

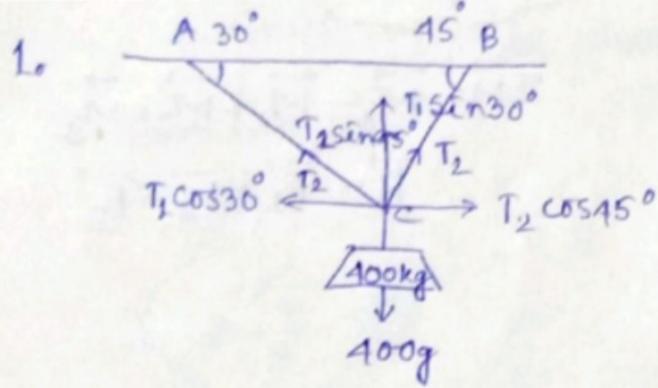
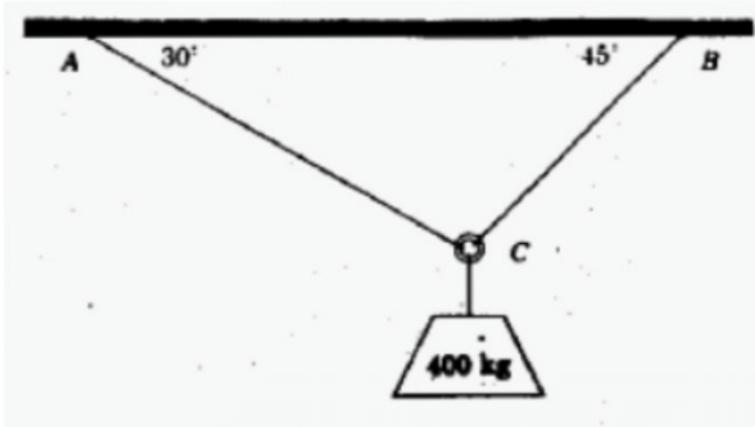
PROBLEM SHEET 1

[EQUILIBRIUM]

New Problem Sheet No. 1

(Equilibrium)

1. Determine the tension in cables CA and CB.
 Ans. $T_{CA} = 2870 \text{ N}$, $T_{CB} = 3520 \text{ N}$.



Since the system is in equilibrium

$$T_1 \sin 30^\circ + T_2 \sin 45^\circ = 100g \quad \dots \text{(1)}$$

$$T_1 \cos 30^\circ = T_2 \cos 45^\circ \quad \dots \text{(2)}$$

From (2) \Rightarrow

$$T_1 = \frac{2}{\sqrt{3}} \times \frac{1}{\sqrt{2}} T_2$$

From (1) \Rightarrow

$$T_1 \times \frac{1}{2} + T_2 \times \frac{1}{\sqrt{2}} = 100 \times 9.8$$

$$\text{or } \frac{2}{\sqrt{3}} \times \frac{1}{\sqrt{2}} T_2 \times \frac{1}{2} + \frac{T_2}{\sqrt{2}} = 3920$$

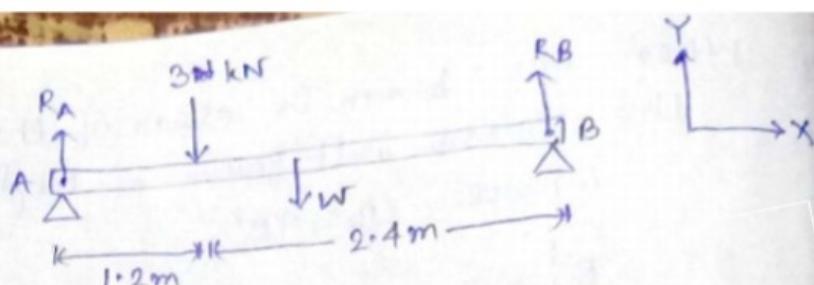
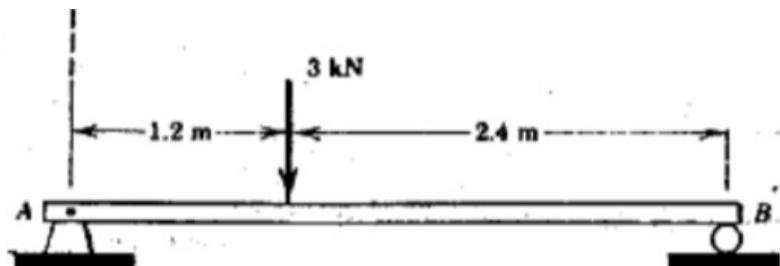
$$\text{or } \frac{T_2}{\sqrt{2}} \left(\frac{1}{\sqrt{3}} + 1 \right) = 3920$$

$$\text{or } T_2 = 3521 \text{ N [Ans]}$$

$$T_1 = 2869 \text{ N [Ans]}$$

2. The uniform bar has a mass per unit length of 60 kg/m. Determine the reactions at the supports.

Ans. $A_y = 3060 \text{ N}$, $B_y = 2060 \text{ N}$.



$$\begin{aligned} \text{Total mass} &= 60 \times (1.2 + 2.4) \text{ kg} \\ &= 216 \text{ kg} \end{aligned}$$

Considering the moments wrt the point A \Rightarrow

$$-3 \times 1.2 - 216 \times \frac{10}{1000} \times 1.8 + R_B \times 3.6 = 0$$

$$\text{or } R_B = 2080 \text{ N. [Ans]}$$

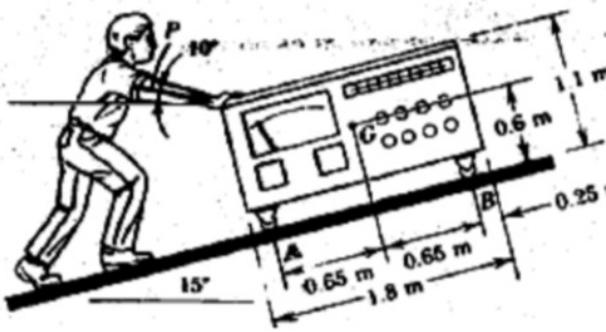
Considering the moments wrt the point B \Rightarrow

$$1.8 \times 216 \times 10 + 3000 \times 2.4 - R_A \times 3.6 = 0$$

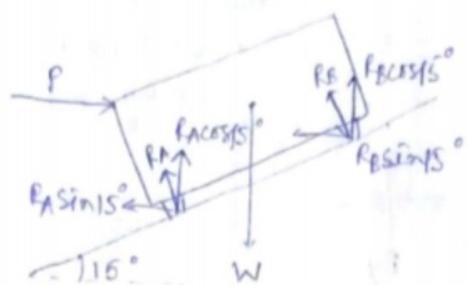
$$\text{or } R_A = 3080 \text{ N. [Ans]}$$

3. A man pushes the 40 kg machine with mass centre at G up an incline at a steady speed. Determine the required force magnitude P and the normal reaction forces at A and B. Neglect the small effects of friction.

Ans. $P = 112.1 \text{ N}$, $N_B = 219 \text{ N}$, $N_A = 207 \text{ N}$.

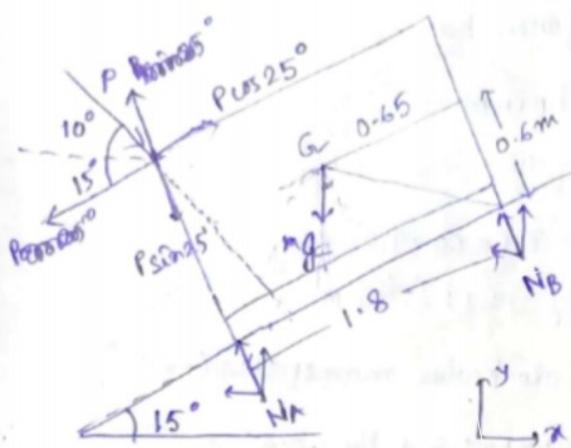


3.



As per the diagram,

$$N_A \sin 15^\circ + N_B \sin 15^\circ = P \dots \dots \dots (1)$$



$$\sum F_x = +P \cos 10^\circ - N_B \sin 15^\circ - N_A \sin 15^\circ = 0$$

$$\text{or } P \cos 10^\circ = (N_B + N_A) \sin 15^\circ$$

$$\text{or } P \times 3.805 = N_B + N_A$$

$$\sum F_y = N_A \cos 15^\circ + N_B \cos 15^\circ - P \sin 10^\circ - 40 \times 9.81 = 0$$

$$\text{or } (N_A + N_B) \cos 15^\circ = P \sin 10^\circ + 392.4$$

$$\text{or } P \times 3.675 - P \times 0.174 = 392.4$$

$$\text{or } P = 112.08 \text{ N}$$

$$\approx 112.1 \text{ N. (Ans)}$$

$\sum M_G = 0$; considering ↑ free,

$$-N_A \times 0.65 + N_B \times 0.65 + P \cos 25^\circ \times (1.1 - 0.6) + P \sin 25^\circ \times 0.08 = 0$$

~~$$\text{or } (N_B - N_A) \times 0.65 = P [0.8288 + 0.380]$$~~

~~$$\text{or } N_B - N_A = 939.07 \text{ N.}$$~~

~~$$\therefore N_B = 219 \text{ N}$$~~

$$\text{or } N_B - N_A = \frac{112.1}{0.65} (0.453 - 0.380)$$

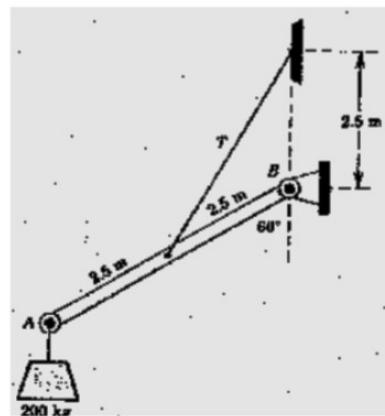
$$\text{or } N_B - N_A = 12.53 \dots$$

$$\therefore N_B = \frac{1}{2} (939.07) \text{ N} = 219.53 \text{ N.} \quad \left. \right\} \text{[Ans]}$$

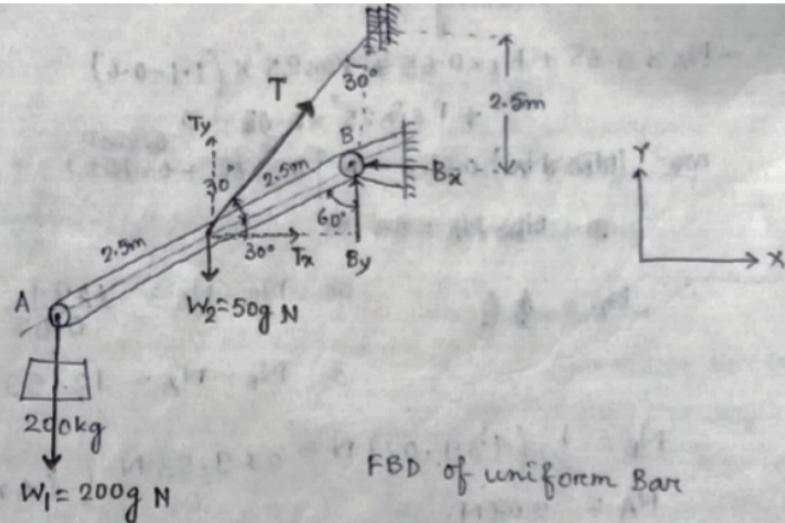
$$N_A = 207 \text{ N.}$$

4. The uniform bar AB has a mass of 50 kg and supports the 200-kg load at A. Calculate the tension in the supporting cable and the magnitude F_B of the force supported by the pin at B.

Ans. $T = 7650 \text{ N}$, $F_B = 5660 \text{ N}$



4.



From the conditions of equilibrium

$$(i) \sum F_x = 0 \quad \text{or} \quad T \cos 60^\circ - B_x = 0 \quad \dots \dots \dots (i)$$

$$(ii) \sum F_y = 0 \quad \text{or} \quad B_y + T \sin 60^\circ - 200g - 50g = 0$$

$$\text{or} \quad \frac{\sqrt{3}}{2} T + B_y = 2452.5 \quad \dots \dots \dots (ii)$$

$$(iii) \sum M_B = 0, \text{ taking } \uparrow + \text{ve}$$

$$\text{or} \quad 200 \times 9.81 \times 5 \cos 30^\circ + 50 \times 9.81 \times 2.5 \cos 30^\circ \\ + T \times 2.5 \sin 30^\circ = 0.$$

$$\text{or} \quad T = 7646.13 \text{ N} \quad [\text{Ans}]$$

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From, (i) \Rightarrow

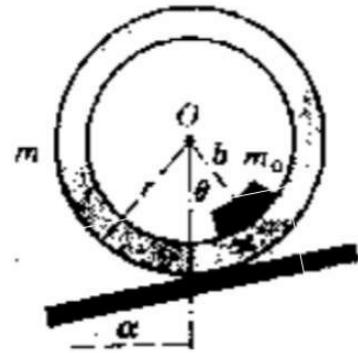
$$B_x = 7646.13 \times \frac{1}{2} \\ = 3823.065 \text{ N}$$

From, (ii) $\Rightarrow B_y = -4169.25 \text{ N} \quad [\text{Ans}]$

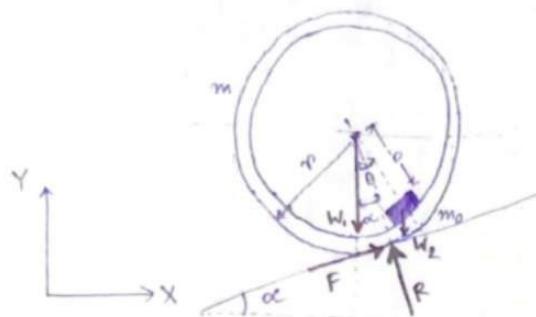
$$\therefore F_B = \sqrt{B_x^2 + B_y^2} \\ = 5656.7 \text{ N} \quad [\text{Ans}]$$

5. A uniform ring of mass m and radius r carries an eccentric mass m_0 at a radius b and is in an equilibrium position on the incline, which makes an angle α with the horizontal. If the contacting surfaces are rough enough to prevent slipping, write the expression for the angle θ which defines the equilibrium position.

$$\text{Ans. } \theta = \sin^{-1} \left[\frac{r}{b} \left(1 + \frac{m}{m_0} \right) \sin \alpha \right]$$



5.



Derive the expression of θ .

FBD of ring and the eccentric mass

From the conditions of equilibrium:-

$$i) \sum F_x = 0 \therefore F \cos \alpha - R \sin \alpha = 0$$

$$\text{or } F = R \tan \alpha \dots\dots\dots (i)$$

$$ii) \sum F_y = 0 \therefore R \cos \alpha + F \sin \alpha - mg - m_0 g = 0$$

$$\text{or } F \sin \alpha + R \cos \alpha = (m + m_0) g \dots\dots\dots (ii)$$

$$iii) \sum M_O = 0; \quad \uparrow (\text{towards center})$$

$$\text{or } f \times r - m_0 g b \sin \alpha = 0.$$

$$\text{or } F r = m_0 g b \sin \alpha \dots\dots\dots (iii)$$

From eqn (ii) \Rightarrow

$$R \tan \alpha \sin \alpha + R \cos \alpha = (m + m_0) g$$

$$\text{or } \frac{R}{\cos \alpha} = (m + m_0) g$$

$$\text{or } R = (m + m_0) g \cos \alpha$$

From eqn (i) \Rightarrow

$$F = (m + m_0) g \sin \alpha$$

From eqn (iii) \Rightarrow

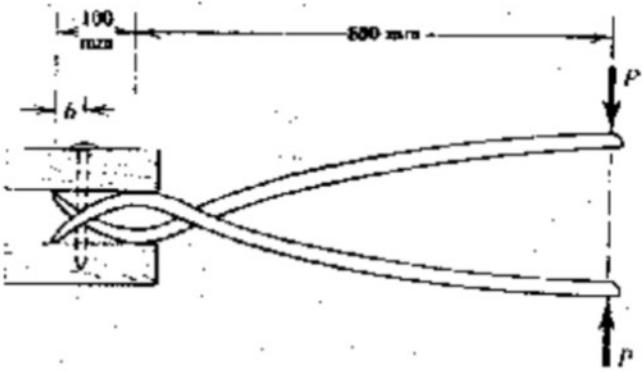
$$(m + m_0) g \sin \alpha \cdot r = m_0 g b \sin \alpha.$$

$$\text{or } R \sin \alpha = \left(1 + \frac{m}{m_0} \right) \frac{r}{b} \sin \alpha.$$

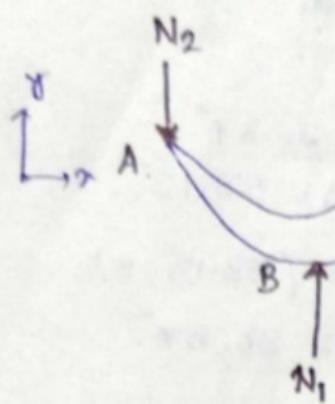
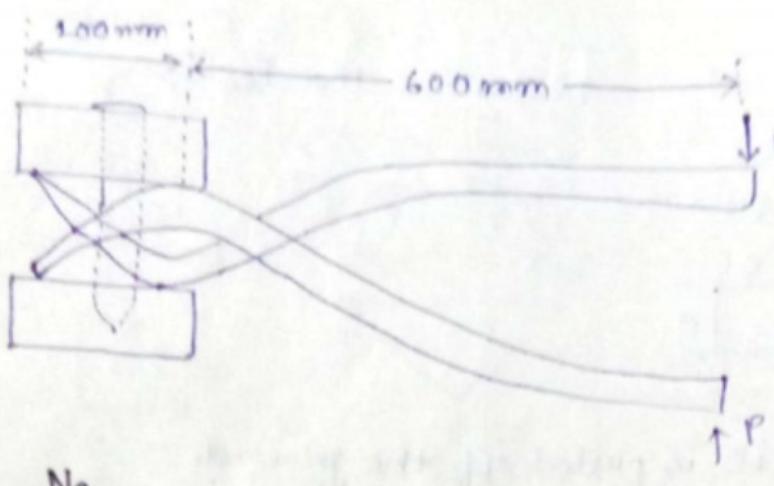
$$\text{or } \therefore \theta = \sin^{-1} \left[\frac{r}{b} \left(1 + \frac{m}{m_0} \right) \sin \alpha \right] \quad (\text{Proved}).$$

6. The two planks are connected by a large spike. If a force $P = 120 \text{ N}$ is required on the handle of each crowbar to loosen the spike, calculate the corresponding tension T in the spike. Also find the value of b which will eliminate any tendency to bend the spike. State any assumptions which you make.

Ans. $T = 1560 \text{ N}$, $b = 53.8 \text{ mm}$



6.



FBD of crowbar

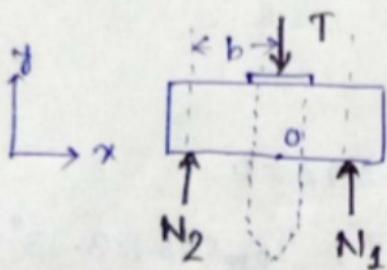
$$\begin{aligned} \sum M_B &= 0 \quad \text{(+ve)} \\ \text{or } N_2(100) - P(600) &= 0 \\ \text{or } N_2 &= 6 \times P \\ &= 720 \text{ N.} \end{aligned}$$

$$\sum F_y = 0$$

$$\text{or } N_2 + P = N_1$$

$$\text{or } 720 + 120 = N_1$$

$$\text{or } N_1 = 840 \text{ N.}$$



FBD of plank

$$\sum F_y = 0$$

$$\text{or } T = N_2 + N_1$$

$$\text{or } T = (720 + 840) \text{ N}$$

$$= 1560 \text{ N. [Ans]}$$

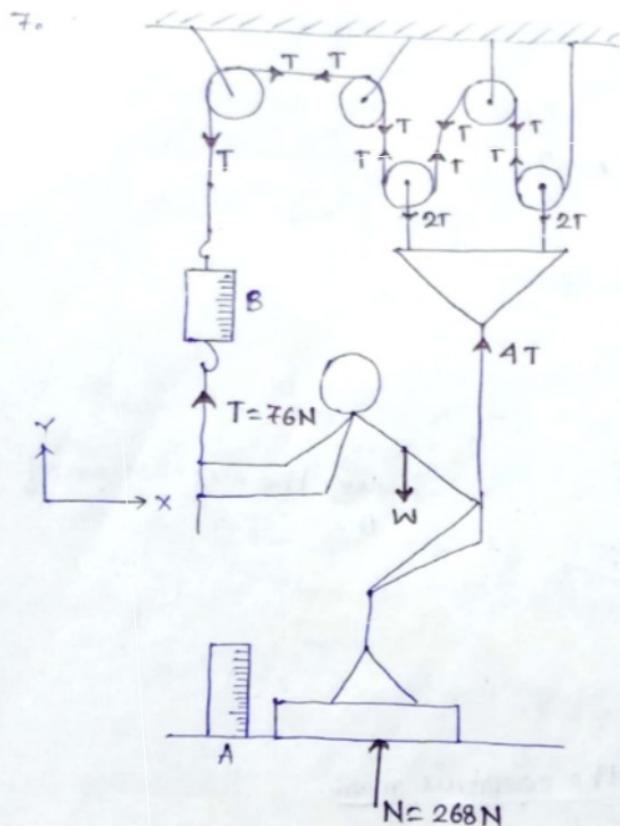
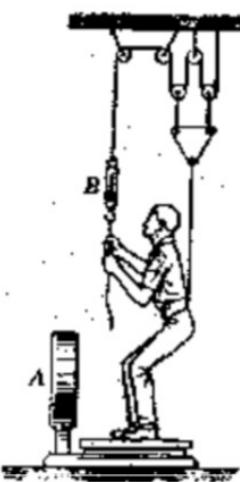
$$\sum M_O = 0$$

$$\text{or } N_1(100 - b) - N_2 b = 0$$

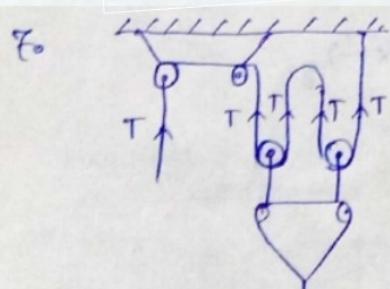
$$\text{or } b = 53.8 \text{ mm [Ans]}$$

7. A former student of mechanics wishes to weigh himself but has access only to a scale A with capacity limited to 400 N and a small 80-N spring dynamometer B. With the rig shown he discovers that when he exerts a pull on the rope so that B registers 76 N, the scale A reads 268 N. What are his correct weight and mass m?

Ans. W = 648 N, m = 66.1 kg



$$\begin{aligned}
 \sum F_y &= 0 \\
 \text{on } T + 4T - W + N &= 0 \\
 \text{or } W &= (5T + N) \\
 \text{on } W &= (5 \times 76 + 268) \text{ Newton} \\
 &= 648 \text{ N} \quad [\text{Ans}] \\
 m &= \frac{648}{9.81} \text{ kg} \\
 &= 66.1 \text{ kg} \quad [\text{Ans}]
 \end{aligned}$$



So, the apparent weight is
(W - 5T) N.

Now, the pull he exerts on the
rope is 76 N.

$$\therefore T = 76 \text{ N}$$

$$\text{Hence } W - 5 \times 76 = 268$$

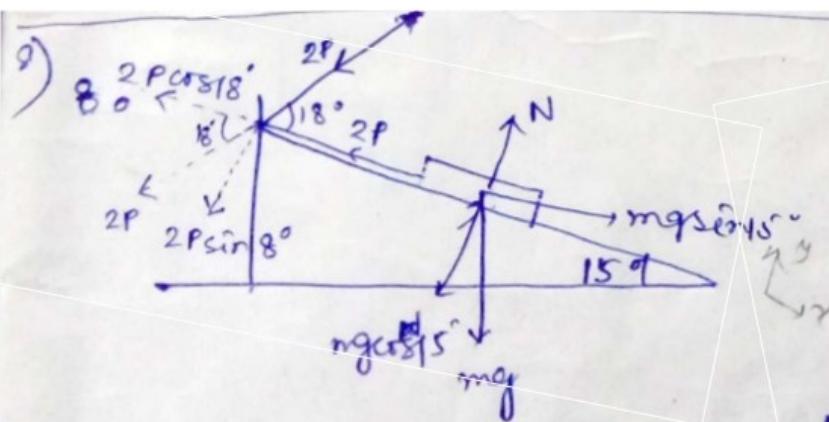
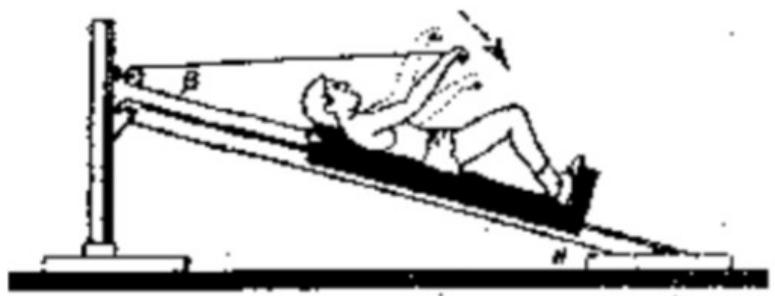
$$\text{on } W = 268 + 5 \times 76$$

$$= 648 \text{ N} \quad [\text{Ans}]$$

$$\therefore \text{The actual mass} = \frac{648}{9.8} \text{ kg} = 66.1 \text{ kg} \quad [\text{Ans}]$$

8. The exercise machine consists of a lightweight cart which is mounted on small rollers so that it is free to move along the inclined ramp. Two cables are attached to the cart – one for each hand. If the hands are together so that the cables are parallel and if each cable lies essentially in a vertical plane, determine the force P which each hand must exert on its cable in order to maintain an equilibrium position. The mass of the person is 70 kg, the ramp angle is 15° and the angle β is 18° . In addition, calculate the force R which the ramp exerts on the cart.

Ans. $P = 45.5 \text{ N}$, $R = 691 \text{ N}$



$$\sum F_x = 0$$

$$mg \sin 15^\circ = 2P + 2P \cos 18^\circ$$

$$\text{or } 2P(1 + \cos 18^\circ) = 177.73$$

$$\text{or } P = 45.5 \text{ N}$$

$$\sum F_y = 0$$

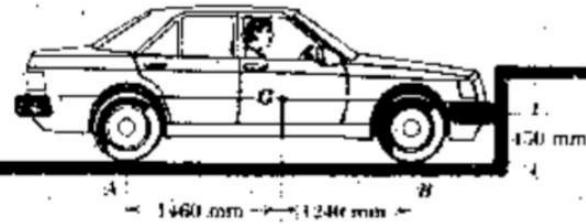
$$\text{or } N - mg \cos 15^\circ - 2P \sin 18^\circ = 0$$

$$\text{or } N = 691.42 \text{ N}$$

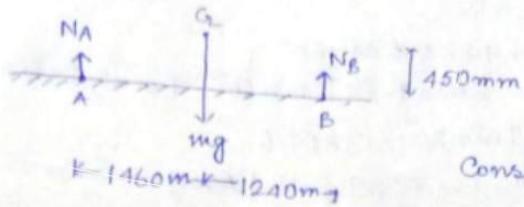
9. The rear-wheel-drive car has a mass of 1400 kg with mass center at G. First calculate the normal forces under the front and rear wheel pairs when the car is at normal test. Then repeat your calculations for the case shown where the front bumper is tested by causing the car to push, via its rear wheels, against the fixed barrier with a 2500-N force. Neglect friction at the bumper-barrier interface, but not at the tire-ground interface.

Ans. Normally, $N_A = 6310 \text{ N}$, $N_B = 7430 \text{ N}$

Under test, $N_A = 6720 \text{ N}$, $N_B = 7010 \text{ N}$



First Case



Considering the system is in equilibrium

$$\sum M_G = 0$$

$$\text{or } +N_A \times 1460 = +N_B \times 1240$$

$$\text{or } 73N_A = 62N_B$$

Now,

$$N_A + N_B = mg$$

$$\text{or } N_A + N_B = 1400 \times 9.8$$

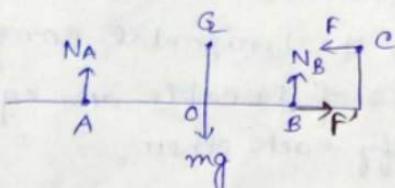
$$\text{or } \frac{62}{73}N_B + N_B = 1400 \times 9.8$$

$$\text{or } N_B = 7418.96$$

$$\approx 7420 \text{ N.}$$

$$\therefore N_A = 6300 \text{ N} \quad [\text{Ans}].$$

Second Case



Considering the system is in equilibrium,

$$\sum M_O = 0$$

$$\text{or } -N_A \times 1460 + N_B \times 1240 + F \times 450 = 0$$

$$\text{or } N_A \times 1460$$

$$= N_B \times 1240 + 2500 \times 450$$

$$\text{or } 146N_A - 124N_B = 112500$$

$$\text{Now, } N_A + N_B = mg$$

$$\text{or } N_A + N_B = 13720 \quad \dots \text{(W)}$$

$$\text{Now, } (1) + 124(W) \Rightarrow$$

$$270N_A = 112500 + 1701280$$

$$\text{or } N_A = 6717.70$$

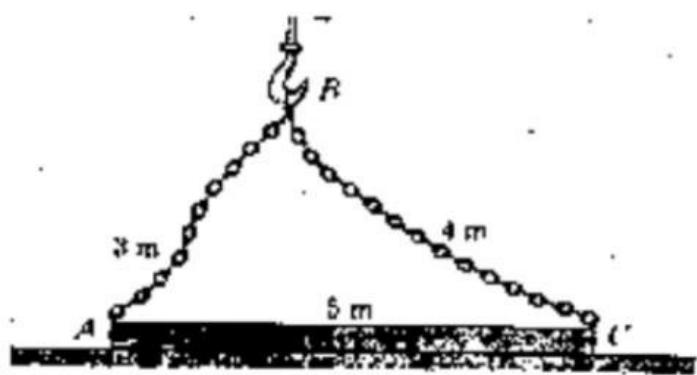
$$\approx 6720 \text{ N.}$$

$$\therefore N_B = 7000 \text{ N.}$$

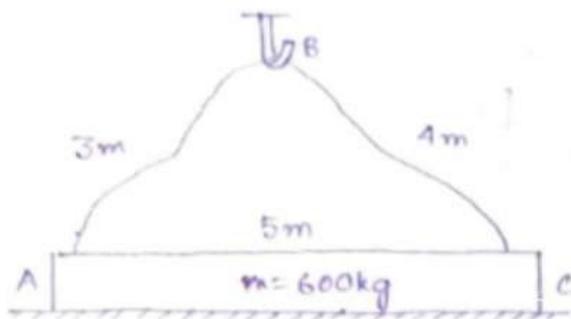
[Ans]

10. The 5-m uniform steel beam has a mass of 600 kg and is to be lifted from the ring at B with the two chains, AB of length 3 m, and CB of length 4 m. Determine the tension in chain AB when the beam is clear of the platform.

Ans. $T = 3.53 \text{ kN}$

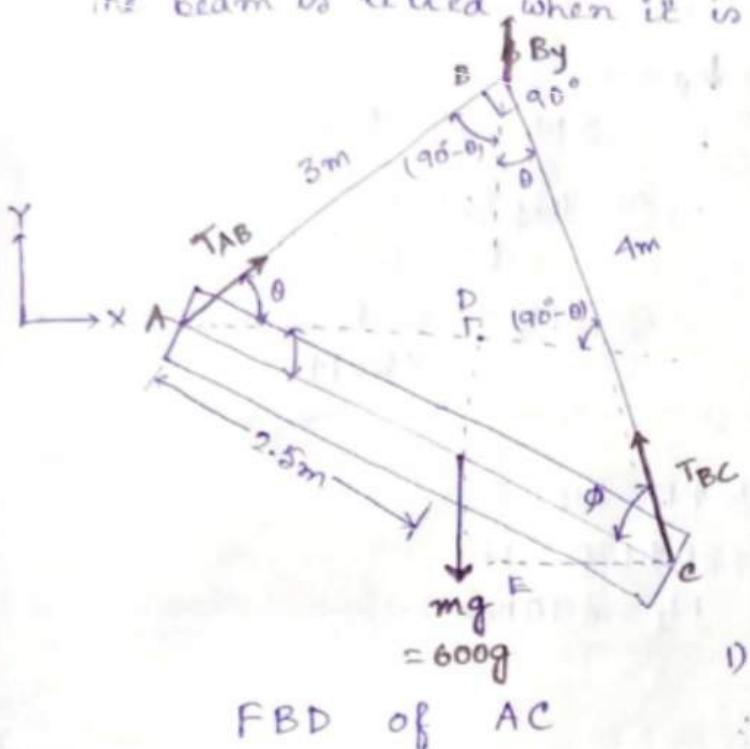


10.



