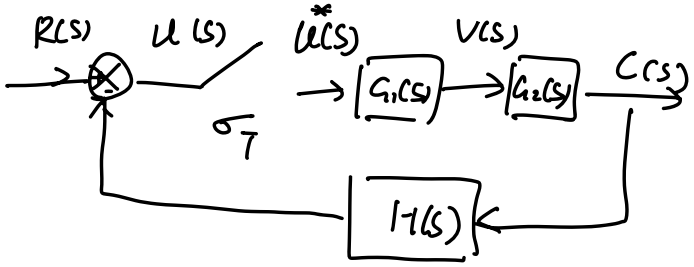
21-S1-Q2

Q: (a) show pulse transfer function

Solution



$$G_p(z) = \frac{C(z)}{R(z)}$$

$$U(S) = R(S) - C(S)H(S) = R(S) - CH(S)$$

$$G_1(S) = \frac{V(S)}{U^*(S)} \qquad G_2(S) = \frac{C(S)}{V(S)}$$

消去V(S)

$$G_1(S)\,U^*(S) = \frac{C(S)}{G_2(S)}$$

$$C(S) = U^*(S)\,G_1G_2(S)$$

$$C^*(S) = U^*(S)\,G_1G_2^*(S)$$

$$C(z) = U(z)\,G_1G_2(z)$$

$$U^*(S) = R^*(S) - CH^*(S)$$

$$U(z) = R(z) - CH(z)$$

$$C(z) = [R(z) - CH(z)]\,G_1G_2(z)$$

$$= R(z)\,G_1G_2(z) - CH(z)\,G_1G_2(z)$$

$$G_p(z) = \frac{C(z)}{R(z)} = G_1G_2(z) - \frac{CH(z)\,G_1G_2(z)}{R(z)}$$

X

Q(b) $G_1(s) = \frac{1-e^{-Ts}}{s}$ $G_2(s) = \frac{10(0.5s+1)}{s^2}$ $H(s) = 1$

$T = 0.2$

Solution

$G_p(z) = Z\left\{ \frac{1-e^{-Ts}}{s} \cdot \frac{10(0.5s+1)}{s^2} \right\}$ — X ↑ X

Solution - V2

(a) $G_1G_2(z) = \frac{C(z)}{R(z) - H(z)C(z)}$

$C(z) = G_1G_2(z)R(z) - H(z)C(z)G_1G_2(z)$

$[1 + H(z)G_1G_2(z)]C(z) = G_1G_2(z)R(z)$

$\frac{C(z)}{R(z)} = \frac{G_1G_2(z)}{1 + H(z)G_1G_2(z)}$

$E(s) = R(s) - C(s)H(s)$

$C(s) = E^*(s)G_1G_2(s)$

$E(s) = R(s) - E^*(s)G_1G_2H(s)$

$E^*(s) = R^*(s) - E^*(s)G_1G_2H^*(s)$

$E^*(s) = \frac{R^*(s)}{1 + G_1G_2H^*(s)}$

$C^*(s) = E^*(s)G_1G_2^*(s)$

$\frac{C(z)}{R(z)} = \frac{G_1G_2(z)}{1 + G_1G_2H(z)}$

(b) $\frac{C(z)}{R(z)} = \frac{Z\left\{\frac{1-e^{-Ts}}{s}\frac{10(0.5s+1)}{s^2}\right\}}{1 + Z\left\{\frac{1-e^{-Ts}}{s}\frac{10(0.5s+1)}{s^2}\right\}}$

其中 $Z\left\{\frac{1-e^{-Ts}}{s}\frac{10(0.5s+1)}{s^2}\right\}$

$= 10(1-z^{-1})Z\left\{\frac{0.5s+1}{s^3}\right\}$

其中 $Z\left\{\frac{0.5s+1}{s^3}\right\}$

$$=0.5Z\left\{\frac{1}{s^2}\right\}+\frac{1}{2}Z\left\{\frac{2}{s^3}\right\}$$

$$=\frac{1}{2}\frac{0.2z^{-1}}{(1-z^{-1})^2}+\frac{1}{2}\frac{0.2^2z^{-1}(1+z^{-1})}{(1-z^{-1})^3}$$

$$=\frac{0.2z^{-1}(1-z^{-1})+0.04z^{-1}(1+z^{-1})}{2(1-z^{-1})^3}$$

$$S_D \quad Z\left\{\frac{1-e^{-Ts}}{s}\frac{10(0.5s+1)}{s^2}\right\}$$

$$=10(1-z^{-1})Z\left\{\frac{0.5s+1}{s^3}\right\}$$ 符号不用，不能合并同类项

$$=10(1-z^{-1})\frac{0.2z^{-1}(1-z^{-1})+0.04z^{-1}(1+z^{-1})}{2(1-z^{-1})^3}$$

$$=\frac{z^{-1}(1-z^{-1})+0.2z^{-1}(1+z^{-1})}{(1-z^{-1})^2}$$

$$=\frac{1.2z^{-1}}{(1-z^{-1})}$$

$$=\frac{(z-1)+0.2(z+1)}{(z-1)^2}$$

$$=\frac{1.2}{z-1}$$

$$=\frac{z-1+0.2z+0.2}{(z-1)^2}$$

$$\frac{C(z)}{R(z)}=\frac{\frac{1.2}{z-1}}{1+\frac{1.2}{z-1}}$$

$$=\frac{1.2z-0.8}{(z-1)^2}$$

$$= \frac{1.2}{z-1+1.2}$$

$$= \frac{1.2}{z+0.2}$$

$$\frac{C(z)}{R(z)} = \frac{\frac{1.2z-0.8}{(z-1)^2}}{1+\frac{1.2z-0.8}{(z-1)^2}}$$

$$= \frac{1.2z-0.8}{(z-1)^2+1.2z-0.8}$$

$$= \frac{1.2z-0.8}{z^2-2z+1+1.2z-0.8}$$

$$= \frac{1.2z-0.8}{z^2-0.8z+0.2}$$

(c) Jury Test

$z^2 - 0.8z + 0.2$

	z^0	z^1	z^2
1	0.2	−0.8	1

① $|0.2| < 1$ ✓

② $P(1) = 1 - 0.8 + 0.2 = 0.4 > 0$ ✓

③ $P(-1) = 1 + 0.8 + 0.2 = 2 > 0$ ✓ $n = 2$, even

Hence, the CE is stable