Advanced Programming - Appunti

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Introduction

1.1 19 - Settembre

1.1.1 Info and Contact

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1.1.2 Framwork

A software **framework** is a collection of common code providing generic functionality that can selectively overryidden or specialized by user code prvoiding specific functionality.

When using *framworks* htere is an **inversion of control**. Differently from what happens when using libraries, the program-flow is dictated by the framework, not the caller.

1.1.3 Design Patterns

A design pattern is a general reusable solution to a commonly occurring problem within a given context in software design. A design pattern is characterized by:

- ♦ Name
- ⋄ Problem Addressed
- ♦ Context Used to determine applicability
- ♦ Forces Constraints or issues that the solution must address
- \diamond **Solution** It must resolve all *forces*

1.2 20 - Settembre

Useful tool, to see preprocessor output, compiling, ecc.

1.2.1 Programming Languages

A PL is defined via syntax, semantics and pragmatics¹.

Syntax

Used by the compiler for *scanning* and *parsing*. The *lexical* grammar defines the syntax of token (e.g. "for" blocks, constants)

Semantics

Semantics might be described using natural language, which even if precise, allows amibguousity. Formal approches to semantics definition are:

- 1. Denotational Mapping every syntactic entity with a mathematical entity
- 2. Operational Defining a computation relation in a form $e \Rightarrow v$, where e is a program
- 3. Axiomatic Based on Hoare-triples $Precondition \land Program \Rightarrow Postcondition$

However, they rarely scale to fully-fledged programming languages.

¹the way in which the PL is intended to be used in practice

Pragmatics

Pragmatics include coding conventions, guidelines for elegant code, etc.

1.2.2 Programming Paradigms

Paradigms belong to languages pragmatics, not to the way the language is defined, i.e. not syntax nor semantics.

- 1. Imperative
- 2. Object-oriented
- 3. Concurrent Processes, communication, ...
- 4. Functional
- 5. Logic Assertions, relations, strange sorceries...

Modern PLs, provide constructs and solutions to program in all these paradigms

1.2.3 Implementing PLs

- ♦ Programs written in **L** must be *executable*
- \diamond Every language L implicitly defines and Abstract Machine M_L having L as a Machine Language
- \diamond Implementing M_L on an existing host machine M_O via compilation or interpretation (or both) makes programs written in **L** executables

An **Abstract Machine** M_L for L is a collection of data structures and algorithms which can perform the storage and execution of programs written in L.

Viceversa, M defines a language L_M including all programs which can be executed by the interpreter M.

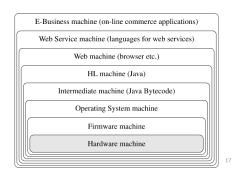
There is a bidirectional correspondance between machines and languages components.



In computer science one of the main focuses is abstraction, as can be seen in this hierarchical scheme.

Hierarchies of Abstract Machines

- Implementation of an AM with another can be iterated, leading to a hierarchy (onion skin model)
- Example:



A software component is a unit of composition with contractually specified interfaces and explicit context dependencies only. A software component can be deployed independently and is subject to composition by third parties.

Implementing PLs - Wrap Up

- $\diamond~L$ High-level programming language
- $\diamond M_L$ Abstract machine for L
- \diamond M_O host machine

Pure Interpretation

...PIC HERE

 M_L is interpreted over M_O . It isn't very efficient, mainly because of fetch-decode phases

Pure Compilation

...PIC HERE

L programs are translated into L_O , the machine laguange of M_O , hence, M_L is not realized at all and the programs are directly executed on M_O .

Compilation is more efficient than Interpretation, but produced code is larger

Both

...PIC HERE

All real languages use both interpretation and compilation,

Some languages, e.g. Java, use an intermediate Abstract Machine, called a *Virtual Machine*, which increases *Portability* and *Interoperability*.

JVM

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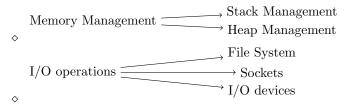
2.1 Runtime System

Every language defines and **execution model**, which is (partially) implemented by a **runtime system**, providing runtime **support** needed by both *compiled* and *interpreted* programs.

A Runtime system includes (eventually):

- ♦ Code:
 - in the executing program generated by the compiler
 - running in other threads/processes]
- ♦ Language libraries
- \diamond Operating system functionalities
- ♦ The interpreter/virtual machine itself

Runtime support can be needed for various reasons:



- ♦ Intercation with runtime environment
- ♦ Parallel execution (threads/processes)
- ♦ Dynamic binding type checking
- Dynamic loading and linking of modules
- Debugging
- ♦ ¿Code Generation?
- ♦ ¿Verification and Monitoring?

2.1.1 JRE

The Java Runtime Environment includes JVM and JCL (Java Class Library).

2.2 JVM

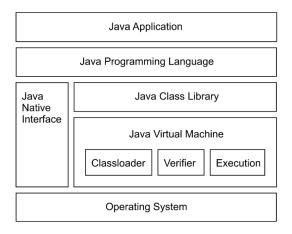
JVM is an abstract machine, defined by the documentation, which omits details on stuff like memory layout of runtime data area, garbage-collection, internal optimization, and even the representation of the null constant. The JVM specification, instead, defines precisely a machine indipendent "class file format" that all JVM implementations must support; it also imposes strong synctatic and structural constraints on the code in a class file.

The \mathbf{JVM} is not register-based, instead it is a multi-threaded \mathbf{stack}^1 based machine. Id est the \mathbf{JVM} pops intructions

¹Not to be confused with the stack of activation records!

from the top of **operand stack** of the current frame, and pushes their result on the top of the **operand stack**. The **operand stack** is used to:

- Pass arguments to functions
- ♦ Return results from a function
- ♦ Store intermediate values while evaluating expressions
- ♦ Store local variables



2.2.1 Data types

.class file are platform independent external representations, which are represented internally by the JVM using simpler data types, which are implementation dependent.

⋄ Primitive types

- Numerica integral
- Numeric floating point
- boolean (support only for arrays)
- internal (for exception handling)

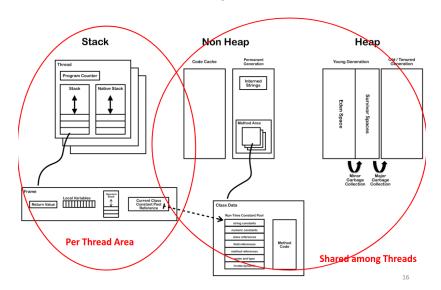
⋄ Reference types

- Class types
- Array types
- Interface types

No type information on local variables at runtime, there are only operand types specified by opcodes e.g. iadd, fadd,

2.2.2 Threads

There are some runtime areas of the JVM related to a single thread, while others are shared among threads



All Java Programs are multithreaded, since there is at least a main thread running the user's program, and many

daemons:

- ♦ Garbage collector
- ♦ Signal Dispatching
- Compilation
- ♦ ¿ ... ?

JVM doesn't poses strong implementation constraints, by defining a precise abstract consistency model, including volatiles, allowing non-atomic longs and doubles, distinguishing working-memory and general store.

2.2.3 per-thread Data Areas

- pc pointer to next instruction in method area undefined if current method is native
- ♦ **Java stack**: stack of *frames* (or *activation records*)
- ♦ Native stack: used for invocation of natve functions through the Java Native Interface (JNI)

Considering the **structure** of **frames**, each one is composed by:

- ♦ Local Variable Array (32 bits) containing:
 - 1. Reference to this
 - 2. Method parameters
 - 3. Local variables
- ♦ Operand stack
- ♦ Reference to Constant Pool of current class

Differently from C/C++, where the **linking** phase is done before running an executable, java computes linking **dynamically** at **runtime**; this is achieved using **symbolic** references, which can be resolved using *static* (eager) or *late* (lazy) resolution.

Since the execution of a Java program must **not** depend on the JVM implementation, the JVM always behaves as if the implementation implies *lazy* resolution, even if the actual implementation provides static resolution instead.

