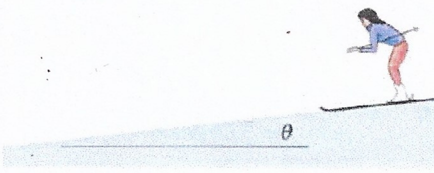
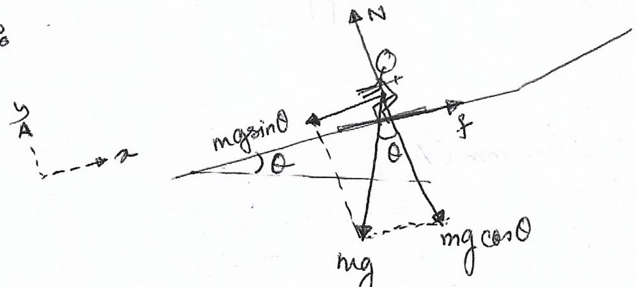
Problem Sheet 2 (Friction)

1. The designer of a ski resort wishes to have a portion of a beginner's slope on which the skier's speed will remain fairly constant. Tests indicate the average coefficients of friction between skis and snow to be $\mu_s = 0.10$ and $\mu_d = 0.08$. What should be the slope angle θ of the constant speed section.

   Ans $\theta = 4.57^0$



Soln:



To ski with constant speed, the skier must be in equilibrium;

$\therefore f - mg\sin\theta = 0$ ($\because \Sigma F_x = 0$)

$\Rightarrow \mu_d N = mg\sin\theta$ ($\because$ determining the angle, $f = \mu_d N$) while skiing

$\Sigma F_y = 0 \Rightarrow N - mg\cos\theta = 0$

$\Rightarrow N = mg\cos\theta$

$\Rightarrow \mu_d = \dfrac{mg\sin\theta}{N}$

$\Rightarrow \mu_d = \dfrac{mg\sin\theta}{mg\cos\theta}$

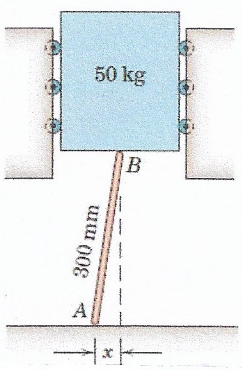
$\Rightarrow \mu_d = \tan\theta$

$\Rightarrow \theta = \tan^{-1}(\mu_d)$

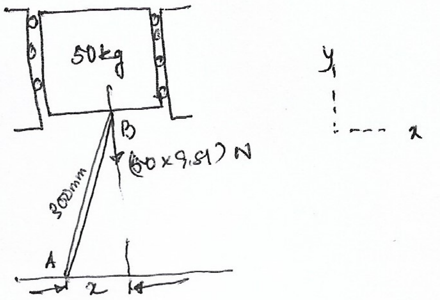
$\Rightarrow \theta = \tan^{-1}(0.08)$

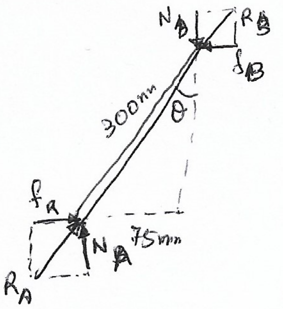
$\Rightarrow \theta = 4.574^\circ$ (Ans)

2. The light bar is used to support the 50-kg block in its vertical guides. If the coefficient of static friction is 0.30 at the upper end of the bar and 0.40 at the lower end of the bar, find the friction force acting at each end for x = 75 mm. Also find the maximum value of x for which the bar will not slip. Ans: 86.2 mm



Soln:





$N_A = W = 50\times9.81 = 490.5$N

In Equilibrium,

$\Sigma F_y = 0$

$\Rightarrow N_A = N_B$ --- (I)

$\Sigma F_x = 0$

$\Rightarrow f_B = f_A$ --- (II)

$\therefore R_A = R_B$ from (I) & (II)

When $x = 75$,

$\tan\theta = \dfrac{f_A}{N_A}$; $\theta = \sin^{-1}\left(\dfrac{75}{300}\right)$

$= 14.48^\circ$ (Ans)

We know friction angle $\phi = \tan^{-1}\mu_s$

at A friction angle (ϕ_A) $= \tan^{-1}(0.4) = 21.8^\circ$

at B friction angle (ϕ_B) $= \tan^{-1}(0.3) = 16.7^\circ$

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As $\theta < \phi_A$ and $\theta < \phi_B$, hence the bar is not slipping.