Determine the $T_{AB} = ?$

The beam is tilted when it is pulled off the ground.



FBD of AC

$$\angle BCA = \phi = \sin^{-1}\left(\frac{3}{5}\right) = 36.87^\circ,$$

$$\angle BAC = (90^\circ - 36.87^\circ) = 53.13^\circ.$$

$$AD = 3\cos\theta = 2.5 \cos(53.13^\circ - \theta)$$

$$\text{By solving, } \theta = 36.87^\circ.$$

$$\begin{aligned} \angle BCE &= \angle BCA + 53.13^\circ - 36.87^\circ \\ &= 53.13^\circ. \end{aligned}$$

Now,

$$(i) \sum F_x = 0$$

$$\therefore T_{AB} \cos 36.87^\circ - T_{BC} \cos 53.13^\circ = 0$$

$$\text{or } T_{AB} \cos 36.87^\circ = T_{BC} \cos 53.13^\circ$$

$$\text{or } T_{BC} = T_{AB} \times 1.333 \dots \dots \dots (1)$$

$$(ii) \sum F_y = 0$$

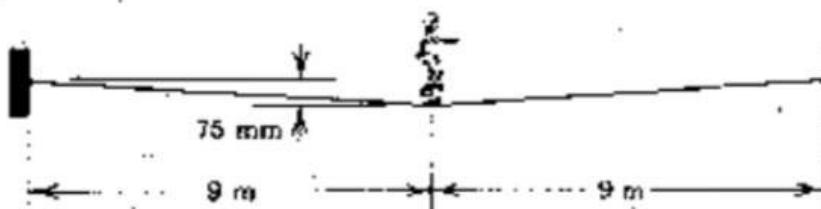
$$\text{or } T_{AB} \sin 36.87^\circ + T_{BC} \sin 53.13^\circ - 600g = 0$$

$$\text{or } T_{AB} \sin 36.87^\circ + T_{AB} \times 1.333 \times \sin 53.13^\circ = 600 \times 9.81$$

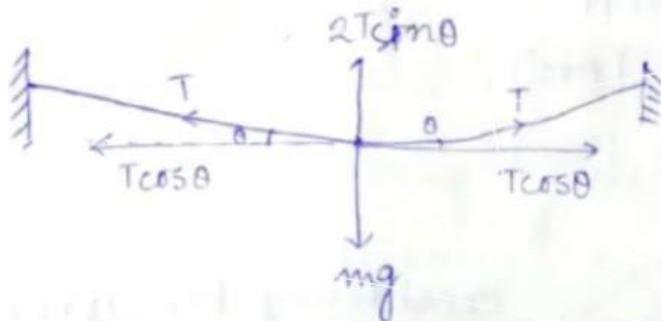
$$\text{or } T_{AB} = 3.53 \text{ kN}.$$

11. A 50-kg acrobat pedals her unicycle across the taut but slightly elastic cable. If the deflection at the center of the 18-m span is 75 mm, determine the tension in the cable. Neglect the effects of the weights of the cable and unicycle.

Ans. $T = 29.4 \text{ kN}$



11.



In the diagram we can see, when the acrobat is in the mid-point of the cable, the horizontal components of the tension in the two parts of the cable are equal and opposite. So, they nullify each other.

Now, as the system is in equilibrium

$$2T \sin \theta = mg$$

$$\text{or } 2T \tan \theta \approx mg \quad (\text{since } \theta \text{ is very small})$$

$$\text{or } 2 \times T \times \frac{0.075}{9} = 50 \times 9.8$$

$$\text{or } T = 29400 \text{ N}$$

$$= 29.4 \text{ kN} \quad [\text{Ans}]$$

12. The uniform 30-kg bar with end rollers is supported by the horizontal and vertical surfaces and by the wire AC. Calculate the tension T in the wire and the reactions against the rollers at A and at B.

Ans. 73.6 N, B = 196.2 N, T = 295 N

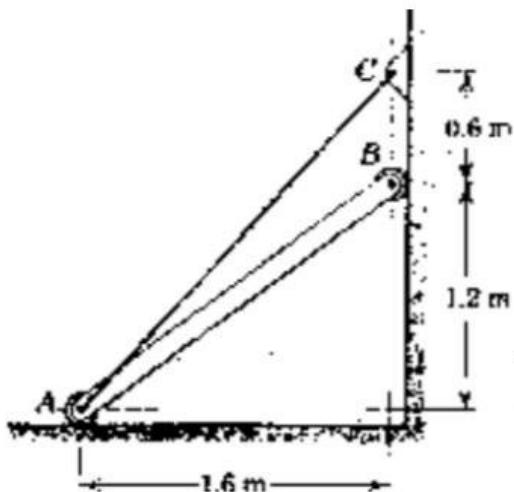


Diagram showing free body diagram of the bar. At A, there is a reaction force F_A upwards and to the left at an angle α to the horizontal, and a reaction force F_{Ax} to the left. At B, there is a reaction force F_B to the left and a reaction force F_{By} downwards. At C, there is a tension force T along the wire AC.

$W = 30 \times 9.81 \text{ N} = 294.3 \text{ N}$

Since, the system is in equilibrium

$$\sum M_A = 0$$

$$\text{or } -W \times 0.8 + F_B \times 1.2 = 0$$

$$\text{or } F_B = \frac{294.3 \times 8}{12} = 196.2 \text{ N [Ans]}$$

$$\sum M_C = 0$$

$$\text{or } -F_B \times 0.6 - F_A \times 1.6 + W \times 0.8 = 0$$

$$\text{or } F_A \times 1.6 = 294.3 \times 0.8 - 196.2 \times 0.6$$

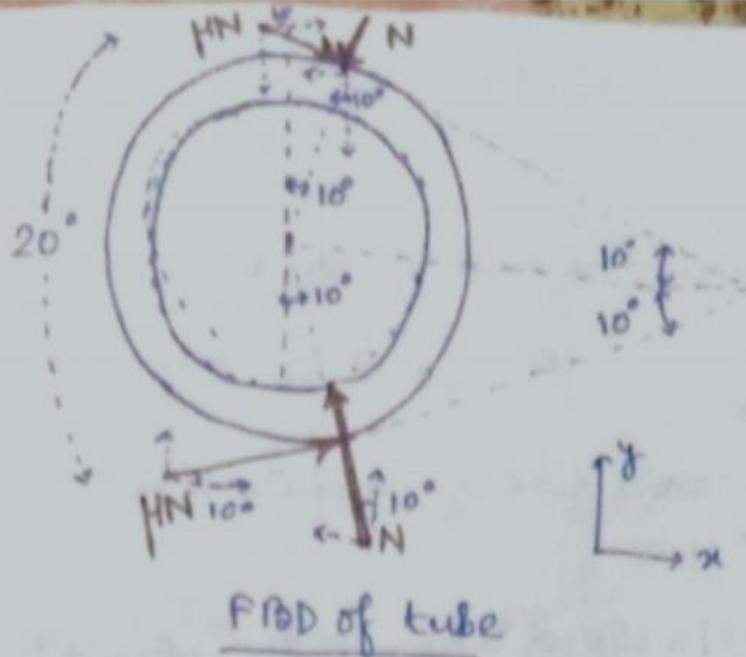
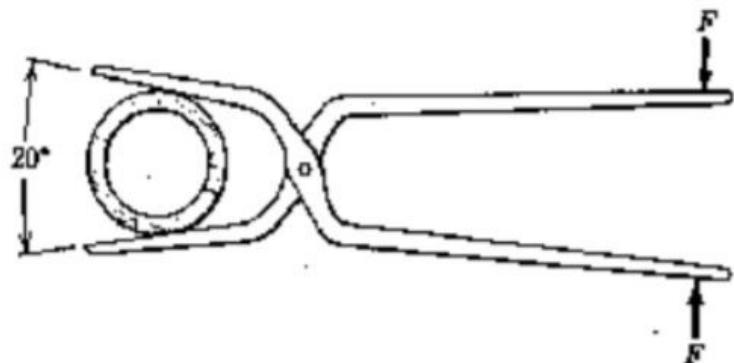
$$\text{or } F_A = 73.57 \approx 73.6 \text{ N [Ans]}$$

PROBLEM SHEET 2

[FRICTION]

3. The tongs are used to handle hot steel tubes that are being heat-treated in an oil bath. For a 200 mm jaw opening, what is the minimum coefficient of static friction between the jaws and the tube that will enable the tongs to grip the tube without slipping.

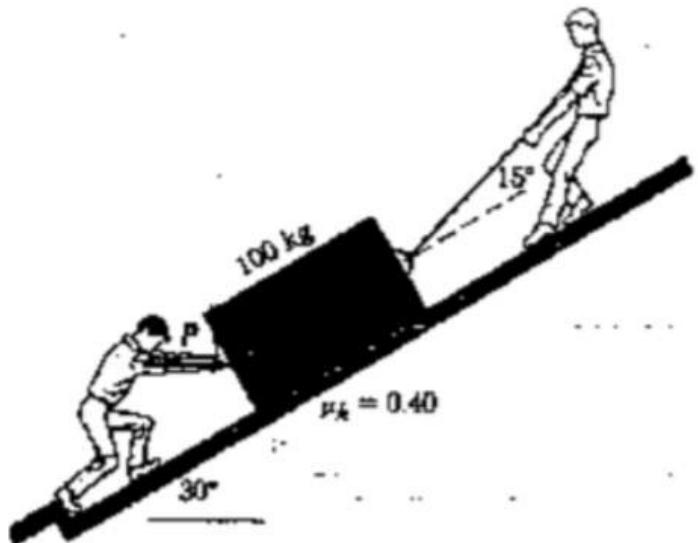
Ans. $\mu_s = 0.176$



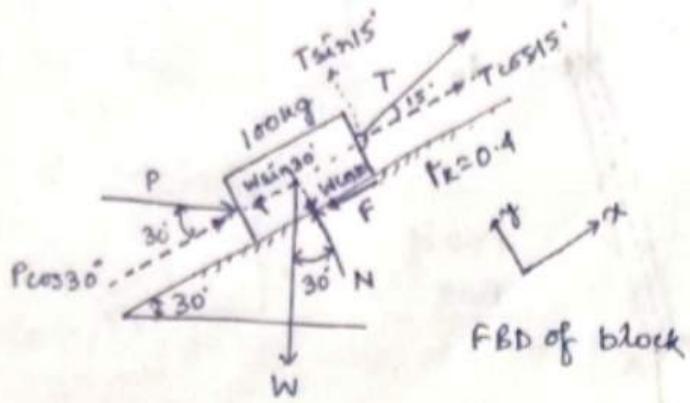
$$\begin{aligned} \sum F_x &= 0 \\ \text{on } -N\sin 10^\circ - N\sin 10^\circ &+ \mu N\cos 10^\circ + \mu N\cos 10^\circ = 0 \\ \text{or } \mu &= \tan 10^\circ \\ &= 0.176 \text{ (Ans)} \end{aligned}$$

4. Two men are sliding a 100-kg crate up an incline. If the lower man pushes horizontally with a force of 500 N and if the coefficient of kinetic friction is 0.4, determine the tension T which the man must exert in the rope to maintain motion of the crate.

Ans. $T = 465 \text{ N}$



15)



$$P = 500 \text{ N}$$

$$W = 100 \times 9.81 \text{ N} \\ = 981 \text{ N}$$

$$F = \mu_k N = (0.4 \text{ N}) \text{ Newton}$$

from the conditions
of equilibrium,

$$(i) \sum F_x = 0$$

$$\therefore P \cos 30^\circ + T \cos 15^\circ - \mu_k N - W \sin 30^\circ = 0$$

$$\text{or } 0.4 \text{ N} = 0.966 T - 57.49$$

$$\text{or } N = 2.415 T + 143.725 \dots (i)$$

$$(ii) \sum F_y = 0$$

$$\therefore T \sin 15^\circ - W \cos 30^\circ - P \sin 30^\circ + N = 0$$

$$\text{or } N = W \cos 30^\circ + P \sin 30^\circ - T \sin 15^\circ$$

$$\text{or } N = 1099.57 - 0.259 T \dots (ii)$$

From (i), (ii) \Rightarrow

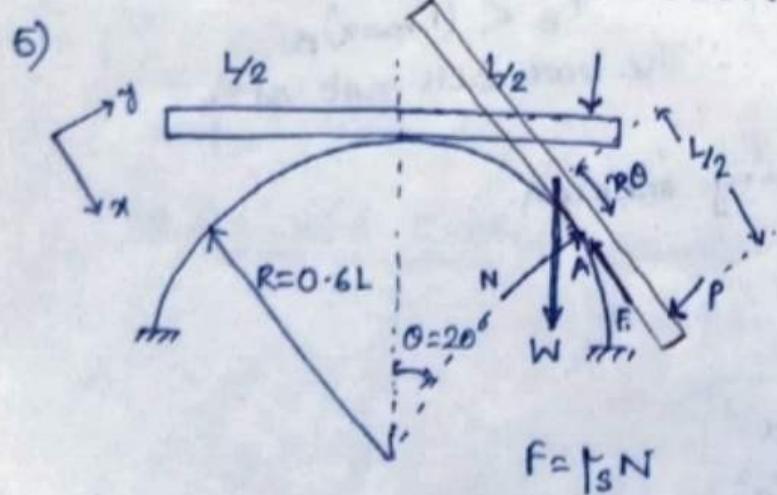
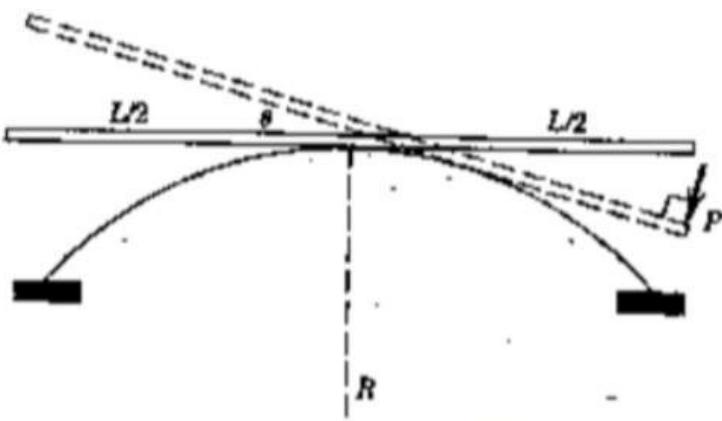
$$2.415 T - 143.725 = 1099.57 - 0.259 T$$

$$\text{or } 2.674 T = 1243.295$$

$$\text{or } T = 465 \text{ N (Approx)}$$

5. The uniform slender rod of mass m and length L is initially at rest in a centered horizontal position on the fixed circular surface of radius $R = 0.6L$. If a force normal to the bar is gradually applied to its end until the bar begins to slip at the angle $\theta = 20^\circ$, determine the coefficient of static friction μ_s .

Ans. $\mu_s = 0.212$



Find μ_s .

$$\text{i) } \sum F_x = 0$$

$$\text{or } W \sin \theta - F = 0$$

$$\text{or } mg \sin \theta = f_s N \dots \dots \text{(i)}$$

$$\text{ii) } \sum F_y = 0$$

$$N - mg \cos \theta - P = 0$$

$$\text{or } N = P + mg \cos \theta \dots \dots \text{(ii)}$$

$$\text{iii) } \sum M_A = 0 \therefore +mg \cos \theta \times R\theta - P(\frac{L}{2} - R\theta) = 0$$

$$\text{or } mg \cos \theta \times 0.6L \times \theta - P(\frac{L}{2} - 0.6L\theta) = 0$$

$$\text{or } mg \cos \theta \times 0.6 \times 0.6 - P(\frac{1}{2} - 0.6\theta) = 0 \dots \text{(iii)}$$

From (i) & (ii) \Rightarrow

$$mg \sin \theta = f_s (P + mg \cos \theta)$$

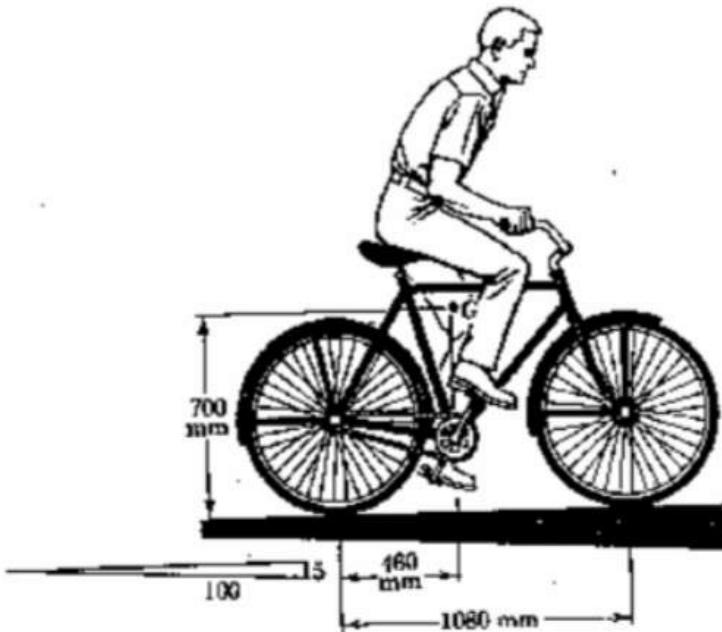
$$\text{or } mg \sin \theta = \mu_s \left[mg \cos \theta + \frac{0.6 mg \cos \theta \times \theta}{0.5 - 0.6\theta} \right] = f_s$$

$$\text{or } \mu_s \sin \theta = \mu_s \mu_s \cos \theta \left[1 + \frac{0.6\theta}{0.5 - 0.6\theta} \right]$$

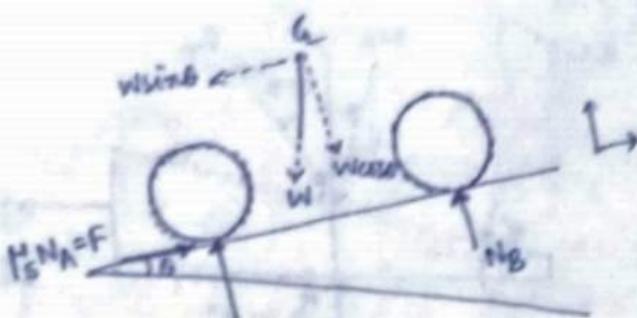
$$\text{or } \mu_s = 0.212 \text{ (Ans)}$$

7. A man pedals his bicycle up a 5 percent grade on a slippery road at a steady speed. The man and bicycle have a combined mass of 82 kg with mass center at G. If his rear wheel is on the verge of slipping, determine the coefficient of friction μ_s between the rear tire and the road. If the coefficient of friction were doubled, what would be the friction force acting on the rear wheel?

Ans. $\mu_s = 0.082$, $F = 40.2 \text{ N}$



7. i)



Now $\tan\theta = \frac{5}{100} = 0.05$
 $\Rightarrow \theta = 2.882^\circ$

Considering the system in eq-

i) $\sum F_x = 0$

or $\mu_s N_A + (-W \sin\theta) = 0$

or $\mu_s N_A = 40.17 \approx 40.2 \text{ --- (i)}$

ii) $\sum F_y = 0$

or $N_A + N_B = W \cos\theta$

or $N_A + N_B = 803.42 \text{ --- (ii)}$

iii) $\sum M_G = 0; \uparrow +ve$

or $-N_A \times 460 + N_B (1080 - 460)$

+ $B \mu_s N_A \times 700 = 0$

or $-460 N_A + 620 (803.42 - N_A)$

+ $402 \times 700 = 0$

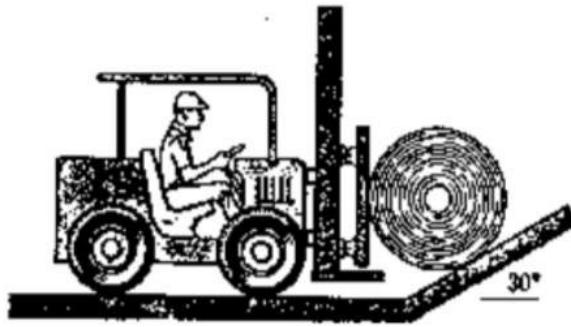
or $N_A = 487.28 \text{ --- (iii)}$

Now, from (i), (ii) $\Rightarrow \mu_s = \frac{40.2}{487.28} = 0.082 \text{ [Ans]}$

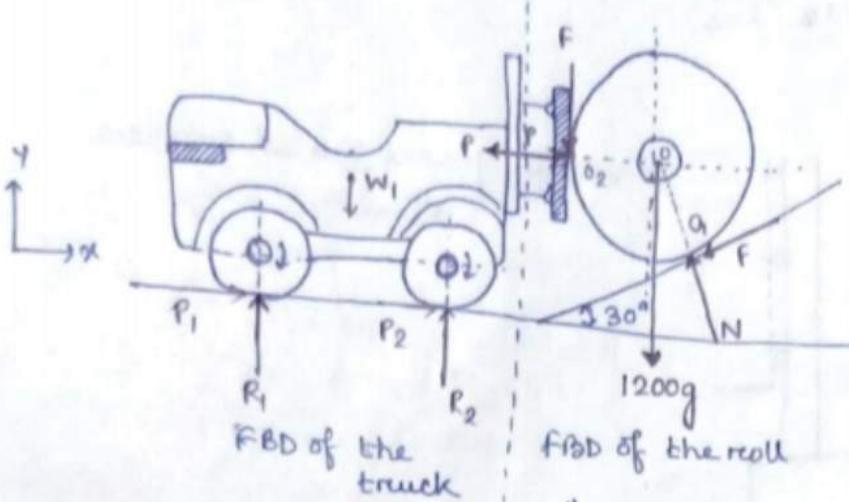
ii) Now, even if we double the coefficient of friction in between the road and the wheel, the friction force would remain same = 40.2 N , but the wheel would not be at the verge of slipping.

8. The industrial truck is used to move the solid 1200-kg roll of paper up the 30° incline. If the coefficients of static and kinetic friction between the roll and the vertical barrier of the truck and between the roll and the incline are both 0.40, compute the required tractive force P between the tires of the truck and the horizontal surface.

$$\text{Ans. } P = 22.1 \text{ kN}$$



8.



$$P_1 + P_2 = P \text{ (Tractive Force)}$$

$$m = 1200 \text{ kg}$$

$$\mu = 0.4$$

Determine tractive force P .

* Only one point of contact it can slide otherwise it rolls.

Let us assume the slides at O_1 $\Rightarrow F = \mu N$
(i.e. roller slips)

i) $\sum F_x = 0 \quad \therefore F = (P - \frac{N}{2}) \frac{2}{\sqrt{3}}$

or $P - N \sin 30^\circ - F \cos 30^\circ = 0 \quad \dots \text{(1)}$

ii) $\sum F_y = 0$
or $-1200g - F \sin 30^\circ + N \cos 30^\circ - f = 0$

or $N \frac{\sqrt{3}}{2} - \frac{3F}{2} - 1200 \times 9.81 = 0$

or $N \frac{\sqrt{3}}{2} - \frac{3}{2} (P - \frac{N}{2}) \frac{2}{\sqrt{3}} - 1200 \times 9.81 = 0$

or $N = P + \frac{1200 \times 9.81}{\sqrt{3}}$

$\therefore N > P$

$\therefore \mu N > \mu P$

So, the roller will slip at point O_2 first.

$$\therefore F = \mu P$$

Now, $\sum M_{O_1} = 0 ; \text{ Q(tve)}$

or $P \times \cancel{y} \cos 60^\circ - F (\cancel{y} + \cancel{x} \cos 60^\circ) - 1200 \times 9.81 \times \cancel{y} \cos 60^\circ = 0$

or $P \times \cos 30^\circ - 0.4 P (1 + \cos 60^\circ) - 1200 \times 9.81 \times \cos 60^\circ = 0$

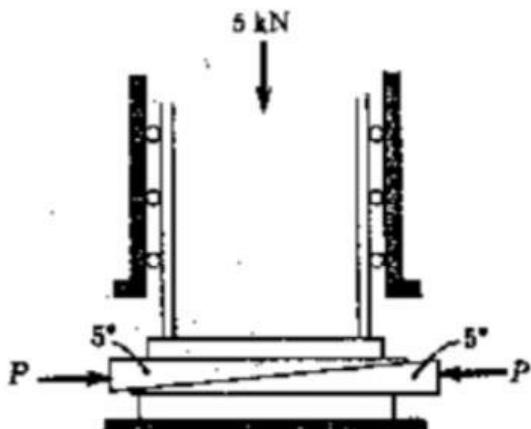
or $P = 22103 \text{ N} = 22.103 \text{ kN} \text{ (Ans)}$

$$P - N \times \frac{1}{2} - 0.40 P \times \frac{\sqrt{3}}{2} = 0$$

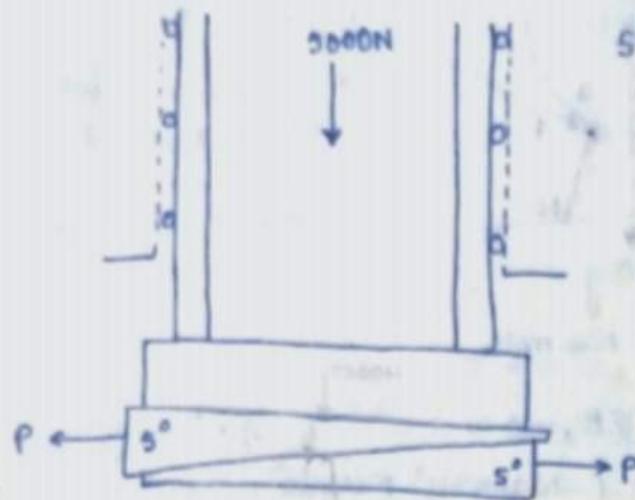
or $1.30 \text{ F} = N$

9. The two 5° wedges shown are used to adjust the position of the column under a vertical load of 5 kN. Determine the magnitude of forces P required to lower the column if the coefficient of friction for all surfaces is 0.40.

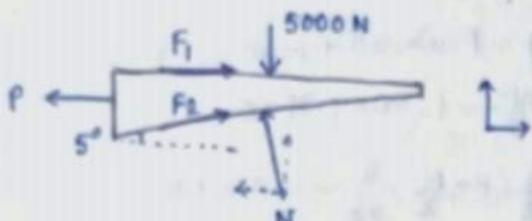
Ans. $P = 3.51$ kN



9)



5° wedge, $\mu = 0.4$ for all surfaces.
Find P .



$$\begin{aligned} 2F_f &= 0 \\ \text{or } 5000 \times 0.40 + 0.4N \cos 5^\circ - P &= 0 \\ \text{or } P - 0.4N \sin 5^\circ &= 2000 \quad \dots \dots (1) \end{aligned}$$

$$2F_N = 0$$

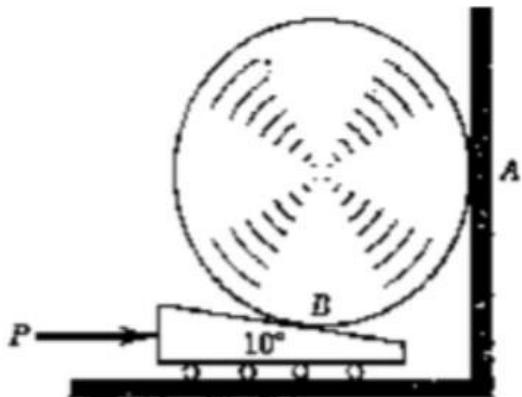
$$\text{or } 0.4N \sin 5^\circ + N \cos 5^\circ - 5000 = 0$$

$$\text{or } N = 4849.39 \text{ N}$$

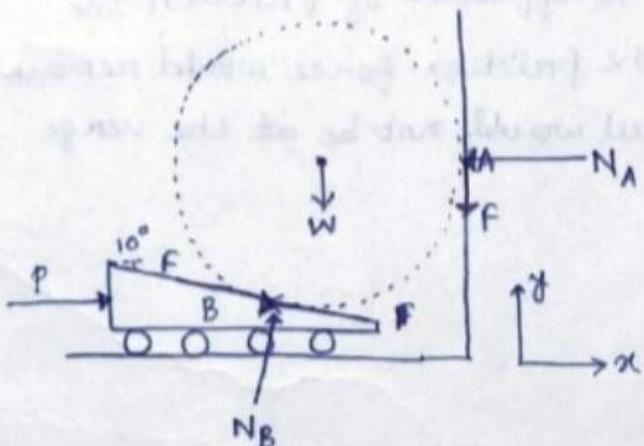
$$\begin{aligned} \therefore P &= 2000 + 0.311 \times 4849.39 \\ &= 3508.16 \text{ N} \\ &= 3.51 \text{ kN} \end{aligned}$$

10. Calculate the horizontal force P on the light 10^0 wedge necessary to initiate movement of the 40-kg cylinder. The coefficient of static friction for both pairs of contacting surfaces is 0.25. Also determine the friction force F_B at point B.

Ans. $P = 98.6 \text{ N}$, $F_B = 24.6 \text{ N}$



10)



FBD of the Cylinder

$$\sum F_x = 0$$

$$\text{or } -N_A + F \cos 10^\circ + N_B \sin 10^\circ = 0 \quad \dots \text{(v)}$$

$$\sum F_{xy} = 0$$

$$\text{or } N_B \cos 10^\circ - F \sin 10^\circ - F = 0$$

$$\text{or } N_B = (1.192F + 398.4) \quad \dots \text{(vi)}$$

From, (v) \Rightarrow

$$-N_A + 0.985F + 1.192F \times 0.174 = 0 \\ + 392.4 \times 0.174$$

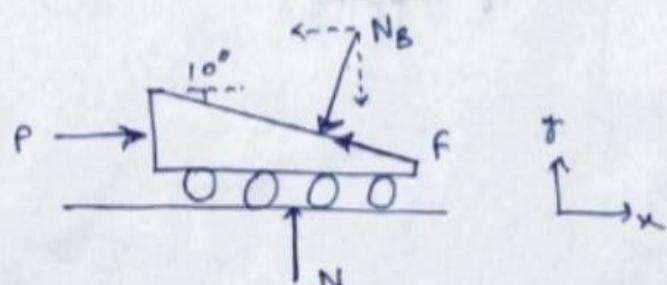
$$\text{or } N_A = (1.192F + 68.14) \quad \dots \text{(vii)}$$

$$\text{So, } N_B > N_A$$

$$\mu N_B > \mu N_A$$

So, it will slip first at A

$$F = \mu N_A = 0.25 N_A \quad \dots \text{(viii)}$$



FBD of wedge

$$\sum F_x = 0$$

$$\therefore P - F \cos 10^\circ - N_B \sin 10^\circ = 0$$

$$\text{or } P = F \cos 10^\circ + N_B \sin 10^\circ \quad \dots \text{(ix)}$$

$$\sum F_y = 0$$

$$\text{or } N + F \sin 10^\circ - N_B \cos 10^\circ = 0$$

$$\text{or } N = N_B \cos 10^\circ - F \sin 10^\circ \quad \dots \text{(x)}$$

Now, From (vii), (viii) \Rightarrow

$$N_A = 1.192 \times 0.25 N_A + 68.14$$

$$\text{or } N_A = 98.0 \text{ N} = 98.0 \text{ N}$$

$$F = 24.6 \text{ N (Ans)}$$

From, (vii) \Rightarrow

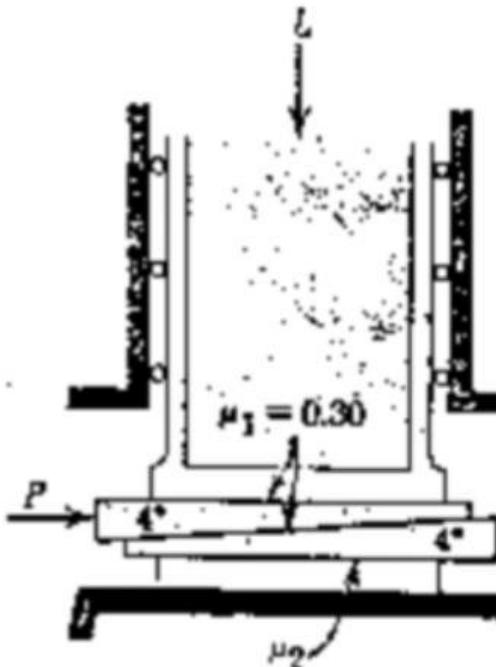
$$N_B = 427.32 \text{ N}$$

From (viii) \Rightarrow

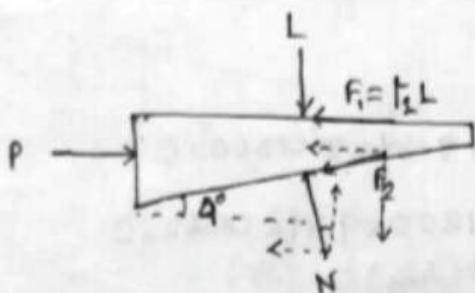
$$P = 98.0 \text{ N (Ans)}$$

11. The two wedges are used to position the vertical column under a load L . What is the least value of the coefficient of friction μ_2 for the bottom pair of surfaces for which the column may be raised by applying a single horizontal force P to the upper wedge?

