Example 5.5

Q: 设计补偿器 $G_D(w)$

phase margin is 50° $\quad$ w plane

gain margin 10 dB $\quad$ $K_v = 2\ \text{sec}^{-1}$

$T = 0.2$ sec

Solution $\quad G_{ZAS}(z) = Z\left\{ \frac{1-e^{-Ts}}{s} \cdot \frac{k}{s(s+1)} \right\}$

$$= (1-z^{-1})\, Z\left\{ \frac{k}{s^2(s+1)} \right\}$$

查表#13 $\quad = (1-z^{-1})\, k \dfrac{[(T-1+e^{-T}) + (1-e^{-T}-Te^{-T})z^{-1}]z^{-1}}{(1-z^{-1})^2(1-e^{-T}z^{-1})}$

$$= \frac{k[0.01873 + 0.01752\, z^{-1}]z^{-1}}{(1-z^{-1})(1-0.8187z^{-1})}$$

$$= 0.01873\, \frac{k(z+0.9356)}{(z-1)(z-0.8187)}$$

$$z = \frac{1+\frac{wT}{2}}{1-\frac{wT}{2}} = \frac{1+0.1w}{1-0.1w}$$

$$G_{ZAS}(w) = G_{ZAS}(z)\Big|_{z=\frac{1+0.1w}{1-0.1w}}$$

$$= 0.01873\, \frac{k\left(\frac{1+0.1w}{1-0.1w} + 0.9356\right)}{\left(\frac{1+0.1w}{1-0.1w} - 1\right)\left(\frac{1+0.1w}{1-0.1w} - 0.8187\right)}$$

$$= 0.01873 \frac{K(1+0.1w+0.9356-0.09356w)(1-0.1w)}{\underbrace{(1+0.1w-1+0.1w)}_{0.2w}(1+0.1w-0.8187+0.08187w)}$$

$0.18187w + 0.1813$

$$= 0.01873 \frac{K(0.00644w+1.09356)(1-0.1w)}{0.1813+0.38187w}$$

$0.2w(0.18187w+0.1813)$

Δ Δ

提Δ出去

$$= \boxed{\frac{K(0.00644w+1.09356)(1-0.1w)}{20.3881w+9.6797}} \times$$

$$\approx \frac{K(\frac{w}{300}+1)(1-\frac{w}{10})}{w(w+1)}$$

? 应该是算错了

$$\frac{0.5149K(0.00644w+1.09356)\cdots}{w(w+1)}$$

$0.00331\underset{5}{8}w + 0.9966$ ≈ 1

$0.00333 = \frac{1}{300}$

$$G_D(w) = \frac{Tw+1}{\alpha Tw+1} \qquad 0<\alpha<1$$

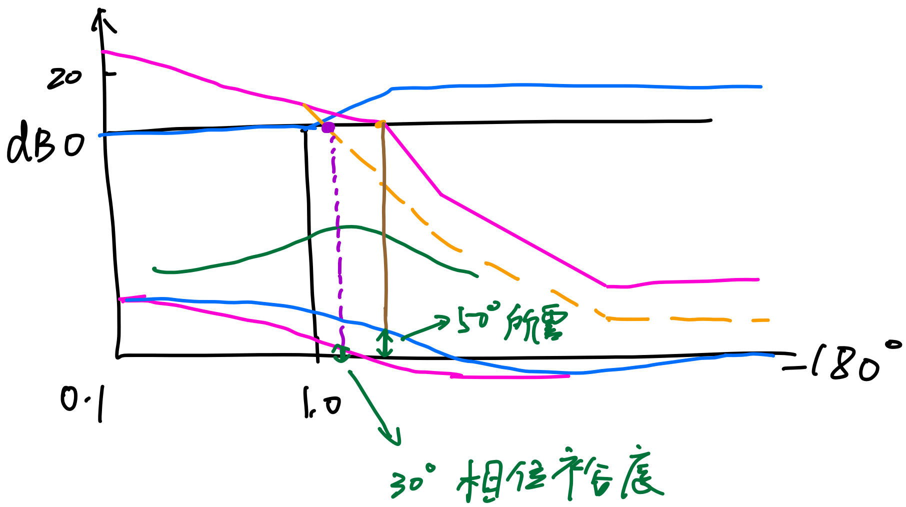
$$G_D(w)G_{ZAS}(w) = \frac{Tw+1}{\alpha Tw+1} \frac{K(\frac{w}{300}+1)(1-\frac{w}{10})}{w(w+1)}$$

