22 - S1 - Q1

Q: (a) continuous? $T=0.5$ Z? $\lim_{k\to\infty} x(kT) = ?$

Solution

Justify: Yes, because it is defined for all real values of t and the function x(t) provides a specific value for every time t in the real number domain, fulfilling the definition of a continuous-time signal

① defined over a continuous range of time

② amplitude of continuous range of value or discrete values

$$Z\left\{e^{-2t}\sin\left(\frac{\pi}{2}t\right)+1\right\}$$

$$= Z\left\{e^{-2t}\sin\left(\frac{\pi}{2}t\right)\right\} + Z\{1\}$$

$$= Z\left\{\sin\frac{\pi}{2}t\right\}\Big|_{z=ze^{2T}} + \frac{1}{1-z^{-1}}$$

$$= \frac{z^{-1}\sin\frac{\pi}{2}T}{1-2z^{-1}\cos\frac{\pi}{2}T+z^{-2}}\Bigg|_{z=ze^{2T}} + \frac{1}{1-z^{-1}}$$

$$= \frac{z^{-1}\frac{\sqrt{2}}{2}}{1-2z^{-1}\frac{\sqrt{2}}{2}+z^{-2}}\Bigg|_{z=ez} + \frac{1}{1-z^{-1}}$$

$$= \frac{\frac{\sqrt{2}}{2e} z^{-1}}{1 - \frac{\sqrt{2}}{e} z^{-1} + \frac{1}{e^2} z^{-2}} + \frac{1}{1-z^{-1}}$$

$$= \frac{0.2601 z^{-1}}{1 - 0.5203 z^{-1} + 0.1353 z^{-2}} + \frac{1}{1-z^{-1}}$$

$$= \frac{0.2601 z^{-1}(1-z^{-1}) + 1 - 0.5203 z^{-1} + 0.1353 z^{-2}}{(1 - 0.5203 z^{-1} + 0.1353 z^{-2})(1-z^{-1})}$$

$$= \frac{1 - 0.2602 z^{-1} - 0.1248 z^{-2}}{(1 - 0.5203 z^{-1} + 0.1353 z^{-2})(1-z^{-1})}$$

the pole at $0.2601 \pm j0.2601$, 1 within the unit circle and with a possible exception of a simple pole at $z=1$, fulfill the final Value Theorem

$$\lim_{k\to\infty} x(k) = \lim_{z\to 1} (1-z^{-1}) X(z)$$

$$= \lim_{z\to 1} \frac{1 - 0.2602 z^{-1} - 0.1248 z^{-2}}{1 - 0.5203 z^{-1} + 0.1353 z^{-2}}$$

$$= 1$$

(b) Solve ? $\lim_{k\to\infty} x(kT)$?

Solution

apply z transform

$$x(k+2) - x(k) = 1(k)$$

$$z^2 X(z) - z^2 x(0) - z x(1) - X(z) = \frac{1}{1-z^{-1}}$$

let $k=-2$, $x(0) - x(-2) = 1(-2) \Rightarrow x(0) = 0$

let $k=-1$, $x(1) - x(-1) = 1(-1) \Rightarrow x(1) = 0$

So $(z^2-1)\, X(z) = \frac{1}{1-z^{-1}}$

$$X(z) = \frac{1}{(1-z^{-1})(z^2-1)}$$

$$= \frac{z}{(z-1)(z^2-1)}$$

$$\frac{X(z)}{z} = \frac{1}{(z-1)(z^2-1)} = \frac{1}{(z-1)(z-1)(z+1)} = \frac{1}{(z-1)^2(z+1)}$$

$$= \frac{A}{z-1} + \frac{B}{(z-1)^2} + \frac{C}{z+1}$$

$$\Rightarrow A(z-1)(z+1) + B(z+1) + C(z-1)^2$$

$$= A(z^2-1) + B(z+1) + C(z^2-2z+1)$$

$$= (A+C)z^2 + (B-2C)z + (-A+B+C)$$

$$\begin{cases} A+C=0 \\ B-2C=0 \\ -A+B+C=1 \end{cases} \Rightarrow \begin{cases} A=-\frac{1}{4} \\ B=\frac{1}{2} \\ C=\frac{1}{4} \end{cases}$$

$$\frac{X(z)}{z} = \frac{-\frac{1}{4}}{z-1} + \frac{\frac{1}{2}}{(z-1)^2} + \frac{\frac{1}{4}}{z+1}$$

$$X(z) = -\frac{1}{4}\frac{1}{1-z^{-1}} + \frac{1}{2}\frac{z^{-1}}{(1-z^{-1})^2} + \frac{1}{4}\frac{1}{1+z^{-1}}$$

apply inverse z transform

$$x(kT) = -\frac{1}{4}1(k) + \frac{1}{2}k + (-1)^k.$$

$$X(z) = \frac{z}{(z-1)(z^2-1)}$$

the pole at $z=1$, $z=-1$, $z=1$

$z=-1$ is not within the unit circle,

so it can't apply the Final value Theorem

$$x(kT) = -\frac{1}{4}1(k) + \frac{1}{2}k + (-1)^k.$$

$\lim_{k\to\infty} \frac{1}{2}k \to \infty$, so $x(kT) \to \infty$