2.2.4 shared data areas

Heap

- Memory for objects and arrays
- ♦ No explicit deallocation, it is demanded to the garbage collection.

Non-Heap

Memory for objects never deallocated

- ♦ Method area
- ♦ Interned strings
- ♦ Code cache for JIT

Just In Time (JIT) compilation refers to profiling as "hot" code areas of bytecode which may be executed many times, and storing the compiled native code in a cache in the Non-heap memory.

Method-area

Here class files are loaded. For each class a classloader reference and the following info from the class file are stored:

- ♦ Runtime Constant Pool
- ♦ Field data
- ♦ Method data
- ♦ Method code

Method area is shared among threads! Access to it must be thread safe.

This should a **permanent** area of the memory, but it may be **edited** when a new class is loaded or when a symbolic link is resolved by dynamic linking.

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Constant Pool

Contains constants and symbolic references for dynamic binding. It is possible to see the constant pool of a compiled .class file using the command:

```
| javap -v name.class
```

Displaying something resembling to:

```
#1 = Methodref
                          #6.#14
                                          // java/lang/Object."<init>":() V
#2 = Fieldref
                          #15.#16
                                          // java/lang/System.out:Ljava/io/PrintStream;
#3 = String
                          #17
                                          // Hello World
                                          // java/io/PrintStream.println:(Ljava/lang/
#4 = Methodref
                          #18.#19
    String;) V
#5 = Class
                          #20
                                          // com/baeldung/jvm/ConstantPool
#6 = Class
                          #21
                                          // java/lang/Object
#7 = Utf8
                          <init>
#8 = Utf8
                          () V
#9 = Utf8
                          Code
#10 = Utf8
                          LineNumberTable
#11 = Utf8
                          sayHello
#12 = Utf8
                          SourceFile
```

2.2.5 Loading

Loading is finding the binary representation of a class or interface type with a given name and creating a class or interface from it.

Class (or Interface) C creation is triggered by other classes **referencing** C or by methods (e.g. reflection). If not previously loaded, loader.loadClass is invoked.

There are 4 Classloaders:

- 1. Bootstrap CL: loads basic Java APIs
- 2. Extension CL: loads classes from standard Java extension APIs
- 3. System CL: loads application classes from *classpath* (default application CL)
- 4. User Defined CLs: can be used for:
 - runtime classes reloading
 - ♦ loading network, encrypted files or on-the-fly generated classes
 - supporting separation between different groups of loaded classes as required by web servers

Runtime Constant Pool

2.2.6 Linking

Linking includes verification, preparation, resolution.

- 1. **Verification** multiple checks at runtime, e.g. operand stack under/overflows, validity of variable uses and stores, validity arguments type. Details later on
- 2. **Preparation** Allocation of storage
- 3. Resolution² resolve symbol references by loading referred classes/interfaces

Verification is a relevant part of JVM Specification, it is described in 170pp over a total of 600pp. When a class file is loaded there is a *first* verification pass to check formatting, there is a *second* one when a class file is linked regarding only not instruction-dependant checks. During the linking phase there is a data-flow analysis on each method (*third check*), and lastly (*fourth check*) when a method is invoked for the first time.

2.2.7 Initialization

<clinit> initialization method is invoked on classes and interfaces to initialize class variables; it also executes static
initializers. <init> initialization method instead is used for instances.

 $^{^2}$ Optional, it may be postponed till first use by an instruction

JVM Instr Set & JIT

3.1 Instruction Set

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Let's consider the instructions **format**. Each instr may have different "forms" supporting different kinds of operands. For example there are different forms of iload (i.e. push).

Runtime memory contains - Local variable array (frame) - Operand stack (frame) - Object fields (heap) - Static fields (method area)

Note that Java instructions are explicitly typed through opCodes, e.g. dload,iload,fload.

opCodes are bytes, allowing only for 256 distinct ones; hence it is impossible to have for each instruction on opCode per type. The JVM specification indicates a selection of which types to support for each op instruction, and not supported types have to be converted; resulting in the Instruction Set Architecture to present non-orthogonality. Types like byte, char and short are usually converted to int when performing computations.

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3.1.1 Invoking methods

invokevirtual causes the allocation of a new frame, pops the arguments from the stack into the local variables of the caller (putting this in 0), and passes the control to it by changing the pc.

- ♦ A resolution of the symbolic link is performed
- ireturn pushes the top of the current stack to the stack of the caller, and passes the control to it. Similarly for dreturn, freturn ...
- ⋄ return just passes the control to the caller

There are 4 others kinds of method invocation:

- ♦ invokestatic: call methods with static modifier; this is not passed
- ♦ invokespecial: call constructors, private methods or *superclass* methods; *this* is always passed
- invokeinterface: identical to invokevirtual, but used when the method is declared in an interface, thus a
 different lookup is required
- ⋄ invokedynamic: introduced in Java 7 to support dynamic typing¹

 (\ldots)

3.2 JIT

AOT Ahead of Time Compilation leads to better performance in general, exploiting hardware features and variables allocation without runtime lookup; While Interpretation facilitates interactive debugging and testing: it allows

¹lambda functions related?

command-line invocation.

JIT aims to get the advantages of both.

JIT differs from AOT since it runs in the same process of the application and competes with the app for resources, thus compilation time for JIT is more relevant than for an AOT Compiler. Besides, a JIT compiler doesn't verify classes at compile time, it is a task performed by the JVM at load time. JIT can exploit new optimization possibilities, e.g. deoptimization and speculation. A JIT takes bytecode as input and outputs machine code that the CPU executes directly.

Wrapping up:

- ♦ Code starts executing interpreted with no delay
- ♦ Methods that are found commonly executed (hot) are JIT compiled
- ♦ Once compiled code is available, the execution switches to it.

To identify hot methods, there is a **threshold** on two per-method counters:

- 1. Times the method is invoked
- 2. Times a brach back to the start of a loop is taken in the method

A tradeoff between "fast-to-start-but-slow-to-execute" interpreter vs "slow-to- start-but-fast-to-execute" compiled code is managed by a multi tier system.

(...)

3.2.1 Deoptimization and Speculation

Usually method executions pass in three phases:

Interpreter → Low tier compiler ← Optimizing compiter

But sometimes deoptimization can happen, i.e.:

Interpreter \longrightarrow Low tier compiler \longrightarrow Optimizing compiter

Component-Based software

2 - Ottobre

Component software indicates **composite systems** made of **software components**. In short, component software allows reuse, improving reliability¹ and reducing costs.

Bertrand Meyer suggests some guidelines regarding Object-Oriented software construction (1997):

- 1. modular
- 2. reliable
- 3. efficient
- 4. portable
- 5. timely

4.1 Definitions

A software component is a unit of composition with contractually specified interfaces and explicit context dependencies only. A software component can be deployed independently and is subject to composition by third parties.

A contract, A specification attached to an interface (component specification) that mutually binds the clients and providers of the components.

Context dependencies are specification of the deployment environment and run-time environment. This goes beyond the simple interfaces required and provided which are specified in the contract. Context dependencies include required tools, platform and resources.

Deployed independently means that a component can be plugged or unplugged from an architecture, even at runtime in some cases. It is common-practice to deploy "small components" called connectors, to resolve situations where two components supposed to interact do not provide identical interfaces, creating the need for a intermediary.

composition by third party means that a component may interact with third parties components without knowing the internals of such components.

4.2 Concepts of Component Model

- ♦ Component interface describes the operations implemented and exposed by the component
- ♦ Composition mechanism How components can be composed to work together to accomplish a task
- ♦ Component platform A platfom for the development of the components

Concepts should be laguage/paradigm agnostic, laying the ground for language interoperability.

The ancestors of Components are **Modules**, whose support has been introduced in Java 9, but isn't very common. Some concepts related to modules can be found in more modern notions such as classes, components and packages. For example, objects inside a module are visible to each other, but not visible from outside unless exported. Modules

 $^{^{1}}$ Industries may even require to use certified components

worked — pretty much like classes — as abstraction mechanism \rightarrow collection of data with operations defined on them. In OOP the concept of **inheritance**, unknown in modules, is introduced.