Ans. $\mu_2 = 0.378$



11.



$$\sum F_x = 0$$

$$\text{or } P - N \sin 4^\circ - \mu_1 N \cos 4^\circ - f_1 L = 0$$

$$\text{or } P - N(\sin 4^\circ + \mu_1 \cos 4^\circ) = 0.3 L \quad \dots \dots (1)$$

$$\sum F_y = 0$$

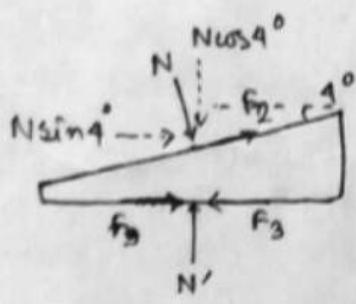
$$\text{or } -L - f_1 N \sin 4^\circ + N \cos 4^\circ = 0$$

$$\text{or } N(\mu_1 \cos 4^\circ - f_1 \sin 4^\circ) = L$$

$$\text{or } N = \frac{L}{0.977} = 1.023 L \quad \dots \dots (2)$$

$$\text{From, (1), } P = 0.3 L + 1.023 L(0.369)$$

$$= 0.677 L \quad \dots \dots (3)$$



$$\text{Now, } \sum F_x = 0$$

$$N \sin 4^\circ + 0.3069 L \times \cos 4^\circ = f_2 N'$$

$$\text{or } f_2 L = 1.023 L \sin 4^\circ + 0.3069 L$$

$$\sum F_y = 0 \quad \text{or } f_2 = 0.378 \quad (\text{Ans})$$

$$\text{or } -N \cos 4^\circ + f_2 \sin 4^\circ + N' = 0$$

$$\text{or } N' = 1.021 L - 0.021 L$$

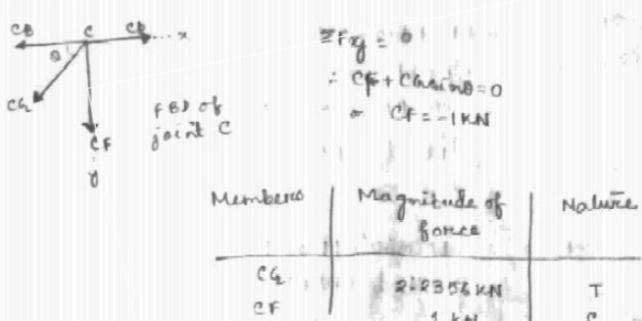
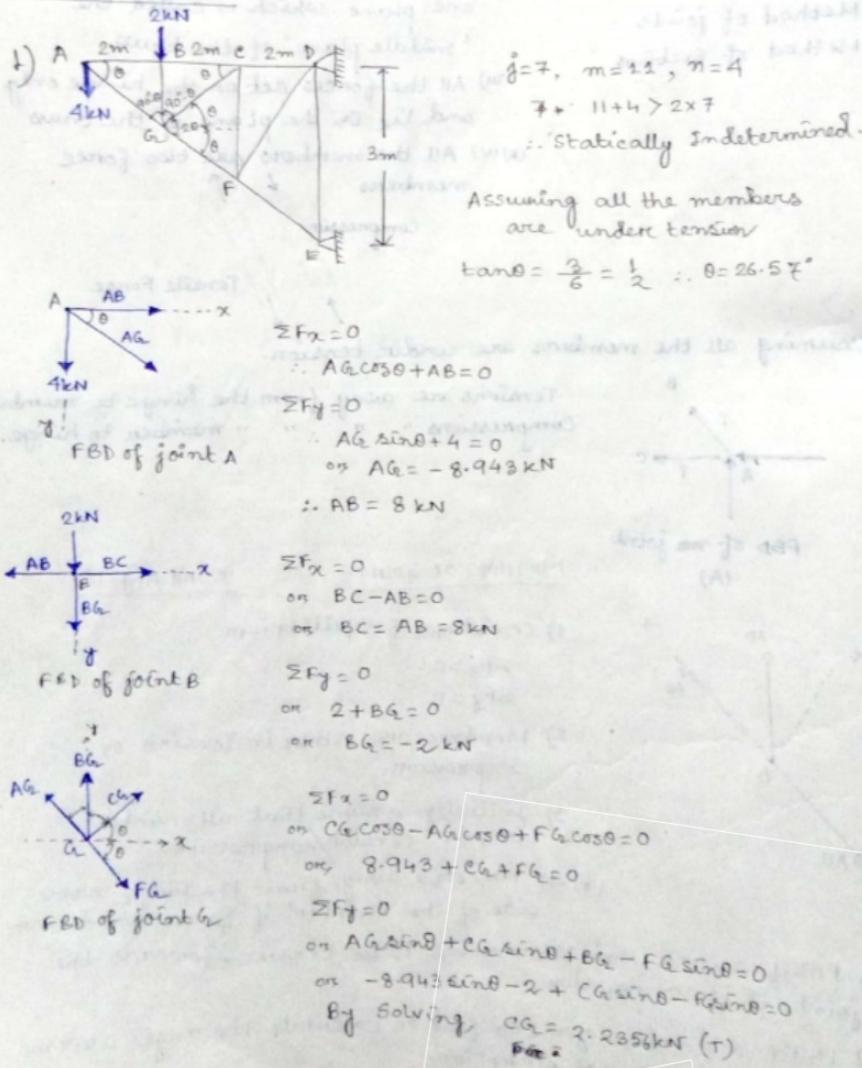
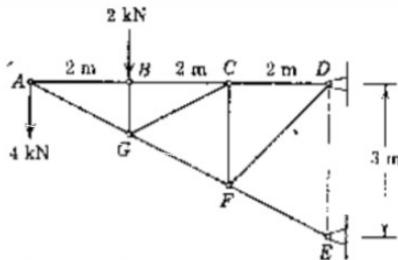
$$= L$$

PROBLEM SHEET 3.1

[TRUSS]

1. Calculate the forces in members CG and CF for the truss shown. If the 2-kN force acting on the truss were removed, identify by inspection those members in which the forces are zero. On the other hand, if the 2-kN force were applied at G instead of B, would there be any zero force members?

Ans CG = 2.24 kN (T), CF = 1 kN (C)



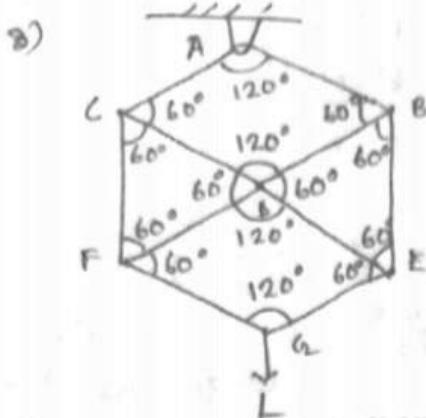
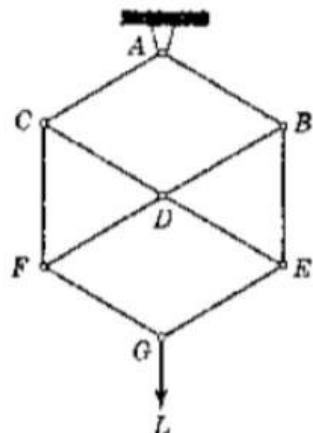
If 2 kN is removed, the BG would become zero.

If 2 kN force is applied at G instead of B \Rightarrow

$BG = 0$.

If 2 kN force is applied at G instead of B \Rightarrow

3. Solve for the forces in members BE and BD of the truss which supports the load L. All interior angles are 60° or 120° .



i)

FBD of G

$\sum F_x = 0 \quad \therefore -EG\sin 60^\circ + EB\cos 60^\circ = 0$
 $\therefore EG\cos 60^\circ = EB\sin 60^\circ$
 $\therefore EG = EB$

$\sum F_y = 0 \quad \therefore -EG\cos 60^\circ + EB\sin 60^\circ = 0$
 $\therefore EB = L \quad (\text{Ans})$

FBD of E

$\sum F_x = 0 \quad \therefore -ED\sin 60^\circ + EB\cos 60^\circ = 0$
 $\therefore ED\cos 60^\circ = EB\sin 60^\circ$
 $\therefore ED = EB$

$\sum F_y = 0 \quad \therefore -EG\cos 60^\circ + ED\sin 60^\circ = 0$
 $\therefore ED = EG$

$\therefore ED = EG = EB = L$

FBD of G

$\sum F_x = 0 \quad \therefore -GF\sin 60^\circ + GE\cos 60^\circ = 0$
 $\therefore GF\sin 60^\circ = GE\cos 60^\circ$
 $\therefore GF = GE$

$\sum F_y = 0 \quad \therefore -GE\cos 60^\circ + GF\sin 60^\circ = 0$
 $\therefore GE = GF = L$

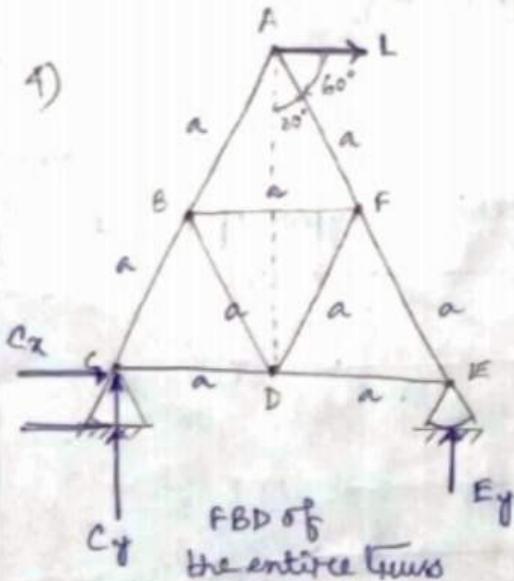
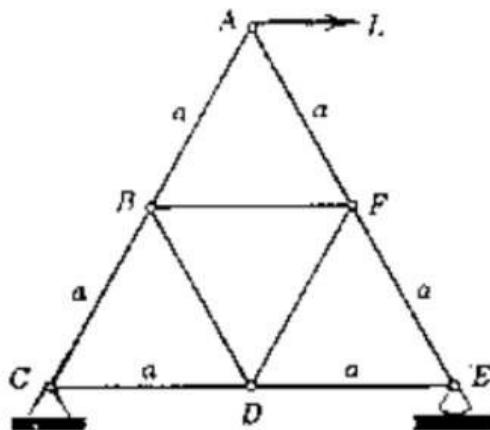
FBD of B

$\sum F_x = 0 \quad \therefore -BA\sin 60^\circ + BD\sin 60^\circ = 0$
 $\therefore BA = BD$

$\sum F_y = 0 \quad \therefore BA\cos 60^\circ - BD\cos 60^\circ - BE = 0$
 $\therefore -BD\cos 60^\circ - BE = 0$
 $\therefore BD = -BE \quad (\text{Ans})$

4. The equiangular truss is loaded and supported as shown. Determine the forces in all members in terms of the horizontal load L.

Ans. AB = BC = L (T), AF = EF = L (C)
 $DE = CD = L/2$ (T), BF = DF = BD = 0



$$\text{i) } \sum M_C = 0 \quad \text{on } +\text{ve}$$

$$E_y \times 2a - L(2 \times \frac{\sqrt{3}a}{2}) = 0$$

$$\text{or } E_y = \frac{\sqrt{3}L}{2}$$

$$\text{ii) } \sum F_y = 0$$

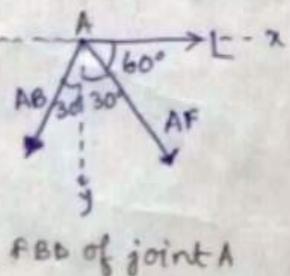
$$\text{or } C_y + E_y = 0$$

$$\text{or } C_y = -E_y = -\frac{\sqrt{3}}{2}L$$

$$\text{iii) } \sum F_x = 0$$

$$\text{or } C_x + L = 0$$

$$\text{or } C_x = -L$$



$$\sum F_x = 0$$

$$\text{or } L + AF \cos 60^\circ - AB \cos 60^\circ = 0 \quad \dots \dots \text{v}$$

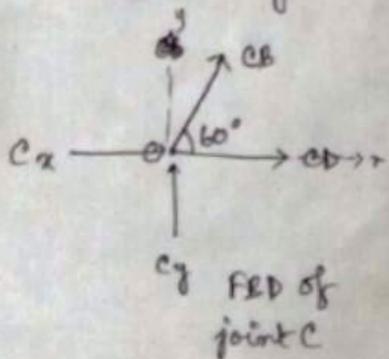
$$\sum F_y = 0$$

$$\text{or } AF \sin 60^\circ + AB \sin 60^\circ = 0$$

$$\text{or } AF = -AB \quad \dots \dots \text{vi}$$

By Solving

$$\therefore \boxed{AB = L, AF = -L}$$



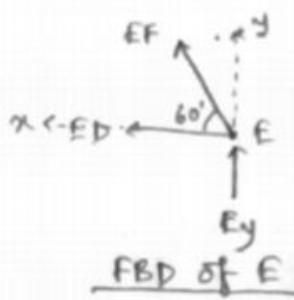
$$\sum F_x = 0 \quad \therefore C_x + CD + CB \cos 60^\circ = 0$$

$$\text{or } -L + CD + \frac{CB}{2} = 0 \quad \dots \dots \text{vii}$$

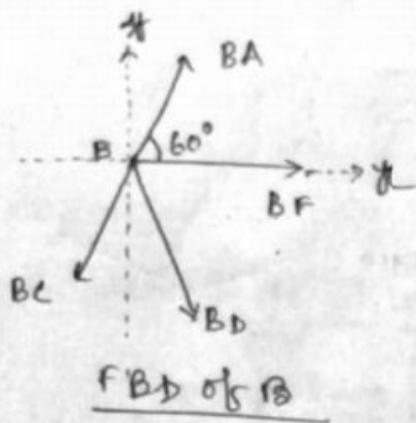
$$\sum F_y = 0, \quad C_y + CB \sin 60^\circ = 0 \quad \dots \dots$$

$$\text{or } CB = (-\frac{\sqrt{3}}{2}L) \times (-\frac{2}{\sqrt{3}}) = L \quad [\text{Ans}]$$

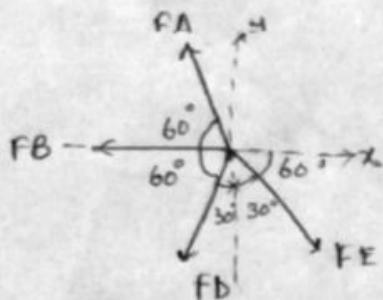
$$\boxed{CD = \frac{L}{2}, CB = L}$$



$$\begin{aligned}\sum F_x &= 0 \quad \therefore ED + EF \cos 60^\circ = 0 \\ \sum F_y &= 0 \quad \therefore Ey + EF \sin 60^\circ = 0 \\ \text{or } EF &= -\frac{\sqrt{3}}{2} L \times \frac{L}{\sqrt{3}} = -L \\ \therefore ED &= \frac{L}{2}, EF = -L\end{aligned}$$



$$\begin{aligned}\sum F_x &= 0 \quad \therefore BA \cos 60^\circ + BF + BD \cos 60^\circ - BC \cos 60^\circ = 0 \\ \text{or } L \times \frac{1}{2} + BF + BD \frac{1}{2} - \frac{1}{2} &= 0 \\ \text{or } BF + \frac{BD}{2} &= 0 \\ BA \sin 60^\circ - BC \sin 60^\circ - BD \sin 60^\circ &= 0 \\ \text{or } \frac{L}{2} - \frac{L}{2} - BD &= 0 \\ \text{or } BD &= 0 \quad \therefore BF = 0, BD = 0\end{aligned}$$

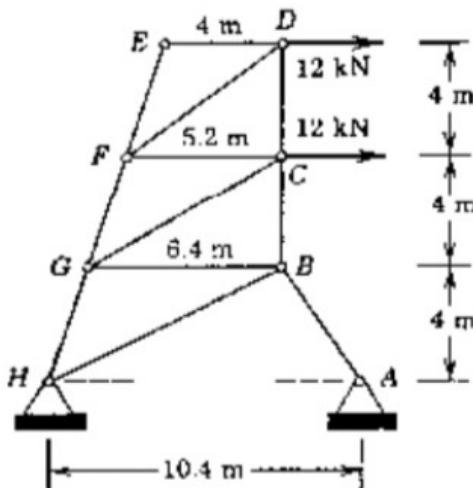


$$\begin{aligned}\sum F_x &= 0 \quad \therefore FA \cos 60^\circ - FB - FD \cos 60^\circ + FE \cos 60^\circ = 0 \\ \text{or } \frac{L}{2} - 0 - \frac{FD}{2} - \frac{L}{2} &= 0 \\ \text{or } FD &= 0\end{aligned}$$

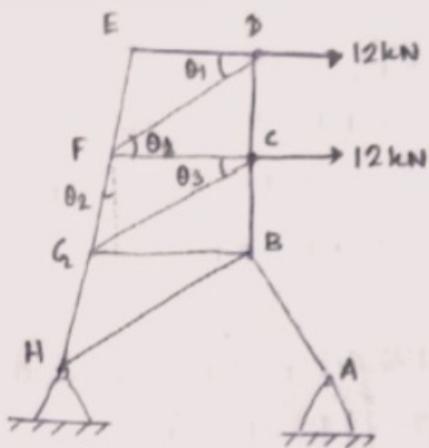
Members	Magnitude of Forces	Types of Forces
AB	L	(T)
AE		
BC	L	(T)
BD	0	
CD	L/2	
DE	L/2	(T)
DF	0	
FE	L	(T)
FA	L	(C)
BF	0	(C)

5. Calculate the forces in members CF, CG, EF of the loaded truss.

Ans. CG = 25.0 kN (T)



5.

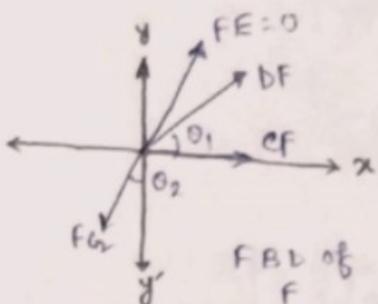


④

$$\begin{aligned} \text{FBD of } E: & \quad \text{DE} \rightarrow x, \quad \text{EF} \downarrow y \\ & \text{i) } \sum F_y = 0 \therefore EF = 0 \\ & \text{ii) } \sum F_x = 0 \therefore DE = 0 \\ & \tan \theta_1 = \frac{4}{5.2} \Rightarrow \theta_1 = 37.57^\circ \end{aligned}$$

$$\begin{aligned} \tan \theta_2 &= \frac{6.4 - 5.2}{4} \therefore \theta_2 = 16.69^\circ \\ \tan \theta_3 &= \frac{4}{6.4} \therefore \theta_3 = 32^\circ \end{aligned}$$

⑤

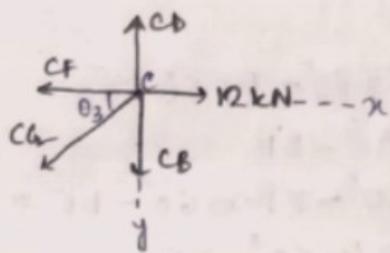


⑥

$$\begin{aligned} \text{FBD of } D: & \quad \text{DE} \leftarrow x, \quad \text{DF} \downarrow y, \quad \text{DC} \rightarrow z \\ & \sum F_x = 0 \quad \text{on } 12 - DE - DF \cos \theta_1 = 0 \\ & \text{or } DE + DF \cos \theta_1 = 12 \\ & \text{or } DF = 15.14 \text{ kN} \end{aligned}$$

$$\begin{aligned} \sum F_y &= DF \sin \theta_1 - FG \cos \theta_2 = 0 \therefore FG = 9.64 \text{ kN} \\ \sum F_x &= CF + DF \cos \theta_1 - FG \sin \theta_2 = 0 \\ \text{or } CF &= -9.23 \text{ kN} \end{aligned}$$

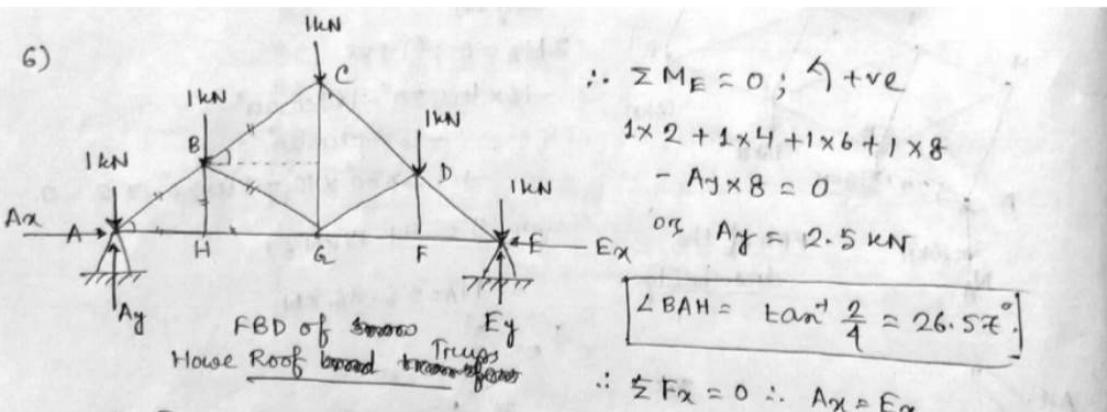
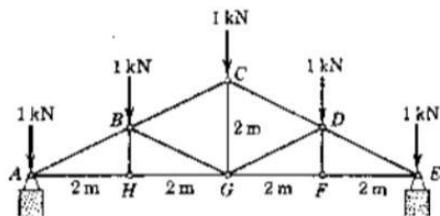
⑦



$$\begin{aligned} \sum F_x &= 0 \\ \text{or } -CG \cos \theta_3 - CF + 12 &= 0 \\ \text{or } CG &= 25.03 \text{ kN (T)} \end{aligned}$$

6. A snow load transfers the forces shown to upper joints of a Howe roof truss respectively. Neglect any horizontal reaction at the supports and solve for the forces in all members and compare them.

Ans. For Howe roof truss AB = DE = 3.35 kN (C), BC = CD = 2.24 N (C), AH = EF = 3.00 kN (T), BH = DF = 0, GH = FG = 3.0 kN (T), BG = DG = 1.12 kN (C)



7)

$\sum F_x = 0$
 $\therefore AB \cos 26.57 + AH = 0$
 $\therefore AH = 3.00 \text{ kN}$

$\sum F_y = 0$
 $\therefore A_y + AB \sin 26.57 - 1 = 0$
 $\therefore AB = -3.35 \text{ kN}$

$\therefore AH = 3.00 \text{ kN}$

$\sum F_y = 0$
 $\therefore HB = 0$

$\sum F_x = BC \cos 26.57 + BG \cos 26.57 - BA \cos 26.57 = 0$
 $\therefore BC + BG = BA$
 $\therefore BC + BG = -3.35 \text{ kN}$

$\sum F_y = BC \sin 26.57 - BG \sin 26.57 - EA \sin 26.57 - BH = 0$
 $\therefore BC - BG - EA = + 2.28$
 $\therefore BC - BG = \frac{2.28}{\sin 26.57} = -1.12 \text{ kN}$

$\therefore BC = -2.24 \text{ kN}$ $\therefore BG = -1.12 \text{ kN}$

From the symmetry,
 $AB = DE = -3.35 \text{ kN}$ $BC = CB = -2.24 \text{ kN}$
 $AH = EF = 3.00 \text{ kN}$
 $BH = DF = 0 \text{ kN}$
 $BG = DG = -1.12 \text{ kN}$
 $HG = GF = 3.00 \text{ kN}$

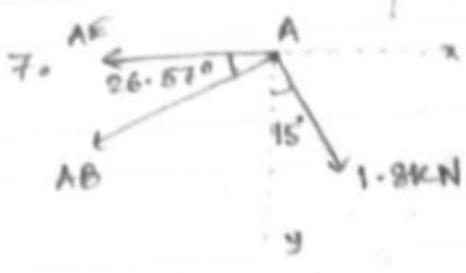
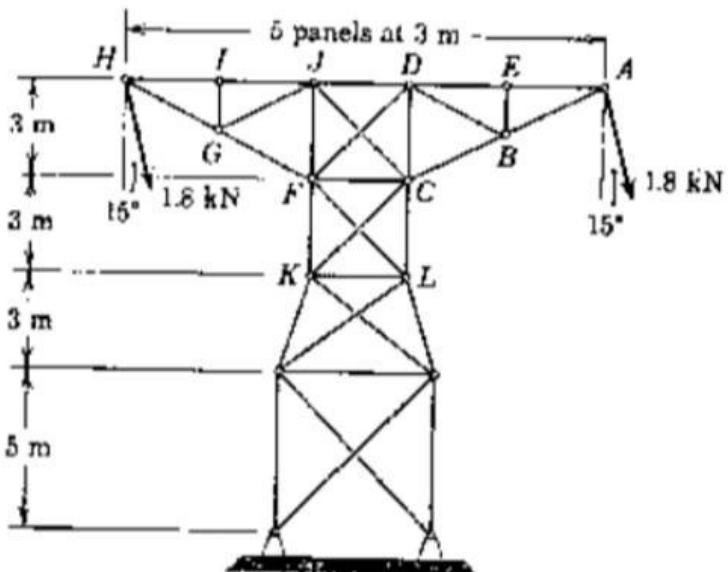
NOW,

$\sum F_x = CB \cos 26.57 - CB \cos 26.57 = 0$
 $\therefore CB = CB = -2.24 \text{ kN}$

$\sum F_y = CB \sin 26.57 - CG \sin 26.57 - CG = 0$
 $\therefore CG = -1.00 \text{ kN (Ans)}$

7. The tower for a transmission line is modeled by the truss shown. The crossed members in the center sections of the truss may be assumed capable of supporting tension only. For the loads of 1.8 kN applied in the vertical plane, compute the forces induced in members AB, DB and CD.

