

Lecture Example 1.5

$$Q \quad x(k+1) = \begin{bmatrix} 1.35 & 0.55 \\ -0.45 & 0.35 \end{bmatrix} x(k) + \begin{bmatrix} 0.5 \\ 0.5 \end{bmatrix} u(k)$$

$$y(k) = [1 \ -1] x(k)$$

Find zeros and poles and transfer function

Solution

$$\begin{aligned} [zI - A]^{-1} &= \begin{bmatrix} z - 1.35 & -0.55 \\ 0.45 & z - 0.35 \end{bmatrix}^{-1} \\ &= \frac{\begin{bmatrix} z - 0.35 & 0.55 \\ -0.45 & z - 1.35 \end{bmatrix}}{(z - 1.35)(z - 0.35) + 0.55 \times 0.45} \\ &= \frac{\begin{bmatrix} z - 0.35 & 0.55 \\ -0.45 & z - 1.35 \end{bmatrix}}{z^2 - 1.7z + 0.72} \end{aligned}$$

$$\frac{Y(z)}{U(z)} = C [zI - A]^{-1} B + D$$

$$= [1 \ -1] \frac{\begin{bmatrix} z - 0.35 & 0.55 \\ -0.45 & z - 1.35 \end{bmatrix}}{z^2 - 1.7z + 0.72} \begin{bmatrix} 0.5 \\ 0.5 \end{bmatrix}$$

$$= \frac{1}{z^2 - 1.7z + 0.72} \begin{bmatrix} -z - 0.35 + 0.45 & 0.55 - z + 1.35 \end{bmatrix} \begin{bmatrix} 0.5 \\ 0.5 \end{bmatrix}$$

$$= \frac{1}{z^2 - 1.7z + 0.72} \begin{bmatrix} -z + 0.1 & -z + 1.9 \end{bmatrix} \begin{bmatrix} 0.5 \\ 0.5 \end{bmatrix}$$

$$= \frac{1}{z^2 - 1.7z + 0.72} (-0.5z + 0.65 - 0.5z + 0.95)$$

$$= \frac{1-z}{z^2 - 1.7z + 0.72}$$

为什么传递函数算出来不一样?

poles : $\det [zI - A] = 0$

$$\begin{vmatrix} z - 1.35 & -0.55 \\ 0.45 & z - 0.35 \end{vmatrix} = 0$$

$$(z - 1.35)(z - 0.35) + 0.55 \times 0.45 = 0$$

$$z^2 - 1.7z + 0.72 = 0$$

$$z_1 = 0.9 \quad z_2 = 0.8$$

zeros : $\det \begin{bmatrix} z_0 I - A & -B \\ C & D \end{bmatrix} = 0$

$$\begin{vmatrix} z - 1.35 & -0.55 & -0.5 \\ 0.45 & z - 0.35 & -0.5 \\ 1 & -1 & 0 \end{vmatrix} = 0$$

why $\frac{1}{2} \det = 1$
对出零点

$$0.55 \times 0.5 + 0.45 \times 0.5 - (Z - 0.35)0.5 - 0.5(Z - 1.35)$$

$$= 0.5 - 0.5Z + 0.5 \times 0.35 - 0.5Z + 0.5 \times 1.35$$

$$= -Z + 0.5(1 + 0.35 + 1.35)$$

$$= -Z + 1.35$$

$$\text{So } Z_0 = 1.35$$