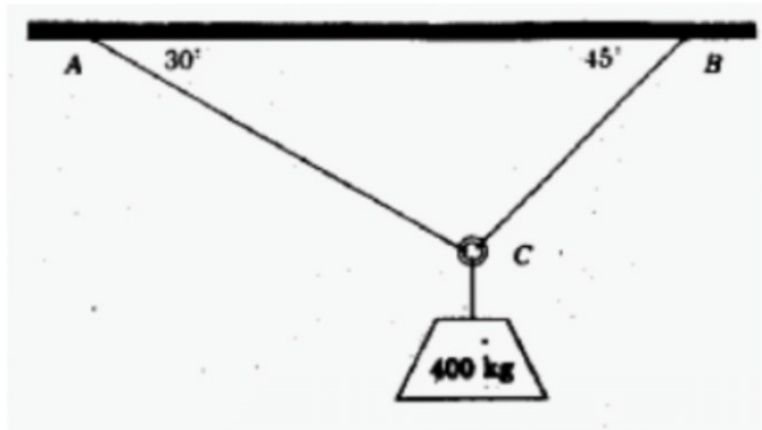
PROBLEM SHEET 1 [EQUILIBRIUM]

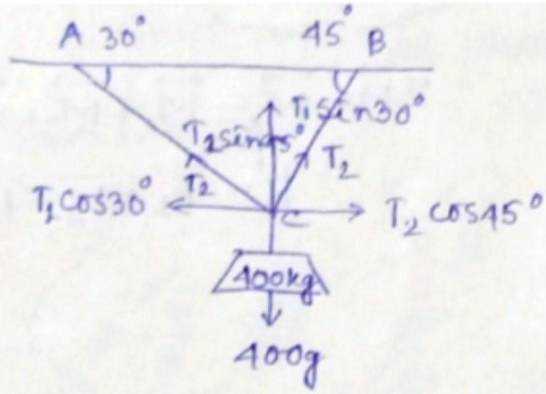
New Problem Sheet No. 1
(Equilibrium)

1. Determine the tension in cables CA and CB.

Ans. $T_{CA} = 2870$ N, $T_{CB} = 3520$ N.



1.



Since the system is in equilibrium

$$T_1 \sin 30° + T_2 \sin 45° = 400g \text{ --- (1)}$$

$$T_1 \cos 30° = T_2 \cos 45° \text{ --- (ii)}$$

From (ii) ⇒

$\cancel{T_1 = 0.866 T_2}$ $T_1 = \frac{2}{\sqrt{3}} \times \frac{1}{\sqrt{2}} T_2$

From (1) ⇒

$$T_1 \times \frac{1}{2} + T_2 \times \frac{1}{\sqrt{2}} = 400 \times 9.8$$

or $\frac{\cancel{2}}{\sqrt{3}} \times \frac{1}{\sqrt{2}} \times T_2 \times \frac{1}{\cancel{2}} + \frac{T_2}{\sqrt{2}} = 3920$

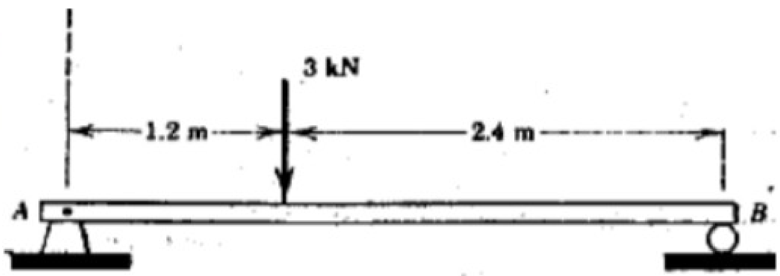
or $\frac{T_2}{\sqrt{2}}\left(\frac{1}{\sqrt{3}} + 1\right) = 3920$

or $T_2 = 3511$ N [Ans]

$T_1 = 2869$ N [Ans]

2. The uniform bar has a mass per unit length of 60 kg/m. Determine the reactions at the supports.

Ans. $A_y = 3060$ N, $B_y = 2060$ N.



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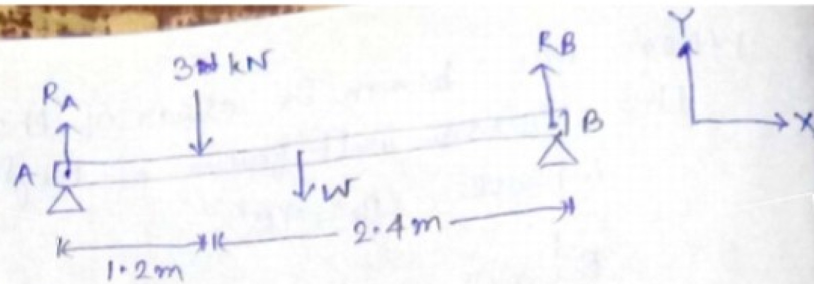


Total mass = $60\times(1.2+2.4)$ kg

= 216 kg

Considering the moments wrt the point A ⇒

$$-3\times1.2\times1000 - 216\times\cancel{1.8}\,10\times1.8 + R_B\times3.6 = 0$$

or $R_B = 2080$ N. [Ans]

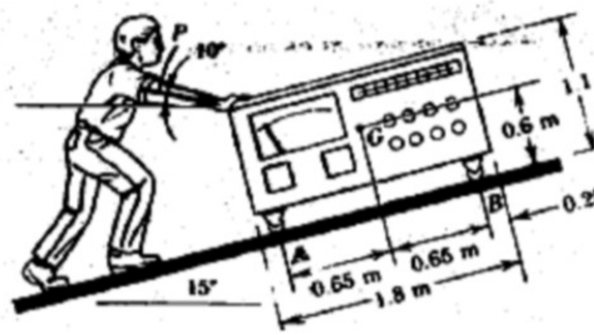
Considering the moments wrt the point B ⇒

$$1.8\times216\times10 + 3000\times2.4 - R_A\times3.6 = 0$$

or $R_A = 3080$ N. [Ans]

3. A man pushes the 40 kg machine with mass centre at G up an incline at a steady speed. Determine the required force magnitude P and the normal reaction forces at A and B. Neglect the small effects of friction.

Ans. P = 112.1 N, N_B = 219 N, N_A = 207 N.



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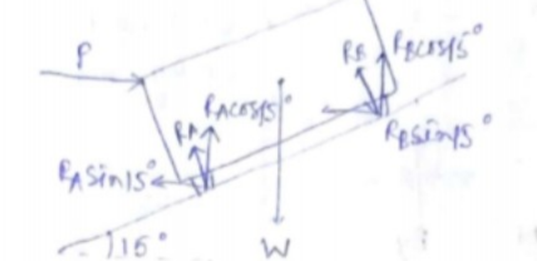
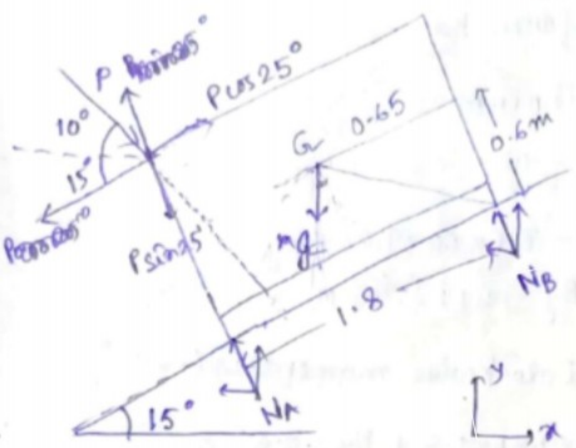


As per the diagram,

$R_A \sin 15° + R_B \sin 15° = P$ ---- (i)



$\Sigma F_x = +P\cos 10° - N_B \sin 15° - N_A \sin 15° = 0$

or $P\cos 10° = (N_B + N_A)\sin 15°$

or $P \times 3.805 = N_B + N_A$

$\Sigma F_y = N_A \cos 15° + N_B \cos 15° - P \sin 10° - 40 \times 9.81 = 0$

or $(N_A + N_B)\cos 15° = P\sin 10° + 392.4$

or $P \times 3.675 - P \times 0.174 = 392.4$

or $P = 112.08$ N

≈ 112.1 N. (Ans)

$\Sigma M_G = 0$; considering ↺ +ve,

$-N_A \times 0.65 + N_B \times 0.65 + P\cos 25° \times (1.1 - 0.6) + P\sin 25° \times 0.08 = 0$

~~[illegible]~~

or $N_B - N_A = \frac{112.1}{0.65}(0.453 - 0.380)$

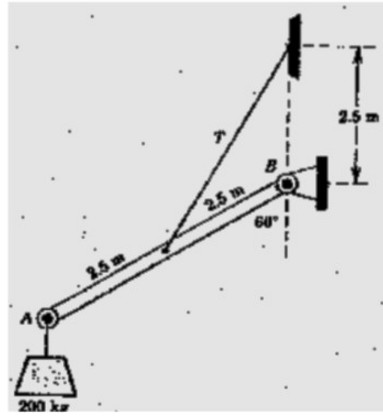
or $N_B - N_A \approx 12.53 \ldots$

∴ $N_B = \frac{1}{2}(439.07)$ N $\approx$ 219.53 N.

N_A = 207N. } [Ans]

4. The uniform bar AB has a mass of 50 kg and supports the 200-kg load at A. Calculate the tension in the supporting cable and the magnitude F_B of the force supported by the pin at B.

Ans. $T = 7650$ N, $F_B = 5660$ N



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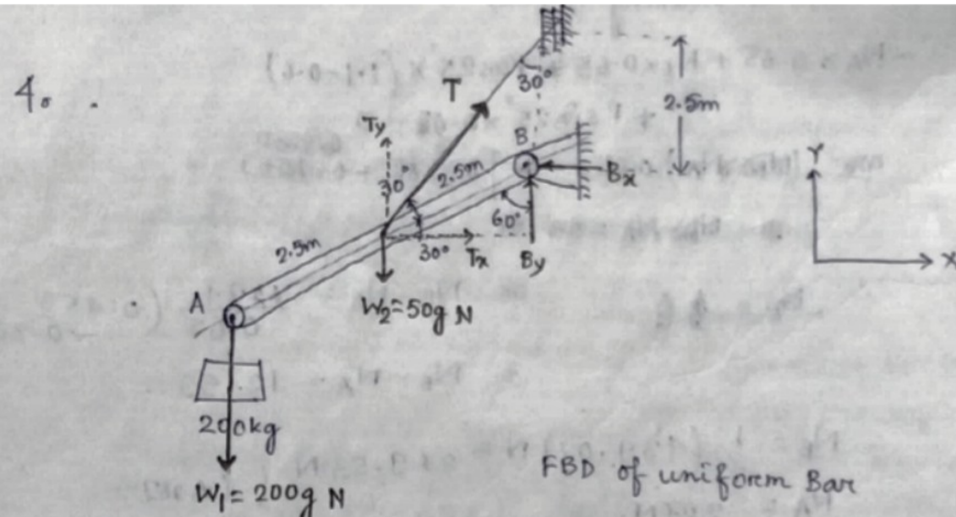

FBD of uniform Bar

From the conditions of equilibrium

i) $\Sigma F_x = 0$ or, $T\cos 60^\circ - B_x = 0$ ......(i)

ii) $\Sigma F_y = 0$ or, $B_y + T\sin 60^\circ - 200g - 50g = 0$

or, $\frac{\sqrt{3}}{2}T + B_y = 2452.5$ ......(ii)

iii) $\Sigma M_B = 0$, taking ↻ +ve

or, $200 \times 9.81 \times 5\cos 30^\circ + 50 \times 9.81 \times 2.5\cos 30^\circ - T \times 2.5\sin 30^\circ = 0.$

or, $T = 7646.13$ N [Ans]

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From, (i) ⇒

$B_x = 7646.13 \times \frac{1}{2}$

$= 3823.065$ N

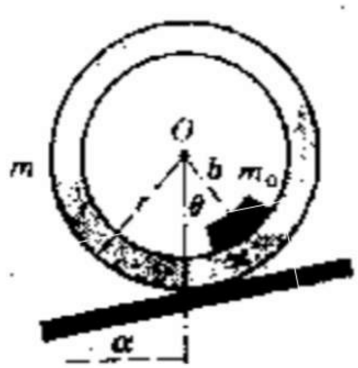
From, (ii) ⇒ $B_y = -4169.25$ N [Ans]

$\therefore F_B = \sqrt{B_x^2 + B_y^2}$

$= 5656.7$ N [Ans]

5. A uniform ring of mass m and radius r carries an eccentric mass m_0 at a radius b and is in an equilibrium position on the incline, which makes an angle α with the horizontal. If the contacting surfaces are rough enough to prevent slipping, write the expression for the angle θ which defines the equilibrium position.

$$Ans.\theta = \sin^{-1}\left[\frac{r}{b}\left(1+\frac{m}{m_0}\right)\sin\alpha\right]$$



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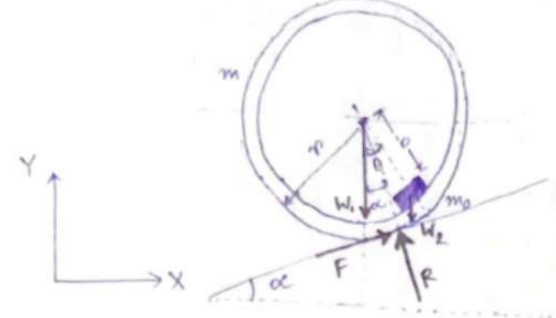


Derive the expression of θ.

FBD of ring and the eccentric mass

From the conditions of equilibrium :-

1) $\Sigma F_x = 0$ $\therefore$ $F\cos\alpha - R\sin\alpha = 0$

or $F = R\tan\alpha$ ....(i)

ii) $\Sigma F_y = 0$ $\therefore$ $R\cos\alpha + F\sin\alpha - mg - m_0 g = 0$

or $F\sin\alpha + R\cos\alpha = (m+m_0)g$ ------(ii)

iii) $\Sigma M_0 = 0$; $\circlearrowleft$(+ve)

or $+F\times r - m_0 g \times b\sin\theta = 0$.

or $Fr = m_0 g b\sin\theta$ -----(iii)

From eqn (ii) ⇒

$$R\tan\alpha\sin\alpha + R\cos\alpha = (m+m_0)g$$

$$\text{or } \frac{R}{\cos\alpha} = (m+m_0)g$$

$$\text{or } R = (m+m_0)g\cos\alpha$$

From eqn (i) ⇒

$$F = (m+m_0)g\sin\alpha$$

From eqn (iii) ⇒

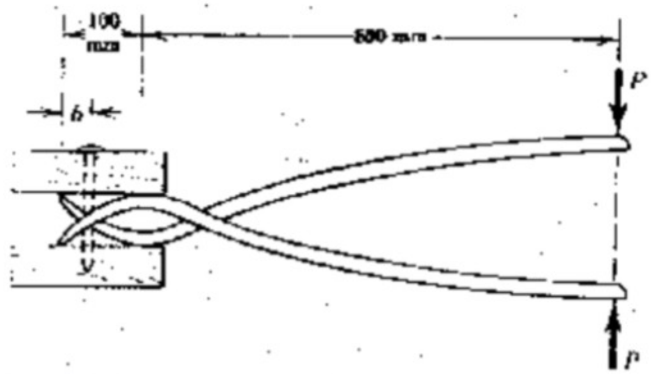
$$(m+m_0)\cancel{g}\sin\alpha\, r = m_0\cancel{g}b\sin\theta.$$

$$\text{or } \sin\theta = \left(1+\frac{m}{m_0}\right)\frac{r}{b}\sin\alpha.$$

$$\therefore \theta = \sin^{-1}\left[\frac{r}{b}\left(1+\frac{m}{m_0}\right)\sin\alpha\right]$$ (Proved).

6. The two planks are connected by a large spike. If a force P = 120 N is required on the handle of each crowbar to loosen the spike, calculate the corresponding tension T in the spike. Also find the value of b which will eliminate any tendency to bend the spike. State any assumptions which you make.

Ans. T = 1560 N, b = 53.8 mm



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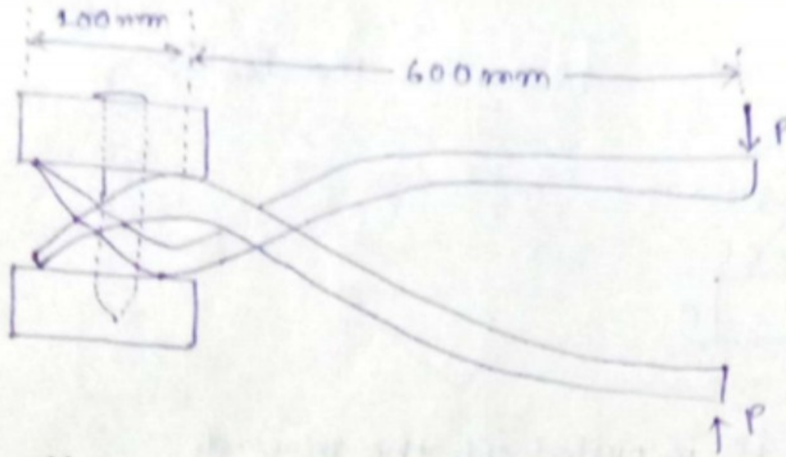
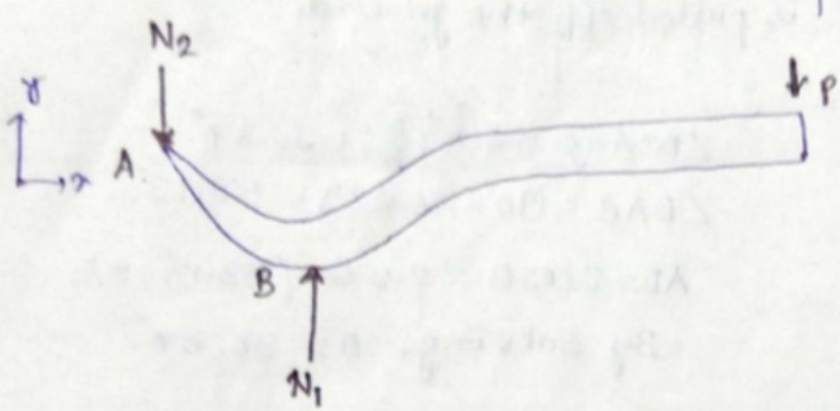




FBD of crowbar

$\Sigma M_B = 0$ ↺+ve

or $N_2(100) - P(600) = 0$

or $N_2 = 6 \times P$

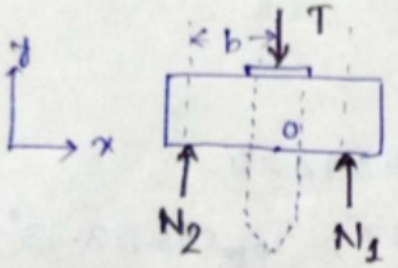
$= 720 N.$

$\Sigma F_y = 0$

or $N_2 + P = N_1$

or $720 + 120 = N_1$

or $N_1 = 840 N$



FBD of plank

$\Sigma F_y = 0$

or $T = N_2 + N_1$

or $T = (720 + 840) N$

$= 1560 N.$ [Ans]

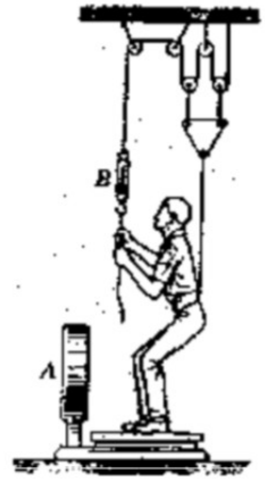
$\Sigma M_0 = 0$

or $N_1(100 - b) - N_2 b = 0$

or $b = 53.8 mm$ [Ans]

7. A former student of mechanics wishes to weigh himself but has access only to a scale A with capacity limited to 400 N and a small 80-N spring dynamometer B. With the rig shown he discovers that when he exerts a pull on the rope so that B registers 76 N, the scale A reads 268 N. What are his correct weight and mass m?

Ans. W= 648 N, m= 66.1 kG



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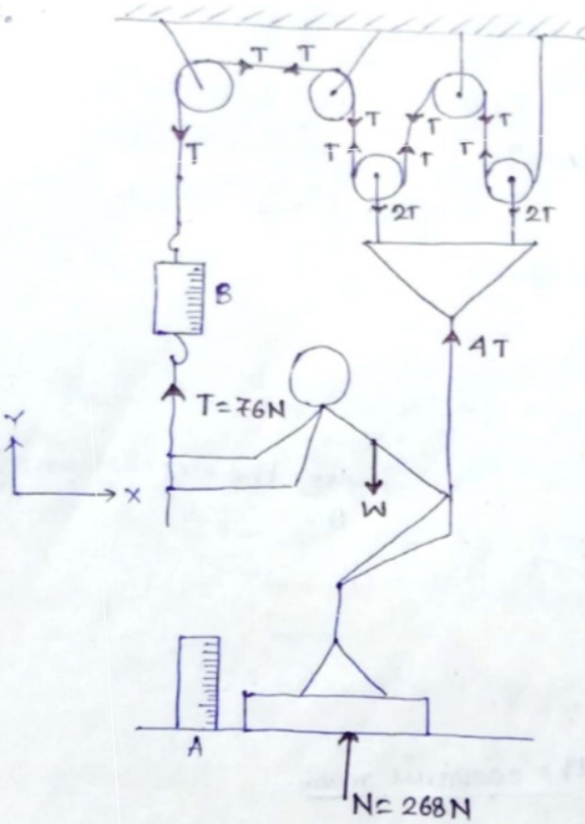


FBD of the student

$\Sigma F_y = 0$

or $T + 4T - W + N = 0$

or $W = (5T + N)$

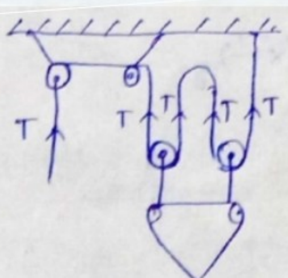
or $W = (5 \times 76 + 268)$ Newton

$= 648$ N [Ans]

$\therefore m = \frac{648}{9.81}$ kg

$= 66.1$ kg [Ans]

7.



So, the apparent weight is $(W - 5T)$ N.

Now, the pull he exerts on the rope is 76 N.

$\therefore T = 76$ N

A/Q $\Rightarrow$ $W - 5 \times 76 = 268$

or $W = 268 + 5 \times 76$

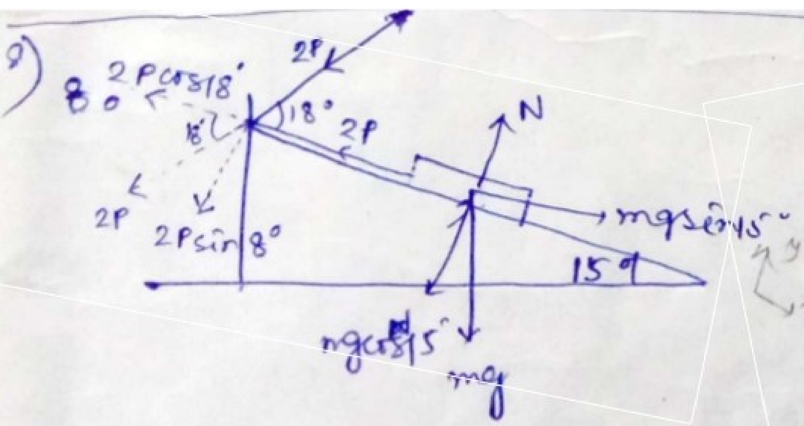
$= 648$ N [Ans]

$\therefore$ The actual mass $= \frac{648}{9.8}$ kg $= 66.1$ kg [Ans]

8. The exercise machine consists of a lightweight cart which is mounted on small rollers so that it is free to move along the inclined ramp. Two cables are attached to the cart – one for each hand. If the hands are together so that the cables are parallel and if each cable lies essentially in a vertical plane, determine the force P which each hand must exert on its cable in order to maintain an equilibrium position. The mass of the person is 70 kg, the ramp angle is 15^0 and the angle β is 18^0. In addition, calculate the force R which the ramp exerts on the cart.

Ans. P = 45.5 N, R = 691 N

8)



$\Sigma F_x = 0$

$mg\sin 15^\circ = 2P + 2P\cos 18^\circ$

or $2P(1+\cos 18^\circ) = 177.73$

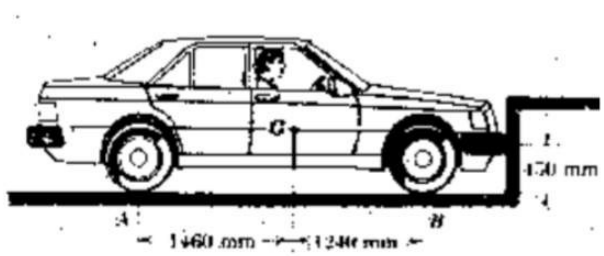
or $P = 45.5\ N$

$\Sigma F_y = 0$

or $N - mg\cos 15^\circ - 2P\sin 18^\circ = 0$

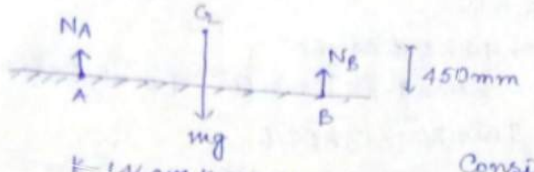
or $N \simeq 691.42\ N$

9. The rear-wheel-drive car has a mass of 1400 kg with mass center at G. First calculate the normal forces under the front and rear wheel pairs when the car is at normal test. Then repeat your calculations for the case shown where the front bumper is tested by causing the car to push, via its rear wheels, against the fixed barrier with a 2500-N force. Neglect friction at the bumper-barrier interface, but not at the tire-ground interface.



Ans. Normally, $N_A = 6310$ N, $N_B = 7430$ N
Under test, $N_A = 6720$ N, $N_B = 7010$ N

9. # First Case



Considering the system is in equilibrium

$\Sigma M_G = 0$

or $+ N_A \times 146\cancel{0} = + N_B \times 124\cancel{0}$

or $73 N_A = 62 N_B$

Now, $N_A + N_B = mg$

or $N_A + N_B = 1400 \times 9.8$

or $\frac{62}{73} N_B + N_B = 1400 \times 9.8$

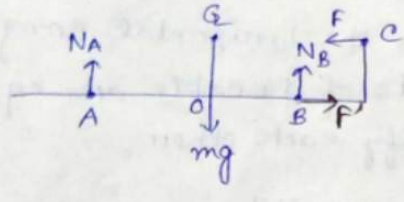
or $N_B = 7418.96$

≈ 7420 N.

$\therefore N_A = 6300$ N [Ans].

Second Case

Considering the system is in equilibrium,



$\Sigma M_O = 0$

or $-N_A \times 1460 + N_B \times 1240 + F \times 450 = 0$

or $N_A \times 146\cancel{0}$

$= N_B \times 124\cancel{0} + 2500 \times 45\cancel{0}$

$\therefore$ or $146 N_A - 124 N_B = 112500$ ---(1)

Now, $N_A + N_B = mg$

or $N_A + N_B = 13720$ ---(ii)

Now, (1) + 124 (ii) $\Rightarrow$

$270 N_A = 112500 + 1701280$

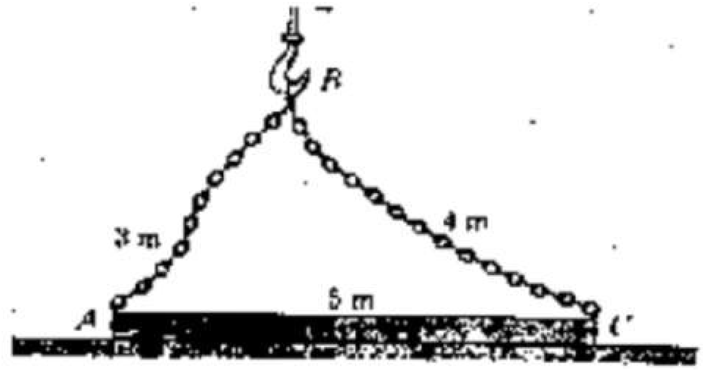
or $N_A = 6717.70$

≈ 6720 N. [Ans]

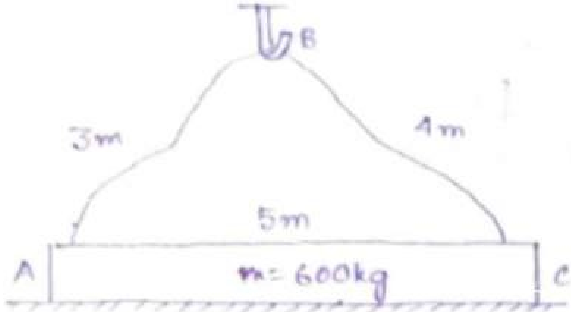
$\therefore N_B = 7000$ N.

10. The 5-m uniform steel beam has a mass of 600 kg and is to be lifted from the ring at B with the two chains, AB of length 3 ,m, and CB of length 4 m. Determine the tension in chain AB when the beam is clear of the platform.

Ans. T = 3.53 kN

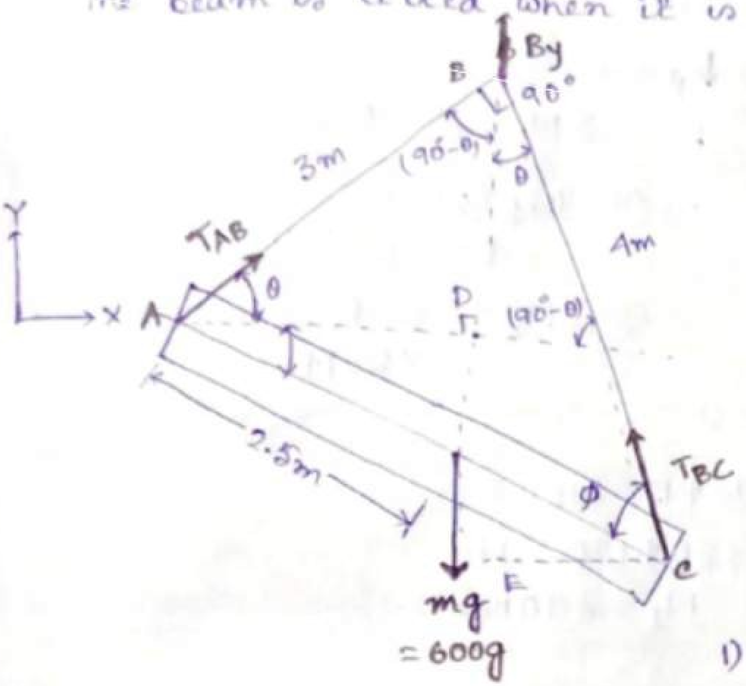


10.



Determine the $T_{AB} = ?$

The beam is tilted when it is pulled off the ground.



FBD of AC

$\angle BCA = \phi = \sin^{-1}\left(\frac{3}{5}\right) = 36.87^\circ$.

$\angle BAC = (90^\circ - 36.87^\circ) = 53.13^\circ$.

$AD = 3\cos\theta = 2.5\cos(53.13^\circ - \theta)$

By solving, $\theta = 36.87^\circ$.

$\angle BCE = \angle BCA + 53.13^\circ - 36.87^\circ$

$= 53.13^\circ$.

Now,

1) $\Sigma F_x = 0$

$\therefore T_{AB}\cos\theta - T_{BC}\cos 53.13^\circ = 0$

or $T_{AB}\cos 36.87^\circ = T_{BC}\cos 53.13^\circ$

or $T_{BC} = T_{AB} \times 1.333$ ------(1)

(ii) $\Sigma F_y = 0$

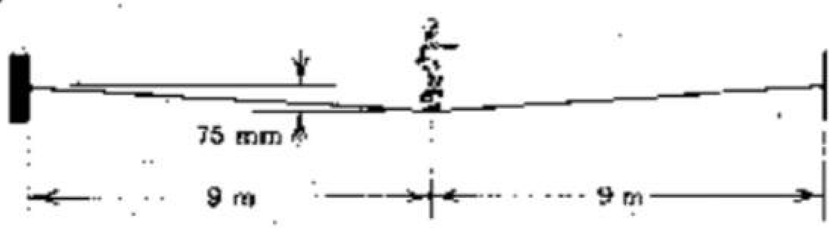
or $T_{AB}\sin\theta + T_{BC}\sin 53.13^\circ - 600g = 0$

or $T_{AB}\sin 36.87^\circ + T_{AB} \times 1.333 \times \sin 53.13^\circ = 600 \times 9.81$

or $T_{AB} = 3.53\,kN$.

11. A 50-kg acrobat pedals her unicycle across the taut but slightly elastic cable. If the deflection at the center of the 18-m span is 75 mm, determine the tension in the cable. Neglect the effects of the weights of the cable and unicycle.

