List of Abbreviations

OBR   Office Budget Responsibility

ODE   Ordinary Differential Equation

GDP   Gross Domestic Product

CPI   Consumer Price Index

SDE   Stochastic Differential Equation

1 Project Proposal

The aim of this project is to design and develop a mathematical model of the most important macroeconomic indicators in the Russian national economy. The model will be mainly inspired from the macroeconomic model designed and used by the Office of Budget Responsibility (OBR) in the UK (OBR 2019b). Such model is important for various reasons. The OBR uses its model for forecasting[1]. They prepare and publish economic forecasts for the purposes of forecasting the public finances (OBR 2019a). Examine figure 1 for an example about their forecasting.
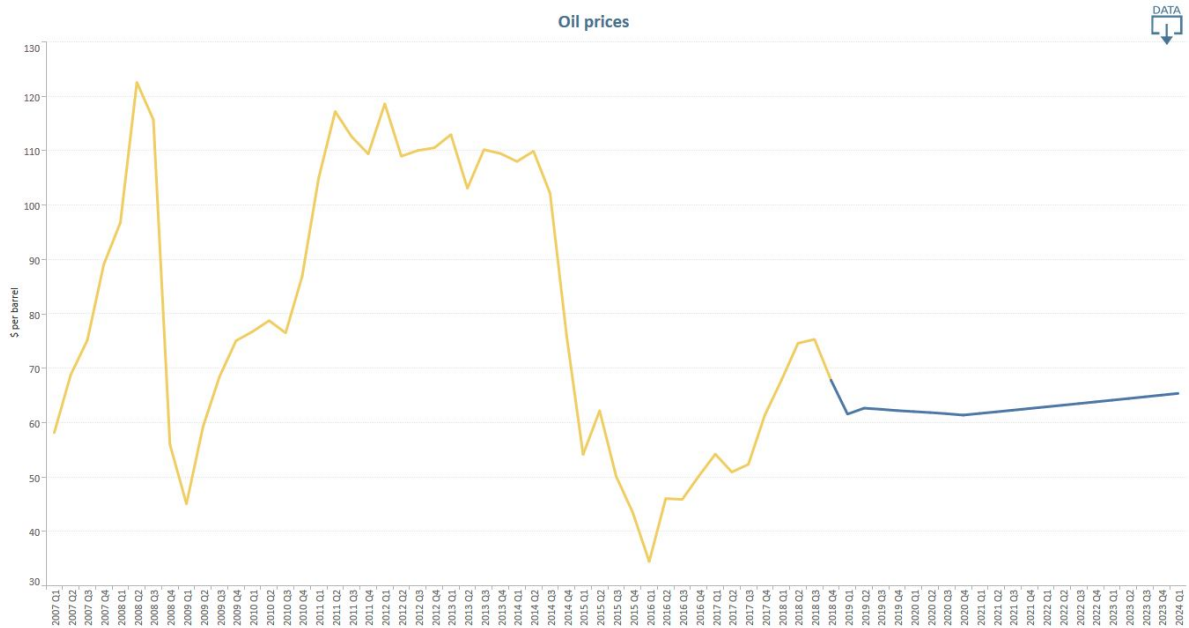

Figure 1: UK oil price forecasting by OBR[2]. The yellow part corresponds to the outturn (real) price recorded, while the blue part corresponds to the forecasting (predicted) price. Q_i on the horizontal axis corresponds the ith quarter of the year.

1.1 Project Plan

Table 2 describes the plan that will be used to accomplish this project.

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[1] https://obr.uk/forecasts-in-depth

[2] https://obr.uk/forecasts-in-depth/the-economy-forecast/conditioning-assumptions/#oilprices

Iteration	Week	Tasks
N/A	1	N/A
	2	
1	3	Write project proposal and plan. Review OBR model. Review common macroeconomic indicators.
	4	Design a system of ordinary differential equations (ODEs) to provide dynamics for the model. Review macroeconomic indicators in more depth.
2	5	
	6	
3	7	Review and collect Russian economic statistics.
	8	Mid-term presentation.
4	9	Implement the model and numerically solve the ODEs system.
	10	
5	11	Evaluate the model.
	12	Improve the model, or enhance the model (e.g. introduce stochastic terms).
6	13	Iterate between improving and evaluating the model.
	14	
N/A	15	Final presentation.