4.3 Components and Programming Concepts

Components can be anything and can contain anything, they can be *collections of* classes, objects, functions/algorithms, data structures.

¡Note that
$$OOP \neq COP!$$

OOP isn't focused on reuse, instead its focus in onto appropriate domain and problem representation.

- ♦ **Component Specification** describes the behavior (as a set of *Interfaces*) of a set of Component Objects and defines a unit of implementation.
- \diamond Component Implementation is a realization of Component Specification which can be independently deployed².
- ♦ **Installed Component** is an installed (i.e. *deployed*) copy of a *Component Implementation*. A Component Implementation is deployed by registering it with the runtime environment
- ♦ Component Object is an instance of a Component Implementation. It is a runtime concept, an object with its own data and unique identity. Ideally, it is the "thing that performs the implemented behaviour". An Installed Component may have multiple Component Objects

Some examples of successful components are Plugin architectures, Microsoft's Visual Basic, Operating Systems, Java Beans, and others. It is clear that components can be purchased by independent providers and deployed by the clients, and that multiple components can coexist in the same installation. Besides, components exist on a level on abstraction where they directly mean something to the deploying client.

Recalling the comparison with modules, while modules are usually seen as part of a program, components are parts of a system.

 $^{^2}$ It does **not** mean that it cannot have dependencies nor that it must be a single file

Java Beans

5.1 3 - Ottobre

"A Java Bean is a reusable software component that can be manipulated visually in a builder tool."

Typically a Bean has a GUI representation but is not necessary. What is necessary instead for a class to be recognized as a bean is that it:

- has a public constructor with no arguments
- implements java.io.Serializable
- ♦ is in a jar file with a manifest file that contains: Java-Bean: True

Beans can be **assembled** to build a new bean or application, writing clue code to wire beans together. Connection-oriented programming is based on the **Observer** or (Publish-Subscribe) paradigm. Observers come into play when there is a 1:N dependency between objects and one of them changes state, creating the need for the others to be notified and updated. Beans must be able to run in a design environment allowing the user to customize aspect and behaviour. Beans provide support for some standard features:

- 1. **Properties** e.g. color. **Bounded** properties generate an *event* of type PropertyChangeEvent, while **constrained** can only change value if none of the registered *observers* "poses a veto", by raising an *exception* when they receive the PropertyChangeEvent object.
- 2. **Events**: The **Observer** pattern is based on *Events* and *Events listeners*. An *event* is an object created by an *event source* and propagated to the registered *event listeners*. Sometimes event **adaptors** can be placed between source and listener, which might implement queuing mechanism, filter events, demuxing from many sources to a single listener.
 - ⋄ Design Patterns for Events

```
public void add<EventListType>(<EventListType> a)
public void remove<EventListType>(<EventListType> a)
```

- 3. Customization
- 4. Persistence
- 5. **Introspection**: process of analyzing a bean to determine capabilities. There are implicit methods based on reflection, naming conventions and design patterns, but can be simplified by explicitly defining info for the builder tool in the <BeanName>BeanInfo class. Such class allows exposition of features, specifying customizer class, segregate feats in normal/expert mode, and some other stuff.
 - Design Patterns for Simple Methods

```
public <PropertyType > get <PropertyName > ();
public void set <PropertyName > (<PropertyType > a);
```

Design Patterns for Simple Methods

```
public java.awt.Color getSpectrum (int index);
public java.awt.Color[] getSpectrum ();
public void setSpectrum (int index, java.awt.Color color);
public void setSpectrum (java.awt.Color[] colors);
```

Reflection

9 - Ottobre

6.1 Introduction and Definitions

Reflection is the ability of a program to manipulate as data something representing the state of the program during its own execution. Another dimension of reflection is if a program is allowed to **read only**, or also to **change** itself.

- ♦ **Introspection** is the ability of a program to observe and therefore reason about its own state
- ♦ **Intercession** is the ability for a program to modify its own execution state or alter its own interpretation or meaning
- Reification is the mechanism of encoding execution state into data, which is needed by both introspection and intercession

Structural reflection is concerned with the ability of the **language** to provide a complete *reification* of both the *program* executed and its *abstract data types*.

Behavioral reflection is concerned instead with the reification of its 1 semantics & implementation (processor) and the data and implementation of the run-time system.

6.2 Uses and drawbacks

6.2.1 Uses

♦ Class Browsers need to be able to enumerate the number of classes

- ♦ Visual Development Environments can exploit type info available in reflection to aid the developer in writing correct code
- ♦ Debuggers need to be able to examine private members on classes
- ♦ Test Tools exploit reflection to ensure a high level of code coverage in a test suite
- Extensibility Features an app may make use of external, user-defined classes by creating instances of extensibility objects.

6.2.2 Drawbacks

- ♦ Performance Overhead
- ♦ Security Restrictions
- ⋄ Exposure of internals

6.3 Reflection in Java

Java supports **introspection** and **reflexive invocation**, but not *code modification*.

¹referred to a language

6.3.1 Introspection

The JVM mantains for every type an associated object of type java.lang.Class which "reflects" the type it represents, acting as entry point for reflection, since it provides all info needed:

- ♦ Class name and modifiers
- \diamond Extended superclasses and implemented inferfaces
- ♦ Methods, fields, constructors, etc.

To retrieve such java.lang.Class object it is sufficient to do Object.getClass(). Class objects are constructed automatically by the JVM as classes are loaded.

Using java.util.reflect.* it is possible also to retrieve class **Members** i.e. *fields, constructors* and *methods*. The extensive java.util.reflect.* API provides many *methods* to achieve this which will not be reported here. There is a class for each Member

- ♦ java.util.reflect.Field: access type info and set/get values.
- ♦ java.util.reflect.Method: type info for parameters and return type; invoking method on a given object.
- java.util.reflect.Constructor: note that constructors have no return values and invocation creates a new
 instance of the given class.

6.3.2 Program Manipulation

By now we have talked only about **introspection** in java, but reflection can be used also to create objects of a type not known at compile time, or to access members (access fields or invoke methods) unknown at compile time.

Annotations

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In java, static, private,... modifiers are *meta-data* describing properties of program elements. Annotations can be understood as (user-) definable modifiers. They are composed by one or two parts:

- 1. name
- 2. finite number of attributes i.e. name=value. There may be no attributes.

The syntax is the following:

constExp are expression which can be evaluated at *compile time*. Besides, attributes have a *type*, thus the supplied values have to convertible to that type.

Annotations can be applied to almost any syntactic element, from packages to parameters and any type use.

7.1 Defining annotations

```
@interface InfoCode {
    String author ();
    String date ();
    int ver () default 1;
    int rev () default 0;
    String [] changes () default {};
}
```

This defines the custom annotation InfoCode, imposing some fields possibly with default values. It can be used as follows:

```
@InfoCode(author="Beppe", date="10/12/07")
   public class C {
   public static void m1() { /* ... */ }
   @InfoCode(author="Gianni",
        date="4/8/08", ver=1, rev=2)
   public static void m2() { /* ... */ }
}
```

Polymorphism

Polymorphism basically means "many forms", where forms are **types**. Thus there may be polymorphic function names, or polymorphic types.

There are many "flavors" of polymorphism, many variations. Two main kinds opposed to each other are *ad hoc* and *universal* polymorphism, which however, may coexist:

- ♦ ad hoc PM indicates that a single function name denotes different algorithms, determined by the actual types.
- ♦ universal PM indicates a single algorithm (solution) applicable to objects of different types.

When PM is taken into account, it is crucial to consider when happens the **binding** between a function name and the actual code to be executed:

- ⋄ compile time; static/early binding
- linking time
- execution time; late/dynamic binding

In general the earlier the binding happens, the better (for debugging reasons). If the binding spans over more phases (e.g. overriding in Java), as a convention we consider the **binding time** the last phase.

