22-S1 -Q3

Q : state - space

Solution

$$\begin{bmatrix} \dot{x}_1(t) \\ \dot{x}_2(t) \\ \dot{x}_3(t) \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & -\frac{k_4}{k_5} & \frac{k_3}{k_5} \\ 0 & -k_2 & -k_1 \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \\ x_3(t) \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} u(t)$$

$$y(t) = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \\ x_3(t) \end{bmatrix}$$

$$y(t) = \phi(t) = x_1(t) \Rightarrow C = [1\ 0\ 0]$$

$$u(t) - k_2 x_2(t) = \dot{x}_3(t) + k_1 x_3(t)$$

$$\Rightarrow \dot{x}_3(t) = -k_2 x_2(t) - k_1 x_3(t) + u(t)$$

$$k_3 x_3(t) = k_4 x_2(t) + k_5 \dot{x}_2(t)$$

$$\dot{x}_2(t) = -\frac{k_4}{k_5} x_2(t) + \frac{k_3}{k_5} x_3(t)$$

$$\dot{x}_1(t) = x_2(t)$$

(b) (i) ZOH $T=0.1$ discretised?

$$A_d = \Phi(t=T) = \mathcal{L}^{-1}\{[sI-A]^{-1}\}\Big|_{t=T}$$

$$[sI-A]^{-1} = \begin{bmatrix} s+2 & 0 \\ -3 & s+3 \end{bmatrix}^{-1} = \frac{1}{(s+2)(s+3)} \begin{bmatrix} s+3 & 0 \\ 3 & s+2 \end{bmatrix}$$

$$= \begin{bmatrix} \frac{1}{s+2} & 0 \\ \frac{3}{(s+2)(s+3)} & \frac{1}{s+3} \end{bmatrix}$$

$$A_d = \begin{bmatrix} e^{-2T} & 0 \\ 3[e^{-2T} - e^{-3T}] & e^{-3T} \end{bmatrix} = \begin{bmatrix} e^{-0.2} & 0 \\ 3e^{-0.2} - 3e^{-0.3} & e^{-0.3T} \end{bmatrix}$$

$$B_d = \int_0^T \Phi(\tau) B \, d\tau$$

$$= \int_0^T \begin{bmatrix} e^{-2\tau} & 0 \\ 3[e^{-2\tau} - e^{-3\tau}] & e^{-3\tau} \end{bmatrix} \begin{bmatrix} 2 \\ 0 \end{bmatrix} d\tau$$

$$= \int_0^T \begin{bmatrix} 2e^{-2\tau} \\ 6[e^{-2\tau} - e^{-3\tau}] \end{bmatrix} d\tau$$

$$2\int_0^T e^{-2\tau} d\tau = \frac{2}{-2}\left[e^{-2\tau} \Big|_0^T \right] = -[e^{-2T} - 1] = 1 - e^{-2T}$$

$$6\left[\int_0^T e^{-2\tau} - \int_0^T e^{-3\tau}\right] = 6[\cancel{1 - e^{-2T}} - \cancel{1 + e^{-3T}}]$$

前面有系数2

$$\cancel{= 6(e^{-3T} - e^{-2T})}$$

$$\int_0^T e^{-2\tau} = \frac{1}{2}(1 - e^{-2T}) \qquad \int_0^T e^{-3\tau} = \frac{e^{-3\tau}}{-3}\Big|_0^T$$

$$6\int_0^T e^{-2\tau} - 6\int_0^T e^{-3\tau}\Big] = 3(1-e^{-2T}) + 2(e^{-3T}-1) \qquad = -\tfrac{1}{3}(e^{-3T}-1)$$

$$= 3 - 3e^{-2T} + 2e^{-3T} - 2$$

$$B_d = \begin{bmatrix} 1-e^{-0.2} \\ 6(e^{-0.3}-e^{-0.2}) \end{bmatrix} \qquad = -3e^{-0.2} + 2e^{-0.3} + 1$$