Table 2: Project plan.

1.2   Iterations

This sections describes the actual work done in each iteration.

1.2.1   Iteration 1

- Did relevant reading of chapters 6 and 7 from (Samuelson and Nordhaus 2009).

- Reviewed OBR.

- Wrote project proposal.

- Wrote project plan.

1.2.2   Iteration 2

- Did relevant reading of chapters 19 and 20 from (Samuelson and Nordhaus 2009).

- Wrote proposed model section.

- Familiarizing myself by reviewing some Russian economic statistics.

2   Proposed Model

This section describes the proposed model on a high-level. The first subsection describes the economic indicators involved in the model. The second subsection describes the ODEs system proposed to model the dynamics of these indicators.

Basically, the model is a set of differential equations that describe the inter-relationships between different macroeconomic indicators. We are using the term *macroeconomic indicators* to refer to the factors, features, or variables that we would like to model. Such as annual GDP growth, unemployment rate, and similar indicators. The proposed model is a system of ODEs that tries to capture the dynamics of such indicators. Each equation in the system may involve: an indicator value, a function of some indicator, a change (differential) of some indicator, and so on. This will be clarified shortly in section 2.2.

2.1   Macroeconomic Indicators

We classify the indicators used in our model into two categories: (a) International indicator (items 1-5), and (b) Russia-specific indicators (items 6-11).

1. Annual GDP

2. Base Interest Rate (from the central bank)

3. Inflation Rate (correlated with CPI)

4. Unemployment Rate

5. Average Wage (i.e average income per person)

6. USD/RUB (exchange rate of US dollar for ruble)

7. Brent Crude Oil Price

8. MOEX Equity Index

9. State-funded investments (per year)

10. Defense Spendings

11. Population Growth Rate

We refer to these vectors using a vector $\vec{x} = (x_1, \cdots, x_n)$ where in $n = 11$ and each x_i refers to the corresponding item. For example, x_4 is unemployment rate.

2.2 Mathematical Model

The proposed model is encapsulated by the following equation which represents a system of stochastic differential equations (SDEs):

$$d\vec{x} = A\vec{x}\,dt + \Sigma\,d\vec{W}_t \tag{1}$$

$\vec{x}$ is defined in the previous section. A is an $n \times n$ matrix of constant coefficients (not dependent on x). t is time. Σ is an $n \times n$ matrix. $\vec{W}_t$ is a vector of n independent Brownian motion terms.

If we omit the second term for the moment, by setting $\Sigma = 0$ for example. We can see that the system is simply reduced to a linear system of ODEs. The second term introduces stochasticity in our model by using Brownian motion terms.

Σ corresponded to a correlation matrix. We consider the value of $\Sigma^T\Sigma$ to be of the form $B(x)$. That is, $\Sigma^T\Sigma = B(x)$, Where $B(x)$ is an $n \times n$ matrix that is dependent on x. We consider an element $B_{ij}(x)$ of this matrix to have the form $B_{ij}(x) = b_{ij} \cdot x + c_{ij}$. b_{ij} is a vector of length n; the dot $\cdot$ is the dot product; and the components of the vector b_{ij} along with c_{ij} are constants independent of x. This model is called an affine model (**citation needed**). By this formulation, it can be shown that $\Sigma = \sqrt{B}$ (**citation needed**).

We start with a reduced simplified model where we set $b_{ij} = 0$. This basically corresponds to setting B as a constant matrix independent of x. Which corresponds to Σ being independent of x. After the implementation and calibration of this reduced model, we reconsider the original model if time is available.

References

OBR (2019a). *Office of Budget Responsibility: The Economy Forecast.* URL: https://obr.uk/forecasts-in-depth/the-economy-forecast (visited on 02/10/2020).

— (2019b). *Office of Budget Responsibility: The Macroeconomic Model.* URL: https://obr.uk/topics/obr-macroeconomic-model (visited on 02/10/2020).

Samuelson, Paul and William Nordhaus (2009). *Economics.* 19th Edition. McGraw-Hill Education.