Ans. T = 29.4 kN



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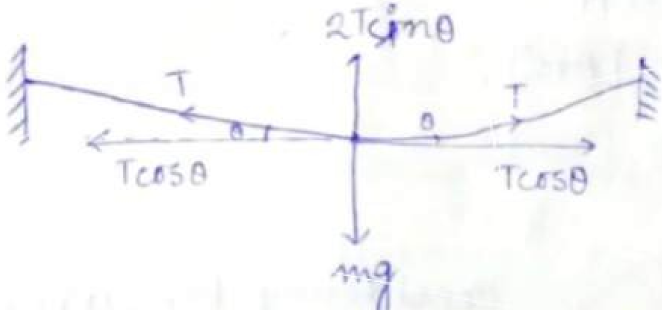


In the diagram we can see, when the acrobat is in the mid-point of the cable, the horizontal components of the tension in the two parts of the cable are equal and opposite. So, they nullify each other.

Now, as the system is in equilibrium

$$2T\sin\theta = mg$$

or $2T\tan\theta \approx mg$ (Since θ is very small)

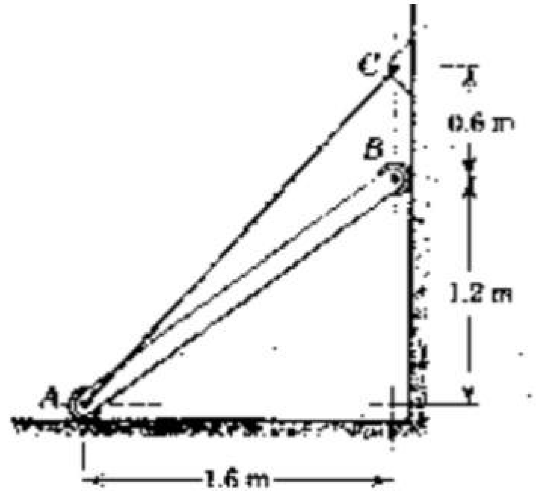
or $2 \times T \times \frac{0.075}{9} = 50 \times 9.8$

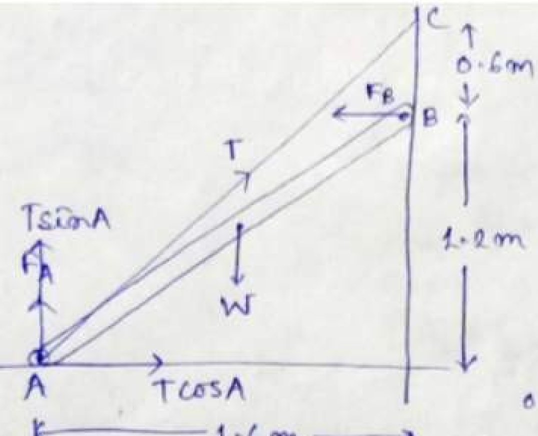
or $T = 29400$ N

$= 29.4$ kN [Ans]

12. The uniform 30-kg bar with end rollers is supported by the horizontal and vertical surfaces and by the wire AC. Calculate the tension T in the wire and the reactions against the rollers at A and at B.

Ans. 73.6 N, B = 196.2 N, T = 295 N





$W = 30 \times 9.81$ N
$= 294.3$N

Since, the system is in equilibrium

$\Sigma M_A = 0$

or $-W \times 0.8 + F_B \times 1.2 = 0$

or $F_B = \frac{294.3 \times 8}{12}$

$= 196.2$ N [Ans]

$\Sigma M_C = 0$

or $-F_B \times 0.6 - F_A \times 1.6 + W \times 0.8 = 0$

or $F_A \times 1.6 = 294.3 \times 0.8 - 196.2 \times 0.6$

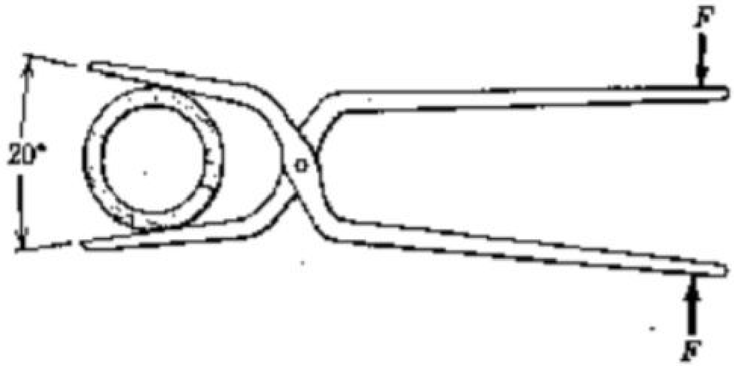
or $F_A = 73.57$

≈ 73.6 N [Ans]

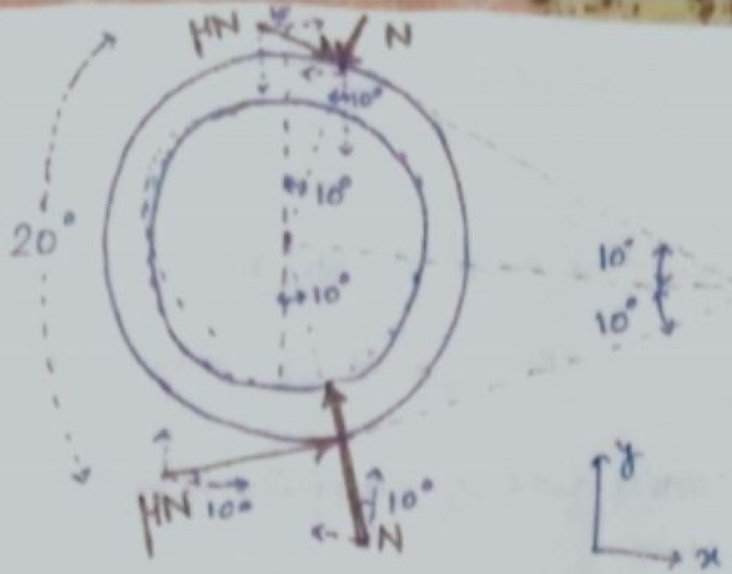
PROBLEM SHEET 2
[FRICTION]

3. The tongs are used to handle hot steel tubes that are being heat-treated in an oil bath. For a 200 jaw opening, what is the minimum coefficient of static friction between the jaws and the tube that will enable the tongs to grip the tube without slipping.

Ans. $\mu_s = 0.176$



3. 14)



FBD of tube

$$\Sigma F_x = 0$$

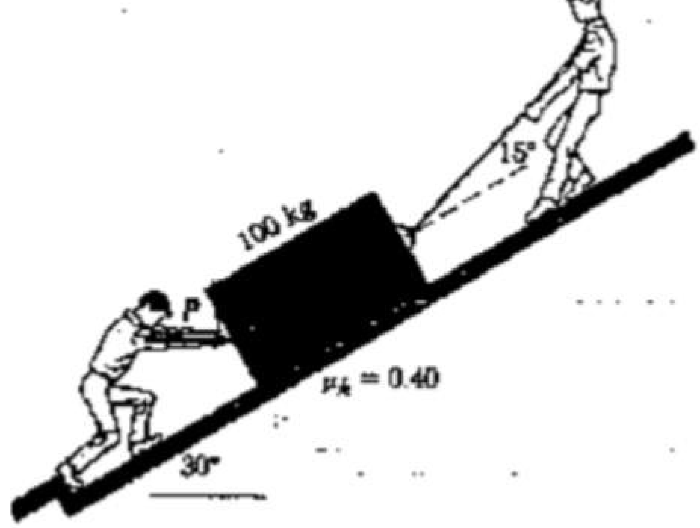
or $-N\sin 10^\circ - N\sin 10^\circ + \mu N\cos 10^\circ + \mu N\cos 10^\circ = 0$

or $\mu = \tan 10^\circ$

$= 0.176$ (Ans)

4. Two men are sliding a 100-kg crate up an incline. If the lower man pushes horizontally with a force of 500 N and if the coefficient of kinetic friction is 0.4, determine the tension T which the man must exert in the rope to maintain motion of the crate.

Ans. T = 465 N



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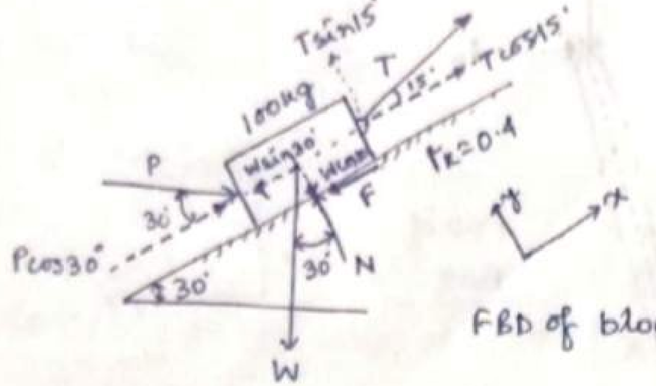


FBD of block

$P = 500N$

$W = 100 \times 9.81\ N$

$= 981N$

$F = \mu_k N = (0.4N)$ Newton

From the conditions of equilibrium,

i) $\Sigma F_x = 0$

$\therefore P\cos 30^\circ + T\cos 15^\circ - \mu_k N - W\sin 30^\circ = 0$

or $0.4N = 0.966T - 57.49$

or $N = 2.415T - 143.725 \cdots (i)$

ii) $\Sigma F_y = 0$

$\therefore T\sin 15^\circ - W\cos 30^\circ - P\sin 30^\circ + N = 0$

or $N = W\cos 30^\circ + P\sin 30^\circ - T\sin 15^\circ$

or $N = 1099.57 - 0.259T \cdots (ii)$

From (i), (ii) $\Rightarrow$

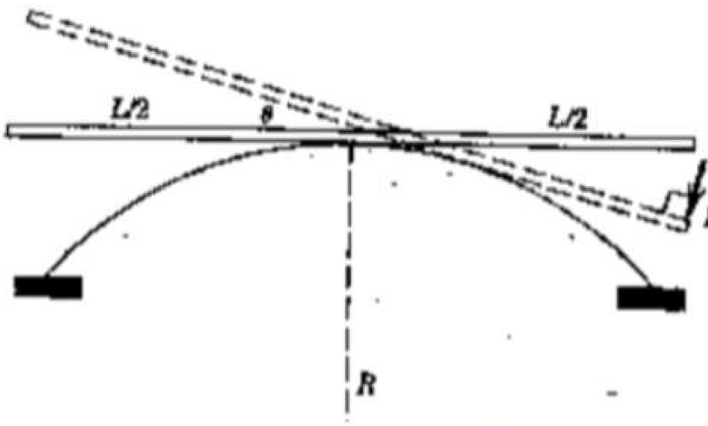
$2.415T - 143.725 = 1099.57 - 0.259T$

or $2.674T = 1243.295$

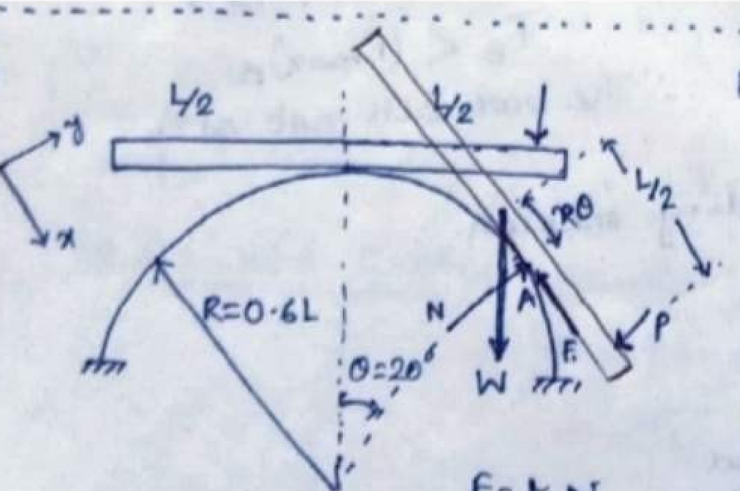
or $T = 465\ N$ (Approx)

5. The uniform slender rod of mass m and length L is initially at rest in a centered horizontal position on the fixed circular surface of radius $R = 0.6L$. If a force normal to the bar is gradually applied to its end until the bar begins to slip at the angle $\theta = 20^0$, determine the coefficient of static friction μ_s.

Ans. $\mu_s = 0.212$



5)



$F = \mu_s N$

Find μ_s.

i) $\Sigma F_x = 0$

or $W\sin\theta - F = 0$

or $mg\sin\theta = \mu_s N$ ------ (i)

ii) $\Sigma F_y = 0$

$N - mg\cos\theta - P = 0$

or $N = P + mg\cos\theta$ ----(ii)

iii) $\Sigma M_A = 0$ $\therefore + mg\cos\theta \times R\theta - P(L/2 - R\theta) = 0$

or $mg\cos\theta \times 0.6L \times \theta - P(\frac{L}{2} - 0.6L\theta) = 0$

or $mg\cos\theta \times L\theta \times 0.6 - P(\frac{1}{2} - 0.6\theta) = 0$ --- (iii)

From (i) & (ii) $\Rightarrow$

$mg\sin\theta = \mu_s(P + mg\cos\theta)$

or $mg\sin\theta = \mu_s\left[mg\cos\theta + \frac{0.6\, mg\cos\theta \times \theta}{0.5 - 0.6\theta}\right]$

or $\cancel{mg}\sin\theta = \mu_s \cancel{mg}\cos\theta\left[1 + \frac{0.6\theta}{0.5 - 0.6\theta}\right]$

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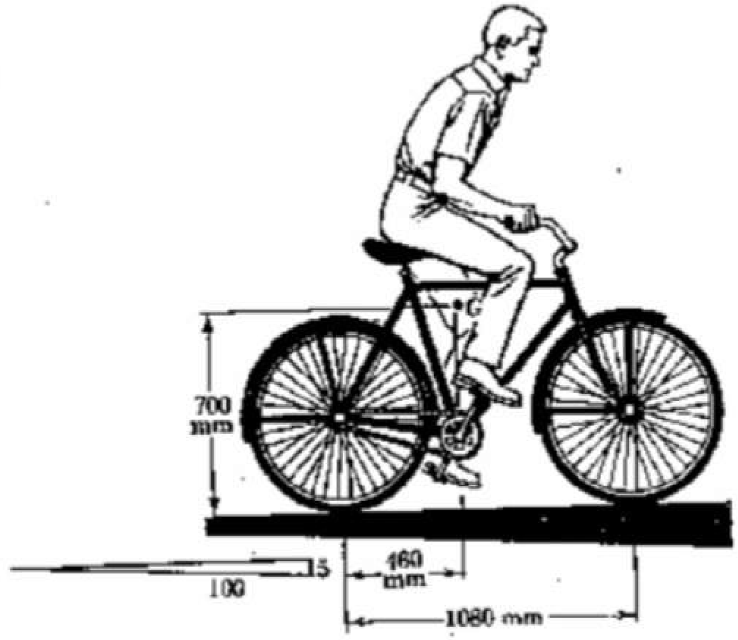
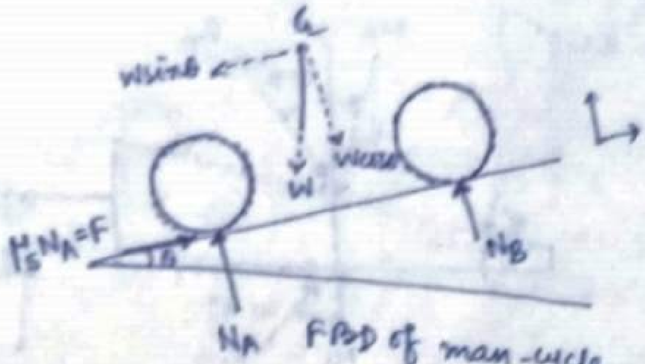
or $\mu_s = 0.212$ (Ans)

7. A man pedals his bicycle up a 5 percent grade on a slippery road at a steady speed. The man and bicycle have a combined mass of 82 kg with mass center at G. If his rear wheel is on the verge of slipping, determine the coefficient of friction μ_s between the rear tire and the road. If the coefficient of friction were doubled, what would be the friction force acting on the rear wheel?

Ans. $\mu_s = 0.082$, $F = 40.2$ N



7. 1)



FBD of man-cycle

Now $\tan\theta = \frac{5}{100} = 0.05$

$\Rightarrow \theta = 2.862°$

Considering the system in eq–

i) $\Sigma F_x = 0$

or $\mu_s N_A + (-W\sin\theta) = 0$

or $\mu_s N_A = 40.17 \approx 40.2$ ---- (i)

ii) $\Sigma F_y = 0$

or $N_A + N_B = W\cos\theta$

or $N_A + N_B = 803.42$ ---- (ii)

iii) $\Sigma M_G = 0$; ↺ +ve

or $-N_A \times 460 + N_B(1080 - 460) + \mu_s N_A \times 700 = 0$

or $-460 N_A + 620(803.42 - N_A) + 40.2 \times 700 = 0$

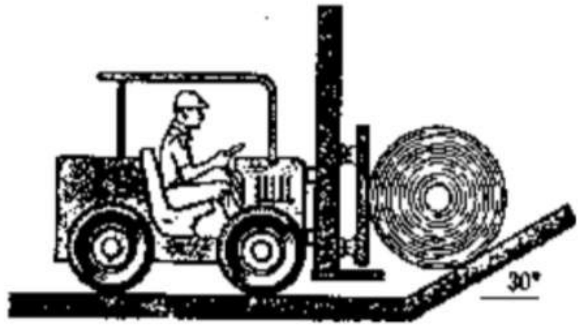
or $N_A = 487.28$ ---- (iii)

Now, from (i), (iii) $\Rightarrow$ $\mu_s = \frac{40.2}{487.28} = 0.082$ [Ans]

ii) Now, even if we double the co-efficient of friction in between the road and the wheel, the friction force would remain same $= 40.2$ N, but the wheel would not be at the verge of slipping.

8. The industrial truck is used to move the solid 1200-kg roll of paper up the 300 incline. If the coefficients of static and kinetic friction between the roll and the vertical barrier of the truck and between the roll and the incline are both 0.40, compute the required tractive force P between the tires of the truck and the horizontal surface.

Ans. P = 22.1 kN



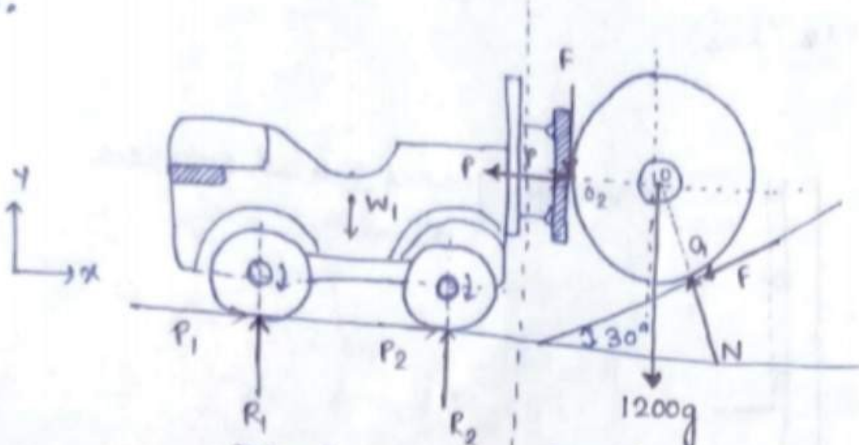
8.

$m = 1200\,kg$

$\mu = 0.4$

Determine tractive force P.

* Only one point of contact it can slide, otherwise it rolls.



FBD of the truck

FBD of the roll

$P_1 + P_2 = P$ (Tractive Force)

i) $\Sigma F_x = 0$ $\qquad \therefore F = (P - N/2)\frac{2}{\sqrt{3}}$

or $P - N\sin 30^\circ - F\cos 30^\circ = 0$ ....(1)

ii) $\Sigma F_y = 0$

or $-1200g - F\sin 30^\circ + N\cos 30^\circ - F = 0$

or $N\frac{\sqrt{3}}{2} - \frac{3F}{2} - 1200\times 9.81 = 0$

or $N\frac{\sqrt{3}}{2} - \frac{3}{2}\left(P - \frac{N}{2}\right)\frac{2}{\sqrt{3}} - 1200\times 9.81 = 0$

or $N = P + \frac{1200\times 9.81}{\sqrt{3}}$

$\therefore N > P$

$\therefore \mu N > \mu P$

So, the roller will slip at point O_2 first.

$\therefore F = \mu P$

Now, $\Sigma M_{O_1} = 0$; (+ve)

or $P\times \cancel{r}\cos 30^\circ - F(\cancel{r} + \cancel{r}\cos 60^\circ) - 1200\times 9.81\times \cancel{r}\cos 60^\circ = 0$

or $P\times \cos 30^\circ - 0.4P(1 + \cos 60^\circ) - 1200\times 9.81\times \cos 60^\circ = 0$

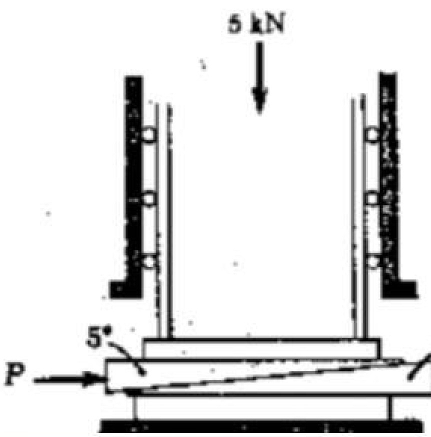
or $P = 22103\,N = 22.103\,kN$ (Ans)

$P - N\times\frac{1}{2} - 0.40P\times\frac{\sqrt{3}}{2} = 0$

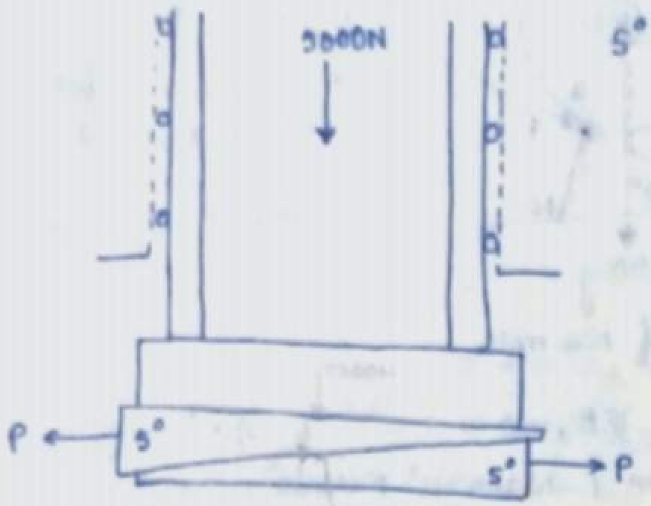
or $1.307P = N$

9. The two 5^0 wedges shown are used to adjust the position of the column under a vertical load of 5 kN. Determine the magnitude of forces P required to lower the column if the coefficient of friction for all surfaces is 0.40.

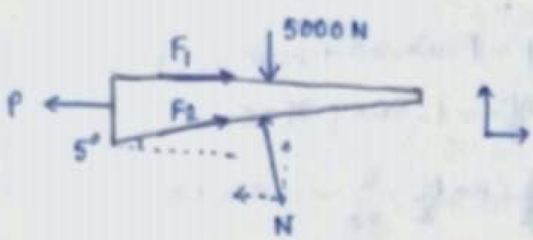
Ans. P = 3.51 kN



9)



5° wedge, $\mu = 0.4$ for all surfaces.
Find P.



$\Sigma F_x = 0$

or $5000 \times 0.40 + 0.4N\cos 5^\circ - N\sin 5^\circ - P = 0$

or $P - \cancel{0.4}\ 0.311 N = 2000$ ....(1)

$\Sigma F_y = 0$

or $0.4N\sin 5^\circ + N\cos 5^\circ - 5000 = 0$

or $N = 4849.39$ N

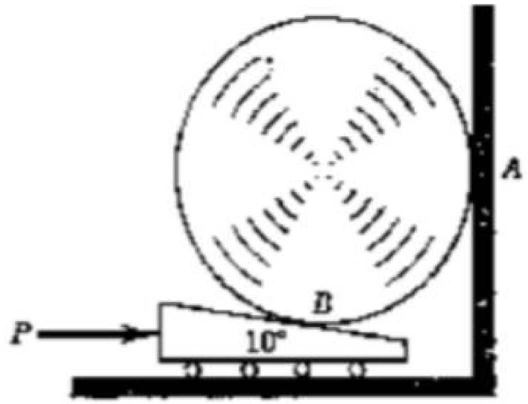
$\therefore P = 2000 + 0.311 \times 4849.39$

$= 3508.16$ N

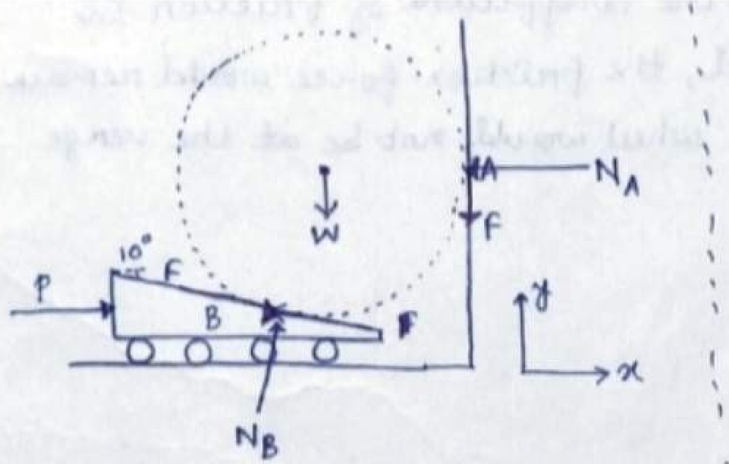
$= 3.51$ kN

10. Calculate the horizontal force P on the light 10^0 wedge necessary to initiate movement of the 40-kg cylinder. The coefficient of static friction for both pairs of contacting surfaces is 0.25. Also determine the friction force FB at point B.

Ans. P = 98.6 N, F_B = 24.6 N



10)



FBD of the Cylinder

$\Sigma F_x = 0$

or $-N_A + F\cos 10^\circ + N_B \sin 10^\circ = 0$ ----(i)

$\Sigma F_y = 0$

or $N_B \cos 10^\circ - F\sin 10^\circ - F - W = 0$

or $N_B = (1.192F + 398.4)$ ----(ii)

From, (i) ⇒

$-N_A + 0.985F + 1.192F \times 0.174 + 392.4 \times 0.174 = 0$

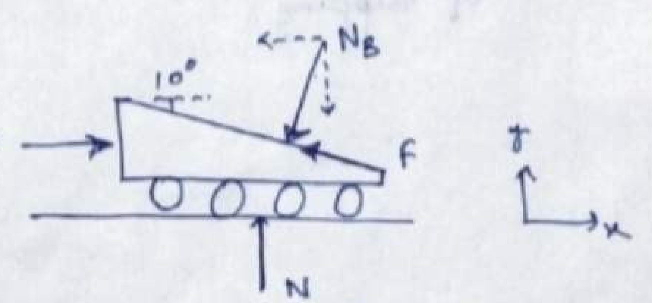
or $N_A = (1.192F + 68.14)$ ----(iii)

So, $N_B > N_A$

$\mu N_B > \mu N_A$

So, it will slip first at A

$F = \mu N_A = 0.25 N_A$ ----(vi)



FBD of wedge

$\Sigma F_x = 0$

$\therefore P - F\cos 10^\circ - N_B \sin 10^\circ = 0$

or $P = F\cos 10^\circ + N_B \sin 10^\circ$ ---(iv)

$\Sigma F_y = 0$

or $N + F\sin 10^\circ - N_B \cos 10^\circ = 0$

or $N = N_B \cos 10^\circ - F\sin 10^\circ$ ---(v)

Now, from (iii),(vi) ⇒

$N_A = 1.192 \times 0.25 N_A + 69.37$

or $N_A = 98.74 N = 98.76 N$

$F = 24.68 N$ (Ans)

From, (ii) ⇒

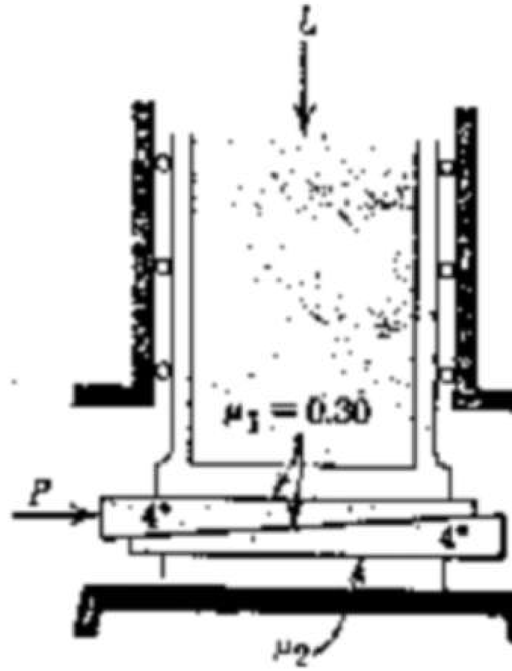
$N_B = 427.77 N$

From (iv) ⇒

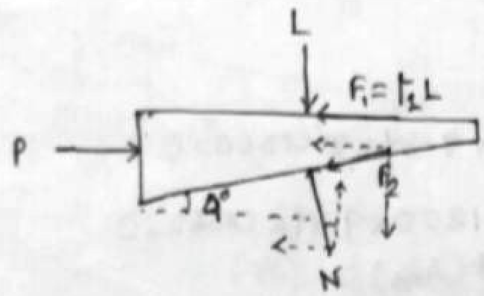
$P = 98.64 N$ (Ans)

11. The two wedges are used to position the vertical column under a load L. What is the least value of the coefficient of friction μ_2 for the bottom pair of surfaces for which the column may be raised by applying a single horizontal force P to the upper wedge?

Ans. $\mu_2 = 0.378$



11.



$\Sigma F_x = 0$

or $P - N\sin 4^\circ - \mu_2 N \cos 4^\circ - \mu_1 L = 0$

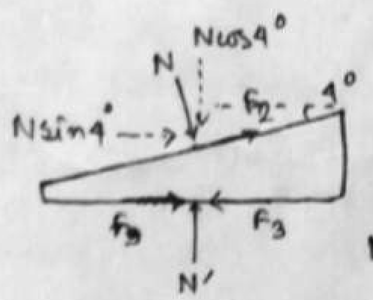
or $P - N(\sin 4^\circ + \mu_2 \cos 4^\circ) = 0.3L$ ----- (i)

$\Sigma F_y = 0$

or $-L - \mu_2 N \sin 4^\circ + N\cos 4^\circ = 0$

or $N(\cancel{\mu_2}\cos 4^\circ - \mu_2 \sin 4^\circ) = L$

or $N = L/0.977 = 1.023L$ -- (ii)



From, (i), $P = 0.3L + 1.023L(0.369)$

$= 0.677L$ .....(iii)

Now, $\Sigma F_x = 0$

$N\sin 4^\circ + 0.3069L \times \cos 4^\circ = \mu_2 N'$

or $\mu_2 \cancel{L} = 1.023\cancel{L}\sin 4^\circ + 0.3069\cancel{L}\cos 4^\circ$

or $\mu_2 = 0.378$. (Ans)

$F_2 = 0.3N$

$= 0.3069L$

$N = 1.023L$

$\Sigma F_y = 0$

or $-N\cos 4^\circ + F_2 \sin 4^\circ + N' = 0$

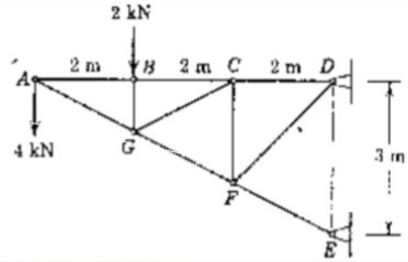
or $N' = 1.021L - 0.021L$

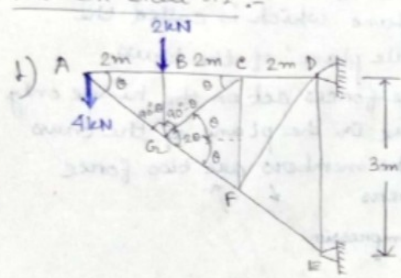
$= L$

PROBLEM SHEET 3.1
[TRUSS]

1. Calculate the forces in members CG and CF for the truss shown. If the 2-kN force acting on the truss were removed, identify by inspection those members in which the forces are zero. On the other hand, if the 2-kN force were applied at G instead of B, would there be any zero force members?

Ans CG = 2.24 kN (T), CF = 1 kN (C)





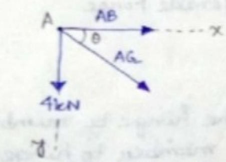
$j=7,\ m=11,\ n=4$

$\therefore\ 11+4 > 2\times 7$

$\therefore$ Statically Indetermined.