8.1 Classification

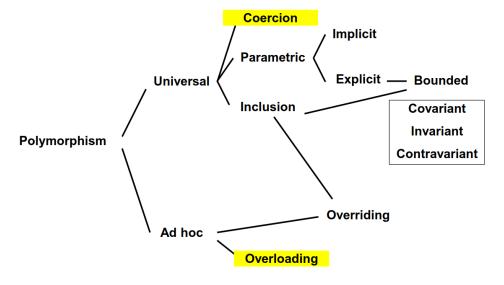


Figure 8.1: Polymorphism classification

8.1.1 Overloading

Overloading is present in every language for basic operators + - *..., and sometimes is supported for user-defined functions, and in some languages it is even allowed the overloading of primitive operator by user-defined functions.

Since this falls under the **ad hoc** polymorphism family, the code to be executed is determined by the type of the arguments; the binding can either happen at *compile* or at *runtime*, depending on the typing of the language, whether it is static or dynamic.

```
// C language doesn't allow overloading for user-defined functions
int sqrInt(int x) { return x * x; }
double sqrDouble(double x) { return x * x; }

// Overloading in Java & C++
int sqr(int x) { return x * x; }
double sqr(double x) { return x * x; }
```

Haskell introduces type classes for handling overloading in presence of type inference

8.2 Coercion

Coercion is the automatic (implicit) conversion of an object to a different type, opposed to casting which is explicit instead. Coercion allows a code snippet to be applied of arguments of different (convertible) types. Sometimes coercion is allowed only if there is no **information loss**.

```
double sqrt(double x){...}
double d = sqrt(5) // applied to int
```

8.3 Inclusion Polymorphism

Inclusion polymorphism is also known as *subtyping polymorphism* or *inheritance*. It is ensured by *Barbara Liskov*'s **substitution principle**:

A subtype object can be used in any context where a supertype object is expected

Methods and fields defined in a superclass may be invoked and accessed by subclasses if not redefined (see *Overriding*).

8.4 Overriding

In Java a method m of a class A can be redefined in a subclass B of A.

Overriding introduces ad hoc polymorphism in the universal polymorphism of inheritance. Notice that overriding requires the final binding to happen at runtime: it happens through the lookup done by invokevirtual in the JVM.

8.5 C++ v Java

```
class A {
  public:
    virtual void onFoo() {}
    virtual void onFoo(int i) {}
};
class B : public A {
  public:
    virtual void onFoo(int i) {}
};
class C : public B {};
int main() {
    C* c = new C();
    c->onFoo();
    // Compile error - doesn't exist
}
```

The equivalent code in Java compiles, because in java invokes the function onFoo() with no arguments defined in the superclass A. In C++ instead, the function onFoo(int i) defined in B is found and stops the search, but there is arguments type mismatch, thus it doesn't compile. This happens because in C++ the method lookup is based on the method name, not on its signature.

8.6 C++ Templates

They are similar to *Generics* in Java, they are used as function and class templates each concrete instantiation produces a copy of the generic code, specialized for that type: monomorphization. In java Generics, instead, **type erasure** happens at runtime, i.e. type variables T are replaced by **Object** variables.

Templates support parametric polymorphism and type parameters can also be primitive types (unlike Java generics)

```
template <class T> // or <typename T>
T sqr(T x) { return x * x; }
```

Assuming to invoke sqr(T x) on variables of different types, the compiler will generated a specific code for each type used. This works even on user-defined types; check the following code for an example:

It is important to check for type ambiguosity; in the following example, it is highlighted a case where it's not clear whether it is i to be converted to long or m to i.

8.6.1 Macros

Macros can be exploited to achieve *polymorphism* and can have the same effect of the templates, but notice that macros are executed by the preprocessor¹ and are only **textual substition**, there is no parsing, no static analysis checks or whatsoever.

```
#define sqr(x) ((x) * (x))
int a = 2;
int aa = sqr(a++); // int aa = ((a++) * (a++));
// value of aa? aa contains 6 :(
#define fact(n) (n == 0) ? 1 : fact(n-1) * n
// compilation fails because fact is not defined
```

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8.6.2 Specialization

A template can be **specialized** by defining a template with the same name but with more specific parameters (*partial specialization*) or with no parameters (*full specialization*). This is kinda similar to *Overriding*, leaving to the compiler the choice of the most appropriate template.

 $^{^1\}mathrm{Macro}$ expansion can be seen using the option -E when compiling

Templates can be used by a compiler to generate temporary source code, which is merged by the compiler with the rest of the source code and then compiled.

Template compilation happens on demand: the code of a template is not compiled until an instantiation is required, however in case of fully-specialized template, the compiler treats the template as a function, thus it generates its code **regardless** whether it is ever used or not.

Note that in C/C++ while method *prototypes* usually are in a separate .h file, the compiler needs the template declaration and definition in the same place to instatiate it.

Generics

Generics are instance of *Universal Polymorphism* with explicit parameters (see Fig 8.1).

9.1 Methods

```
public static <T> T getFirst(List<T> list)
```

Invocations of generic methods must instantiate all type parameters, either explicitly or implicitly. Some sort of type inference is applied in case of implicit instantiation.

```
class NumList < E extends Number > {
    void m(E arg) {
        arg.intValue(); // OK, since...
        // Number and its subtypes support intValue()
    }
}
```

Type parameters can also be **bounded** as in the above example, allowing methods (and fields) defined in the **bound** to be invoked on objects of the type parameter T.

There may be various kinds of type bounds:

```
<TypeVar extends SuperType>
   // UPPER bound; SuperType and any of its subtype are ok.
<TypeVar extends ClassA & InterfaceB & InterfaceC & ...>
   // MULTIPLE UPPER bounds
<TypeVar super SubType>
   // LOWER bound; SubType and any of its supertype are ok
```

Unlike C++ where *overloading* is resolved and can **fail** after instantiating a template, in Java **type checking** ensures that overloading will succeed.

9.2 Inheritance and Arrays

There are two major issues which came up along with generics. The first one regards **inheritance**; consider the following example:

Since Integer is a *subtype* of Number, is List<Integer> *subtype* of List<Number>?

NO!

In a formal way, *subtyping is invariant* for Generic classes. Informally, given A,B concrete types, MyClass<A> has no relationship to MyClass, even if A,B have one.

On the other hand if A extends B and are *generic* classes, then A<C> extends B<C> for any type C. For example, ArrayList<Integer> extends List<Integer>.

Note that the common parent of MyClass and MyClass<A> is MyClass<?>.

Let's now discuss **covariance** and **contravariance**, with the aid of a few examples.

```
List < Integer > lisInt = new ...;
List < Number > lisNum = new ...;
lisNum = lisInt; // ??? - Reassign pointer
lisNum.add(new Number(...)); // NOT ALLOWED
listInt = lisNum; // ??? - Reassign pointer
Integer n = lisInt.get(0); // NOT ALLOWED
```

List<Integer> is neither a subtype or a supertype of List<Number>, thus the above operations aren't allowed. However there are read-only and write-only situations where they may be allowed.

```
RO_List < Integer > lisInt = new ...;
RO_List < Number > lisNum = new ...;
lisNum = lisInt; // ???
Number n = lisNum.get(0); // OK
```

It is ok to read a supertype starting from a subtype.

covariance is safe if the type is read-only

```
WO_List<Integer> lisInt = new ...;
WO_List<Number> lisNum = new ...;
lisInt = lisNum; // ???
lisInt.add(new Integer(...)); // OK
```

It is ok to write a **subtype** in the place of from a **supertype**.

contravariance is safe if the type is write-only

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Other languages

In the case of C#, generic classes can be marked with the keyword out (covariant) or in (contravariant), otherwise the class is invariant. In **Scala** the same happens, but with the + or - operators.

Let's now discuss **arrays**.

Let A extends B, then A[] extends B[] even if instead Array<A> is not related to Array.

Thus, arrays in Java are covariant.

