

Lect 1 Example. 1.6

Q. discrete time. space model & transfer function

Solution 先求 Φ , 再在这个基础上求 Θ function

$$\textcircled{1} x((k+1)T) = \Phi(T)x(kT) + \Theta(T)u(kT)$$

$$\Phi(T) = L^{-1} \left\{ [sI - A]^{-1} \right\}$$

$$[sI - A]^{-1} = \begin{bmatrix} s & -1 \\ 0 & s \end{bmatrix}^{-1} = \frac{1}{s^2} \begin{bmatrix} s & 1 \\ 0 & s \end{bmatrix} = \begin{bmatrix} \frac{1}{s} & \frac{1}{s^2} \\ 0 & \frac{1}{s} \end{bmatrix}$$

$$\Phi(T) = L^{-1} \left\{ \begin{bmatrix} \frac{1}{s} & \frac{1}{s^2} \\ 0 & \frac{1}{s} \end{bmatrix} \right\} = \begin{bmatrix} 1 & T \\ 0 & 1 \end{bmatrix}$$

$$\Theta(T) = \left[\int_0^T \Phi(\gamma) d\gamma \right] B$$

$$= \left[\int_0^T \begin{bmatrix} 1 & \gamma \\ 0 & 1 \end{bmatrix} d\gamma \right] \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} T & \frac{1}{2}T^2 \\ 0 & T \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} \frac{1}{2}T^2 \\ T \end{bmatrix}$$

$$x((k+1)T) = \begin{bmatrix} 1 & T \\ 0 & 1 \end{bmatrix} x(kT) + \begin{bmatrix} \frac{1}{2}T^2 \\ T \end{bmatrix} u(kT) \quad \checkmark$$

② transfer function

$$\frac{Y(z)}{U(z)} = C [zI - A]^{-1} B + D$$

$$= [1 \ 0] \begin{bmatrix} z-1 & -T \\ 0 & z-1 \end{bmatrix}^{-1} \begin{bmatrix} \frac{1}{2}T^2 \\ T \end{bmatrix}$$

$$= \frac{1}{(z-1)^2} [1 \ 0] \begin{bmatrix} z-1 & T \\ 0 & z-1 \end{bmatrix} \begin{bmatrix} \frac{1}{2}T^2 \\ T \end{bmatrix}$$

$$= \frac{1}{(z-1)^2} [z-1 \quad T] \begin{bmatrix} \frac{1}{2}T^2 \\ T \end{bmatrix}$$

$$= \frac{1}{(z-1)^2} \left(\frac{(z-1)T^2}{2} + T^2 \right)$$

$$= \frac{T^2}{2(z-1)} + \frac{T^2}{(z-1)^2}$$

$$= T^2 \left[\frac{z-1}{2(z-1)^2} + \frac{2}{2(z-1)^2} \right]$$

$$= \frac{T^2}{2} \frac{z+1}{(z-1)^2} \quad \checkmark$$