23-S1-Q2

Q: $G_{ZAS}(z) = \dfrac{0.5z + 0.2}{(z-0.5)(z-0.4)}$

(a) PI $C(z) = 1 + \dfrac{1}{1-z^{-1}}$ stable? e_{ss}?

Solution $G_{cl}(z) = \dfrac{C(z)\,G_{ZAS}(z)}{1 + C(z)G_{ZAS}(z)}$

$$C(z) = \frac{2 - z^{-1}}{1 - z^{-1}} = \frac{2z - 1}{z - 1}$$

$$C(z)G_{ZAS}(z) = \frac{(2z-1)(0.5z+0.2)}{(z-1)(z-0.5)(z-0.4)}$$

$$= \frac{(z+0.4)}{(z-1)(z-0.4)}$$

X 不能約分, ref. PPT 19

$$G_{cl}(z) = \frac{\dfrac{(z+0.4)}{(z-1)(z-0.4)}}{\dfrac{(z-1)(z-0.4) + (z+0.4)}{(z-1)(z-0.4)}}$$

$$= \frac{z + 0.4}{z^2 - 1.4z + 0.4 + z + 0.4}$$

$$= \frac{z + 0.4}{z^2 - 0.4z + 0.8}$$

$G_{cl}(z) = \dfrac{G(z)}{1 + G(z)H(z)}$

CE: $1 + G(z)H(z) = 0$

$$C(z)G_{ZAS}(z) = \frac{(2z-1)(0.5z+0.2)}{(z-1)(z-0.5)(z-0.4)}$$

$$G_{CL}(z) = \frac{C(z)G_{ZAS}(z)}{1+C(z)G_{ZAS}(z)}$$

CE:

$$1 + \frac{(2z-1)(0.5z+0.2)}{(z-1)(z-0.5)(z-0.4)} = 0$$

$$(z-1)(z^2-0.9z+0.2) + (z^2+0.4z-0.5z-0.2)$$

$$= z^3 - 0.9z^2 + 0.2z - z^2 + 0.9z - 0.2$$

$$+ z^2 - 0.1z - 0.2$$

$$= z^3 - 0.9z^2 + z - 0.4 = 0$$

Jury Test

z^0	z^1	z^2	z^3
a_3			a_0
-0.4	1	-0.9	1
1	-0.9	1	-0.4
-0.84	0.5	-0.62	

$|-0.4| < 1$ ✓

$P(1) = 1 - 0.9 + 1 - 0.4 = 0.7 > 0$ ✓

$P(-1) = -1 - 0.9 - 1 - 0.4 < 0$ ✓ $n = 3$ ○

$|b_2| > |b_0|$ $0.84 > 0.62$ ✓ Stable

$$e_{ss} = \lim_{z \to 1} \left[(1 - z^{-1}) \frac{1}{1 + C(z) G_{ZAS}(z)} R(z) \right]$$

$$R(z) = \frac{1}{1 - z^{-1}}$$

$$e_{ss} = \lim_{z \to 1} \frac{1}{1 + \frac{(2z - 1)(0.5z + 0.2)}{(z - 1)(z - 0.5)(z - 0.4)}}$$

$$= 0$$

(b) $R(z) = \frac{1}{1 - z^{-1}}$

$$U(z) = G_{cl}(z) \frac{R(z)}{G_{ZAS}(z)}$$

$$G_{ZAS}(z) = \frac{0.5z + 0.2}{(z - 0.5)(z - 0.4)}$$

$$= \frac{(0.5 + 0.2z^{-1}) z^{-1}}{(1 - 0.5z^{-1})(1 - 0.4z^{-1})}$$

$$U(z) = G_{cl}(z) \frac{(1 - 0.5z^{-1})(1 - 0.4z^{-1})}{(1 - z^{-1})(0.5 + 0.2z^{-1}) z^{-1}}$$

$$G_{cl}(z) = K z^{-1} (0.5 + 0.2z^{-1})$$

$$G_{cl}(1) = K(0.5+0.2) = 0.7K = 1$$

$$\Rightarrow K = 1.4286$$

$$C(z) = \frac{1}{G_{zAS}(z)} \frac{G_{cl}(z)}{1 - G_{cl}(z)}$$

$$= \frac{(z-0.5)(z-0.4)}{0.5z+0.2} \frac{1.4286 z^{-1}(0.5+0.2z^{-1})}{1 - 1.4286 z^{-1}(0.5+0.2z^{-1})}$$

$$= \frac{(z-0.5)(z-0.4)}{0.5z+0.2} \frac{1.4286 \quad (0.5z+0.2)}{z^2 - 1.4286(0.5z \quad +0.2)}$$

$$= \frac{(z-0.5)(z-0.4)\,1.4286\,(0.5z+0.2)}{(0.5z+0.2)(z^2 - 0.78573z - 0.28572)}$$

$$= \frac{1.4286(z-0.5)(z-0.4)}{(z-1)(z+0.2857)}$$

(c) $G_{zAS}(z) = \dfrac{0.5z+0.2}{(z-1.5)(z-0.4)}$

This plant has an unstable pole at $z = 1.5$

not possible, because unstable poles

cannot be canceled using a causal controller