Assuming all the members are under tension

$\tan\theta = \frac{3}{6} = \frac{1}{2}$ $\therefore\ \theta = 26.57°$



FBD of joint A

$\Sigma F_x = 0$

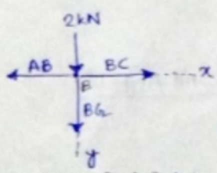
$\therefore\ AG\cos\theta + AB = 0$

$\Sigma F_y = 0$

$AG\sin\theta + 4 = 0$

or $AG = -8.943$ kN

$\therefore\ AB = 8$ kN



FBD of joint B

$\Sigma F_x = 0$

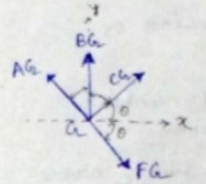
or $BC - AB = 0$

or $BC = AB = 8$ kN

$\Sigma F_y = 0$

or $2 + BG = 0$

or $BG = -2$ kN



FBD of joint G

$\Sigma F_x = 0$

or $CG\cos\theta - AG\cos\theta + FG\cos\theta = 0$

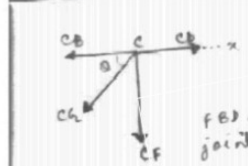
or, $8.943 + CG + FG = 0$

$\Sigma F_y = 0$

or $AG\sin\theta + CG\sin\theta + BG - FG\sin\theta = 0$

or $-8.943\sin\theta - 2 + CG\sin\theta - FG\sin\theta = 0$

By Solving $CG = 2.2356$ kN (T)



FBD of joint C

$\Sigma F_y = 0$

$\therefore\ CF + CG\sin\theta = 0$

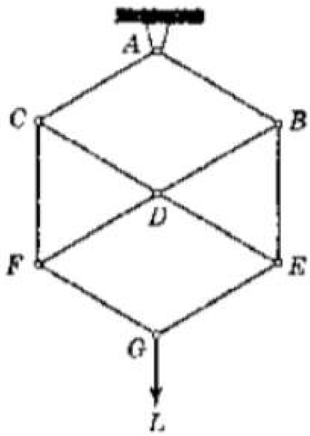
or $CF = -1$ kN

Members	Magnitude of force	Nature
CG	2.2356 kN	T
CF	1 kN	C

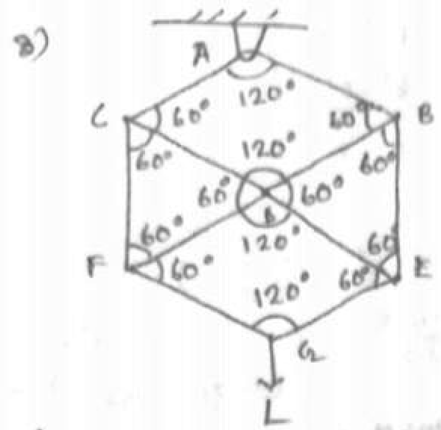
If 2kN is removed, the BG would become zero.
CG " "
CF " "
DF " "

If 2kN force is applied at G instead of B ⇒
BG = 0.

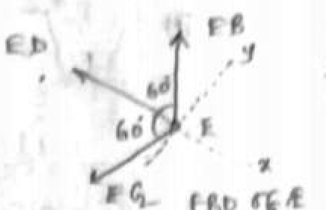
3. Solve for the forces in members BE and BD of the truss which supports the load L. All interior angles are 60° or 120°.



3)



iv)



FBD of E

$\Sigma F_x = 0$ $\therefore -ED - EB\cos 60^\circ - EG\cos 60^\circ = 0$

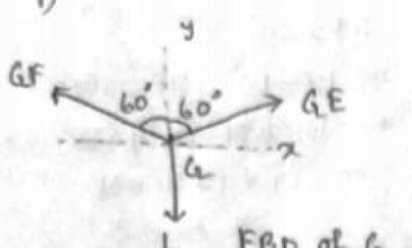
or $ED + \frac{EB}{2} = -\frac{L}{2}$

$\Sigma F_y = 0$ $\therefore -EG\sin 60^\circ + EB\sin 60^\circ = 0$

or $\boxed{EB = L}$ (Ans)

$\boxed{ED = -L}$

i)



FBD of G

$\Sigma F_x = 0$

$\therefore GE\sin 60^\circ = GF\sin 60^\circ$

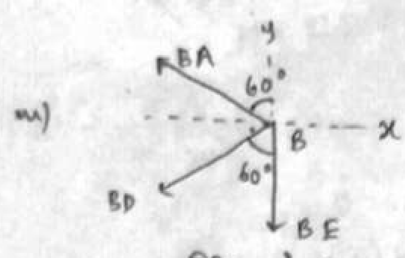
or $GE = GF$

$\Sigma F_y = 0$

$\therefore GE\cos 60^\circ + GF\cos 60^\circ = L$

or $\boxed{GE = GF = L}$

iii)



FBD of B

$\Sigma F_x = 0$ $\therefore -BA\sin 60^\circ - BD\sin 60^\circ = 0$

or $BA = -BD$

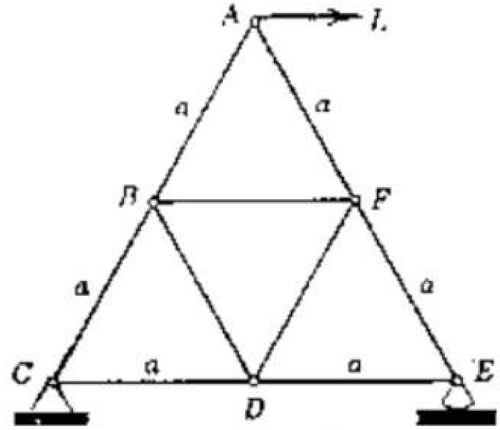
$\Sigma F_y = 0$ $\therefore BA\cos 60^\circ - BD\cos 60^\circ - BE = 0$

or $-2BD\cos 60^\circ - BE = 0$

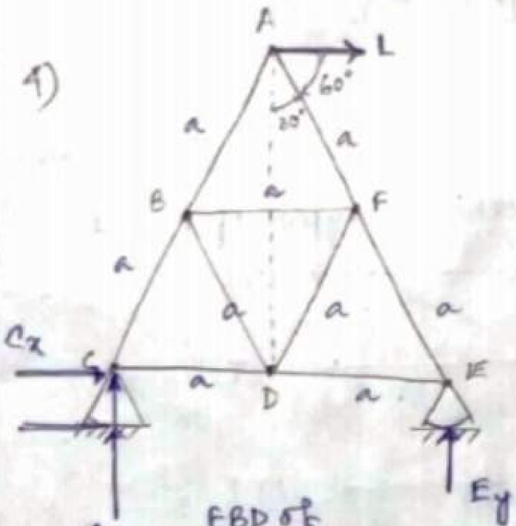
or $\boxed{BD = -L}$ (Ans)

4. The equiangular truss is loaded and supported as shown. Determine the forces in all members in terms of the horizontal load L.

Ans. AB = BC = L (T), AF = EF = L (C)
DE = CD = L/2 (T), BF = DF = BD = 0



4)



FBD of the entire truss

i) $\Sigma M_C = 0$ ↺ +ve

$E_y \times 2a - L(2 \times \frac{\sqrt{3}}{2} a) = 0$

or $E_y = \frac{\sqrt{3}L}{2}$

ii) $\Sigma F_y = 0$

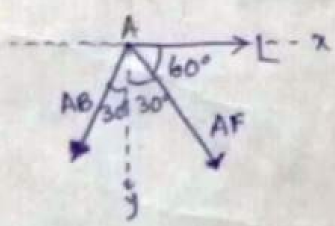
or $C_y + E_y = 0$

or $C_y = -E_y = -\frac{\sqrt{3}}{2}L$

iii) $\Sigma F_x = 0$

or $C_x + L = 0$

or $C_x = -L$



FBD of joint A

$\Sigma F_x = 0$

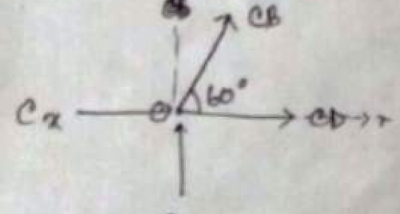
or $L + AF\cos 60° - AB\cos 60° = 0$ ----(i)

$\Sigma F_y = 0$

or $AF\sin 60° + AB\sin 60° = 0$

or $AF = -AB$ ---- (ii)

By Solving $\therefore$ $\boxed{AB = L,\ AF = -L}$



FBD of joint C

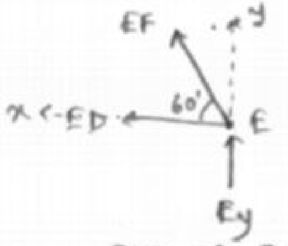
$\Sigma F_x = 0$ $\therefore$ $C_x + CD + CB\cos 60° = 0$

or $-L + CD + \frac{CB}{2} = 0$ ---- (iii)

$\Sigma F_y = 0$, $C_y + CB\sin 60° = 0$ ----

or $CB = (-\frac{\sqrt{3}}{2}L) \times (-\frac{2}{\sqrt{3}}) = L$ [Ans]

$\boxed{CD = \frac{L}{2},\ CB = L}$



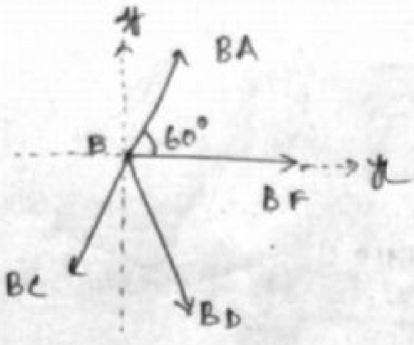
FBD of E

$\Sigma F_x = 0 \quad \therefore \quad ED + EF\cos 60^\circ = 0$

$\Sigma F_y = 0 \quad \therefore \quad E_y + EF\sin 60^\circ = 0$

or $EF = -\frac{\sqrt{3}}{2} L \times \frac{2}{\sqrt{3}} = -L$

$\therefore \boxed{ED = \frac{L}{2}, \; EF = -L}$



FBD of B

$\Sigma F_x = 0 \quad \therefore \quad BA\cos 60^\circ + BF + BD\cos 60^\circ - BC\cos 60^\circ = 0$

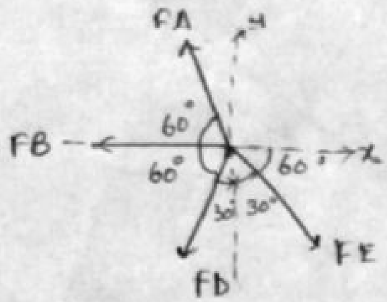
or $L \times \frac{1}{2} + BF + BD\frac{1}{2} - \frac{1}{2} = 0$

or $BF + \frac{BD}{2} = 0$

$\Sigma F_y = 0 \quad BA\sin 60^\circ - BC\sin 60^\circ - BD\sin 60^\circ = 0$

or $\frac{L}{\cancel{2}} - \frac{L}{\cancel{2}} - BD = 0$

or $BD = 0 \quad \therefore \boxed{BF = 0, \; BD = 0}$



$\Sigma F_x = 0 \quad \therefore \; -FA\cos 60^\circ - FB - FD\cos 60^\circ + FE\cos 60^\circ$

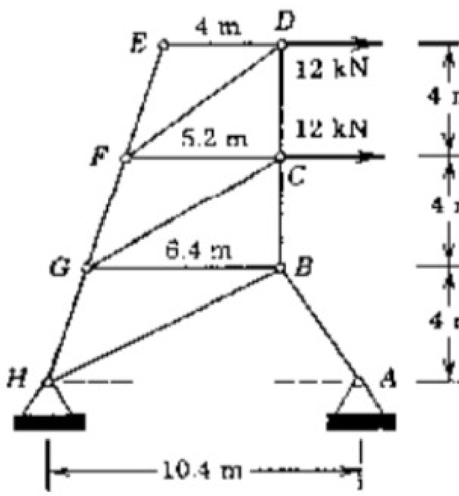
or $\frac{L}{2} - 0 - \frac{FD}{2} - \frac{L}{2} = 0$

or $\boxed{FD = 0}$

Members	Magntitude of Forces	Types of Forces
AB	L	(T)
BC	L	(T)
BD	0	
CD	L/2	(T)
DE	L/2	(T)
DF	0	
FE	L	(C)
FA	L	(C)
BF	0	

5. Calculate the forces in members CF, CG, EF of the loaded truss.

Ans. CG = 25.0 kN (T)



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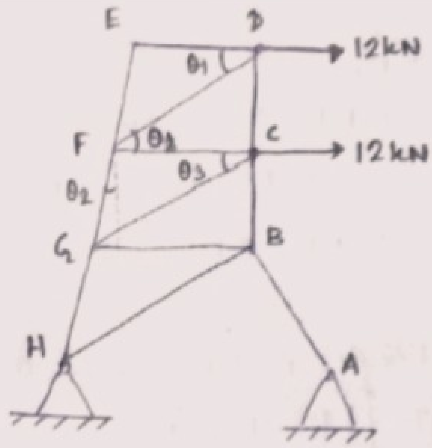
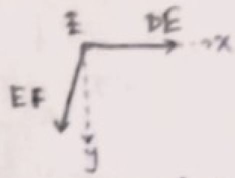




FBD of E

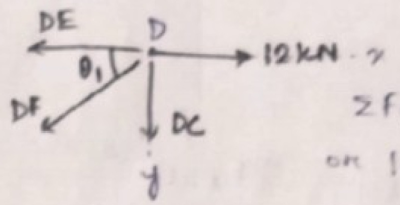
1) $\Sigma F_y = 0 \therefore EF = 0$

ii) $\Sigma F_x = 0 \quad \therefore DE = 0$

$\tan\theta_1 = \frac{4}{5.2} \Rightarrow \theta_1 = 37.57^\circ$

$\tan\theta_2 = \frac{6.4 - 5.2}{4}$; $\theta_2 = 16.69^\circ$

$\tan\theta_3 = \frac{4}{6.4}$, $\theta = 32^\circ$



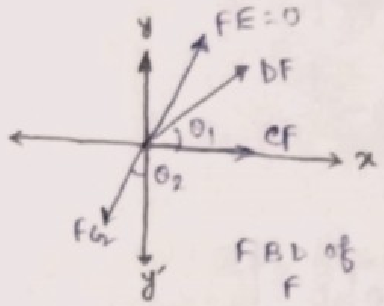
FBD of D

$\Sigma F_x = 0$

or $12 - DE - DF\cos\theta_1 = 0$

or $DE + DF\cos\theta_1 = 12$

or $DF = 15.14$ kN

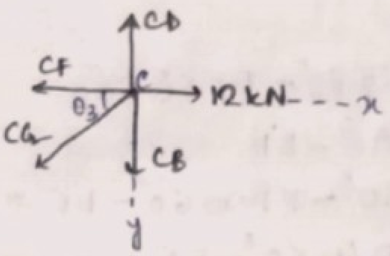


FBD of F

$\Sigma F_y = DF\sin\theta_1 - FG\cos\theta_2 = 0 \quad \therefore FG = 9.64$ kN

$\Sigma F_x = CF + DF\cos\theta_1 - FG\sin\theta_2 = 0$

or $CF = -9.23$ kN

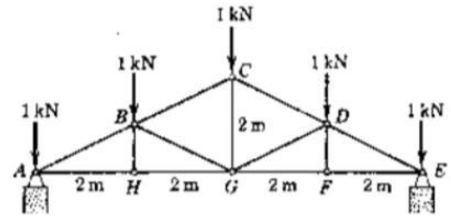


$\Sigma F_x = 0$

or $-CG\cos\theta_3 - CF + 12 = 0$

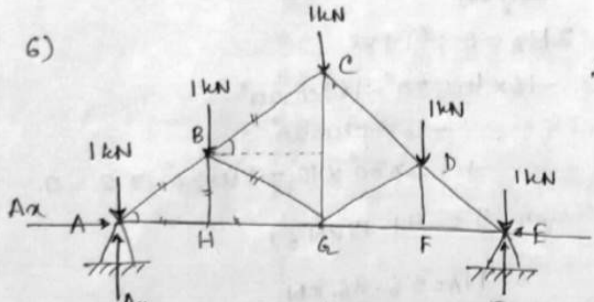
or $CG = 25.03$ kN (T)

6. A snow load transfers the forces shown to upper joints of a Howe roof trusses respectively. Neglect any horizontal reaction at the supports and solve for the forces in all members and compare them.



Ans. For Howe roof truss AB = DE = 3.35 kN (C), BC = CD = 2.24 N (C), AH = EF = 3.00 kN (T), BH = DF = 0, GH = FG = 3.0 kN (T), BG = DG = 1.12 kN (C)

6)



FBD of ~~snow~~ Howe Roof ~~board~~ Truss ~~transfers~~

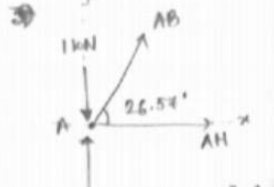
$\therefore \Sigma M_E = 0$; ↺ +ve

$1\times 2 + 1\times 4 + 1\times 6 + 1\times 8 - A_y \times 8 = 0$

or $A_y = 2.5$ kN

$\boxed{\angle BAH = \tan^{-1}\frac{2}{4} = 26.57^\circ}$

$\therefore \Sigma F_x = 0 \therefore A_x = E_x$



FBD of A

$\Sigma F_x = 0$

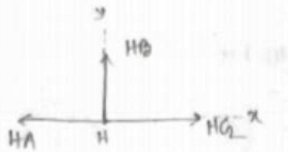
$\therefore AB\cos 26.57 + AH = 0$

$\Sigma F_y = 0$

or $A_y + AB\sin 26.57^\circ - 1 = 0$

or $\boxed{AB = -3.35 \text{ kN}}$

$\therefore \boxed{AH = 3.00 \text{ kN}}$



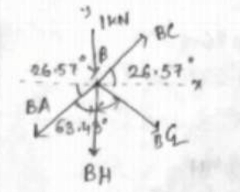
FBD of H

$\Sigma F_x = 0$

$\therefore HA = \boxed{HG = 3.00 \text{kN}}$

$\Sigma F_y = 0$

$\therefore \boxed{HB = 0}$



FBD of B

$\Sigma F_x = BC\cos 26.57 + BG\cos 26.57^\circ - BA\cos 26.57^\circ = 0$

or $BC + BG = BA$

or $BC + BG = -3.35$ kN

$\Sigma F_y = BC\sin 26.57^\circ - BG\sin 26.57^\circ - BA\sin 26.57^\circ - BH - 1 = 0$

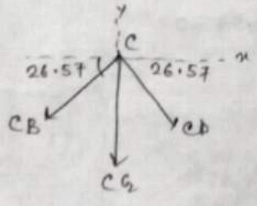
or $BC - BG - BA = +2.23$

or $BC - BG = -1.12$ kN

$\therefore \boxed{BC = -2.24 \text{kN}}$ $\boxed{BG = -1.12 \text{kN}}$

From the symmetry, AB = DE = −3.35 kN BC = CD = −2.24 kN
AH = EF = 3.00 kN
BH = DF = 0 kN
BG = DG = −1.12 kN
HG = GF = 3.00 kN

Now,



$\Sigma F_x = CD\cos 26.57^\circ - CB\cos 26.57^\circ = 0$

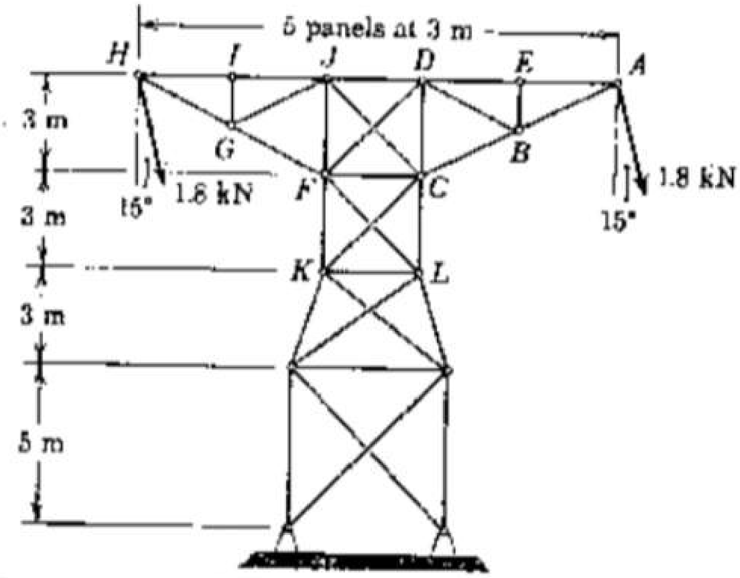
or $CD = CB = -2.24$ kN

$\Sigma F_y = -CB\sin 26.57^\circ - CD\sin 26.57^\circ - CG = 0$

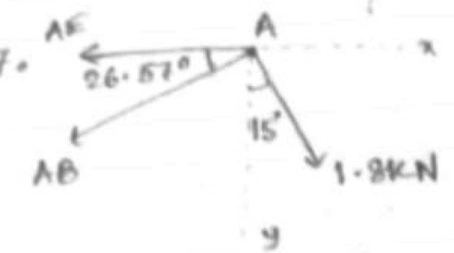
or $CG = 1.00$ kN (Ans)

7. The tower for a transmission line is modeled by the truss shown. The crossed members in the center sections of the truss may be assumed capable of supporting tension only. For the loads of 1.8 kN applied in the vertical plane, compute the forces induced in members AB, DB and CD.

Ans. AB = 3.89 kN (C), DB = 0,
CD = 0.93 kN (C)



7.



FBD of A

$\Sigma F_x = 0$

$\therefore -AE - AB\cos 26.57^\circ + 1.8\sin 15^\circ = 0$

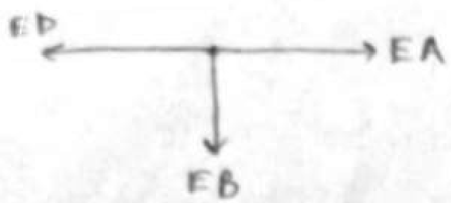
or $AE + AB\cos 26.57^\circ = 0.466..$

$\Sigma F_y = 0$

$\therefore 1.8 \times \cos 15^\circ + AB\sin 26.57^\circ = 0$

or $\boxed{AB = -3.89\,kN}$.

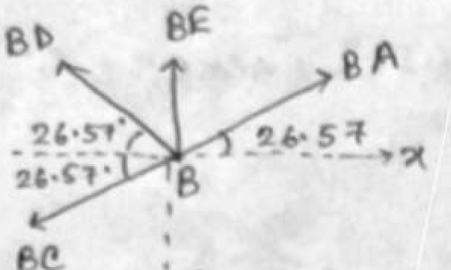
$\therefore AE = 3.945\,kN$



FBD of E

$\Sigma F_x = \boxed{ED = EA = 3.945\,kN}$

$\Sigma F_y = 0 = EB$



$\Sigma F_x = BA\cos 26.57^\circ - BD\cos 26.57^\circ - BC\cos 26.57^\circ = 0$

$\therefore BA = BD + BC$

or $BD + BC = -3.89\,kN$.

$\Sigma F_y \Rightarrow BA\sin 26.57 + BD\sin 26.57^\circ = BC\sin 26.57^\circ$
$= 0$

$\therefore BA + BD = BC$

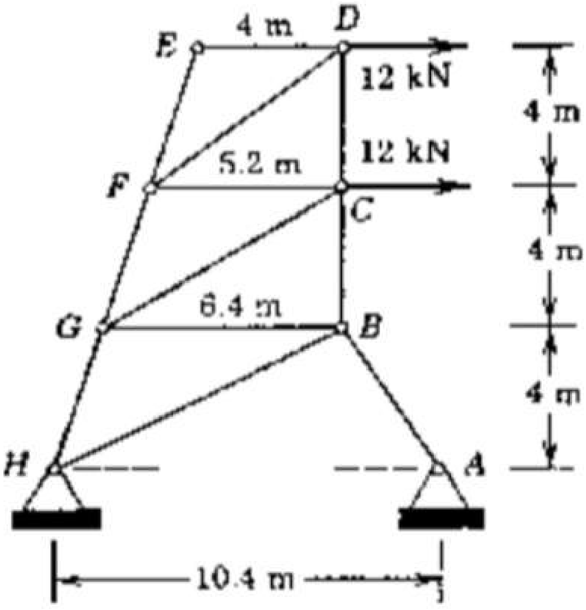
or $-3.89 + BD = -3.89$

or $\boxed{BD = 0\,kN}$

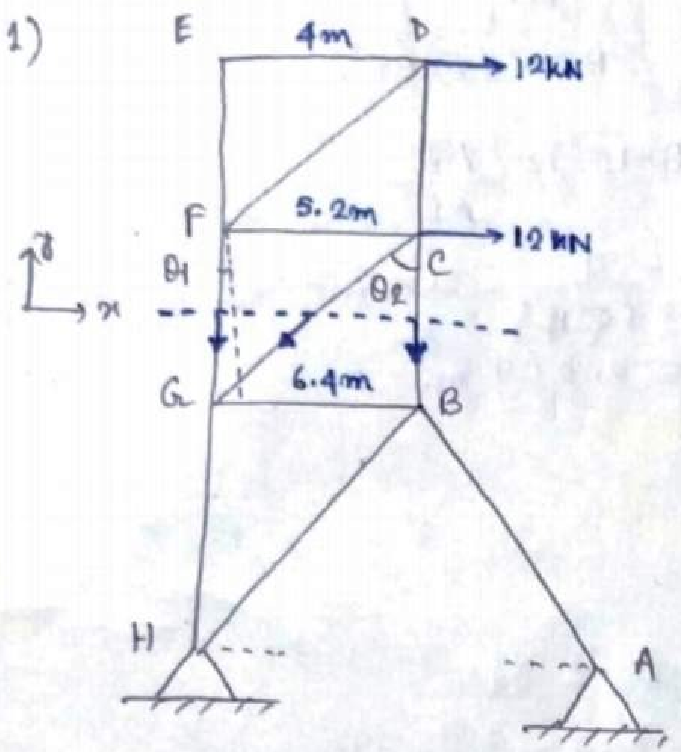
PROBLEM SHEET 3.2
[TRUSS]

1. Solve for the force in member CG from an equilibrium equation which contains that force as the only unknown.

Ans CG = 25.0 kN (T)



1)



FBD of the truss

Find out $F_{CG} = ?$

$\tan\theta_1 = \frac{6.4-5.2}{4}$; $\theta_1 = 16.69^\circ$

$\tan\theta_2 = \frac{6.4}{4}$; $\theta_2 = 58^\circ$.

$\Sigma F_x = 0$

or $12+12-FG\sin\theta_1 - CG\sin\theta_2 = 0$

or $CG\sin\theta_2 + FG\sin\theta_1 = 24$ ---- (i)

$\Sigma F_y = 0$

or $-FG\cos\theta_1 - CG\cos\theta_2 - CB = 0$

or $CG\cos\theta_2 + F_G\cos\theta_1 = -CB$ ----- (ii)

$\Sigma M_G = 0$; +ve

or $BC\times 6.4 + 12\times 4 + 12\times 8 = 0$

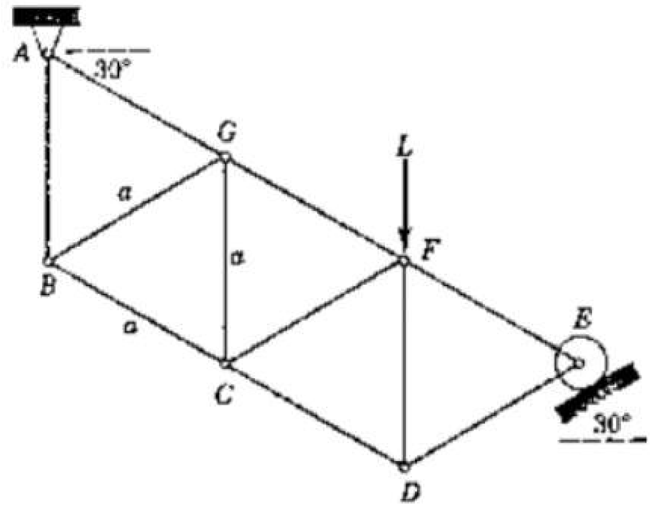
or $BC = -22.5$ kN ---- (iii)

By solving (i), (ii),

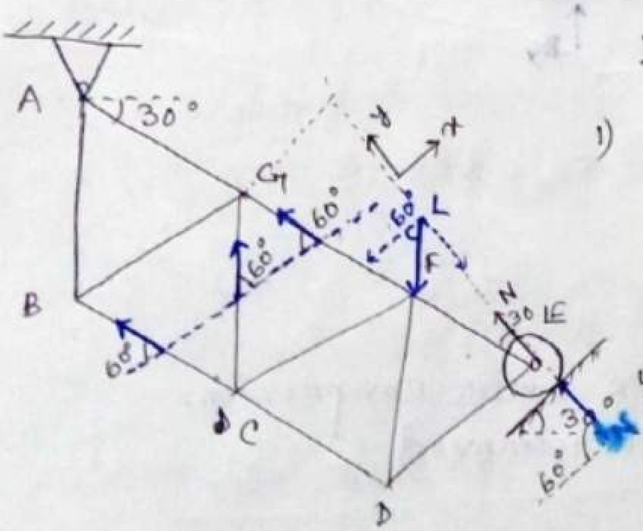
$CG = 25.06$ kN (T)

2. Determine the forces in members BC and CG of the truss loaded as shown.

Ans. BC = CG = L/3 (T)



2)



Determine $F_{BC} = ?$ $F_{CG} = ?$

1) $\Sigma F_x = 0$

or $-CB\cos60° + CG\cos60° - FG\cos60° - L\cos60° = 0$

or $CG = CB + FG + L$ ----- (1)

ii) $\Sigma F_y = 0$

or $CB\sin60° + CG\sin60° + FG\sin60° + N - L\sin60° = 0$

or $CB + CG + FG + \frac{N}{\sin60°} - L = 0$ ---- (ii)

or $CG + CB + FG + \frac{2}{\sqrt{3}} \times \frac{2L}{\sqrt{3}} - L = 0$

or $CB + CG + FG = -\frac{L}{3}$.

or $CG - L + CG = -\frac{L}{3}$ [From (1)]

or $2CG = \frac{2L}{3}$

or $CG = L/3$ [Ans]

iii) $\Sigma M_G = 0$; ↺ +ve

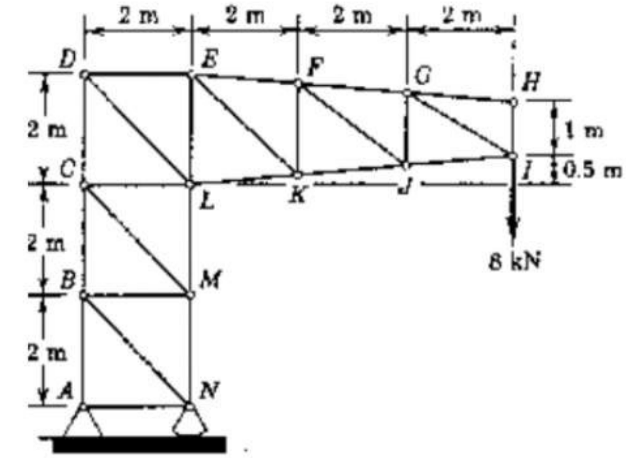
or $-BC \times a\frac{\sqrt{3}}{2} \mp L \times 2 \times a\frac{\sqrt{3}}{2} + N \times 2a \times \sin30° = 0$

or $-BC \times \frac{a\sqrt{3}}{2} + L \times a\sqrt{3} + aN = 0$

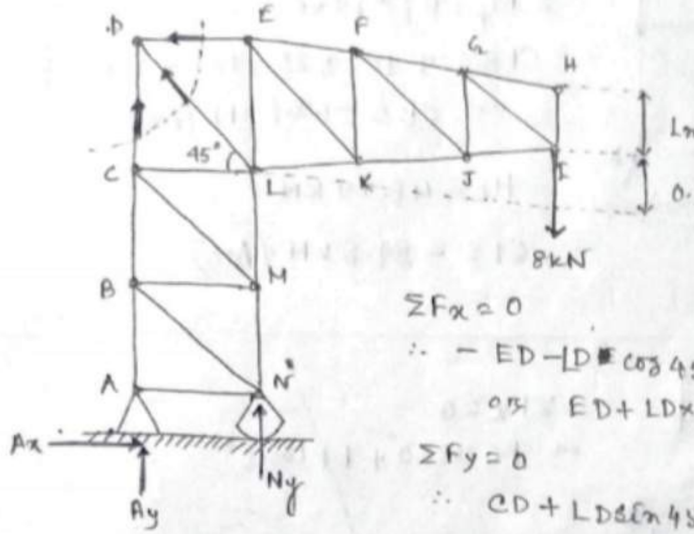
or $BC\frac{a\sqrt{3}}{2} = La\sqrt{3} + a \times \frac{2L}{\sqrt{3}}$

or BC =

3. Determine the forces in members DE and DL.



3.
?



