

Lect 3 Example 3-4

$$Q: \frac{Y(z)}{U(z)} = \frac{z+0.2}{(z+0.8)(z+0.2)} = \frac{z+0.2}{z^2+z+0.16}$$

controllable? observable?

Solution

① pole-zero cancellations. $\rightarrow Cx$
or $0x$
or $Cx \ 0x$

② CCF $\begin{matrix} 0 & 1 \\ & \lambda \end{matrix}$

$$A_c = \begin{bmatrix} 0 & 1 \\ -0.16 & -1 \end{bmatrix} \quad B_c = \begin{bmatrix} 0 \\ 1 \end{bmatrix} \quad C_c = [0.2 \ 1] \quad d_c = 0$$

$$W_c = [B_c \ A_c B_c] = \begin{bmatrix} 0 & 1 \\ 1 & -1 \end{bmatrix} \quad |W_c| = -1 \neq 0$$

controllable.

$$W_o = \begin{bmatrix} C_c \\ C_c A_c \end{bmatrix} = \begin{bmatrix} 0.2 & 1 \\ -0.16 & -0.8 \end{bmatrix} \quad |W_o| = -0.16 + 0.16 = 0$$

unobservable.

$$C_c A_c = [0.2 \ 1] \begin{bmatrix} 0 & 1 \\ -0.16 & -1 \end{bmatrix} = [0.16 \ -0.8]$$

③ OCF $p \lambda$

$$A_o = \begin{bmatrix} 0 & -0.16 \\ 1 & -1 \end{bmatrix} \quad B_o = \begin{bmatrix} 0.2 \\ 1 \end{bmatrix} \quad C_o = [0 \ 1] \quad d_o = 0$$

$$W_o = \begin{bmatrix} C_o \\ C_o A_o \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ \textcircled{0} & -1 \end{bmatrix} \quad C_o A_o = [0 \ 1] \begin{bmatrix} 0 & -0.16 \\ \textcircled{0} & -1 \end{bmatrix} = [\textcircled{0} \ -1]$$

$$|W_o| = \textcircled{0} - 1 \quad \textcircled{0} \text{unobservable.}$$

$$W_c = [B_o \ A_o B_o] = \begin{bmatrix} 0.2 & -0.16 \\ 1 & -0.8 \end{bmatrix} \quad A_o B_o = \begin{bmatrix} 0 & -0.16 \\ 1 & -1 \end{bmatrix} \begin{bmatrix} 0.2 \\ 1 \end{bmatrix}$$

$$|W_c| = -0.16 + 0.16 = 0 \quad = [-0.16 \ -0.8]^T$$

'un controllable