However there is a counterpart, since this allows rule-breaking assignments which are allowed by the compiler but which lead to a runtime ArrayStoreException. This happens because the dynamic type of an array is checked at runtime. Knowing this, for each array update, a runtime check is performed by the JVM which throws the exception if needed.

After compilation Generic are all **type-erasured** to Object or to their first *bound*, if present. This choice has been made mainly for compatibility with legacy code, leading all instances of the same generic type to have the same type at runtime; i.e.

```
List < String > lst1 = new ArrayList < String > ();
List < Integer > lst2 = new ArrayList < Integer > ();
assert (lst1.getClass() == lst2.getClass())
```

9.2.1 Generic Arrays

What about arrays of generics? Such arrays in Java are **not allowed**, because every array update needs a runtime check which is impossible to perform on generics, since at runtime generics are all of the same type due to type-erasure.

Wildcards 9.3

Wildcards are strongly related to the topic of covariance and contravariance.

As briefly mentioned before, wildcards are the only relationship between generic classes.

To use wildcards, the PECS principle is applied: Producer Extends, Consumer Super.

- ♦ ? extends T to get values from a *Producer*: covariance allowed
- ♦ ? super T to insert values into a Consumer: contravariance allowed
- ♦ Never use ? when both insertion and retrieving is needed, T is sufficient and way more appropriate.

Wildcards improve type-safety, allowing a program to fail at *compile-time* instead of *runtime*.

```
List<Apple> apples = new ArrayList<Apple>();
List<? extends Fruit> fruits = apples;
fruits.add(new Strawberry()); // COMPILING FAILS
```

Generics Limitations 9.4

♦ Cannot instantiate Generics with primitive types:

```
ArrayList<int> a = ...
                                                            // compile error
♦ Cannot create instances of type parameters
```

♦ Cannot declare static fields whose types are type parameters

```
public class C<T>{ public static T local; ...}
```

Because static fields are represented in the unique representation of the class in the dedicated static memory area of the JVM for classes

Cannot use casts or instanceof with parameterized types

```
mylist instanceof ArrayList < Integer >
                                                // fails
                                                // OK
mylist instanceof ArrayList <?>
```

- ♦ Cannot Create arrays of parameterized types
- ♦ Cannot create, catch, or throw objects of parameterized types
- ♦ Cannot overload a method where the formal parameter types of each overload erase to the same raw type.

```
public class Example {
                                              // does not compile
   public void print(Set<String> strSet) { }
   public void print(Set<Integer> intSet) { } }
```

Standard Templates Library

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The goal of STL is to represent algorithms in as general form as possible without compromising efficiency. There is an extensive use of **templates**, **overloading** and **iterators**, which are used for decoupling algorithms from containers, and can be seen as an abstraction of pointers.

STL is very different from the Java Collection Library since it does **not** use dynamic binding and is **not** object oriented; instead the STL uses only static binding and inlining.

10.1 Main Entities

- ♦ Container collection of typed objects
- ♦ Iterator Generalization of pointer or address; used to step through the elements of collections
- ♦ Algorithm initializing, sorting, searching, and transforming contents of containers
- ♦ Adaptor Convert from one form to another e.g. iterator from updatable container; or stack from list
- ♦ Function Object Form of closure (class with "operator()" defined)
- ♦ **Allocator** encapsulation of a memory pool

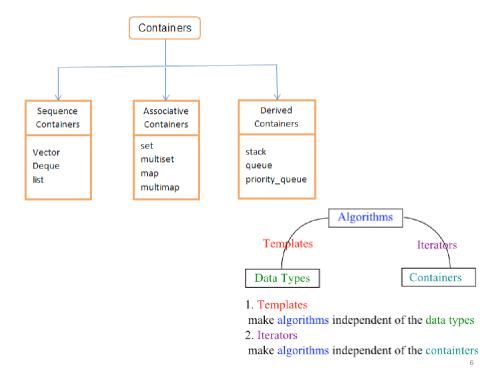


Figure 10.1: TIKZ RE-DO: STL Containers

10.2 Iterators

Since algorithms cannot be used *directly* on different kinds of collections, Iterators come in handy by providing a **uniform**, **linear** access to elements of different collections.

In **Java** iterators are supported by the JCF^1 through the interface Interface<T>. They are related to an **instance** of a class and are usually defined as *nested classes*, more precisely *non-static private member classes*. Collections equipped with iterators must **implements** Iterable<T> interface.

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In C++ there is no next/hasNext() function, standard ++ -- operators are used instead. In case of arrays pointers can be trivially used, since int v[] is no different int *v². In the case of vector instead, an actual iterator may be instantiated, but the operator ++ stays the same.

```
vector < int > vec;
vector < int > :: iterator v = vec.begin();
while( v != vec.end()) {
    cout << "value of v = " << *v << endl;
    v++;
}</pre>
```

Every class in C++ has its own iterator; more specifically, containers define and expose a type named iterator in the container's **namespace**, allowing the semantic value of **iterator** to change according to the context.

10.2.1 C++ iterators implementation

Typically iterators are implemented as **struct** and provide a visit of the container, retaining information about the **state** of the visit, e.g. pointer to next element, remaining elements, and so on. Note that in case of *trees* or *graphs* the visit's state may not be trivial to be represented.

```
template <class T>
 struct v_iterator {
            T*v;
            int sz;
            v_{int} = v_{int} + v_{i
             // != implicitly defined
            bool operator == (v_iterator& p) { return v == p->v; }
            T operator*() { return *v; }
             v_iterator& operator++() { // Pre-increment
                         if (sz)
                                    ++v, --sz;
                         else
                                    v = NULL;
                         return *this;
            }
             v_iterator operator++(int) { // Post-increment!
                         v_iterator ret = *this;
                                                                                          // call pre-increment
                         ++(*this);
                        return ret;
            }
};
```

	insert/erase				
Container	beginning	middle	end		
vector	linear	linear	amortized constant		
list	constant	constant	constant		
deque	amortized constant	linear	amortized constant		

Table 10.1: Guaranteed time complexity for iterators

To achieve transparency to third-party algorithms STL assumes *constant* time for every operation, and allows 5 types of operators:

 $^{^1} Java\ Collection\ Framework$

²At least from an "accessing values" point of view, there are some differences in terms of static/dynamic allocation of memory.

- ♦ Formard iterators only dereference and pre/post increment
- ♦ Input (and Output) iterators same as formard iterators but with possible issues when dereferencing
- ♦ Bidirectional iterators dereference, pre/post increment and decrement
- \diamond Random access iterators same as Bidirectional but allow also integer sum (p+n) and difference (p-q)

Each category defines only the functions which take constant time. Not all iterators are defined for all containers, e.g. since random access takes linear time on lists, there is no random access iterator on lists.

10.2.2 Invalidation

When a container is *modified*, iterators *may* become **invalid**: no "exception" is thrown, iterators can still be used, but their behaviour is **undefined**. *Not* every operation invalidates iterators, it depends on the operation and on the container.

The main limiting aspect of STL's iterators is that they provide a **linear view** of the container, allowing the definition of operations only on one-dimensional containers; thus, if it is needed to access the organization of the container (e.g. tree custom visist), the only way-to-go is to define a custom iterator which behaves as desired.

10.3 C++ specific features

10.3.1 Inheritance

STL relies on typedefs combined with namespaces to implement genericity, the programmer always refers to container ::iterator to know the type of the iterator. Note that there is no relation among iterators for different containers (!), if not a semantically abstract one. The reason for this is **performance**: without *inheritance*, types are resolved at compile time and the compiler may produce better and optimized code. On the other hand sacrificing inheritance may lead to lower expressivity and lack of type-checking; in fact, STL relies only on coding conventions: when the programmer uses a wrong iterator the compiler complains of a bug in the library.

10.3.2 Inlining

C++ standard has the notion of **inlining** which is a form of semantic macros. Inline methods should be available in header files and can be labelled *inline* or defined within class definition, invocation on such methods is type-checked and then it is replaced by the method body. The compiler tends to (automatically?) inline methods with small bodies and without iteration; it is able to determine types at compile time and usually does inlining of function objects.