$\Sigma F_x = 0$

$\therefore \; -ED - LD\cos 45° + A_x = 0$

or $ED + LD \times \frac{1}{\sqrt{2}} = A_x$ ----(i)

$\Sigma F_y = 0$

$\therefore \; CD + LD\sin 45° - 8 + A_y + N_y = 0$

or $CD + \frac{LD}{\sqrt{2}} + A_y + N_y = 8$ ----(ii)

$\Sigma M_A = 0$; ↺ +ve

$LD\cos 45° \times 4 + LD\sin 45° \times 2 + ED \times 6 - 8 \times 8 + N_y \times 2 = 0$

or $\frac{2LD}{\sqrt{2}} + \frac{LD}{\sqrt{2}} + 6ED \times 3 - 32 + N_y = 0$

or $\frac{3}{\sqrt{2}} LD + 3ED + N_y = 32$ ----(iii)

$\Sigma M_D = 0$; ↺ +ve

$A_x \times 6 + N_y \times 2 - 8 \times 8 = 0$

or $3A_x + N_y = 32$ ----(iv)

$\Sigma M_N = 0$; ↺ +ve

$-A_y \times 2 + LD\cos 45° \times 4 - CD \times 2 - 8 \times 6 = 0$

or $A_y + CD + 24 = \frac{2LD}{\sqrt{2}}$ ----(v)

From, (ii) ⇒ $\frac{2LD}{\sqrt{2}} - 24 + N_y + \frac{LD}{\sqrt{2}} = 8$

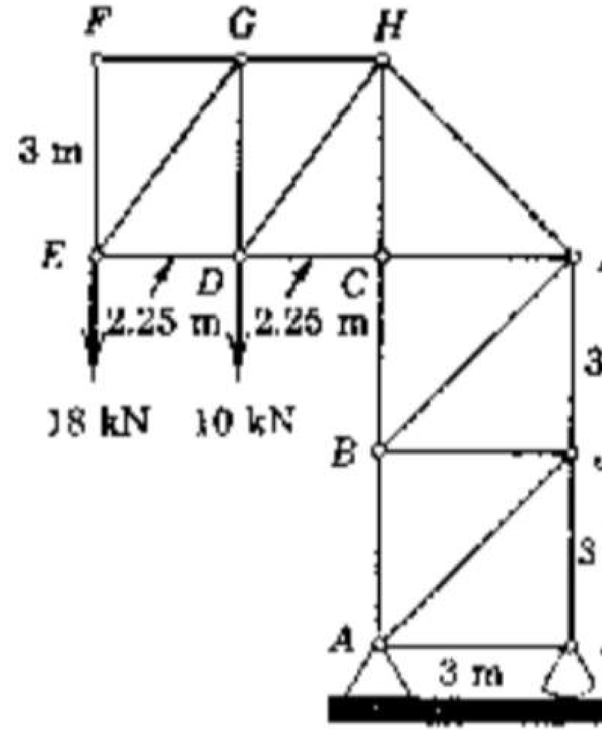
or $\frac{3LD}{\sqrt{2}} + N_y = 32$ ----(vi)

From, (iii), (vi) ⇒ $\boxed{ED = 0}$

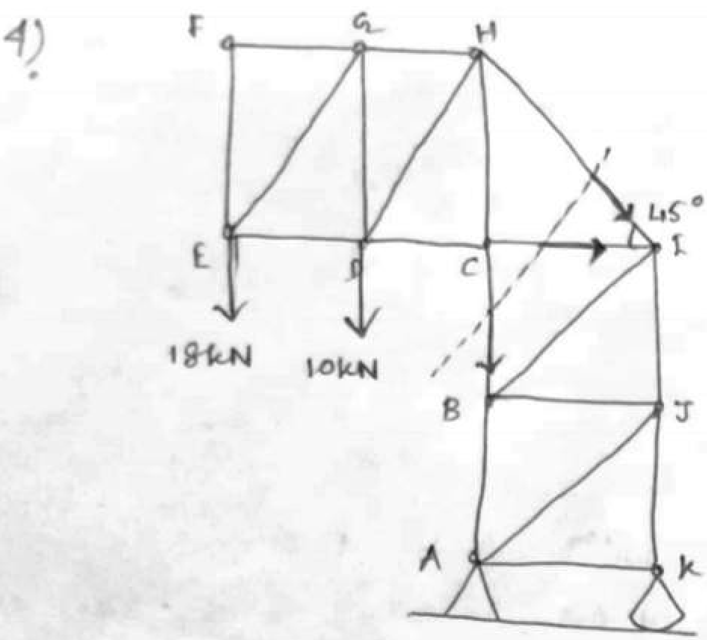
From, (iv) ⇒ $3 \times \frac{LD}{\sqrt{2}} + 32 - 31$

4. Determine the forces in members BC and CI.

Ans. BC = 62.5 kN (C), CI = 34.5 kN (C)



4)



$\Sigma F_x = 0$

or $CI + HI\cos 45^\circ = 0$ ---(i)

$\Sigma F_y = 0$

or $-HI\sin 45^\circ - 18 - 10 - CB = 0$

or $\frac{HI}{\sqrt{2}} + CB + 28 = 0$ ---(ii)

$\Sigma M_I = 0$; ↺ +ve

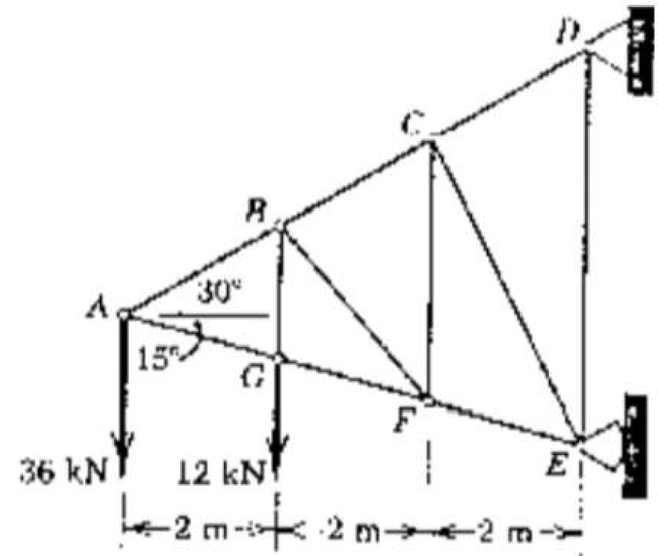
$CB \times 3 + 10 \times 5.25 + 18 \times 7.5 = 0$

or $CB = -62.5$ kN (Ans)

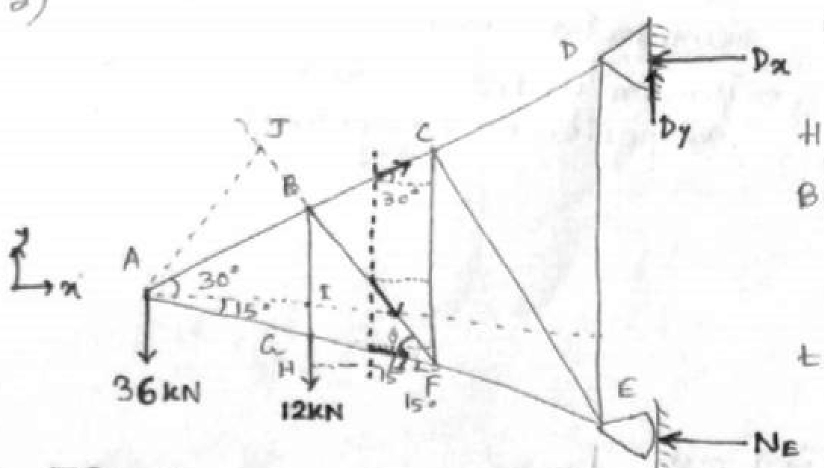
$HI = 48.79$ kN

$CI = -34.5$ kN (Ans)

5. Determine the force in the member BF.



5)



$F_{BF} = ?$

$HF = 2m$

$BH = BI + IG + GH$

$= 2\tan 30° + 2\tan 15° + 2\tan 15°$

$= 2.2265\ m.$

$\tan\theta = \frac{BH}{HF} = \frac{2.2265}{2}$

$\therefore \theta = 48.06°$

$\Sigma F_x = 0$

$\therefore BC\cos 30° + GF\cos 15° + BF$

$\sin(\theta - 15°) = \frac{AJ}{AF}$

or $AJ = \frac{4}{\cos 15°} \times \sin 33.06°$

$= 2.259\ m.$

$\Sigma M_A = 0$; ↻ +ve.

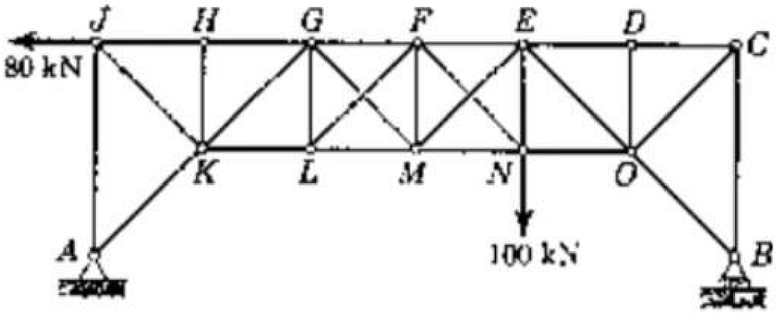
or $BF \times AJ + 12 \times 2 = 0$

or $BF = -\frac{24}{2.259}$

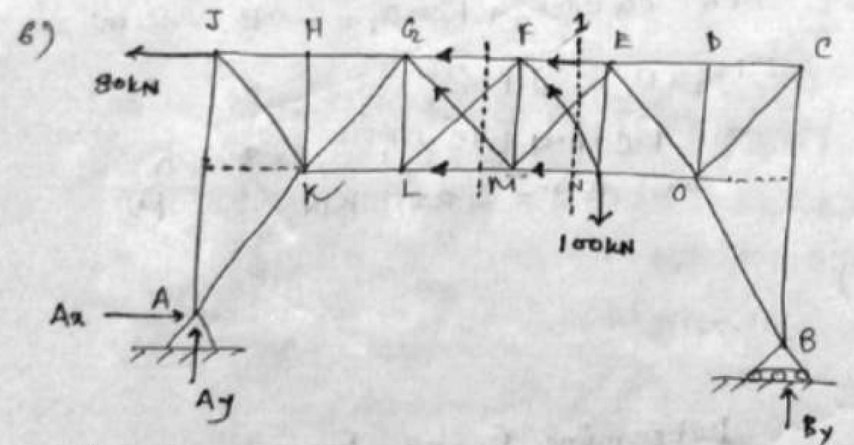
$= -10.62\ kN$ (Ans)

6. The truss shown is composed of 450 right triangles. The crossed members in the center two panels are slender tie rods incapable of supporting compression. Retain the two rods which are under tension and compute the magnitude of their tensions. Also find the force in the member MN.

Ans. FN = GM = 84.8 kN (T), MN = 20 kN (T)



6)



For the entire truss:

$\Sigma M_A = 0$, ↺+ve

$B_y \times 6a + 80 \times 2a - 100 \times 4a = 0$

From Section (1) or $B_y = 40$ kN (Ans)

Cross Member, FN is under tension, EN is under compression. So, EN is removed

$\Sigma F_y = 0 \quad \therefore FN \sin 45^\circ = 60$

or $FN = 84.85$ kN (T)

From Section (2) $\Sigma F_y = 0$, $GM \sin 45^\circ = 60$

or $GM = 84.85$ kN (T)

From section (1),

$\Sigma M_F = 0$; ↺+ve

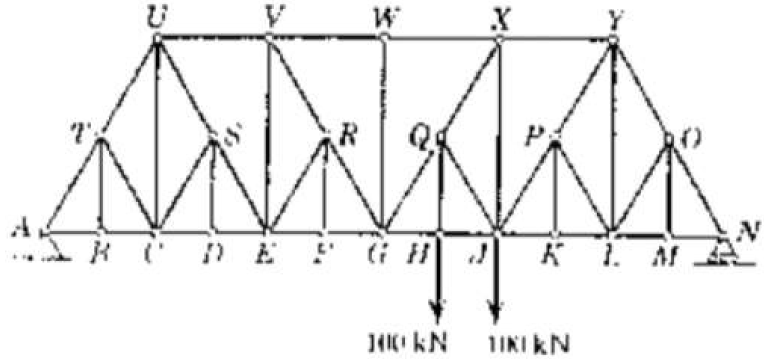
or $-MN \times a - 100 \times a + B_y \times 3a = 0$

or $MN = 3 \times 40 - 100$

$= 20$ kN (T) Ans.

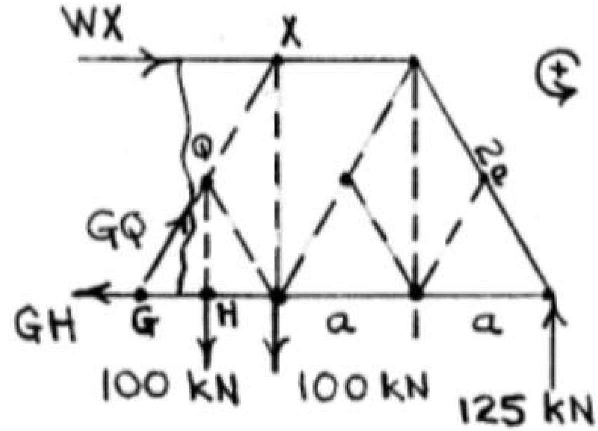
7. Find the force in member JQ for the Baltimore truss where all angles are 300, 600, 900 or 1200.

Ans. JQ = 57.7 kN (C)



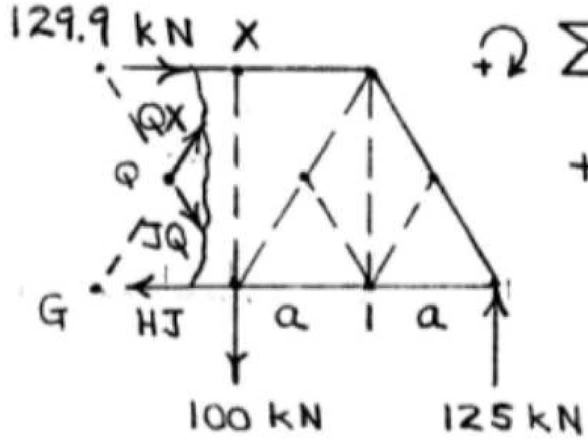
4/48 From truss as a whole, $\Sigma M_A = 0$ gives

$N = 125$ kN.



$$\Sigma M_G = 0: \ 125(3a) - 100\left(\frac{3a}{2}\right) - WX\left(2a\frac{\sqrt{3}}{2}\right) = 0$$

$$WX = 129.9 \text{ kN C}$$

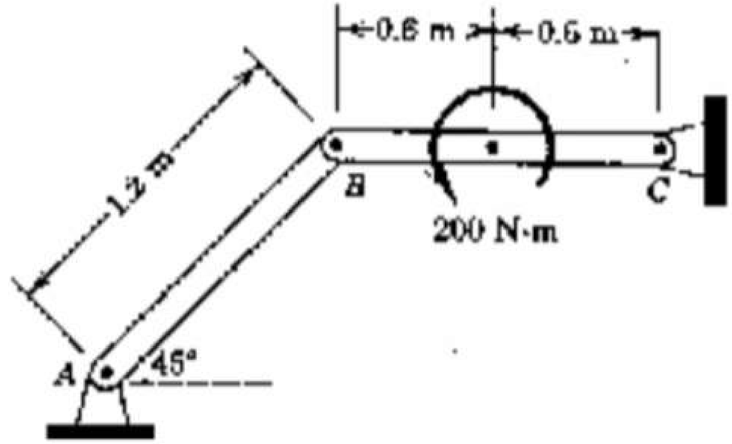


$$\Sigma M_G = 0: \ 100(a) + 129.9(a\sqrt{3}) + JQ\left(a\frac{\sqrt{3}}{2}\right) - 125(3a) = 0$$

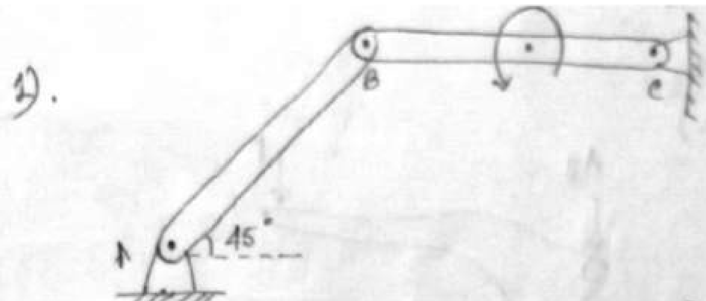
$$\underline{JQ = 57.7 \text{ kN C}}$$

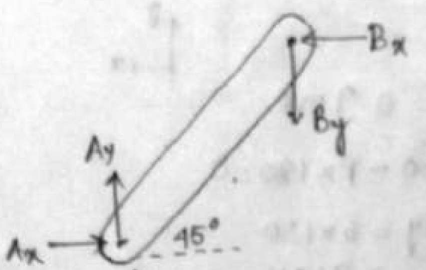
PROBLEM SHEET 4
[FRAMES]

1. Determine the magnitude of the pin reaction at C.
   Ans C = 236 N

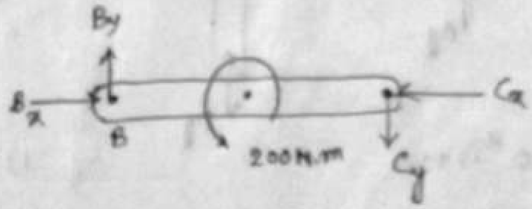


1).





FBD of AB



FBD of BC

$\Sigma M_A = 0$ ↺+ve

$\therefore B_x \times 1.2\sin 45^\circ - B_y \times 1.2\cos 45^\circ = 0$

or $B_x = B_y = 166.67\,N$

i) $\Sigma M_C = 0$ ↺+ve

$\therefore 200 - B_y \times 1.2 = 0$

or $B_y = 166.67\,N$.

ii) $\Sigma F_y = 0$ $\therefore B_y = C_y = 166.67\,N$

iii) $\Sigma F_x = 0$ $\therefore B_x = C_x = 166.67\,N$

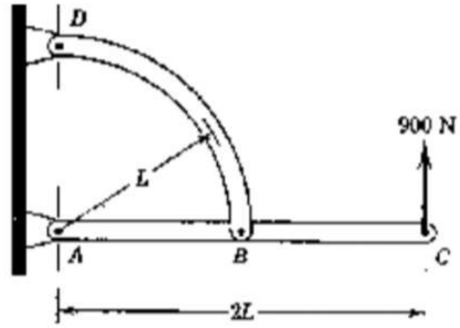
$\therefore C = \sqrt{C_x^2 + C_y^2}\,N$

$= 166.67\sqrt{2}\,N$

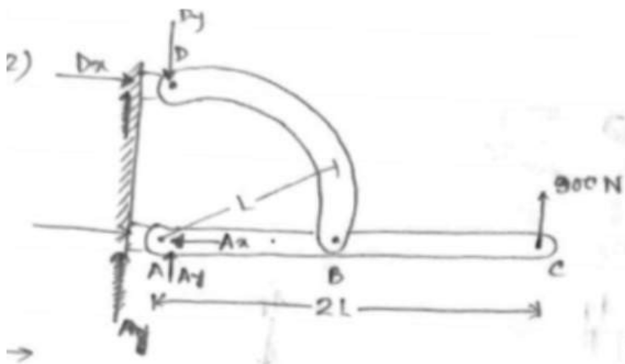
$= 235.71\,N$

$\approx 236\,N$.

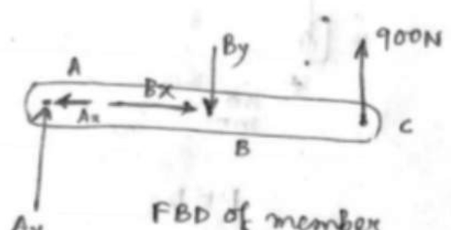
2. Determine the magnitude of the pin reaction at A for the frame loaded by the 900-N force.



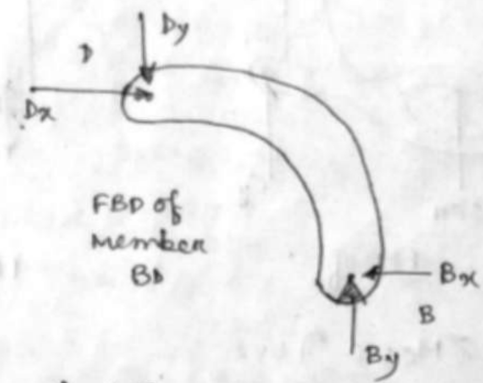
2)



Find Pin reaction at A.



FBD of member ABC



FBD of member BD

i) $\Sigma M_B = 0$; ↺+ve

$900 \times L - A_y \times L = 0$

$\therefore \boxed{A_y = 900 \text{ N}}$

ii) $\Sigma F_x = 0$

$\therefore -A_x + B_x = 0$

or $B_x = A_x$

iii) $\Sigma F_y = 0$

$A_y - B_y + 900 = 0$

or $\boxed{B_y = 1800 \text{ N.}}$

So, $\boxed{B_x = A_x = 1800 \text{N}}$

i) $\Sigma M_D = 0$; ↺+ve

$B_y \times L - B_x \times L = 0$

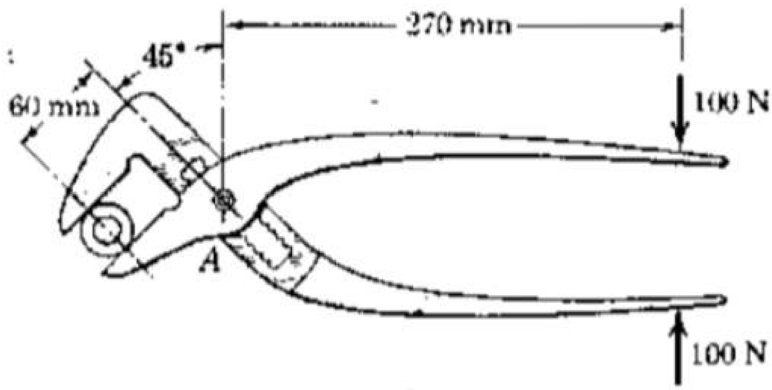
$\therefore B_y = B_x$

Reaction at A $= \sqrt{1800^2 + 900^2}$ N

$= 900\sqrt{5}$ N

$= 2012$ N (Ans)

3. Compute the force supported by the pin at A for the slip-joint pliers under a grip of 100 N.
   Ans. 525 N



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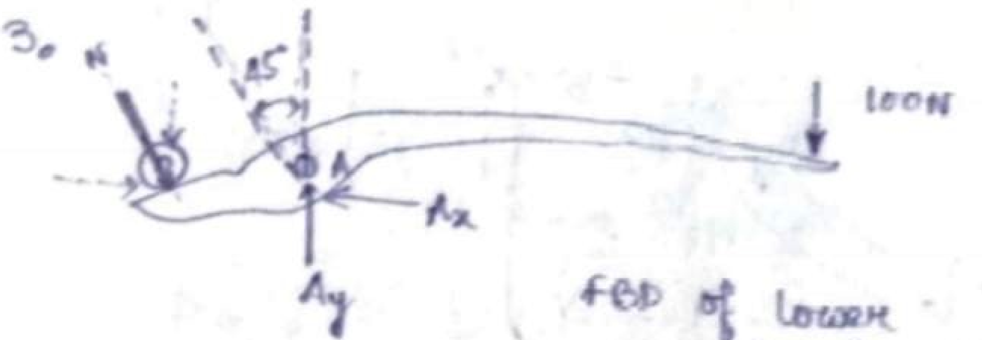


FBD of lower member

$\Sigma F_x = 0 \therefore N \sin 45^\circ - A_x = 0$

or $A_x = \frac{N}{\sqrt{2}}$

$\Sigma F_y = 0 \therefore -N \cos 45^\circ - 100 + A_y = 0$

or $A_y = \frac{N}{\sqrt{2}} + 100.$

Now, $\Sigma M_A = 0$ ↺ +ve

$\therefore -100 \times 270 + N \times 60 = 0$

or $N = \frac{\cancel{100} \times \cancel{270}}{\cancel{60}}$

$= 450 \text{ N}$

$\therefore A_x = \frac{450}{\sqrt{2}} \text{ N} = 318.198 \text{ N}$

$A_y = 418.198 \text{ N}$

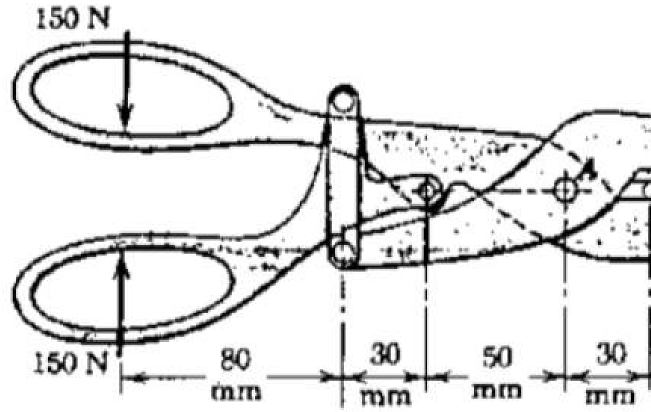
$\therefore A = \sqrt{A_x^2 + A_y^2} \text{ N}$

$= 525.4898 \text{ N}$

$\simeq 526 \text{ N}$. (Ans)

4. Compound-lever snips, shown in the figure, are often used in place of regular tinners' snips when large cutting forces are required. For the gripping force of 150 N, what is the cutting force P at a distance of 30 mm along the blade from the pin at A.

Ans. P = 1467 N



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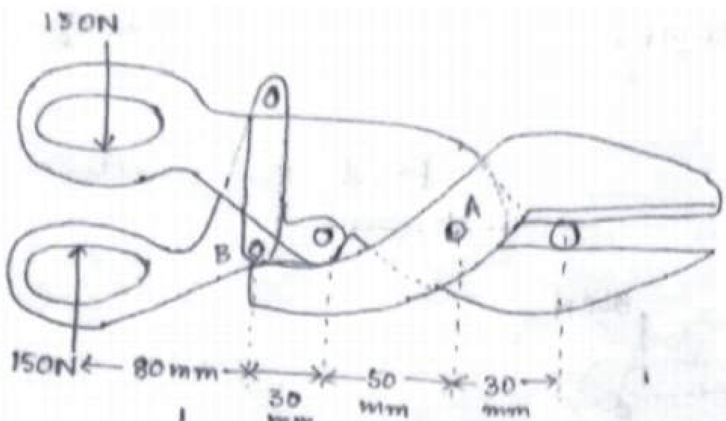
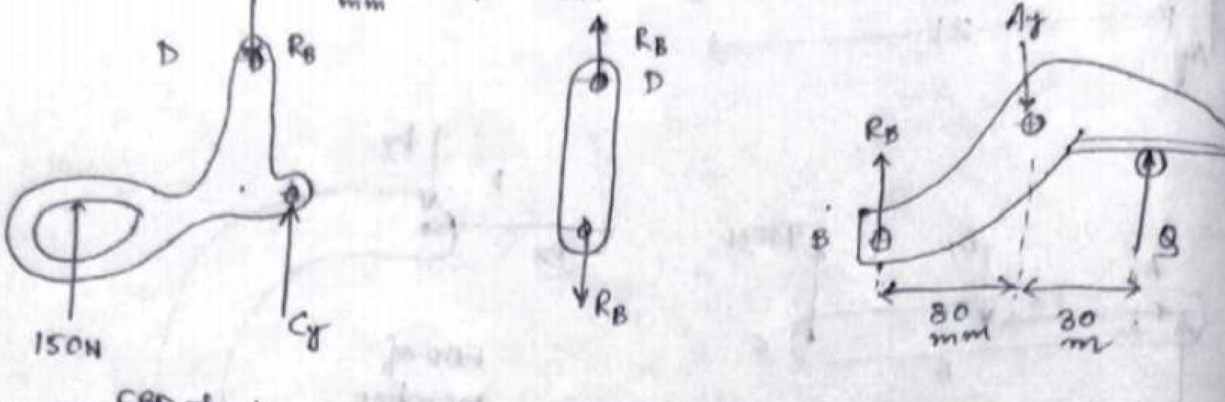




FBD of D

FBD

FBD

$\Sigma M_C = 0$ ↺ +ve

$-150 \times (80 + 30) + R_B \times 30 = 0$

or $R_B = 550.N$

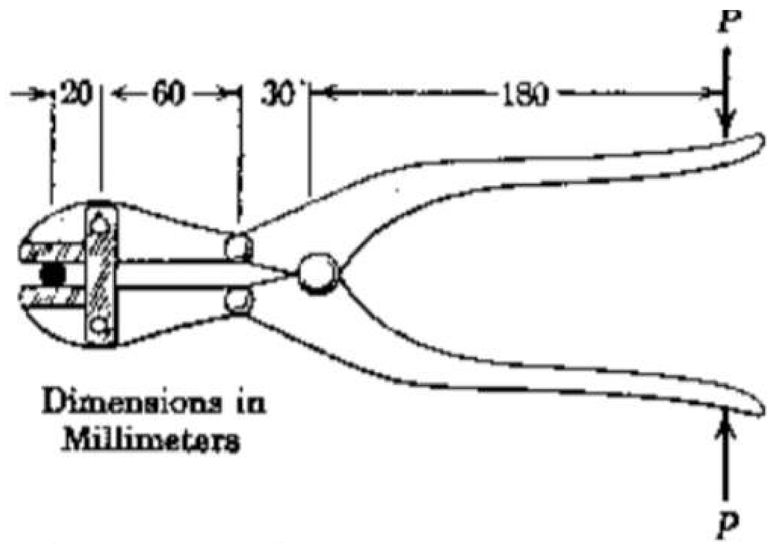
$\Sigma M_A = 0;$

$Q \times 30 - R_B \times 80 = 0$

or $Q = \frac{550 \times 80}{30}$

$= 1467 N$

5. The A small bolt cutter operated by hand for cutting small bolts and rods is shown in the sketch. For a hand grip P = 150 N, determine the force Q developed by each jaw on the rod to be cut.



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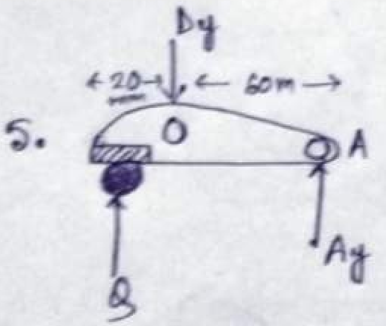
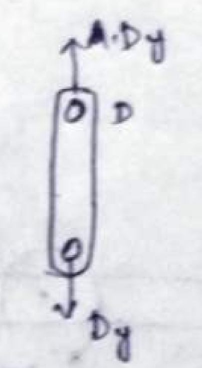
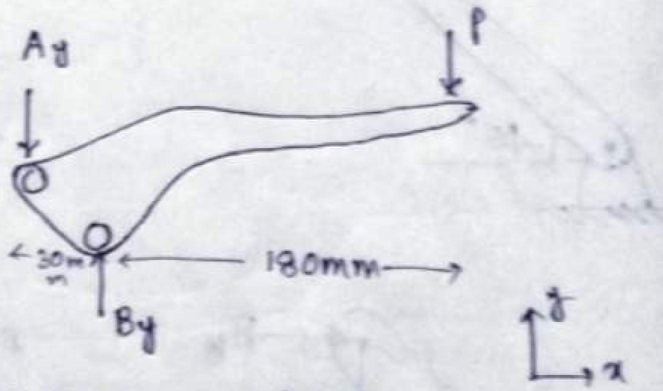






$\Sigma M_D = 0$ +ve

$A_y \times 60 - Q \times 20 = 0$

or $Q = 3A_y$

or $Q = (3 \times 900)$ N

$= 2700$ N

i) $\Sigma M_B = 0$ +ve

$\therefore A_y \times 30 - P \times 180 = 0$

or $A_y = 6 \times 150$

$= 900$ N

ii) $\Sigma F_y = 0$

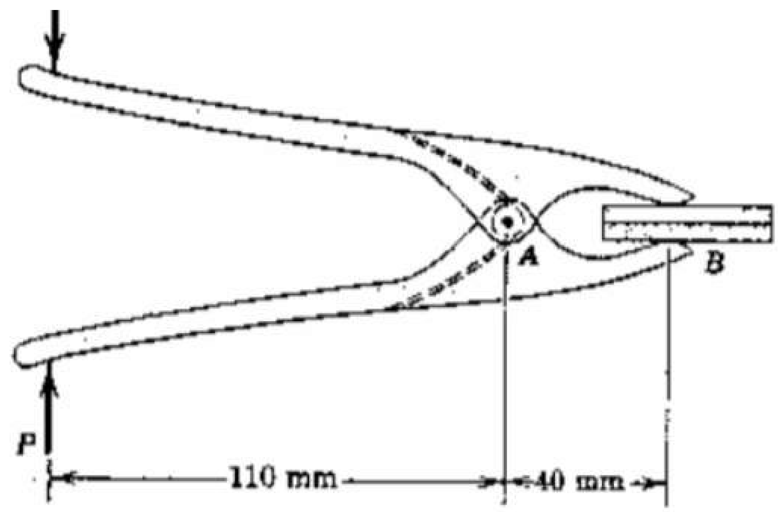
$\therefore A_y + P = B_y$

or $B_y = 900 + 150$

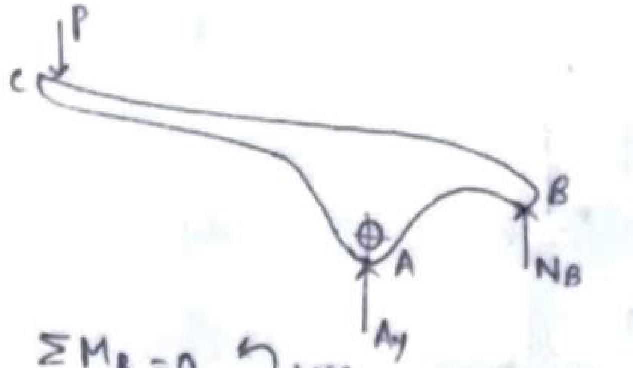
$= 1050$ N

6. In the spring clamp shown, an internal spring is coiled around the pin A and the spring ends bear against the inner surfaces of the handle halves in order to provide the desired clamping force. In the position shown, a force of magnitude P = 25 N is required to release the clamp. Determine the compressive force at B if P = 0.

