Example 5.5

Q: 设计补偿器 $G_D(z)$

phase margin is 50° w plane

gain margin 10 dB $K_v = 2\ sec^{-1}$

$T = 0.2$ sec

Solution $G_{ZAS}(z) = Z\left\{\frac{1-e^{-Ts}}{s}\cdot\frac{k}{s(s+1)}\right\}$

$= (1-z^{-1})\, Z\left\{\frac{k}{s^2(s+1)}\right\}$

查表#13 $= (1-z^{-1})\, k\, \frac{[(T-1+e^{-T})+(1-e^{-T}-Te^{-T})z^{-1}]z^{-1}}{(1-z^{-1})^2(1-e^{-T}z^{-1})}$

$= \frac{k[0.01873+0.01752\,z^{-1}]z^{-1}}{(1-z^{-1})(1-0.8187\,z^{-1})}$

$= 0.01873\ \frac{k(z+0.935)}{(z-1)(z-0.8187)}$

$z = \frac{1+\frac{wT}{2}}{1-\frac{wT}{2}} = \frac{1+0.1w}{1-0.1w}$

$G_{ZAS}(w) = G_{ZAS}(z)\Big|_{z=\frac{1+0.1w}{1-0.1w}}$

$= 0.01873\ \frac{k\left(\frac{1+0.1w}{1-0.1w}+0.935\right)}{\left(\frac{1+0.1w}{1-0.1w}-1\right)\left(\frac{1+0.1w}{1-0.1w}-0.8187\right)}$

$$= 0.01873 \frac{k(1+0.1w+0.9356-0.09358w)(1-0.1w)}{(1+0.1w-1+0.1w)(1+0.1w-0.8187+0.08187w)}$$

$$= 0.01873 \frac{k(0.00644w+1.09356)(1-0.1w)}{0.1813+0.38187w}$$

$$= \frac{k(0.00644w+1.09356)(1-0.1w)}{20.3881w+9.6797} \times$$

$$\approx \frac{k\left(\frac{w}{300}+1\right)\left(1-\frac{w}{10}\right)}{w(w+1)} \quad ?$$

$$G_D(w) = \frac{Tw+1}{\alpha Tw+1} \qquad 0<\alpha<1$$

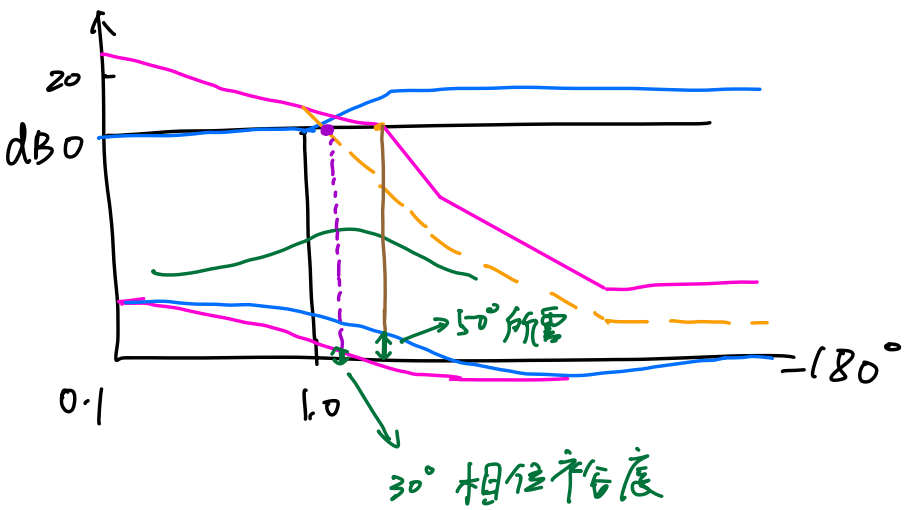
$$G_D(w)G_{ZAS}(w) = \frac{Tw+1}{\alpha Tw+1} \frac{k\left(\frac{w}{300}+1\right)\left(1-\frac{w}{10}\right)}{w(w+1)}$$

$$K_v = 2 = \lim_{w\to 0}\{w[G_D(w)\,G_{ZAS}(w)]\} = k \quad ?$$

So $k=2$

$$G(w) = 2G_{ZAS}(w) = \frac{2\left(\frac{w}{300}+1\right)\left(1-\frac{w}{10}\right)}{w(w+1)} \quad ?$$

Bold plot



$$\phi = 50^\circ - 30^\circ + 8^\circ = 28^\circ$$?

$$\alpha = \frac{1-\sin\phi}{1+\sin\phi} = 0.361$$

$$20\log|G(j\omega)| = 10\log\alpha = -4.425\,dB$$?

$$\Rightarrow \omega_g' = 1.7\ rad/s$$?

$$\omega_g' = \frac{1}{\sqrt{\alpha}T} = 1.7 \Rightarrow T = 0.9790$$?

$$G_D(w) = \frac{Tw+1}{\alpha Tw+1} = \frac{0.9790w+1}{0.3534w+1}$$

$$G(z) = G_D(w)\Big|_{w=\frac{2z-1}{Tz+1}}$$

$$= \frac{0.9790\left(\frac{2z-1}{0.2z+1}\right)+1}{0.3534\left(\frac{2z-1}{0.2z+1}\right)+1}$$

$$= \frac{2.3798z - 1.9378}{z - 0.5569}$$