Example 4.6

Q: 闭环极点 (close loop pole?)

$$x(k+1) = \begin{bmatrix} 1 & 0.0952 \\ 0 & 0.905 \end{bmatrix} x(k) + \begin{bmatrix} 0.00484 \\ 0.0952 \end{bmatrix} u(k)$$

$$y(k) = [1 \quad 0]\, x(k)$$

Controller poles $0.888 \pm j\, 0.173$

observer poles $0.819 \quad 0.819$

Solution

① $\bar{x}(k+1) = A\bar{x}(k) + Bu(k) + L_o(y(k) - C\bar{x}(k))$

$= [A - L_oC]\,\bar{x}(k) + Bu(k) + L_o y(k)$

② 求 L_o

$\alpha_o(z) = (z - 0.819)^2 = z^2 - 1.638z + 0.670761$

$\alpha_o(A) = A^2 - 1.638A + 0.670761 I_2 = \begin{bmatrix} 0.03276 & 0.02542 \\ 0 & 0.007396 \end{bmatrix}$

$W_o = \begin{bmatrix} C \\ CA \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 1 & 0.0952 \end{bmatrix} \qquad W_o^{-1} = \begin{bmatrix} 1 & 0 \\ -10.5042 & 10.5042 \end{bmatrix}$

$L_o = \alpha_c(A)\, W_o^{-1} \begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 0.2670 \\ 0.07769 \end{bmatrix}$

$[A - L_o C] = \begin{bmatrix} 1 & 0.0952 \\ 0 & 0.905 \end{bmatrix} - \begin{bmatrix} 0.2670 & 0 \\ 0.07769 & 0 \end{bmatrix} = \begin{bmatrix} 0.733 & 0.0952 \\ -0.07769 & 0.905 \end{bmatrix}$

$$\bar{x}(k+1)=\begin{bmatrix}0.733 & 0.0952\\ -0.07769 & 0.805\end{bmatrix}\bar{x}(k)+\begin{bmatrix}0.00484\\ 0.0952\end{bmatrix}u(k)+\begin{bmatrix}0.2670\\ 0.07769\end{bmatrix}y(k)$$

③ $u(k)=-K\bar{x}(k)$

④ 求K

$$K=[0\ 1]\,W_c^{-1}\alpha_c(A)$$

$$\alpha_c(z)=[z-(0.888+j0.173)][z-(0.888-j0.173)]$$

$$=z^2-1.776z+0.8185$$

$$\alpha_c(A)=\begin{bmatrix}0.0425 & 0.01228\\ 0 & 0.03025\end{bmatrix}$$

$$W_c=[B\ AB]=\begin{bmatrix}0.00484 & 0.0139\\ 0.0952 & 0.08616\end{bmatrix}$$

$$W_c^{-1}=\begin{bmatrix}-95.0715 & 15.3377\\ 105.0465 & -5.3406\end{bmatrix}$$

$$K=[0\ 1]\begin{bmatrix}-95.0715 & 15.3377\\ 105.0465 & -5.3406\end{bmatrix}\begin{bmatrix}0.0425 & 0.01228\\ 0 & 0.03025\end{bmatrix}$$

$$=[105.0465\ \ -5.3406]\begin{bmatrix}0.0425 & 0.01228\\ 0 & 0.03025\end{bmatrix}$$

$$=[4.4645\ \ 1.1284]$$

$$u(k)=[4.4645\ \ 1.1284]\bar{x}(k)$$

⑤

$$\bar{x}(k+1) = \begin{bmatrix} 0.733 & 0.0952 \\ -0.07769 & 0.905 \end{bmatrix} \bar{x}(k) + \begin{bmatrix} 0.00484 \\ 0.0952 \end{bmatrix} u(k) + \begin{bmatrix} 0.2670 \\ 0.07769 \end{bmatrix} y(k)$$

$$\begin{bmatrix} 0.7114 & 0.8974 \\ -0.5027 & 0.7976 \end{bmatrix}$$

$$= \begin{bmatrix} 0.7546 & 0.1007 \\ 0.3473 & 1.0124 \end{bmatrix} \bar{x}(k) + \begin{bmatrix} 0.2670 & 0 \\ 0.07769 & 0 \end{bmatrix} x(k)$$

$$\downarrow [1\ 0]x(k)$$

⑥

$$x(k+1) = Ax(k) + Bu(k)$$

$$\pi(k+1) = \begin{bmatrix} 1 & 0.0952 \\ 0 & 0.905 \end{bmatrix} x(k) + \begin{bmatrix} 0.00484 \\ 0.0952 \end{bmatrix} u(k)$$

$$\pi(k+1) = \begin{bmatrix} 1 & 0.0952 \\ 0 & 0.905 \end{bmatrix} x(k) + \begin{bmatrix} 0.00484 \\ 0.0952 \end{bmatrix} \begin{bmatrix} -4.4645 & -1.1284 \end{bmatrix} \bar{x}(k)$$

$$\pi(k+1) = \begin{bmatrix} 1 & 0.0952 \\ 0 & 0.905 \end{bmatrix} x(k) + \begin{bmatrix} -0.02161 & -0.005461 \\ -0.4250 & -0.1074 \end{bmatrix} \bar{x}(k)$$

$$\begin{bmatrix} x(k+1) \\ \bar{x}(k+1) \end{bmatrix} = \begin{bmatrix} 1 & 0.0952 & -0.02161 & -0.005461 \\ 0 & 0.905 & -0.4250 & -0.1074 \\ 0.2670 & 0 & 0.7546 & 0.1007 \\ 0.07769 & 0 & 0.3473 & 1.0124 \end{bmatrix} \begin{bmatrix} x(k) \\ \bar{x}(k) \end{bmatrix}$$

$$\begin{bmatrix} 0.7114 & 0.8974 \\ -0.5027 & 0.7976 \end{bmatrix}$$