10.3.3 Memory management

STL abstract from the specific memory model using a concept named **allocators**. All the information about the memory model is encapsulated in the **Allocator** class. Each container is parametrized by such an allocator to let the implementation be unchanged when switching memory models.

10.3.4 Potential Problems

The problem may be error checking: almost all facilities of the compiler fail with STL resulting in lengthy error messages that ends with error within the library

Functional Programming

Functional Programming languages radicate their roots in the Church's model of computing known as lambda calculus. Such model is based on the notion Λ – parametrized expressions, with the focus on defining mathematical functions in a constructive and effective way. The computation proceeds by substituting parameters into expressions.

Functional programming languages such as *Lisp*, *Scheme*, *FP*, *ML*, *Miranda* and *Haskell* aim to implement Church's lambda calculus in the form of a programming language which does everything needed by **composing functions**, thus no *mutable state* and no *side effects*.

 ${\rm FPL^1}$ needs some key features which are often absent in *imperative* languages:

- \diamond 1st-class order and **high-order** functions: Functions can be *denoted*, passed as *arguments* to other functions, returned as result of function invocation
- ♦ Recursion opposed to "control variables"
- ♦ Powerful list facilities: Recursive functions exploit recursive definition of lists
- Polymorphism typically universal parametric implicit, which plays a key role when handling containers/collections.
- ♦ Fully general aggregates: there is a wide use of tuples and records, besides, data structures cannot be modified (no state!), they have to be re-created.
- Structured function returns allow to pass more meaningful information to the caller, avoiding the need for "side-effects".
- ♦ Gargabe collection

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11.1 FP language families

- 1. **LISP**: currently most used for AI after Python. Original LISP is no longer used, the current standard is $Common\ LISP$ which introduced statical scope opposed to the dynamic one of $Original\ LISP$; another version is called Scheme
- 2. ML: Common languages of this family are *Standard ML*, *Caml*, *OCaml*, *F*#. These are compiled languages, but intended for interactive use. ML results from the combination of Lisp and Algol-like features, including Garbage collection, Abstract data types, Module system and Exceptions
- 3. Haskell: Many features are shared with ML languages, but with some differences.
 - Type inference, Implicit parametric polymorphism, Ad hoc polymorphism (overloading) with type classes
 - ♦ Lazy evaluation, Tail recursion and continuations
 - \diamond **Purely functional** \rightarrow precise management of side effects

 $^{^1}$ Short for $Functional\ Programming\ Languages$

11.2 Haskell basics

```
♦ Unit
♦ Booleans
                                                           o Patterns
♦ Integers
                                                           o Declarations
♦ Strings
                                                     Other types
                                                          o Functions
♦ Reals
                                                          • Polymorphism
♦ Tuples
                                                          • Type declarations
♦ Lists
                                                           o Type Classes
⋄ Records
                                                           • Monads

    Exceptions
```

Note that basic types are written with the first letter Uppercased.

Haskell provides an interactive read-eval-print interpreter (ghci): many examples are available in the lecture's slides, here we will discuss only some more interesting ones.

Variables (names) are bound to expressions, without evaluating them (because of lazy evaluation); the scope of the binding is the rest of the session.

Moving onto anonymous functions i.e. $\x \rightarrow \dots$ lambda notation

To declare explicit functions instead, the syntax is quite simple

```
f (x,y) = x+y --argument must match pattern (x,y)

reverse xs = -- linear, tail recursive
   let rev ([], accum ) = accum
   rev ( y:ys, accum ) = rev ( ys, y:accum )
   in rev ( xs, [] )
```

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11.3 More on Haskell features

Let's recall that Haskell is a lazy language, thus functions and data constructor don't evaluate arguments until they actually need them.

```
myData = [1,2,3,4,5,6,7]

twiceData = [2 * x | x <- myData]

-- [2,4,6,8,10,12,14]
```

Datatype declearations

```
data Color = Red | Yellow | Blue
data Atom = Atom String | Number
data List = Nil | Cons (Atom, List)

-- General form:
data <name> = <clause> | ... | <clause>
<clause> ::= <constructor> | <contructor> <type>

-- also possible to define Recursive data types
data Tree = Leaf Int | Node (Int, Tree, Tree)

Node(4, Node(3, Leaf 1, Leaf 2), Node(5, Leaf 6, Leaf 7))

-- it is possible to use constructors in pattern matching
sum (Leaf n) = n
sum (Node(n,t1,t2)) = n + sum(t1) + sum(t2)
```

Besides it is possible to match different cases with a specific case statement; note that Indendation in case statement MATTERS

```
data Exp = Var Int | Const Int | Plus (Exp, Exp)

case e of
   Var n -> ...
   Const n -> ...
   Plus(e1,e2) -> ...

-- Indendation in case statement MATTERS
```

11.3.1 Function Types

 $f :: A \rightarrow B \text{ means that:}$

$$\forall x \in Af(x) = \begin{cases} \exists y = f(x) \in B \\ run \ for ever \end{cases}$$

In other words, if f(x) terminates, then $f(x) \in B$. In ML, functions with type A \to B can throw an exception or have other effects, but **not** in Haskell.

11.3.2 Loops and Recursion

In FP for and while iterative loops are replaced by **recusive** subroutines calling themselves directly or indirectly (mutual recursion).

```
length' [] = 0
length' (x:s) = 1 + length'(s)
   -- definition using guards and pattern matching
   -- take' n lst returns first n elements of a list
```

```
take' :: (Num i, Ord i) => i -> [a] -> [a]
take' n _
| n <= 0 = []
take' _ [] = []
take' n (x:xs) = x : take' (n-1) xs</pre>
```

11.3.3 Higher-Order functions

Functions that take other functions as arguments or return a function as a result are higher-order functions.

```
applyTwice :: (a \rightarrow a) \rightarrow a \rightarrow a
                                     -- function as arg and res
applyTwice f x = f (f x)
> applyTwice (+3) 10 => 16
> applyTwice (++ " HAHA") "HEY" => "HEY HAHA HAHA"
> applyTwice (3:) [1] => [3,3,1]
applyTwice, f = f.f
                                      -- equivalent definition
:t(.)
> (.) :: (b -> c) -> (a -> b) -> a -> c
   -- define the operator |> which inverts the order between function and argument
   (|>) a f = f a
   (|>) :: t1 -> (t1 -> t2) -> t2
   -- Seems dull right?
   -- Look at the following example
   -- Here, the order of invocation is the same,
   -- but the second "infix" form is (might be) more readable
   > length ( tail ( reverse [1,2,3]))
      2
   > [1,2,3] |> reverse |> tail |> length
(+) :: Num a => a -> a -> a
> let f = (+) 5 // partial application
>:t f ==> f :: Num a => a -> a
> f 4 ==> 9
elem :: (Eq a, Foldable t) => a -> t a -> Bool
> let isUpper = ('elem' ['A'..'Z'])
>:t isUpper ==> isUpper :: Char -> Bool
> isUpper 'A' ==> True
> isUpper '0' ==> False
```

Combinators

map combinator applies argument function to each element in a collection.

```
map :: (a -> b) -> [a] -> [b]
map _ [] = []
map f (x:xs) = f x : map f xs
```

filter takes a collection and a boolean predicate, and returns the collection of the elements satisfying the predicate. It is defined as follows:

And can be applied in the following way

```
> filter (>3) [1,5,3,2,1,6,4,3,2,1]
[5,6,4]
> filter (==3) [1,2,3,4,5]
[3]
> filter even [1..10]
[2,4,6,8,10]
> let notNull x = not (null x)
in filter notNull [[1,2,3],[],[3,4,5],[2,2],[],[]]
[[1,2,3],[3,4,5],[2,2]]
```

reduce (foldl,foldr): takes a collection, an initial value, and a function, and combines the elements in the collection according to the function.

```
-- folds values from end to beginning of list
foldr :: Foldable t => (a -> b -> b) -> b -> t a -> b
foldr f z [] = z
foldr f z (x:xs) = f x (foldr f z xs)
-- folds values from beginning to end of list
foldl :: Foldable t => (b -> a -> b) -> b -> t a -> b
foldl f z [] = z
foldl f z (x:xs) = foldl f (f z x) xs
-- variants for non-empty lists
foldr1 :: Foldable t => (a -> a -> a) -> t a -> a
foldl1 :: Foldable t => (a -> a -> a) -> t a -> a
```

