Lect 3 example 3-6

Q: ZOH → controllability and observability

$$\dot{x}(t) = \begin{bmatrix} 0 & w \\ -w & 0 \end{bmatrix} x(t) + \begin{bmatrix} 0 \\ w \end{bmatrix} u(t)$$

请问状态方程是怎么得到的？

$$y(t) = \begin{bmatrix} 1 & 0 \end{bmatrix} x(t)$$

→ 位置
速度.

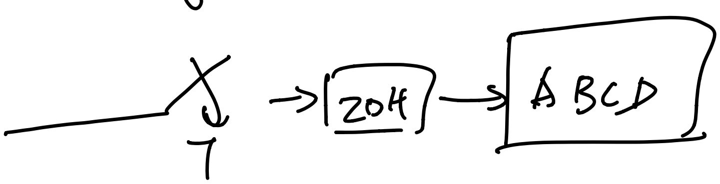
$$H(s) = \frac{w^2}{s^2 + w^2}$$

→ 自然频率

请问H(s)是怎么得到的？

$$= C\,[sI - A]^{-1} B$$

Summary



$$A_d = \Phi(t = T) \qquad \Phi(t) = \mathcal{L}^{-1}\{(sI - A)^{-1}\}$$

$$B_d = \int_0^T \Phi(\tau)\, B \, d\tau$$

$$C = C$$

$$W_{cd} = \begin{bmatrix} B_d & A_d B_d & \cdots & A_d^{n-1} B_d \end{bmatrix}$$

$$W_{od} = \begin{bmatrix} C_d \\ C_d A_d \\ \vdots \\ C_d A_d^{n-1} \end{bmatrix}$$

Solution: ① 先算状态 matrix $\Phi(t)$

$$(sI-A)^{-1} = \begin{bmatrix} s & -w \\ w & s \end{bmatrix}^{-1} = \frac{1}{s^2+w^2}\begin{bmatrix} s & w \\ -w & s \end{bmatrix}$$

$$\Phi(T) = L^{-1}\{(sI-A)^{-1}\} = L^{-1}\left\{\begin{bmatrix} \frac{s}{s^2+w^2} & \frac{w}{s^2+w^2} \\ -\frac{w}{s^2+w^2} & \frac{s}{s^2+w^2} \end{bmatrix}\right\}$$

$$= \begin{bmatrix} \cos wT & \sin wT \\ -\sin wT & \cos wT \end{bmatrix} = A_d$$

$$B_d = \int_0^T \Phi(t) B \, dt$$

$$= \int_0^T \begin{bmatrix} \cos wt & \sin wt \\ -\sin wt & \cos wt \end{bmatrix}\begin{bmatrix} 0 \\ w \end{bmatrix} dt$$

$$= \int_0^T \begin{bmatrix} w \sin wt \\ w \cos wt \end{bmatrix} dt = \begin{bmatrix} 1-\cos wT \\ \frac{\sin wT}{\cancel{w}} \end{bmatrix}$$

其中 $\int_0^T w \sin wt \, dt$

$$= w\int_0^T \sin wt \, dt$$

$$= w \times \left(-\frac{\cos wt}{w}\Big|_0^T\right) = -(\cos wT - \cos 0) = 1-\cos wT$$

其中 $\int_0^T \omega \cos\omega t \, dt$

不要忘了乘下来

$$= \omega \frac{\sin\omega t}{\omega} \Big|_0^T$$

$$= \frac{\sin\omega T}{\omega} \quad \sin\omega T$$

$$A_d = \begin{bmatrix} \cos\omega T & \sin\omega T \\ -\sin\omega T & \cos\omega T \end{bmatrix} \qquad B_d = \begin{bmatrix} 1-\cos\omega T \\ \frac{\sin\omega T}{\cancel{\omega}} \end{bmatrix}$$

$$C_d = C = [1 \quad 0]$$

$$|W_c| = [B_d \quad A_d B_d] = \left| \begin{bmatrix} 1-\cos\omega T & \cos\omega T(1-\cos\omega T) + \frac{\sin^2\omega T}{\cancel{\omega}} \\ \frac{\sin\omega T}{\cancel{\omega}} & \sin\omega T(\cos\omega T - 1) + \frac{\sin\omega T \cos\omega T}{\cancel{\omega}} \end{bmatrix} \right|$$

$$A_d B_d = \begin{bmatrix} \cos\omega T(1-\cos\omega T) + \frac{\sin^2\omega T}{\cancel{\omega}} \\ \sin\omega T(\cos\omega T - 1) + \frac{\sin\omega T \cos\omega T}{\cancel{\omega}} \end{bmatrix}$$

$$|W_c| = -s(c-1)^2 + \frac{sc(1-c)}{\cancel{\omega}} - \frac{sc(1-c)}{\cancel{\omega}} - \frac{s^3}{\cancel{\omega^2}}$$

$$= -s(c-1)^2 - \frac{s^3}{\cancel{\omega^2}}$$

$$= -s(c-1)^2 - s(1-c^2)$$

$$= -s(c^2-2c+1) - s + sc^2$$

$$= -sc^2 + 2sc - s - s + sc^2$$

$$= 2s(c-1)$$

$$= 2\sin\omega T\,(\cos\omega T - 1)$$

$\omega T = k\pi$, $k \in N$, $|W_c| = 0$, uncontrollable

$$|W_o| = \begin{vmatrix} C_d \\ C_d A_d \end{vmatrix} = \begin{vmatrix} 1 & 0 \\ \cos\omega T & \sin\omega T \end{vmatrix} = \sin\omega T$$

$\omega T = k\pi$, $k \in N$

$|W_o| = 0$

unobeservable

$$C_d A_d = [1 \quad 0] \begin{bmatrix} \cos\omega T & \sin\omega T \\ -\sin\omega T & \cos\omega T \end{bmatrix}$$

$$= [\cos\omega T \quad \sin\omega T]$$