Ans. B = 68.8 N



6)



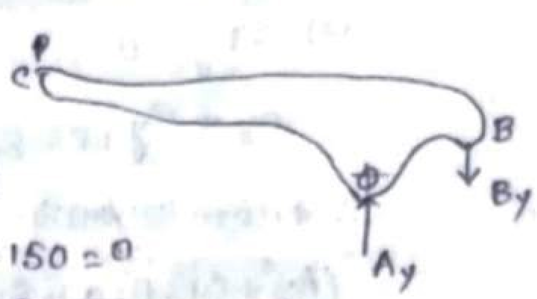
$\Sigma M_B = 0$ ↺ +ve

$\therefore\ 25 \times 150 - A_y \times 40 = 0$

or $A_y = \dfrac{25 \times 15\cancel{0}}{4\cancel{0}}$

$= 93.75\text{N}$

Now, if $P = 0$



$\Sigma M_C = 0$ ↺ +ve

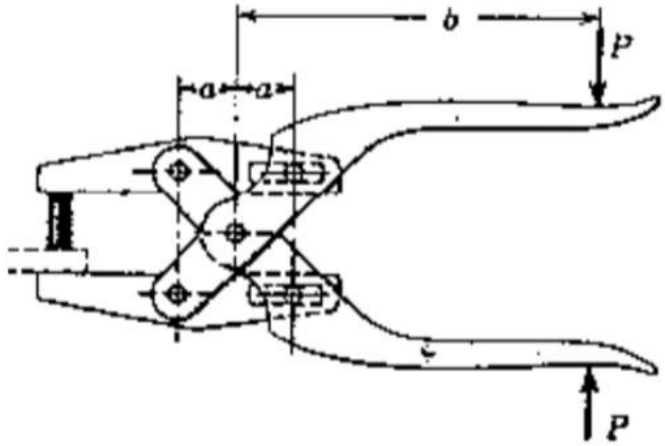
$A_y \times 110 - B_y \times 150 = 0$

or $B_y = \dfrac{93.75 \times 11\cancel{0}}{15\cancel{0}}$

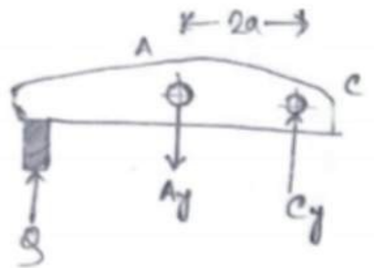
$= 68.75\text{N}$

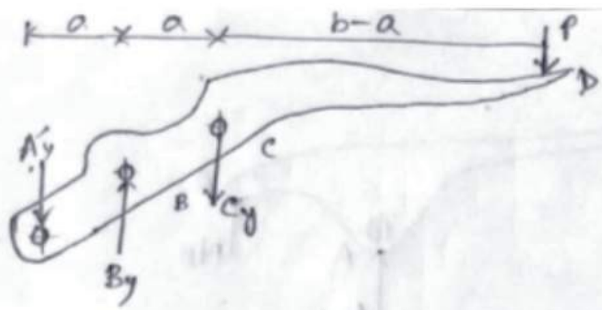
7. For the paper punch shown find the punching force Q corresponding to a hand grip P.

Ans. $Q = P(b/a)$



7.





i) $\Sigma M'_A = 0$ +ve

$B_y \times a - C_y \times 2a - P \times (b+a) = 0$ ---(i)

ii) $\Sigma M_C = 0$ +ve

$A'_y \times 2a - B_y \times a - P(b-a) = 0$ ---(ii)

iii) $\Sigma F_y = 0$

$\therefore$ $A'_y + C_y + P = B_y$ ---(iii)

$\therefore$ from (i), (iii) $\Rightarrow$

$(A'_y + C_y + P)a - C_y \times 2a = P(b+a)$

or $A'_y a - C_y \times a = P \times b$ ... (iv)

$\therefore$ from (ii) (iii) $\Rightarrow$

$A'_y \times 2a - a(A'_y + C_y + P) = P(b-a)$

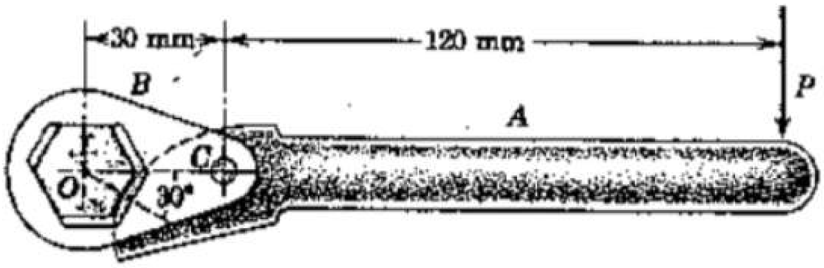
or $A'_y a - C_y \times a = Pb$.

iv) $\Sigma M_D = 0$ +ve

$A'_y (b+a) - B_y (b) + C_y \times (b-a) = 0$

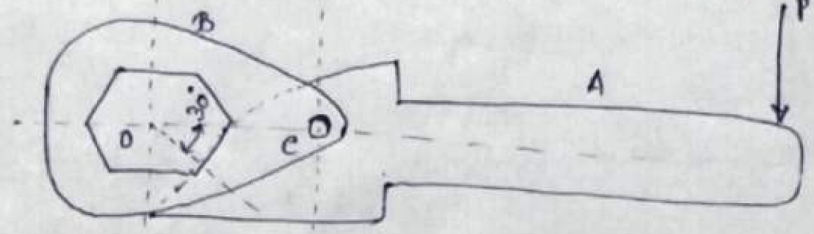
or $A'_y (b+a) - b($

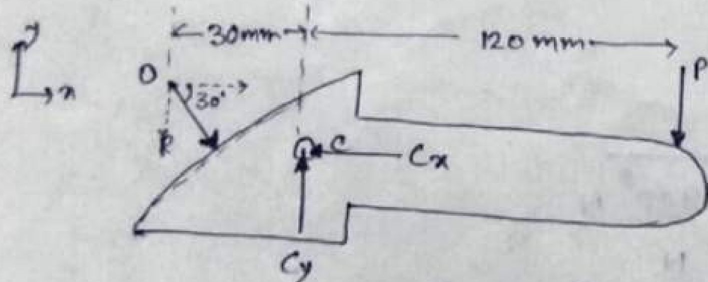
8. The special box wrench with head B swiveled at C to the handle A will accommodate a range of sizes of hexagonal bolt heads. For the nominal size shown where the center O of the bolt and the pin C are in line with the handle, compute the magnitude of the force supported by the pin at C if P = 160 N. Assume the surface of the bolt head to be smooth.



Ans. C = 1367 N

8.





O, C are in Same line

FBD of member

i) ΣM_O; ↺ +ve

$C_y \times 30 - P(30+120) = 0$

or $C_y = \frac{P \times 50}{10}$

or $C_y = \frac{160 \times 50}{10} = 800N$

iii) ΣM_C; ↺ +ve

or $R\sin 30° \times 30 - P \times 120 = 0$

or $R = \frac{160 \times 1\cancel{20}^{4} \times 2}{3\cancel{0}}$

$= 1280N$

ii) $\Sigma F_x = 0$

$\therefore R\cos 30° - C_x = 0$

or $C_x = R\cos 30°$.

$\therefore C_x = 1280\cos 30°$

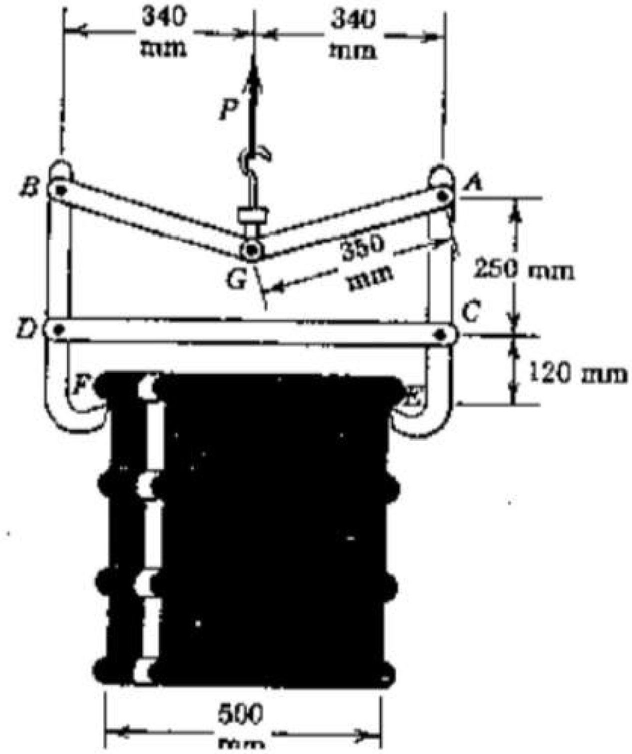
$= 1109$ N (Ans)

$\therefore C = \sqrt{C_x^2 + C_y^2}$ N

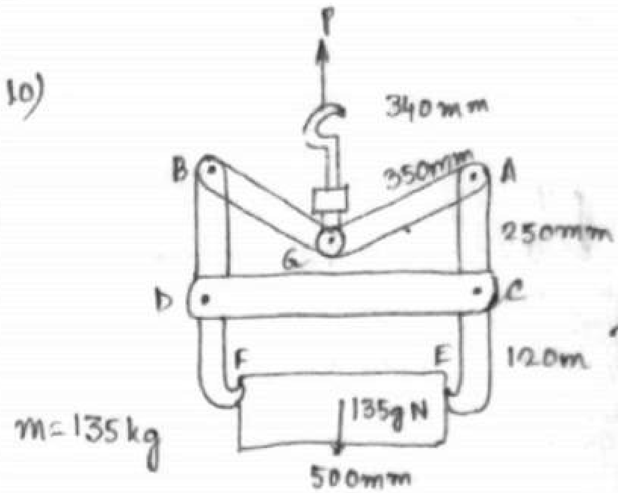
$= 1367$N (Ans)

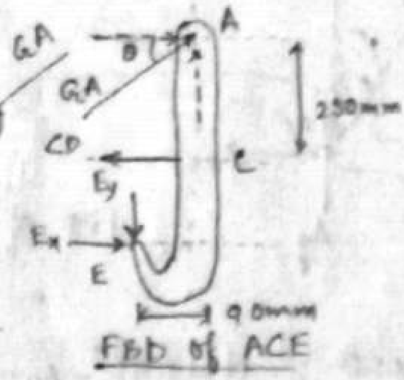
10. A lifting device for transportating 135-kg steel drums is shown. Calculate the magnitude of the force exerted on the drum at E and F.

Ans. E = F = 5.19 N



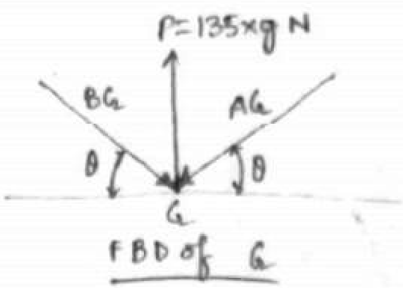
10)





$\cos\theta = \frac{340}{350}$

$\therefore \theta = 13.73$



$\Sigma F_y = 0$

$GA \sin\theta - E_y = 0$

or $E_y = 2790 \sin\theta$

$= 662.21$ N.

$\Sigma M_C = 0$, ↻ +ve

$GA \times 2$

$\cos 30^\circ \times 250$

$GA \cos 30^\circ \times 250 - E_y \times 90$

$- E_x \times 120 = 0$

or $E_x = 5150$.

$\Sigma F_y = 0$; $\Sigma F_x = 0$

$P - 2GA \sin\theta = 0$; or $GB \cos\theta - GA \cos\theta = 0$

or $GA = 2790$ N ; $\therefore GB = GA$

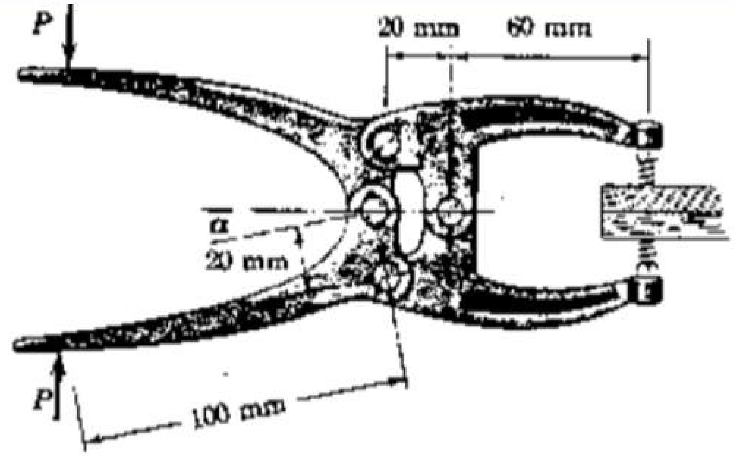
$\therefore E = F = \sqrt{E_x^2 + E_y^2}$

$= 5192$ N

$= 5.19$ kN (Ans).

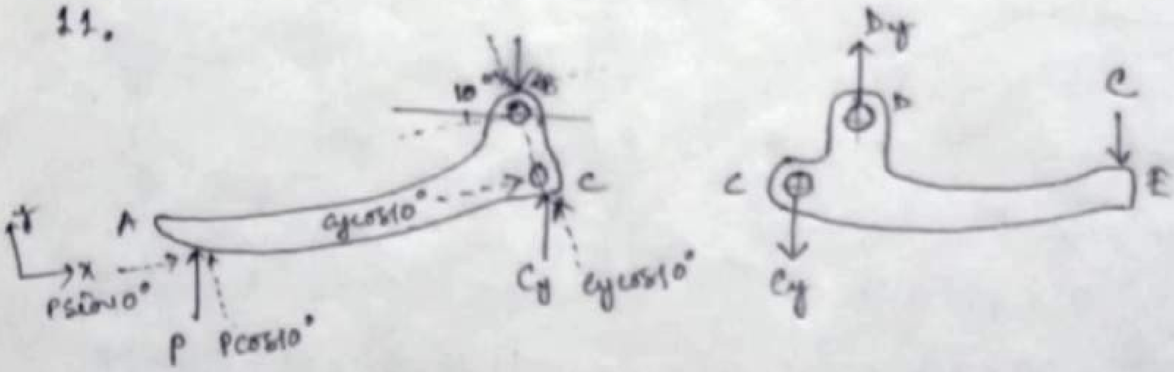
11. The toggle pliers are used for a variety of clamping purposes. For the handle position given by $\alpha = 10^0$ and for a handle grip of $P = 150$ N, Calculate the clamping force C produced.

Ans. C = 1368 N



11)

11.



$\Sigma M_B = 0$ (+ve

$\therefore -P\times \sin 10^\circ \times 20 + P\times \cos 10^\circ \times 100 \mp C_y \sin 10^\circ \times 20 = 0$

or $C_y = 27.36 \times 150$

$= 4103.46$ N.

$\Sigma M_D = 0$ +ve

$C_y \times 20 - C \times 60 = 0$

or $C = \frac{C_y}{3}$

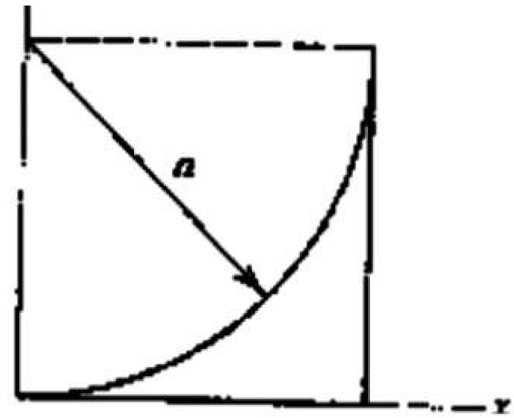
$= \frac{4103.46}{3}$ N

$= 1367.82$ N.

≈ 1368 N.

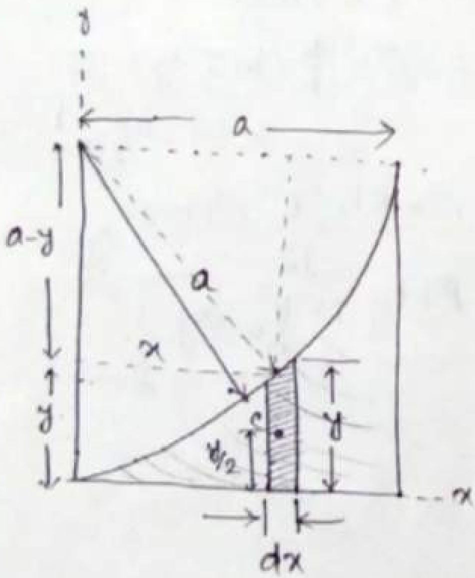
PROBLEM SHEET 5.1
[CENTERS OF MASS]

3. Locate the centroid of the area shown in the figure by direct integration. (Caution: observe carefully the proper sign of the radical involved).



Ans. $\bar{x} = \dfrac{2a}{3(4-\pi)}$, $\bar{y} = \dfrac{10-3\pi}{3(4-\pi)}a$

3.



Find $\bar{x}, \bar{y}$.

$(a-y)^2 + x^2 = a^2$

or $a - y = \sqrt{a^2 - x^2}$

or $y = a - \sqrt{a^2 - x^2}$

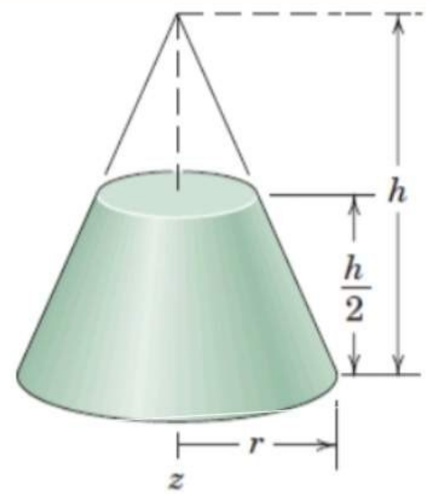
Now, $dA = y\,dx$

$$\bar{x} = \frac{\int x_c\,dA}{\int dA} = \frac{\int x \cdot y\,dx}{\int y\,dx} = \frac{\int_0^a x\left(a - \sqrt{a^2 - x^2}\right)dx}{\int_0^a \left(a - \sqrt{a^2 - x^2}\right)dx} = \frac{2a}{3(4-\pi)}$$

$$\bar{y} = \frac{\int y_c\,dA}{\int dA} = \frac{\int_0^a \frac{y}{2} \times y\,dx}{\int_0^a y\,dx} = \frac{\frac{1}{2}\int_0^a \left(a - \sqrt{a^2 - x^2}\right)^2 dx}{\int_0^a \left(a - \sqrt{a^2 - x^2}\right)dx}$$

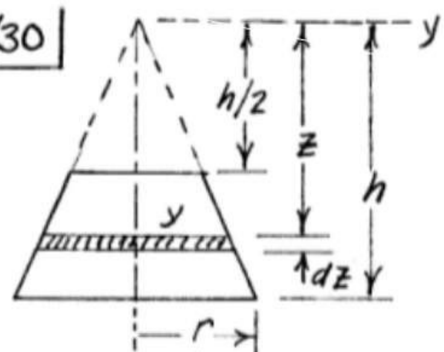
$$= \frac{(10 - 3\pi)a}{3(4-\pi)}$$ [Ans].

4. Calculate the distance $\bar{h}$ measured from the base to the centroid of the volume of the frustum of the right-circular cone.



$$\text{Ans. } \bar{h} = \frac{11}{56} h$$

5/30



$dV = \pi y^2 dz$ where $y = \frac{r}{h} z$

$$= \pi \frac{r^2}{h^2} z^2 dz$$

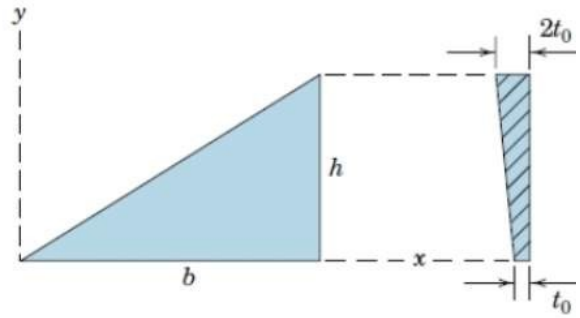
$$V = \pi \frac{r^2}{h^2} \int_{h/2}^{h} z^2 dz = \frac{7\pi r^2 h}{24}$$

$$\int z_c \, dV = \int_{h/2}^{h} z \, \pi \frac{r^2}{h^2} z^2 dz = \frac{15}{64} \pi r^2 h^2$$

$$\bar{z} = \int z_c \, dV / V = \frac{15}{64} \pi r^2 h^2 \Big/ \frac{7}{24} \pi r^2 h = \frac{45}{56} h$$

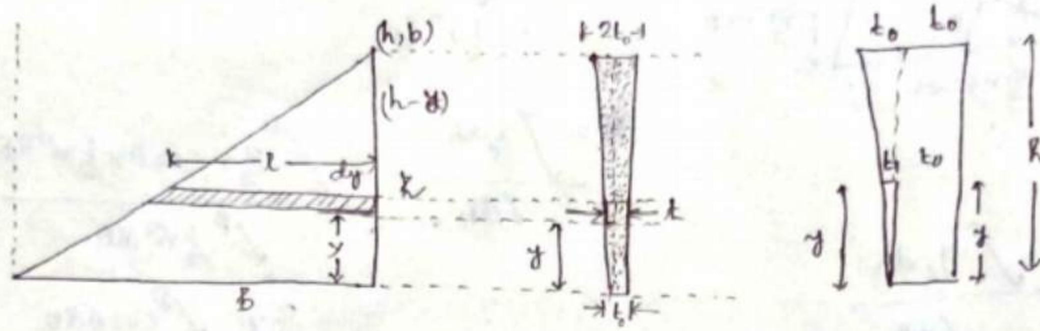
$$\bar{h} = h - \bar{z} = \underline{\frac{11}{56} h}$$

5. The thickness of the triangular plate varies linearly with y from a value t_0 along its base $y = 0$ to $2t_0$ at $y = h$. Determine the y-coordinate of the center of the mass of the plate.



$$Ans.\ \bar{y} = \frac{3h}{8}$$

5.



$$\bar{y} = \frac{\int y_c\, dm}{\int dm}$$

$$= \frac{\int_0^h y\,\cancel{\rho}\,\cancel{b}\,\cancel{t_0}\left(1-\frac{y^2}{h^2}\right)dy}{\int_0^h \cancel{\rho}\,\cancel{b}\,\cancel{t_0}\left(1-\frac{y^2}{h^2}\right)dy}$$

$$= \frac{\int_0^h \left(y - \frac{y^3}{h^2}\right)dy}{\int_0^h \left(1 - \frac{y^2}{h^2}\right)dy}$$

$$= \frac{\left[\frac{y^2}{2} - \frac{y^4}{4h^2}\right]_0^h}{\left[h - \frac{y^3}{3h^2}\right]_0^h}$$

$$= \frac{\frac{h^2}{2} - \frac{h^2}{4}}{h - \frac{h}{3}}$$

$$= \frac{\frac{h^2}{4}}{\frac{2h}{3}}$$

$$= \frac{3h}{8}$$

$$\frac{t_1}{y} = \frac{t_0}{h}$$

$$\text{or } t_1 = t_0 \cdot \frac{y}{h}$$

$$\text{Now } t = t_0 + t_1 = t_0\left(1 + \frac{y}{h}\right)$$

$$\frac{h - y}{l} = \frac{h}{b}$$

$$\text{or } l = \frac{b}{h}(h - y)$$

$$dA = \frac{b}{h}(h-y)\,dy$$

$$\therefore dm = \rho \times dA \times t$$

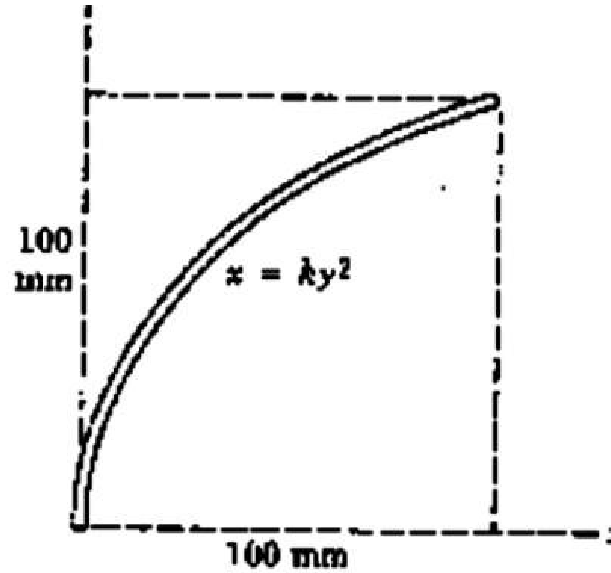
$$= \rho \times \frac{b}{h}(h-y)\,dy \times t_0\left(1 + \frac{y}{h}\right)$$

$$= \frac{\rho \times b\,t_0 (h-y)(h+y)}{h^2}\,dy$$

$$\boxed{dm = \rho\, b\, t_0\left(1 - \frac{y^2}{h^2}\right)dy}$$

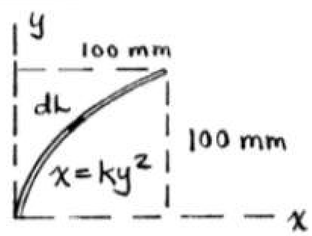
6. The homogeneous slender rod has a uniform cross section and is bent into the shape shown. Calculate the y-coordinate of the mass center of the rod. (reminder: A differential arc length is

$$dL = \sqrt{(dx)^2 + (dy)^2} = \sqrt{1 + \left(\frac{dx}{dy}\right)^2}\, dy$$



Ans. $y = 57.4$ mm

5/33



$$x = ky^2 = y^2/100, \quad \frac{dx}{dy} = \frac{y}{50}$$

$$L = \int dL = \int_0^{100} \sqrt{1 + (dx/dy)^2}\, dy = \int_0 \sqrt{1 + \frac{y^2}{50^2}}\, dy$$

$$= \frac{1}{50} \int_0^{100} \sqrt{50^2 + y^2}\, dy = \frac{1}{50 \cdot 2} \left[y\sqrt{50^2+y^2} + 50^2 \ln\left(y + \sqrt{50^2+y^2}\right) \right]_0^{100}$$

$$= 147.9 \text{ mm}$$

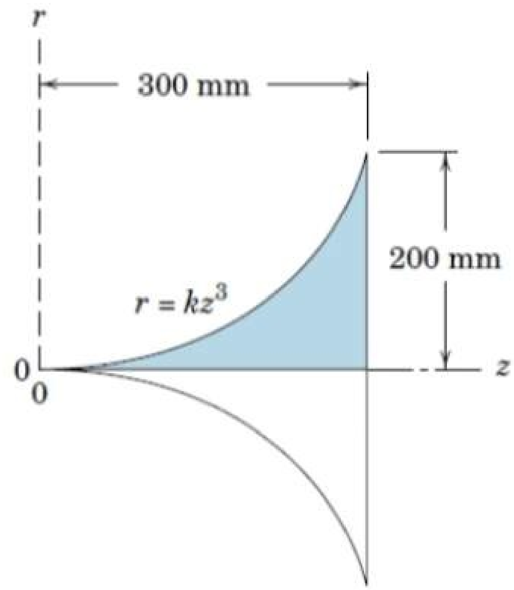
$$\int y_c\, dL = \frac{1}{50} \int_0^{100} y\sqrt{50^2+y^2}\, dy = \frac{1}{50}\,\frac{1}{3}\left(50^2+y^2\right)^{3/2} \Big|_0^{100}$$

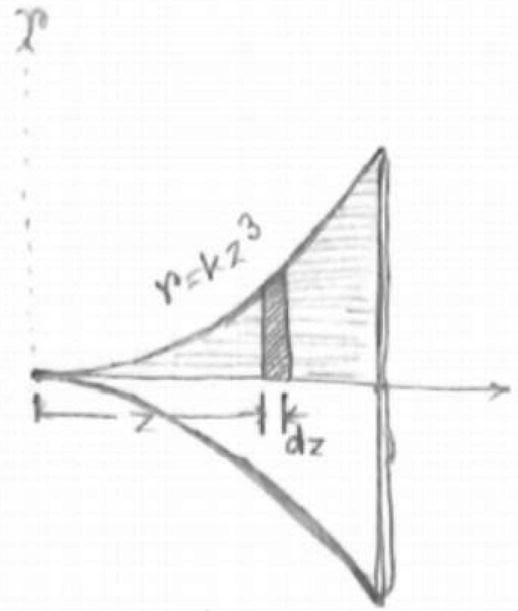
$$= 8480 \text{ mm}^2$$

$$\bar{y} = \frac{\int y_c\, dL}{L} = \frac{8480}{147.9} = \underline{57.4 \text{ mm}}$$

7. Locate the mass center of the homogeneous solid body whose volume is determined by revolving the shaded area through 360^0 about the z-axis.

Ans. $\bar{z} = 263$ mm





$dm = \rho \pi r^2 dz$
$= \rho \pi k^2 z^6 dz$

$$\bar{z} = \frac{\int z\,dm}{\int dm}$$

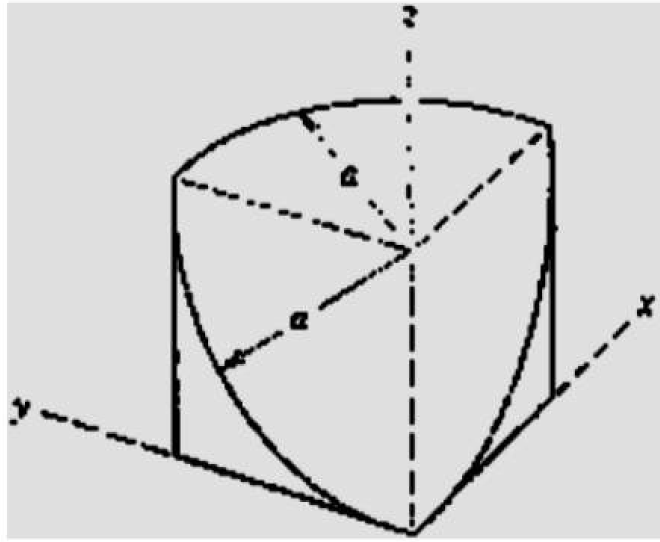
$$\bar{z} = \frac{\int_0^{300} z \times \cancel{\rho} \times \cancel{\pi} \cancel{k^2} z^6\,dz}{\int_0^{300} \cancel{\rho}\,\cancel{\pi}\,\cancel{k^2} z^6\,dz}$$

$$= \frac{\left[z^8/8\right]_0^{300}}{\left[\frac{z^7}{7}\right]_0^{300}}$$

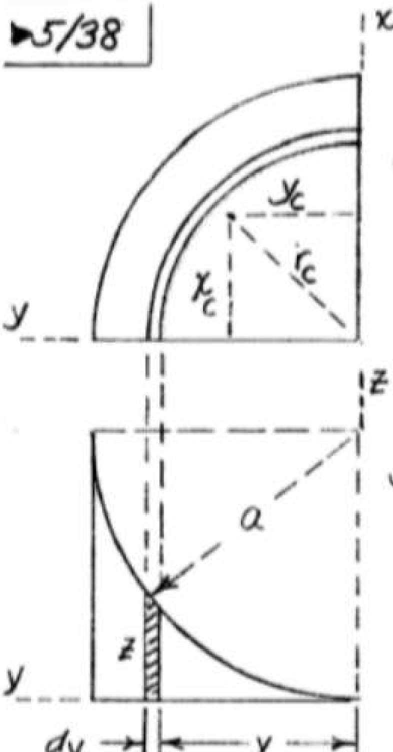
$$= \left(\frac{7}{8} \times 300\right) \text{ mm.}$$

$= 262.5$ mm. (Ans)

8. Determine the coordinates of the centroid of volume obtained by revolving the shaded area about the z-axis through the 90^0 angle.



