

Let 1 Example 1.4

Q. $T = 0.1$

$$\dot{x}(t) = \begin{bmatrix} 0 & 1 \\ 0 & -1 \end{bmatrix} x(t) + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u(t)$$

$$y(t) = \begin{bmatrix} 1 & 0 \end{bmatrix} x(t) \quad \text{for discrete-time state space.}$$

Solution.

$$[sI - A]^{-1} = \begin{bmatrix} s & -1 \\ 0 & s+1 \end{bmatrix}^{-1} = \frac{1}{s(s+1)} \begin{bmatrix} s+1 & 1 \\ 0 & s \end{bmatrix}$$

$$= \begin{bmatrix} \frac{1}{s} & \frac{1}{s(s+1)} \\ 0 & \frac{1}{s+1} \end{bmatrix}$$

$$\Phi(t) = L^{-1} \begin{bmatrix} \frac{1}{s} & \frac{1}{s(s+1)} \\ 0 & \frac{1}{s+1} \end{bmatrix} = \begin{bmatrix} 1 & 1 - e^{-t} \\ 0 & e^{-t} \end{bmatrix}$$

$$\Phi(T) = \begin{bmatrix} 1 & 1 - e^{-T} \\ 0 & e^{-T} \end{bmatrix} = \begin{bmatrix} 1 & 1 - e^{-0.1} \\ 0 & e^{-0.1} \end{bmatrix} = \begin{bmatrix} 1 & 0.0952 \\ 0 & 0.9048 \end{bmatrix}$$

$$\theta(T) = \int_0^T \Phi(y) dy B$$

$$= \begin{bmatrix} t & t + e^{-t} \\ 0 & -e^{-t} \end{bmatrix} \Big|_0^{0.1} \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

$$= \left(\begin{bmatrix} 0.1 & 0.1 + e^{-0.1} \\ 0 & -e^{-0.1} \end{bmatrix} - \begin{bmatrix} 0 & 1 \\ 0 & -1 \end{bmatrix} \right) \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

...

$$= \begin{bmatrix} 0.1 & -0.9 + e^{-0.1} \\ 0 & -e^{-0.1} + 1 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} -0.9 + e^{-0.1} \\ 1 - e^{-0.1} \end{bmatrix} = \begin{bmatrix} 0.00484 \\ 0.0952 \end{bmatrix}$$

$$x((k+1)T) = \Phi(T) x(kT) + \Theta(T) u(kT)$$

$$= \begin{bmatrix} 1 & 0.0952 \\ 0 & 0.9048 \end{bmatrix} x(kT) + \begin{bmatrix} 0.00484 \\ 0.0952 \end{bmatrix} u(kT)$$

$$y(kT) = [1 \ 0] x(kT)$$

最后的离散状态模型，如果有T具体数字，要将T消除？
 如果T没有给出具体数字，
 请问是按照没有T的来写还是按照有T的写？

Example 1.4 A servomotor has a continuous-time state space representation as shown below.

Sample the system with a sampling period of $T = 0.1$ sec and obtain a discrete-time state space model of the motor.

连续模型

$$\dot{\mathbf{x}}(t) = \begin{bmatrix} 0 & 1 \\ 0 & -1 \end{bmatrix} \mathbf{x}(t) + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u(t)$$

$$y(t) = \begin{bmatrix} 1 & 0 \end{bmatrix} \mathbf{x}(t)$$

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状态转移矩阵

$$\Rightarrow [s\mathbf{I} - \mathbf{A}]^{-1} = \begin{bmatrix} \frac{1}{s} & \frac{1}{s(s+1)} \\ 0 & \frac{1}{s+1} \end{bmatrix}$$

$$\Rightarrow \Phi(t) = \mathcal{L}^{-1} \left\{ [s\mathbf{I} - \mathbf{A}]^{-1} \right\} = \begin{bmatrix} 1 & 1 - e^{-t} \\ 0 & e^{-t} \end{bmatrix}$$

$$\Rightarrow \Phi(T) = \begin{bmatrix} 1 & 1 - e^{-T} \\ 0 & e^{-T} \end{bmatrix} = \begin{bmatrix} 1 & 0.0952 \\ 0 & 0.905 \end{bmatrix}$$

$$\Theta(T) = \left[\int_0^T \Phi(\tau) d\tau \right] \begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 0.00484 \\ 0.0952 \end{bmatrix}$$

$$\begin{aligned} \Rightarrow \mathbf{x}(k+1) &= \begin{bmatrix} 1 & 0.0952 \\ 0 & 0.905 \end{bmatrix} \mathbf{x}(k) \\ &\quad + \begin{bmatrix} 0.00484 \\ 0.0952 \end{bmatrix} u(k) \quad \dots(2.11) \end{aligned}$$

$$y(k) = \begin{bmatrix} 1 & 0 \end{bmatrix} \mathbf{x}(k) \quad \dots(2.12)$$