EE 416 – Image Processing and Computer Vision **(UH Manoa, Fall 2020)**

Homework Assignment #6

November 12, 2020

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Problem 1, a)

$$
\begin{aligned}
E[||x-x^*||^2] =& E[(x-y^T\theta)^2] \\
=& E[x^2 - 2\theta^T yx + \theta^T yy^T\theta] \\
=& E[x^2] - 2\theta^T E[yx] + \theta^T E[yy^t]\theta \\
=& \sigma^2 - 2\theta^T r_{y,x} + \theta^T C_y \theta
\end{aligned}
$$

Problem 1, b)

$$
\begin{aligned}
\operatorname*{argmin}_{\theta} E[||x-x^*||^2] =& \frac{\partial}{\partial\theta} E[||x-x^*||^2] \\
=& \frac{\partial}{\partial\theta}[\sigma^2 - 2\theta^T r_{y,x} + \theta^T C_y\theta] \\
=& -2r_{y,x} + 2C_y\theta \\
=& 0 \\
\text{Therefore} & \\
\theta =& C_y^{-1} r_{y,x}
\end{aligned}
$$

Problem 1, c)

$$
\begin{aligned}
C_y =& E[yy^T] \\
=& E[(ax+n)(ax+n)^T] \\
=& E[x^2aa^T + xan^T + xna^T + nn^T] \\
=& E[x^2]aa^T + E[xn^T]a + E[xn]a^T + E[nn^T] \\
\text{Since } n \text{ is independent.} & E[xn] = E[xn^T] = 0 \\
=& \sigma^2 aa^T + 0 + 0 + C_n \\
=& \sigma^2 aa^T + C_n
\end{aligned}
$$

Problem 1, d)

$$
\begin{aligned}
r_{y,x} =& E[yx] \\
=& E[(ax+n)(x)] \\
=& E[ax^2 + nx] \\
=& aE[x^2] + E[n]E[x] \\
=& \sigma^2 a + 0 \\
=& \sigma^2 a
\end{aligned}
$$

Problem 1, e)

$$
\begin{aligned}
x^* =& y^T \theta \\
=& y^T C_y^{-1} r_{y,x} \\
=& (ax+n)^T (\sigma^2 a a^T + C_n)^{-1} \sigma^2 a
\end{aligned}
$$

Problem 2, a)

$$
\begin{aligned}
E[MSE_2(\hat{\theta})] =& E[\frac{1}{M} \sum_{n \in S_2} ||x_n - f(y_n, \hat{\theta})||_2^2] \\
=& \frac{1}{M} E[\sum_{n \in S_2} ||x_n - f(y_n, \hat{\theta})||_2^2] \\
=& \frac{1}{M} \sum_{n \in S_2} E[||x_n - f(y_n, \hat{\theta})||_2^2]
\end{aligned}
$$

Since when we know that $\hat{\theta} > \theta^*$.

$$
\begin{aligned}
\frac{1}{M} \sum_{n \in S_2} E[||x_n - f(y_n, \hat{\theta})||_2^2] \geq& \frac{1}{M} \sum_{n \in S_2} E[||x_n - f(y_n, \theta^*)||_2^2] \\
E[MSE_2(\hat{\theta})] \geq& E[MSE_2 \theta^*]
\end{aligned}
$$

Thus, $MSE_2(\theta^*)$ is smaller.

Problem 2, b)

We will need to know the distribution of x_n and y_n. This values are usually not given/available.

Problem 2, c)

I think L needs to be at least larger than p, where p is the dimension of $\theta(\theta \in R^p)$. Is also important to note that if $L \Rightarrow \infty$, the better $\hat{\theta}$ becomes.

Problem 2, d)

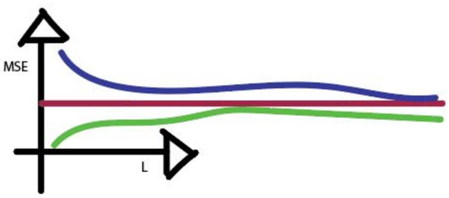

Figure 1: Problem 2-d

Blue Line (Top line) is $MSE_2\hat{\theta}$
Purple Line (Middle Line) is $MSE_2(\theta^*)$
Green Line (Bottom Line) is $MSE_1(\hat{\theta})$

Problem 3, e)

The PSNR for the moving average filter (22.8478) is worst than the learned filter (24.9928).