EE 416 – Image Processing and Computer Vision **(UH Manoa, Fall 2020)**

Homework Assignment #2

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Problem 1, a)

Is this linear system?

Let:

$$\begin{aligned} g[m,n] =& \sum_{k-1}^{1}\sum_{l-1}^{1} x[m-k, n-l] \\ y_1[m,n] =& x_1[m,n] + \lambda(x_1[m,n] - \frac{1}{9}g_1[m,n]) \\ y_2[m,n] =& x_2[m,n] + \lambda(x_2[m,n] - \frac{1}{9}g_2[m,n]) \end{aligned}$$

Lets check if scaling property holds:

$$\begin{aligned} x_s[m.n] =& \alpha x_1[m,n] \\ g_s[m,n] =& \alpha g_1[m,n] \\ y_s[m.n] =& \alpha y_1[m,n] \\ y_s[m,n] =& x_s[m,n] + \lambda(x_s[m,n] - \frac{1}{9}g_s[m,n]) \\ =& \alpha x_1[m,n] + \lambda(\alpha x_1[m,n] - \frac{1}{9}\alpha g_1[m,n]) \\ =& \alpha[x_1[m,n] + \lambda(x_1[m,n] - \frac{1}{9}g_1[m,n])] \\ =& \alpha y_1[m,n] \end{aligned}$$

Thus, the scaling property is true.

Lets check if additive property holds:

$$\begin{aligned} x_s[m.n] =& x_1[m,n] + x_2[m,n] \\ g_s[m,n] =& g_1[m,n] + g_2[m,n] \\ y_s[m.n] =& y_1[m,n] + y_2[m,n] \\ y_s[m,n] =& x_s[m,n] + \lambda(x_s[m,n] - \frac{1}{9}g_s[m,n]) \\ =& x_1[m,n] + x_2[m,n] + \lambda(x_1[m,n] + x_2[m,n] - \frac{1}{9}(g_1[m,n] + g_2[m,n])) \\ =& x_1[m,n] + \lambda(x_1[m,n] - \frac{1}{9}g_1[m,n]) + x_2[m,n] + \lambda(x_2[m,n] - \frac{1}{9}g_2[m,n]) \\ =& y_1[m,n] + y_2[m,n] \end{aligned}$$

Thus, the additive property is true.

Since the scale and additive property are true, this implies that $y[m,n]$ is a linear system.

Is this a space invariant system?

$$
\begin{aligned}
x_s[m.n] =& x_1[m-m_0, n-n_0] \\
g_s[m,n] =& g_1[m-m_0, n-n_0] \\
y_s[m.n] =& y_1[m-m_0, n-n_0] \\
y_s[m,n] =& x_s[m,n] + \lambda(x_s[m,n] - \frac{1}{9}g_s[m,n]) \\
=& x_1[m-m_0, n-n_0] + \lambda(x_1[m-m_0, n-n_0] - \frac{1}{9}g_1[m-m_0, n-n_0]) \\
=& y_1[m-m_0, n-n_0]
\end{aligned}
$$

Thus, it is space invariant.

To conclude, $y[m,n]$ is a linear system and space invariant.

Problem 1, b)

If 2D impulse response:

$$
\begin{aligned}
x[m,n] =& \delta[m,n] \\
g[m,n] =& \sum_{k-1}^{1}\sum_{l-1}^{1}\delta[m-k, n-l]
\end{aligned}
$$

Thus:

$$
h[m,n] = \delta[m,n] + \lambda(\delta[m,n] - \frac{1}{9}g[m,n])
$$

We also know that $\delta[m,n] = 1$, when $m \wedge n = 0$. Therefore:

$$
h[m,n] = \begin{cases} 1 + \frac{8}{9}\lambda & \text{if } m \wedge n = 0 \\ -\lambda\frac{1}{9} & \text{if } -1 \leq m \wedge n \leq 1\text{,but no } m \wedge n = 0 \\ 0 & \text{otherwise} \end{cases}
$$