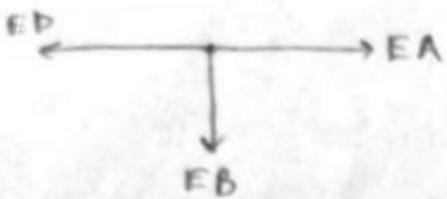
Ans. AB = 3.89 kN (C), DB = 0,
CD = 0.93 kN (C)



FBD of A

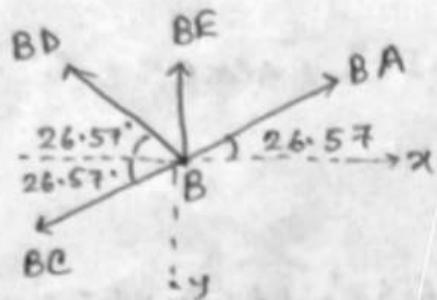
$$\begin{aligned}\sum F_x &= 0 \\ \therefore -AE - AB \cos 26.57^\circ + 1.8 \sin 15^\circ &= 0 \\ \text{or } AE + AB \cos 26.57^\circ &= 0.466 \dots\end{aligned}$$

$$\begin{aligned}\sum F_y &= 0 \\ \therefore 1.8 \times \cos 15^\circ + AB \sin 26.57^\circ &= 0 \\ \text{or } AB &= -3.89 \text{ kN} \\ \therefore AE &= 3.945 \text{ kN}\end{aligned}$$



FBD of E

$$\begin{aligned}\sum F_x &= ED = EA = 3.945 \text{ kN} \\ \sum F_y &= 0 = EB\end{aligned}$$



$$\begin{aligned}\sum F_x &= BA \cos 26.57^\circ - BD \cos 26.57^\circ \\ &\quad - BC \cos 26.57^\circ = 0\end{aligned}$$

$$\therefore BA = BD + BC$$

$$\text{or } BD + BC = -3.89 \text{ kN}.$$

$$\begin{aligned}\sum F_y &\rightarrow BA \sin 26.57^\circ + BD \sin 26.57^\circ \\ &= BC \sin 26.57^\circ \\ &= 0\end{aligned}$$

$$\therefore BA + BD = BC$$

$$\text{or } -3.89 + BD = -3.89$$

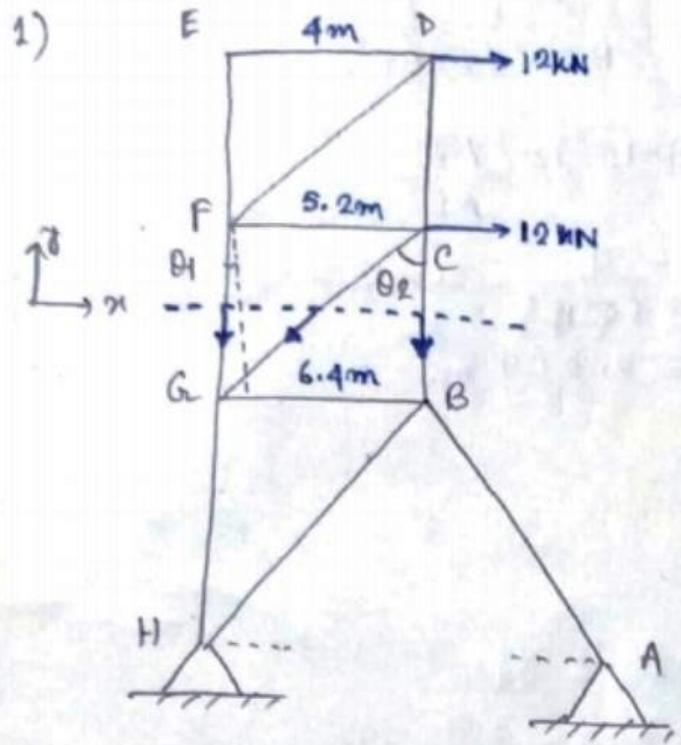
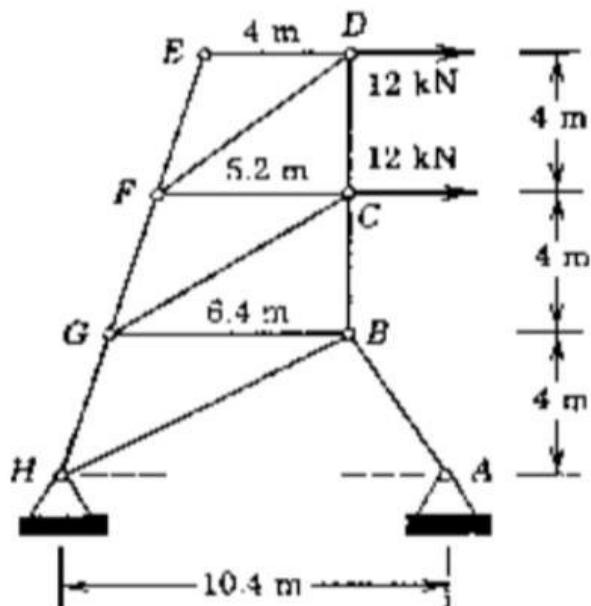
$$\text{or } BD = 0 \text{ kN}$$

PROBLEM SHEET 3,2

[TRUSS]

1. Solve for the force in member CG from an equilibrium equation which contains that force as the only unknown.

Ans CG = 25.0 kN (T)



FBD of the truss

By solving (1), (4),

$$CG = 25.06 \text{ kN (T)}$$

Find out $F_{CG} = ?$

$$\tan \theta_1 = \frac{6.4 - 5.2}{4} ; \theta_1 = 16.69^\circ$$

$$\tan \theta_2 = \frac{6.4}{4} ; \theta_2 = 58^\circ$$

$$\sum F_x = 0$$

$$\text{on } 12 + 12 - FG \sin \theta_1 - CG \sin \theta_2 = 0$$

$$\text{on } CG \sin \theta_2 + FG \sin \theta_1 = 24 \quad \dots \dots (1)$$

$$\sum F_y = 0$$

$$\text{on } - FG \cos \theta_1 - CG \cos \theta_2 - CB = 0$$

$$\text{on } CG \cos \theta_2 + FG \cos \theta_1 = - CB \quad \dots \dots (2)$$

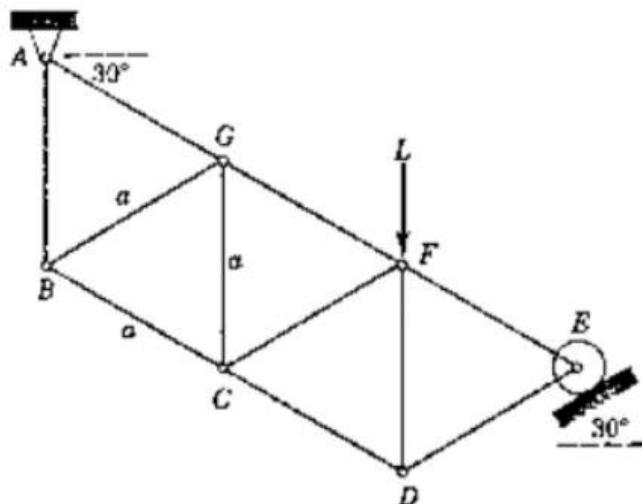
$$\sum M_G = 0 ; \rightarrow +ve$$

$$\text{on } BC \times 6.4 + 12 \times 4 + 12 \times 8 = 0$$

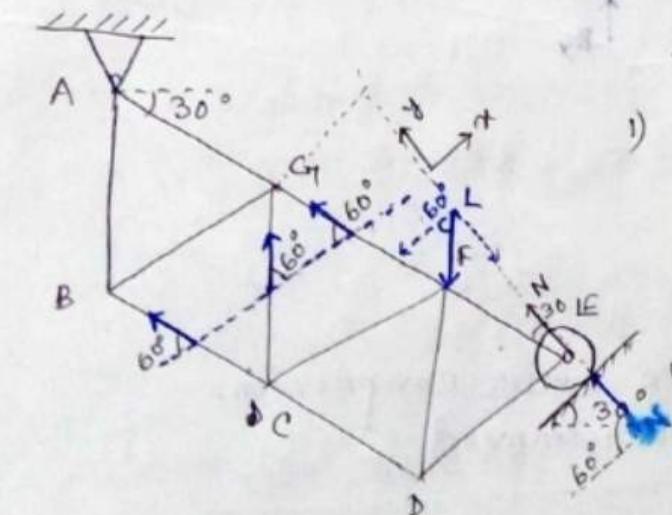
$$\text{on } BC = - 22.5 \text{ kN} \quad \dots \dots (3)$$

2. Determine the forces in members BC and CG of the truss loaded as shown.

Ans. $BC = CG = L/3$ (T)



2)



Determine $F_{BC} = ?$ $F_{CG} = ?$

$$1) \sum F_x = 0$$

$$\text{on } -CB\cos 60^\circ + CG\cos 60^\circ - FG\cos 60^\circ - NL\cos 60^\circ - L\cos 60^\circ = 0$$

$$\text{or } CG = CB + FG + L \quad \dots \dots (1)$$

$$2) \sum F_y = 0$$

$$\text{on } CB\sin 60^\circ + CG\sin 60^\circ + FG\sin 60^\circ + N\sin 60^\circ - L\sin 60^\circ = 0$$

$$\text{on } CB + CG + FG + N - L = 0 \quad \dots \dots (ii)$$

$$iii) \sum M_G = 0; \uparrow \text{ +ve}$$

$$\text{on } -BC \times a\frac{\sqrt{3}}{2} + L \times 2 \times a\frac{\sqrt{3}}{2} = 0$$

$$\text{or } +N \times 2a \times \sin 30^\circ$$

$$\text{on } -BC \times a\frac{\sqrt{3}}{2} + L \times a\sqrt{3} + aN = 0$$

$$\text{or } BC \frac{a\sqrt{3}}{2} = L a\sqrt{3} + a \times \frac{2L}{\sqrt{3}}$$

$$\text{or } BC =$$

$$\text{on } CG + CB + FG + \frac{2}{\sqrt{3}} \times \frac{2L}{\sqrt{3}} - L = 0$$

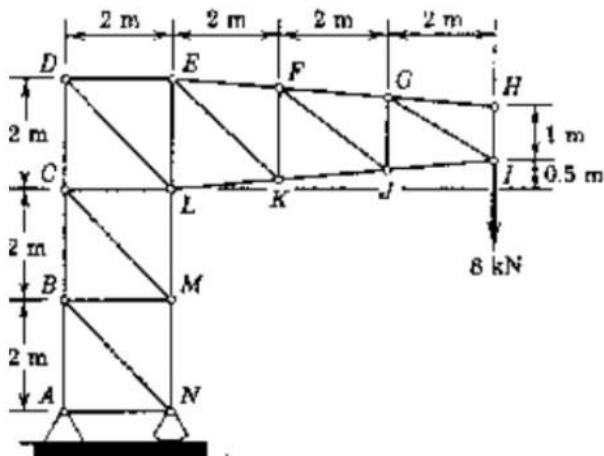
$$\text{on } CB + CG + FG = -\frac{L}{3}$$

$$\text{or } CG - L + CB = -\frac{L}{3} \quad [\text{From (ii)}]$$

$$\text{on } 2CG = \frac{2L}{3}$$

$$\text{on } CG = \frac{L}{3} \quad [\text{Ans}]$$

3. Determine the forces in members DE and DL.



Free body diagram of the truss with coordinate axes:

$\Sigma F_x = 0$

$$-ED - LD \cos 45^\circ + Ax = 0$$

$$\text{or } ED + LD \cdot \frac{1}{\sqrt{2}} = Ax \quad \dots \dots (1)$$

$\Sigma F_y = 0$

$$CD + LD \sin 45^\circ - 8 + Ay + Ny = 0$$

$$\text{or } CD + \frac{LD}{\sqrt{2}} + Ay + Ny = 8 \quad \dots \dots (2)$$

$\Sigma M_A = 0; \text{ counter-clockwise} +ve$

$$LD \cos 45^\circ \times 4 + LD \sin 45^\circ \times 2 + ED \times 6 - 8 \times 8 + Ny \times 2 = 0$$

$$\text{or } \frac{2LD}{\sqrt{2}} + \frac{LD}{\sqrt{2}} + 6ED - 32 + Ny = 0$$

$$\text{or } \frac{3}{\sqrt{2}} LD + 6ED + Ny = 32 \quad \dots \dots (3)$$

$\Sigma M_D = 0; \text{ counter-clockwise} +ve$

$$Ax \times 6 + Ny \times 2 - 8 \times 8 = 0$$

$$\text{or } 3Ax + Ny = 32 \quad \dots \dots (4)$$

$\Sigma M_N = 0; \text{ counter-clockwise} +ve$

$$-Ay \times 2 + LD \cos 45^\circ \times 4 - CD \times 2 - 8 \times 6 = 0$$

$$\text{or } Ay + CD + 24 = \frac{2LD}{\sqrt{2}} \quad \dots \dots (5)$$

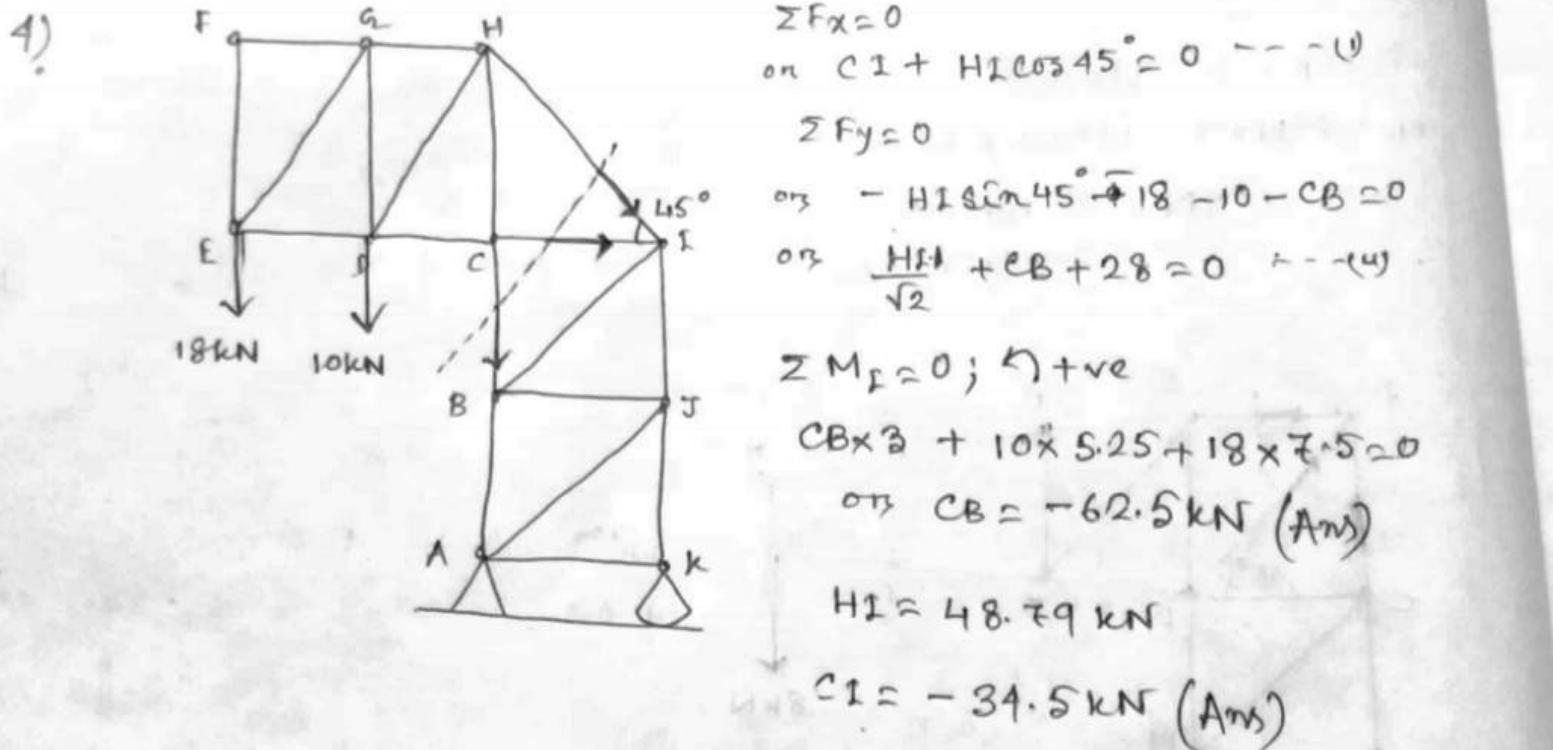
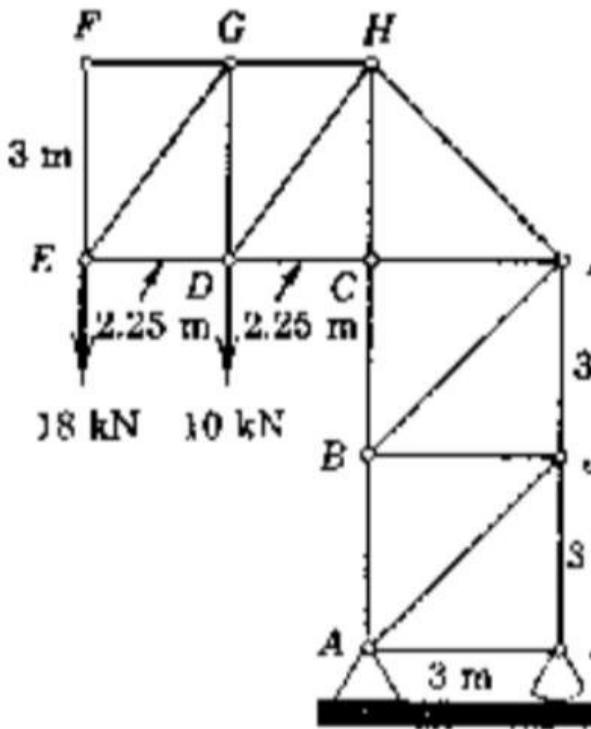
From, (4) $\Rightarrow \frac{2LD}{\sqrt{2}} - 24 + Ny + \frac{LD}{\sqrt{2}} = 8$

$$\text{or } \frac{3LD}{\sqrt{2}} + Ny = 32 \quad \dots \dots (6)$$

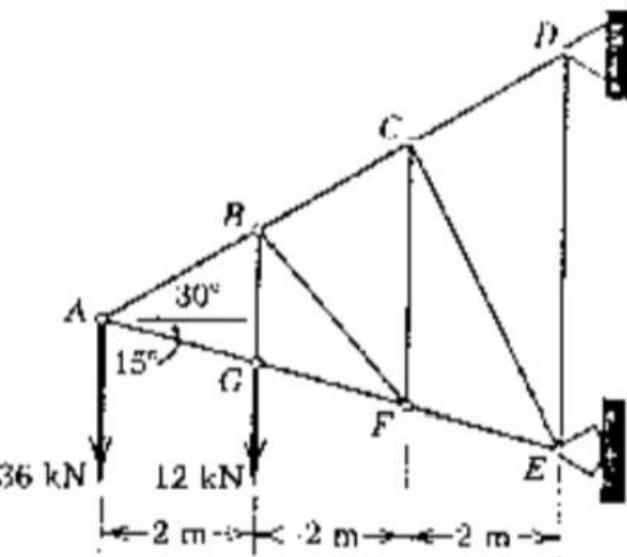
From, (5), (6) $\Rightarrow \boxed{ED = 0}$

From, (6) $\Rightarrow 3 \times \frac{LD}{\sqrt{2}} + 32 = 32$

4. Determine the forces in members BC and CI.
- Ans. BC = 62.5 kN (C), CI = 34.5 kN (C)



5. Determine the force in the member BF.



5)

Diagram shows the truss structure with coordinate axes x and y . Joint A is at the bottom left, joint D is at the top right, and joint E is at the bottom right. Joint C is above E. Joint B is above A. Joint F is between C and E. A vertical member connects A and C. Horizontal dimensions: AB = BC = 2m, CE = 2m, EF = 2m. Vertical dimensions: AG = 2m, GC = 2m, GE = 2m. Horizontal distances from A to B and from C to E are each 2m. A horizontal force of 36 kN acts downwards at A. A horizontal force of 12 kN acts downwards at C. Angles: $\angle BAG = 30^\circ$, $\angle GAC = 15^\circ$, $\angle AGE = 30^\circ$.

$F_{BF} = ?$

$HF = 2m$

 $BH = BZ + ZG + GH$
 $= 2t \tan 30^\circ + 2t \tan 15^\circ + 2t \tan 15^\circ$
 $= 2.2265 m.$
 $\tan \theta = \frac{BH}{HF} = \frac{2.2265}{2}$
 $\therefore \theta = 48.06^\circ$

$\sum F_x = 0$

 $\therefore BC \cos 30^\circ + GF \cos 15^\circ + BF$

$\sum M_A = 0; \text{ J +ve.}$

on $BF \times AJ + 12 \times 2 = 0$

or $BF = -\frac{24}{2.259}$

 $= -10.62 \text{ kN (Ans)}$

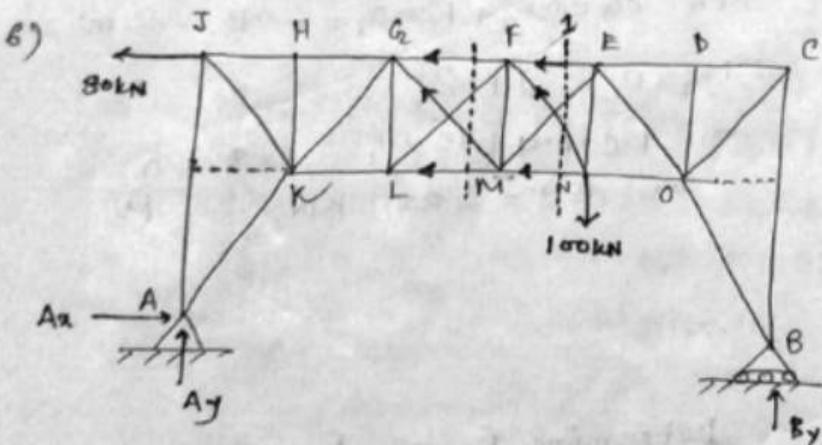
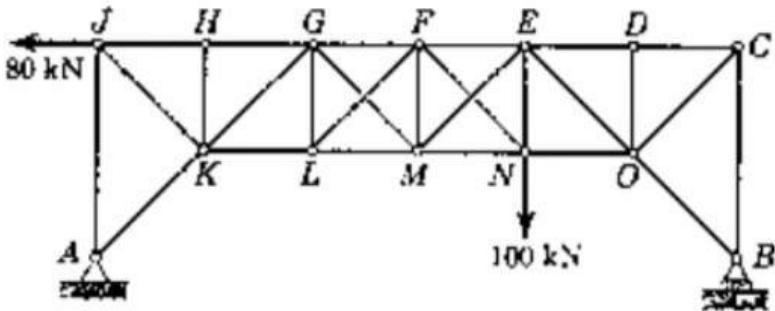
$\sin(\theta - 15^\circ) = \frac{AJ}{AF}$

or $AJ = \frac{4}{\cos 15^\circ} \times \sin 33.06^\circ$

 $= 2.259 \text{ m.}$

6. The truss shown is composed of 450 right triangles. The crossed members in the center two panels are slender tie rods incapable of supporting compression. Retain the two rods which are under tension and compute the magnitude of their tensions. Also find the force in the member MN.

Ans. $FN = GM = 84.8 \text{ kN (T)}$, $MN = 20 \text{ kN (T)}$



For the entire truss:

$$\sum M_A = 0, \uparrow +ve$$

$$By \times 6a + 80 \times 2a - 100 \times 4a = 0$$

From section (1) \uparrow $By = 40 \text{ kN}$ (Ans)

Cross Member, FN is under tension, EN is under compression.
So, EN is removed

$$\sum F_y = 0 \therefore FN \sin 45^\circ = 60$$

$$\therefore FN = 84.85 \text{ kN (T)}$$

From section (2) $\sum F_y = 0, GM \sin 45^\circ = 60$

$$\therefore GM = 84.85 \text{ kN (T)}$$

Scanned by CamScanner

From section (1),

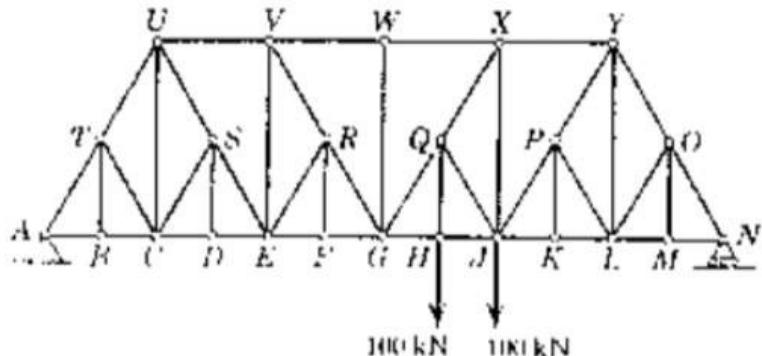
$$\sum M_F = 0; \uparrow +ve$$

$$\therefore -MN \times d - 100 \times a + By \times 3a = 0$$

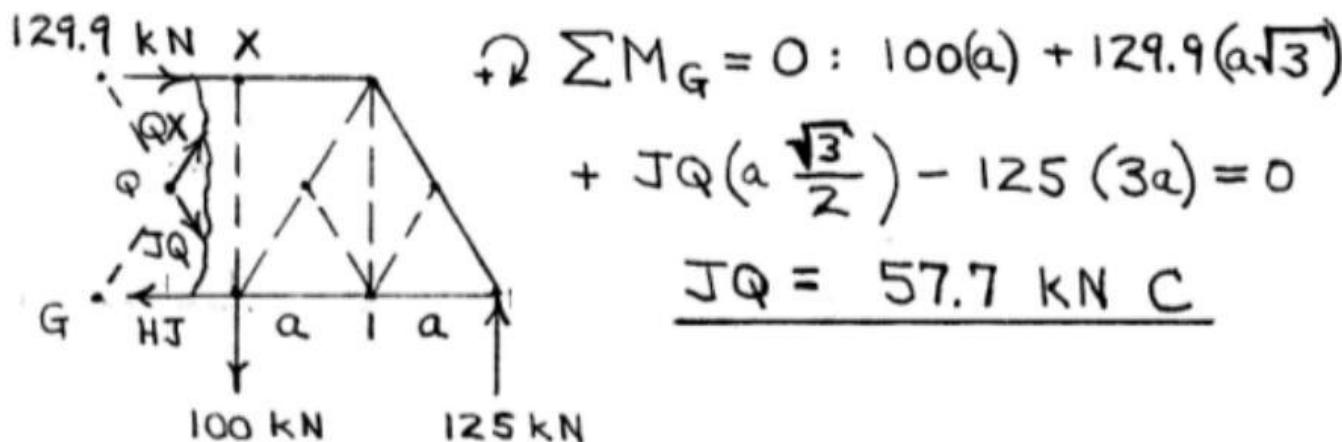
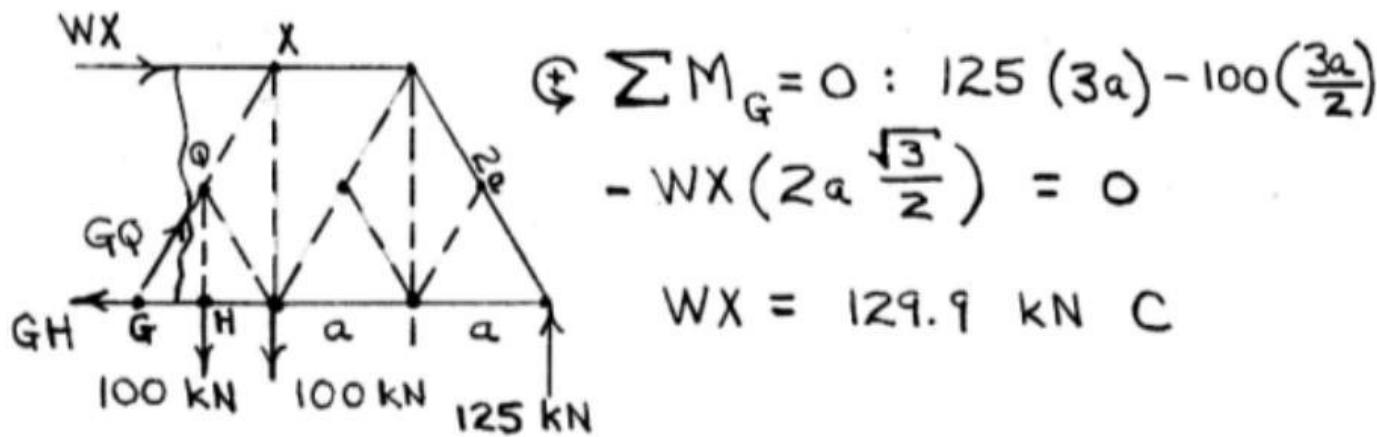
$$\therefore MN = 3 \times 40 - 100$$

$$= 20 \text{ kN (T)} \text{ Ans.}$$

7. Find the force in member JQ for the Baltimore truss where all angles are 300, 600, 900 or 1200.
 Ans. JQ = 57.7 kN (C)



4/48 From truss as a whole, $\sum M_A = 0$ gives
 $N = 125 \text{ kN}$.

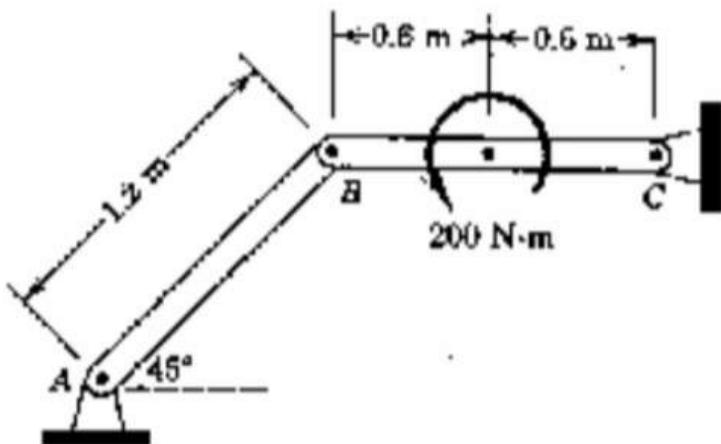


PROBLEM SHEET 4

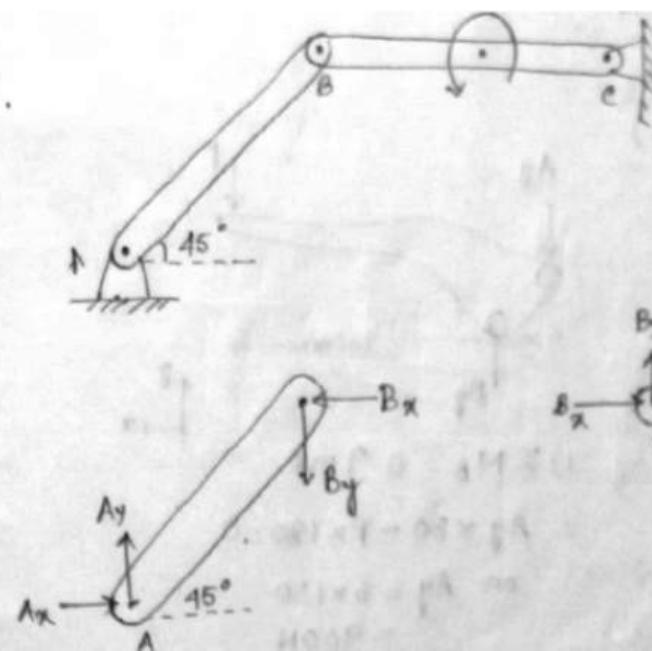
[FRAMES]

1. Determine the magnitude of the pin reaction at C.

Ans C = 236 N



Q.



