21 − S1 − Q3

Q: (a) discrect ZOH?

$$\begin{bmatrix} \dfrac{dx_1(t)}{dt} \\ \dfrac{dx_2(t)}{dt} \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u(t)$$

$$y(t) = \begin{bmatrix} 1 & 1 \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix}$$

Solution

$$A_d = \Phi(t=T) = L^{-1}\{[sI-A]^{-1}\}\Big|_{t=T}$$

$$[sI-A]^{-1} = \begin{bmatrix} s & -1 \\ 1 & s \end{bmatrix}^{-1}$$

$$= \frac{1}{s^2+1} \begin{bmatrix} s & 1 \\ -1 & s \end{bmatrix}$$

$$= \begin{bmatrix} \frac{s}{s^2+1} & \frac{1}{s^2+1} \\ -\frac{1}{s^2+1} & \frac{s}{s^2+1} \end{bmatrix}$$

$$A_d = L^{-1}\{[sI-A]^{-1}\}\Big|_{t=T} = \begin{bmatrix} \cos T & \sin T \\ -\sin T & \cos T \end{bmatrix}$$

#15 w4
#14 w4

$$B_d = \int_0^T \Phi(\tau) B \, d\tau$$

$$= \int_0^T \begin{bmatrix} \cos\tau & \sin\tau \\ -\sin\tau & \cos\tau \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix} d\tau$$

$$\int_0^T \cos\tau = \sin\tau \Big|_0^T$$

$$\int_0^T \sin\tau = 1 - \cos T$$

$$= \int_0^T \begin{bmatrix} \sin\tau \\ \cos\tau \end{bmatrix} d\tau$$

$$\int_0^T \sin\tau = -\cos\tau \Big|_0^T$$
$$= -[\cos T - \cos 0]$$
$$= -[(-1) - 1]$$
$$= 2$$

$\cos T \neq \cos \pi$

$$= \cancel{\begin{bmatrix} 2 \\ 0 \end{bmatrix}} = \begin{bmatrix} 1 - \cos T \\ \sin T \end{bmatrix}$$

$$C_d = C = [1 \quad 1] \qquad D_d = 0$$

$$\int_0^T \cos\tau = \sin\tau \Big|_0^T$$
$$= \overset{\sin T}{0} - 0 = 0$$

So

$$x(k+1) = \begin{bmatrix} \cos T & \sin T \\ -\sin T & \cos T \end{bmatrix} x(k) + \begin{bmatrix} \overset{1-\cos T}{2} \\ \underset{\sin T}{0} \end{bmatrix} u(k)$$

$$y(k) = [1 \quad 1] \, x(k) \qquad k = 0, 1, 2, 3 \ldots$$

(b) Q $$x(k+1) = \begin{bmatrix} 1.5 & 2 \\ -2 & 0 \end{bmatrix} x(k) + \begin{bmatrix} 1 \\ -0.5 \end{bmatrix} u(k)$$

$$y(k) = [1 \quad 0] \, x(k)$$

(i) $x(0) = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$ $\quad x(2) = \begin{bmatrix} 1 \\ \beta \end{bmatrix}$ $\quad u(0) = ?$ $\quad u(1) = ?$

(ii) $\dfrac{Y(z)}{U(z)} = ?$

Solution (i) $x(1) = \begin{bmatrix} 1 \\ -0.5 \end{bmatrix} u(0)$

$$x(2) = \begin{bmatrix} 1.5 & 2 \\ -2 & 0 \end{bmatrix} x(1) + \begin{bmatrix} 1 \\ -0.5 \end{bmatrix} u(1)$$

$$\begin{bmatrix} 1 \\ \beta \end{bmatrix} = \begin{bmatrix} 1.5 & 2 \\ -2 & 0 \end{bmatrix} \begin{bmatrix} 1 \\ -0.5 \end{bmatrix} u(0) + \begin{bmatrix} 1 \\ -0.5 \end{bmatrix} u(1)$$

$$\begin{bmatrix} 1 \\ \beta \end{bmatrix} = \begin{bmatrix} 0.5 \\ -2 \end{bmatrix} u(0) + \begin{bmatrix} 1 \\ -0.5 \end{bmatrix} u(1)$$

$u(0) = ?$ $u(1) = ?$ Solution $\begin{cases} 1 = \frac{1}{2} u(0) + u(1) & \text{①} \\ \beta = -2u(0) - 0.5u(1) & \text{②} \end{cases}$

