Example 2-15

Q PF E z^{-1} $\quad X(z) = \dfrac{z^2+z+2}{(z-1)(z^2-z+1)}$ ①

Solution: PFE

$$X(z) = \frac{A}{(z-1)} + \frac{Bz+C}{z^2-z+1}$$

$$= \frac{Az^2 - Az + A + (Bz+C)(z-1)}{(z-1)(z^2-z+1)}$$

$$= \frac{Az^2 - Az + A + Bz^2 - Bz + Cz - C}{(z-1)(z^2-z+1)}$$

$$\Rightarrow \begin{cases} A+B = 1 \\ -A-B+C = 1 \\ A-C = 2 \end{cases} \Rightarrow \begin{cases} A = 4 \\ B = -3 \\ C = 2 \end{cases}$$

$$X(z) = \frac{4}{z-1} + \frac{-3z+2}{z^2-z+1}$$

$$= \frac{4z^{-1}}{1-z^{-1}} + \frac{-3z^{-1}+2z^{-2}}{1-z^{-1}+z^{-2}}$$

卡住了.
很难想

提公 z^{-1}

$$= z^{-1}\left[\frac{4}{1-z^{-1}} + \frac{-3+2z^{-1}}{1-z^{-1}+z^{-2}}\right]$$

由于必须要出现 z^{-1} 在分子，才能凑 $\cos\omega kT$ $\sin\omega kT$

根据分母 $\cos\omega T = \frac{1}{2}$ $\sin\omega T = \frac{\sqrt{3}}{2}$

凑 $1 - z^{-1}\cos\omega t = 1 - \frac{1}{2}z^{-1}$

$-3+2z^{-1} = -3(1-\frac{1}{2}z^{-1}) + \frac{1}{2}z^{-1}$

$$= z^{-1}\left[\frac{4}{1-z^{-1}} - 3\frac{1-0.5z^{-1}}{1-z^{-1}+z^{-2}} + \frac{0.5z^{-1}}{1-z^{-1}+z^{-2}}\right]$$

延迟一拍

每个都乘 $1(k)$

$4\times 1(k) - 3\cos\frac{k\pi}{3}1(k) + \frac{1}{\sqrt{3}}\sin\frac{(k-1)\pi}{3}1(k)$

$x(kT) = 4\times 1(k-1) - 3\cos\frac{(k-1)\pi}{3}1(k-1) + \frac{1}{\sqrt{3}}\sin\frac{(k-1)\pi}{3}1(k-1)$

$$\frac{X(z)}{z} = \frac{z^2+z+2}{z(z-1)(z^2-z+1)}$$

$$= \frac{A}{z} + \frac{B}{z-1} + \frac{Cz+D}{z^2-z+1}$$

$$= \frac{A(z-1)(z^2-z+1) + Bz(z^2-z+1) + (Cz+D)z(z-1)}{z(z-1)(z^2-z+1)}$$

$$= \frac{A(z^3-z^2+z-z^2+z-1) + B(z^3-z^2+z) + (Cz+D)(z^2-z)}{z(z-1)(z^2-z+1)}$$

$$= \frac{A(z^3-2z^2+2z-1) + B(z^3-z^2+z) + Cz^3 - Cz^2 + Dz^2 - Dz}{z(z-1)(z^2-z+1)}$$

$$\Rightarrow \begin{cases} A+B+C = 0 \\ -2A-B-C+D = 1 \\ 2A+B-D = 1 \\ -A = 2 \end{cases} \Rightarrow \begin{cases} A = -2 \\ B = 4 \\ C = -2 \\ D = -1 \end{cases}$$

$$\frac{X(z)}{z} = \frac{-2}{z} + \frac{4}{z-1} + \frac{-2z-1}{z^2-z+1}$$

$$X(z) = -2 + \frac{4}{1-z^{-1}} + \frac{-2z^2-z}{z^2-z+1}$$

$$X(z) = -2 + \frac{4}{1-z^{-1}} + \frac{-2-z^{-1}}{1-z^{-1}+z^{-2}}$$

$$x(kT) = -2\,\delta_0(k) + 4 + Z^{-1}\left\{\frac{-2-z^{-1}}{1-z^{-1}+z^{-2}}\right\}$$

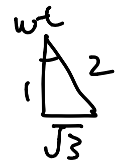
$$Z^{-1}\left\{\frac{-2-z^{-1}}{1-z^{-1}+z^{-2}}\right\}$$

#15 $$\frac{1-z^{-1}\cos wT}{1-2z^{-1}\cos wT+z^{-2}} = \frac{1-\frac{1}{2}z^{-1}}{1-z^{-1}+z^{-2}}$$

let $-2\cos wT = -1$

$\cos wT = \frac{1}{2}$

$\sin wT = \frac{\sqrt{3}}{2}$



$wT = 60° = \frac{\pi}{3}$

#14 $$\frac{z^{-1}\sin wT}{1-2z^{-1}\cos wT+z^{-2}} = \frac{\frac{\sqrt{3}}{2}z^{-1}}{1-z^{-1}+z^{-2}}$$

$$\mathcal{Z}^{-1}\left\{\frac{-2-z^{-1}}{1-z^{-1}+z^{-2}}\right\} = 2\cos wkT - \frac{8\sqrt{3}}{3}\sin wkT$$

$$= 2\cos\frac{\pi}{3}k - \frac{8\sqrt{3}}{3}\sin\frac{\pi}{3}k.$$

$$x(kT) = -2\delta_0(k) + 4 - 2\cos\frac{k\pi}{3} - \frac{4}{\sqrt{3}}\sin\frac{k\pi}{3},\ k \geq 0$$