FBD of AB

$$\sum M_A = 0 \quad ? + ve$$

$$\therefore B_x \times 1.2 \sin 45^\circ - B_y \times 1.2 \cos 45^\circ = 0$$

$$\text{on } B_x = B_y = 166.67 \text{ N}$$

$$i) \sum M_C = 0 \quad ? + ve$$

$$\therefore 200 - B_y \times 1.2 = 0$$

$$\text{on } B_y = 166.67 \text{ N.}$$

$$ii) \sum F_y = 0 \quad \therefore B_y = C_y = 166.67 \text{ N}$$

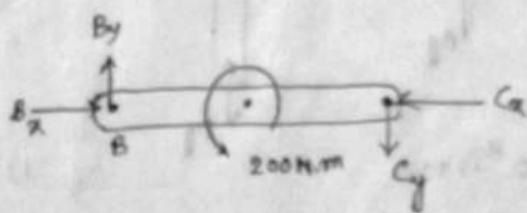
$$iii) \sum F_x = 0 \quad \therefore B_x = C_x = 166.67 \text{ N}$$

$$\therefore C = \sqrt{C_x^2 + C_y^2} \text{ N}$$

$$= 166.67\sqrt{2} \text{ N}$$

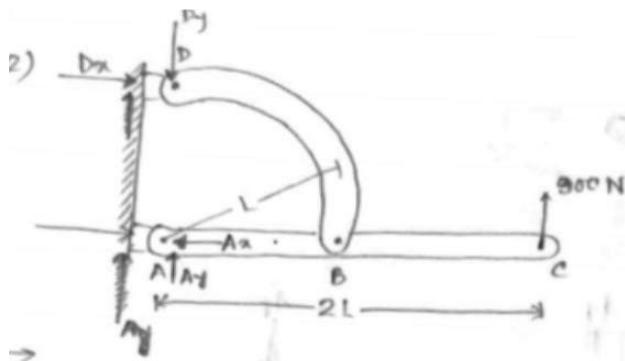
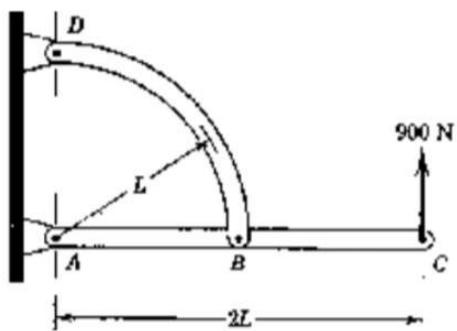
$$= 236.73 \text{ N}$$

$$\approx 236 \text{ N.}$$

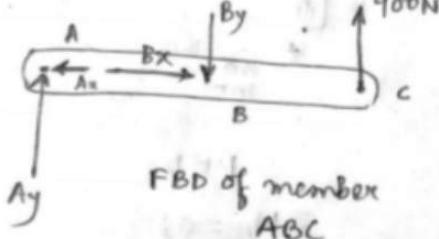


FBD of BC

2. Determine the magnitude of the pin reaction at A for the frame loaded by the 900-N force.



Find Pin reaction at A.



$$\text{D) } \sum M_B = 0; \uparrow +\text{ve}$$

$$900 \times L - A_y \times L = 0$$

$$\therefore \boxed{A_y = 900 \text{ N}}$$

$$\text{u) } \sum F_x = 0$$

$$\therefore -A_x + B_x = 0$$

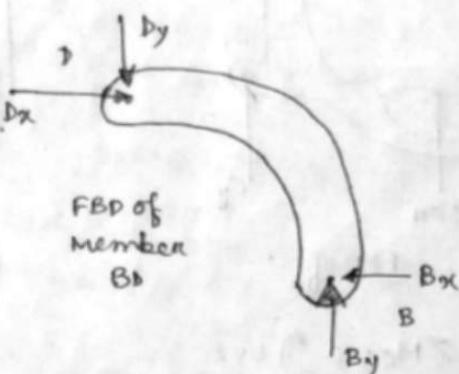
$$\text{or } B_x = A_x$$

$$\text{u) } \sum F_y = 0$$

$$A_y - B_y + 900 = 0$$

$$\text{or } \boxed{B_y = 1800 \text{ N.}}$$

$$\text{So, } \boxed{B_x = A_x = 1800 \text{ N}}$$



$$\text{i) } \text{MF}_B \sum M_B = 0; \uparrow +\text{ve}$$

$$By \times L - Bx \times L = 0$$

$$\therefore \boxed{By = Bx}$$

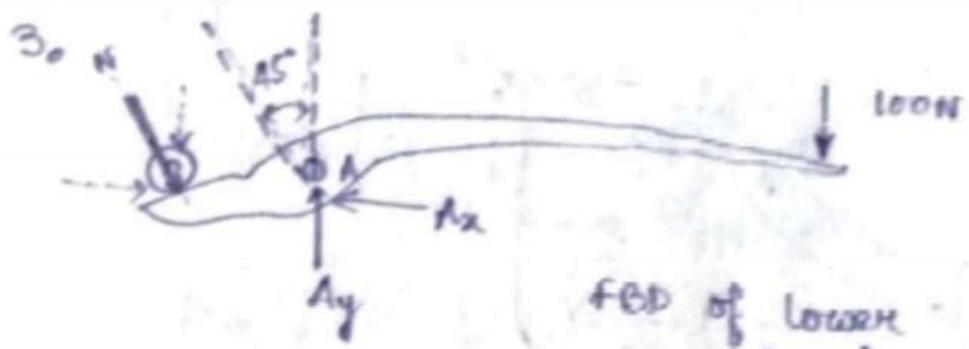
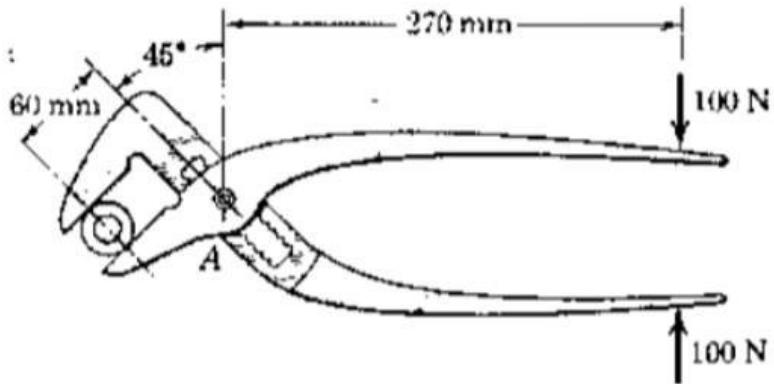
$$\text{Reaction at A} = \sqrt{1800^2 + 900^2} \text{ N}$$

$$= 900\sqrt{5} \text{ N}$$

$$= 2012 \text{ N (Ans)}$$

3. Compute the force supported by the pin at A for the slip-joint pliers under a grip of 100 N.

Ans. 525 N



$$\sum F_x = 0 \therefore N \sin 45^\circ - A_x = 0$$

$$\text{on } A_x = \frac{N}{\sqrt{2}}$$

$$\sum F_y = 0 \therefore N \cos 45^\circ - 100 + A_y = 0$$

$$\text{on } A_y = \frac{N}{\sqrt{2}} + 100.$$

$$\text{Now, } \sum M_A = 0 \quad ? + ve$$

$$\therefore -100 \times 210 + N \times 60 = 0$$

$$\text{or } N = \frac{100 \times 210}{60} = 350 \text{ N}$$

$$= 450 \text{ N}$$

$$\therefore A_x = \frac{450}{\sqrt{2}} \text{ N} = 318.198 \text{ N}$$

$$A_y = 418.198 \text{ N}$$

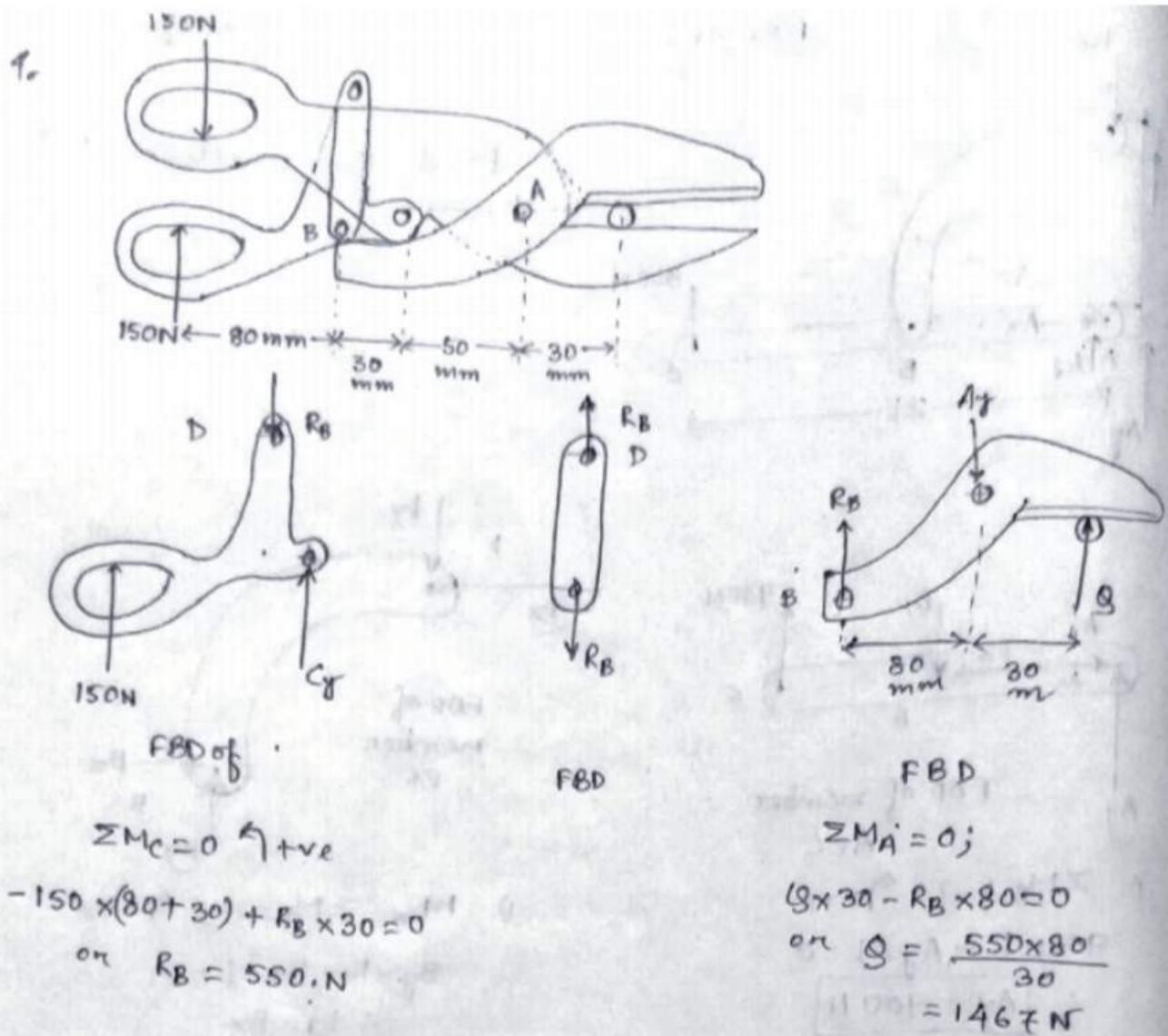
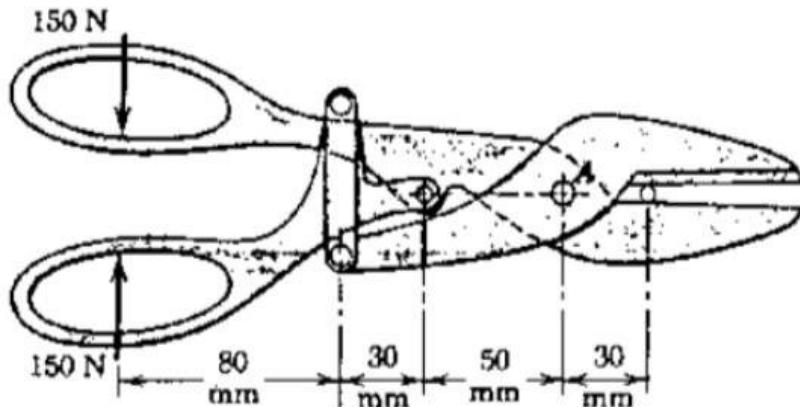
$$\therefore A = \sqrt{A_x^2 + A_y^2} \text{ N}$$

$$= 525.4898 \text{ N}$$

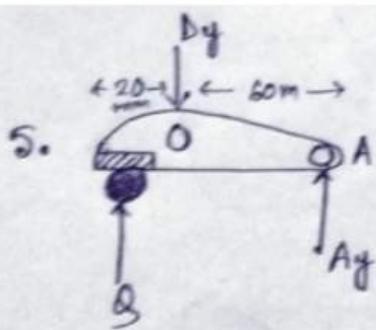
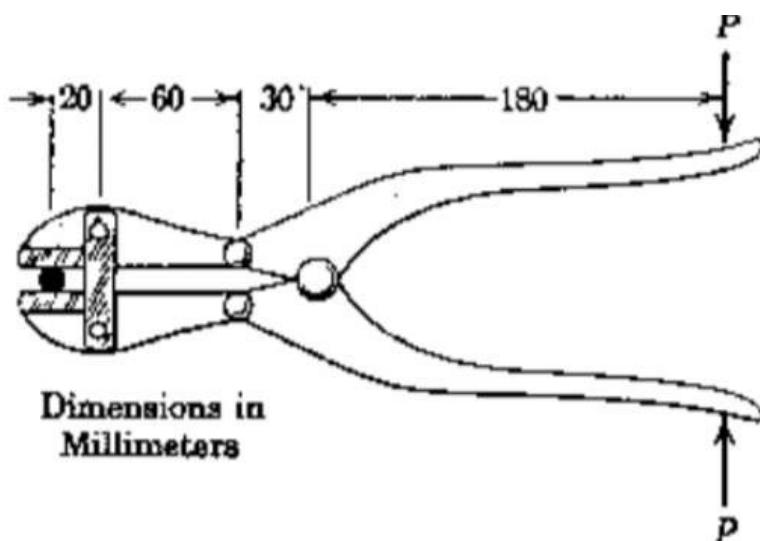
$$\approx 526 \text{ N. (Ans)}$$

4. Compound-lever snips, shown in the figure, are often used in place of regular tinnings' snips when large cutting forces are required. For the gripping force of 150 N, what is the cutting force P at a distance of 30 mm along the blade from the pin at A.

Ans. $P = 1467 \text{ N}$



5. The A small bolt cutter operated by hand for cutting small bolts and rods is shown in the sketch. For a hand grip $P = 150 \text{ N}$, determine the force Q developed by each jaw on the rod to be cut.



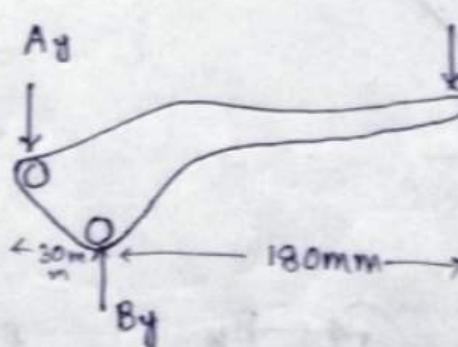
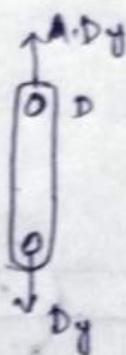
$$\sum M_D = 0 \quad \text{(clockwise)} +ve$$

$$Ay \times 60 - Q \times 20 = 0$$

$$\therefore Q = 3Ay$$

$$\begin{aligned} \text{or } Q &= (3 \times 150) \text{ N} \\ &= 450 \text{ N} \end{aligned}$$

$\sim 180 \text{ mm}$ (mm)



$$i) \sum M_B = 0 \quad \text{(clockwise)} +ve$$

$$\therefore Ay \times 30 - P \times 180 = 0$$

$$\begin{aligned} \text{or } Ay &= 6 \times 150 \\ &= 900 \text{ N} \end{aligned}$$

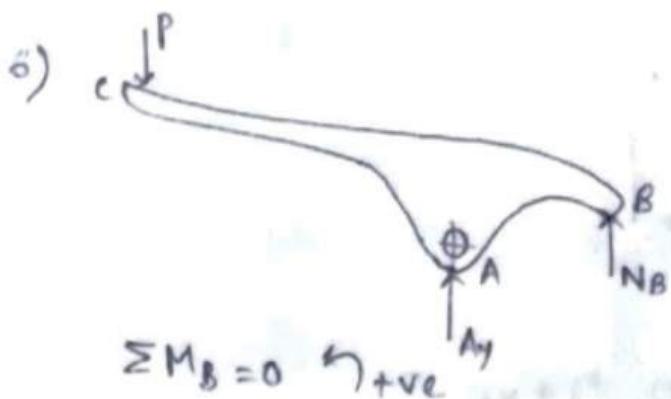
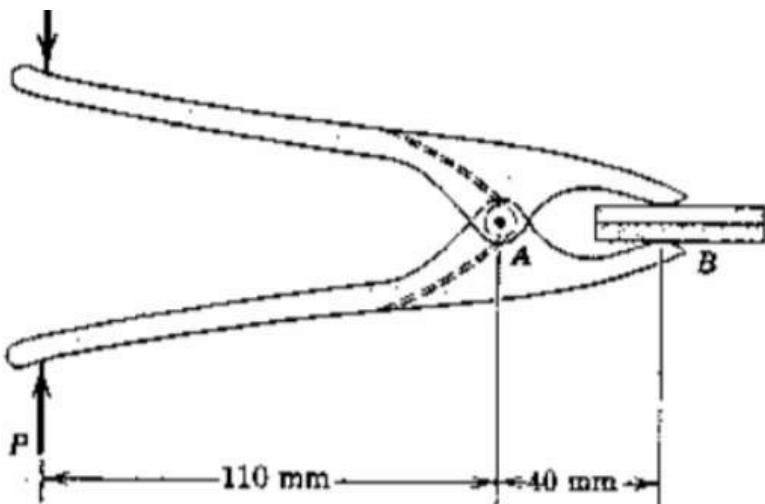
$$ii) \sum F_y = 0$$

$$\therefore Ay + P = By$$

$$\begin{aligned} \text{or } By &= 900 + 150 \\ &= 1050 \text{ N} \end{aligned}$$

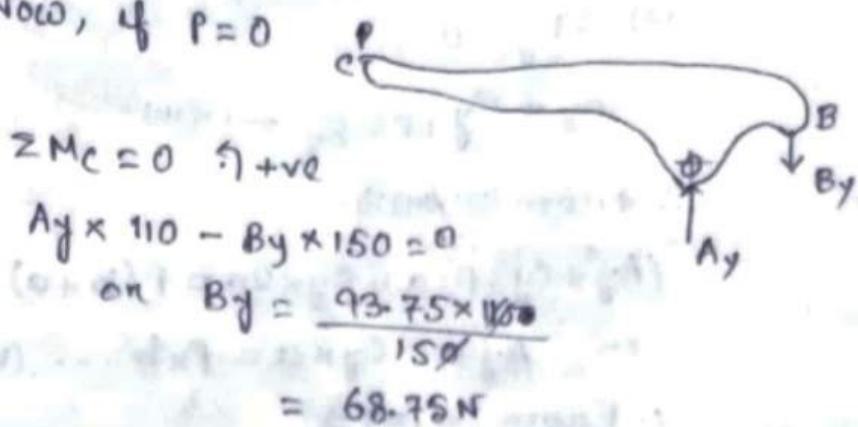
6. In the spring clamp shown, an internal spring is coiled around the pin A and the spring ends bear against the inner surfaces of the handle halves in order to provide the desired clamping force. In the position shown, a force of magnitude $P = 25$ N is required to release the clamp. Determine the compressive force at B if $P = 0$.

Ans. B = 68.8 N



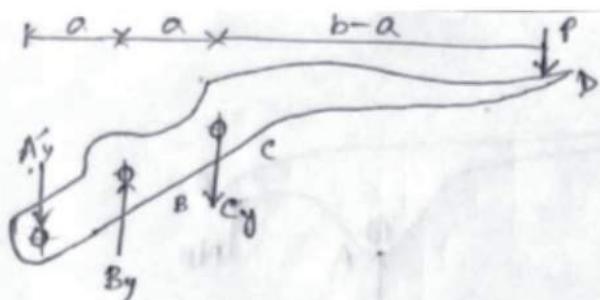
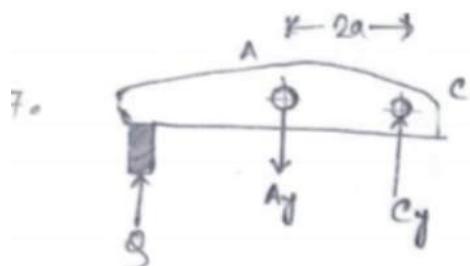
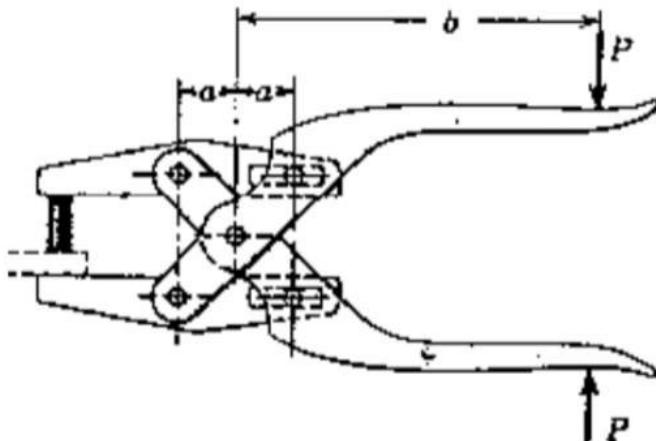
$$\sum M_B = 0 \quad \text{clockwise} \\ 25 \times 150 - Ay \times 40 = 0 \\ \text{on } Ay = \frac{25 \times 150}{40} \\ = 93.75 \text{ N}$$

Now, if $P = 0$



7. For the paper punch shown find the punching force Q corresponding to a hand grip P.

$$\text{Ans. } Q = P(b/a)$$



$$i) \sum M_A = 0 \quad \text{q + ve}$$

$$By \times a - Cy \times 2a - P \times (b+a) = 0 \quad \dots (1)$$

$$ii) \sum M_c = 0 \quad \text{q + ve}$$

$$Ay \times 2a - By \times a - P(b-a) = 0 \quad \dots (2)$$

$$iii) \sum F_y = 0$$

$$\therefore Ay' + Cy + P = By \quad \dots (3)$$

\therefore from (1), (2) \Rightarrow

$$(Ay' + Cy + P)a - Cy \times 2a = P(b+a)$$

$$\text{on } Ay'a - Cy \times a = Pb \quad \dots (4)$$

\therefore from (3), (4) \Rightarrow

$$Ay' \times 2a - a(Ay' + Cy + P) = Pb(b-a)$$

$$\text{on } Ay'a - Cy \times a = Pb.$$

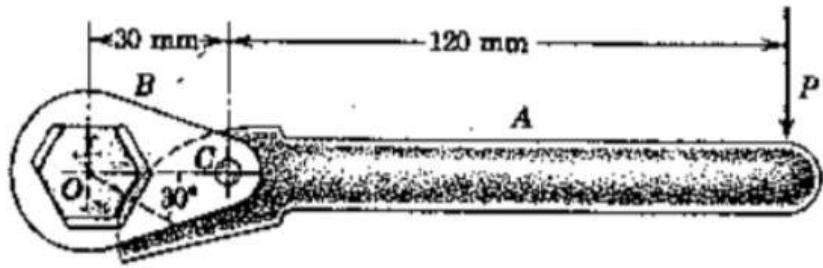
$$v) \sum M_B = 0 \quad \text{q + ve}$$

$$Ay'(b+a) - By(b) + Cy \times (b-a) = 0$$

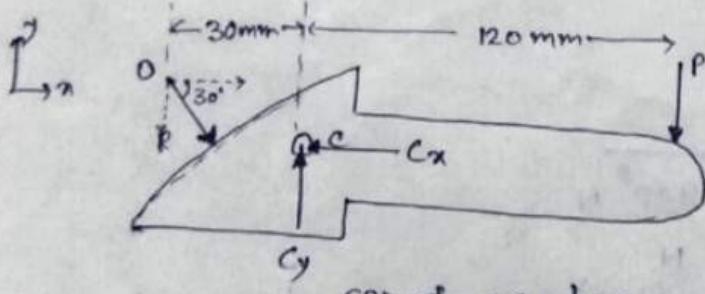
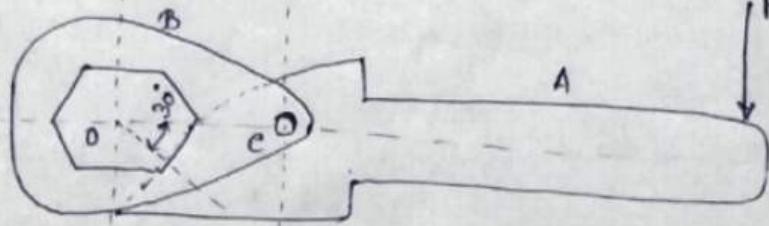
$$\text{on } Ay'(b+a) - b(-$$

8. The special box wrench with head B swiveled at C to the handle A will accommodate a range of sizes of hexagonal bolt heads. For the nominal size shown where the center O of the bolt and the pin C are in line with the handle, compute the magnitude of the force supported by the pin at C if $P = 160 \text{ N}$. Assume the surface of the bolt head to be smooth.

$$\text{Ans. } C = 1367 \text{ N}$$



8.



O, C are in
Same line.

$$i) \sum M_O; \leftarrow +ve$$

$$C_y \times 30 - P(30+120) = 0$$

$$\text{on } C_y = \frac{P \times 50}{30}$$

$$\text{on } C_y = \frac{160 \times 50}{30} = 800 \text{ N}$$

$$ii) \sum M_C; \leftarrow +ve$$

$$o) R \sin 30^\circ \times 30 - P \times 120 = 0$$

$$\text{on } R = \frac{160 \times 120 \times 2}{30^\circ} \\ = 1280 \text{ N}$$

$$iii) \sum F_x = 0$$

$$\therefore R \cos 30^\circ - C_x = 0$$

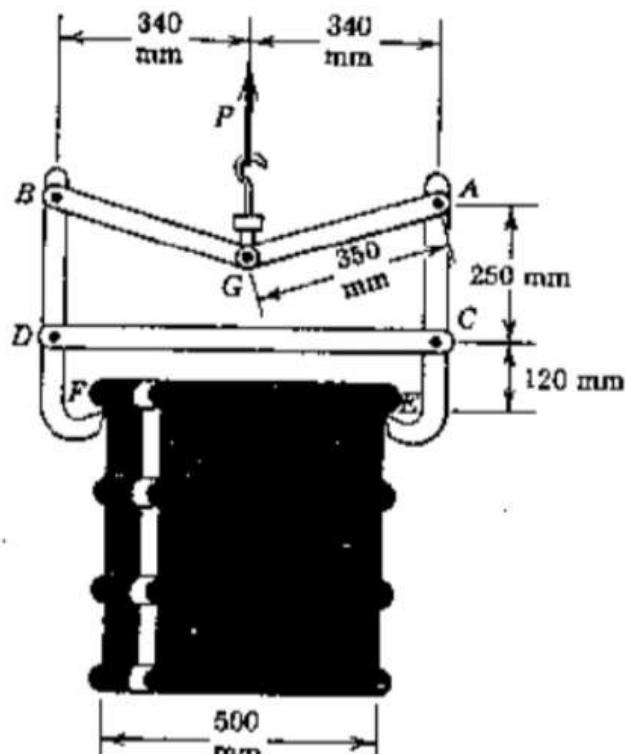
$$\text{on } C_x = R \cos 30^\circ$$

$$\therefore C_x = 1280 \cos 30^\circ \\ = 1109 \text{ N (Ans)}$$

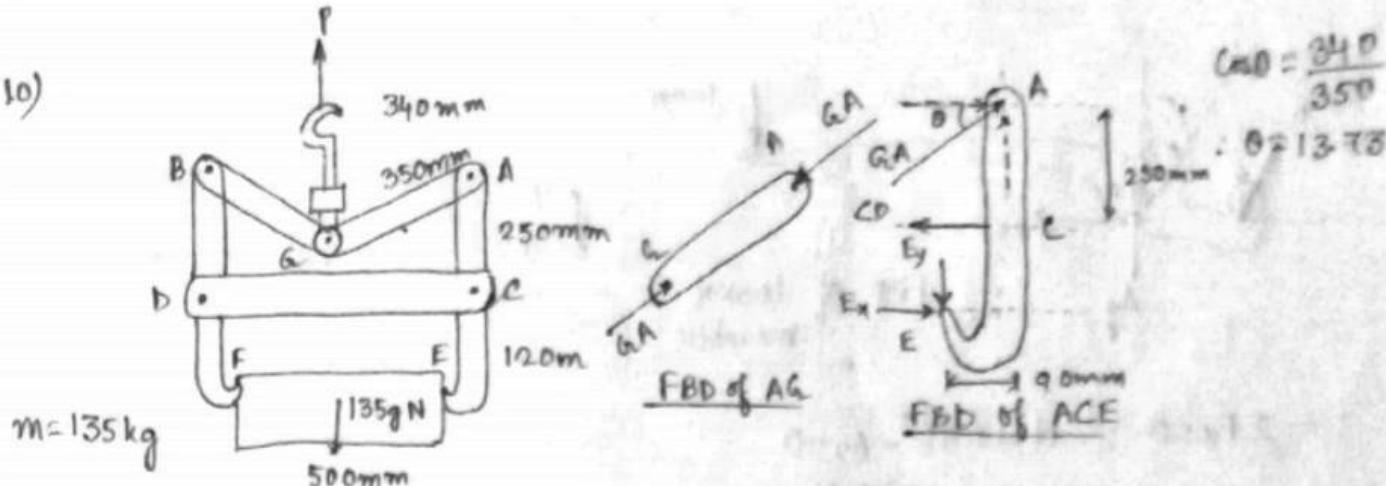
$$\therefore C = \sqrt{C_x^2 + C_y^2} \text{ N} \\ = 1367 \text{ N (Ans)}$$

10. A lifting device for transporting 135-kg steel drums is shown. Calculate the magnitude of the force exerted on the drum at E and F.

Ans. $E = F = 5.19 \text{ N}$



10)



$$\sum F_y = 0$$

$$GA \sin \theta - E_y = 0$$

$$\text{on } E_y = 2790.5 \text{ N} \\ = 662.21 \text{ N.}$$

$$\sum M_c = 0, \text{ J + v.}$$

$$GA \times 2 \\ \cos 30^\circ \times 250$$

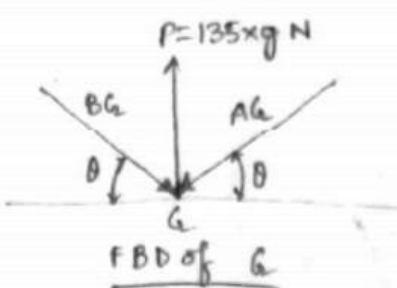
$$GA \cos 30^\circ \times 250 - E_y \times 90 \\ - E_x \times 120 = 0$$

$$\text{on } E_x = 5150.$$

$$E = F = \sqrt{E_x^2 + E_y^2}$$

$$= 5192 \text{ N}$$

$$= 5.19 \text{ kN (Ans.)}$$



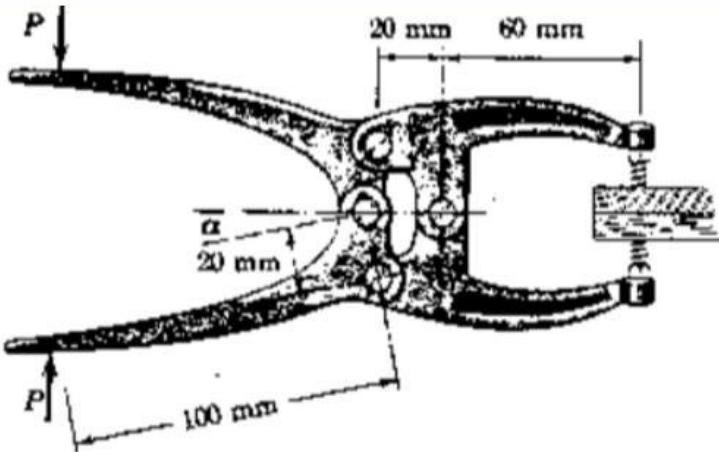
$$\sum F_y = 0 ; \sum F_x = 0$$

$$P - 2G \sin \theta = 0 \quad \text{or} \quad G_B \cos \theta - G_A \cos \theta = 0$$

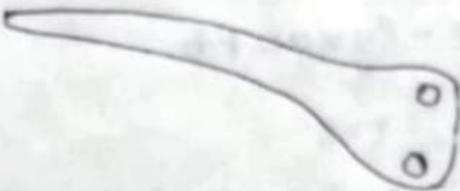
$$\text{on } G_A = 2790 \text{ N} \quad \therefore G_B = G_A$$

11. The toggle pliers are used for a variety of clamping purposes. For the handle position given by $\alpha = 10^\circ$ and for a handle grip of $P = 150 \text{ N}$, Calculate the clamping force C produced.

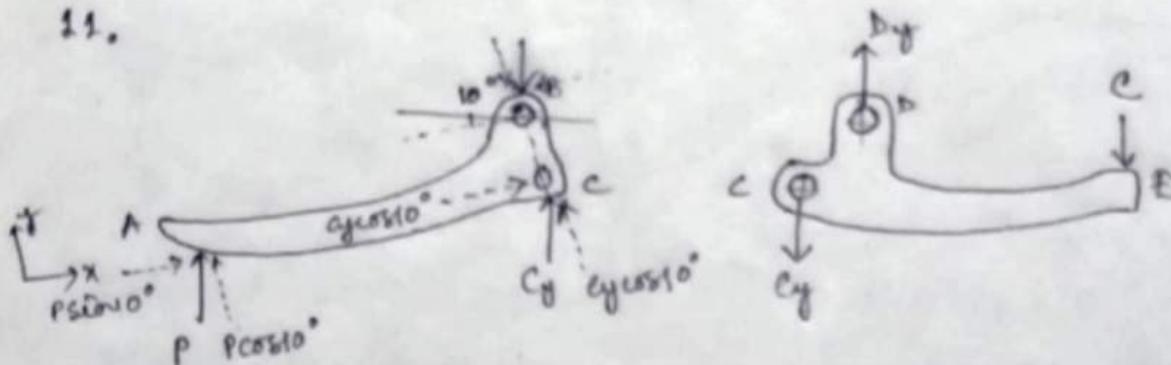
Ans. $C = 1368 \text{ N}$



11)



11.



$$\sum M_B = 0 \rightarrow +ve$$

$$\begin{aligned} -P \times \sin 10^\circ \times 80 + P \times \cos 10^\circ \times 100 + C_y \sin 10^\circ \times 20 &= 0 \\ \text{or } C_y &= 27.36 \times 150 \\ &= 4103.46 \text{ N.} \end{aligned}$$

$$\sum M_B = 0 \rightarrow +ve$$

$$C_y \times 20 - C \times 60 = 0$$

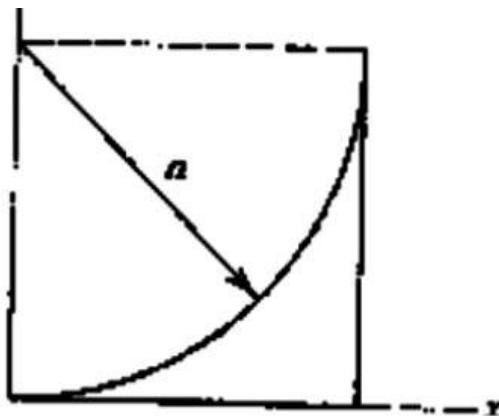
$$\begin{aligned} \text{or } C &= \frac{C_y}{3} \\ &= \frac{4103.46}{3} \text{ N} \\ &= 1367.82 \text{ N.} \end{aligned}$$

$$\approx 1368 \text{ N.}$$

PROBLEM SHEET 5.1

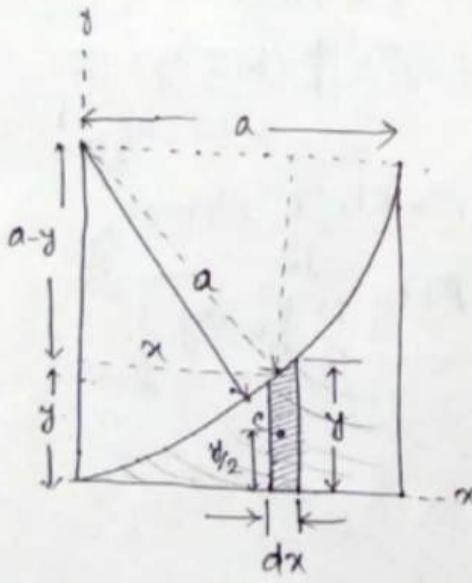
[CENTERS OF MASS]

3. Locate the centroid of the area shown in the figure by direct integration. (Caution: observe carefully the proper sign of the radical involved).



$$\text{Ans. } x = \frac{2a}{3(4-\pi)}, y = \frac{10-3\pi}{3(4-\pi)}a$$

3.



