Lect 1 Example 1.2

Q. 求 servomotor 的 space model 4 transport Function

$$e_m(t) = K_b w(t) = K_b \frac{d\theta(t)}{dt} \qquad J \quad B$$

$$T(t) = K_T i(t) = J\frac{d^2\theta(t)}{dt^2} + B\frac{d\theta(t)}{dt}$$

$$e(t) = i(t) R_a + e_m(t)$$

$$x_1(t) = \theta(t)$$

$$x_2(t) = \frac{d\theta(t)}{dt} = \dot{x}_1(t)$$

$$y(t) = x_1(t)$$

Solution：状态：$x_1(t)$ $x_2(t)$ 外力 $e(t)$

输出 $y(t)$

$$\dot{x}_1(t) = x_2(t) = \frac{d\theta(t)}{d(t)}$$

$$\dot{x}_2(t) = \frac{d\theta^2(t)}{dt^2}$$

$$\stackrel{1.8}{=} \frac{1}{J}\left[K_T i(t) - B x_2(t)\right]$$

$$\stackrel{1.9}{=} \frac{1}{J}\left[K_T \frac{e(t) - e_m(t)}{R_a} - B x_2(t)\right]$$

$$\overset{1.7}{=} \frac{1}{J}\left[K_T \frac{e(t) - K_b x_2(t)}{R_a} - B x_2(t) \right]$$

$$\overset{\text{分开}x_2(t)\text{和}e(t)}{=} -\frac{K_T K_b + B R_a}{J R_a} x_2(t) + \frac{K_T}{J R_a} e(t)$$

$$\begin{bmatrix} \dot{x}_1(t) \\ \dot{x}_2(t) \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 0 & -\frac{K_T K_b + B R_a}{J R_a} \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} + \begin{bmatrix} 0 \\ \frac{K_T}{J R_a} \end{bmatrix} e(t)$$

$$y(t) = \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix}$$

Transfer Function $\frac{Y(s)}{U(s)} = C\,[sI - A]^{-1} B + D$

$$[sI - A]^{-1} = \begin{bmatrix} 1 & -1 \\ 0 & 1 + \frac{K_T K_b + B R_a}{J R_a} \end{bmatrix}^{-1}$$

$$= \frac{1}{1 + \frac{B R_a + K_T K_b}{J R_a}} \begin{bmatrix} 1 + \frac{B R_a + K_T K_b}{J R_a} & 1 \\ 0 & 1 \end{bmatrix}$$

$$\frac{Y(s)}{U(s)} = \begin{bmatrix} 1 & 0 \end{bmatrix} \frac{1}{1 + \frac{B R_a + K_T K_b}{J R_a}} \begin{bmatrix} 1 + \frac{B R_a + K_T K_b}{J R_a} & 1 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ \frac{K_T}{J R_a} \end{bmatrix} + 0$$

$$= \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} 1 & \frac{1}{1+\frac{BRa+K_TK_b}{JRa}} \\ 0 & \frac{1}{1+\frac{BRa+K_TK_b}{JRa}} \end{bmatrix} \begin{bmatrix} 0 \\ \frac{K_T}{JRa} \end{bmatrix}$$

$$= \begin{bmatrix} 1 & \frac{1}{1+\frac{BRa+K_TK_b}{JRa}} \end{bmatrix} \begin{bmatrix} 0 \\ \frac{K_T}{JRa} \end{bmatrix}$$

$$= \frac{JRa}{JRa+BRa+K_TK_b} \frac{K_T}{JRa}$$

$$= \frac{K_T}{JRa+BRa+K_TK_b}$$