▶5/38



From Sample Problem 5/1, for elemental shell, $x_c = y_c = r_c/\sqrt{2} = \frac{2y}{\pi}$

$$dV = \frac{\pi y}{2}(z\,dy) = \frac{\pi}{2}\left(ay - y\sqrt{a^2-y^2}\right)dy$$

$$V = \frac{\pi}{2}\int_0^a \left(ay - y\sqrt{a^2-y^2}\right)dy$$

$$= \frac{\pi}{2}\left[\frac{ay^2}{2} + \frac{1}{3}\sqrt{(a^2-y^2)^3}\right]_0^a = \frac{\pi}{2}\frac{a^3}{6} = \frac{\pi a^3}{12}$$

$$\int y_c\,dV = \frac{2}{\pi}\int_0^a \frac{\pi}{2}\left(ay^2 - y^2\sqrt{a^2-y^2}\right)dy$$

$$= \left[\frac{ay^3}{3} + \frac{y}{4}\sqrt{(a^2-y^2)^3} - \frac{a^2}{8}\left(y\sqrt{a^2-y^2} + a^2\sin^{-1}\frac{y}{a}\right)\right]_0^a$$

$$= \left[\frac{a^4}{3} + 0 - \frac{a^2}{8}\left(0 + a^2\frac{\pi}{2}\right)\right] = a^4\left[\frac{1}{3} - \frac{\pi}{16}\right]$$

$$\bar{y} = \int y_c\,dV/V = a^4\left(\frac{1}{3} - \frac{\pi}{16}\right)\Big/\frac{\pi a^3}{12} = \left(\frac{4}{\pi} - \frac{3}{4}\right)a = \bar{x}$$

$y^2 + (z-a)^2 = a^2$

$z = a - \sqrt{a^2-y^2}$

(Note sign)

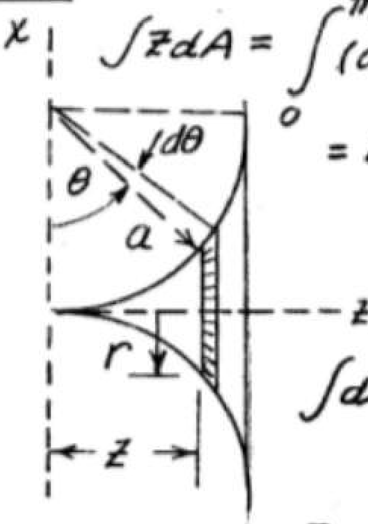
$$\int z_c\,dV = \int \frac{z}{2}\,dV = \int_0^a \frac{a-\sqrt{a^2-y^2}}{2}\,\frac{\pi}{2}\left(ay - y\sqrt{a^2-y^2}\right)dy$$

$$= \frac{\pi a^4}{48}\,; \quad \bar{z} = \int z_c\,dV/V = \frac{\pi a^4}{48}\Big/\frac{\pi a^3}{12} = \frac{a}{4}$$

9. Locate the center of mass of the homogeneous bell-shaped shell of uniform but negligible thickness. Also determine the position of the centroid of the volume within the bell-shaped shell.

Ans. $\bar{z}_{mass} = \dfrac{a}{\pi - 2}$, $\bar{z}_{vol} = \dfrac{a}{2(10 - 3\pi)}$

40



$$dA = 2\pi r\, a\, d\theta = 2\pi a^2(1-\cos\theta)\, d\theta$$

$$\int z\, dA = \int_0^{\pi/2} (a\sin\theta)(2\pi a^2)(1-\cos\theta)\, d\theta$$

$$= 2\pi a^3 \int_0^{\pi/2} (\sin\theta - \sin\theta\cos\theta)\, d\theta$$

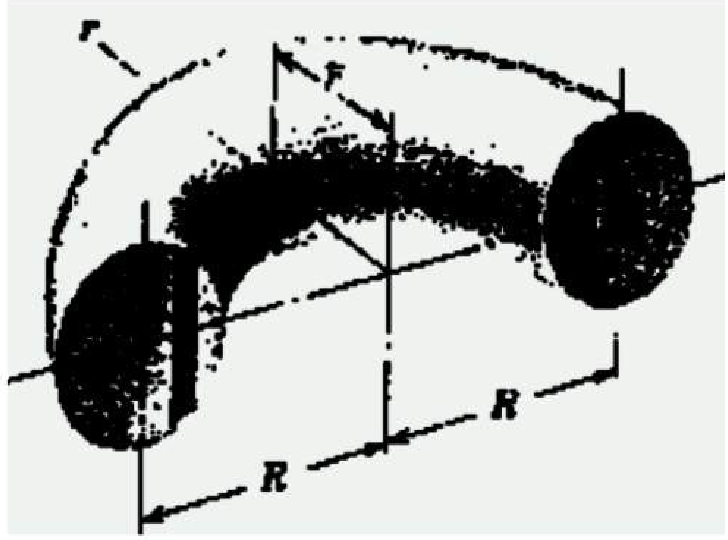
$$= 2\pi a^3\left(1 - \tfrac{1}{2}\right) = \pi a^3$$

$$\int dA = 2\pi a^2 \int_0^{\pi/2} (1-\cos\theta)\, d\theta = 2\pi a^2\left(\frac{\pi}{2} - 1\right)$$

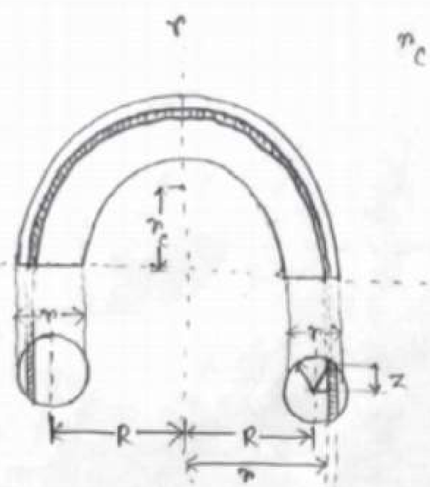
$$\bar{z} = \frac{\int z\, dA}{A} = \frac{\pi a^3}{2\pi a^2\left(\frac{\pi}{2} - 1\right)} = \underline{\frac{a}{\pi - 2}}$$

10. Locate the center of mass G of the steel half ring. (Hint: Choose an element of volume in the form of a cylindrical shell whose intersection with the plane of the ends is shown).

$$\text{Ans. } \bar{r} = \frac{a^2 + 4R^2}{2\square R}$$



10.



$r_c = \frac{2r}{\pi}$

$dA = 2z\,dr$

$dV = \pi r \cdot 2z\,dr$
$= 2\pi r z\,dr$

$z^2 = a^2 - (r-R)^2$

$\therefore z = \sqrt{a^2 - (r-R)^2}$

$\therefore dV = 2\pi r\sqrt{a^2-(r-R)^2}\,dr$

Now, $\bar{r} = \frac{\int r_c\,dm}{\int dm} = \frac{\int r_c\,dV}{\int dV}$

$\therefore \int r_c\,dV = \int \frac{2r}{\pi} \times 2\pi r\sqrt{a^2-(r-R)^2}\,dr$

$= \int 4r^2\sqrt{a^2-(r-R)^2}\,dr$

$= 4\int_{-a}^{a}(u+R)^2\sqrt{a^2-u^2}\,du$ $\quad u = r-R$, $\therefore du = dr$

$= I_1 + I_2 + I_3$

$\therefore I_1 = 4\int_{-a}^{a} u^2\sqrt{a^2-u^2}\,du$
$= \frac{\pi a^4}{2}$

$I_2 = 4\int_{-a}^{a} 2Ru\sqrt{a^2-u^2}\,du = 0.$

$I_3 = 4\int_{-a}^{a} R^2\sqrt{a^2-u^2}\,du = 2\pi a^2R^2.$

$\bar{r} = \frac{\int r_c\,dV}{\int dV} = \frac{\frac{\pi a^4}{2} + 2\pi a^2R^2}{\pi^2 a^2 R} = \frac{a^2+4R^2}{2\pi R}$ (Ans)

$\int dV = 2\pi\int_{-a}^{a}(u+R)\sqrt{a^2-u^2}\,du$

$= 2\pi\int_{-a}^{a} u\sqrt{a^2-u^2}\,du + 2\pi R\int_{-a}^{a}\sqrt{a^2-u^2}\,du$

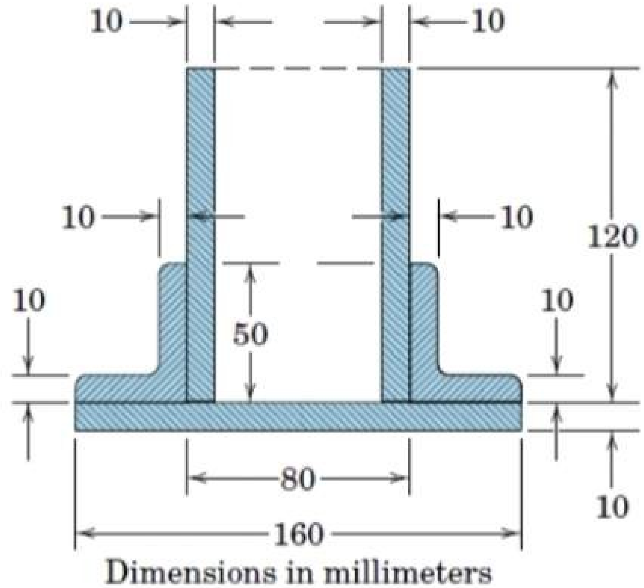
$= 0 + 2\pi R \times \frac{1}{2}\pi a^2$

$= \pi^2 R a^2.$

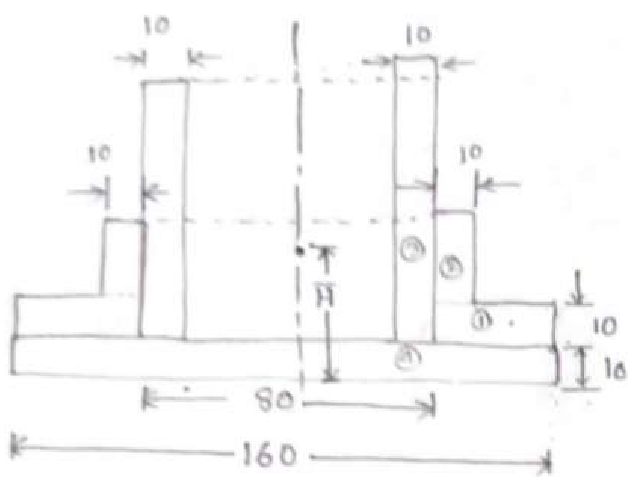
PROBLEM SHEET 5.2

1. Determine the distance $\bar{H}$ from the bottom of the base plate to the centroid of the built-up structural section shown.

Ans. $\bar{H} = 39.3$ mm



1)



Determine $\bar{H}$ from bothom of base plate to the mirucor.

Part	Area(mm^2)	$\bar{h}$(m)	$A\bar{h}$ (mm^3)
1	400×2	15	12000
2	400×2	40	32000
3	120×10×2	70	168000
4	160×10	5	8000

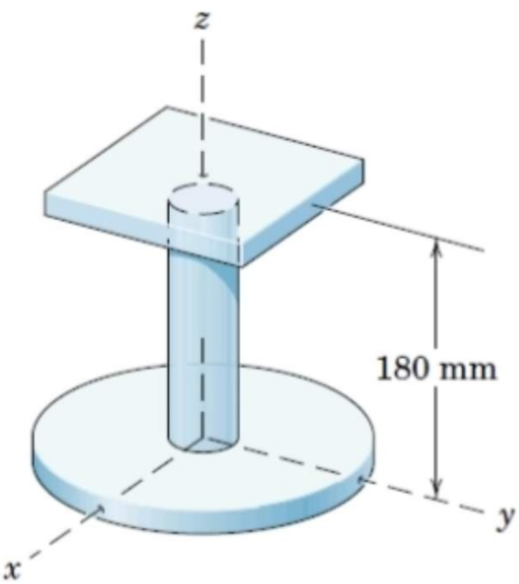
$\Sigma A\bar{h} = 220000$

$\Sigma \bar{h} A = 5600$

$\therefore \bar{H} = \dfrac{\Sigma A\bar{h}}{\Sigma A} = 39.29$ mm (Ans)

2. The rigidly connected unit consists of a 2-kg circular disk, a 1.5-kg round shaft, and a 1-kg square plate. Determine the z-coordinate of the mass center of the unit.

Ans. $\bar{Z} = 70$ mm



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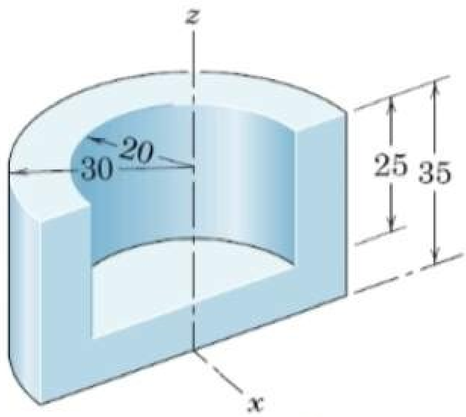
$$\bar{Z} = \frac{\Sigma m\bar{z}}{\Sigma m} = \frac{2(0) + 1.5(90) + 1(180)}{2 + 1.5 + 1} = \underline{70\text{ mm}}$$

4. Calculate the coordinates of the mass center of the metal die casting shown.
Ans. $\bar{X} = -14.71$ mm, $\bar{Z} = 15.17$ mm



Dimensions in millimeters

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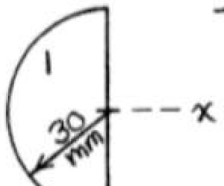


$$V_1 = \frac{\pi}{2}(30)^2(35) = 49\,500 \text{ mm}^3$$

$$\bar{x}_1 = -\frac{4(30)}{3\pi} = -12.73 \text{ mm}$$

$$\bar{z}_1 = 17.5 \text{ mm}$$

Length = 35 mm

$$V_2 = -\frac{\pi}{2}(20)^2(25) = -15\,710 \text{ mm}^2$$

$$\bar{x}_2 = -\frac{4(20)}{3\pi} = -8.49 \text{ mm}$$

$$\bar{z}_2 = \frac{1}{2}(10+35) = 22.5 \text{ mm}$$

Length = 25 mm

$$\bar{X} = \frac{\Sigma V\bar{x}}{\Sigma V} = \frac{49\,500(-12.73) - 15\,710(-8.49)}{49\,500 - 15\,710}$$

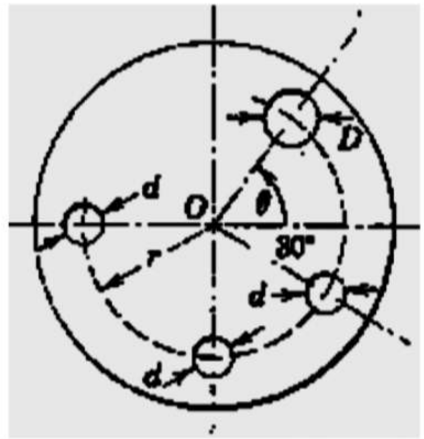
$$= \underline{-14.71 \text{ mm}}$$

$$\bar{Z}_1 = \frac{\Sigma V\bar{z}}{\Sigma V} = \frac{49\,500\,(17.5) - 15\,710\,(22.5)}{49\,500 - 15\,710}$$

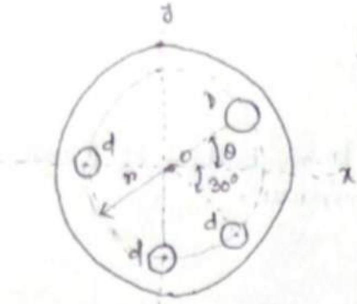
$$= \underline{15.17 \text{ mm}}$$

5. The circular disk rotates about an axis through its center o and has three holes of diameter d positioned as shown. A fourth hole is to be drilled in the disk at the same radius r so that the disk will be in balance (mass center at O). Determine the required diameter D of the new hole and its angular position.

Ans. $D = 1.227d$, $\theta = 84.9^0$



5.



Find out D, θ.

No.	A_i	$\bar{x}_i$	$\bar{y}_i$
1.	πR^2	0	0
2	$\pi \frac{d^2}{4}$	$-r$	0
3.	$\pi \frac{d^2}{4}$	0	$-r$
4.	$\pi \frac{d^2}{4}$	$r\cos 30^\circ$	$-r\sin 30^\circ$
5.	$\frac{\pi}{4} D^2$	$r\cos\theta$	$r\sin\theta$

$$\bar{x} = \frac{A_1\bar{x}_1 \mp A_2\bar{x}_2 - A_3\bar{x}_3 - A_4\bar{x}_4 - A_5\bar{x}_5}{A_1 - (A_2 + A_3 + A_4 + A_5)} = 0$$

$$\therefore \pi R^2 \times 0 = \frac{\pi}{4} d^2(-r) + 0 + \frac{\pi}{4} d^2 r \cos 30^\circ + \frac{\pi}{4} D^2 r \cos\theta$$

$$\text{or } D^2 \cos\theta = 0.13397 d^2 \quad \ldots\ldots (1)$$

$$\bar{y} = 0$$

$$\therefore \sum_{i=2}^{5} A_i \bar{y}_i = 0$$

$$\text{or } +\frac{\pi}{4} d^2 \times 0 + \frac{\pi}{4} d^2(-r) + \frac{\pi}{4} d^2(-r\sin 30^\circ) + \frac{\pi}{4} D^2 r \sin\theta = 0$$

$$\text{or } D^2 \sin\theta = 1.5 d^2 \quad \ldots\ldots (ii)$$

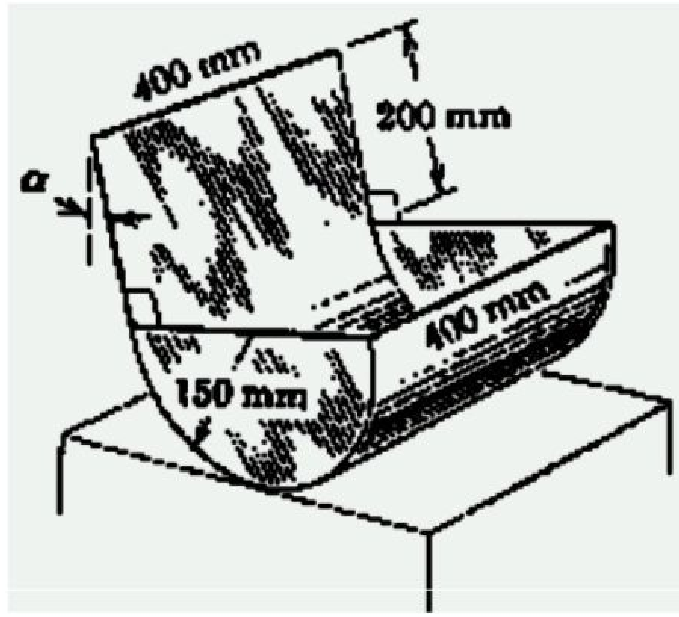
$$\therefore \tan\theta = \frac{1.5}{0.13397} \qquad \therefore \theta = 84.89^\circ$$

$$\therefore D^2 \cos\theta = 0.13397 d^2$$

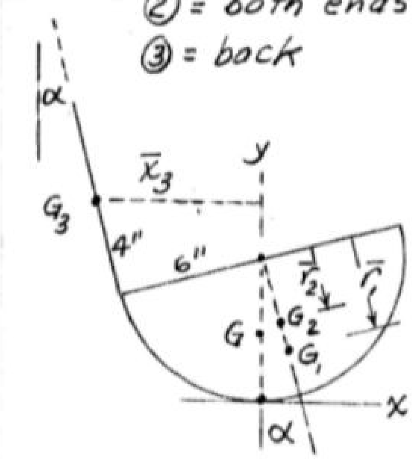
or $D = 1.227d$ (Ans).

6. A cylindrical container with an extended rectangular back and semicircular ends is all fabricated from the same sheet-metal stock. Calculate the angle ⊠ made by the back with the vertical when the container rests in an equilibrium position on a horizontal surface.

Ans. $\alpha = 39.6^0$



▸5/69 | ① = semicircular shell
② = both ends
③ = back



$A_1 = \pi(6)(16) = 301.6 \text{ in.}^2$

$A_2 = 2(\pi \cdot 6^2/2) = 113.1 \text{ in.}^2$

$A_3 = 8(16) \quad = 128 \text{ in}^2$

$\Sigma A = 542.7 \text{ in}^2$

$\bar{r}_1 = \frac{2(6)}{\pi} = 3.82 \text{ in}$

$\bar{r}_2 = \frac{4(6)}{3\pi} = 2.55 \text{ in.}$

$\bar{x}_1 = 3.82 \sin\alpha, \; \bar{x}_2 = 2.55 \sin\alpha$

$\bar{x}_3 = -6\cos\alpha - 4\sin\alpha$

$\bar{X} = \frac{\Sigma A\bar{x}}{\Sigma A}$; $0 = 301.6(3.82\sin\alpha) + 113.1(2.55\sin\alpha)$
$+ 128(-6\cos\alpha - 4\sin\alpha) = 0$

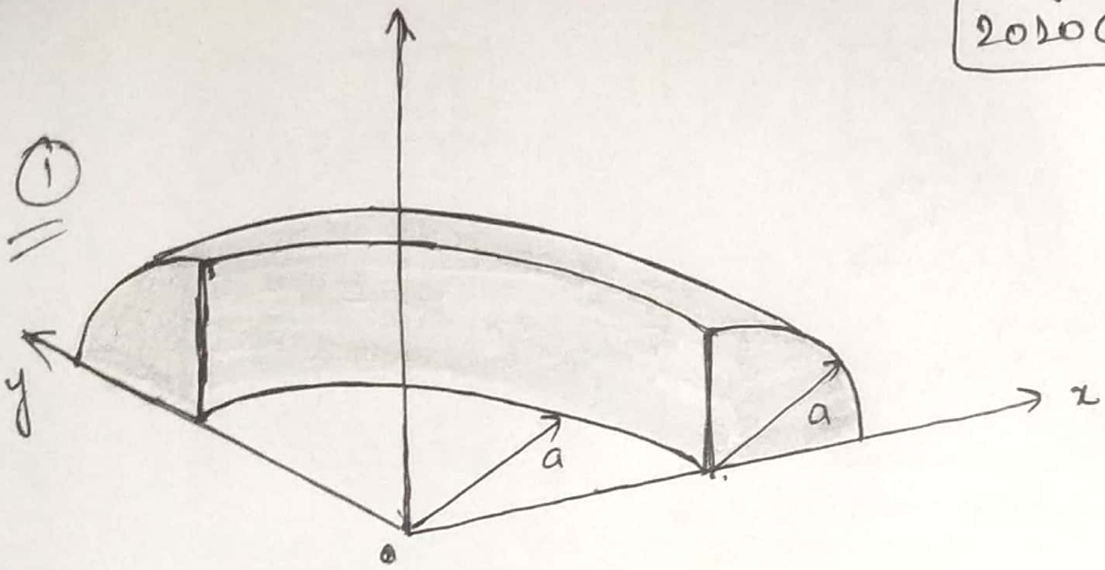
$928.0\sin\alpha = 768\cos\alpha, \quad \tan\alpha = \frac{768}{928} = 0.828$

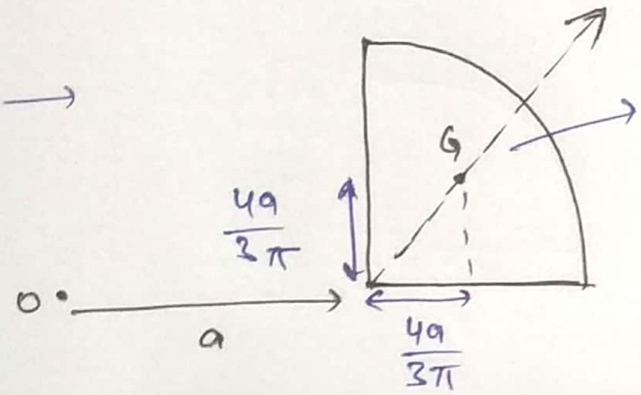
$\underline{\alpha = 39.6°}$

sheet 5.3

Satyen Jha
2020CEB073

1





Quarter circular plate

$$A = \frac{\pi a^2}{8}$$

Total distance travelled by centre of mass of this plate is $\frac{\pi}{2} \times r$ where

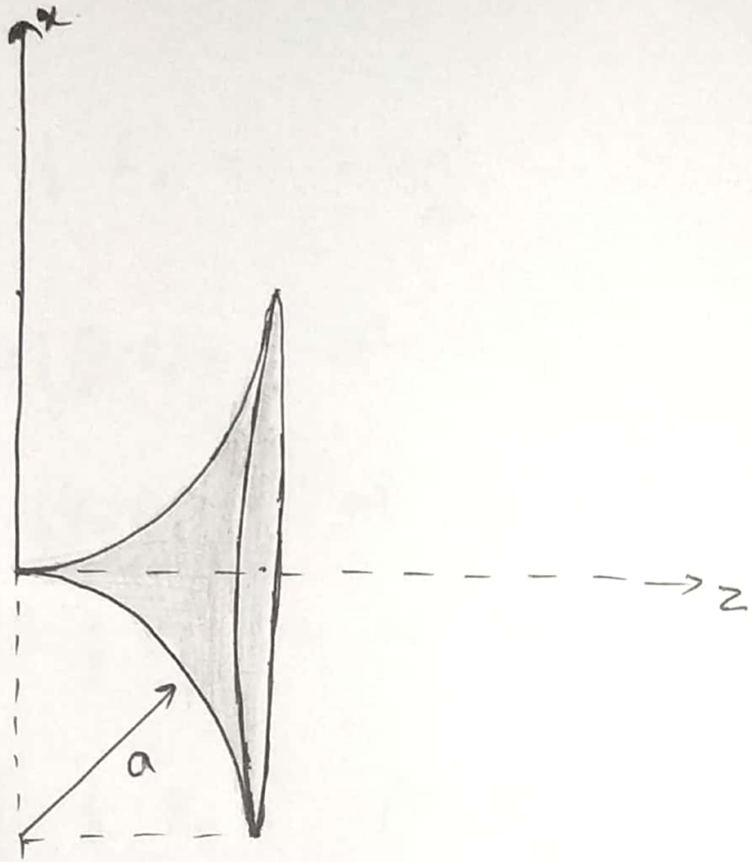
$$r = a + \frac{4a}{3\pi} = a\left(\frac{4+3\pi}{3\pi}\right)$$

So, from pappus theorem (2nd one)

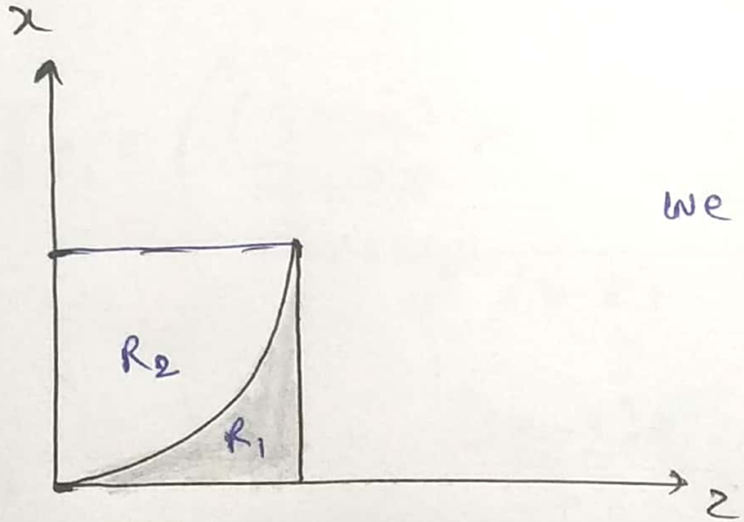
$$V = \frac{\pi}{2} r \times A = \frac{\cancel{\pi}}{2} \times \frac{\pi a^2}{8} \times a\left(\frac{4+3\pi}{3\cancel{\pi}}\right)$$

$$\Rightarrow \boxed{V = \frac{\pi a^3 (3\pi + 4)}{48}}$$

3



$V = ?$



we need to find y_{cm} of R_1 plate

Now;

$$\text{Area of } R_2 = -\frac{\pi a^2}{4}$$

$$\text{Area of } (R_2+R_1) = a^2$$

centroid of (R_2+R_1) is at $\left(\frac{a}{2}, \frac{a}{2}\right)$

centroid of R_2 is at $\left(\frac{4a}{3\pi}, a - \frac{4a}{3\pi}\right)$

So; $\bar{y}$ of $R_1 = \dfrac{\left(a - \frac{4a}{3\pi}\right) \times -\frac{\pi a^2}{4} + a^2 \times \frac{a}{2}}{a^2\left(\frac{4-\pi}{4}\right)}$

$$\Rightarrow \bar{y}_{R_1} = \frac{\left(\frac{(3\pi-4)}{4\times 3\cancel{\pi}} \times -\cancel{\pi}a^3 + \frac{a^3}{2}\right)4}{a^2(4-\pi)}$$

$$\Rightarrow \bar{y}_{R_1} = \frac{2a^3 - \frac{(3\pi-4)a^3}{3}}{(4-\pi)a^2} = \frac{(10-3\pi)}{3(4-\pi)}a.$$

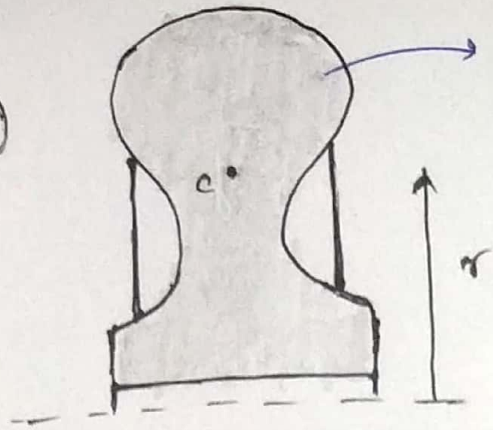
Now; Area of $R_1 = a^2\left(\frac{4-\pi}{4}\right)$

So; using Pappus theorem:-

$$V = 2\pi \times \frac{(10-3\pi)a}{3\cancel{(4-\pi)}} \times a^2\left(\frac{\cancel{4-\pi}}{4}\right)$$

$$\Rightarrow \boxed{V = \frac{\pi a^3}{6}(10-3\pi)}$$

(4)



$A = \frac{1}{2} \times 15,200 \ mm^2$

$V = V$ → using pappus theorem

using density & mass

$\Rightarrow \frac{10 \ kg}{2.69 \ Mg} \ m^3 = \cancel{2}\pi r \times \frac{15,200}{\cancel{2}} \times 10^{-6} \ m^2$

$\Rightarrow \frac{kg}{Mg} \times \frac{10 \ m}{2.69 \times 15,200 \times 3.14} \times 10^{6} = r$

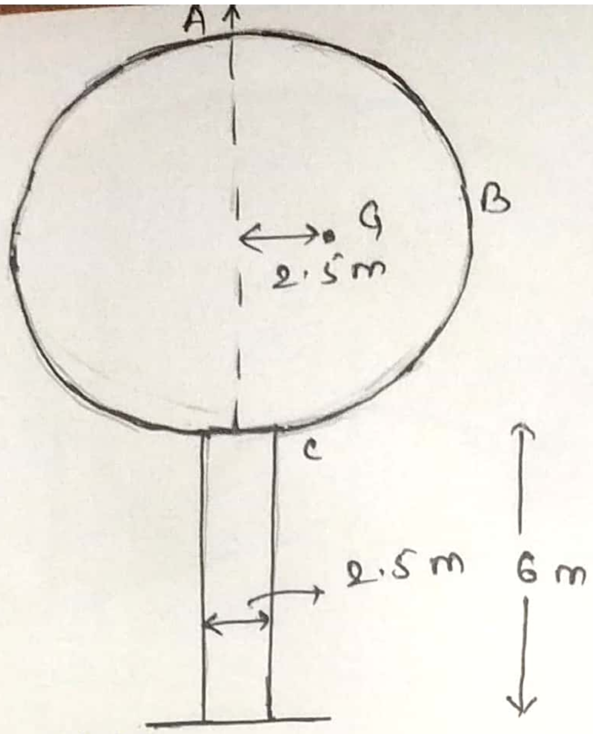
$\Rightarrow r = \frac{10^5 \ m}{2.69 \times 3.14 \times 152} \times 10^{-3}$

$\Rightarrow r = 77.88 \times 10^{-3} \ m$

$\Rightarrow \boxed{r = 77.88 \ mm}$

Scanned with CamScanner

(5)



$l(ABC) = 10\ m$

surface-area

→ ~~volume~~ of tank by using pappus 2nd theorem is:-

$2\pi \times \frac{5}{2} \times 10 = 50\pi\ m^2$

Total area to be painted

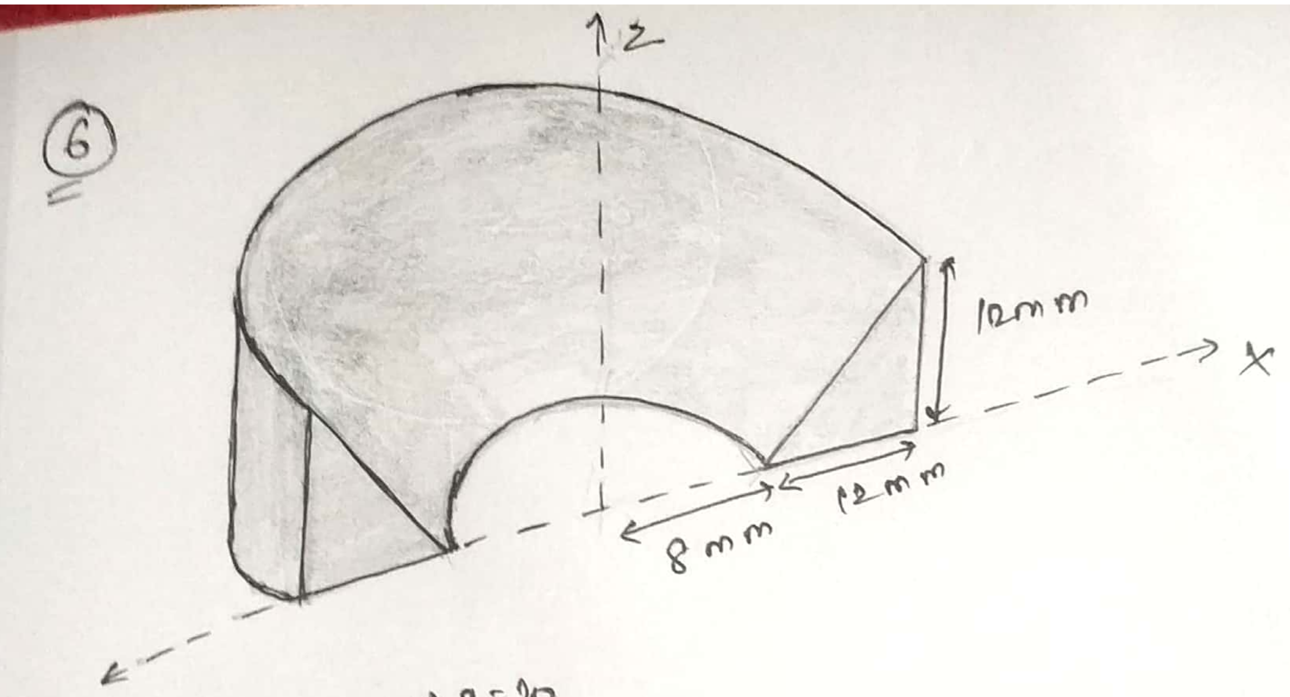
$= 2\left(50\pi + \pi \times 6 \times \frac{5}{2}\right) = 2 \times 65\pi\ m^2 = 130\pi\ m^2$

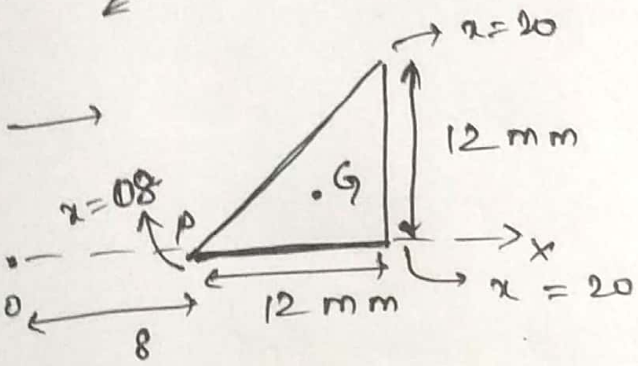
since 2 layers of paint is there.

