19-S1-Q2

Q: (a) $C(z) = ?$

Solution

(a) $G_1G_2(z) = \dfrac{C(z)}{R(z) - CH(z)}$

$$G_1G_2(z) = (1-z^{-1})\, z\left\{\frac{G_2(s)}{s}\right\}$$

$$= (1-z^{-1})\, z\left\{\frac{1}{s^3}\right\}$$

$$= (1-z^{-1})\frac{1}{z}\,\frac{0.1^2 z^{-1}(1+z^{-1})}{(1-z^{-1})^3}$$

$$= \frac{0.005\, z^{-1}(1+z^{-1})}{(1-z^{-1})^2}$$

$$R(z) = \frac{1}{1-z^{-1}}$$

$$G_1G_2(z) = \frac{C(z)}{R(z) - CH(z)}$$

$$C(z) = G_1G_2(z)R(z) - \alpha C(z) G_1G_2(z)$$

$$[1+\alpha G_1G_2(z)]\, C(z) = G_1G_2(z)R(z)$$

$$C(z) = \frac{G_1G_2(z)R(z)}{1+\alpha G_1G_2(z)}$$

others answer?

$$\frac{0.005(z^2+1)}{z^3+(0.005\alpha+1)z^2-z-(1+0.005\alpha)}$$

$$= \frac{\frac{0.005z^{-1}(1+z^{-1})}{(1-z^{-1})}}{1+\alpha\frac{0.005z^{-1}(1+z^{-1})}{(1-z^{-1})^2}}$$

$$= \frac{0.005z^{-1}(1+z^{-1})(1-z^{-1})}{(1-z^{-1})^2+\alpha 0.005z^{-1}(1+z^{-1})}$$

$$= \frac{0.005z^{-1}-0.005z^{-3}}{1+(0.005\alpha-2)z^{-1}+(0.005\alpha+1)z^{-2}} = \frac{0.005(z^2-1)}{z^3+(0.005\alpha-2)z^2+(0.005\alpha+1)z}$$

(b) from $G_{cl}(z)$

$$z^2+(0.005\alpha-2)z+1+0.005\alpha=0$$

$|z_{1,2}|<1 \Rightarrow$ stable

$$|z| = \left|\frac{2-0.005\alpha \pm \sqrt{(0.005\alpha-2)^2-4(1+0.005\alpha)}}{2}\right| < 1$$

$$-2 < 2-0.005\alpha \pm \sqrt{(0.005\alpha-2)^2-4(1+0.005\alpha)} < 2$$

$$\alpha < 0 \quad \text{or} \quad \alpha \geq 1600$$

开环脉冲传递函数 $\alpha G(z) = 0.005\alpha\frac{z+1}{(z-1)^2}$

有2个 $z=1$ 开环极点，type 2 对 step input 的输入误差为 0

(b) try Jury test

$$\frac{C(z)}{R(z)} = \frac{G_1G_2(z)}{1+\alpha G_1G_2(z)}$$

let $1+\alpha\frac{0.005z^{-1}(1+z^{-1})}{(1-z^{-1})^2} = 0$

$$(1-z^{-1})^2 + \alpha 0.005z^{-1} + \alpha 0.005z^{-2} = 0$$

$$1-2z^{-1}+z^{-2}+\alpha 0.005z^{-1} + \alpha 0.005z^{-2} = 0$$

$$1+(\alpha 0.005-2)z^{-1}+(1+\alpha 0.005)z^{-2} = 0$$

$$z^2+(\alpha 0.005-2)z+(1+\alpha 0.005) = 0$$

Jury Test

z^0	z^1	z^2
$1+\alpha 0.005$	$\alpha 0.005-2$	1

$|1+\alpha 0.005| < 1$

$P(1) = 1+0.005\alpha-2+1+0.005\alpha$

$= 0.01\alpha > 0$

$P(-1) = 1-0.005\alpha+2+1+0.005\alpha$

$= 4 > 0$

$n=2$ even

$-1 < 1+0.005\alpha < 1$

$$-2 < 0.005\alpha < 0$$

$$-400$$
$$-2000$$
$$0.005$$

$$\begin{cases} -400 < \alpha < 0 \\ \alpha > 0 \end{cases}$$

So the system is not stable

(a) others answer

$$\frac{z}{z-1}\,\frac{0.005(1+z)}{z^2+2z+1+\alpha 0.005}+\frac{\alpha 0.005 z}{}$$

$$\frac{0.005(z^2+1)}{(z-1)\left[z^2+(0.005\alpha+2)z+(1+0.005\alpha)\right]}$$

$$= z^3+(0.005\alpha+2)z^2+(1+0.005\alpha)z-z^2$$

$$-\left[(0.005\alpha+2)z-(1+0.005\alpha)\right]$$

$$= z^3+(0.005\alpha+1)z^2-z-(1+0.005\alpha)$$

So

$$\frac{0.005(z^2+1)}{z^3+(0.005\alpha+1)z^2-z-(1+0.005\alpha)}$$