21 - S1 - Q4

Q: $A = \begin{bmatrix} \alpha & -1 \\ -4 & 4 \end{bmatrix}$ $B = \begin{bmatrix} 4 \\ 3 \end{bmatrix}$ $C = [2, \ 2]$

(a) (i) $C \rightarrow \alpha$?

Solution $|W_c| = |[B \ \ AB]|$

$$AB = \begin{bmatrix} \alpha & -1 \\ -4 & 4 \end{bmatrix}\begin{bmatrix} 4 \\ 3 \end{bmatrix} = \begin{bmatrix} 4\alpha - 3 \\ -16+12 \end{bmatrix} = \begin{bmatrix} 4\alpha - 3 \\ -4 \end{bmatrix}$$

$$|W_c| = \begin{vmatrix} 4 & 4\alpha - 3 \\ 3 & -4 \end{vmatrix} = -16 - 3(4\alpha - 3)$$

$$= -12\alpha - 7 \neq 0 \quad \Rightarrow \alpha \neq \frac{-7}{12}$$

(ii) Q: $\alpha = 2$ $\quad z_{12} = \pm j0.25 \Rightarrow K$?

Solution

$$K = [0 \ 1] \, W_c^{-1} \alpha_c(A)$$

$$W_c^{-1} = \begin{bmatrix} 4 & 5 \\ 3 & -4 \end{bmatrix}^{-1}$$

$$= \frac{1}{-31}\begin{bmatrix} -4 & -5 \\ -3 & 4 \end{bmatrix}$$

$$= \begin{bmatrix} \frac{4}{31} & \frac{5}{31} \\ \frac{3}{31} & \frac{4}{-31} \end{bmatrix}$$

照抄都能抄错?

$-16-15$
$= -31$

写完一个算式 必须检查两遍

$$\alpha_c(z) = (z - j\,\underset{0.25}{\cancel{0.125}})(z + j\,\underset{0.25}{\cancel{0.125}})$$

$$= z^2 + (\underset{-0.25j}{\cancel{-j0.125}})(\underset{+0.25j}{\cancel{j0.125}}) = z^2 + 0.0625$$

$a = 0 \quad b = -0.125 \quad c = 0 \quad d = 0.125$

$(ac - bd,\ \cancel{bc-ad})$

$= 0.125^2$

$= 0.015625$

$\alpha_c(z) = z^2 + \cancel{0.015625}$ 0.0625

$\alpha_c(A) = A^2 + \cancel{0.015625}\, I_2$ 0.0625

$$= \begin{bmatrix} 2 & -1 \\ -4 & 4 \end{bmatrix}\begin{bmatrix} 2 & -1 \\ -4 & 4 \end{bmatrix}$$

$4+4$

$-8-16=-24$

$-2-4$

$4+16$

0.0625

$$= \begin{bmatrix} 8 & -6 \\ -24 & 20 \end{bmatrix} + \cancel{0.0125625}\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

8.0625

$$= \begin{bmatrix} \cancel{8.01256} & -6 \\ -24 & \cancel{20.01256} \end{bmatrix}$$

20.0625

$K = [0\ 1]\, W_c^{-1} \alpha_c(A)$

8.0625 20.0625

$$= [0\ 1]\begin{bmatrix} \frac{4}{31} & \frac{5}{31} \\ \frac{3}{31} & -\frac{4}{31} \end{bmatrix}\begin{bmatrix} \cancel{8.01256} & -6 \\ -24 & \cancel{20.01258} \end{bmatrix}$$

8.0625

$$= \begin{bmatrix} \frac{3}{31} & -\frac{4}{31} \end{bmatrix}\begin{bmatrix} \cancel{8.01256} & -6 \\ -24 & \cancel{20.01256} \end{bmatrix}$$ 20.0625

$= [3.8714\quad -3.1622]$

$0.09677 \times 8.\sim$

$+(-0.1290)\times(-24)$

$= [3.8770\ ,\ -3.1694]$

法二求K. $|zI - (A - BK)|$

$$A - BK = \begin{bmatrix} 2 & -1 \\ -4 & 4 \end{bmatrix} - \begin{bmatrix} 4 \\ 3 \end{bmatrix} \begin{bmatrix} k_1 & k_2 \end{bmatrix}$$

$$= \begin{bmatrix} 2-4k_1 & -1-4k_2 \\ -4-3k_1 & 4-3k_2 \end{bmatrix}$$

$$\left| zI - (A - BK) \right| = \begin{vmatrix} z-2+4k_1 & 1+4k_2 \\ 4+3k_1 & z-4+3k_2 \end{vmatrix}$$