So; req. volume of paint $= \frac{130\pi}{16} = 25.5$ lts.

Ans

(6)





$A = 72\ mm^2$.

using coordinate geometry $x_G = \frac{20+20+8}{3} = 16$

wrt O

So; centre of Mass (centroid) traverses

a distance of $\pi \times 16 = 16\pi$ mm

So; from pappus Theorem

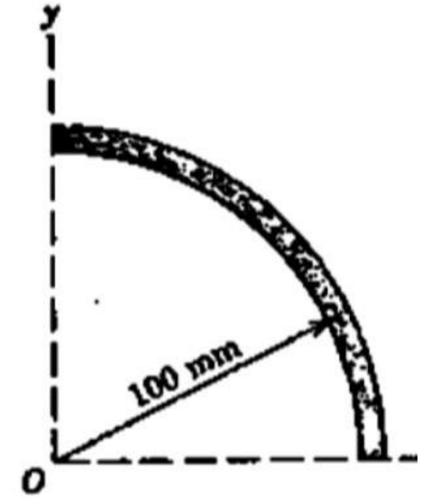
$V = 16\pi \times 72\ mm^3$

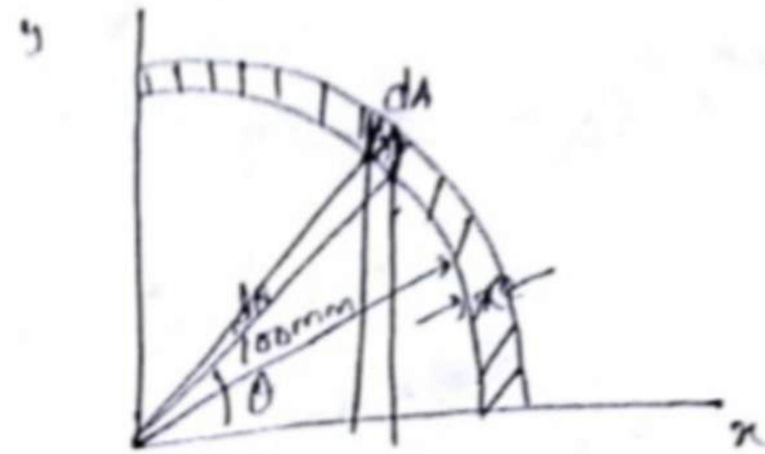
$\Rightarrow \boxed{V = 3617.28\ mm^3}$

PROBLEM SHEET 6.1

1. The thin quarter-circular ring has an area of 1600 mm^2. Determine the moment of inertia of the ring about the x-axis to a close approximation.

Ans $I_x = 8(10^6)\ mm^4$





$dA = r \times d\theta \times x$, now, $A = 1600 mm^2$

$\Rightarrow \frac{\pi \times 100}{2} \times x = 1600$ $\Rightarrow x = \frac{32}{\pi}$ mm

now, $I_x = \int_0^{\pi/2} dA\, y^2$

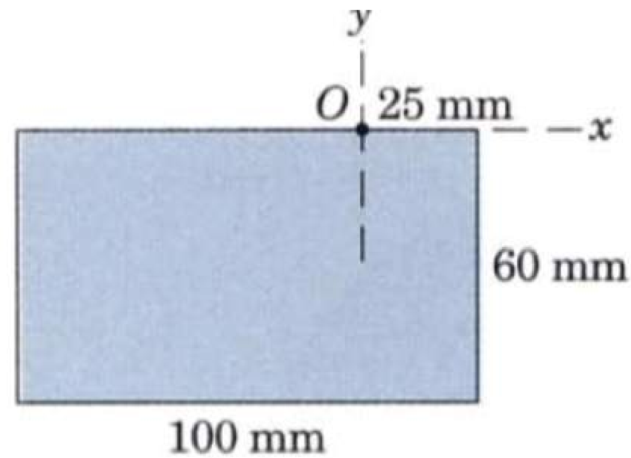
$= \int_0^{\pi/2} r^2 \sin^2\theta \times r \times x\, d\theta$

$= r^3 \times x \times \int_0^{\pi/2} \frac{1-\cos 2\theta}{2} d\theta$

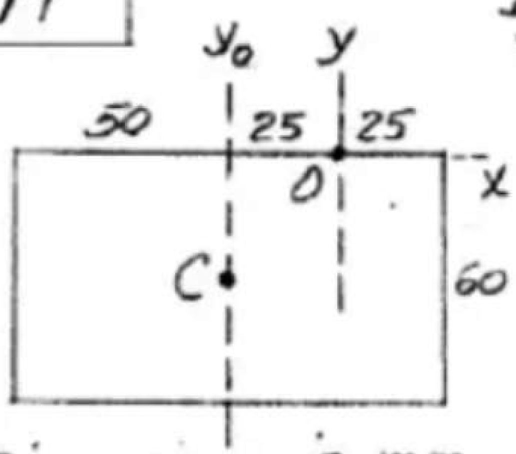
$= \frac{r^3 x}{2} \times \left[\theta - \frac{\sin 2\theta}{2}\right]_0^{\pi/2}$

$= \frac{r^3}{2} x \times \left[\frac{\pi}{2}\right] = 8 \times 10^6 \ mm^4$

3. Calculate the moment of inertia of the rectangular area about the x-axis and find the polar moment of inertia about point O.
Ans. $I_x = 7.2(10^6)\ mm^4$, $I_o = 15.95(10^6)\ mm^4$



A/9



Dimensions in mm

$$I_{y_0} = \frac{1}{12}bd^3 = \frac{1}{12}(60)(100)^3 = 5(10^6)\ mm^4$$

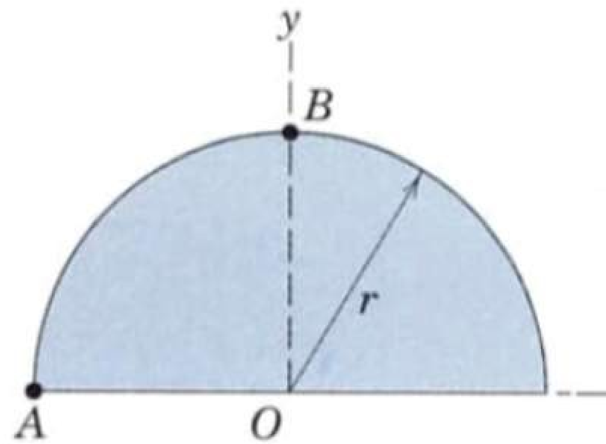
$$I_y = I_{y_0} + Ad^2 = 5(10^6) + 6(10^3)(25)^2$$
$$= 8.75(10^6)\ mm^4$$

$$I_x = \frac{1}{3}bd^3 = \frac{1}{3}(100)(60^3) = \underline{7.2(10^6)\ mm^4}$$

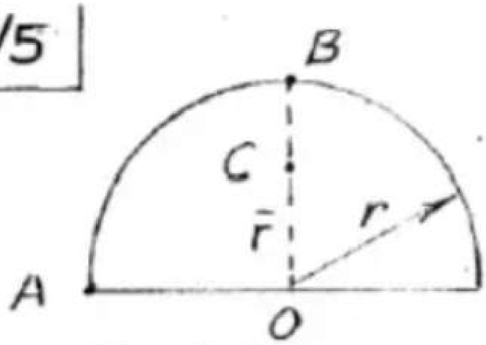
$$I_o = I_x + I_y = (7.2 + 8.75)10^6 = \underline{15.95(10^6)}\ mm^4$$

4. Determine the polar moments of inertia of the semicircular area about points A and B.

Ans . $I_A = \frac{3}{4}$▨r^4, $I_B = r^4\left(\frac{3▨}{4} - \frac{4}{3}\right)$,



A/5



$\bar{r} = 4r/3\pi$

For complete circle

$$I_A = I_O + Ar^2 = \frac{1}{2}Ar^2 + Ar^2$$
$$= \frac{3}{2}Ar^2$$

For half circle

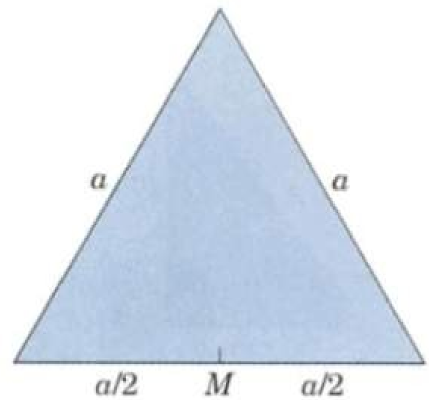
$$I_A = \frac{1}{2}\left(\frac{3}{2}\pi r^4\right) = \underline{\frac{3}{4}\pi r^4}$$

For half circle, $I_O = \frac{1}{4}\pi r^4$

$$I_B = I_C + A(r-\bar{r})^2 = I_O - A\bar{r}^2 + A(r-\bar{r})^2$$
$$= I_O + A(r^2 - 2r\bar{r})$$
$$= \frac{1}{4}\pi r^4 + \frac{\pi r^4}{2}\left(1 - \frac{8}{3\pi}\right) = \underline{r^4\left(\frac{3\pi}{4} - \frac{4}{3}\right)}$$

5. Determine the polar radius of gyration of the area of the equilateral triangle about the midpoint M of its base.

Ans . $k_m = \dfrac{a}{\sqrt{6}}$



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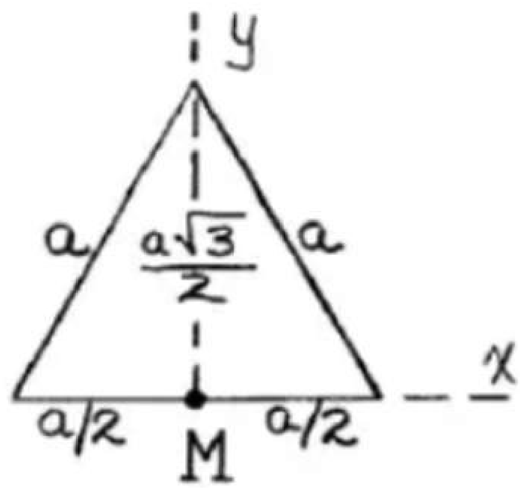


$$I_z = I_x + I_y, \quad I_z = Ak_z^2$$

$$\therefore \quad k_M = \sqrt{(I_x + I_y)/A}$$

$$I_x = \frac{1}{12} bh^3 = \frac{1}{12} a \left(\frac{a\sqrt{3}}{2}\right)^3$$

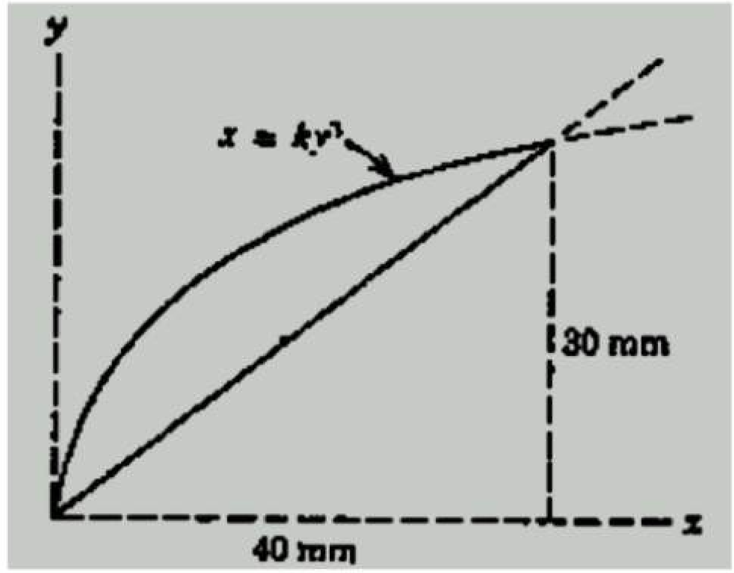
$$= \frac{\sqrt{3}}{32} a^4$$

$$I_y = 2\left(\frac{1}{12} \frac{a\sqrt{3}}{2} \left(\frac{a}{2}\right)^3\right) = \frac{\sqrt{3}}{96} a^4$$

$$k_M = \left[\frac{\frac{\sqrt{3}}{32} a^4 + \frac{\sqrt{3}}{96} a^4}{\frac{a}{2} a \frac{\sqrt{3}}{2}}\right]^{1/2} = \underline{\frac{a}{\sqrt{6}}}$$

6. Calculate the moment of inertia of the shaded area about the x-axis.

Ans. $I_x = 9(10^4)\ mm^4$



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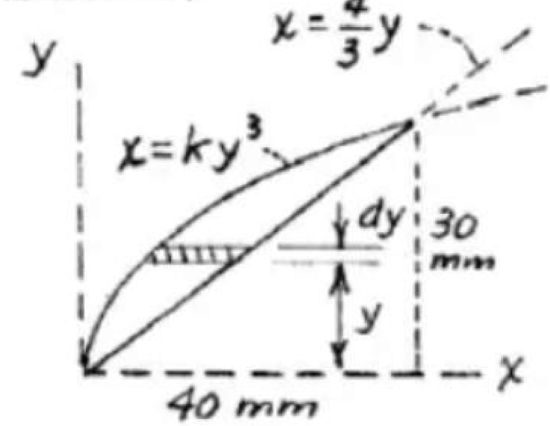


For $x = 40$ mm & $y = 30$ mm, $k = \frac{40}{27(10^3)}$

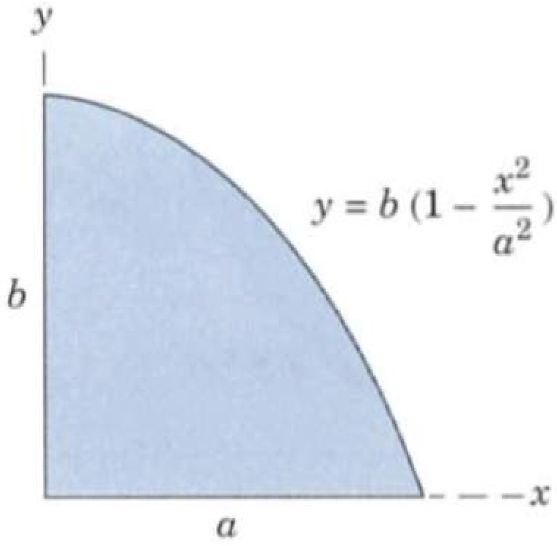
$$dI_x = y^2 dA = y^2\left(\frac{4}{3}y - \frac{4}{2700}y^3\right)dy$$

$$I_x = \int_0^{30}\left(\frac{4}{3}y^3 - \frac{4}{2700}y^5\right)dy$$

$$= \left[\frac{y^4}{3} - \frac{y^6}{4050}\right]_0^{30} = \underline{9(10^4)\ mm^4}$$

7. Determine the moment of inertia of the shaded area about the x-axis using (a) a horizontal strip of area and (b) a vertical strip of area.

Ans. I_x = $16ab^3/105$



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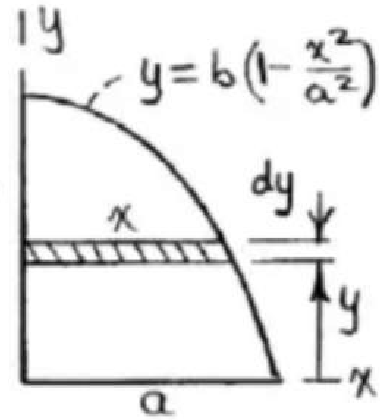
(a) Horizontal strip



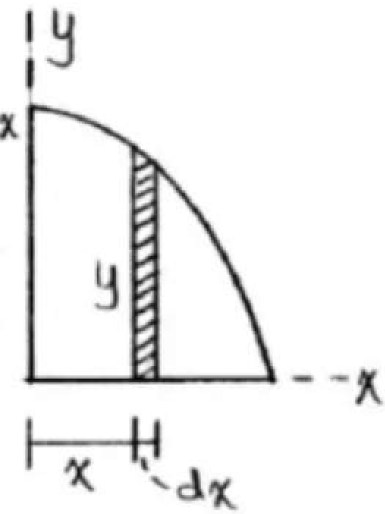
$$I_x = \int y^2 dA = \int_0^b y^2 x\,dy$$

$$= \int_0^b y^2 a\sqrt{1-y/b}\;dy$$

$$= \frac{a}{\sqrt{b}}\int_0^b y^2\sqrt{b-y}\;dy$$

$$= \frac{a}{\sqrt{b}}\,\frac{2}{105(-1)}\left(8b^2+12by+15y^2\right)\sqrt{(b-y)^3}\,\Big|_0^b = \underline{\frac{16ab^3}{105}}$$

(b) Vertical strip



$$I_x = \int_0^a \frac{1}{3}y^2(y\,dx) = \frac{1}{3}\int_0^a b^3\left(1-\frac{x^2}{a^2}\right)^3 dx$$

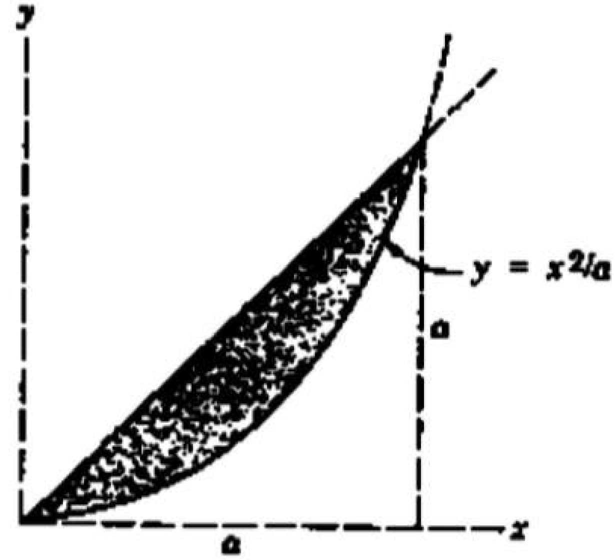
$$= \frac{b^3}{3}\frac{1}{a^6}\int_0^a \left(a^6-3a^4x^2+3a^2x^4-x^6\right)dx$$

$$= \frac{b^3}{3a^6}\left[a^6x-a^4x^3+\frac{3a^2x^5}{5}-\frac{x^7}{7}\right]_0^a$$

$$= \underline{\frac{16ab^3}{105}}$$

8. Determine the moment of inertia of the shaded area about the x– and y–axes. Use the same differential element for both calculations.

Ans. $I_x = a^4/28$, $I_y = a^4/20$



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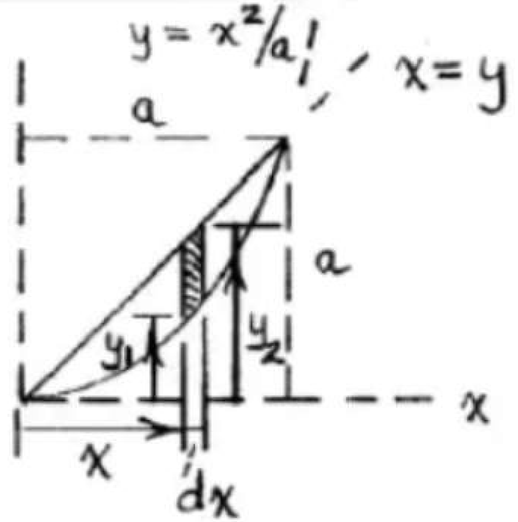


$$dA = (y_2 - y_1)dx = \left(x - \frac{x^2}{a}\right)dx$$

$$I_x = \int \frac{1}{3} y_2^3 dx - \int \frac{1}{3} y_1^3 dx = \frac{1}{3}\int_0^a \left(x^3 - \frac{x^6}{a^3}\right)dx$$

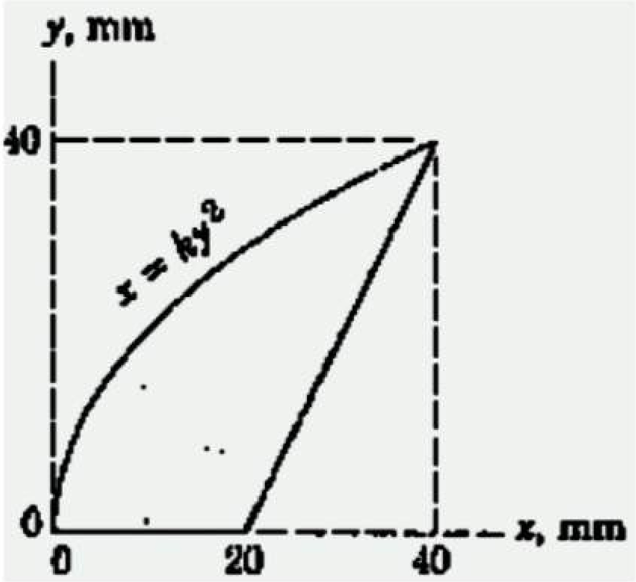
$$= \frac{1}{3}\left[\frac{x^4}{4} - \frac{x^7}{7a^3}\right]_0^a = \frac{a^4}{3}\left(\frac{1}{4} - \frac{1}{7}\right) = \underline{a^4/28}$$

$$I_y = \int x^2 dA = \int x^2\left(x - \frac{x^2}{a}\right)dx = \int_0^a \left(x^3 - \frac{x^4}{a}\right)dx$$

$$= \left[\frac{x^4}{4} - \frac{x^5}{5a}\right]_0^a = \frac{a^4}{4} - \frac{a^4}{5} = \underline{a^4/20}$$

9. Determine the moment of inertia of the shaded area about the y-axis.

Ans. $I_y = 27.8(10^4)\ mm^4$



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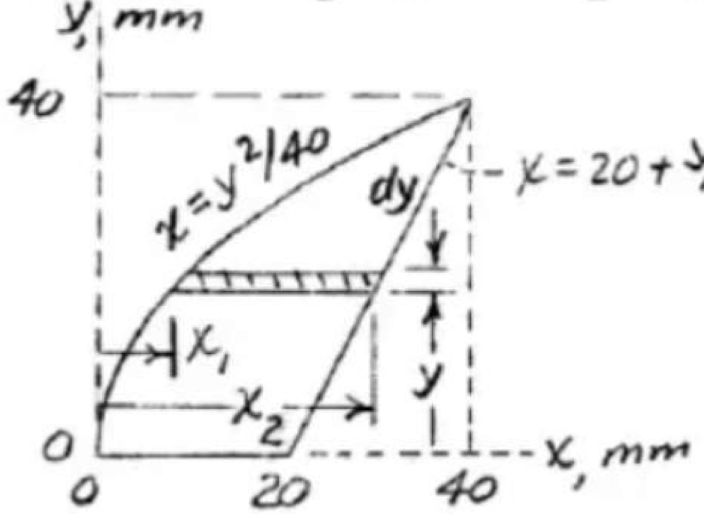
$$dI_y = \frac{1}{3}dy(x_2^3 - x_1^3) = \frac{1}{3}\left[\left(20 + \frac{y}{2}\right)^3 - \left(\frac{y^2}{40}\right)^3\right]dy$$



$$\int_0^{40}\left(20 + \frac{y}{2}\right)^3 dy$$

$$= \frac{1}{2}\left(20 + \frac{y}{2}\right)^4\Big]_0^{40} = 120(10^4)\ mm^4$$

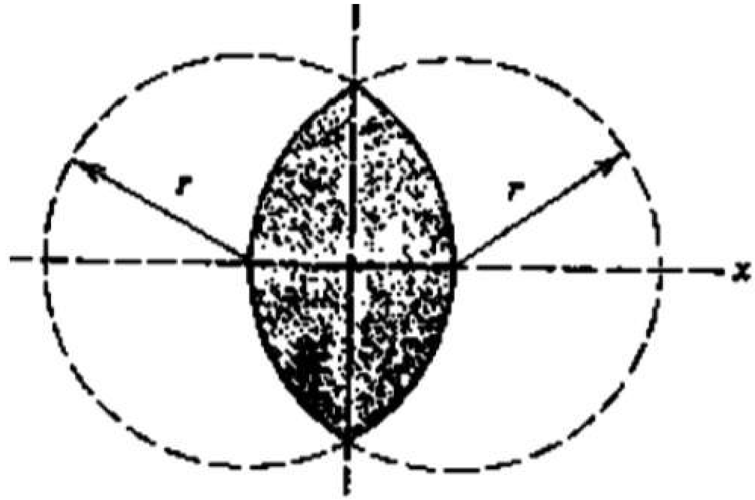
$$\int_0^{40}\frac{y^6}{40^3}dy = \frac{1}{40^3}\frac{y^7}{7}\Big]_0^{40}$$

$$= 36.57(10^4)\ mm^4$$

$$I_y = \frac{1}{3}(120 - 36.6)10^4 = \underline{27.8\,(10^4)\ mm^4}$$

10. Calculate the moment of inertia of the overlapping shaded area of the two circles about the x-axis.

Ans. $I_x = 0.1988r^4$



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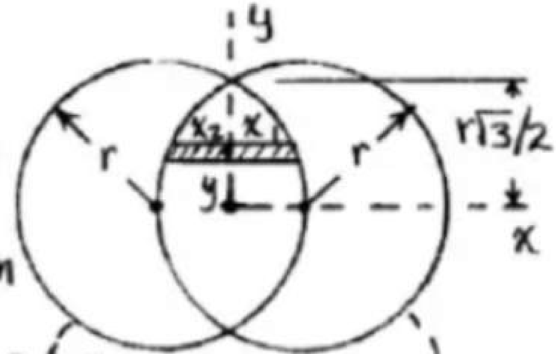
$$x_2 = \frac{r}{2} \pm \sqrt{r^2-y^2} \quad \text{use } (-) \text{ sign}$$

$$x_1 = -\frac{r}{2} \pm \sqrt{r^2-y^2} \quad \text{use } (+) \text{ sign}$$



$$\left(x_1+\frac{r}{2}\right)^2+y^2=r^2$$

$$\left(x_2-\frac{r}{2}\right)^2+y^2=r^2$$

$$(x_1-x_2) = -\frac{r}{2}+\sqrt{r^2-y^2}-\frac{r}{2}+\sqrt{r^2-y^2} = 2\sqrt{r^2-y^2}-r$$

$$dA = \left(2\sqrt{r^2-y^2}-r\right)dy$$

$$I_x = \int y^2\,dA = 2\int_0^{r\sqrt{3}/2} y^2\left(2\sqrt{r^2-y^2}-r\right)dy$$

$$= 4\left\{-\frac{y}{4}\sqrt{(r^2-y^2)^3}+\frac{r^2}{8}\left(y\sqrt{r^2-y^2}+r^2\sin^{-1}\frac{y}{r}\right)\right\}-\frac{2r}{3}y^3\Bigg|_0^{\frac{r\sqrt{3}}{2}}$$

$$= 4\left\{-\frac{r\sqrt{3}}{8}\frac{r^3}{8}+\frac{r^2}{8}\left(\frac{r\sqrt{3}}{2}\frac{r}{2}+r^2\frac{\pi}{3}\right)\right\}-\frac{2\sqrt{3}}{8}r^4-0$$

