lecture 1 Example 1.8

Q $x(k+1)=\begin{bmatrix}0 & 1\\ -2 & -3\end{bmatrix}x(k)+\begin{bmatrix}0\\ 1\end{bmatrix}u(k)$

$y(k)=\begin{bmatrix}3 & 1\end{bmatrix}x(k)$

$x(0)=\begin{bmatrix}0\\ 0\end{bmatrix}$

$u(k)=1$

Solution $x(k)=A^k x(0)+\sum_{i=0}^{k-1}\left(A^{(k-i-1)}Bu(i)\right)$

$=\begin{bmatrix}0 & 1\\ -2 & -3\end{bmatrix}^k\begin{bmatrix}0\\ 0\end{bmatrix}+\sum_{i=0}^{k-1}\left(A^{(k-i-1)}Bu(i)\right)$

$=\sum_{i=0}^{k-1}\left(A^{(k-i-1)}B\right)$

逐推 $x(1)=B=\begin{bmatrix}0\\ 1\end{bmatrix}$

$y(1)=\begin{bmatrix}3 & 1\end{bmatrix}x(1)=\begin{bmatrix}3 & 1\end{bmatrix}\begin{bmatrix}0\\ 1\end{bmatrix}=1$

$x(2)=AB+B=\begin{bmatrix}0 & 1\\ -2 & -3\end{bmatrix}\begin{bmatrix}0\\ 1\end{bmatrix}+\begin{bmatrix}0\\ 1\end{bmatrix}=\begin{bmatrix}1\\ -3\end{bmatrix}+\begin{bmatrix}0\\ 1\end{bmatrix}=\begin{bmatrix}1\\ -2\end{bmatrix}$

$y(2)=\begin{bmatrix}3 & 1\end{bmatrix}\begin{bmatrix}1\\ -2\end{bmatrix}=3-2=1$