Find \bar{x} , \bar{y} .

$$(a-y)^2 + x^2 = a^2$$

$$\text{or } a-y = \sqrt{a^2-x^2}$$

$$\text{or } y = a - \sqrt{a^2-x^2}$$

$$\text{Now, } dA = y dx$$

$$\bar{x} = \frac{\int x_c dA}{\int dA}$$

$$= \frac{\int x \cdot y dx}{\int y dx} = \int_0^a \frac{x(a - \sqrt{a^2-x^2}) dx}{\sqrt{a^2-x^2}}$$

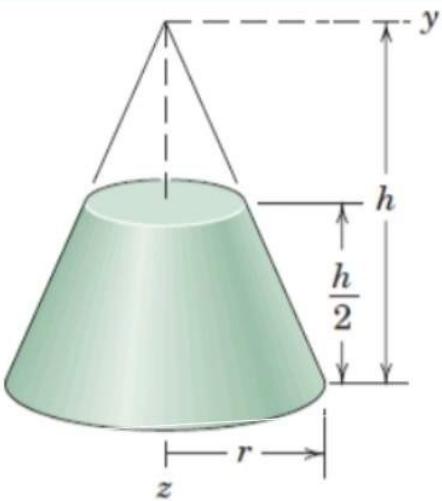
$$\bar{y} = \frac{\int y_c dA}{\int dA}$$

$$= \int_0^a \frac{\frac{1}{2} \times y dx}{\int y dx} = \frac{\frac{1}{2} \int_0^a (a - \sqrt{a^2-x^2})^2 dx}{\int_0^a (a - \sqrt{a^2-x^2}) dx} = \frac{2a}{3(4-\pi)}$$

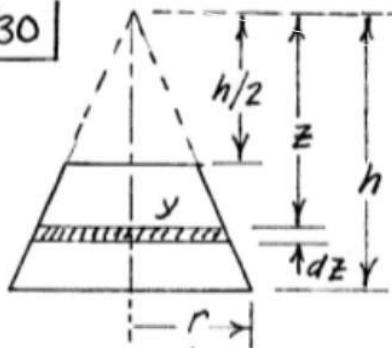
$$= \frac{(10-3\pi)a}{3(4-\pi)} \quad [\text{Ans}]$$

4. Calculate the distance \bar{h} measured from the base to the centroid of the volume of the frustum of the right-circular cone.

$$\text{Ans. } \bar{h} = \frac{11}{56} h$$



5/30 $dV = \pi y^2 dz$ where $y = \frac{r}{h} z$



$$= \pi \frac{r^2}{h^2} z^2 dz$$

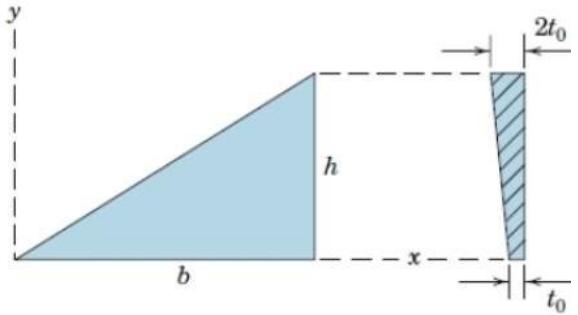
$$V = \pi \frac{r^2}{h^2} \int_{h/2}^h z^2 dz = \frac{7\pi r^2 h}{24}$$

$$\int z_c dV = \int z \pi \frac{r^2}{h^2} z^2 dz = \frac{15}{64} \pi r^2 h^2$$

$$\bar{z} = \int z_c dV / V = \frac{15}{64} \pi r^2 h^2 / \frac{7}{24} \pi r^2 h = \frac{45}{56} h$$

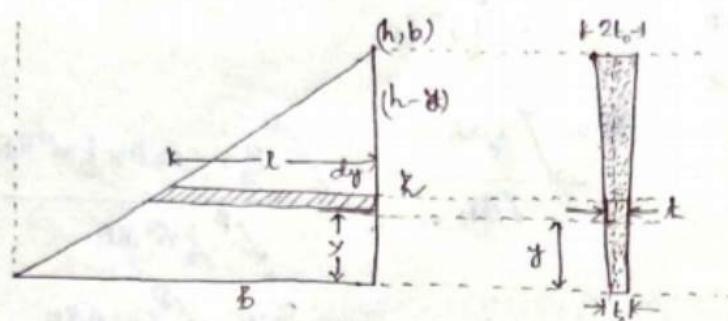
$$\bar{h} = h - \bar{z} = \underline{\underline{\frac{11}{56} h}}$$

5. The thickness of the triangular plate varies linearly with y from a value t_0 at its base $y = 0$ to $2t_0$ at $y = h$. Determine the y -coordinate of the center of the mass of the plate.

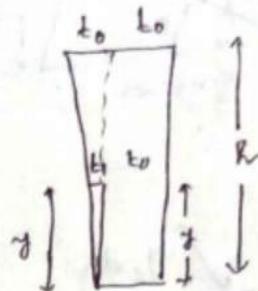


$$\text{Ans. } y = \frac{3h}{8}$$

5.



y



$$\begin{aligned}
 \bar{y} &= \frac{\int y dm}{\int dm} \\
 &= \frac{\int_0^h y \rho b t_0 (1 - \frac{y^2}{h^2}) dy}{\int_0^h \rho b t_0 (1 - \frac{y^2}{h^2}) dy} \\
 &= \frac{\int_0^h (y - \frac{y^3}{h^2}) dy}{\int_0^h (1 - \frac{y^2}{h^2}) dy} \\
 &= \frac{\left[\frac{y^2}{2} - \frac{y^4}{4h^2} \right]_0^h}{\left[h - \frac{y^3}{3h^2} \right]_0^h} \\
 &= \frac{\frac{h^2}{2} - \frac{h^4}{4}}{h - \frac{h^3}{3}} \\
 &= \frac{\frac{h^2}{4}}{\frac{2h}{3}} \\
 &= \frac{3h}{8}.
 \end{aligned}$$

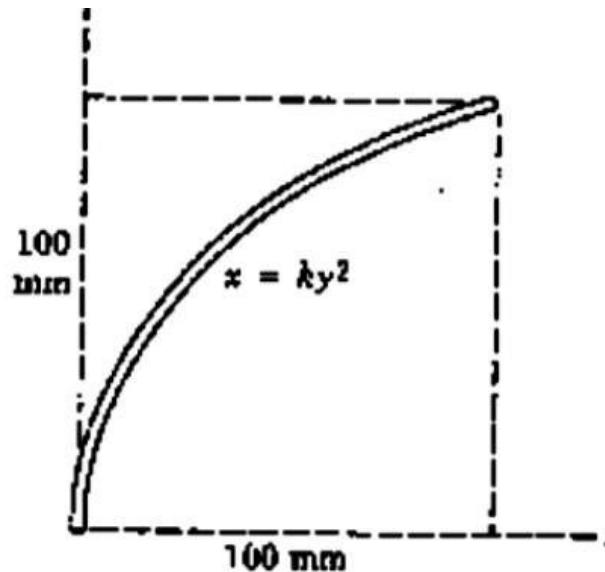
$$\begin{aligned}
 \frac{t_1}{y} &= \frac{t_0}{h} \\
 \text{or } t_1 &= t_0 \cdot \frac{y}{h} \\
 \text{Now } t &= t_0 + t_1 \\
 &= t_0 \left(1 + \frac{y}{h} \right)
 \end{aligned}$$

$$\begin{aligned}
 \frac{h-y}{L} &= \frac{h-y}{h} \\
 \text{or } L &= \frac{h}{h-y} (h-y)
 \end{aligned}$$

$$\begin{aligned}
 dA &= \frac{b}{h} (h-y) dy \\
 \therefore dm &= \rho \times dA \times t \\
 &= \rho \times \frac{b}{h} (h-y) dy \\
 &\quad \times t_0 \left(1 + \frac{y}{h} \right) \\
 &= \rho \times b t_0 (h-y)(h+y) \frac{h^2}{h^2} dy \\
 \boxed{dm} &= \rho b t_0 \left(1 - \frac{4y^2}{h^2} \right) dy
 \end{aligned}$$

6. The homogeneous slender rod has a uniform cross section and is bent into the shape shown. Calculate the y -coordinate of the mass center of the rod. (reminder: A differential arc length is

$$dL = \sqrt{(dx)^2 + (dy)^2} = \sqrt{1 + \left(\frac{dx}{dy}\right)^2} dy$$



Ans. $y = 57.4$ mm

5/33

$x = ky^2 = \frac{y^2}{100}$, $\frac{dx}{dy} = \frac{y}{50}$

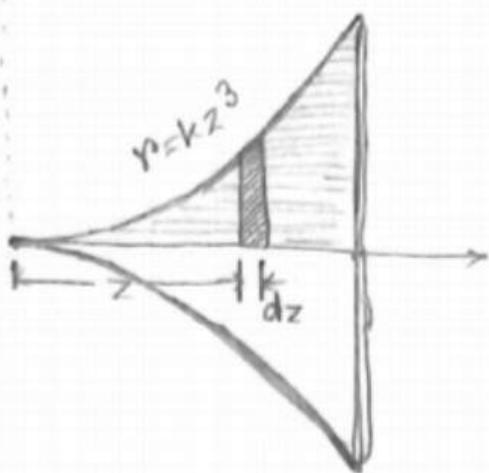
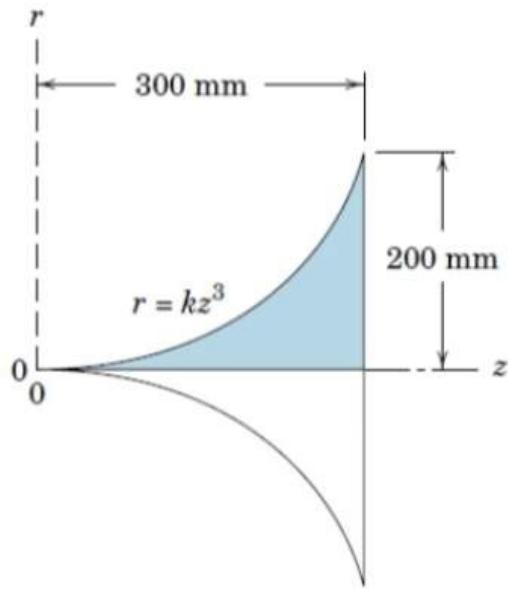
$$\begin{aligned} L &= \int dL = \int_0^{100} \sqrt{1 + \left(\frac{dx}{dy}\right)^2} dy = \int_0^{100} \sqrt{1 + \frac{y^2}{50^2}} dy \\ &= \frac{1}{50} \int_0^{100} \sqrt{50^2 + y^2} dy = \frac{1}{50 \cdot 2} \left[y \sqrt{50^2 + y^2} + 50^2 \ln(y + \sqrt{50^2 + y^2}) \right]_0^{100} \\ &= 147.9 \text{ mm} \end{aligned}$$

$$\begin{aligned} \int y_c dL &= \frac{1}{50} \int_0^{100} y \sqrt{50^2 + y^2} dy = \frac{1}{50} \frac{1}{3} (50^2 + y^2)^{3/2} \Big|_0^{100} \\ &= 8480 \text{ mm}^2 \end{aligned}$$

$$\bar{y} = \frac{\int y_c dL}{L} = \frac{8480}{147.9} = \underline{57.4 \text{ mm}}$$

7. Locate the mass center of the homogeneous solid body whose volume is determined by revolving the shaded area through 360° about the z-axis.

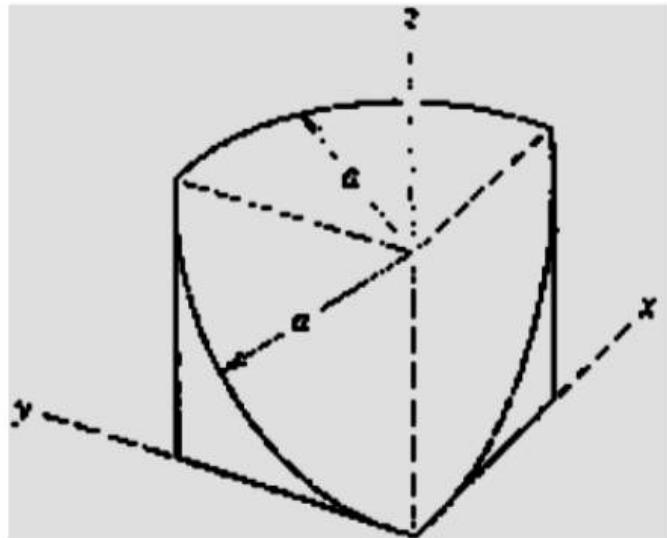
$$\text{Ans. } \bar{z} = 263 \text{ mm}$$



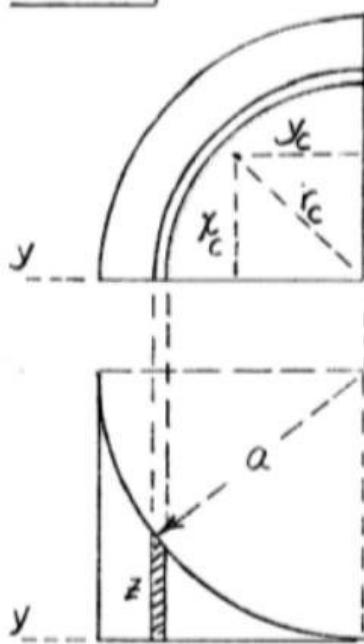
$$\begin{aligned} dm &= \rho \pi r^2 dz \\ &= \rho \pi k^2 z^6 dz \end{aligned}$$

$$\begin{aligned} \bar{z} &= \frac{\int z dm}{\int dm} \\ \bar{z} &= \frac{\int_0^{300} z \times \rho \pi k^2 z^6 dz}{\int_0^{300} \rho \pi k^2 z^6 dz} \\ &= \frac{\left[z^8/8 \right]_0^{300}}{\left[\frac{z^7}{7} \right]_0^{300}} \\ &= \left(\frac{7}{8} \times 300 \right) \text{ mm.} \\ &= 262.5 \text{ mm. (Ans)} \end{aligned}$$

8. Determine the coordinates of the centroid of volume obtained by revolving the shaded area about the z-axis through the 90° angle.



► 5/38



^x From Sample Problem 5/1, for elemental shell, $x_c = y_c = r_c/\sqrt{2} = \frac{2y}{\pi}$

$$dV = \frac{\pi y}{2} (z dy) = \frac{\pi}{2} (ay - y\sqrt{a^2 - y^2}) dy$$

$$V = \frac{\pi}{2} \int_0^a (ay - y\sqrt{a^2 - y^2}) dy$$

$$= \frac{\pi}{2} \left[\frac{ay^2}{2} + \frac{1}{3}\sqrt{(a^2 - y^2)^3} \right]_0^a = \frac{\pi a^3}{2 \cdot 6} = \frac{\pi a^3}{12}$$

$$\int y_c dV = \frac{2}{\pi} \int_0^a \frac{\pi}{2} (ay^2 - y^2\sqrt{a^2 - y^2}) dy$$

$$= \left[\frac{ay^3}{3} + \frac{y}{4}\sqrt{(a^2 - y^2)^3} - \frac{a^2}{8}(y\sqrt{a^2 - y^2} + a^2 \sin^{-1} \frac{y}{a}) \right]_0^a$$

$$= \left[\frac{a^4}{3} + 0 - \frac{a^2}{8}(0 + a^2 \frac{\pi}{2}) \right] = a^4 \left[\frac{1}{3} - \frac{\pi}{16} \right]$$

$$\bar{y} = \int y_c dV / V = a^4 \left(\frac{1}{3} - \frac{\pi}{16} \right) / \frac{\pi a^3}{12} = \left(\frac{4}{\pi} - \frac{3}{4} \right) a = \bar{x}$$

$$y^2 + (z-a)^2 = a^2$$

$$z = a - \sqrt{a^2 - y^2}$$

(Note sign)

$$\int z_c dV = \int \frac{z}{2} dV = \int_0^a \frac{a - \sqrt{a^2 - y^2}}{2} \frac{\pi}{2} (ay - y\sqrt{a^2 - y^2}) dy$$

$$= \frac{\pi a^4}{48}; \quad \bar{z} = \int z_c dV / V = \frac{\pi a^4}{48} / \frac{\pi a^3}{12} = \frac{a}{4}$$

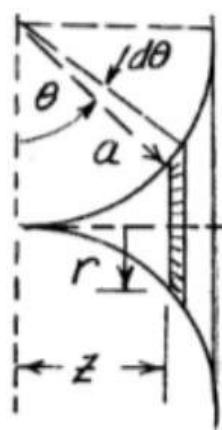
9. Locate the center of mass of the homogeneous bell-shaped shell of uniform but negligible thickness. Also determine the position of the centroid of the volume within the bell-shaped shell.

$$\text{Ans. } z_{\text{mass}} = \frac{a}{\pi - 2}, z_{\text{vol}} = \frac{a}{2(10 - 3\pi)}$$



[40] $dA = 2\pi r ad\theta = 2\pi a^2(1-\cos\theta) d\theta$

$$\begin{aligned} \int z dA &= \int_0^{\pi/2} (a \sin\theta) (2\pi a^2)(1-\cos\theta) d\theta \\ &= 2\pi a^3 \int_0^{\pi/2} (\sin\theta - \sin\theta \cos\theta) d\theta \\ &= 2\pi a^3 \left(1 - \frac{1}{2}\right) = \pi a^3 \end{aligned}$$

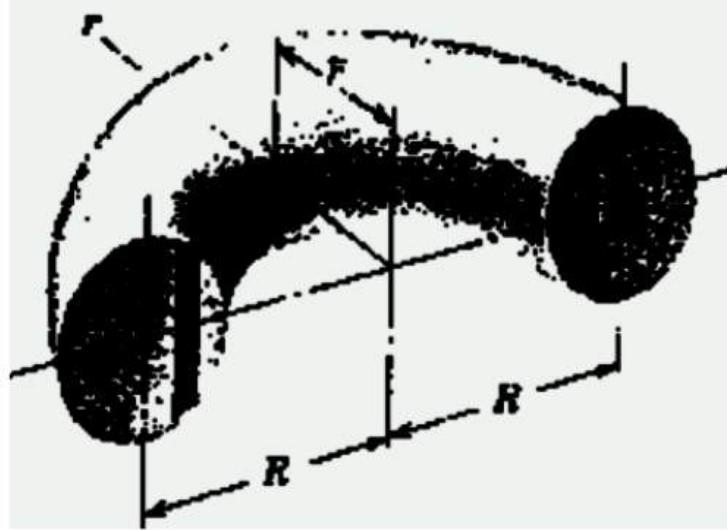


$$\int dA = 2\pi a^2 \int_0^{\pi/2} (1-\cos\theta) d\theta = 2\pi a^2 \left(\frac{\pi}{2} - 1\right)$$

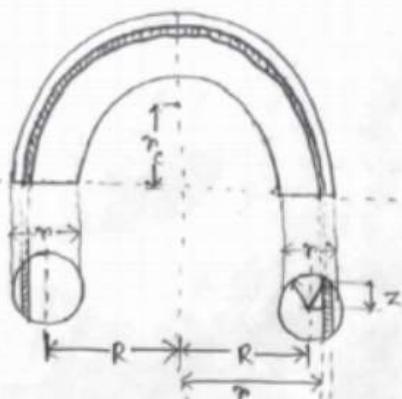
$$\bar{z} = \frac{\int z dA}{A} = \frac{\pi a^3}{2\pi a^2 \left(\frac{\pi}{2} - 1\right)} = \frac{a}{\pi - 2}$$

10. Locate the center of mass G of the steel half ring. (Hint: Choose an element of volume in the form of a cylindrical shell whose intersection with the plane of the ends is shown).

$$\text{Ans. } r = \frac{a^2 + 4R^2}{2\pi R}$$



10.



$$n_c = \frac{2\pi}{\pi}$$

$$dA = 2z dm$$

$$dV = \pi n_c 2z dm$$

$$= 2\pi n_c z dm$$

$$z^2 = a^2 - (r-R)^2$$

$$\therefore z = \sqrt{a^2 - (r-R)^2}$$

$$\therefore dV = 2\pi n_c \sqrt{a^2 - (r-R)^2} dr$$

$$\text{Now, } \bar{r} = \frac{\int r_c dm}{\int dm} = \frac{\int r_c dv}{\int dv}$$

$$\therefore \int r_c dv = \int \frac{2\pi}{\pi} \times 2\pi n_c \sqrt{a^2 - (r-R)^2} dr$$

$$= \int 4n_c^2 \sqrt{a^2 - (r-R)^2} dr$$

$$= 4 \int_{-a}^a (u+R)^2 \sqrt{a^2 - u^2} du$$

$$u = 2R - R \\ du = dr$$

$$= I_1 + I_2 + I_3$$

$$\therefore I_1 = 4 \int_{-a}^a u^2 \sqrt{a^2 - u^2} du$$

$$= \frac{\pi a^4}{2}$$

$$I_2 = 4 \int_{-a}^a 2Ru \sqrt{a^2 - u^2} du = 0.$$

$$I_3 = 4 \int_{-a}^a R^2 \sqrt{a^2 - u^2} du = 2\pi a^2 R^2.$$

$$\bar{r} = \frac{\int r_c dv}{\int dv} = \frac{\frac{\pi a^4}{2} + 2\pi a^2 R^2}{\frac{4\pi a^2 R^2}{2}} = \frac{a^2 + 4R^2}{2\pi R} \quad (\text{Ans})$$

$$\int dv = 2\pi \int_{-a}^a (u+R) \sqrt{a^2 - u^2} du$$

$$= 2\pi \int_{-a}^a u \sqrt{a^2 - u^2} du + 2\pi R \int_{-a}^a \sqrt{a^2 - u^2} du$$

$$= 0 + 2\pi R \times \frac{1}{2} \pi a^2$$

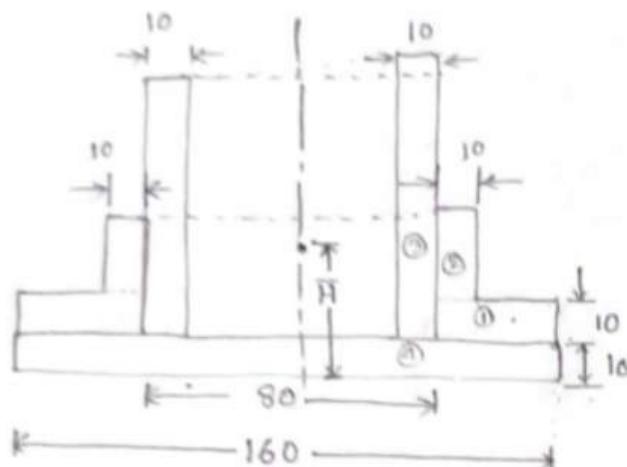
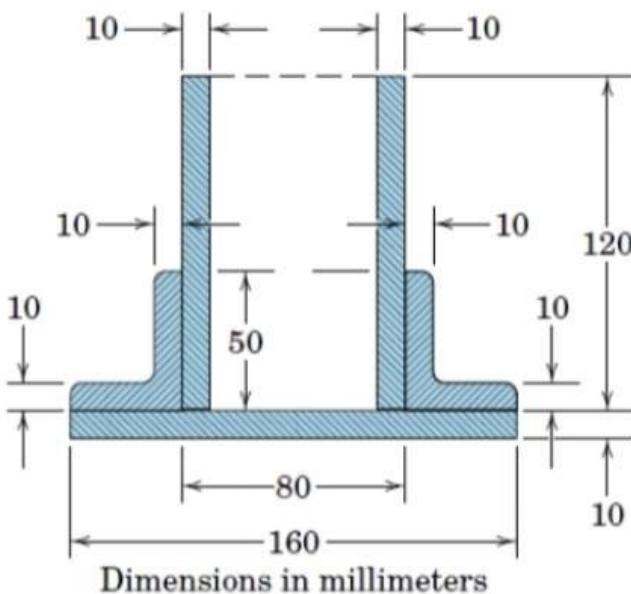
$$= \pi^2 R a^2.$$

PROBLEM SHEET

5.2

1. Determine the distance \bar{H} from the bottom of the base plate to the centroid of the built-up structural section shown.

Ans. $\bar{H} = 39.3 \text{ mm}$



Dimensions are
in mm

Part	Area(mm^2)	$\bar{z}_i(\text{m})$	$A\bar{z}_i(\text{mm}^3)$
1	400×2	15	12000
2	400×2	40	32000
3	$120 \times 10 \times 2$	70	168000
4	160×10	5	8000

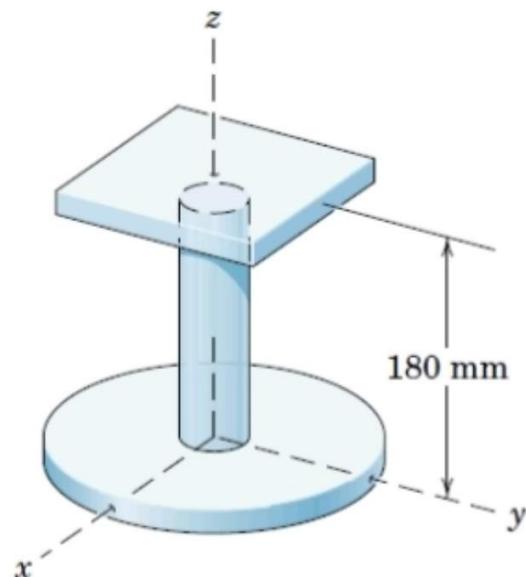
$$\sum A\bar{z}_i = 220000$$

$$\sum \bar{z}_i A = 5600$$

$$\therefore \bar{H} = \frac{\sum A\bar{z}_i}{\sum A} = 39.29 \text{ mm (Ans)}$$

2. The rigidly connected unit consists of a 2-kg circular disk, a 1.5-kg round shaft, and a 1-kg square plate. Determine the z-coordinate of the mass center of the unit.

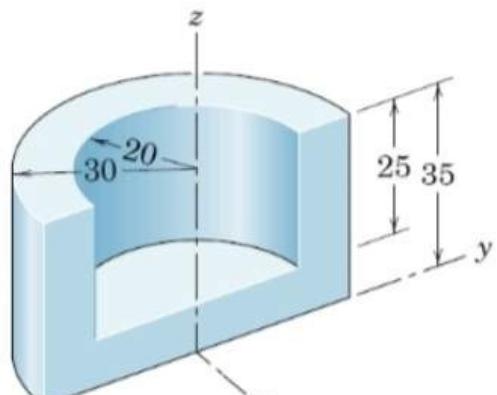
$$\text{Ans. } \bar{z} = 70 \text{ mm}$$



$$\boxed{\frac{5/53}{\bar{z}}} = \frac{\sum m \bar{z}}{\sum m} = \frac{2(0) + 1.5(90) + 1(180)}{2 + 1.5 + 1} = \underline{70 \text{ mm}}$$

4. Calculate the coordinates of the mass center of the metal die casting shown.

Ans. $\bar{X} = -14.71 \text{ mm}$, $\bar{Z} = 15.17 \text{ mm}$



Dimensions in millimeters

5/64



Length = 35 mm



Length = 25 mm

$$\left\{ \begin{array}{l} V_1 = \frac{\pi}{2} (30)^2 (35) = 49500 \text{ mm}^3 \\ \bar{x}_1 = -\frac{4(30)}{3\pi} = -12.73 \text{ mm} \\ \bar{z}_1 = 17.5 \text{ mm} \end{array} \right.$$

$$V_2 = -\frac{\pi}{2} (20)^2 (25) = -15710 \text{ mm}^3$$

$$\bar{x}_2 = -\frac{4(20)}{3\pi} = -8.49 \text{ mm}$$

$$\bar{z}_2 = \frac{1}{2}(10+35) = 22.5 \text{ mm}$$

$$\bar{X} = \frac{\sum V \bar{x}}{\sum V} = \frac{49500(-12.73) - 15710(-8.49)}{49500 - 15710}$$

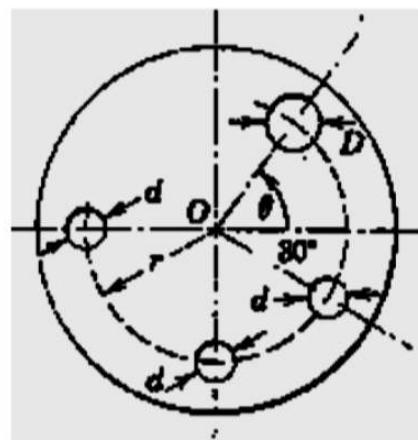
$$= \underline{-14.71 \text{ mm}}$$

$$\bar{Z} = \frac{\sum V \bar{z}}{\sum V} = \frac{49500(17.5) - 15710(22.5)}{49500 - 15710}$$

$$= \underline{15.17 \text{ mm}}$$

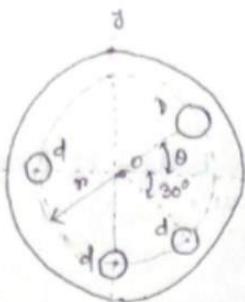
5. The circular disk rotates about an axis through its center O and has three holes of diameter d positioned as shown. A fourth hole is to be drilled in the disk at the same radius r so that the disk will be in balance (mass center at O). Determine the required diameter D of the new hole and its angular position.

$$\text{Ans. } D = 1.227d, \theta = 84.9^\circ$$



5.