$$0.02544 \qquad = 0.02544$$

$$C_d = C = [1 \quad 0]$$

$$x(k+1) = \begin{bmatrix} e^{-0.2} & 0 \\ 3e^{-0.2} - 3e^{-0.3} & e^{-0.3T} \end{bmatrix} x(k) + \begin{bmatrix} 1-e^{-0.2} \\ 6(e^{-0.3}-e^{-0.2}) \end{bmatrix} u(k)$$

$$0.02544$$

$$y(k) = [1 \quad 0]\, x(k)$$ 请写出最终小数

$$x(k+1) = \begin{bmatrix} 0.8187 & 0 \\ 0.2337 & 0.7408 \end{bmatrix} x(k) + \begin{bmatrix} 0.1813 \\ \cancel{-0.4875} \end{bmatrix} u(k)$$

0.02544

$\cancel{0.23375}$

(ii) $\dfrac{Y(z)}{U(z)} = C[zI - \cancel{A}]^{-1} B + D$

discretised state must use $A_d\ B_d\ C_d$ not $A\ B\ C$

$$= [1 \quad 0]\begin{bmatrix} \frac{1}{z+2} & 0 \\ \frac{3}{(z+2)(z+3)} & \frac{1}{z+3} \end{bmatrix}\begin{bmatrix} 2 \\ 0 \end{bmatrix}$$

$$= \left[\frac{1}{z+2} \quad 0\right]\begin{bmatrix} 2 \\ 0 \end{bmatrix}$$

$$= \frac{2}{z+2} \qquad z = -2 \qquad |z| > 1 \quad \text{no stable}$$

stable poles : find K ? let $\alpha_c(z) = z$

$$|zI - A + BK| = \begin{vmatrix} z+2+2k_1 & 2k_2 \\ -3 & z+3 \end{vmatrix}$$

$$\frac{Y(z)}{U(z)} = C_d[zI - A_d]^{-1} B_d = [1 \quad 0]\begin{bmatrix} z - e^{-2T} & 0 \\ -3e^{-2T} + 3e^{-3T} & z - e^{-3T} \end{bmatrix}^{-1}\begin{bmatrix} 1-e^{-2T} \\ 6e^{-3T} - 6e^{-2T} \end{bmatrix}$$

pole : $|zI - A_d| = (z - e^{-2T})(z - e^{-3T})$

two poles : $z_1 = e^{-2T}$ $z_2 = e^{-3T}$ for $T > 0$, $z_{1,2} \in (0,1)$

Stable

So $T > 0$

$$= \cancel{z^2 + (5+2k_1)z + 6 + 6k_1 + 6k_2}$$

$$\cancel{\text{let} \begin{cases} 5+2k_1 = 0 \\ 6+6k_1+6k_2 = 0 \end{cases}} \Rightarrow \begin{cases} k_1 = -\frac{5}{2} \\ k_2 = \frac{3}{2} \end{cases}$$

(c) Q : State space model ?

$$x(k+1) = A\,x(k) + B\,u(k)$$

$$y(k) = C\,x(k) + d\,u(k)$$

$$x_3 = u(k)$$

$$u(k+1) = -a\,u(k) + r(k) + y(k)$$

$$\begin{bmatrix} x_1(k+1) \\ x_2(k+1) \\ x_3(k+1) \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ -1 & 0 & -a \end{bmatrix} \begin{bmatrix} x_1(k) \\ x_2(k) \\ x_3(k) \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} r(k)$$

$$\begin{bmatrix} y_1(k) \\ y_2(k) \\ y_3(k) \end{bmatrix} = [1\ 0\ 0] \begin{bmatrix} x_1(k) \\ x_2(k) \\ x_3(k) \end{bmatrix}$$

$$x_3(k+1) = -a\,x_3(k) + r(k) - y(k) \quad ①$$

$$x_1(k+1) = x_1(k) + x_2(k) + u(k) = x_1 + x_2 + x_3$$

$$x_2(k+1) = x_2(k) + u(k) = x_2(k) + x_3(k)$$

$$y(k) = x_1(k) \quad ②$$

①② : $x_3(k+1) = -x_1(k) - a\,x_3(k) + r(k)$