Problem 1, c)

$$
\begin{aligned}
g[m,n] =& \sum_{k-1}^{1}\sum_{l-1}^{1}\delta[m-k,n-l] \\
H(e^{i\mu},e^{i\nu}) =& \sum_{m,n=-\infty}^{\infty} h[m,n]e^{-i(\mu m+\nu n)} \\
=& \sum_{m,n=-\infty}^{\infty} \delta[m,n]+\lambda(\delta[m,n]-\frac{1}{9}g[m,n])e^{-i(\mu m+\nu n)} \\
=&(1+\lambda)(e^{-i(\mu 0+\nu 0)})-\lambda\frac{1}{9}[\\
&\quad [(e^{-i(\mu(-1)+\nu(-1))})+(e^{-i(\mu(-1)+\nu(0))})+(e^{-i(\mu(-1)+\nu(1))})]+ \\
&\quad [(e^{-i(\mu(0)+\nu(-1))})+(e^{-i(\mu(0)+\nu(0))})+(e^{-i(\mu(0)+\nu(1))})]+ \\
&\quad [(e^{-i(\mu(1)+\nu(-1))})+(e^{-i(\mu(1)+\nu(0))})+(e^{-i(\mu(1)+\nu(1))})]] \\
=&1+\lambda-\lambda\frac{1}{9}[\\
&\quad [(e^{i(\mu+\nu)})+(e^{i\mu})+(e^{i(\mu-\nu)})]+ \\
&\quad [(e^{i(\nu)})+(e^{0})+(e^{i(-\nu)})]+ \\
&\quad [(e^{-i(\mu-\nu)})+(e^{-i\mu})+(e^{-i(\mu+\nu)})] \\
=&1+\lambda-\lambda\frac{1}{9}[\\
&\quad [e^{i\mu}((e^{i\nu})+(1)+(e^{-i\nu}))]+ \\
&\quad [(e^{i\nu})+(1)+(e^{-i\nu})]+ \\
&\quad [e^{-i\mu}((e^{i\nu})+(1)+(e^{-i\nu}))] \\
=&1+\lambda-\lambda\frac{1}{9}((e^{i\mu})+(1)+(e^{-i\mu}))((e^{i\mu})+(1)+(e^{-i\nu})) \\
=&1+\lambda-\lambda\frac{1}{9}(2cos(\mu)+1)(2cos(\nu)+1)
\end{aligned}
$$

Problem 1, d)

High-Pass Filter
We can observe that the the filter will be approximately $\lambda\frac{8}{9}(2cos(\mu)+1)(2cos(\nu)+1)$.
This clearly shows that is a high pass filter as this filter will sharpen an image.

Problem 1, e)

Low-Pass Filter.
We can observe that the the filter will be approximately $\frac{1}{9}(2cos(\mu)+1)(2cos(\nu)+1)$.
This clearly shows that is a low pass filter as this filter will blur an image.

Problem 2, a)

$$
\begin{aligned}
H(e^{i\mu}, e^{i\nu}) &= \sum_{m,n=-\infty}^{\infty} h[m,n] e^{-i(\mu m + \nu n)} \\
&= \sum_{m,n=-2}^{2} \frac{1}{25} e^{-i(\mu m + \nu n)} + 0 \\
&= \frac{1}{25}(e^{i2\mu} + e^{i\mu} + e^{0} + e^{-i\mu} + e^{-i2\mu})(e^{i2\nu} + e^{i\nu} + e^{0} + e^{-i\nu} + e^{-i2\nu}) \\
&= \frac{1}{25}(2cos(2\mu) + 2cos(\mu) + 1)(2cos(2\nu) + 2cos(\nu) + 1)
\end{aligned}
$$

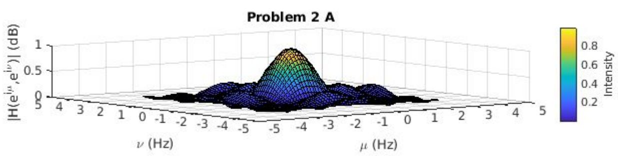

Figure 1: Problem 2-a)

Problem 2, b)

$$
\begin{aligned}
G(e^{i\mu}, e^{i\nu}) &= \sum_{m,n=-\infty}^{\infty} g[m,n] e^{-i(\mu m + \nu n)} \\
&= \sum_{m,n=-\infty}^{\infty} (\delta[m,n] + \lambda\delta[m,n] - \lambda h[m,n]) e^{-i(\mu m + \nu n)} \\
&= 1 + \lambda - \lambda H(e^{i\mu}, e^{i\nu}) \\
H(e^{i\mu}, e^{i\nu}) &= \frac{1}{25}(2cos(2\mu) + 2cos(\mu) + 1)(2cos(2\nu) + 2cos(\nu) + 1)
\end{aligned}
$$

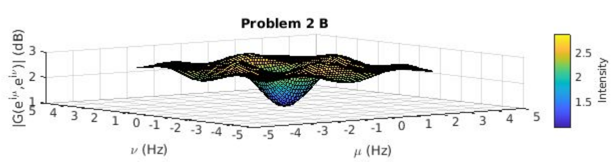

Figure 2: Problem 2-b)