Now $R_A = R_B = W \tan\theta$

$= 50 \times 9.81 \times \tan 14.5^\circ$

$= 126.65\ N$ (Ans)

As x is increased from 75 mm, the bar will slip at B position with friction angle $\phi_B = 16.7^\circ$

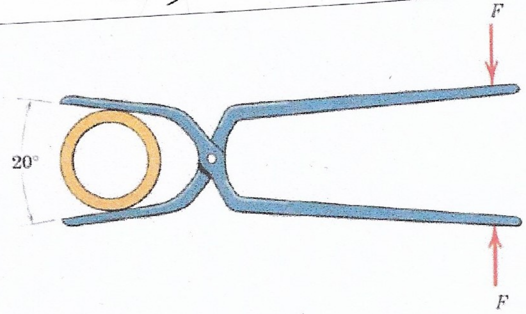
$\therefore$ At that position, x will x_{max} without slipping

$\therefore \dfrac{x_{max}}{300} = \sin 16.7^\circ$

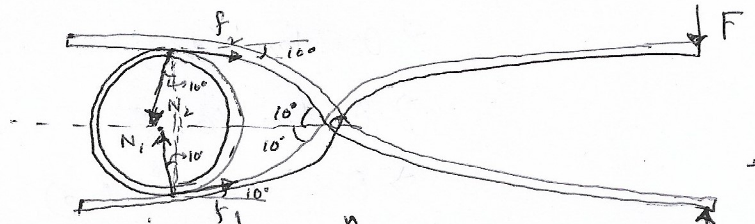
$\Rightarrow x_{max} = 300 \times \sin 16.7^\circ = 86.2\ mm$ (Ans)

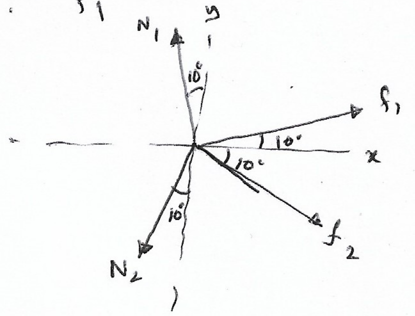
3. The tongs are used to handle hot steel tubes that are being heat-treated in an oil bath. For a 20^0 jaw opening, what is the minimum coefficient of static friction between the jaws and the tube that will enable the tongs to grip the tube without slipping.

Ans. $\mu_s = 0.176$



Soln:





In Equilibrium,

$\Sigma F_x = 0$

$\Rightarrow f_1 \cos 10^\circ + f_2 \cos 10^\circ = N_1 \sin 10^\circ + N_2 \sin 10^\circ$

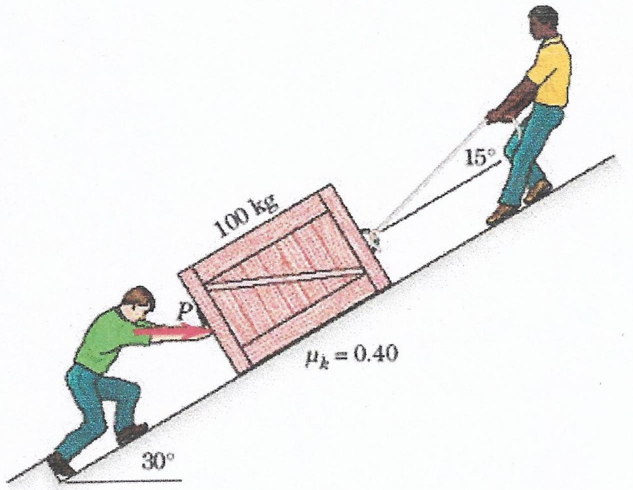
$\Rightarrow \mu N_1 \cos 10^\circ + \mu N_2 \cos 10^\circ = N_1 \sin 10^\circ + N_2 \sin 10^\circ$

$\Rightarrow \mu = \dfrac{\sin 10^\circ}{\cos 10^\circ}$

$\Rightarrow \mu = 0.176$ (Ans)

4. Two men are sliding a 100-kg crate up an incline. If the lower man pushes horizontally with a force of 500 N and if the coefficient of kinetic friction is 0.4, determine the tension T which the man must exert in the rope to maintain motion of the crate.

