22-S1-Q4

Q (a) (i) K ? $\rightarrow$ deadbeat control

$$x(k+1) = [A - BK]\, x(k)$$

$$[A - BK] = \begin{bmatrix} 1 & 0 \\ -1 & 1 \end{bmatrix} - \begin{bmatrix} 1 \\ 1 \end{bmatrix} K$$

$$K = [0 \ 1] W_c^{-1} \alpha_c(A)$$

$$W_c^{-1} = [B \ \ AB]^{-1}$$

$$AB = \begin{bmatrix} 1 & 0 \\ -1 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}^{-1}$$

$$= -\begin{bmatrix} 0 & -1 \\ -1 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 1 \\ 1 & -1 \end{bmatrix}$$

$$\alpha_c(z) = z^2 = A^2 = \begin{bmatrix} 1 & 0 \\ -1 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ -1 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ -2 & 1 \end{bmatrix}$$

$$K = [0 \ 1] \begin{bmatrix} 0 & 1 \\ 1 & -1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ -2 & 1 \end{bmatrix} = [1 \ -1] \begin{bmatrix} 1 & 0 \\ -2 & 1 \end{bmatrix} = [3 \ -1]$$

$$[A - BK] = \begin{bmatrix} 1 & 0 \\ -1 & 1 \end{bmatrix} - \begin{bmatrix} 1 \\ 1 \end{bmatrix} [3 \ -1]$$

$$= \begin{bmatrix} 1 & 0 \\ -1 & 1 \end{bmatrix} - \begin{bmatrix} 3 & -1 \\ 3 & -1 \end{bmatrix}$$

$$= \begin{bmatrix} -2 & 1 \\ -4 & 2 \end{bmatrix}$$

$\exists q > 1$, $[A-BK]^q = 0$, $x(k) = [A-BK]^q x(0)$

$$[A-BK]^2 = \begin{bmatrix} -2 & 1 \\ -4 & 2 \end{bmatrix} \begin{bmatrix} -2 & 1 \\ -4 & 2 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} = 0 \quad q = 2.$$

(ii) $\hat{x}_1(k)$ 是什么? 还没学到 尝试写

$$\begin{bmatrix} \hat{x}_1(k) \\ \hat{x}_2(k) \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} x_1(k) \\ x_2(k) \end{bmatrix}$$

$$u(k) = -\hat{K} \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} x_1(k) \\ x_2(k) \end{bmatrix}$$

状态转移矩阵

定义 $x(k) = p\,w(k)$ ... lecture 2-5-P4

$$x(k) = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}^{-1} \hat{x}(k)$$

$$p = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}^{-1} = \begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix}$$

$$A_w = p^{-1} A p = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ -1 & 1 \end{bmatrix} \begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 1 \\ -1 & 1 \end{bmatrix} \begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 1 \\ -1 & 2 \end{bmatrix}$$

$$B_w = P^{-1} B = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$$

$$C_w = CP = \begin{bmatrix} 1 & 1 \end{bmatrix} \begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \end{bmatrix}$$

$$\hat{x}(k+1) = \begin{bmatrix} 0 & 1 \\ -1 & 2 \end{bmatrix} \hat{x}(k) + \begin{bmatrix} 2 \\ 1 \end{bmatrix} u(k)$$

$$\hat{y}(k) = \begin{bmatrix} 1 & 0 \end{bmatrix} \hat{x}(k)$$

$$\hat{K} = \begin{bmatrix} 0 & 1 \end{bmatrix} \hat{W}_c^{-1} \alpha_c(A_w)$$

$$\hat{W}_c^{-1} = \begin{bmatrix} B_w & A_w B_w \end{bmatrix}^{-1} = \begin{pmatrix} 2 & 1 \\ 1 & 0 \end{pmatrix}^{-1} = -\begin{bmatrix} 0 & -1 \\ -1 & 2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 1 & -2 \end{bmatrix}$$

$$A_w B_w = \begin{bmatrix} 0 & 1 \\ -1 & 2 \end{bmatrix} \begin{bmatrix} 2 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

$$\alpha_c(z) = z^2$$

$$\alpha_c(A_w) = A_w^2 = \begin{bmatrix} 0 & 1 \\ -1 & 2 \end{bmatrix} \begin{bmatrix} 0 & 1 \\ -1 & 2 \end{bmatrix} = \begin{bmatrix} -1 & 2 \\ -2 & 3 \end{bmatrix}$$

$$\hat{K} = \begin{bmatrix} 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & 1 \\ 1 & -2 \end{bmatrix} \begin{bmatrix} -1 & 2 \\ -2 & 3 \end{bmatrix}$$

$= [1 \ -2]\begin{bmatrix} -1 & 2 \\ -2 & 3 \end{bmatrix}$

$= [3 \ -4]$

(c) optimal feedback gains minimize J
obtain values of $u^*(k)$ as a function of $x(0)$
$k=0,1,2$

Solution $A=5$ $B=1$ $N=3$ $Q=8$ $r=1$

initialize $S(3)=0$

let $k=2$ $K(2)=(1\times 0\times 1+1)^{-1}\times 1\times 0\times 5 = 0$

$S(2)=[5-K(2)]^T\times 0\times[5-K(2)]+K^2(2)+8$
$=8$

let $k=1$ $K(1)=(1\times 8\times 1+1)^{-1}\times 1\times 8\times 5$

$=\frac{40}{9}=4.4444$

$S(1)=(5-1\times\frac{40}{9})\times 8\times(5-1\times\frac{40}{9})+(\frac{40}{9})^2+8$

$=\frac{272}{9}=30.2222$

let $k=0$

$K(0)=(\frac{272}{9}+1)^{-1}\times\frac{272}{9}\times 5$

$=\frac{1360}{281}=4.8399$

$$S(0) = \left(5 - \frac{1360}{281}\right)^2 \times \frac{272}{9} + \left(\frac{1360}{281}\right)^2 + 8$$

$$= \frac{9048}{281} = 32.1993$$

$$u^*(0) = -K(0)\ X(0) = -\frac{1360}{281} X(0) = -4.8399\ X(0)$$

$$X^*(1) = A X(0) + B u^*(0) = 5X(0) + u^*(0)$$

$$= \frac{45}{281} X(0)$$

$$= 0.1601\ X(0)$$

$$u^*(1) = -K(1)\ X(1) = -\frac{44}{9} \times \frac{45}{281} X(0)$$

$$= -\frac{220}{281} X(0)$$

$$= -0.7829\ X(0)$$

$$X^*(2) = 5X(1) + u^*(1)$$

$$= \left(5 \times \frac{45}{281} - \frac{220}{281}\right) X(0)$$

$$= \frac{5}{281} X(0)$$

$$= 0.01779\ X(0)$$

$$u^*(2) = -K(2)\ X(2) = 0$$

Summary $u^*(0) = -4.8399\ X(0)$

$u^*(1) = -0.7829\ X(0)$

$u^*(2) = 0$