolate partition 3 from a hash, we call `mask(hash, 3*5)` which shifts the hash right 15 places, then isolates the first 5 bits of the result.

The Clojure implementation has `shift` as a multiple of 5—instead of the number of times to multiply by 5—presumably for performance reasons. We conjecture that it is faster to pay an incremental cost of adding 5 every time you descend one layer, instead of an extra multiplication every time `mask` is called.

Isolating bits Each node maintains a 32-bit bitmap, which contains a 1 where the node's array has an entry.

To index into this bitmap, we use the `bitpos` function. The return value is a 32-bit integer with exactly 1 bit set to 1.

```
// returns a 32-bit integer with exactly 1 bit
// set to 1
int bitpos(int hash, int shift) {
  return 1 << mask(hash, shift);
}
```

The return value can then be used bit *and*ed with the bitmap to return the value of the desired bit in the bitmap.

Array indexing For non-resizable nodes (like `ArrayNode`), indexing into the next level of the trie is the result of a `mask`. Resizable nodes (like `BitmapIndexedNode`) are more involved, and require further bit manipulations.

To retrieve the next array index, we count the number of 1's below the given bit in the bitmap (assuming the given bit is set to 1). This number i is the number of nodes *before* the node of interest—thus indexes $2i$ and $2i + 1$ contain the key and value of interest. To demonstrate this, say we have a bitmap with the first 8 bits 1011 0010, and otherwise zeros. Since four bits are 1, we have four nodes in an array of size $4 \cdot 2 = 8$.

0	:a	1
2	:b	2
4	:c	3
6	:e	5

To lookup the fourth node with key `:e`, `mask` would return the binary number 1000 0000. Since the 8th bit is set to 1 in our bitmap, we count the number of 1's below the 8th bit—there are 3—so the desired key and value are at indexes $2 \cdot 3 = 6$ and $2 \cdot 3 + 1 = 7$, respectively.

This approach also holds when inserting a new node. Say we are inserting a new entry for the 7th bit in the bitmap. Since there are 3 existing entries below the 7th bit in the bitmap, we insert the new entry in the 4th position of the array, moving the existing entry to the 5th position.

0	:a	1
2	:b	2
4	:c	3
⋮		
6	:e	5

:d	4

0	:a	1
2	:b	2
4	:c	3
6	:d	4
8	:e	5

Now, updating the bit map to 1111 0010 with the 7th bit set to 1 keeps the bit counting invariant—now the 8th entry is after the 4th entry because there are 4 bits set to 1 before the 8th bitmap bit.

The bit counting is calculated by the `index` function.

```
// returns the number of 1's in bitmap before the
// 'bit'th bit
int index(int bit, int bitmap) {
  return Integer.bitCount(bitmap & (bit - 1));
}
```

In the implementation, `bit` is a 32-bit integer with exactly 1 bit set to 1. By decrementing `bit`, we aquire a useful mask to isolate all the necessary bits in the bitmap. For example, if bitmap was `1000`—that is, isolating the 4th bit—decrementing it results in `0111`. Bit *and*ing `0111` with bitmap then isolates the 1st-3rd bits, which we can then use to count the number of 1's below `bit` in `bitmap`.

3.2 Memory layout

The HAMT data structure has a very compact representation that dynamically expands to make room for 0-16 keys at each level. For 17-32 keys, a full array for 32 keys is allocated once and does not expand further.

For presentational purposes,

PersistentHashMap The enclosing class for the Clojure HAMT is the `PersistentHashMap`, which we briefly summarize. It contains a nullable root node, and has a special field to store `null` entries in the map—INodes use `null` as an indicator.

It provides three methods: `assoc` to associate a new key/value pair, `dissoc` to dissociate an entry by its key, and `find` locate an entry by its key.

INode There are various kinds of nodes in `PersistentHashMap`, all implementing the `INode` interface. It provides a similar interface as above.

```
  Bitmap
00000000 00000000 00000000 00000000
```

BitmapIndexedNode The `BitmapIndexedNode` is the meat of the HAMT data structure. It maintains an expandable array for storing up to 16 nodes, where even array indices i contain keys and $i + 1$ have their associated value or subtrie. The bitmap is a 32-bit integer that indexes into the array via some bit manipulation and class invariants.

For sub-tries with n nodes, where $n \leq 16$, we only need to allocate an array of size $2 * n$. This is where the trick of counting number of 1's below the current bit comes into play. This number tells us where to index into this array to find the node in question.