Ans. T = 465 N



Soln:

$W = 100 \times 9.81 = 981\ N$

$P = 500\ N$, $\mu_K = 0.4$

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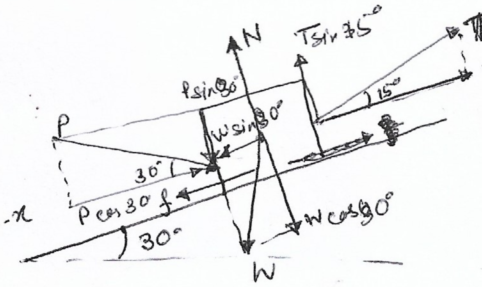
In Equilibrium,

$\Sigma F_x = 0$

$\Rightarrow P\cos 30^\circ - W\sin 30^\circ - f + T\cos 15^\circ = 0$

$\Rightarrow -\mu_K N + T\cos 15^\circ = 981\sin 30^\circ - 500\cos 30^\circ$

$\Rightarrow T\cos 15^\circ - 0.4N = 57.49\ N$ --- (1)



$\Sigma F_y = 0$

$\Rightarrow N + T\sin 15^\circ - P\sin 30^\circ - W\cos 30^\circ = 0$

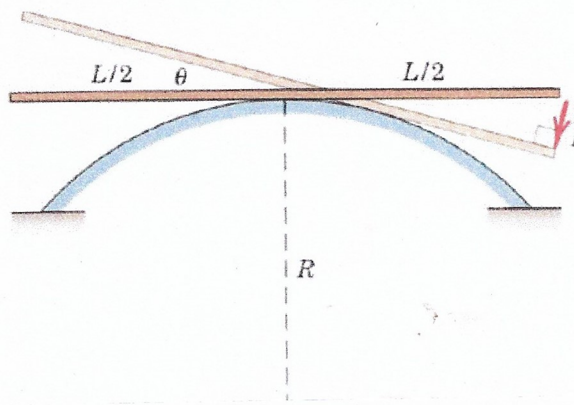
$\Rightarrow N = 500\sin 30^\circ + 981\cos 30^\circ - T\sin 15^\circ$

Putting (2) in (1);

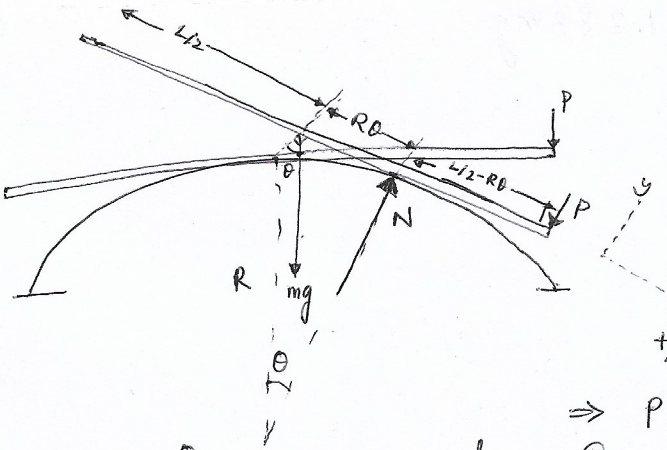
$T\cos 15^\circ - (1099.57 - T\sin 15^\circ) \times 0.4 = 57.49$

$\Rightarrow T(\cos 15^\circ + 0.4\sin 15^\circ) = 57.49 + 0.4 \times 1099.57$

5. The uniform slender rod of mass m and length L is initially at rest in a centered horizontal position on the fixed circular surface of radius R = 0.6L. If a force normal to the bar is gradually applied to its end until the bar begins to slip at the angle θ = 20°, determine the coefficient of static friction μ_s.
Ans. $\mu_s = 0.212$



Soln:



In Equilibrium,

$\Sigma F_x = 0$

$\Rightarrow mg\sin\theta - \mu_s N = 0$ --- (1)

$\Sigma F_y = 0$

$\Rightarrow N - mg\cos\theta - P = 0$ --- (2)

$+\circlearrowright \Sigma M_0 = 0$

$\Rightarrow P(L/2 - R\theta) - mg\cos\theta . R\theta = 0$ --- (3)

from (3);