Find out D, θ .



$$\text{No. } A_i \quad \bar{x}_i \quad \bar{y}_i$$

$$1. \pi R^2 \quad 0 \quad 0$$

$$2. \frac{\pi d^2}{4} \quad -r \quad 0$$

$$3. \frac{\pi d^2}{4} \quad 0 \quad -r$$

$$4. \frac{\pi d^2}{4} \quad r \cos 30^\circ \quad -r \sin 30^\circ$$

$$5. \frac{\pi D^2}{4} \quad r \cos \theta \quad r \sin \theta$$

$$\bar{x} = \frac{A_1 \bar{x}_1 + A_2 \bar{x}_2 + A_3 \bar{x}_3 + A_4 \bar{x}_4 + A_5 \bar{x}_5}{A_1 + (A_2 + A_3 + A_4 + A_5)} = 0$$

$$\therefore \pi R^2 \times 0 = \frac{\pi}{4} d^2 (-r) + 0 + \frac{\pi}{4} d^2 r \cos 30^\circ + \frac{\pi}{4} D^2 r \cos \theta$$

$$\text{or } D^2 \cos \theta = 0.13397 d^2 \dots \dots (i)$$

$$\bar{y} = 0$$

$$\therefore \sum_{i=2}^5 A_i \bar{y}_i = 0$$

$$\text{or } +\frac{\pi}{4} d^2 \times 0 + \frac{\pi}{4} d^2 (-r) + \frac{\pi}{4} d^2 (-r \sin 30^\circ) + \frac{\pi}{4} D^2 r \sin \theta = 0$$

$$\text{or } D^2 \sin \theta = 1.5 d^2 \dots \dots (ii)$$

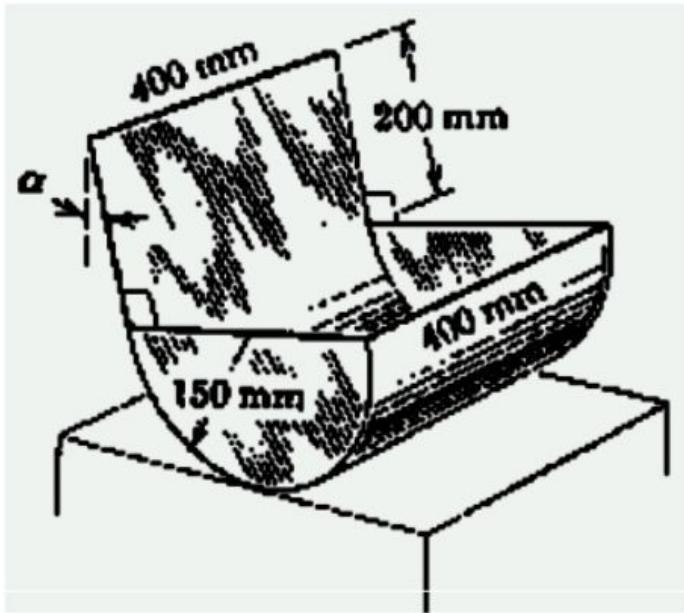
$$\therefore \tan \theta = \frac{1.5}{0.13397} \therefore \theta = 84.89^\circ$$

$$\therefore D^2 \cos 30^\circ = 0.13397 d^2$$

$$\text{or } D = 1.227d \text{ (Ans.)}$$

6. A cylindrical container with an extended rectangular back and semicircular ends is all fabricated from the same sheet-metal stock. Calculate the angle α made by the back with the vertical when the container rests in an equilibrium position on a horizontal surface.

$$\text{Ans. } \alpha = 39.6^\circ$$

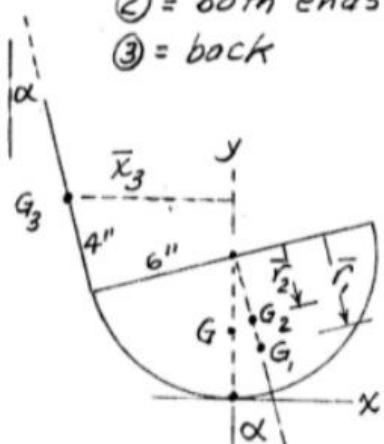


► 5/69

① = semicircular shell

② = both ends

③ = back



$$A_1 = \pi(6)^2/16 = 301.6 \text{ in}^2$$

$$A_2 = 2(\pi \cdot 6^2/2) = 113.1 \text{ in}^2$$

$$A_3 = 8(1/16) = 128 \text{ in}^2$$

$$\Sigma A = 542.7 \text{ in}^2$$

$$\bar{r}_1 = \frac{2(6)}{\pi} = 3.82 \text{ in}$$

$$\bar{r}_2 = \frac{4(6)}{3\pi} = 2.55 \text{ in.}$$

$$\bar{x}_1 = 3.82 \sin \alpha, \bar{x}_2 = 2.55 \sin \alpha$$

$$\bar{x}_3 = -6 \cos \alpha - 4 \sin \alpha$$

$$\bar{X} = \frac{\Sigma A \bar{x}}{\Sigma A}; 0 = 301.6(3.82 \sin \alpha) + 113.1(2.55 \sin \alpha) + 128(-6 \cos \alpha - 4 \sin \alpha) = 0$$

$$928.0 \sin \alpha = 768 \cos \alpha, \tan \alpha = \frac{768}{928} = 0.828$$

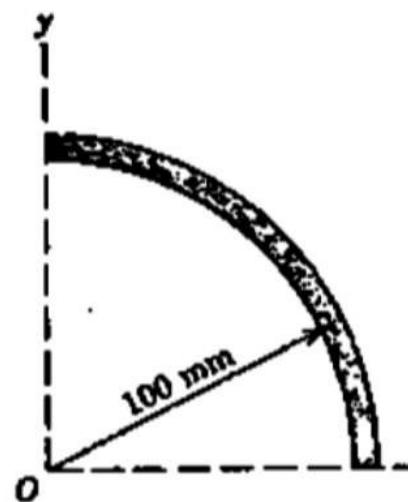
$$\underline{\alpha = 39.6^\circ}$$

PROBLEM SHEET

6.1

1. The thin quarter-circular ring has an area of 1600 mm^2 . Determine the moment of inertia of the ring about the x-axis to a close approximation.

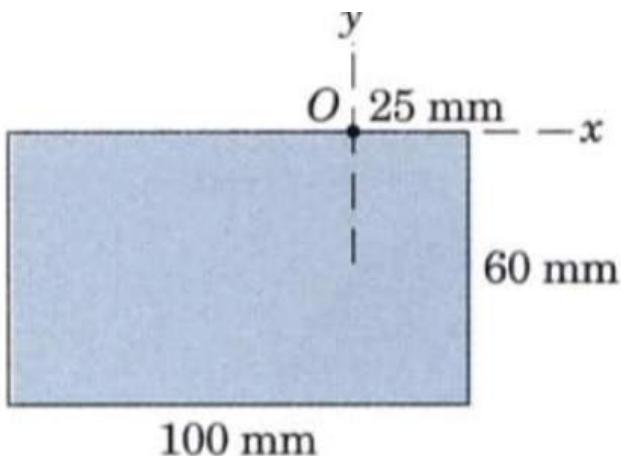
$$\text{Ans } I_x = 8(10^6) \text{ mm}^4$$



$$dA = \gamma x d\theta \times x, \text{ now, } A = 1600 \text{ mm}^2$$

$$\therefore \frac{\pi x 100 x x}{2} = 1600 \\ \Rightarrow x = \frac{32}{\pi} \text{ mm}$$

$$\begin{aligned} \text{now, } I_x &= \int_0^{\pi/2} dA y^2 \\ &= \int_0^{\pi/2} \gamma x^2 \sin^2 \theta x \gamma x x d\theta \\ &= \gamma x^3 x \int_0^{\pi/2} \frac{1 - \cos 2\theta}{2} d\theta \\ &= \frac{\gamma^3 x^4}{2} \left[\theta - \frac{\sin 2\theta}{2} \right]_0^{\pi/2} \\ &= \frac{\gamma^3 x^4}{2} \left[\frac{\pi}{2} \right] = 8 \times 10^6 \text{ mm}^4 \end{aligned}$$



3. Calculate the moment of inertia of the rectangular area about the x-axis and find the polar moment of inertia about point O.

$$\text{Ans. } I_x = 7.2(10^6) \text{ mm}^4, I_o = 15.95(10^6) \text{ mm}^4$$

A/9

Dimensions in mm

$$I_{y_0} = \frac{1}{12} b d^3 = \frac{1}{12} (60)(100)^3 = 5(10^6) \text{ mm}^4$$

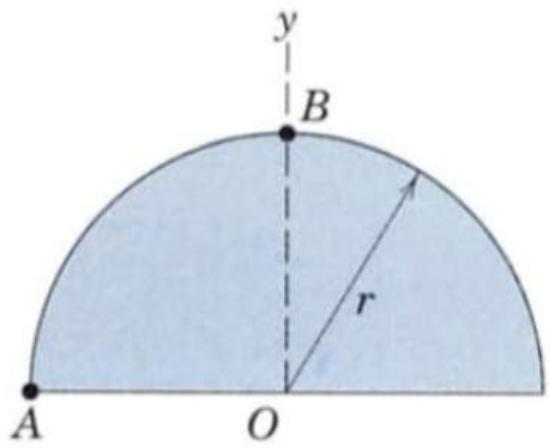
$$I_y = I_{y_0} + A d^2 = 5(10^6) + 6(10^3)(25)^2 = 8.75(10^6) \text{ mm}^4$$

$$I_x = \frac{1}{3} b d^3 = \frac{1}{3} (100)(60)^3 = 7.2(10^6) \text{ mm}^4$$

$$I_o = I_x + I_y = (7.2 + 8.75)10^6 = \underline{\underline{15.95(10^6) \text{ mm}^4}}$$

4. Determine the polar moments of inertia of the semicircular area about points A and B.

$$\text{Ans. } I_A = \frac{3}{4} \pi r^4, I_B = r^4 \left(\frac{3\pi}{4} - \frac{4}{3} \right),$$



A/5

For complete circle

$$I_A = I_0 + Ar^2 = \frac{1}{2}Ar^2 + Ar^2$$

$$= \frac{3}{2}Ar^2$$

For half circle

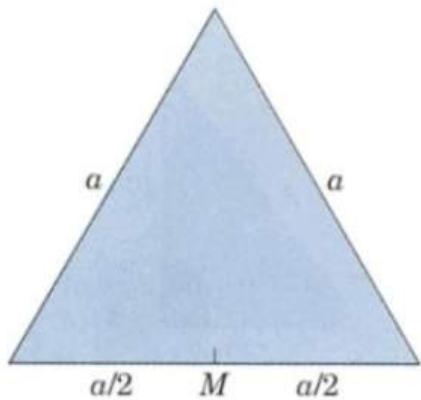
$$I_A = \frac{1}{2} \left(\frac{3}{2} \pi r^4 \right) = \underline{\underline{\frac{3}{4} \pi r^4}}$$

For half circle, $I_0 = \frac{1}{4} \pi r^4$

$$\begin{aligned}
 I_B &= I_C + A(r - \bar{r})^2 = I_0 - A\bar{r}^2 + A(r - \bar{r})^2 \\
 &= I_0 + A(r^2 - 2r\bar{r}) \\
 &= \frac{1}{4}\pi r^4 + \frac{\pi r^4}{2} \left(1 - \frac{8}{3\pi} \right) = \underline{\underline{r^4 \left(\frac{3\pi}{4} - \frac{4}{3} \right)}}
 \end{aligned}$$

5. Determine the polar radius of gyration of the area of the equilateral triangle about the midpoint M of its base.

$$\text{Ans. } k_M = \frac{a}{\sqrt{6}}$$



A/15

$$I_z = I_x + I_y, \quad I_z = A k_z^2$$

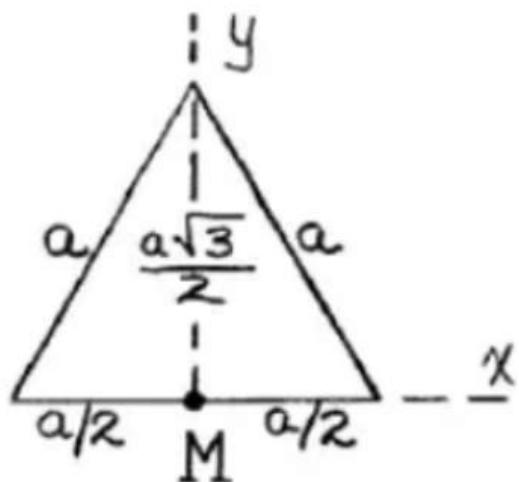
$$\therefore k_M = \sqrt{(I_x + I_y)/A}$$

$$I_x = \frac{1}{12} b h^3 = \frac{1}{12} a \left(\frac{a\sqrt{3}}{2}\right)^3$$

$$= \frac{\sqrt{3}}{32} a^4$$

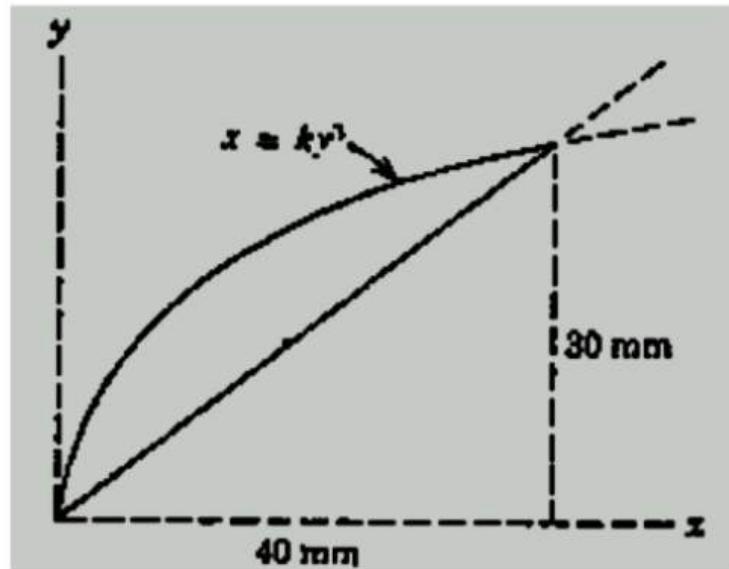
$$I_y = 2 \left(\frac{1}{12} \frac{a\sqrt{3}}{2} \left(\frac{a}{2}\right)^3 \right) = \frac{\sqrt{3}}{96} a^4$$

$$k_M = \left[\frac{\frac{\sqrt{3}}{32} a^4 + \frac{\sqrt{3}}{96} a^4}{\frac{a}{2} a \frac{\sqrt{3}}{2}} \right]^{1/2} = \frac{a}{\sqrt{6}}$$



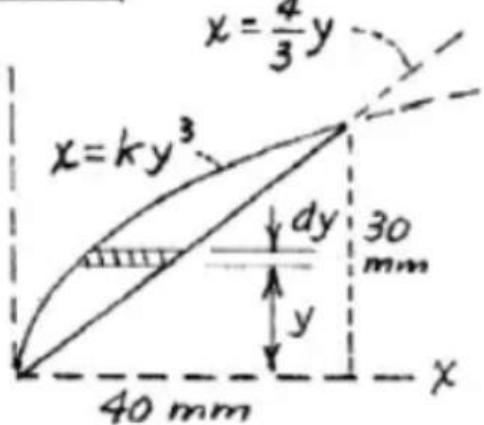
6. Calculate the moment of inertia of the shaded area about the x-axis.

$$\text{Ans. } I_x = 9(10^4) \text{ mm}^4$$



A/17

$$\text{For } x = 40 \text{ mm} \text{ & } y = 30 \text{ mm}, k = \frac{40}{27(10^3)}$$



$$dI_x = y^2 dA = y^2 \left(\frac{4}{3}y - \frac{4}{2700}y^3 \right) dy$$

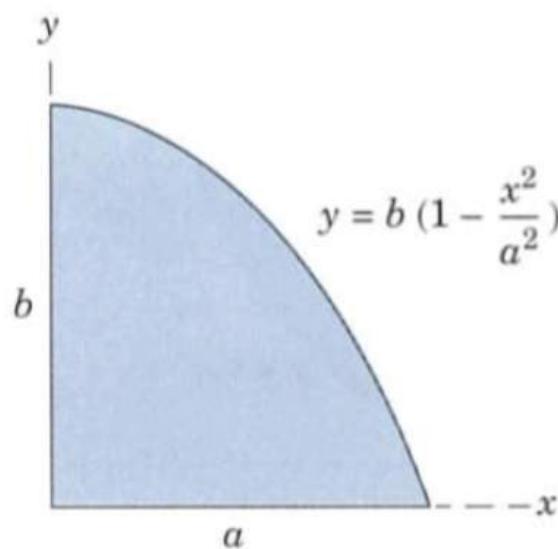
$$I_x = \int_0^{30} \left(\frac{4}{3}y^3 - \frac{4}{2700}y^5 \right) dy$$

$$= \left[\frac{y^4}{3} - \frac{y^6}{4050} \right]_0^{30} = \underline{\underline{9(10^4) \text{ mm}^4}}$$

7. Determine the moment of inertia of the shaded area about the x-axis using (a) a horizontal strip of area and (b) a vertical strip of area.

$$\text{Ans. } I_x =$$

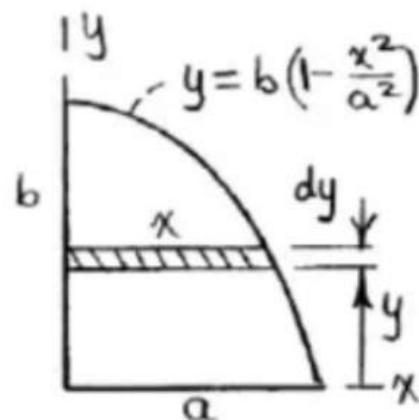
$$16ab^3/105$$



A/19

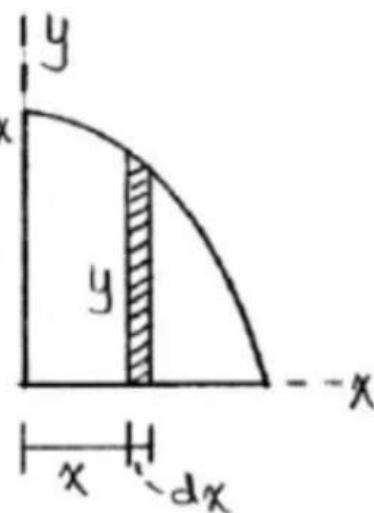
(a) Horizontal strip

$$\begin{aligned} I_x &= \int y^2 dA = \int_0^b y^2 x dy \\ &= \int_0^b y^2 a \sqrt{1 - y/b} dy \\ &= \frac{a}{\sqrt{b}} \int_0^b y^2 \sqrt{b-y} dy \\ &= \frac{a}{\sqrt{b}} \left. \frac{2}{105(-1)} (8b^2 + 12by + 15y^2) \sqrt{(b-y)^3} \right|_0^b = \frac{16ab^3}{105} \end{aligned}$$



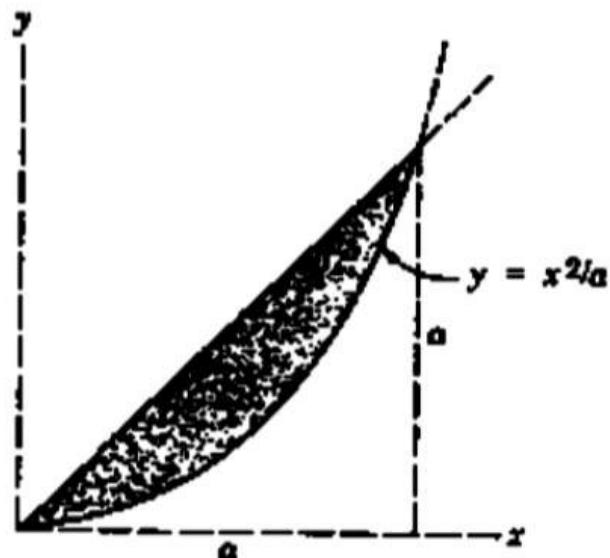
(b) Vertical strip

$$\begin{aligned} I_x &= \int_0^a \frac{1}{3} y^2 (y dx) = \frac{1}{3} \int_0^a b^3 (1 - \frac{x^2}{a^2})^3 dx \\ &= \frac{b^3}{3} \frac{1}{a^6} \int_0^a (a^6 - 3a^4 x^2 + 3a^2 x^4 - x^6) dx \\ &= \frac{b^3}{3a^6} \left[a^6 x - a^4 x^3 + \frac{3a^2 x^5}{5} - \frac{x^7}{7} \right]_0^a \\ &= \frac{16ab^3}{105} \end{aligned}$$

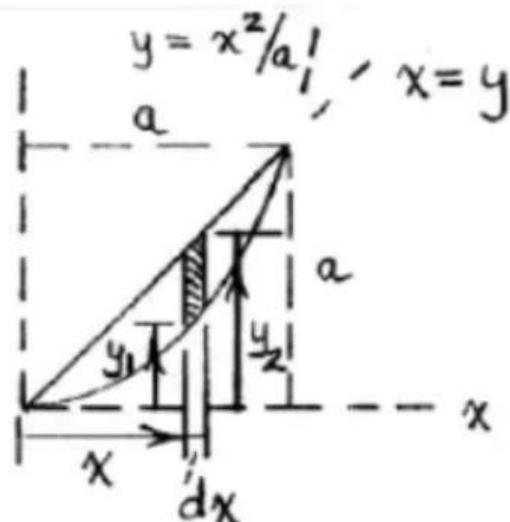


8. Determine the moment of inertia of the shaded area about the x - and y -axes. Use the same differential element for both calculations.

$$\text{Ans. } I_x = a^4/28, I_y = a^4/20$$



A/21

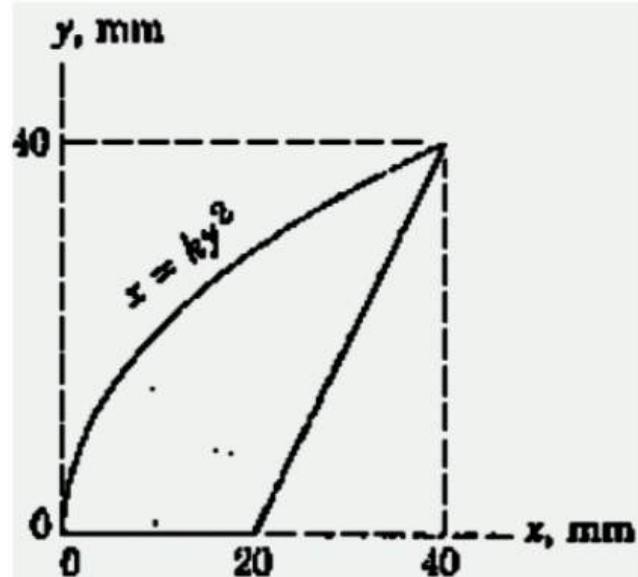


$$\begin{aligned} dA &= (y_2 - y_1)dx \\ &= \left(x - \frac{x^2}{a}\right)dx \end{aligned}$$

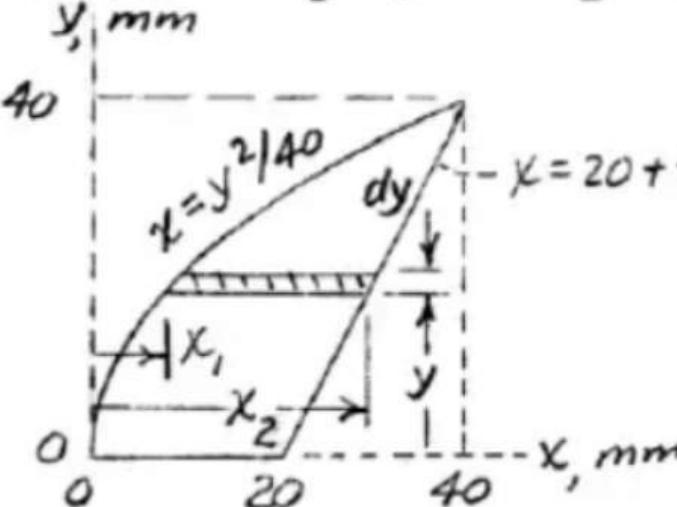
$$\begin{aligned} I_x &= \int \frac{1}{3} y_2^3 dx - \int \frac{1}{3} y_1^3 dx = \frac{1}{3} \int_0^a \left(x^3 - \frac{x^6}{a^3}\right) dx \\ &= \frac{1}{3} \left[\frac{x^4}{4} - \frac{x^7}{7a^3} \right]_0^a = \frac{a^4}{3} \left(\frac{1}{4} - \frac{1}{7}\right) = \underline{\underline{a^4/28}} \end{aligned}$$

$$\begin{aligned} I_y &= \int x^2 dA = \int x^2 \left(x - \frac{x^2}{a}\right) dx = \int_0^a \left(x^3 - \frac{x^4}{a}\right) dx \\ &= \left[\frac{x^4}{4} - \frac{x^5}{5a} \right]_0^a = \frac{a^4}{4} - \frac{a^4}{5} = \underline{\underline{a^4/20}} \end{aligned}$$

9. Determine the moment of inertia of the shaded area about the y -axis.
 Ans. $I_y = 27.8(10^4) \text{ mm}^4$



$$\boxed{A/2.7} \quad dI_y = \frac{1}{3} dy (x_2^3 - x_1^3) = \frac{1}{3} \left[(20 + \frac{y}{2})^3 - (\frac{y^2}{40})^3 \right] dy$$



$$= \frac{1}{2} (20 + \frac{y}{2})^4 \Big|_0^{40} = 120(10^4) \text{ mm}^4$$

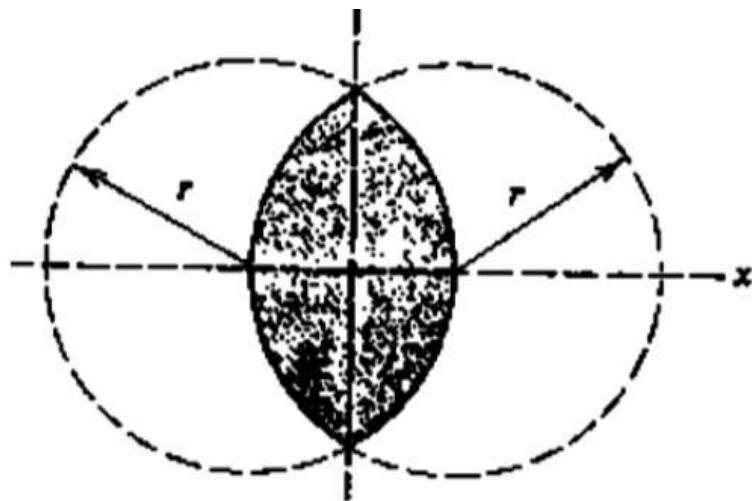
$$= \int_{0}^{40} \frac{y^6}{40^3} dy = \frac{1}{40^3} \frac{y^7}{7} \Big|_0^{40} = 36.57(10^4) \text{ mm}^4$$

$$I_y = \frac{1}{3} (120 - 36.57) 10^4 = \underline{\underline{27.8(10^4) \text{ mm}^4}}$$

10. Calculate the moment of inertia of the overlapping shaded area of the two circles about the x-axis.

Ans. $I_x =$

$0.1988r^4$

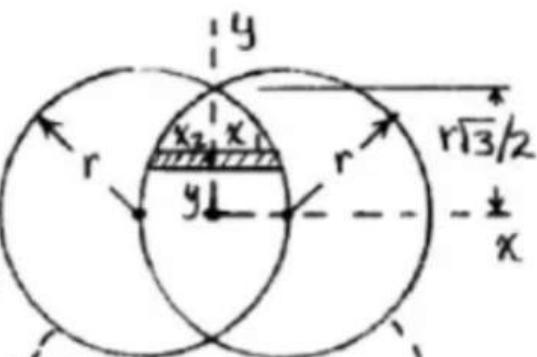


► A/31

$$x_2 = \frac{r}{2} \pm \sqrt{r^2 - y^2} \quad \text{use (-) sign}$$

$$x_1 = -\frac{r}{2} \pm \sqrt{r^2 - y^2} \quad \text{use (+) sign}$$

$$(x_1 + \frac{r}{2})^2 + y^2 = r^2 \quad (x_2 - \frac{r}{2})^2 + y^2 = r^2$$



$$(x_1 - x_2) = -\frac{r}{2} + \sqrt{r^2 - y^2} - \frac{r}{2} - \sqrt{r^2 - y^2} = 2\sqrt{r^2 - y^2} - r$$

$$dA = (2\sqrt{r^2 - y^2} - r) dy$$

$$I_x = \int y^2 dA = 2 \int_0^{r\sqrt{3}/2} y^2 (2\sqrt{r^2 - y^2} - r) dy$$

$$= 4 \left\{ -\frac{y}{4} \sqrt{(r^2 - y^2)^3} + \frac{r^2}{8} \left(y \sqrt{r^2 - y^2} + r^2 \sin^{-1} \frac{y}{r} \right) \right\} - \frac{2r^3}{3} y \Big|_0^{\frac{r\sqrt{3}}{2}}$$

$$= 4 \left\{ -\frac{r\sqrt{3}}{8} \frac{r^3}{8} + \frac{r^2}{8} \left(\frac{r\sqrt{3}}{2} \frac{r}{2} + r^2 \frac{\pi}{3} \right) \right\} - \frac{2\sqrt{3}}{8} r^4 - 0$$

$$= \frac{r^4}{2} \left\{ -\frac{\sqrt{3}}{8} + \frac{\sqrt{3}}{4} + \frac{\pi}{3} - \frac{\sqrt{3}}{2} \right\} = \frac{r^4}{2} \left\{ \frac{\pi}{3} - \frac{3\sqrt{3}}{8} \right\}$$

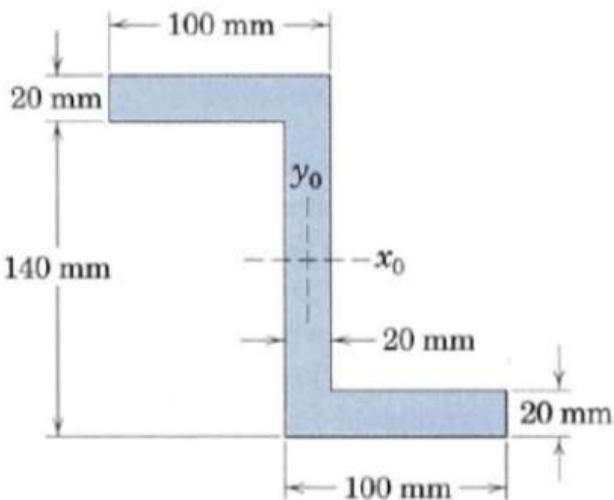
$$= 0.1988 r^4$$

PROBLEM SHEET

6.2

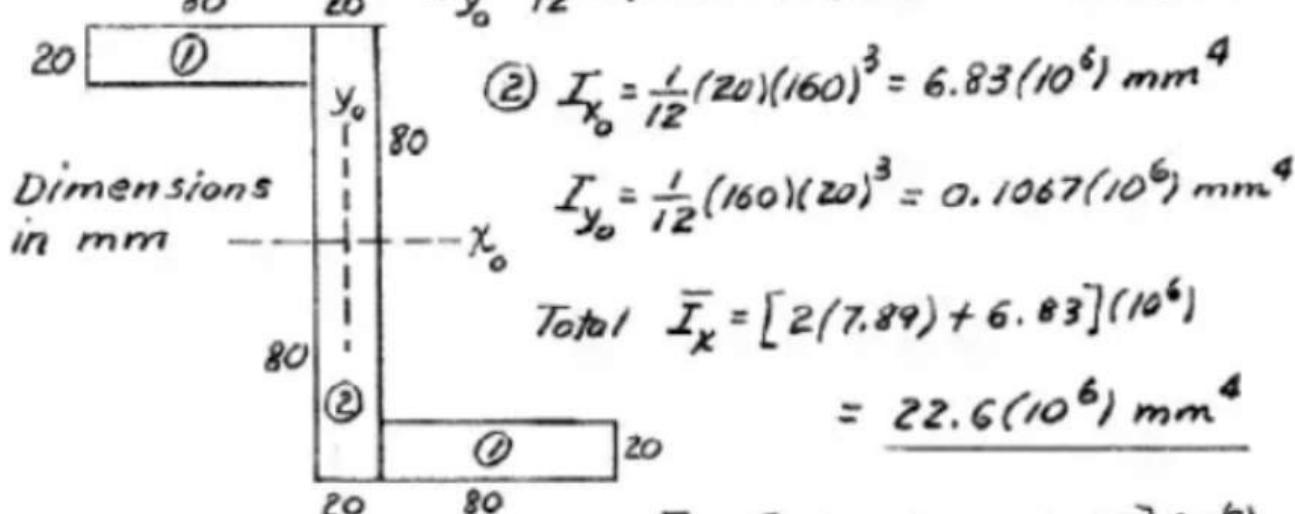
2. Determine the moments of inertia of the Z-section about its centroidal x_0 - and y_0 -axes.