0	a	6
2	b	3
⋮		
4	c	9

c	d

When we need to insert a new node, we allocate a new array of size $2 * (n + 1)$, and we "fit" the new key at position $2 * idx$ and the new value at position $2 * idx + 1$. The bitmap is still valid!

Take the bitmap `0010 0010` which has the 2nd bit's entry at position 0-1 in the array, and the 6th bit's entry at position 2. If we insert a new entry at bit 5, `0011 0010`, we extend our array to have 3*2 elements, and insert the new entry at position 2-3, bumping the 6th bit to positions 4-5. Notice the bitmap still holds—the 2nd bit has zero 1's below it, so it is at entry $0 * 2 = 0$; the 5th bit has a single 1 bit below it, so its index is $1 * 2 = 2$; the 6th bit has two 1 bits below it, so its index is $2 * 2 = 4$.

This continues until we need a 17th entry, at which point we convert to a `ArrayNode`.

ArrayNode An ArrayNode allocates a length 32 array with a subnode in every element, rather than every second element in a `BitmapIndexedNode`. Now the `mask` function truly returns the index into the array, so `bitpos` is no longer needed.

3.3 Search

Search is straightforward, and works as described in Section 2.

3.4 Insertion

Hash collisions In the case of two non-equal keys hashing to the same value, the implementation then resorts to a linear search amongst the keys to find a match. The `HashCollisionNode` plays this role.

3.5 Deletion

Deletion is slightly more complicated than Bagwell's original HAMT. Since Clojure's HAMT is persistent and immutable, it must copy and alter the node containing the deleting leaf. The rest of the trie can be reused, making this still a cheap operation.

4. Reimplementation in Clojure

We reimplemented [1] the `PersistentHashMap` class from the Clojure standard library in Clojure—it was originally written in Java. This is not the first reimplementation. ClojureScript features similar pure-Clojure port in its standard library, and several other languages have ported their own versions based on Clojure's original implementation.

Our decision to port to Clojure was mainly educational. We extended the implementation with a graphical visualization of the underlying trie (Figure 2).

4.1 Implementation Challenges

We came across several challenges in porting from Java to Clojure.

Automatic widening before bit operations While Clojure and Java share the same bit layout for numbers, two's complement, Clojure defaults to 64-bit integers, while the Java implementation used 32-bit integers. We came across several situations where we intended 32-bit bit operations, but Clojure upcasted the input to 64-bits. This was a problem especially for negative numbers—in the two's complement representation, the most significant bit is 1, so the output of a bit-operation was wildly incorrect if interpreting the full 64-bits as an answer.

Abstract classes Clojure provides poor support for extending abstract classes. This meant we either had to reimplement the abstract class, which was often long, or use proxies, a less elegant feature of Clojure. We decided to use proxies to save effort, but we think it detracts from the readability of the implementation.

Mutation Clojure does not expose plain Java variables, so several style and performance decisions were made faithfully port local loops with complicated mutation. We decided to use Clojure's volatiles, which are variables that guarantee ordering constraints (like a `volatile` Java variable).

Autoboxing and object identity The Java implementation often compares two primitive `int` values with `==`, which uses object identity—written `clojure.core/identical?` in Clojure. Care was needed to reason about when autoboxing was happening, so a literal transliteration of Java's `==` to `clojure.core/identical?` was actually correct. Sometimes, object identity was important, such as comparing two nodes. Other times, it was better to port to `clojure.core/=`, which features more consistent numeric comparisons.

[1] `https://github.com/frenchy64/optimized-kw-maps`

4.2 Tests

We used the excellent `collection-check` [2] library to fuzz test our implementation. It successfully narrowed down several bugs, and greatly increased our confidence that the implementation was correct.

4.3 Visualization

We also implemented graphical trie visualization via `rhizome` [3]. See Figure 3 for an example for a representative trie.

5. Experiments

We prototyped an optimization based on just-in-time compilation techniques. Leveraging Clojure's `case` statement, we created specialized hash array mapped tries for specific keysets. We decide which keysets based on runtime frequency.

Maps with keyword keys are very common in Clojure. A type system for Clojure has special support for such maps [4] and the `clojure.spec` library[4] has exposes special primitives to generate and verify such maps.

5.1 Approach

To prototype this approach, we repurposed Clojure's `defrecord` construct, which creates maps specialized maps on a known keyset. Instead of statically compiling only known keysets, our approach effectively compiles records at runtime based the most frequent keyword keysets for that particular run.

5.2 Evaluation

To evaluate our approach, we developed a prototype that is a simple wrapper around the existing `PersistentHashMap` class that intercepts operations on keyword keys.

To minimize the overhead of compiling new classes, we delegate compilation to a separate thread of execution, using shared data to communicate which keysets to specialize.