$(N - mg\cos\theta)(L/2 - R\theta) - mg\cos\theta . R\theta = 0$

$\Rightarrow N.L/2 - NR\theta - mg\cos\theta . L/2 + \cancel{mg\cos\theta . R\theta} - \cancel{mg\cos\theta . R\theta} = 0$

$\Rightarrow N(L/2 - R\theta) = mg\cos\theta . L/2$

$\Rightarrow N = \dfrac{mg\cos\theta . L . \cancel{2}}{\cancel{2}(L - 2R\theta)} = \dfrac{mg\cos\theta . L}{L - 2R\theta}$

from (1) $\Rightarrow N = \dfrac{mg\sin\theta}{\mu_s}$

from (2) $\Rightarrow \dfrac{mg\sin\theta}{\mu_s} - mg\cos\theta = P$

$\therefore N = \dfrac{mg\cos\theta . L}{L - 2\times 0.6L\,\theta} = \dfrac{mg\cos\theta}{(1 - 1.2\theta)}$

Now, $P = \dfrac{mg\cos\theta}{(1 - 1.2\theta)} - mg\cos\theta$

from (1) $\Rightarrow \mu_s N = mg\sin\theta$

$\Rightarrow \mu_s = \dfrac{mg\sin\theta \times (1 - 1.2\theta)}{mg\cos\theta} = (1 - 1.2\theta)\tan\theta$

$= \left\{1 - 1.2\times\left(\dfrac{20}{180}\right)\pi\right\} \times \tan 20^\circ$

$= 0.2115 \simeq 0.212$ (Ans)

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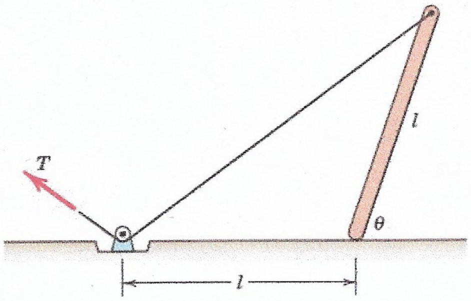
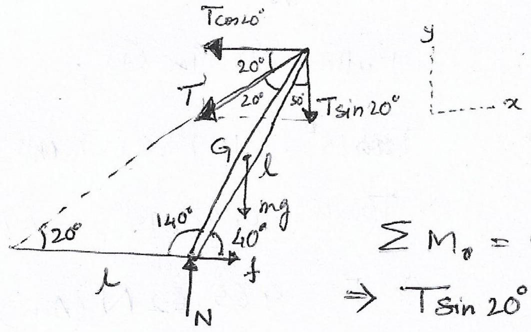
6. The uniform slender rod is slowly lowered from the upright position ($\theta = 90^0$) by means of the cord attached to its upper end and passing over the small fixed pulley. If the rod is observed to slip at its lower end when $\theta = 40^0$, determine the coefficient of static friction at the horizontal surface.

Ans. $\mu_s = 0.761$



Soln:



$\sum F_x = 0$

$\Rightarrow f = T\cos 20^\circ$ ...... ①

$\sum F_y = 0$

$\Rightarrow N = T\sin 20^\circ + mg$ .... ②

$\sum M_o = 0$

$\Rightarrow T\sin 20^\circ \times l\cos 40^\circ + mg\frac{l}{2}\cos 40^\circ - T\cos 20^\circ \times l\sin 40^\circ = 0$

$\Rightarrow T = 1.120\ mg$ .... ③

from ③ value of T in terms of mg,

from ① $\Rightarrow f = 1.120\ mg \times \cos 20^\circ$

from ② $\Rightarrow N = 1.120\ mg\ \sin 20^\circ + mg$

$= mg(1.120 \sin 20^\circ + 1)$

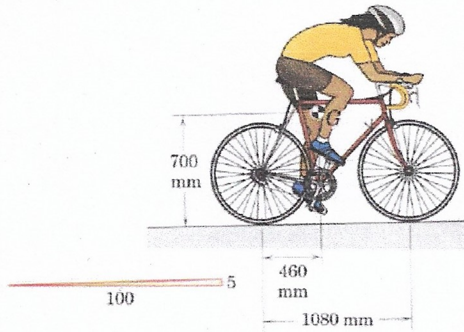
$= 1.383\ mg$

$\mu_s = \frac{f}{N} = \frac{1.120 \times \cos 20^\circ}{1.383}$

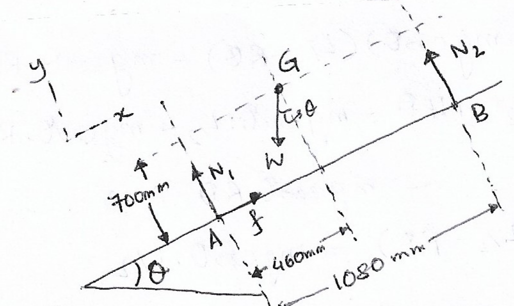
$= 0.761$ (Ans)

