Lect3 Example 10

Q: discrete time? CCA?

$$\dot{x}(t) = \begin{bmatrix} -1 & 0 \\ 1 & 0 \end{bmatrix} x(t) + \begin{bmatrix} 1 \\ 0 \end{bmatrix} u(t)$$

$$y(t) = \begin{bmatrix} 1 & 0 \end{bmatrix} x(t)$$

state feedback controller?

$$u(k) = -\begin{bmatrix} k_1 & k_2 \end{bmatrix} \begin{bmatrix} x_1(k) \\ x_2(k) \end{bmatrix}$$

let pole at $-p_1$, $-p_2$

$\Phi(T) = \Phi(t=T) = \mathcal{L}^{-1}\{(sI-A)^{-1}\}\big|_{t=T}$

状态转移矩阵是 S 非 Z

Solution $A_d = \mathcal{L}^{-1}\{\Phi(T)\}$

$$(\cancel{z}sI - A)^{-1}$$

$$= \begin{bmatrix} \cancel{z}s+1 & 0 \\ -1 & \cancel{z}s \end{bmatrix}^{-1}$$

$$= \frac{1}{\cancel{z}s(\cancel{z}s+1)} \begin{bmatrix} \cancel{z}s & 0 \\ 1 & \cancel{z}s+1 \end{bmatrix}$$

$$= \begin{bmatrix} \frac{1}{\cancel{z}s+1} & 0 \\ \frac{1}{\cancel{z}s(\cancel{z}s+1)} & \frac{1}{\cancel{z}s} \end{bmatrix}$$

$$\Phi(t) = \mathcal{L}^{-1}\{(\cancel{z}sI - A)^{-1}\}$$

$$= \begin{bmatrix} e^{-t} & 0 \\ 1-e^{-t} & 1 \end{bmatrix}$$

$$A_d = \Phi(T) = \begin{bmatrix} e^{-T} & 0 \\ 1-e^{-T} & 1 \end{bmatrix}$$

$$B_d = \int_0^T \Phi(\tau) B \, d\tau$$ ☆背

$$= \int_0^T \begin{bmatrix} e^{-\tau} & 0 \\ 1-e^{-\tau} & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix} d\tau$$

$$= \int_0^T \begin{bmatrix} e^{-\tau} \\ 1-e^{-\tau} \end{bmatrix} d\tau$$

$$= \begin{bmatrix} 1-e^{-T} \\ T-1+e^{-T} \end{bmatrix}$$

$$-e^{-\tau}\Big|_0^T$$

$$= -(e^{-T} - e^{0})$$

$$= 1 - e^{-T}$$

$$\int 1-e^{-\tau} = \int 1 - \int e^{-\tau}$$

$$= T - 1 + e^{-T}$$

$$C_d = C = [1 \ \ 0]$$

$$x(k+1) = \begin{bmatrix} e^{-T} & 0 \\ 1-e^{-T} & 1 \end{bmatrix} x(k) + \begin{bmatrix} 1-e^{-T} \\ T-1+e^{-T} \end{bmatrix} u(k)$$

$$y(k) = [1 \ \ 0] \, x(k)$$

化为 CCA，先求特征多项式

$$|zI - A_d| = \begin{vmatrix} z-e^{-T} & 0 \\ e^{-T}-1 & z-1 \end{vmatrix}$$

$$= z^2 - (1+e^{-T})z + e^{-T}$$

$$A_c = \begin{bmatrix} 0 & 1 \\ e^{-T} & -(1+e^{-T}) \end{bmatrix} \quad B_c = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

$$C_c = C = \begin{bmatrix} 1 & 0 \end{bmatrix}$$

CCA: $x(k+1) = \begin{bmatrix} 0 & 1 \\ e^{-T} & -(1+e^{-T}) \end{bmatrix} x(k) + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u(k)$

$$y(k) = \begin{bmatrix} 1 & 0 \end{bmatrix} x(k)$$

求 K：希望 $\alpha_c(z) = [z-(-p_1)][z-(-p_2)]$

$$= (z+p_1)(z+p_2)$$

$$= z^2 + (p_1+p_2)z + p_1 p_2$$

特征多项式

$$\alpha(z) = |zI - A + BK|$$

$$BK = \begin{bmatrix} 0 \\ 1 \end{bmatrix} \begin{bmatrix} k_1 & k_2 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ k_1 & k_2 \end{bmatrix}$$

$$= \begin{vmatrix} z & -1 \\ -e^{-T}+k_1 & z+1+e^{-T}+k_2 \end{vmatrix}$$

$$= z^2 + (1+e^{-T}+k_2)z + (k_1 - e^{-T})$$

令 $1+e^{-T}+k_2=p_1+p_2$

$k_1-e^{-T}=p_1p_2$

可得出 $k_1=p_1p_2+e^{-T}$

$k_2=p_1+p_2-1-e^{-T}$

$$k_{i+1}=\beta_{n-i}-a_i$$

$k_1=\beta_2-a_0=p_1p_2+e^{-T}$

$k_2=\beta_1-a_1=p_1+p_2-(1+e^{-T})$

α_c A_c