We tested three kinds of maps in our benchmark.

Plain map This is the Clojure implementation of hash array mapped trie described in Section 3.

Record This is a map constructed with Clojure's `defrecord` form. It is specialized at compile time for a specific known set of keyword entries.

Optimized map This is our prototype implementation that specializes maps at runtime.

Figure 5 contains the benchmark code. The purpose of the benchmark is to simulate a set of map associations followed by many lookups, since our optimization only triggers after an association operation. In this version of the benchmark, we perform 20 associations, each with 400,000 lookups on keyword entries. Each iteration has a set number of keyword entries, and we vary the

5.3 Results

Figure 4 plots the results. The *Record* is hand-optimized for the particular keyword entries in the benchmark, and outperforms the *Plain map* as expected. Our *Optimized map* compiles its own specialized map for the set of keywords, and consistently beats *Plain map* for 1,500 extra keys and under. It also stays competitive with the hand-tuned *Record* for 1,500 and under.

After 1,500 keys, there is an unknown problem with our optimized implementation. Our hand-written benchmarking scheme

[2] `https://github.com/ztellman/collection-check`

[3] `https://github.com/ztellman/rhizome`

[4] `http://clojure.org/guides/spec`

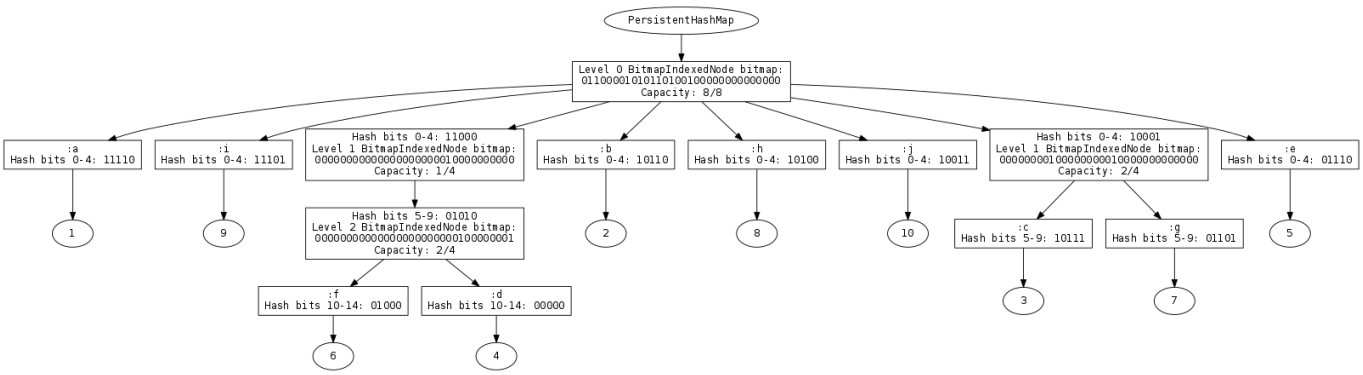


Figure 3. Trie visualization in Clojure port, for map {`:a 1 :b 2 :c 3 :d 4 :e 5 :f 6 :g 7 :h 8 :i 9 :j 10`} (See Appendix B for corresponding hashes).

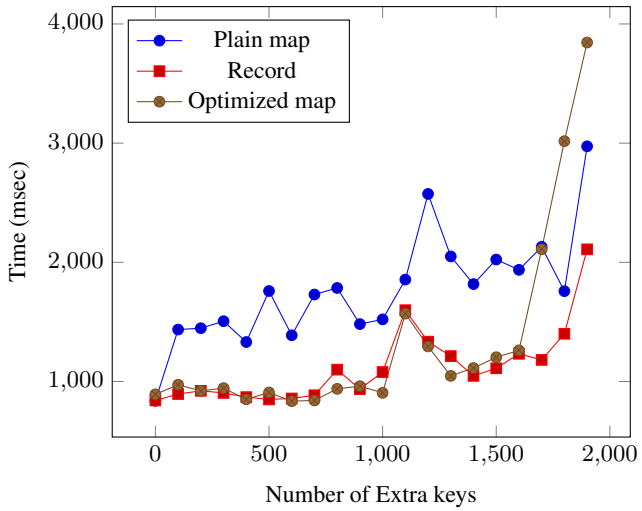


Figure 4. Running time for keyword keys benchmark, varying the number of extra entries in the map.

```
(defn exercise-bench [f n]
  (loop [i 20
         m (f (into {:a 1 :b 2 :c 3 :d 4
                     :e 5 :f 6 :g 7 :h 8}
                    (map #(vector % %) (range n))))]
    (when-not (zero? i)
      (dotimes [_ 100000]
        (+ (:a m) (:b m) (:c m) (:d m)
           (:e m) (:f m) (:g m) (:h m)))
      (recur (dec i) (update m :a inc)))))
```

Figure 5. Benchmark code that takes a function f that returns the kind of map we are currently benchmarking (eg., plain, record, or optimized), and a number n, the number of extra keys to fill the map with.

does not compensate for garbage collection and is rather crude, so we would like to revisit this benchmark in the future to investiate this issue.

