Example 5.7

Q design a digital controller?

$$G(s) = \frac{1}{10s+1} e^{-5s}$$

- → unit step → 0 steady state error
- settling time 10s
- 5% error tolerance
- no overshoot

$$T = 1s$$

Solution

$$G_{ZAS}(z) = (1-z^{-1}) Z\left\{\frac{G(s)}{s}\right\}$$

$$= (1-z^{-1}) Z\left\{\frac{1}{(10s+1)s} e^{-5s}\right\}$$

$$= (1-z^{-1}) z\left\{\frac{1}{(10s+1)s}\right\}$$

e^{-5s} → z^{-5}

s与z的变换 $z = e^{sT}$

$T = 1$

$z = e^{s}$

$z^{-5} = e^{-5s}$

其中 $Z\left\{\frac{0.1}{(s+0.1)s}\right\}$

$$= \frac{(1-e^{-0.1T})z^{-1}}{(1-z^{-1})(1-e^{-0.1T}z^{-1})}$$

$$G_{ZAS}(z) = (1-z^{-1}) \frac{(1-e^{-0.1T})z^{-1}}{(1-z^{-1})(1-e^{-0.1T}z^{-1})}$$

$$= \frac{(1-e^{-0.1})z^{-1}}{1-e^{-0.1}z^{-1}}$$

$$= \frac{0.09516}{z - 0.9048} z^{-5}$$

5%, error tolerance

$t_s = \frac{3}{\zeta \omega_n} = 10 \cdot 5 = 5$ $t_s = 10s$

$\Rightarrow \omega_n = 0.6$

$\zeta = 1$

$G_{cl}(z) = \frac{K}{z - 0.5} z^{-5}$?✓

setting $G_{cl}(1) = 1$?✓稳定

$G_{cl}(1) = \frac{K}{0.5} = 1 \Rightarrow K = 0.5$

$$C(z) = \frac{1}{G_{ZAS}(z)} \frac{G_{cl}(z)}{1 - G_{cl}(z)}$$

代数

$$= \frac{z - 0.9048}{0.09516 z^{-5}} \frac{\frac{0.5}{z - 0.5} z^{-5}}{1 - \frac{0.5}{z - 0.5} z^{-5}}$$

$$= \frac{z - 0.9048}{0.09516 z^{-5}} \frac{\frac{0.5}{z - 0.5} z^{-5}}{\frac{(z - 1)}{z - 0.5}}$$

$$= \frac{0.5 z^5 (z - 0.9048)}{0.09516 (z - 1)}$$

$$\frac{z - 0.9048}{0.09516} \frac{0.5 z^5}{z^6 - 0.5 z^5 - 0.5} = \frac{5.2543 (z - 0.9048) z^5}{z^6 - 0.5 z^5 - 0.5}$$

no overshoot ?

$\zeta = -\zeta \omega_n \pm j \omega_n \sqrt{1 - \zeta^2}$

$= -\omega_n$

? a pole at $z = 0.5$ ↓ 0.1

? mapped continuous pole $-a$?

$z = e^{sT}$

$= e^{s}$

$= e^{-0.7} \approx 0.5$

$e^{-\frac{\pi \zeta}{\sqrt{1 - \zeta^2}}} = 0 \Rightarrow \zeta \to 1$

$$= \frac{5.2543 z^{5} (z - 0.9048)}{(z-1) \times z^{6} - 0.5 z^{5} - 0.5}$$

?