

7.3.7 Q7 RSA

$$p=3 \quad q=7$$

(i) 2 first possible public keys e .

(ii) Bob : $d \leftarrow e=5$

(iii) $M=8 \rightarrow C$

Solution (i) $\textcircled{1} n = p \times q = 3 \times 7 = 21$

$$\phi(n) = (p-1) \times (q-1) = 2 \times 6 = 12$$

Factor 2 : 2

6 : 2, 3

prime : 2 3 5 7 9 11

X X \checkmark \checkmark \checkmark \checkmark

the first two possible values for public key are 5 and 7

(ii) $d = \frac{k\phi(n)+1}{e}$

$$d = \frac{12k+1}{5}$$

For $k=1$, $d = \frac{12+1}{5} = \frac{13}{5}$

since the result not a whole number
it doesn't satisfy the condition

For $k=2$, we have

$$d = \frac{12 \times 2 + 1}{5} = \frac{24+1}{5} = \frac{25}{5} = 5$$

Bob's private key = 5

(iii) Encryption in RSA

$$C = m^e \bmod n$$

$$= 8^5 \bmod 21$$

$$= 8$$

The ciphertext = 8