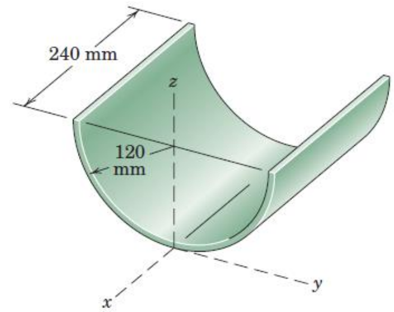
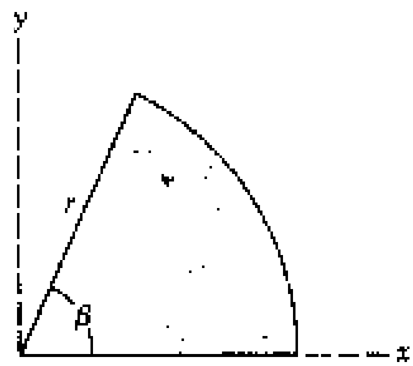
New Problem Sheet No. 5.1
(Centers of Mass and Centroids of Simple Geometric Figures)

1. Specify the x- and z-coordinates of the center of mass of the semi-cylindrical shell.

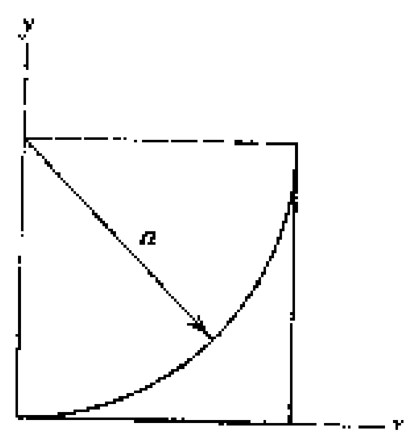
   Ans x =120 mm, $\bar{z}$ = 43.6 mm





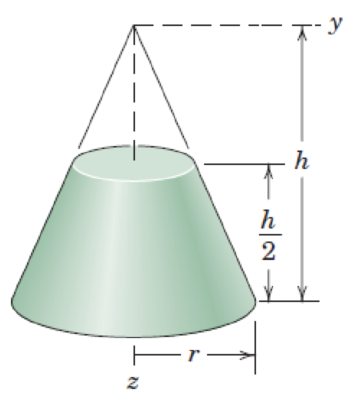
2. Determine the coordinates of the centroid of the area of the circular sector using the coordinates shown.

$$Ans.\bar{x} = \frac{2r}{3\beta}\sin\beta, \bar{y} = \frac{2r}{3\beta}(1-\cos\beta)$$



3. Locate the centroid of the area shown in the figure by direct integration. (Caution: observe carefully the proper sign of the radical involved).

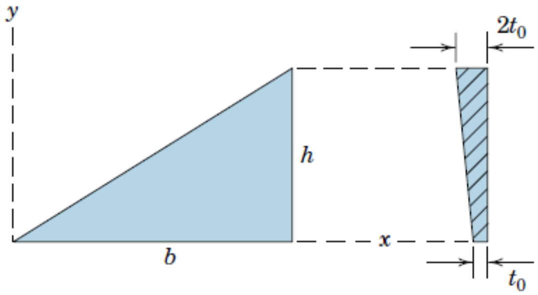
$$Ans.\bar{x} = \frac{2a}{3(4-\pi)}, \bar{y} = \frac{10-3\pi}{3(4-\pi)}a$$



4. Calculate the distance $\bar{h}$ measured from the base to the centroid of the volume of the frustum of the right-circular cone.

$$Ans.\bar{h} = \frac{11}{56}h$$

5. The thickness of the triangular plate varies linearly with y from a value t_0 along its base y = 0 to $2t_0$ at y = h. Determine the y-coordinate of the center of the mass of the plate.

