Lect3 Example 3.8

Q $T = 0.1$

$$x(k+1) = \begin{bmatrix} 1 & 0.0952 \\ 0 & 0.905 \end{bmatrix} x(k) + \begin{bmatrix} 0.00484 \\ 0.0952 \end{bmatrix} u(k)$$

$$y(k) = [1 \;\; 0]\, x(k)$$

state feedback gain matrix ?

poles: $(z-p_1)(z-p_2) = 0$

$0.888 \pm j\,0.173 = 0.905 \angle \pm 0.193$

$$\zeta = \frac{-\ln r}{\sqrt{\ln^2 r + \theta^2}} = 0.46 \qquad T = \frac{-T}{\ln r} = 1 \text{ sec.}$$

Solution: ① C → 可K

$$|W_c| = |B \;\; AB| = \begin{vmatrix} 0.00484 & 0.0139 \\ 0.0952 & 0.0862 \end{vmatrix} \Rightarrow |W_c| \neq 0$$

② $|zI - A + BK|$

$$= \begin{vmatrix} z - 1 + 0.00484k_1 & -0.952 + 0.00484k_2 \\ 0.0952k_1 & z - 0.905 + 0.0952k_2 \end{vmatrix}$$

$$BK = \begin{bmatrix} 0.00484k_1 & 0.00484k_2 \\ 0.0952k_1 & 0.0952k_2 \end{bmatrix}$$

$[\,]_{I\to}$

$$= z^2 + (0.00484k_1 + 0.0952k_2 - 1.905)z + (0.00484k_1 - 0.0952k_2 + 0.905)$$

$\alpha_c(z) = z^2 - 1.776z + 0.819$

怎么得的？ $0.888 \pm j0.173$

0.905 ± 0.193

$\alpha(z) \equiv \alpha_c(z)$

$$\begin{cases} 0.00484k_1 + 0.0952k_2 - 1.905 = -1.776 \\ 0.00484k_1 - 0.0952k_2 + 0.905 = 0.819 \end{cases}$$

$$\Rightarrow \begin{cases} k_1 = 4.52 \\ k_2 = 1.12 \end{cases}$$

$$u(k) = -kX = -[4.52 \quad 1.12] \begin{bmatrix} x_1(k) \\ x_2(k) \end{bmatrix}$$

$$= -4.52\, x_1(k) - 1.12\, x_2(k)$$

Solve Problem:

$0.888 + j0.173$ $\quad$ $0.888 - j0.173$ $\quad$ 为 p_1 p_2

$$\alpha_c(z) = [z - (0.888 + j0.173)][z - (0.888 - j0.173)]$$

$$= z^2 - [(0.888 - j0.173) + (0.888 + j0.173)]z$$

$$+ (0.888 + j0.173)(0.888 - j0.173)$$

$$= z^2 - 1.776z$$

$a = 0.888 \quad b = 0.173 \quad c = 0.888 \quad d = -0.173$

$(ac - bd,\ bc + ad)$

$$= (0.888^2 + 0.173^2,\ 0.888 \times 0.173 - 0.888 \times 0.173)$$

$$= 0.8185$$

$$\alpha(c) = z^2 - 1.776z + 0.8185$$