Example 2.5

Q: $\mathcal{Z}(e^{-at}\sin\omega t) = ?$ $\mathcal{Z}(e^{-at}\cos\omega t) = ?$

Solution

$$\mathcal{Z}(e^{-at}\sin\omega t) = X(ze^{aT})$$

$$= \mathcal{Z}(\sin\omega t)\Big|_{z=ze^{aT}}$$

$$= \frac{(ze^{aT})^{-1}\sin\omega T}{1 - 2(ze^{aT})^{-1}\cos\omega T + (ze^{aT})^{-2}}$$

$$= \frac{ze^{aT}\sin\omega T}{z^2e^{2aT} - 2ze^{aT}\cos\omega T + 1}$$

$$\mathcal{Z}(e^{-at}\cos\omega t) = \mathcal{Z}(\cos\omega t)\Big|_{z=ze^{aT}}$$

$$= \frac{z^2 - z\cos\omega T}{z^2 - 2z\cos\omega T + 1}\Bigg|_{z=ze^{aT}}$$

$$= \frac{z^2e^{2aT} - ze^{aT}\cos\omega T}{z^2e^{2aT} - 2ze^{aT}\cos\omega T + 1}$$