19-S1-Q2

Q: (a) $C(Z) = ?$

Solution

(a) $$G_1G_2(Z) = \frac{C(Z)}{R(Z) - CH(Z)}$$

$$G_1G_2(Z) = (1-Z^{-1})\,\mathcal{Z}\left\{\frac{G_2(s)}{s}\right\}$$

$$= (1-Z^{-1})\,\mathcal{Z}\left\{\frac{1}{s^3}\right\}$$

$$= (1-Z^{-1})\frac{1}{2}\,\frac{0.1^2 Z^{-1}(1+Z^{-1})}{(1-Z^{-1})^3}$$

$$= \frac{0.005 Z^{-1}(1+Z^{-1})}{(1-Z^{-1})^2}$$

$$R(Z) = \frac{1}{1-Z^{-1}}$$

$$G_1G_2(Z) = \frac{C(Z)}{R(Z) - CH(Z)}$$

$$C(Z) = G_1G_2(Z)R(Z) - \alpha C(Z)G_1G_2(Z)$$

$$[1+\alpha G_1G_2(Z)]\,C(Z) = G_1G_2(Z)R(Z)$$

$$C(Z) = \frac{G_1G_2(Z)R(Z)}{1+\alpha G_1G_2(Z)}$$

$$= \frac{\dfrac{0.005 z^{-1}(1+z^{-1})}{(1-z^{-1})}}{1+\alpha\dfrac{0.005 z^{-1}(1+z^{-1})}{(1-z^{-1})^2}}$$

$$\frac{0.005(z^2+1)}{z^3+(0.005\alpha+1)z^2-z-(1+0.005\alpha)}$$

$$= \frac{0.005 z^{-1}(1+z^{-1})(1-z^{-1})}{(1-z^{-1})^2+\alpha 0.005 z^{-1}(1+z^{-1})}$$

$$= \frac{0.005 z^{-1} - 0.005 z^{-3}}{1+(0.005\alpha-2)z^{-1}+(0.005\alpha+1)z^{-2}} = \frac{0.005(z^2-1)}{z^3+(0.005\alpha-2)z^2+(0.005\alpha+1)z}$$

(b) from $G_{cc}(z)$

$$z^2+(0.005\alpha-2)z+1+0.005\alpha=0$$

$$|z_{1,2}|<1 \implies \text{stable}$$

$$|z| = \left|\frac{2-0.005\alpha \pm\sqrt{(0.005\alpha-2)^2-4(1+0.005\alpha)}}{2}\right| < 1$$

$$-2 < 2-0.005\alpha \pm\sqrt{(0.005\alpha-2)^2-4(1+0.005\alpha)} < 2$$

$$\alpha<0 \quad \text{or} \quad \alpha \geqslant 1600$$

开环脉冲传递函数 $\alpha G(z) = 0.005\alpha\dfrac{z+1}{(z-1)^2}$

有2个 $z=1$ 开环极点，type 2 对 step input 的输入误差为0

(b) try Jury test

$$\frac{C(z)}{R(z)} = \frac{G_1G_2(z)}{1+\alpha G_1G_2(z)}$$

$$1+\alpha\frac{0.005z^{-1}(1+z^{-1})}{(1-z^{-1})^2}$$

$$\frac{z}{z-1} \quad \frac{0.005(1+z)}{z^2+\underline{2z}+1+\alpha 0.005+\underline{\alpha 0.005 z}}$$

$$\frac{0.005(z^2+1)}{(z-1)\left[z^2+(0.005\alpha+2)z+(1+0.005\alpha)\right]}$$

$$= z^3+(0.005\alpha+2)z^2+(1+0.005\alpha)z-z^2$$

$$-(0.005\alpha+2)z-(1+0.005\alpha)$$

$$= z^3+(0.005\alpha+1)z^2-z-(1+0.005\alpha)$$

so $$\frac{0.005(z^2+1)}{z^3+(0.005\alpha+1)z^2-z-(1+0.005\alpha)}$$