$2\text{②} + \text{①}: 1 + 2\beta = -\frac{7}{2} u(0)$

$u(0) = -\frac{2+4\beta}{7}$

$u(1) = 1 - \frac{1}{2} \times \left(-\frac{2+4\beta}{7}\right)$

$= 1 + \frac{1+2\beta}{7}$

$= \frac{8+2\beta}{7}$

(ii) $\frac{Y(z)}{U(z)} = C[zI - A]^{-1}B + D$

$$= [1 \quad 0] \begin{bmatrix} z-1.5 & -2 \\ 2 & z \end{bmatrix}^{-1} \begin{bmatrix} 1 \\ -0.5 \end{bmatrix}$$

$$= [1 \quad 0] \frac{1}{(z-1.5)z+4} \begin{bmatrix} z & 2 \\ -2 & z-1.5 \end{bmatrix} \begin{bmatrix} 1 \\ -0.5 \end{bmatrix}$$

$$= \frac{1}{z^2 - 1.5z + 4} [z \quad 2] \begin{bmatrix} 1 \\ -0.5 \end{bmatrix}$$

$$= \frac{z-1}{z^2 - 1.5z + 4}$$

(c) Q state ?

$$x(k+1) = A\,x(k) + B\,u(k) \qquad \text{①}$$

$$y(k) = C\ x(k) + d\,u(k) \quad ②$$

$$u(k) = r(k) - y(k) \quad ③$$

联立②③，消去$u(k)$

$$y(k) = C\ x(k) + r(k) - y(k)$$

$$y(k) = \frac{1}{2}C\,x(k) + \frac{1}{2}r(k) \quad ④$$

联立③④，消去$y(k)$

$$u(k) = r(k) - \frac{1}{2}C\,x(k) - \frac{1}{2}r(k)$$

$$= -\frac{1}{2}C\,x(k) + \frac{1}{2}r(k) \quad ⑤$$

联立①⑤ 消$u(k)$

$$x(k+1) = A\,x(k) + B\left[-\frac{1}{2}C\,x(k) + \frac{1}{2}r(k)\right]$$

$$= A\,x(k) - \frac{1}{2}BC\,x(k) + \frac{1}{2}Br(k)$$

$$= \left[A - \frac{1}{2}BC\right]x(k) + \frac{1}{2}Br(k)$$

$$= \begin{bmatrix} \frac{1}{2} & -\frac{1}{2} \\ -\frac{1}{2} & \frac{1}{2} \end{bmatrix} x(k) + \begin{bmatrix} \frac{1}{2} \\ \frac{1}{2} \end{bmatrix} r(k)$$

$$\left[A - \frac{1}{2}BC\right] = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} - \frac{1}{2}\begin{bmatrix} 1 \\ 1 \end{bmatrix}\begin{bmatrix} 1 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} \end{bmatrix}$$

$$= \begin{bmatrix} \frac{1}{2} & -\frac{1}{2} \\ -\frac{1}{2} & \frac{1}{2} \end{bmatrix}$$

$$y(k) = \frac{1}{2} C\, x(k) + \frac{1}{2} r(k)$$

$$= \frac{1}{2} [1 \quad 1]\, x(k) + \frac{1}{2} r(k)$$

So $x(k+1) = \begin{bmatrix} \frac{1}{2} & -\frac{1}{2} \\ -\frac{1}{2} & \frac{1}{2} \end{bmatrix} x(k) + \begin{bmatrix} \frac{1}{2} \\ \frac{1}{2} \end{bmatrix} r(k)$

$$y(k) = [\tfrac{1}{2} \quad \tfrac{1}{2}]\, x(k) + \tfrac{1}{2} r(k)$$

Bd重算

$$B_d = \int_0^T \begin{bmatrix} \sin\tau \\ \cos\tau \end{bmatrix} d\tau$$

$$\int_0^T \sin\tau \, d\tau = -\cos\tau \Big|_0^T = -[\cos T - \cos 0]$$

$$= 1 - \cos T$$

$$\int_0^T \cos\tau \, d\tau = \sin\tau \Big|_0^T = \sin T - \sin 0$$

$$= \sin T$$

$$Bd = \begin{bmatrix} 1 - \cos T \\ \sin T \end{bmatrix}$$