21-S1-05

Q $\zeta = 0.8 \quad \omega_n = 5$ rad/s

$T = 1$ s

Solution $\omega_d = \omega_n\sqrt{1-\zeta^2} = 3$ rad/s

$$s = \frac{2}{T}\frac{z-1}{z+1}$$

$$G(s) = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$$

$$= \frac{25}{s^2 + 2\times 0.8 \times 5\, s + 25}$$

$$= \frac{25}{s^2 + 8s + 25}$$

$$G(z) = G(s)\Big|_{s=\frac{2}{T}\frac{z-1}{z+1}} = 2\frac{z-1}{z+1}$$

$$= \frac{25}{4\frac{(z-1)^2}{(z+1)^2} + 16\frac{z-1}{z+1} + 25}$$

$$= \frac{25(z+1)^2}{4(z-1)^2 + 16(z^2-1) + 25(z+1)^2}$$

$$= \frac{25(z^2+2z+1)}{4(z^2-2z+1) + 16z^2 - 16 + 25(z^2+2z+1)}$$

$$= \frac{25(z^2+2z+1)}{(4+16+25)z^2+(-8+50)z+(4-16+25)}$$

$$= \frac{25(z^2+2z+1)}{45z^2+42z+13}$$

$$= \frac{0.5556(z+1)^2}{z^2+0.9333z+0.2889}$$

$$G_{ZAS}(z) = \frac{0.6z+0.4}{(z-0.5)(z-0.4)}$$

(b) Show C(z) must contain an integrator:

$G_{ZAS}(z)$ is a type 0 system, due to it doesn't has a pole at $z=0$, $z=1$, so the plane don't have an integrator. For achieve the zero steady-state error, the system must be Type 1, which means have an integrator. So, C(z) must include an integrator, which means has a pole at $z=1$

$$R(z) = \frac{1}{1-z^{-1}} \qquad G_{cl}(1) = 1$$

$$U(z) = \frac{Y(z)}{G_{ZAS}(z)} = \frac{Y(z)}{R(z)}\frac{R(z)}{G_{ZAS}(z)} = G_{cl}(z)\frac{R(z)}{G_{ZAS}(z)}$$

$$= G_{cl}(z)\frac{\frac{1}{1-z^{-1}}}{\frac{(0.6+0.4z^{-1})z^{-1}}{(1-0.5z^{-1})(1-0.4z^{-1})}}$$

$$= G_{CL}(z)\,\frac{(1-0.5z^{-1})(1-0.4z^{-1})}{z^{-1}(0.6+0.4z^{-1})(1-z^{-1})}$$

$C(z)$ must contain $(1-z^{-1})$

$E(z)=\dfrac{U(z)}{C(z)}$ $\Rightarrow G_{CL}(z)=Kz^{-1}(0.6+0.4z^{-1})$

参考Example 5.12，$C(z)$ 消 $(1-z^{-1})$，$G_{CL}(z)$ 消其它

$G_{CL}(1)=1$ $\quad G_{CL}(1)=K=1 \Rightarrow K=1$

$\Rightarrow G_{CL}(z)=z^{-1}(0.6+0.4z^{-1})$

$$C(z)=\frac{1}{G_{ZAS}(z)}\,\frac{G_{CL}(z)}{1-G_{CL}(z)}$$

$$=\frac{(z-0.5)(z-0.4)}{0.6z+0.4}\,\frac{z^{-1}(0.6+0.4z^{-1})}{1-z^{-1}(0.6+0.4z^{-1})}$$

$$=\frac{(z-0.5)(z-0.4)}{0.6z+0.4}\,\frac{(0.6z+0.4)}{z^2-0.6z-0.4}$$

$$=\frac{(z-0.5)(z-0.4)}{0.6z+0.4}\,\frac{(0.6z+0.4)}{(z-1)(z+0.4)}$$

$$=\frac{(z-0.5)(z-0.4)}{(z-1)(z+0.4)}$$

(c) standard programming $C(z)$

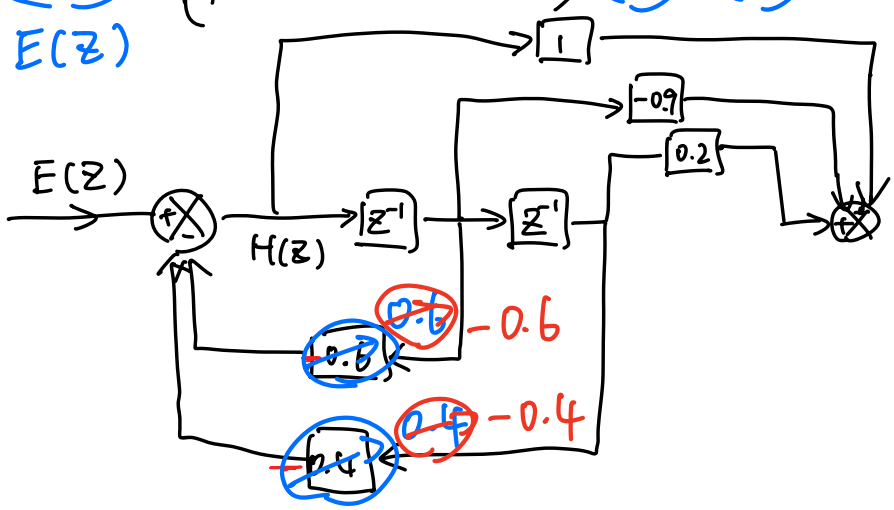
$$C(z) = \frac{z^2 - 0.9z + 0.2}{z^2 - 0.6z - 0.4}$$

$$= \frac{1 - 0.9z^{-1} + 0.2z^{-2}}{1 - 0.6z^{-1} - 0.4z^{-2}}$$

$$C(z) = \frac{U(z)}{H(z)} \frac{H(z)}{E(z)} = (1 - 0.9z^{-1} + 0.2z^{-2}) \frac{1}{1 - 0.6z^{-1} - 0.4z^{-2}}$$

$$U(z) = (1 - 0.9z^{-1} + 0.2z^{-2}) H(z) = H(z) - 0.9z^{-1}H(z) + 0.2z^{-2}H(z)$$

$$H(z) = (1 - 0.6z^{-1} - 0.4z^{-2}) E(z) = E(z) - 0.6z^{-1}H(z) - 0.4z^{-2}H(z)$$



不管上面是正还是负，都按照系数写

$$H(z) = E(z) + 0.6z^{-1}H(z) + 0.4z^{-2}H(z)$$