Example 4.8

Q k_r $\lim_{k\to\infty} y(k)$

$$\alpha_c(z) = z^2 - 2\times 0.8882 + 0.888^2 + 0.173^2$$

$\alpha_c(A)$ ✓ W_c^{-1} ✓ $K = [0\ 1]\, W_c^{-1}\, \alpha_c(A)$ ✓

$K = [4.52 \quad 1.12]$

Solution $\frac{Y(z)}{R(z)} = k_r\, C\,[zI - A - BK]^{-1} B$

$$= k_r [1\ 0] \begin{bmatrix} z - 1.02188 & -0.1006 \\ -0.4303 & z - 1.0116 \end{bmatrix}^{-1} \begin{bmatrix} 0.00484 \\ 0.0952 \end{bmatrix}$$

$$-BK = \begin{bmatrix} -0.02188 & -0.00542 \\ -0.4303 & -0.1066 \end{bmatrix}$$

$$\frac{Y(z)}{R(z)} = k_r [1\ 0] \frac{\begin{bmatrix} z - 1.0116 & 0.1006 \\ 0.4303 & z - 1.02188 \end{bmatrix} \begin{bmatrix} 0.00484 \\ 0.0952 \end{bmatrix}}{(z - 1.02188)(z - 1.0116) - 0.04329}$$

$$= k_r \frac{\begin{bmatrix} z - 1.0116 & 0.1006 \end{bmatrix} \begin{bmatrix} 0.00484 \\ 0.0952 \end{bmatrix}}{(z - 1.02188)(z - 1.0116) - 0.04329}$$

$$= k_r \frac{0.00484z - 0.004896 + 0.009577}{(z - 1.02188)(z - 1.0116) - 0.04329}$$

$$= k_r \frac{0.00484z + 0.004681}{(z - 1.02188)(z - 1.0116) - 0.04329}$$

$$= k_r \frac{0.00484z + 0.004681}{z^2 - 2.03348z + 0.9904}$$

$$\lim_{k\to\infty} y(k) = \lim_{z\to 1} (z-1)\, Y(z)$$

$$= \lim_{z\to 1} (z-1)\, k_r \frac{0.00484z + 0.00468}{z^2 - 2.03348z + 0.9904} R(z)$$

$$1 = k_r \frac{(0.00484z + 0.00468)(z-1)}{z^2 - 2.03348z + 0.9904}$$

$$z - 1 \;\overline{)\; z^2 - 2.03348z + 0.9904 \;\big|\; z - 1.03348}$$

$$z^2 - z$$

$$-1.03348z + 0.9904$$

$$-1.03348z + 1.03348$$

$$-0.04308$$

$$1 = k_r \frac{0.00484z + 0.00468}{z - 1.03348}$$

$$k_r = -3.5164$$

why $k_r = 4.52$?

$$z - 1 \;\overline{)\; z^2 - 1.776z + 0.819 \;\big|\; z - 0.776}$$

$$z^2 - z$$

$$-0.776z + 0.819$$

$$-0.776z + 0.776$$

$$0.043$$

$$k_r = \frac{z - 0.776}{0.00484z + 0.00468}$$

$$= 23.5294$$