ICS 483 – Computer Vision **(UH Manoa, Spring 2019)**

Homework Assignment #1 - Theory Problems

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Problem 1

Let $p = \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$ and $p^{'} = (S \cdot R \cdot T) \cdot p \therefore$

$$\begin{aligned}
p^{'} &= \begin{bmatrix} s_x & 0 & 0 \\ 0 & s_y & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} cos\theta & -sin\theta & 0 \\ sin\theta & cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & t_x \\ 0 & 1 & t_y \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix} \\
&= \begin{bmatrix} s_x & 0 & 0 \\ 0 & s_y & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} cos\theta & -sin\theta & t_x^{'} \\ sin\theta & cos\theta & t_y^{'} \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix} \\
&= \begin{bmatrix} S^{'} & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} R^{'} & t^{'} \\ 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix} \\
&= \begin{bmatrix} S^{'}R^{'} & S^{'}t^{'} \\ 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}
\end{aligned}$$

$$Where$$

$$\begin{aligned}
S^{'} &= \begin{bmatrix} S_x & 0 \\ 0 & S_y \end{bmatrix} \\
R^{'} &= \begin{bmatrix} cos\theta & -sin\theta \\ sin\theta & cos\theta \end{bmatrix} \\
t^{'} &= \begin{bmatrix} t_x cos\theta - t_y sin\theta \\ t_x sin\theta + t_y cos\theta \end{bmatrix}
\end{aligned}$$

Thus, we can observed that the SRT matrix and the TRS matrix from the lecture are different since:

$$SRT \neq TRS$$

$$\begin{bmatrix} S^{'}R^{'} & S^{'}t^{'} \\ 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix} \neq \begin{bmatrix} R^{'}S^{'} & t \\ 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}, \quad t = \begin{bmatrix} t_x \\ t_y \end{bmatrix}$$

Problem 2

$$A = U\Sigma V^T \Rightarrow A^T = V\Sigma^T U^T$$
$$\therefore$$
$$AA^T = U\Sigma V^T V\Sigma^T U^T \Leftrightarrow$$
$$\Leftrightarrow AA^T = U\Sigma\Sigma^T U^T$$
$$\Leftrightarrow AA^T U = U\Sigma\Sigma^T U^T U$$
$$\Leftrightarrow AA^T U = U\Sigma\Sigma^T$$

Thus, we can observed that $\Sigma\Sigma^T$ must be the eigenvalue matrix of AA^T. Therefore each $\sigma^2 = \lambda(A^T A)$.

Assuming that A is a diagonal Matrix, it implies that the square root of eigenvalues are the singular values since $AA^T = \sigma_i^2 \frac{u_i}{v_i} = \sigma_i^2 \frac{u_i}{u_i}$ where σ_i are the singular value and $u_i = v_i$ for this case.

Finally, we can conclude that AA^T are the columns of U.

Problem 3

A (Total area of sensor) $= 512x512$ pixels
C (Center) $= (256, 256)$
$P(1, 2, 8) \rightarrow (356, 456)$ (In meters and pixels) $Q(-3, -1, 16)$

From the projection of P we deduce the following:
$x - axis \rightarrow 1 : 100(356 - 256 = 100)$
$y - axis \rightarrow 2 : 200(456 - 256 = 200)$

We can observe that $Q_z = 16$ is 2 time bigger than $P_z = 8$. This implies that P_x and P_y are 4 time bigger than Q_x and Q_y.

Therefore,
$x - axis \rightarrow -3 : -75 \because for P_x\ 1 : 100 \therefore Q_y\ 1 : 25 \rightarrow (256 - 75 = 181)$
$y - axis \rightarrow -1 : -25 \because for P_y\ 1 : 100 \therefore Q_y\ 1 : 25 \rightarrow (256 - 25 = 231)$

Projection of Q is $(231, 181)$