Lect3 Example 3-12 $u(0)$? $u(1)$?

Q ①写出状态方程 ②化为离散状态 ③deadbeat K? $x(0)=\begin{bmatrix}1\\1\end{bmatrix}$

$$Y(s)=\frac{1}{s^2}U(s) \Rightarrow \frac{d^2y(t)}{dt^2}=u(t)$$

$$x_1(t)=y(t)$$

$$x_2(t)=\frac{y(t)}{dt}=\dot{x}_1(t)$$

Solution ①

$$\begin{bmatrix}\dot{x}_1(t)\\ \dot{x}_2(t)\end{bmatrix}=\begin{bmatrix}0 & 1\\ 0 & 0\end{bmatrix}\begin{bmatrix}x_1(t)\\ x_2(t)\end{bmatrix}+\begin{bmatrix}0\\1\end{bmatrix}u(t)$$

$$y(t)=\begin{bmatrix}1 & 0\end{bmatrix}\begin{bmatrix}x_1(t)\\ x_2(t)\end{bmatrix}$$

$$\frac{d^2y(t)}{dt^2}=u(t) \Rightarrow \ddot{x}_1(t)=u(t)$$

$$\Rightarrow \dot{x}_2(t)=u(t)$$

② 离散

$$A_d=\Phi(T)=\Phi(t=T)=L^{-1}\{[sI-A]^{-1}\}\Big|_{t=T}$$

$$[SI-A]^{-1} = \begin{bmatrix} s & -1 \\ 0 & s \end{bmatrix}^{-1} = \frac{1}{s^2}\begin{bmatrix} s & 1 \\ 0 & s \end{bmatrix} = \begin{bmatrix} \frac{1}{s} & \frac{1}{s^2} \\ 0 & \frac{1}{s} \end{bmatrix}$$

$$A_d = L^{-1}\left\{\begin{bmatrix} \frac{1}{s} & \frac{1}{s^2} \\ 0 & \frac{1}{s} \end{bmatrix}\right\}\Bigg|_{t=T} = \begin{bmatrix} 1 & T \\ 0 & 1 \end{bmatrix}$$

$$B_d = \int_0^T \Phi(\tau) B \, d\tau = \int_0^T \begin{bmatrix} 1 & \tau \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 0 \\ 1 \end{bmatrix} d\tau$$

$$= \int_0^T \begin{bmatrix} \tau \\ 1 \end{bmatrix} d\tau = \begin{bmatrix} \frac{T^2}{2} \\ T \end{bmatrix}$$

$$C_d = C = [1 \; 0]$$

多检查,别照着抄抄错

$$x(k+1) = \begin{bmatrix} 1 & T \\ 0 & 1 \end{bmatrix} x(k) + \begin{bmatrix} \frac{T^2}{2} \\ T \end{bmatrix} u(k)$$

$$y(k) = [1 \quad 0] \, x(k)$$

⑤ 定义 $\cancel{|\lambda_i[A-BK]| < 1}$

~~或~~ $\cancel{x(k) = [A-BK]^k x(0) = 0, \; k \to \infty}$

$\lambda_i[A-BK] = 0$ 这才是 deadbeat 定义

设 $u(k) = -Kx(k)$

已知 $x(0) = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$ 求K? $u(0)$? $u(1)$?

$$\frac{T^3}{2} - \frac{3T^3}{2} = -T^3$$

$$K = [0\ 1]\, W_c^{-1} \alpha_c(A)$$

$$W_c^{-1} = [B \quad AB]^{-1} = \begin{bmatrix} \frac{T^2}{2} & \frac{3T^2}{2} \\ T & T \end{bmatrix}^{-1} = \frac{1}{-T^3} \begin{bmatrix} T & -\frac{3T^2}{2} \\ -T & \frac{T^2}{2} \end{bmatrix}$$

$$= \begin{bmatrix} -\frac{1}{T^2} & \frac{3}{2T} \\ \frac{1}{T^2} & -\frac{1}{2T} \end{bmatrix}$$

$$\alpha_c(A) = z^2 = A^2 = \begin{bmatrix} 1 & T \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & T \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 2T \\ 0 & 1 \end{bmatrix}$$

$$k = [0\ 1] \begin{bmatrix} \frac{T^2}{2} & \frac{3T^2}{2} \\ T & T \end{bmatrix} \begin{bmatrix} 1 & 2T \\ 0 & 1 \end{bmatrix}$$

$$= [T \quad T] \begin{bmatrix} 1 & 2T \\ 0 & 1 \end{bmatrix}$$

$$= [T \quad 2T^2 + T]$$

$$k = [0\ 1] \begin{bmatrix} -\frac{1}{T^2} & \frac{3}{2T} \\ \frac{1}{T^2} & -\frac{1}{2T} \end{bmatrix} \begin{bmatrix} 1 & 2T \\ 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} \frac{1}{T^2} & -\frac{1}{2T} \end{bmatrix} \begin{bmatrix} 1 & 2T \\ 0 & 1 \end{bmatrix}$$

$$[A-BK]^2 = \begin{bmatrix} \frac{1}{2} & \frac{T}{4} \\ -\frac{1}{T} & -\frac{1}{2} \end{bmatrix} \begin{bmatrix} \frac{1}{2} & \frac{T}{4} \\ -\frac{1}{T} & -\frac{1}{2} \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} \frac{1}{T^2} & \frac{3}{2T} \end{bmatrix}$$

deadbeat $[A-BK]^n = 0$

$$[A-BK] = \begin{bmatrix} 1 & T \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} \frac{T^2}{2} \\ T \end{bmatrix} \begin{bmatrix} \frac{1}{T^2} & \frac{3}{2T} \end{bmatrix}$$

$q = 2$

$x(2) = 0$

$x(3) = 0.$

$\cdots$

$$= \begin{bmatrix} 1 & T \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} \frac{1}{2} & \frac{3}{4}T \\ \frac{1}{T} & \frac{3}{2} \end{bmatrix}$$

$$= \begin{bmatrix} \frac{1}{2} & \frac{T}{4} \\ -\frac{1}{T} & -\frac{1}{2} \end{bmatrix}^{\star}$$

$\star$ 核心证明

some $q > 0$ $x(k) = 0$

$k > 1$

$$u(0) = -Kx(0) = -\begin{bmatrix} \frac{1}{T^2} & \frac{3}{2T} \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix} = -\left(\frac{1}{T^2} + \frac{3}{2T}\right)$$

$$u(1) = -Kx(1) = -K[A-BK]x(0)$$

$$x(k) = [A-BK]^k x(0)$$

$$u(1) = -\begin{bmatrix} \frac{1}{T^2} & \frac{3}{2T} \end{bmatrix} \begin{bmatrix} \frac{1}{2} & \frac{T}{4} \\ -\frac{1}{T} & -\frac{1}{2} \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} \frac{1}{T^2} & \frac{1}{2T} \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix} = \frac{1}{T^2} + \frac{1}{2T}$$