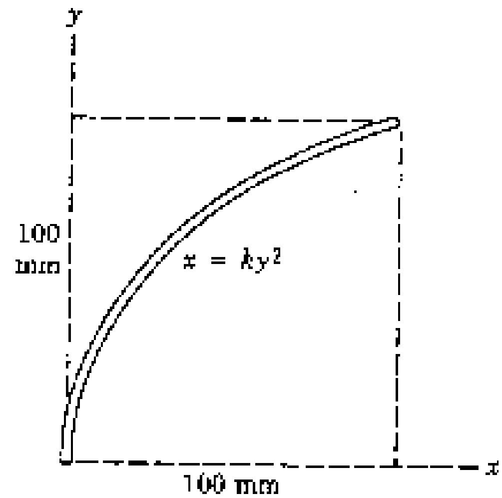


$$Ans.\ \bar{y} = \frac{3h}{8}$$

6. The homogeneous slender rod has a uniform cross section and is bent into the shape shown. Calculate the y-coordinate of the mass center of the rod. (reminder: A differential arc length is

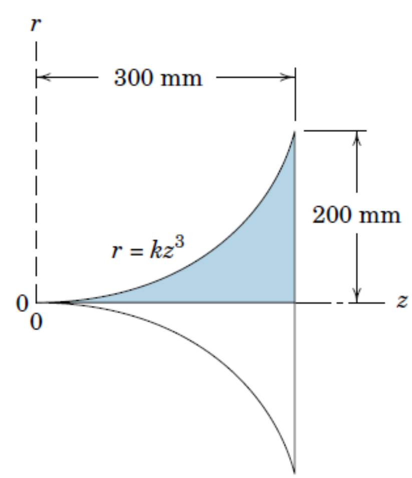
$$dL = \sqrt{(dx)^2 + (dy)^2} = \sqrt{1 + \left(\frac{dx}{dy}\right)^2}\, dy$$

Ans. y = 57.4 mm

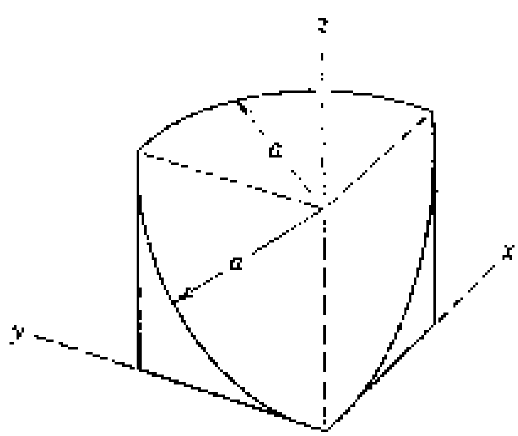


7. Locate the mass center of the homogeneous solid body whose volume is determined by revolving the shaded area through 360^0 about the z-axis.

Ans. $\bar{z}$ = 263 mm



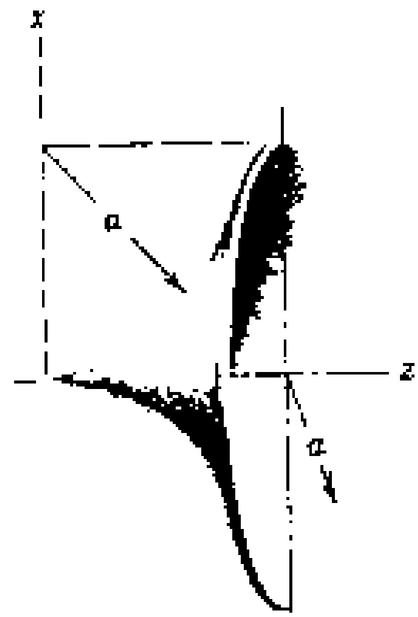
8. Determine the coordinates of the centroid of volume obtained by revolving the shaded area about the z-axis through the 90^0 angle.



$$Ans.\ \bar{x} = \bar{y} = \left(\frac{4}{\pi} - \frac{3}{4}\right)a,\ \bar{z} = \frac{a}{4}$$

9. Locate the center of mass of the homogeneous bell-shaped shell of uniform but negligible thickness. Also determine the position of the centroid of the volume within the bell-shaped shell.

$$Ans.\ \bar{z}_{mass} = \frac{a}{\pi - 2}, \bar{z}_{vol} = \frac{a}{2(10 - 3\pi)}$$



10. Locate the center of mass G of the steel half ring. (Hint: Choose an element of volume in the form of a cylindrical shell whose intersection with the plane of the ends is shown).

$$Ans.\ \bar{r} = \frac{a^2 + 4R^2}{2\pi R}$$