5.4 Interpretation

Our optimization stays mostly competitive with hand-tuned records. However, there are no wins in our predicted primary use-case: small maps with only keyword keys. Our benchmark tested an unusual case: keyword lookups on a hybrid keyword and integer keyed map. We constructed it like this to ensure our implementation beat plain maps in expected cases.

We hope our approach can be improved to cater to our original vision of speeding up small keyword maps—our initial experiments show no speedup in this case.

6. Related Work

Recently, Steindorfer and Vinju [6] created Heterogeneous Hash Array Mapped Tries. They are interested in creating a product line of HAMT implementations [7].

Our approach to speeding up the hash-map implementation is related to *storage strategies* by Bolz et. al [3]. They observe that dynamically typed languages often use heterogeous collections as homogeneous, and present optimizations to take advantage of this fact.

7. Future directions

In future work, we plan to devise several benchmarks that test real-world usages of small keyword maps. We will devise several more optimization strategies based on the optimization described in Section 5, and evaluate them using these new benchmarks.

Furthermore, we plan to override Clojure's default hash map implementation in a custom version of Clojure, and run benchmarks over entire programs. We plan to find programs that extensively use the "tagged map" idiom of using plain maps as records—we conjecture these are more likely to see speedups if our optimizations satisfy the micro-benchmarks.

8. Conclusion

Clojure has a general purpose hash-map implementation, but programmers are encouraged to hash-maps primarily as records, which are typically small maps with less than 32 entries with only keyword keys.

In this paper, we prototyped an optimization that caters to this usecase, by compiling specialized hash-maps for specific keyword keysets discovered at runtime. Our preliminary investigation found

we could duplicate performance for hand-tuned records with hybrid keyword and integer keyed maps. In the future, we plan to investigate optimizations and benchmarks that better target small keyword maps.

9. Dedication

This paper is dedicated to the memory of Phil Bagwell. Thank you for sharing your gifts.

References

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Appendices

A. Remark on unsigned bit arithmetic on the JVM

Clojure's implementation of HAMT is implemented on the JVM, which only has signed 32-bit integers. The HAMT implementation, however, treats hashes as arbitrary strings of 32-bits, so we need to emulate unsigned arithmetic operations.

The JVM represents integers using 2's complement, which we will briefly describe. To calculate the corresponding negative number for a positive number, simply invert all the bits and add one.

For example, the number -4 can be derived by taking 4, flipping the bits, and adding 1 (below).

```
0000 0000 0000 0000 0000 0000 0000 0100
  flip bits
1111 1111 1111 1111 1111 1111 1111 1011
  add one
1111 1111 1111 1111 1111 1111 1111 1100
```

This representation is mostly transparent for our purposes—except for the bit shift right operation. The JVM exposes two operations bit shift right operations: signed (`>>` in Java, `bit-shift-right` in Clojure), and unsigned (`>>>` in Java, `unsigned-bit-shift-right` in Clojure).

The difference is, `>>` preserves the most significant bit, while `>>>` replaces it with 0.

```
1000 1101 >>  1 = 1100 0110    //signed
1000 1101 >>> 1 = 0100 0110    //unsigned
```

We always want *unsigned* bit operations, because no bits are special in a hash, or in a bitmap.

B. Hashes for examples

```
              6    5     4     3     2     1     0
(hash :a) = 10 00000 10110 11110 10111 11000 11110
(hash :b) = 01 01100 00101 10001 11100 11010 10110
(hash :c) = 10 01011 01110 01111 10100 10111 10001
(hash :d) = 01 11010 11000 11001 00000 01010 11000
(hash :e) = 01 01001 00101 01000 11111 10110 01110
(hash :f) = 10 10000 01100 11011 01000 01010 11000
(hash :g) = 01 10011 11001 10010 01001 01101 10001
(hash :h) = 01 00001 00010 01000 00011 00011 10100
(hash :i) = 10 10110 10101 01100 11110 11000 11101
(hash :j) = 10 10110 01010 11001 00110 01000 10011
```