7. A man pedals his bicycle up a 5 percent grade on a slippery road at a steady speed. The man and bicycle have a combined mass of 82 kg with mass center at G. If his rear wheel is on the verge of slipping, determine the coefficient of friction μs between the rear tire and the road. If the coefficient of friction were doubled, what would be the friction force acting on the rear wheel? (why may we neglect friction under the front wheel?)

Ans. $\mu_s = 0.082$, F = 40.2 N



Soln:



$\tan\theta = \frac{5}{100} = 0.05$

$\therefore \theta = \tan^{-1}(0.05) = 2.86$

$\sum F_x = 0$

$\Rightarrow f = W\sin\theta$

$= \cancel{4.09}\ 40.17$ N

$\sum F_y = 0$

$\Rightarrow N_1 + N_2 = W\cos\theta = 803.42$ N

$\sum M_B = 0$

$\Rightarrow -N_1 \times 1080 + W(\cos\theta \times 620 + \sin\theta \times 700) = 0$

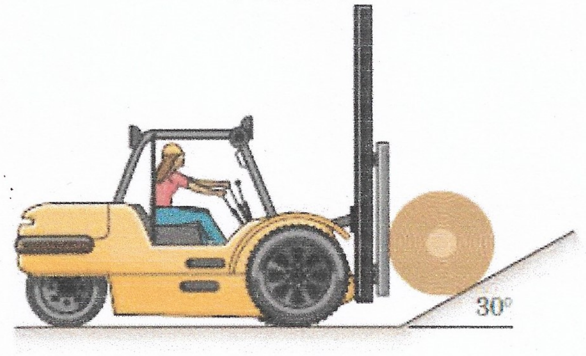
$\Rightarrow N_1 = (49.66 \times 9.81) = 487.16$ N

$\therefore \mu = \frac{f}{N_1} = \frac{40.17}{487.16} = 0.082$ (Ans)

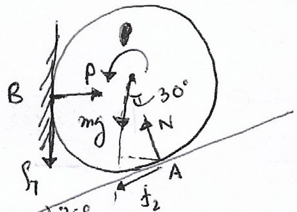
$f = 40.17$ N (Ans)

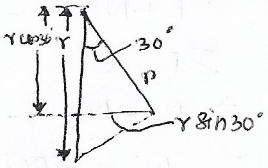
8. The industrial truck is used to move the solid 1200-kg roll of paper up the 300 incline. If the coefficients of static and kinetic friction between the roll and the vertical barrier of the truck and between the roll and the incline are both 0.40, compute the required tractive force P between the tires of the truck and the horizontal surface.

Ans. P = 22.1 kN



Soln:





$\Sigma M_o = 0$

$\Rightarrow f_1 \times r = f_2 \times r$

$\Rightarrow f_1 = f_2 = 0.4P$

$f_1 = \mu P = 0.4P$

In Equilibrium,

$\Sigma F_x = 0$

$N \sin 30^\circ + f_2 \cos 30^\circ - P = 0$

$\Rightarrow N \sin 30^\circ + 0.4 P \cos 30^\circ - P = 0$

$\Rightarrow N = 1.307 P$

$\Sigma F_y = 0$

$\Rightarrow - f_1 - 1200 \times 9.81 - f_2 \sin 30^\circ + N \cos 30^\circ = 0$

$\Rightarrow P = 22.13$ kN (Ans)

OR Moment equilibrium, $\Sigma M_A = 0$

$\Rightarrow f_1 (r + r \sin 30^\circ) - P \times r \cos 30^\circ + mg\, r \sin 30^\circ = 0$