$$= z^2 + (-4+3k_2-2+4k_1)z + (-2+4k_1)(-4+3k_2)$$

$$-(1+4k_2)(4+3k_1)$$

$$-2\times(-4) = 8$$

$$= z^2 + (4k_1+3k_2-6)z + \Big[-8 - 6k_2 - 16k_1 \cancel{+12k_1k_2}$$

$$- 4 - 3k_1 - 16k_2 \cancel{-12k_1k_2}\Big]$$

$$= z^2 + (4k_1+3k_2-6)z + \left[-19k_1 - 22k_2 - 12\right]$$

$$\begin{cases} 4k_1+3k_2-6 = 0 \\ -19k_1-22k_2 \cancel{+12}^{+4} = 0.0625 \end{cases} \qquad \begin{cases} 4k_1+3k_2 = 6 \\ 19k_1+22k_2 = 3.9375 \end{cases}$$

$$\alpha_c(z) = z^2 + 0.0625$$

$k_2 = \dfrac{6-4k_1}{3}$ 代入②式

$$19k_1 + \frac{22}{3}(6-4k_1) = 3.9375$$

$$19k_1 + 44 - \frac{88}{3}k_1 = 3.9375$$

$$\begin{cases} k_1 = \cancel{5.4254}\ 3.8770 \\ k_2 = \cancel{-5.2339}\ -3.1694 \end{cases}$$

$$-\frac{31}{3}k_1 = -\frac{641}{16}$$

$$k_1 = \frac{1923}{496}$$

$$= 3.8770$$

不能代小数

代分数

$$k_2 = \cancel{-\frac{2377}{750}} \quad -\frac{393}{124}$$

$$= \cancel{-3.16933} \quad -3.1694$$

(b)(i)

With $\alpha = 2$, A and C are:

$$A = \begin{bmatrix} 2 & -1 \\ -4 & 4 \end{bmatrix}, \quad C = \begin{bmatrix} 2 & 2 \end{bmatrix}$$

We need $L_0 = \begin{bmatrix} l_1 \\ l_2 \end{bmatrix}$ such that the eigenvalues of $A - L_0C$ are at $z = 0.4 \pm j0.4$. The desired characteristic equation is:

$$z^2 - 0.8z + 0.32 = 0$$

Compute $A - L_0C$:

$$A - L_0C = \begin{bmatrix} 2 - 2l_1 & -1 - 2l_1 \\ -4 - 2l_2 & 4 - 2l_2 \end{bmatrix}$$

Set up the characteristic equation and match coefficients:

$$\begin{cases} 2l_1 + 2l_2 - 6 = -0.8 \\ 4 - 16l_1 - 6l_2 = 0.32 \end{cases}$$

Solving these equations:

1. From the first equation:
$$l_1 + l_2 = 2.6$$
2. Substitute $l_2 = 2.6 - l_1$ into the second equation and solve for l_1, yielding $l_1 \approx -1.192$.
3. Then $l_2 = 2.6 - (-1.192) = 3.792$.

Answer: The estimator gain is $L_0 = \begin{bmatrix} -1.192 \\ 3.792 \end{bmatrix}$.

(b)(ii)

Given $z = 0.4 \pm j0.4$, find the equivalent s-plane poles:

1. Compute the magnitude and angle:

$$|z| = \sqrt{0.4^2 + 0.4^2} = 0.5657, \quad \theta = \arctan\left(\frac{0.4}{0.4}\right) = 45^\circ$$

2. Convert to s-domain using $s = \frac{\ln(z)}{T}$:

$$s = \frac{\ln(0.5657) + j\frac{\pi}{4}}{0.1} = -5.68 \pm j7.854$$

3. Compute the damping ratio:

$$\zeta = \frac{-\mathrm{Re}(s)}{\sqrt{\mathrm{Re}(s)^2 + \mathrm{Im}(s)^2}} = \frac{5.68}{9.688} \approx 0.586$$

Answer: The s-plane poles are at $s = -5.68 \pm j7.854$ with a damping ratio of approximately 0.586.

(c)

Given $A = 1$, $B = 0.8$, $Q = 8$, $r = 1.6$.

Solve the algebraic Riccati equation:

$$0.64S^2 = 8 \times 1.6 + 0.64 \times 8 \times S \implies 0.64S^2 - 5.12S - 12.8 = 0$$

Simplify and solve:

$$S^2 - 8S - 20 = 0 \implies S = 10 \text{ (since } S > 0)$$

Compute the optimal gain:

$$K = \frac{BS}{B^2S + r} = \frac{0.8 \times 10}{0.64 \times 10 + 1.6} = \frac{8}{8} = 1$$

Thus, the optimal control law is $u^*(k) = -x(k)$, and the closed-loop equation is:

$$x(k+1) = x(k) + 0.8(-x(k)) = 0.2x(k)$$

Answer: The optimal control law is $u^*(k) = -x(k)$; the closed-loop state equation is $x(k+1) = 0.2\,x(k)$.