Let's provide some examples:

```
sum' :: (Num a) => [a] -> a
sum' xs = foldl (\acc x -> acc + x) 0 xs
maximum' :: (Ord a) => [a] -> a
reverse' :: [a] -> [a]
reverse' = foldl (\acc x -> x : acc) []
product ' :: (Num a) => [a] -> a
product ' = foldr1 (*)
product ' = foldr (*) 1
-- Notice that product' [] returns 1 !
filter' :: (a -> Bool) -> [a] -> [a]
filter' p = foldr (\x acc -> if p x then x : acc else acc)[]
head' :: [a] -> a
head' = foldr1 (x - \rightarrow x)
last' :: [a] -> a
last' = foldl1 (\ x \rightarrow x)
```

11.3.4 Recursion and Optimization

From a theoretical point of view recursion and iteration are equivalently expressive, and typically one is preferred over the other depending on the problem being faced to make the code more intuitive. In general a procedure call is *much more expensive* than a conditional branch, however FP compilers can perform many optimizations and produce better code, especially for known blocks; for this reason the use of combinators such as map,reduce,filter, foreach,... is strongly encouraged.

Tail-recursive functions are functions in which no operations follow the recursive call(s) in the function, thus the function returns immediately after the recursive call, allowing the compiler to reuse the subroutine's frame on the run-time stack, since the current subroutine state is no longer needed. Besides many compilers instead of re-invoking the function, simply jump to the beginning of the function.

Let's provide the classic example of Fibonacci to illustrate how to convert a normal recursive function to its tail-recursive correspondant one:

```
-- typical Fibonacci

fib = \n -> if n == 0 then 1

else if n == 1 then 1

else fib (n - 1) + fib (n - 2)

fibTR = \n -> let fibhelper (f1, f2, i) =

if (n == i) then f2

else fibhelper (f2, f1 + f2, i + 1)

in fibhelper(0,1,0)
```

Notice that fibTR takes only $\mathcal{O}(n)$ since it builds the Fibonacci sequence starting from 1 to n, while the more canonical approach calculates multiple times the same values, and starts from n until 1 is reached.

foldl is tail-recursive, foldr is not. But because of laziness Haskell has no tail-recursion optimization. Despite this, it provides a more efficient variant of foldl called foldl' where f is evaluated strictly.

Strictly means that the compiler evaluates the accumulator at each step of the folding process, ensuring that intermediate values are not built up as unevaluated thunks i.e. Haskell's term for delayed computations. Due to its strictness fold' is less likely to cause space leaks (see note below), and it generally has better performance for many common folding operations.

[&]quot;Space leaks" mahy happen when a program retains references to data that should no longer be needed, preventing the garbage collector from reclaiming memory. This can lead to inefficient memory usage and, in some cases, cause the program to run out of memory. This (generally) occurs only when handling large data sources or *infinite data structures* (which are allowed in Haskell); space leaks may be very hard to debug, tools like memory profilers and heap profiling can be helpful in identifying them

λ Lambda calculus

Due to Haskell's **laziness**, functions and data constructors don't evaluate their arguments until they need them. In several languages there are forms of lazy evaluations (if-then-else, shortcutting && and ||).

12.1 Syntax

```
| t ::= x | \lambda x.t | t t | (t)

\diamond x variable, name, symbol,...

\diamond \lambda x.t abstraction, defines an anonymous function
```

t t' application of function t to argument t'

We say that an occurrence of x is **free** in a term t if it is not in the body of an abstraction $\lambda x.t$, otherwise it is **bound**; λx instead is a **binder**. Examples $\lambda z.\lambda x.\lambda y.x(yz)$ $(\lambda x.x)x$

Terms without free variables are **combinators**. Identity function: $id = \lambda x.x$ First projection: $fst = \lambda x.\lambda y.x$.

β -Reduction

 β -reduction, i.e. function application, also called **redex**:

$$(\lambda x.t)t' = t[t'/x]$$

$$(\lambda x.x)y \longrightarrow y \tag{12.1}$$

$$(\lambda x. x(\lambda x. x))(ur) \longrightarrow ur(\lambda x. x) \tag{12.2}$$

$$(\lambda x.(\lambda w.xw))(yz) \longrightarrow \lambda w.yzw \tag{12.3}$$

$$(\lambda x.xx)(\lambda x.xx) \longrightarrow (\lambda x.xx)(\lambda x.xx) \tag{12.4}$$

12.2 Functions and lambdas

A definition of a function with a single argument associates a name with a λ -abstraction, while a function with several arguments is equivalent to a sequence of λ -abstractions

```
f x = \langle \exp \rangle -- is equivalent to

f = \lambda x.\langle \exp \rangle

f(x,y) = \langle \exp \rangle -- is equivalent to

f = \lambda x. \lambda y.\langle \exp \rangle

-- Curriend and uncurried functions

curry :: ((a, b) \rightarrow c) \rightarrow a \rightarrow b \rightarrow c

curry f x y = f(x,y)
```

```
uncurry :: (a \rightarrow b \rightarrow c) \rightarrow (a, b) \rightarrow c
uncurry f(x,y) = f x y
```

Well-known functions 12.3

```
• T = \lambda t \cdot \lambda f \cdot t -- first
                                                                    and T F
• F = \lambda t . \lambda f . f -- second
                                                                    \rightarrow (\lambdab.\lambdac.bcF) T F
                                                                    \rightarrow (\lambdac.TcF) F
• and = \lambda b.\lambda c.bcF
                                                                    \rightarrow TFF
                                                                    \rightarrow _{\rm F}
• or = \lambda b \cdot \lambda c \cdot bTc
• not = \lambda x.xFT
• test =\lambda l.\lambda m.\lambda n.lmn
                                                                    not F
                                                                    \rightarrow (\lambdax.xFT) F
 test F u w
                                                                    \rightarrow FFT
 \rightarrow (\lambda1.\lambdam.\lambdan.1mn) F u w
                                                                    \rightarrow T
 \rightarrow (\lambdam.\lambdan.Fmn) u w
 \rightarrow (\lambdan.Fun) w
 → Fuw
 \rightarrow w
```

Figure 12.1: Church Booleans using λ -calculus

$$pair = \lambda f.\lambda s.\lambda b.b \ f \ s$$

$$fst = \lambda p.p \ T$$

$$snd = \lambda p.p \ F$$

fst(pairuw) $(\lambda p.p T)(pair u w)$ \longrightarrow (pair u w) T \longrightarrow $(\lambda f.\lambda s.\lambda b.b \ f \ s) \ u \ wT$ $\longrightarrow (\lambda s. \lambda b. b \ u \ s) \ w \ T$ \longrightarrow $(\lambda b.b \ u \ w) \ T$ $\longrightarrow T u w$ $\longrightarrow u$

$$0 = \lambda s. \lambda z. z \tag{12.5}$$

$$1 = \lambda s. \lambda z. s \ z \tag{12.6}$$

$$2 = \lambda s. \lambda z. s \ (s \ z) \tag{12.7}$$

$$3 = \lambda s. \lambda z. s \ (s(s \ z)) \tag{12.8}$$

 $\begin{array}{ll} \mathbf{\tilde{g}} & 0 = \lambda s. \lambda z.z \\ 1 = \lambda s. \lambda z.s \ z \\ 2 = \lambda s. \lambda z.s \ (s \ z) \\ 3 = \lambda s. \lambda z.s \ (s(s \ z)) \\ & \\ \mathbf{\tilde{g}} \\ Numerals \ n \ \text{takes a function } s \ \text{as argument and returns the } n\text{-}th \ \text{composition of } s \ \text{with itself, } s^n. \end{array}$

e.g. $succ = \lambda n.\lambda s.\lambda z.s(nsz)$

Fix-point Y combinator 12.4

The following fix-point combinator Y, when applied to a function R, returns a fix-point of R, i.e. R(YR) = YR

$$Y = (\lambda y.(\lambda x.y(x\ x))(\lambda x.y(x\ x))) \tag{12.9}$$

$$YR = (\lambda x.R(x x))(\lambda x.R(x x))$$

$$= R((\lambda x.R(x x))(\lambda x.R(x x)))$$

$$= R(YR)$$
(12.10)

12.5 Evaluation ordering

Consider the two following ways of evaluating a redex, but remember that *regardless* of the evaluation order, the evaluation result is only one, and it is **unique**¹.