$\Rightarrow 0.4P(1 + \sin 30^\circ) - P \cos 30^\circ + mg \sin 30^\circ = 0$

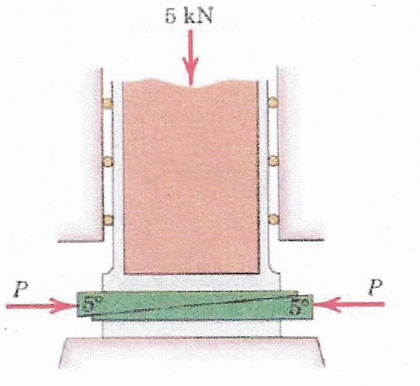
$\Rightarrow 0.266P = 0.5mg$

$\Rightarrow P = 22127$ N $= 22.13$ kN (Ans)

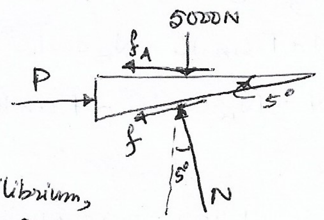
9. The two 5° wedges shown are used to adjust the position of the column under a vertical load of 5 kN. Determine the magnitude of forces P required to ~~lower~~ raise the column if the coefficient of friction for all surfaces is 0.40.

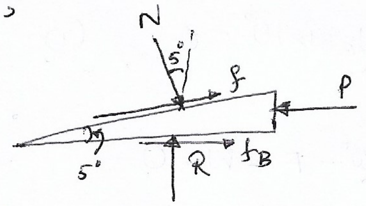
Ans. P = 3.51 kN to lower

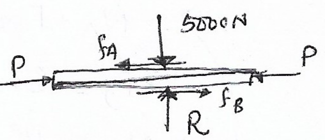
P = 4.53 kN to raise



Soln: Case 1: To raise the load;







In Equilibrium,

$\Sigma F_x = 0$

$\Rightarrow P = f \cos 5^\circ + f_A + N \sin 5^\circ$

$\Sigma F_y = 0$

$\Rightarrow 5000 + f \sin 5^\circ = N \cos 5^\circ$

$\cancel{P = N \sin 5^\circ + 0.4 N \cos 5^\circ + 2000}$

$P = 0.4N \cos 5^\circ + 2000 + N \sin 5^\circ$

$5000 + 0.4 N \sin 5^\circ = N \cos 5^\circ$

In Equilibrium,

$\Sigma F_x = 0$

$\Rightarrow P = N \sin 5^\circ + f \cos 5^\circ + f_B$

$\Sigma F_y = 0$

$\Rightarrow N \cos 5^\circ = R + f \sin 5^\circ$

$\Rightarrow N \cos 5^\circ = 5000 + 0.4 N \sin 5^\circ$

$P = N \sin 5^\circ + 0.4 N \cos 5^\circ + 2000$

$P = N(0.4 \cos 5^\circ + \sin 5^\circ) + 2000$

$= \dfrac{5000\,(0.4 \cos 5^\circ + \sin 5^\circ)}{(\cos 5^\circ - 0.4 \sin 5^\circ)} + 2000 = 4525.84$ N

$= 4.52$ kN (Ans)

$R = 5000$ N

$f_A = f_B = \mu R$

$= (0.4 \times 5000)$ N

$= 2000$ N

$f = \mu N$

$= 0.4 N$

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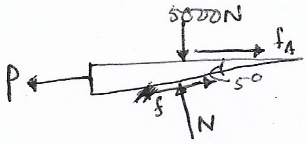
Case 2: To lower the load.



In Equilibrium,

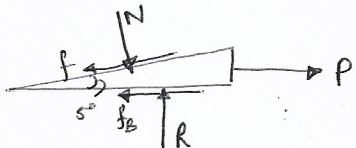
$\Sigma F_x = 0$

$\Rightarrow P - f\cos 5^\circ + N\sin 5^\circ - f_A = 0$

$\Rightarrow P + N(0.4\cos 5^\circ + \sin 5^\circ) = 2000$

$\Sigma F_y = 0$

$\Rightarrow N\cos 5^\circ + f\sin 5^\circ = 5000N$



In Equilibrium,

$\Sigma F_x = 0$

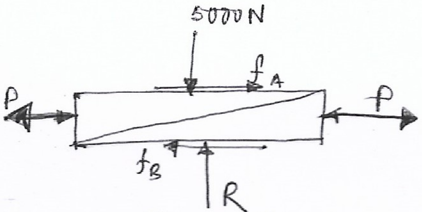
$\Rightarrow P + N\sin 5^\circ - f_B - f\cos 5^\circ = 0$

