

Level 3 Example 3-12

$u(0)$? $u(1)$?

Q①写出状态方程②化为离散状态③deadbeat?

$$Y(s) = \frac{1}{s^2} U(s) \Rightarrow \frac{d^2 y(t)}{dt^2} = u(t) \quad \left\{ \begin{array}{l} x(0) = \begin{bmatrix} 1 \\ 1 \end{bmatrix} \end{array} \right.$$

$$x_1(t) = y(t)$$

$$x_2(t) = \frac{y(t)}{dt} = \dot{x}_1(t)$$

$$\text{Solution } \begin{bmatrix} \dot{x}_1(t) \\ \dot{x}_2(t) \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u(t)$$

$$y(t) = \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix}$$

$$\frac{d^2 y(t)}{dt^2} = u(t) \Rightarrow \ddot{x}_1(t) = u(t)$$

$$\Rightarrow \dot{x}_2(t) = u(t)$$

② 离散

$$A_d = \Phi(T) = \Phi(t=T) = \left[e^{(sI-A)T} \right] \Big|_{t=T}$$

$$[sI - A]^{-1} = \begin{bmatrix} s & -1 \\ 0 & s \end{bmatrix}^{-1} = \frac{1}{s^2} \begin{bmatrix} s & 1 \\ 0 & s \end{bmatrix} = \begin{bmatrix} \frac{1}{s} & \frac{1}{s^2} \\ 0 & \frac{1}{s} \end{bmatrix}$$

$$A_d = L^{-1} \left\{ \begin{bmatrix} \frac{1}{s} & \frac{1}{s^2} \\ 0 & \frac{1}{s} \end{bmatrix} \right\} \bigg|_{t=T} = \begin{bmatrix} 1 & T \\ 0 & 1 \end{bmatrix}$$

$$B_d = \int_0^T \Phi(\tau) B d\tau = \int_0^T \begin{bmatrix} 1 & \tau \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix} d\tau \\ = \int_0^T \begin{bmatrix} \tau \\ 1 \end{bmatrix} d\tau = \begin{bmatrix} \frac{T^2}{2} \\ T \end{bmatrix}$$

$$C_d = C = [1 \ 0]$$

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$$x(k+1) = \begin{bmatrix} 1 & T \\ 0 & 1 \end{bmatrix} x(k) + \begin{bmatrix} \frac{T^2}{2} \\ T \end{bmatrix} u(k)$$

$$y(k) = [1 \ 0] x(k)$$

⑤ ~~定义 $|\lambda_i[A - BK]| < 1$~~

~~或 $x(k) = [A - BK]^k x(0) = 0, k \rightarrow \infty$~~

$\lambda_i[A - BK] = 0$ 这才是 dead beat 定义

已知 $u(k) = -k x(k)$

已知 $x(0) = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$ 求 K ? $u(0)$? $u(1)$?

$$k = [0 \ 1] W_c^{-1} \alpha_c(A) \quad \frac{T^3}{2} - \frac{3T^3}{2} = -T^3$$

$$W_c^{-1} = [B \ AB]^{-1} = \begin{bmatrix} \frac{T^2}{2} & \frac{3T^2}{2} \\ T & T \end{bmatrix}^{-1} = \frac{1}{-T^3} \begin{bmatrix} T & -\frac{3T^2}{2} \\ -T & \frac{T^2}{2} \end{bmatrix}$$

$$= \begin{bmatrix} -\frac{1}{T^2} & \frac{3}{2T} \\ \frac{1}{T^2} & -\frac{1}{2T} \end{bmatrix}$$

$$\alpha_c(A) = z^2 = A^2 = \begin{bmatrix} 1 & T \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & T \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 2T \\ 0 & 1 \end{bmatrix}$$

$$k = [0 \ 1] \begin{bmatrix} \frac{T^2}{2} & \frac{3T^2}{2} \\ T & T \end{bmatrix} \begin{bmatrix} 1 & 2T \\ 0 & 1 \end{bmatrix}$$

$$= [T \ T] \begin{bmatrix} 1 & 2T \\ 0 & 1 \end{bmatrix}$$

$$= [T \ 2T^2 + T]$$

$$k = [0 \ 1] \begin{bmatrix} -\frac{1}{T^2} & \frac{3}{2T} \\ \frac{1}{T^2} & -\frac{1}{2T} \end{bmatrix} \begin{bmatrix} 1 & 2T \\ 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} \frac{1}{T^2} & -\frac{1}{2T} \end{bmatrix} \begin{bmatrix} 1 & 2T \\ 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} \frac{1}{T^2} & \frac{3}{2T} \end{bmatrix}$$

deadbeat $[A-BK]^n = 0$

$$[A-BK] = \begin{bmatrix} 1 & T \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} \frac{T^2}{2} \\ T \end{bmatrix} \begin{bmatrix} \frac{1}{T^2} & \frac{3}{2T} \end{bmatrix}$$

$$= \begin{bmatrix} 1 & T \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} \frac{1}{2} & \frac{3}{4}T \\ \frac{1}{T} & \frac{3}{2} \end{bmatrix}$$

$$= \begin{bmatrix} \frac{1}{2} & \frac{T}{4} \\ -\frac{1}{T} & -\frac{1}{2} \end{bmatrix}^n \quad \text{some } n > 0$$

$$u(0) = -KX(0) = -\begin{bmatrix} \frac{1}{T^2} & \frac{3}{2T} \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix} = -\left(\frac{1}{T^2} + \frac{3}{2T}\right)$$

$$u(1) = -KX(1) = -K[A-BK]X(0)$$

$$X(k) = [A-BK]^k X(0)$$

$$u(1) = -\begin{bmatrix} \frac{1}{T^2} & \frac{3}{2T} \end{bmatrix} \begin{bmatrix} \frac{1}{2} & \frac{T}{4} \\ -\frac{1}{T} & -\frac{1}{2} \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} \frac{1}{T^2} & \frac{1}{2T} \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix} = \frac{1}{T^2} + \frac{1}{2T}$$