$K_v = 2 = \lim_{w\to 0}\{w[G_D(w)\, G_{ZAS}(w)]\} = K$ 公式直接背

So $K=2$

$$G(w) \Rightarrow 2G_{ZAS}(w) = \frac{2(\frac{w}{300}+1)(1-\frac{w}{10})}{w(w+1)}$$

Bold plot



$$\phi = 50^\circ - 30^\circ + 8^\circ = 28^\circ$$ 自选 5−10°都可以

$$\alpha = \frac{1-\sin\phi}{1+\sin\phi} = 0.361$$

$|G(jv_g')| = 0.6$ $\quad 20\log_{10}|G(jw)| = 10\log\alpha = -4.425\,dB$ 增益裕度 相反数

$\Rightarrow v_g' = 1.7\ rad/s$ $\quad |G(jw)| = 10^{-\frac{4.425}{20}} = \cancel{1.6644}$ 0.6

$v_g' = w$?

$$v_g' = \frac{1}{\sqrt{\alpha}\,T} = 1.7 \Rightarrow T = 0.9790$$

$$G_D(w) = \frac{Tw+1}{\alpha T w+1} = \frac{0.9790w+1}{0.3534w+1}$$

$$G_D(z) = G_D(w)\Big|_{w=\frac{2z-1}{Tz+1}}$$

$$= \frac{0.9790\left(\frac{28-1}{0.28+1}\right)+1}{0.3534\left(\frac{28-1}{0.28+1}\right)+1}$$

$$= \frac{2.37988-1.9378}{8-0.5589}$$

$$|G(jv_g')| = 0.6$$

$$|G_{245}(jv_g')| = \left|\frac{2\left(\frac{jv}{300}+1\right)\left(1-\frac{jv}{10}\right)}{jv(jv+1)}\right|$$

$$20\lg\frac{2\sqrt{\left(\frac{v_g'}{300}\right)^2+1}\times\sqrt{1+\left(\frac{v_g'}{10}\right)^2}}{v_g'\sqrt{(v_g')^2+1}} = -4.425$$

幅值公式，参考胡寿松自控

然后用卡西欧求解方程

ALPHA CALC可以打出等号

SHIFT CALC可以要求计算机求解方程

然后填写开始借代的数据，0或者其他

Thought for 1m 11s

To solve for v_g' in the given equation:

$$20\log\left(\frac{2\sqrt{\left(\frac{v_g'}{300}\right)^2+1}\times\sqrt{1+\left(\frac{v_g'}{10}\right)^2}}{v_g'\sqrt{(v_g')^2+1}}\right)=-4.425\text{ dB}$$

We will proceed step by step.

Step 1: Simplify the Equation

Divide both sides by 20 to eliminate the coefficient:

$$\log\left(\frac{2\sqrt{\left(\frac{v_g'}{300}\right)^2+1}\times\sqrt{1+\left(\frac{v_g'}{10}\right)^2}}{v_g'\sqrt{(v_g')^2+1}}\right)=-0.22125$$

Step 2: Remove the Logarithm

Exponentiate both sides to eliminate the logarithm:

$$\frac{2\sqrt{\left(\frac{v_g'}{300}\right)^2+1}\times\sqrt{1+\left(\frac{v_g'}{10}\right)^2}}{v_g'\sqrt{(v_g')^2+1}}=10^{-0.22125}$$

Calculate $10^{-0.22125}$:

$$10^{-0.22125}\approx 0.6009$$

Step 3: Simplify the Fractions

Let $y=v_g'$. The equation becomes:

$$\frac{2\sqrt{\left(\frac{y}{300}\right)^2+1}\times\sqrt{\left(\frac{y}{10}\right)^2+1}}{y\sqrt{y^2+1}}=0.6009$$