$\Rightarrow P + N(\sin 5^\circ - 0.4\cos 5^\circ) = 2000$

$\Sigma F_y = 0$

$\Rightarrow R - N\cos 5^\circ - f\sin 5^\circ = 0$

$\Rightarrow N = \dfrac{5000}{(0.4\cos 5^\circ + \sin 5^\circ)}$



In Equilibrium,

$R = 5000N$

$f_A = f_B = \mu R$

$= 2000N$

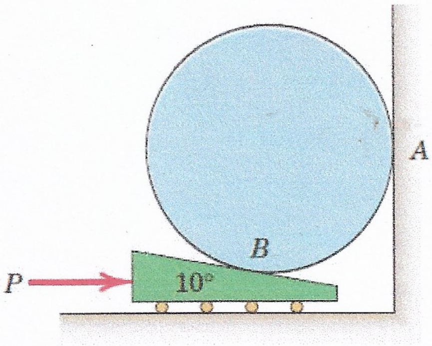
$P = \dfrac{5000\,(0.4\cos 5^\circ - \sin 5^\circ)}{(0.4\cos 5^\circ + \sin 5^\circ)} + 2000$

$= 3509.72$ N

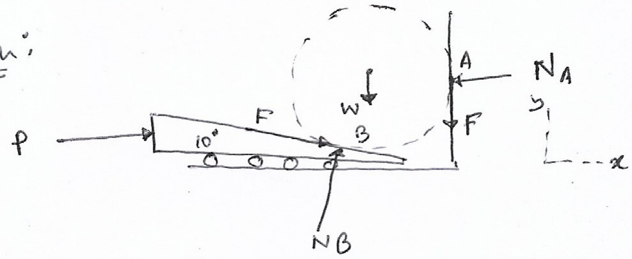
$= 351$ N (Ans)

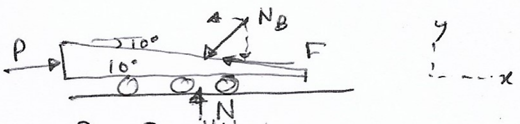
10. Calculate the horizontal force P on the light 10° wedge necessary to initiate movement of the 40-kg cylinder. The coefficient of static friction for both pairs of contacting surfaces is 0.25. Also determine the friction force FB at point B.

Ans. P = 98.6 N, F_B = 24.6 N



Soln:





In Equilibrium

$\Sigma F_x = 0$

$\Rightarrow -N_A + F\cos 10^\circ + N_B\sin 10^\circ = 0$ ---- (i)

$\Sigma F_y = 0$

$\Rightarrow N_B\cos 10^\circ - F\sin 10^\circ - F - W = 0$

$\Rightarrow N_B = 1.192F + 398.4$ ---- (ii)

$-N_A + 0.985F + 1.132F \times 0.174 = 0$
$+ 398.4 \times 0.174$

$\Rightarrow N_A = 1.192F + 68.14$

So $N_B > N_A$

$\Rightarrow \mu_B N_B > \mu N_A$

So, it will slip first at A

$F = \mu N_A = 0.25 N_A$ ---(vi)

In Equilibrium,

$\Sigma F_x = 0$

$\therefore P - F\cos 10^\circ - N_B\sin 10^\circ = 0$

$\Rightarrow P = F\cos 10^\circ + N_B\sin 10^\circ$ ---(iv)

$\Sigma F_y = 0$

$\Rightarrow N + F\sin 10^\circ - N_B\cos 10^\circ = 0$

$\Rightarrow N = N_B\cos 10^\circ - F\sin 10^\circ$ ---(v)

Now,

$N_A = 1.192 \times 0.25 N_A + 62.37$

$\Rightarrow N_A = 98.6$ N (Ans)

$F = 24.6$N (Ans)

$N_B = 427.77$N

$P = 98.68$ N (Ans)

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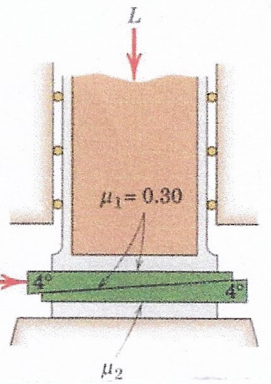
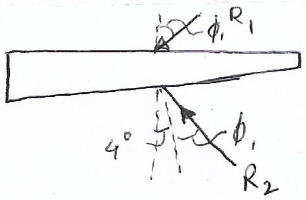
11. The two wedges are used to position the vertical column under a load L. What is the least value of the coefficient of friction μ_2 for the bottom pair of surfaces for which the column may be raised by applying a single horizontal force P to the upper wedge?