Applicative order evaluation implies eager evaluation of arguments before applying them to the function

```
(\lambda x.(+ x x)) (+ 3 2)

→ (\lambda x.(+ x x)) 5

→ (+ 5 5)

→ 10
```

Normal order evaluation implies functions to be evaluated first, and delay argument evaluation only when needed. Note that this may lead to multiple re-evaluations of the same argument.

Haskell realizes **lazy evaluation** by using **call by need** parameter passing: an expression passed as argument is bound to the formal parameter, but it is evaluated *only if* its value is **needed**. Besides, the argument is evaluated *only* the **first time**, using the **memoization** technique: the result is saved and further uses of the argument do not need to re-evaluate it.

Combined with lazy data constructors, this allows to construct potentially infinite data structures and to call infinitely recursive functions without necessarily causing non-termination.

Note: lazy evaluation works fine with purely **functional languages**. Side effects such as IO operations force the programmer to reason about the order in which things happen, which not predictable in lazy languages. We will address this fact when introducing Haskell's IO-Monad.

12.6 Post-lecture Takeaway message

While discussing with the professor after the lecture, an important intuition emerged about evaluation an memoization.

```
a = 5b = a + 3
```

b would evaluate to 8 but it is not evaluated until it is strictly necessarily.

```
a = 5
b = a + 3
a = 2
-- b?
```

Someone may think that due to lazy evaluation, b would now evaluate to 5. However, this is NOT Haskell's case. Due to **memoization**, even if b = a + 3 doesn't get evaluated, the current value of a is memoized and its re-definition doesn't affect b evaluation. Thus this snippet code leads b to be evaluated as 8, regardless of a redefinition.

```
a = 5
b = a + 3
a = 2
b
```

¹Proved by Church and Rosser

Type Inference

Polymorphism in Haskell

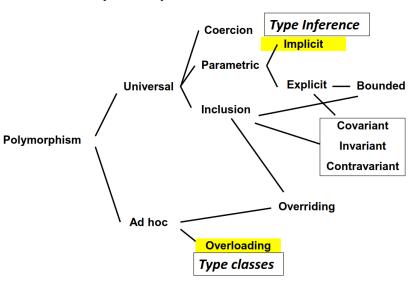


Figure 13.1: Haskell Polymorphism Recap

13.1 Overloading

Haskell allows **overloading** even of **primitive types**: the code to be executed is determined by the type of the arguments, leading to have *early binding* in *statically* typed languages or *late binding* in *dynamically* typed languages.

In Haskell we can write the following, but what is the type?

```
sqr x = x * x
```

When considering overloading besides arithmetic, we find that some functions are fully polymorphic:

```
\texttt{length} \; :: \; \texttt{[w]} \; \to \; \texttt{Int}
```

While others not so much; for example, *membership* works only for types that support equality, while *sorting* works only for types which support *ordering*.

13.2 Type Classes

Type Classes solve many overloading problems concerning arithmetic and equality (and similar properties) support.

implementation summary

The idea is to generalize ML's equippes to arbitrary types and provide concise types to describe overloaded functions, so no exponential blow-up (i.e. defining functions for every possible combination of type arguments).

Type classes allow users to define functions using overloaded operations —e.g. square, squares, and member— and to declare new collections of overloaded functions: equality and arithmetic operators are not privileged built-ins. Haskell's solutions fits perfectly within type inference framework.

The intuition is that a sorting function may allow to be passed a comparison cmp operator as argument, thus making the function parametric.

```
qsort:: (a \rightarrow a \rightarrow Bool) \rightarrow [a] \rightarrow [a] qsort cmp [] = [] qsort cmp (x:xs) = qsort cmp (filter (cmp x) xs) ++ [x] ++ qsort cmp (filter (not.cmp x) xs)
```

Developing this idea, consider rewriting the parabola function to take operators as argument

```
parabola x = (x * x) + x
parabola' (plus, times) x = plus (times x x) x
```

Here the extra parameter is a *dictionary* that provides implementations for the overloaded ops. These implies rewriting calls to pass appropriate implementations for plus and times:

```
y = parabola'(intPlus,intTimes) 10
z = parabola'(floatPlus, floatTimes) 3.14
```

- 1. Type class declarations
 - i. Define a set of operations, give it a name
 - ii. Example: Eq a type class \bullet operations == and \= with type a \rightarrow a \rightarrow Bool
- 2. Type class instance declarations
 - i. Specify the implementations for a particular type
 - ii. For Int instance, == is defined to be integer equality
- 3. Qualified types (or Type Constraints) Concisely express the operations required on otherwise polymorphic type member:: Eq w ⇒ w → [w] → Bool
- 1. Each overloaded symbol has to be introduced in at least one type class
- 2. The compiler translates each function that uses an overloaded symbol into a function with an extra parameter: the dictionary.
- 3. References to overloaded symbols are rewritten by the compiler to lookup the symbol in the dictionary.
- 4. The compiler converts each type class declaration into a dictionary type declaration and a set of selector functions.
- 5. The compiler converts each instance declaration into a dictionary of the appropriate type.
- 6. The compiler rewrites calls to overloaded functions to pass a dictionary. It uses the static, qualified type of the function to select the dictionary.

13.2.1 Compositionality

```
class Eq a where

(==) :: a \rightarrow a \rightarrow Bool

instance Eq Int where

(==) = intEq -- intEq primitive equality

instance (Eq a, Eq b) => Eq(a,b) where

(u,v) == (x,y) = (u == x) && (v == y)

instance Eq a => Eq [a] where

(==) [] [] = True

(==) (x:xs) (y:ys) = x==y && xs == ys

(==) _ = False
```

13.2.2 Compound Translation

13.2.3 Subclasses

A subclass declaration expresses this relationship:

```
class Eq a => Num a where (+) :: a \rightarrow a \rightarrow a (*) :: a \rightarrow a \rightarrow a
```

• With that declaration, we can simplify the type of the function

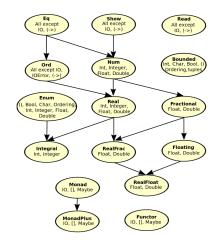


Figure 13.2: Haskell Subclasses relationships

13.2.4 Deriving

For Read, Show, Bounded, Enum, Eq, and Ord, the compiler can generate instance declarations automatically.

```
data Color = Red | Green | Blue
    deriving (Show, Read, Eq, Ord)

Main>:t show
show :: Show a => a → String
Main> show Red
"Red"
Main> Red < Green
True
Main>:t read
read :: Read a => String → a
Main> let c :: Color = read "Red"
Main> c
Red
```

13.2.5 Numeric Literals

```
class Num a where

(+) :: a \rightarrow a \rightarrow a

(-) :: a \rightarrow a \rightarrow a

fromInteger :: Integer \rightarrow a

-- Even literals are overloaded.

-- 1 :: (Num a) => a

...

inc :: Num a => a \rightarrow a

inc x = x + 1
```

Numeric literals can be interpreted as values of any appropriate numeric type, for example: 1 can be an Integer for a Float or a user- defined numeric type.