Examples:

Q: $G(z) = ?$

$Ga(s) \xrightarrow[\text{matched}]{\text{pole-zero}} G(z)$

Solution $Ga(s) = \dfrac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$

$s^2 + 2\zeta\omega_n s + \omega_n^2$

$s_{1,2} = \dfrac{-2\zeta\omega_n \pm \sqrt{4\zeta^2\omega_n^2 - 4\omega_n^2}}{2\times 1}$

$= -\zeta\omega_n \pm \omega_n\sqrt{\zeta^2 - 1}$

$= -\zeta\omega_n \pm j\omega_n\sqrt{1-\zeta^2}$

$\dfrac{-b \pm \sqrt{b^2-4ac}}{2a}$

apply $\omega_d = \omega_n\sqrt{1-\zeta^2}$

$s_{1,2} = -\zeta\omega_n \pm j\omega_d$

$\omega_d = 5\times\sqrt{1-0.5^2} = 4.33\ \text{rad/s}$

$z = e^{sT}$

$(z - e^{s_1 T})(z - e^{s_2 T})$

$= (z - e^{[-\zeta\omega_n + j\omega_d]T})(z - e^{[-\zeta\omega_n - j\omega_d]T})$

$= z^2 - \left(e^{[-\zeta\omega_n + j\omega_d]T} + e^{[-\zeta\omega_n - j\omega_d]T}\right)z$

$+ e^{[-\zeta\omega_n + j\omega_d]T} \times e^{[-\zeta\omega_n - j\omega_d]T}$

$\zeta = 0.5$

$\omega_n = 5$

$\omega_d = \omega_n\sqrt{1-\zeta^2}$

$= 5\times\sqrt{1-\frac{1}{4}} = \frac{5\sqrt{3}}{2}$

$T = 0.1$

$G(1) = Ga(0)$

$e^{jx} + e^{-jx} = 2\cos x \quad e^{jy} - e^{jx} = 2j\sin x$

$$= z^2 - \left[e^{-\zeta w_n T} \left(e^{j w_d T} + e^{-j w_d T} \right) \right] z + e^{-2\zeta w_n T}$$

$$= z^2 - 2e^{-\zeta w_n T} \cos(w_d T) z + e^{-2\zeta w_n T}$$

$G_a(s)$ 有 2 个极点，0 个零点 $n = 2, m = 0$

$n - m - 1 = 2 - 0 - 1 = 1$

-1 阶：

$$G(z) = \alpha \frac{z+1}{z^2 - 2e^{-\zeta w_n T} \cos(w_d T) z + e^{-2\zeta w_n T}}$$

$G(1) = G_a(0) = 1$

$$G(1) = \frac{2\alpha}{1 - 2e^{-0.5 \times 5 \times 0.1} \cos(4.33 \times 0.1) + e^{-2 \times 0.5 \times 5 \times 0.1}} = 1$$

$\alpha = 0.09634$

why PPT = 0.09625 ?

$$G(z) = \frac{0.09634 (z+1)}{z^2 - 1.4139 z + 0.6065}$$

精度问题

why ppt = 1.414 ?

1．4139跟1．414是一样的