Ans. $I_x = 22.6(10^6) \text{ mm}^4$,
 $I_y = 9.81(10^6) \text{ mm}^4$



A/42 | ① $I_{x_0} = \frac{1}{12}(80)(20)^3 + (80)(20)(70)^2 = 7.89(10^6) \text{ mm}^4$

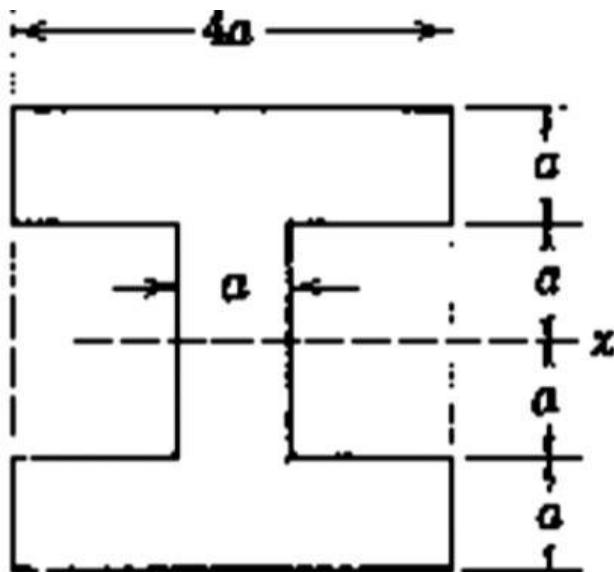
$I_{y_0} = \frac{1}{12}(20)(80)^3 + (20)(80)(50)^2 = 4.85(10^6) \text{ mm}^4$



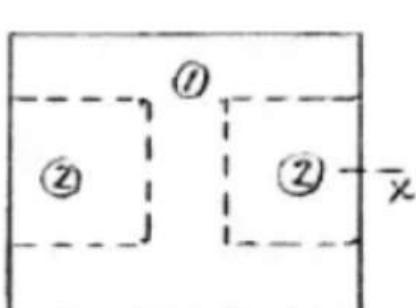
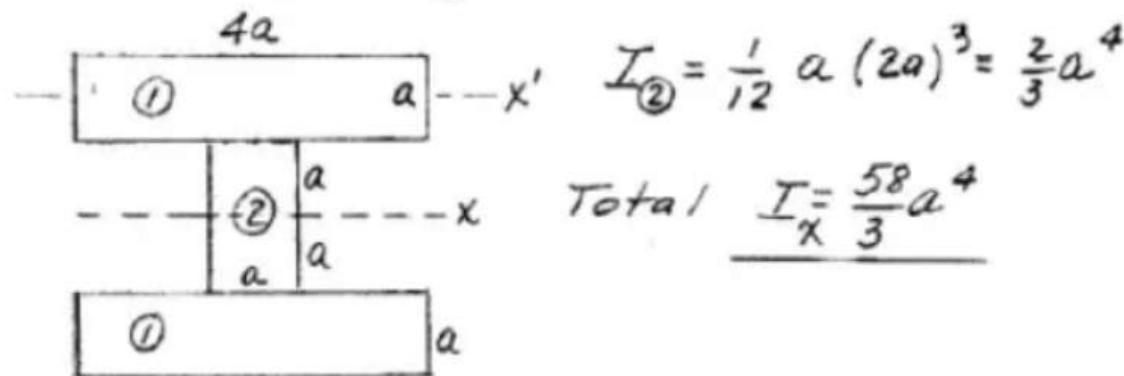
$\bar{I}_y = [2(4.85) + 0.1067](10^6)$
 $= 9.81(10^6) \text{ mm}^4$

3. Determine the moment of inertia of the shaded area about the x-axis.

$$\text{Ans. } I = \frac{58}{3}a^4$$



$$A/43 \quad \text{Sol. I} \quad I_{\text{Total}} = 2 \left[\frac{1}{12} 4a(a^3) + 4a^2 \left(\frac{3a}{2} \right)^2 \right] = \frac{56}{3}a^4$$



Sol. II

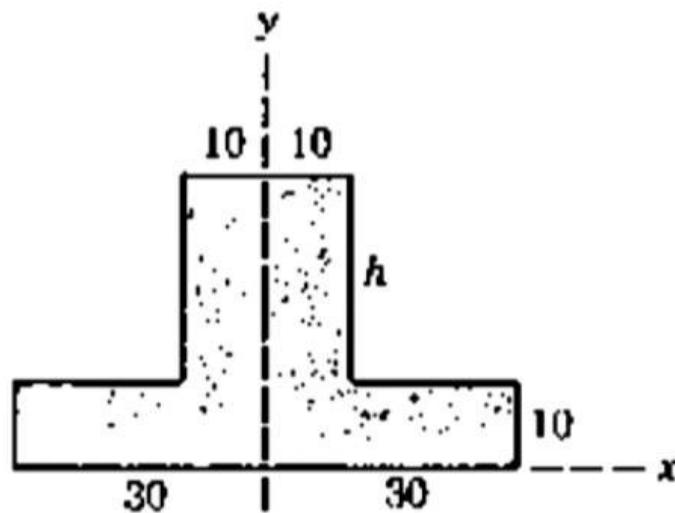
$$I_{\text{①}} = \frac{1}{12} (4a)(4a)^3 = \frac{64}{3}a^4$$

$$I_{\text{②}} = -\frac{1}{12} (3a)(2a)^3 = -2a^4$$

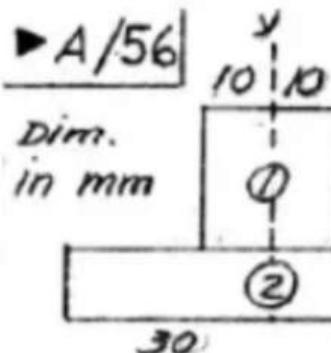
$$I_{\text{Total}} = \left(\frac{64}{3} - \frac{6}{3} \right) a^4 = \frac{58}{3}a^4$$

4. Calculate the value of h for which $I_x = I_y$ for the shaded area shown.

Ans. $h = 20.0 \text{ mm}$



Dimensions in Millimeters



$$\textcircled{1} \quad I_x = \frac{1}{12}(20)h^3 + 20h\left(\frac{h}{2} + 10\right)^2 \\ = 20\left(h^3/3 + 10h^2 + 100h\right) \text{ mm}^4$$

$$I_y = \frac{1}{12}h(20)^3 = \frac{2000h}{3} \text{ mm}^4$$

$$\textcircled{2} \quad I_x = \frac{1}{3}(60)(10)^3 = 20000 \text{ mm}^4$$

$$I_y = \frac{1}{12}(10)(60)^3 = 180000 \text{ mm}^4$$

Thus for equal I_x & I_y totals

$$20\left(h^3/3 + 10h^2 + 100h\right) + 20000 = \frac{2000h}{3} + 180000$$

$$\text{or } h^3 + 30h^2 + 200h = 24000$$

Substitute $h = u - 10$ & get $u^3 = 100u + 24000$ & solve by
Appen. B/4 - 4: Let $p = 100/3$, $q = 12000$ Case II $q^2 - p^3 = (+)$

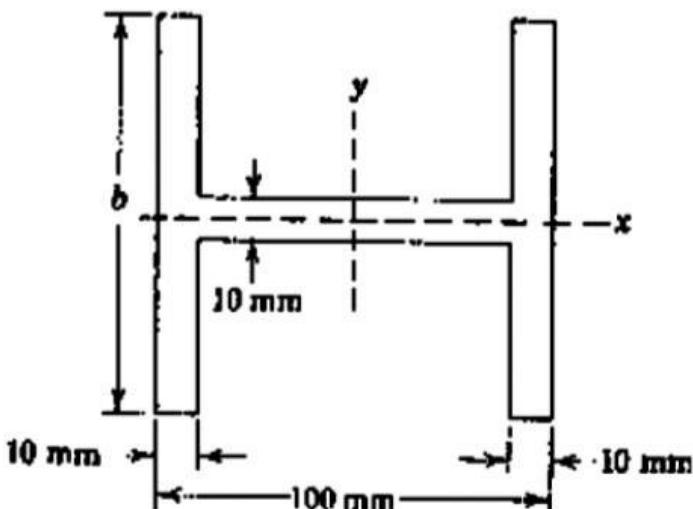
$$(q^2 - p^3)^{1/2} = \sqrt{144(10^6) - 10^6/27} = 11.99846(10^3)$$

$$u = (12000 + 11.998)^{1/3} + (12000 - 11.998.46)^{1/3} = 28.84 + 1.16 = 30.00$$

$$h = 30.00 - 10 = \underline{\underline{20.0 \text{ mm}}}$$

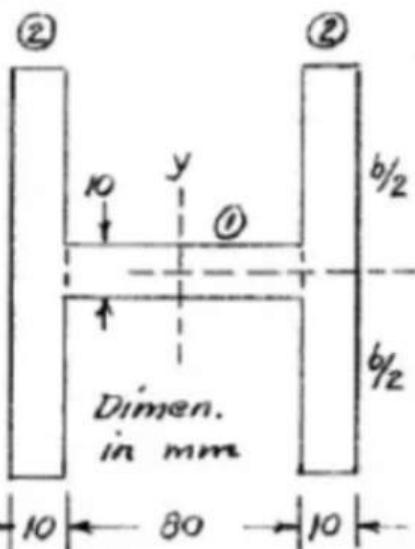
5. For the H-beam section determine the flange width b that will make the moments of inertia about the central x - and y -axes equal.

Ans. $b = 161 \text{ mm}$



A/55

$$\textcircled{1} \quad I_x = \frac{1}{12}(80)(10)^3 = 0.00667(10^6) \text{ mm}^4$$



$$\textcircled{2} \quad I_y = \frac{1}{12}(10)(80)^3 = 0.427(10^6) \text{ mm}^4$$

$$\textcircled{2} \quad I_x = 2 \left[\frac{1}{12}(10)b^3 \right] = 1.667b^3$$

$$I_y = 2 \left[\frac{1}{12}b(10)^3 \right] + (10b)(45)^2 \\ = 0.0407(10^6)b$$

$$\text{Total } I_x = \text{Total } I_y$$

$$(0.00667)(10^6) + 1.667b^3$$

$$= (0.427 + 0.0407)(10^6)$$

$$\text{or } b^3 - 0.0244(10^6)b - 0.252(10^6) = 0$$

Solve by cubic formula ; $\left[\frac{0.252(10^6)}{2} \right]^2 < \left[\frac{0.0244(10^6)}{3} \right]^3$ so 3 real roots

$$\cos u = \frac{q}{p\sqrt{p}} \text{ where } q = \frac{252(10^3)}{2} = 126(10^3), p = \frac{24.4(10^3)}{3} = 8.13(10^3)$$

$$\cos u = 126(10^3) / [8.13(10^3)90.2] = 0.1718, u = 80.11^\circ$$

$$b_1 = 2\sqrt{p} \cos \frac{u}{3} = 2(90.2)(0.8933) = 161.1 \text{ mm} \quad \text{or } b = 161.1 \text{ mm}$$

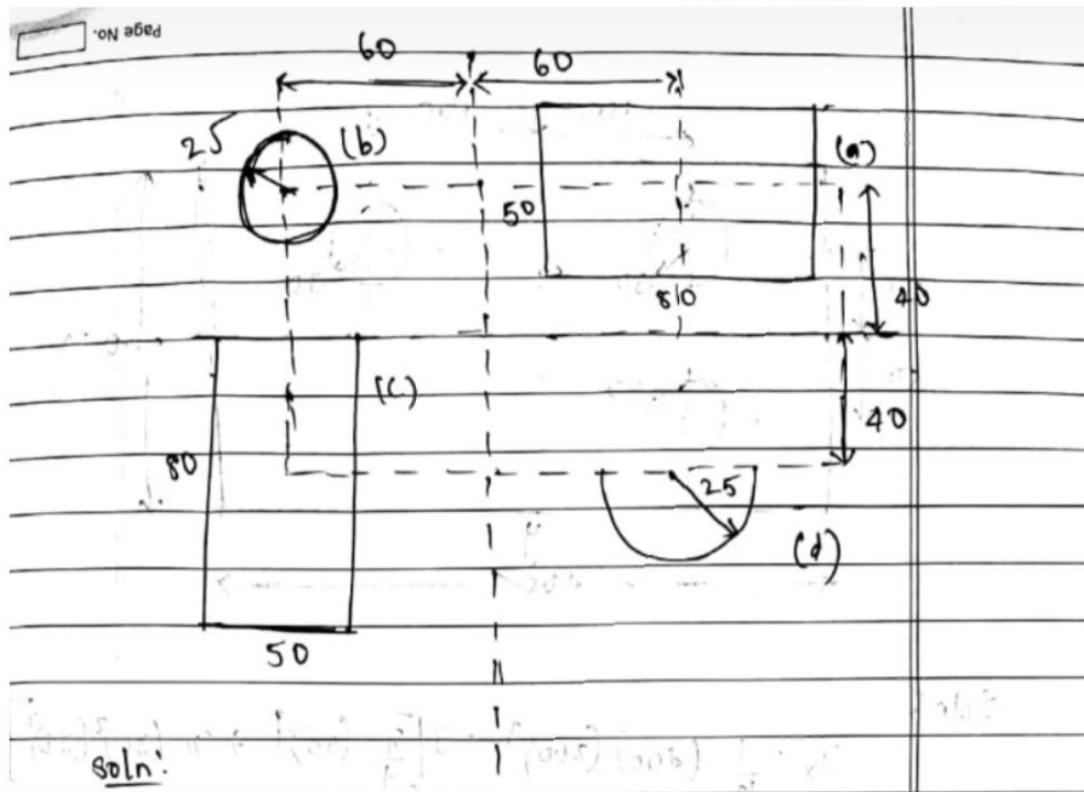
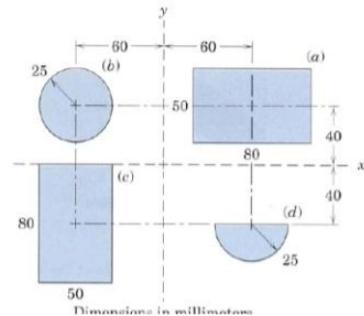
$$b_2 = 2\sqrt{p} \cos \left(\frac{u}{3} + 120^\circ \right) = -1, \quad b_3 = 2\sqrt{p} \cos \left(\frac{u}{3} + 240^\circ \right) = -1$$

PROBLEM SHEET

6.3

1. Determine the product of inertia of each of the four areas about the x-y axes.

Ans. (a) and (c) $I_{xy} = 9.60(10^6) \text{ mm}^4$,
 (b) $I_{xy} = -4.71(10^6) \text{ mm}^4$
 (c) $I_{xy} = -2.98(10^6) \text{ mm}^4$



Soln:

$$(a) \bar{I}_{xy} = \bar{I}_{xy} + d_x d_y A = 0 + (60)(40)(80)(50)$$

$$= 9.60(10^6) \text{ mm}^4$$

$$(b) \bar{I}_{xy} = \bar{I}_{xy} + d_x d_y A = 0 + (-60)(40)(\pi \cdot 25^2)$$

$$= -4.71(10^6) \text{ mm}^4$$

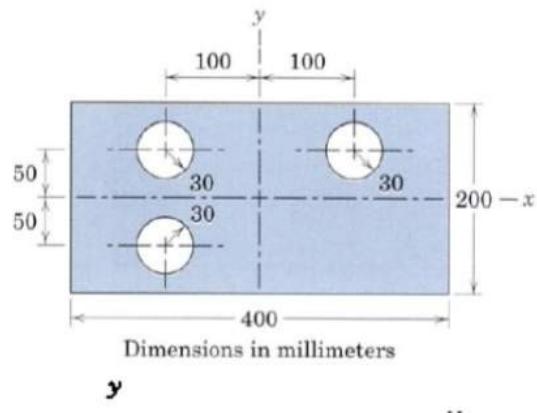
$$(c) \bar{I}_{xy} = \bar{I}_{xy} + d_x d_y A = 0 + (-60)(-40)(80)(50)$$

$$= 9.60(10^6) \text{ mm}^4$$

$$(d) \bar{I}_{xy} = \bar{I}_{xy} + d_x d_y A = 0 + (60)[-40 - \frac{4}{3}\pi(25)]$$

$$\times (\frac{\pi \cdot 25^2}{2})$$

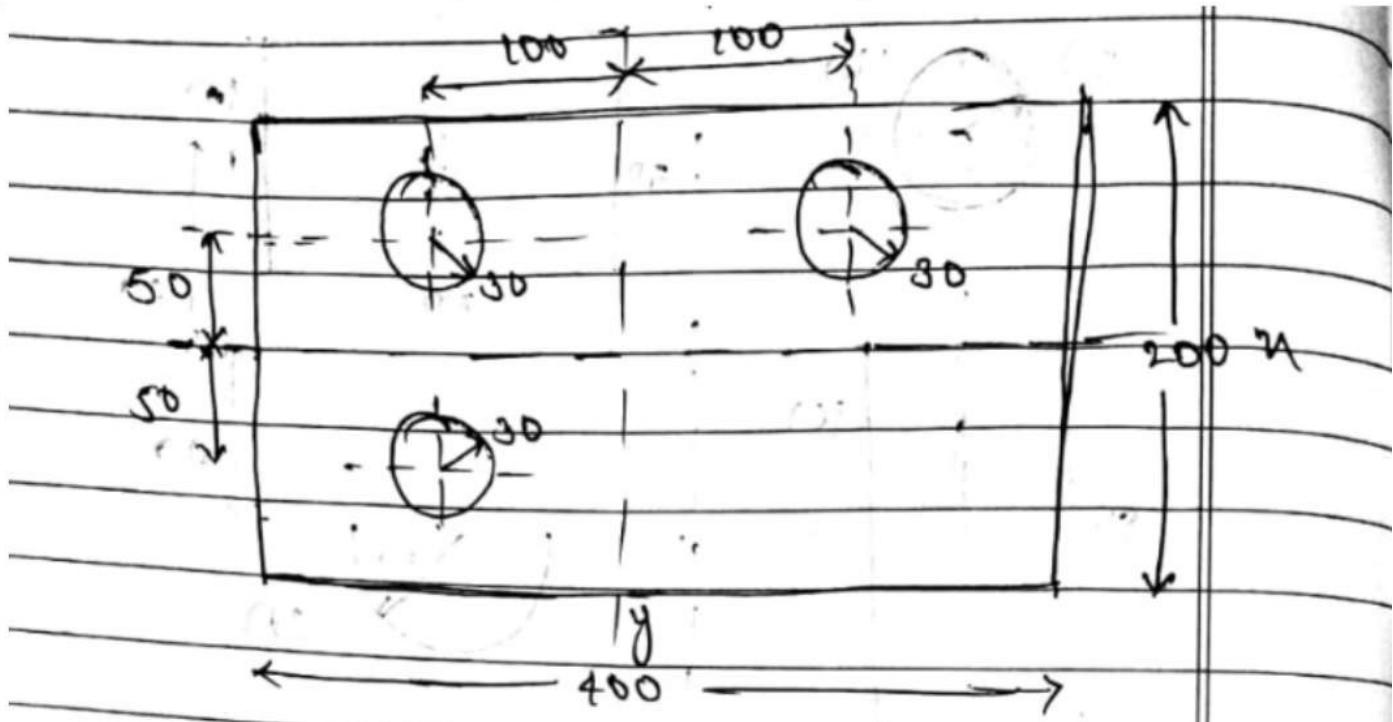
$$= -2.98(10^6) \text{ mm}^4$$



2. Determine I_x , I_y and I_{xy} for the rectangular plate with three equal circular holes.

$$\text{Ans. } I_x = 2.44(10^8) \text{ mm}^4, I_y = 9.80(10^8) \text{ mm}^4$$

$$I_{xy} = -14.14(10^6) \text{ mm}^4$$



Soln:

$$I_x = \frac{1}{12} (400) (200)^3 - 3 \left[\frac{\pi}{4} (30)^4 + \pi (30)^2 (50)^2 \right]$$

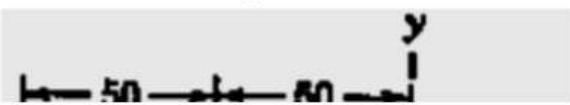
$$= 2.44(10^8) \text{ mm}^4$$

$$I_y = \frac{1}{12} (200) (400)^3 - 3 \left[\frac{\pi}{4} (30)^4 + \pi (30)^2 (100)^2 \right]$$

$$= 9.80(10^8) \text{ mm}^4$$

$$I_{xy} = -\pi (30)^2 [(100)(50) + (-100)(50) + (-100)(-50)]$$

$$= -14.14 (10^6) \text{ mm}^4$$



3. Derive the expression for the product of inertia of the right-triangular area about the x-y axes.

$$\text{Ans. } I_{xy} = \frac{b^2 h^2}{8}$$

First:

$$I_{xy} = \int_0^b \int_0^y xy \, dy \, dx$$

$$= \int_0^b \left[\frac{x}{2} y^2 \right]_0^{\frac{hx}{b}} dx = \int_0^b \frac{h^2}{2b^2} x^3 dx$$

$$= \frac{h^2}{2b^2} \frac{b^4}{4} = \frac{b^2 h^2}{8}$$

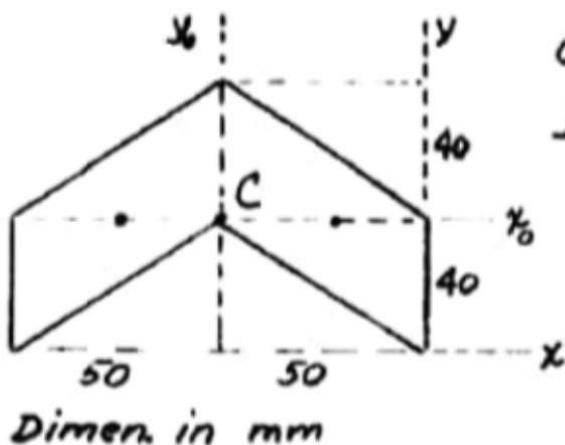
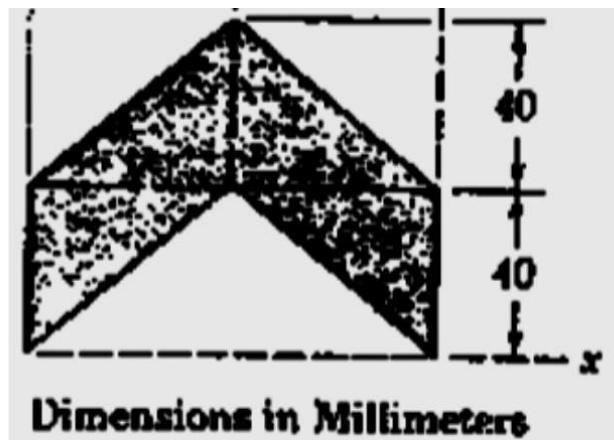
Second: $dI_{xy} = dI_{x_0 y_0} + d_x d_y (dA)$

$$= 0 + \frac{y}{2} x (y \, dx) = \frac{h^2}{2b^2} x^3 dx$$

$$I_{xy} = \frac{h^2}{2b^2} \int_0^b x^3 dx = \frac{h^2}{2b^2} \frac{b^4}{4} = \frac{b^2 h^2}{8}$$

4. Determine the product of inertia of the shaded area with respect to the assigned axes. (hint: locate the centroid of the symmetrical area).

$$\text{Ans. } I_{xy} = -8(10^6) \text{ mm}^4$$

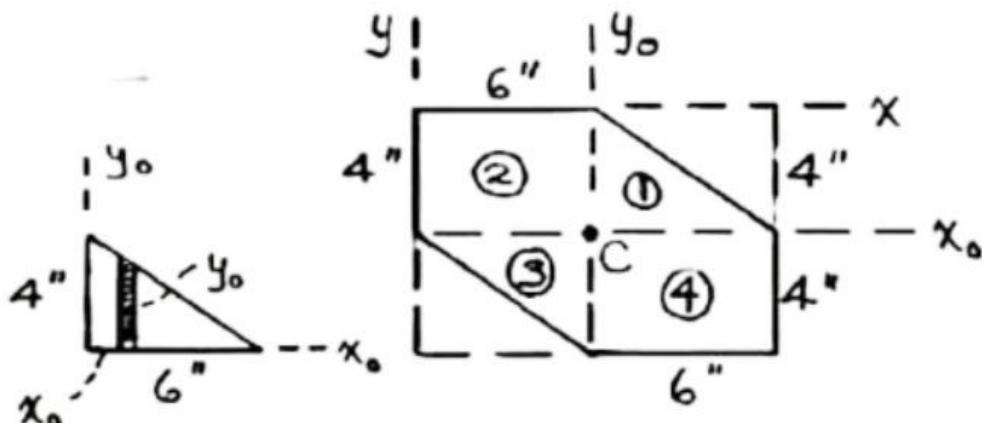
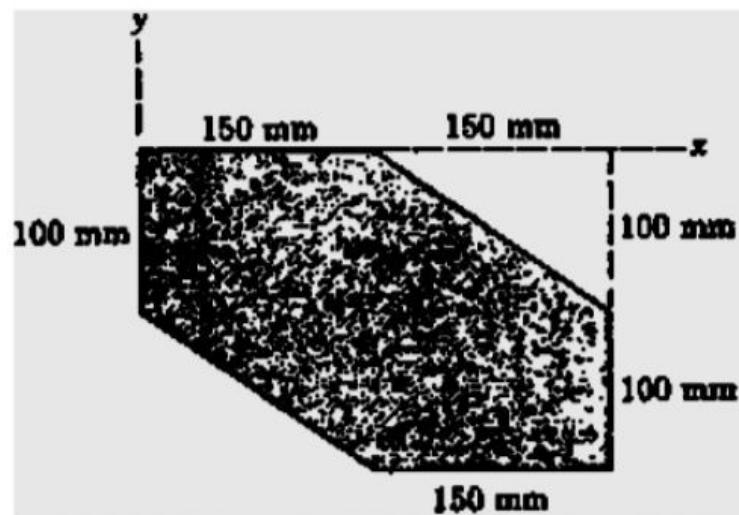


Centroid is at C

$$\begin{aligned}
 I_{xy} &= I_{x_0 y_0} + \bar{x}_0 \bar{y}_0 A \\
 &= 0 + (40)(-50)(40/100) \\
 &= -\underline{\underline{8(10^6) \text{ mm}^4}}
 \end{aligned}$$

5. Calculate the product of inertia of the shaded area about the x - y axes. (Hint: Take advantage of the transfer-of-axes relations).

Ans. $I_{xy} = -769(10^6) \text{ mm}^4$



$$\begin{aligned} \text{Part 1: } I_{x_0 y_0} &= \int_0^6 x_0 \frac{y_0}{2} (y_0 dx_0), \quad y_0 = 4 - \frac{2}{3} x_0 \\ &= \frac{1}{2} \int_0^6 x_0 \left(16 - \frac{16}{3} x_0 + \frac{4}{9} x_0^2 \right) dx_0 \\ &= \frac{1}{2} \left[8x_0^2 - \frac{16}{9} x_0^3 + \frac{4}{27} x_0^4 \right]_0^6 = 24 \text{ in.}^4 \end{aligned}$$

$$\text{Part 3: } I_{x_0 y_0} = 24 \text{ in.}^4$$

$$\text{Part 2: } I_{x_0 y_0} = 4(6)(-3)(+2) = -144 \text{ in.}^4$$

$$\text{Part 4: } I_{x_0 y_0} = -144 \text{ in.}^4$$

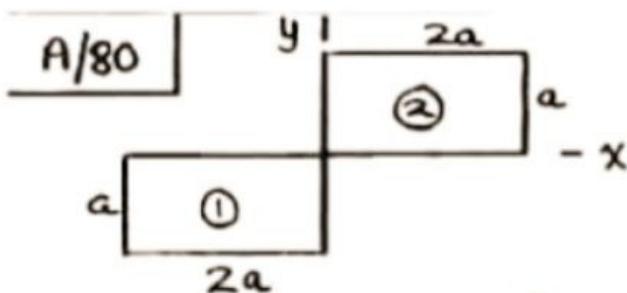
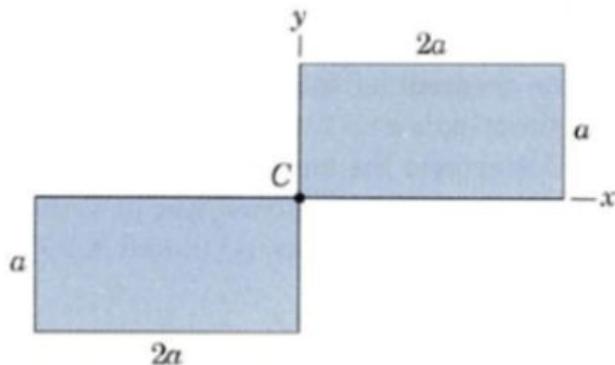
$$\text{Combined: } I_{x_0 y_0} = 2(24) + 2(-144) = -240 \text{ in.}^4$$

$$\text{Combined area} = 2(4)(6) + 2\left(\frac{1}{2}\right)(4)(6) = 72 \text{ in.}^4$$

$$\begin{aligned} \text{So } I_{xy} &= I_{x_0 y_0} + Ad_x dy = -240 + 72(+6)(-4) \\ &= \underline{-1968 \text{ in.}^4} \end{aligned}$$

6. Determine the maximum and minimum moments of inertia with respect to centroidal axes through C for the composite of the two rectangular areas shown. Find the angle α measured from the x-axis to the axis of maximum moment of inertia.

Ans. $I_{\min} = 0.505a^4$, $I_{\max} = 6.16a^4$, $\alpha = 112.5^\circ$



$$\textcircled{1} \quad I_x = \frac{1}{3}(2a)(a^3) = \frac{2}{3}a^4, \quad I_y = \frac{1}{3}(a)(2a)^3 = \frac{8}{3}a^4$$

$$I_{xy} = (2a^2)(a)(\frac{9}{2}) = a^4$$

$$\textcircled{2} \quad I_x = \frac{2}{3}a^4, \quad I_y = \frac{8}{3}a^4, \quad I_{xy} = a^4$$

$$\text{Eq. A/11: } I_{\min} = \frac{I_x + I_y}{2} - \frac{1}{2}\sqrt{(I_x - I_y)^2 + 4I_{xy}^2}$$

$$= \frac{10}{3}a^4 - \frac{1}{2}\sqrt{(-4a^4)^2 + 16a^8} = \underline{\underline{0.505a^4}}$$

$$I_{\max} = \frac{I_x + I_y}{2} + \frac{1}{2}\sqrt{(I_x^2 - I_y^2) + 4I_{xy}^2}$$

$$= \frac{10}{3}a^4 + \frac{1}{2}\sqrt{(-4a^4)^2 + 16a^8} = \underline{\underline{6.16a^4}}$$

$$\text{Eq. A/10: } \tan 2\alpha = \frac{2I_{xy}}{I_y - I_x} = \frac{4a^4}{(\frac{16}{3} - \frac{4}{3})a^4}$$

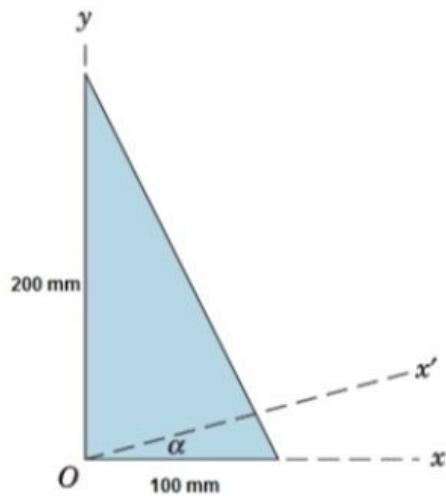
$$2\alpha = 45^\circ \text{ or } 225^\circ$$

$$\alpha = 22.5^\circ \text{ for } I_{\min}$$

$$\text{or } \alpha = \underline{\underline{112.5^\circ}} \text{ for } I_{\max}$$

7. Determine the maximum moment of inertia about an axis through O and the angle α to this axis for the triangular area shown. Also construct the Mohr circle of inertia.

$$\text{Ans. } I_{\max} = 71.7(10^6) \text{ mm}^4, \alpha = -16.85^\circ$$



A/83

$$I_x = \frac{1}{12}bh^3 = \frac{1}{12}4(8^3) = 170.7 \text{ in.}^4$$

$$I_y = \frac{1}{12}b^2h = \frac{1}{12}8(4^2) = 42.7 \text{ in.}^4$$

$$I_{xy} = \frac{b^2h^2}{24} = \frac{1}{24}(4^2)(8^2) = 42.7 \text{ in.}^4$$

Eq. A/11:

$$I_{\max} = \frac{170.7 + 42.7}{2} + \frac{1}{2}\sqrt{(170.7 - 42.7)^2 + 4(42.7)^2}$$

$$= 106.7 + 76.9 = \frac{183.6 \text{ in.}^4}{}$$

$$\text{Eq. A/10: } \tan 2\theta_{cr} = \tan 2\alpha = \frac{2(42.7)}{42.7 - 170.7} = -0.667$$

$$\alpha = -16.85^\circ$$

