EE 416 – Image Processing and Computer Vision **(UH Manoa, Fall 2020)**

Homework Assignment #5

November 3, 2020

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Problem 1, a)

$$
\begin{aligned}
T(x) =& F_d^{-1}(F_X(x)) \\
F_d(x) =& \int \frac{1}{(L-1)-0}(L-1)dx \\
=& x \\
F_d^{-1}(x) =& x \\
F_X(x) =& \int_0^x \frac{2x}{L-1} * (L-1)dx \\
=& x^2|_0^x \\
=& x^2 \\
F_d^{-1}(F_X(x)) =& x^2
\end{aligned}
$$

Therefore, $T(x) = x^2$.

Problem 1, b)

$$
\begin{aligned}
T'(y) =& F_d^{-1}(F_Y(y)) \\
F_d(x) =& \int \frac{3z^2}{(L-1)^3} * (L-1)dz \\
=& \frac{1}{(L-1)^2} * z^3 \\
=& \frac{1}{(L-1)^2} * z^3 \\
F_d^{-1}(x) =& \sqrt[3]{x(L-1)^2} \\
F_X(x) =& \int_0^x \frac{1}{L-1} * (L-1)dx \\
=& x \\
F_d^{-1}(F_X(x)) =& \sqrt[3]{x(L-1)^2}
\end{aligned}
$$

Therefore, $T'(y) = (L-1)\sqrt[3]{y(L-1)^2}$.

Problem 1, c)

$$
\begin{aligned}
f(x) =& \frac{2x}{L-1} \\
F(x) =& \int_0^x \frac{2x}{L-1} dx \\
=& \frac{1}{L-1} x^2 \\
f(z) =& \frac{3z^2}{(L-1)^3} \\
F_d(x) =& \int \frac{3x^2}{(L-1)^3} * (L-1) dx \\
=& \frac{x^3}{(L-1)^2} \\
F_d^{-1}(x) =& \sqrt[3]{x * (L-1)^2} \\
\tilde{T}(x) =& F_d^{-1}(F_X(x)) \\
=& \sqrt[3]{(L-1)x^2}
\end{aligned}
$$

Therefore, $z = cT'(x) = c\sqrt[3]{(L-1)x^2}$.

Problem 2, a)

$$
\begin{aligned}
y =& \theta \\
\frac{\partial}{\partial \theta} \sum_{n=1}^{N} \rho(x_n - \theta) =& 0 \\
\frac{\partial}{\partial \theta} \sum_{n=1}^{N} (x_n - \theta)^2 =& 0 \\
\sum_{n=1}^{N} -2(x_n - \theta) =& 0 \\
\sum_{n=1}^{N} (x_n) - \sum_{n=1}^{N} \theta =& 0 \\
\sum_{n=1}^{N} (x_n) - n\theta =& 0 \\
N\theta =& \sum_{n=1}^{N} (x_n) \\
\theta =& \frac{1}{N} \sum_{n=1}^{N} (x_n)
\end{aligned}
$$

Therefore, $y = \theta = \frac{1}{N} \sum_{n=1}^{N} (x_n)$, which is the mean value.

Problem 2, b)

$$
\begin{aligned}
y =& \theta \\
\frac{\partial}{\partial\theta}\sum_{n=1}^{N}\rho(x_n-\theta) =& 0 \\
\frac{\partial}{\partial\theta}\sum_{n=1}^{N}|x_n-\theta| =& 0 \\
=& \sum_{n=1}^{N} sign(x_n-\theta)
\end{aligned}
$$

Therefore, $y = \theta = \sum_{n=1}^{N} sign(x_n - \theta)$, which is the median.

Problem 2, c)

$$
\begin{aligned}
y =& \theta \\
\frac{\partial}{\partial\theta}\sum_{n=1}^{N}\rho(x_n-\theta) =& 0 \\
\frac{\partial}{\partial\theta}\sum_{n=1}^{N}|x_n-\theta|^{0.5} =& 0 \\
=& 0.5\sum_{n=1}^{N} sign(x_n-\theta)|x_n-\theta|^{-0.5}
\end{aligned}
$$

Therefore, $y = root_\theta\{0.5\sum_{n=1}^{N} sign(x_n - \theta)|x_n-\theta|^{-0.5}\}$.

Problem 2, d)

No, it is not linear because of the $|x_n - \theta|$. It is not homogeneous as well because of the $|x_n - \theta|$.