Step 4: Square Both Sides to Eliminate Square Roots

$$\left(\frac{2\sqrt{\left(\frac{y}{300}\right)^2+1}\times\sqrt{\left(\frac{y}{10}\right)^2+1}}{y\sqrt{y^2+1}}\right)^2=(0.6009)^2$$

Simplify:

$$\frac{4\left(\left(\frac{y}{300}\right)^2+1\right)\left(\left(\frac{y}{10}\right)^2+1\right)}{y^2(y^2+1)}=0.3611$$

Step 5: Cross-Multiply and Simplify

Multiply both sides by $y^2(y^2+1)$:

$$4\left(\left(\frac{y}{300}\right)^2+1\right)\left(\left(\frac{y}{10}\right)^2+1\right)=0.3611y^2(y^2+1)$$

Simplify the fractions:

$$4\left(\frac{y^2}{90,000}+1\right)\left(\frac{y^2}{100}+1\right)=0.3611y^2(y^2+1)$$

Step 6: Expand and Simplify Both Sides

Left Side:

$$4\left(\frac{y^4}{9,000,000}+\frac{901y^2}{90,000}+1\right)$$

Right Side:

$$0.3611y^4+0.3611y^2$$

Multiply both sides by $2,250,000$ to eliminate denominators:

$$y^4+90,100y^2+9,000,000=812,475y^4+812,475y^2$$

Step 7: Rearrange the Equation

Bring all terms to one side:

$$(1-812,475)y^4+(90,100-812,475)y^2+9,000,000=0$$

Simplify coefficients:

$$-812,474y^4-722,375y^2+9,000,000=0$$

Multiply through by -1:

$$812,474y^4+722,375y^2-9,000,000=0$$

Step 8: Solve the Quadratic Equation

Let $z=y^2$:

$$812,474z^2+722,375z-9,000,000=0$$

Divide by $812,474$:

$$z^2+0.889z-11.08=0$$

Use the quadratic formula:

$$z=\frac{-0.889\pm\sqrt{(0.889)^2+4\times 11.08}}{2}$$

Calculate the discriminant:

$$D=(0.889)^2+44.32\approx 45.11$$

Compute z:

$$z=\frac{-0.889\pm 6.72}{2}$$

Discard the negative solution (since y^2 can't be negative):

$$z=\frac{5.831}{2}\approx 2.9155$$

So $y^2=2.9155$ and $y=\sqrt{2.9155}\approx 1.708$.

Conclusion:

$$v_g'\approx 1.708$$

Answer:

An explicit calculation shows that v'_g ≈ 1.708

$$20\log|G(j\omega_g)| = 10\log a = -4.425\,dB$$

$$|G(j\omega_g)| = 10^{\frac{-4.425}{20}} = 0.6008$$

$$G(j\omega_g) = \frac{2\left(\frac{j\omega_g}{300}+1\right)\left(1-\frac{j\omega_g}{10}\right)}{j\omega_g(1+j\omega_g)}$$

$$|G(j\omega_g)|^2 = 0.6008^2 = 0.361$$

$$= \frac{4\left(1+\frac{\omega_g^2}{90000}\right)\left(1+\frac{\omega_g^2}{100}\right)}{\omega_g^2(1+\omega_g^2)}$$

$$\omega_g^2 = x$$

$$|G(j\omega_g)|^2 = \frac{4\left(1+\frac{x}{90000}\right)\left(1+\frac{x}{100}\right)}{x(1+x)} = 0.361$$

$$4\left(1+\frac{x}{100}+\frac{x}{90000}+\frac{x^2}{9000000}\right) = 0.361x + 0.361x^2$$

$$\downarrow$$

$$\cancel{x_1 =}\ 0.3609x^2 + 0.3209x - 4 = 0$$

$$x_1 = 2.91373$$

$$x_2 = -3.8028 \quad \times$$

$$\omega_g = \sqrt{x_1} = 1.7069$$