Ans. $\mu_2 = 0.378$



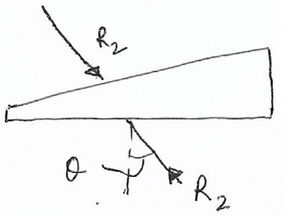
Soln:



$\phi_1 = \tan^{-1}(0.3) = 16.70°$

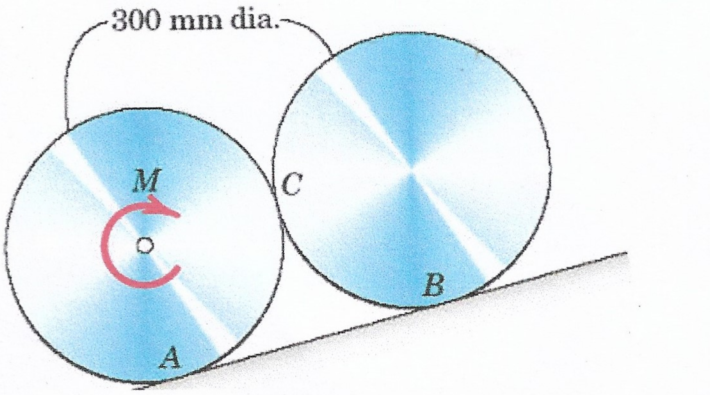
In order for column to be raised, bottom wedge must not move. Therefore μ_2 must be ~~given~~ greater than $\tan\theta$ or

$(\mu_2)_{min} = \tan(4 + 16.70) = 0.378$ (Ans)

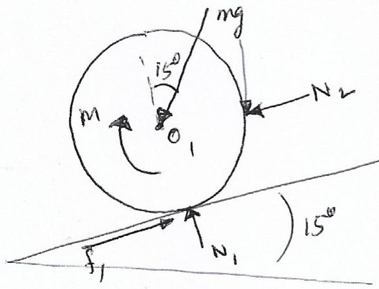


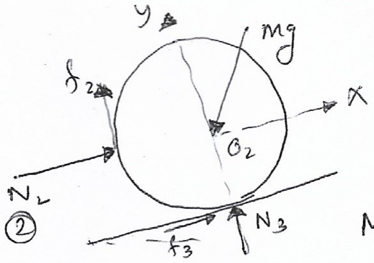
12. Calculate the couple M applied to the lower of the two 20-kg cylinders which will allow them to roll slowly down the incline. The coefficients of static and kinetic friction for all contacting surfaces are $\mu_s = 0.60$ and $\mu_k = 0.50$.

Ans. M = 10.16 N.m



Soln:





$\theta = 15°$

$r = \frac{30 cm}{2} = 15 cm$

$= 0.150$ m

$M = (f_1 - f_2) \times r$

$= (f_1 - f_2) \times 0.150$ N.m.

$mg = 20 \times 9.81 = 196.2 N$

$\mu_s = 0.6, \mu_k = 0.5$

In Equilibrium,

$\Sigma F_x = 0 \Rightarrow f_1 - N - mg\sin\theta = 0$

$\Sigma F_y = 0 \Rightarrow N_1 - f_2 - mg\cos\theta = 0$

$\Sigma F_x = 0 \Rightarrow f_3 + N_2 - mg\sin\theta = 0$

$\Sigma F_y = 0 \Rightarrow N_3 + f_2 - mg\cos\theta = 0$

$\Sigma M_{O_1} = 0 \Rightarrow M + f_2 \times r - f_1 r = 0$

$\Rightarrow M = (f_1 - f_2) \times r$

$\Sigma M_{O_2} = 0 \Rightarrow f_2 \times r = f_3 \times r$

$\Rightarrow f_2 = f_3$

$N_1 = 206.44 N$

$N_2 = 33.85 N$

$N_3 = 172.59 N$

$f_1 = 84.63 N$

$f_2 = f_3 = 16.93 N$

$M = 10.16$ Nm (Ans)

J. Ghosh