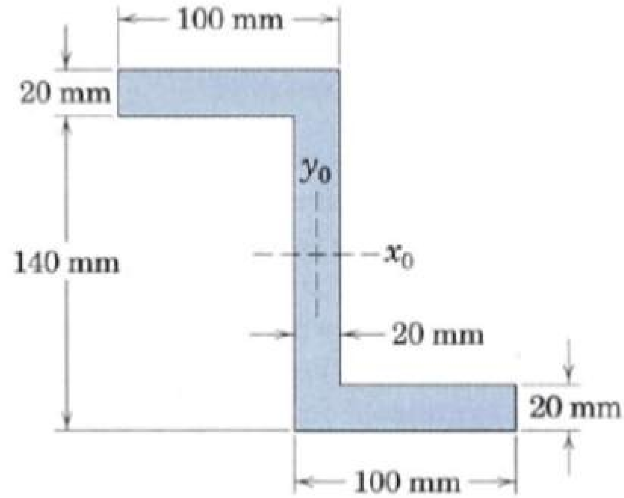
$$= \frac{r^4}{2}\left\{-\frac{\sqrt{3}}{8}+\frac{\sqrt{3}}{4}+\frac{\pi}{3}-\frac{\sqrt{3}}{2}\right\} = \frac{r^4}{2}\left\{\frac{\pi}{3}-\frac{3\sqrt{3}}{8}\right\}$$

$$= 0.1988r^4$$

PROBLEM SHEET
6.2

2. Determine the moments of inertia of the Z-section about its centroidal x_0- and y_0-axes.

Ans. $I_x = 22.6(10^6)$ mm^4,
$I_y = 9.81(10^6)$ mm^4



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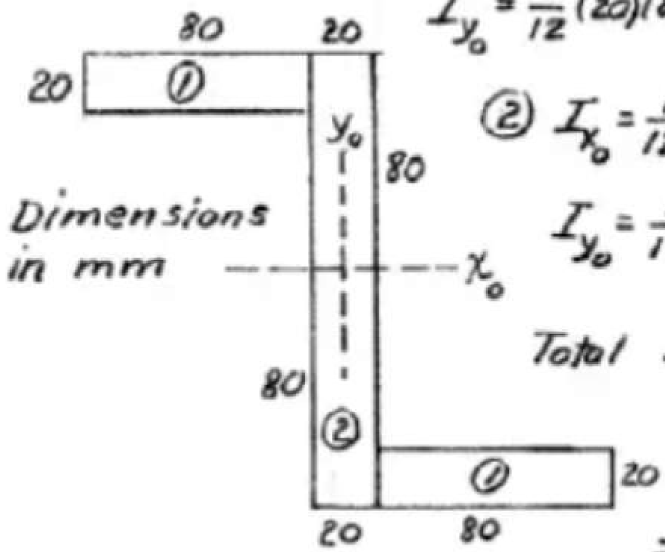
① $I_{x_0} = \frac{1}{12}(80)(20)^3 + (80)(20)(70)^2 = 7.89(10^6)$ mm^4

$I_{y_0} = \frac{1}{12}(20)(80)^3 + (20)(80)(50)^2 = 4.85(10^6)$ mm^4



② $I_{x_0} = \frac{1}{12}(20)(160)^3 = 6.83(10^6)$ mm^4

$I_{y_0} = \frac{1}{12}(160)(20)^3 = 0.1067(10^6)$ mm^4

Total $\bar{I}_x = [2(7.89) + 6.83](10^6)$

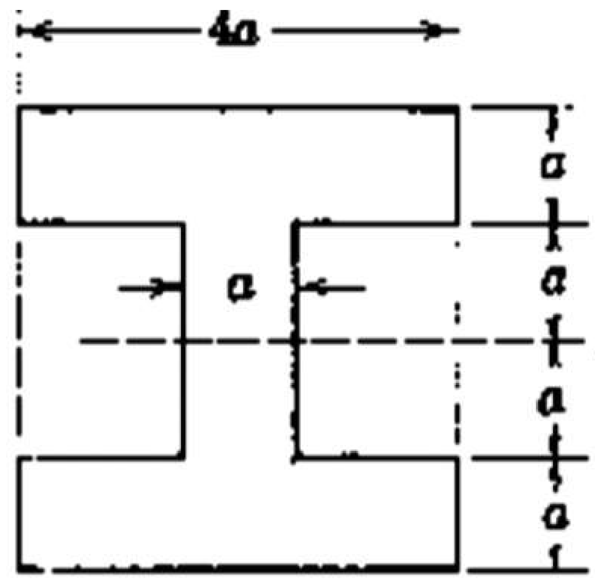
$= \underline{22.6(10^6) \text{ mm}^4}$

$\bar{I}_y = [2(4.85) + 0.1067](10^6)$

$= \underline{9.81(10^6) \text{ mm}^4}$

3. Determine the moment of inertia of the shaded area about the x-axis.

$$Ans.\ I = \frac{58}{3}a^4$$



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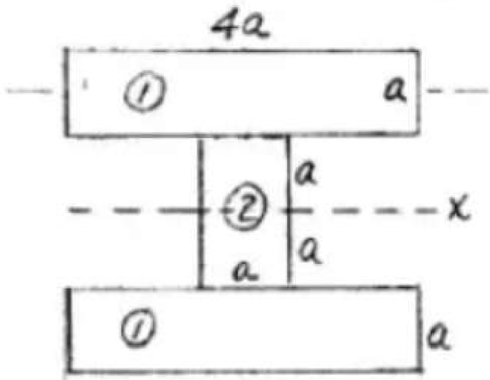
Sol. I

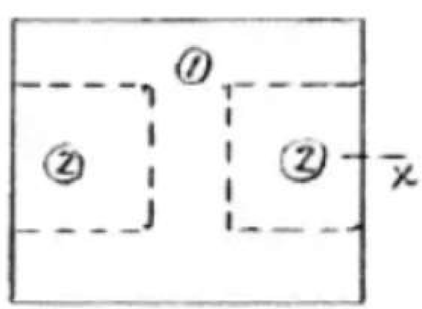
$$I_{①} = 2\left[\frac{1}{12}4a(a^3) + 4a^2\left(\frac{3a}{2}\right)^2\right] = \frac{56}{3}a^4$$



$$I_{②} = \frac{1}{12}a(2a)^3 = \frac{2}{3}a^4$$

Total $I_x = \frac{58}{3}a^4$

Sol. II



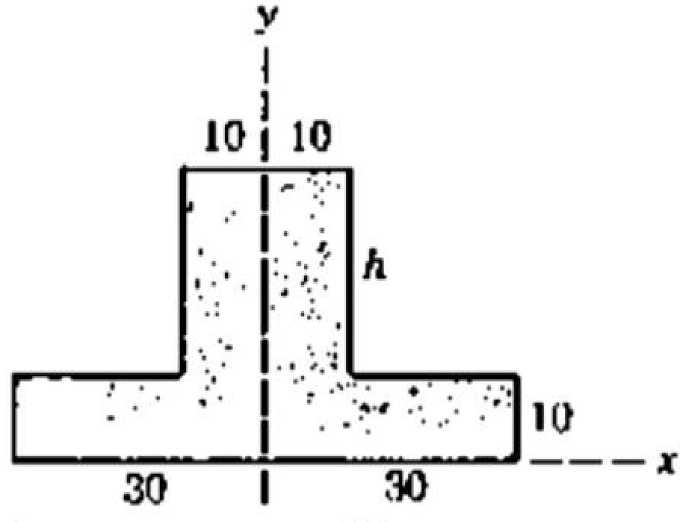
$$I_{①} = \frac{1}{12}(4a)(4a)^3 = \frac{64}{3}a^4$$

$$I_{②} = -\frac{1}{12}(3a)(2a)^3 = -2a^4$$

Total $= \left(\frac{64}{3} - \frac{6}{3}\right)a^4 = \frac{58}{3}a^4$

4. Calculate the value of h for which $I_x = I_y$ for the shaded area shown.

Ans. $h = 20.0$ mm



Dimensions in Millimeters

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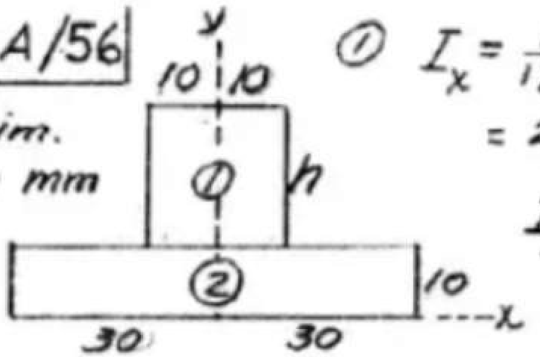


① $I_x = \frac{1}{12}(20)h^3 + 20h\left(\frac{h}{2}+10\right)^2$

$= 20\left(h^3/3 + 10h^2 + 100h\right)$ mm^4

$I_y = \frac{1}{12}h(20)^3 = \frac{2000h}{3}$ mm^4

② $I_x = \frac{1}{3}(60)(10)^3 = 20000$ mm^4

$I_y = \frac{1}{12}(10)(60)^3 = 180\,000$ mm^4

Thus for equal I_x & I_y totals

$20\left(h^3/3 + 10h^2 + 100h\right) + 20000 = \frac{2000h}{3} + 180\,000$

or $h^3 + 30h^2 + 200h = 24000$

Substitute $h = u - 10$ & get $u^3 = 100u + 24000$ & solve by Appen. B/4-4: Let $p = 100/3$, $q = 12000$ Case II $q^2 - p^3 = (+)$

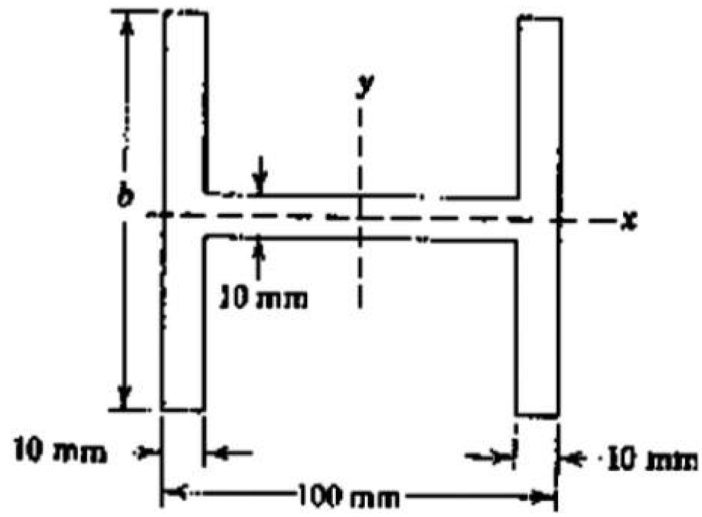
$(q^2 - p^3)^{1/2} = \sqrt{144(10^6) - 10^6/27} = 11.99846\,(10^3)$

$u_1 = (12000 + 11998)^{1/3} + (12000 - 11998.46)^{1/3} = 28.84 + 1.16 = 30.00$

$h = 30.00 - 10 = \underline{20.0 \text{ mm}}$

5. For the H-beam section determine the flange width b that will make the moments of inertia about the central x- and y- axes equal.

Ans. b = 161 mm



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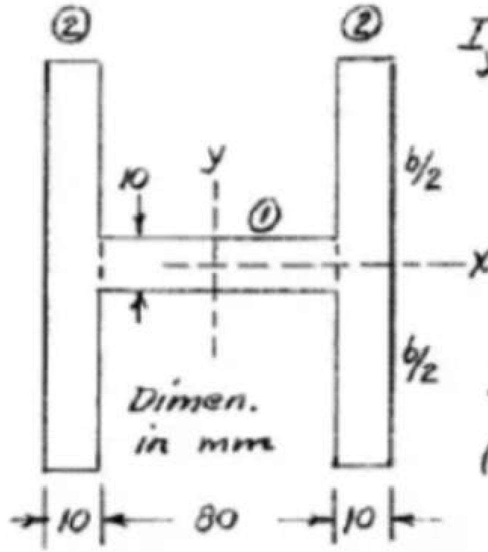


① $I_x = \frac{1}{12}(80)(10)^3 = 0.00667(10^6)\ mm^4$

$I_y = \frac{1}{12}(10)(80)^3 = 0.427(10^6)\ mm^4$

② $I_x = 2\left[\frac{1}{12}(10)b^3\right] = 1.667b^3$

$I_y = 2\left[\frac{1}{12}b(10)^3\right] + (10b)(45)^2\Big]$

$= 0.0407(10^6)b$

Total I_x = Total I_y

$(0.00667)(10^6) + 1.667b^3$

$= (0.427 + 0.0407)(10^6)$

or $b^3 - 0.0244(10^6)b - 0.252(10^6) = 0$

Solve by cubic formula; $\left[\frac{0.252(10^6)}{2}\right]^2 < \left[\frac{0.0244(10^6)}{3}\right]^3$ so 3 real roots

$\cos u = \frac{q}{p\sqrt{p}}$ where $q = \frac{252(10^3)}{2} = 126(10^3)$, $p = \frac{24.4(10^3)}{3} = 8.13(10^3)$

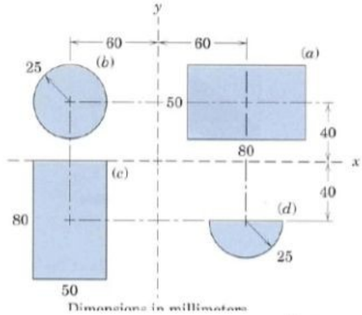
$\cos u = 126(10^3)/[8.13(10^3)90.2] = 0.1718$, $u = 80.11°$

$b_1 = 2\sqrt{p}\cos u/3 = 2(90.2)(0.8933) = 161.1\ mm$ or $\underline{b = 161.1\ mm}$

$b_2 = 2\sqrt{p}\cos\left(\frac{u}{3} + 120°\right) = (-)$, $b_3 = 2\sqrt{p}\cos\left(\frac{u}{3} + 240°\right) = -(\ )$

PROBLEM SHEET
6.3

1. Determine the product of inertia of each of the four areas about the x–y axes.
Ans. (a) and (c) $I_{xy} = 9.60\ (10^6)\ mm^4$,
(b) $I_{xy} = -4.71(10^6)\ mm^4$,
(c) $I_{xy} = -2.98(10^6)\ mm^4$



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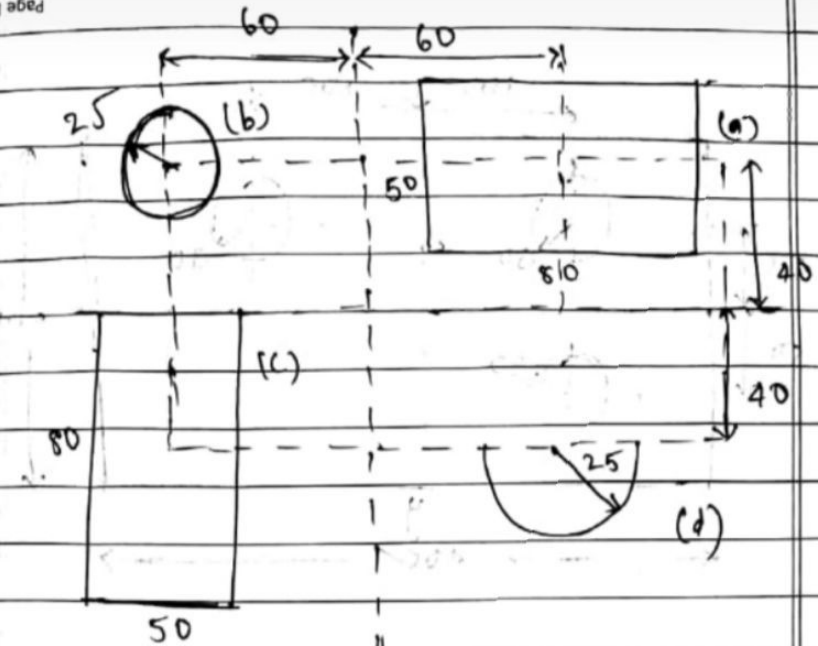


Soln:

(a) $\hat{I}_{xy} = \bar{I}_{xy} + d_x d_y A = 0 + (60)(40)(80)(50)$

$= 9.60(10^6)\ mm^4$

(b) $\hat{I}_{xy} = \bar{I}_{xy} + d_x d_y A = 0 + (-60)(40)(\pi \cdot 25^2)$

$= -4\pi(10^6)\ mm^4$

(c) $\hat{I}_{xy} = \bar{I}_{xy} + d_x d_y A = 0 + (-60)(-40)(80)(50)$

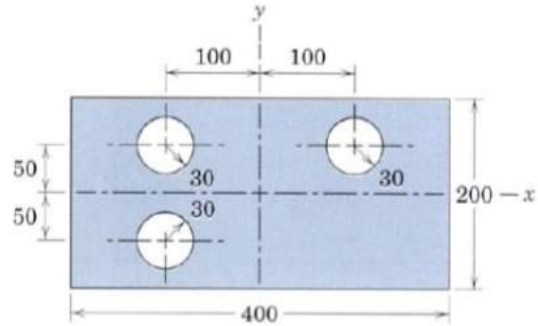
$= 9.60\ (10^6)\ mm^4$

(d) $\hat{I}_{xy} = \bar{I}_{xy} + d_x d_y A = 0 + (60)\left[-40 - \frac{4}{3\pi}(25)\right] \times \left(\frac{\pi \cdot 25^2}{2}\right)$

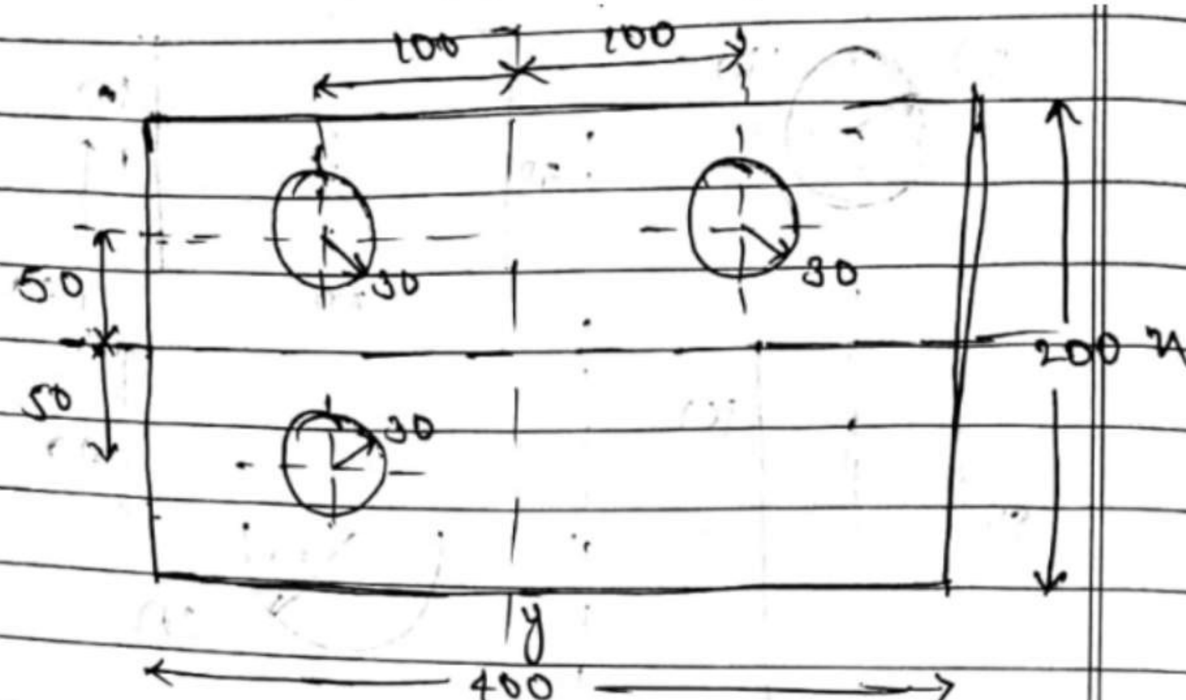
$= -2.98\ (10^6)\ mm^4$

Date

2. Determine I_x, I_y and I_{xy} for the rectangular plate with three equal circular holes.
Ans. $I_x = 2.44(10^8)\ mm^4$, $I_y = 9.80(10^8)\ mm^4$, $I_{xy} = -14.14(10^6)\ mm^4$



Dimensions in millimeters



Soln:

$$I_x = \frac{1}{12}(400)(200)^3 - 3\left[\frac{\pi}{4}(30)^4 + \pi(30)^2(50)^2\right]$$

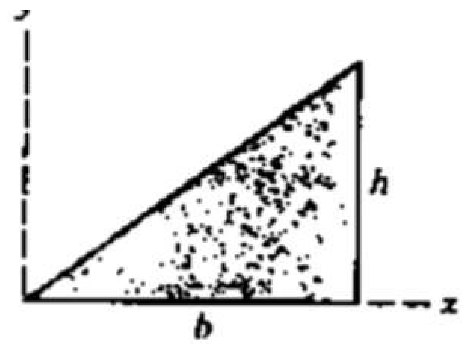
$$= 2.44(10^8)\ mm^4$$

$$I_y = \frac{1}{12}(200)(400)^3 - 3\left[\frac{\pi}{4}(30)^4 + \pi(30)^2(100)^2\right]$$

$$= 9.80(10^8)\ mm^4$$

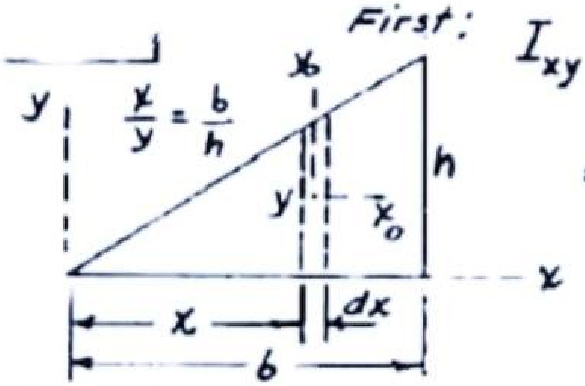
$$I_{xy} = -\pi(30)^2\left[(100)(50) + (-100)(50) + (-100)(-50)\right]$$

$$= -14.14(10^6)\ mm^4$$



3. Derive the expression for the product of inertia of the right-triangular area about the x–y axes.

Ans. $I_{xy} = b^2h^2/8$



First: $I_{xy} = \int_0^b \int_0^y xy\,dy\,dx$

$= \int_0^b \left[\frac{x}{2}y^2\right]_0^{\frac{hx}{b}} dx = \int_0^b \frac{h^2}{2b^2} x^3 dx$

$= \frac{h^2}{2b^2}\,\frac{b^4}{4} = \underline{\frac{b^2h^2}{8}}$

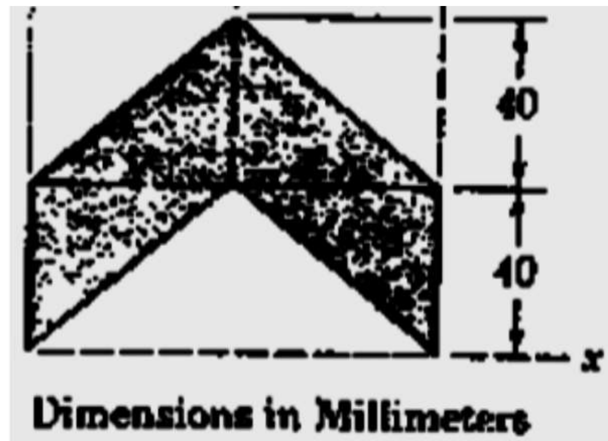
Second: $dI_{xy} = dI_{x_0 y_0} + d_x d_y (dA)$

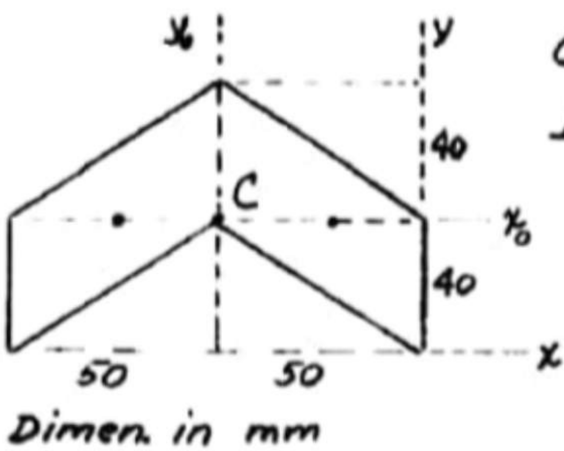
$= 0 + \frac{y}{2}x\,(y\,dx) = \frac{h^2}{2b^2}x^3 dx$

$I_{xy} = \frac{h^2}{2b^2}\int_0^b x^3 dx = \frac{h^2}{2b^2}\,\frac{b^4}{4} = \underline{\frac{b^2h^2}{8}}$

4. Determine the product of inertia of the shaded area with respect to the assigned axes. (hint: locate the centroid of the symmetrical area).

Ans. $I_{xy} = -8(10^6)\ mm^4$





Centroid is at C

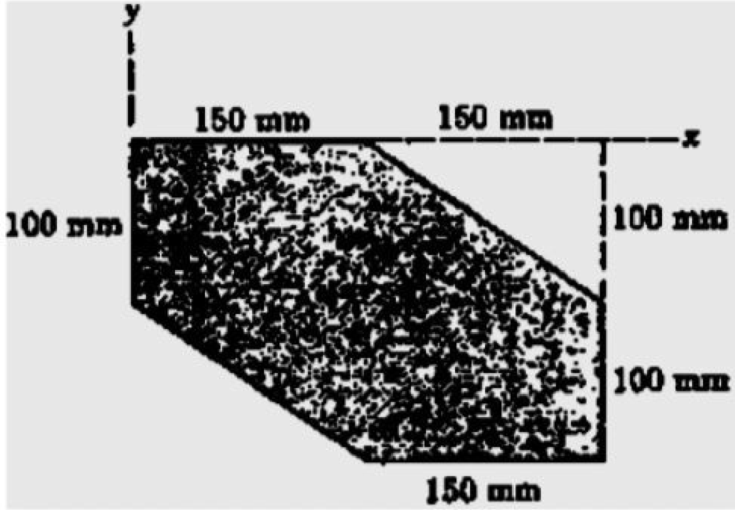
$$I_{xy} = I_{x_0y_0} + d_x d_y A$$

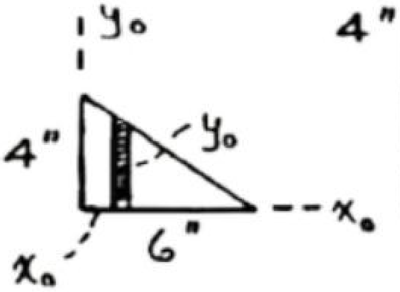
$$= 0 + (40)(-50)(40 \times 100)$$

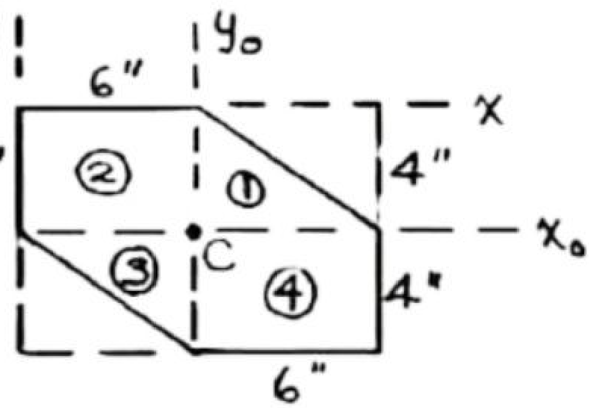
$$= \underline{-8(10^6)\ mm^4}$$

5. Calculate the product of inertia of the shaded area about the x-y axes. (Hint: Take advantage of the transfer-of-axes relations).

Ans. . $I_{xy} = -769(10^6)\ mm^4$







Part 1: $I_{x_0y_0} = \int_0^6 x_0 \frac{y_0}{2}(y_0\,dx_0)$, $y_0 = 4 - \frac{2}{3}x_0$

$= \frac{1}{2}\int_0^6 x_0\left(16 - \frac{16}{3}x_0 + \frac{4}{9}x_0^2\right)dx_0$

$= \frac{1}{2}\left[8x_0^2 - \frac{16}{9}x_0^3 + \frac{1}{9}x_0^4\right]_0^6 = 24\ \text{in.}^4$

Part 3 : $I_{x_0y_0} = 24\ \text{in.}^4$

Part 2 : $I_{x_0y_0} = 4(6)(-3)(+2) = -144\ \text{in}^4$

Part 4 : $I_{x_0y_0} = -144\ \text{in.}^4$

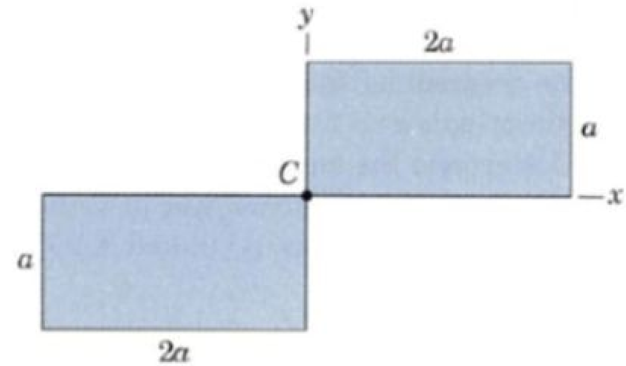
Combined : $I_{x_0y_0} = 2(24) + 2(-144) = -240\ \text{in.}^4$

Combined area $= 2(4)(6) + 2(\frac{1}{2})(4)(6) = 72\ \text{in.}^4$

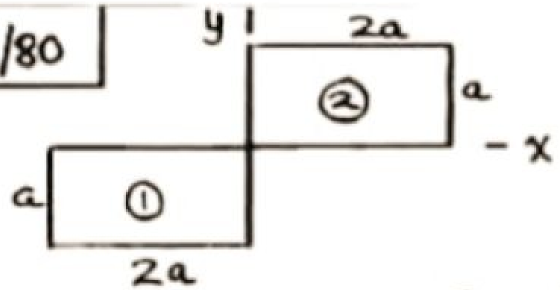
So $I_{xy} = I_{x_0y_0} + A d_x d_y = -240 + 72(+6)(-4)$

$= \underline{-1968\ \text{in.}^4}$

6. Determine the maximum and minimum moments of inertia with respect to centroidal axes through C for the composite of the two rectangular areas shown. Find the angle α measured from the x-axis to the axis of maximum moment of inertia.
Ans. $I_{min} = 0.505a^4$, $I_{max} = 6.16a^4$, $\alpha = 112.5^0$



A/80



① $I_x = \frac{1}{3}(2a)(a^3) = \frac{2}{3}a^4$, $I_y = \frac{1}{3}(a)(2a)^3 = \frac{8}{3}a^4$

$I_{xy} = (2a^2)(a)(a/2) = a^4$

② $I_x = \frac{2}{3}a^4$, $I_y = \frac{8}{3}a^4$, $I_{xy} = a^4$

Eq. A/11 : $I_{min} = \frac{I_x + I_y}{2} - \frac{1}{2}\sqrt{(I_x - I_y)^2 + 4I_{xy}^2}$

$= \frac{10}{3}a^4 - \frac{1}{2}\sqrt{(-4a^4)^2 + 16a^8} = \underline{0.505a^4}$

$I_{max} = \frac{I_x + I_y}{2} + \frac{1}{2}\sqrt{(I_x^2 - I_y^2) + 4I_{xy}^2}$

$= \frac{10}{3}a^4 + \frac{1}{2}\sqrt{(-4a^4)^2 + 16a^8} = \underline{6.16a^4}$

Eq. A/10: $\tan 2\alpha = \frac{2I_{xy}}{I_y - I_x} = \frac{4a^4}{(\frac{16}{3} - \frac{4}{3})a^4}$

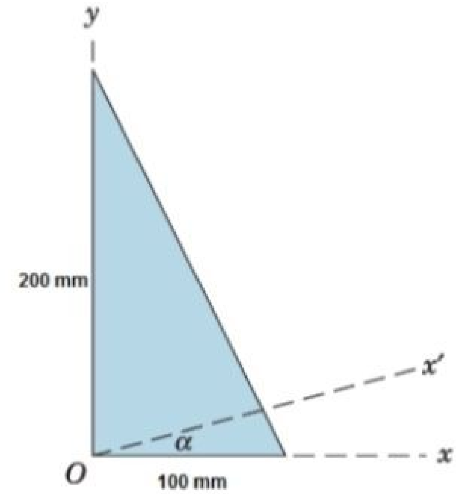
$2\alpha = 45°$ or $225°$

$\alpha = 22.5°$ for I_{min}

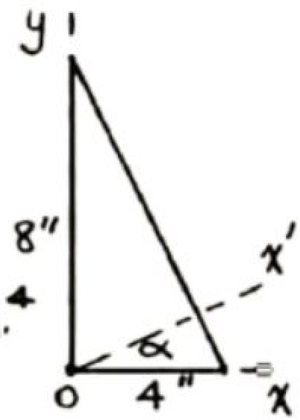
or $\alpha = \underline{112.5°}$ for I_{max}

7. Determine the maximum moment of inertia anout an axis through O and the angle α to this axis for the triangular area shown. Also construct the Mohr circle of inertia.

Ans. $I_{max} = 71.7(10^6)\ mm^4$, $\alpha = -16.85^0$



A/83



$$I_x = \frac{1}{12}bh^3 = \frac{1}{12}4(8^3) = 170.7\text{ in.}^4$$

$$I_y = \frac{1}{12}8(4^3) = 42.7\text{ in.}^4$$

$$I_{xy} = \frac{b^2h^2}{24} = \frac{1}{24}(4^2)(8^2) = 42.7\text{ in.}^4$$

Eq. A/11:

$$I_{max} = \frac{170.7+42.7}{2} + \frac{1}{2}\sqrt{(170.7-42.7)^2+4(42.7)^2}$$

$$= 106.7 + 76.9 = \underline{183.6\text{ in.}^4}$$

Eq. A/10: $\tan 2\theta_{cr} = \tan 2\alpha = \dfrac{2(42.7)}{42.7-170.7} = -0.667$

$$\underline{\alpha = -16.85^\circ}$$