Example 4.1

Q: design an observer

Solution

$$\left|zI - A + L_o C\right| \qquad \begin{bmatrix} l_1 \\ l_2 \end{bmatrix} \begin{bmatrix} 1 & 0 \end{bmatrix} = \begin{bmatrix} l_1 & 0 \\ l_2 & 0 \end{bmatrix}$$

$$= \begin{vmatrix} z-1+l_1 & -T \\ l_2 & z-1 \end{vmatrix}$$

$$= (z-1+l_1)(z-1) + Tl_2$$

$$= z^2 + (l_1 - 2)z + (1 - l_1 + Tl_2) \qquad (1)$$

observer poles $z_{1,2} = 0.4 \pm j0.4$

$$[(z - (0.4 + j0.4)][z - (0.4 - j0.4)]$$

$$= z^2 - 0.8z + 0.32 \qquad (2)$$

let (1) ≡ (2)

$$\begin{cases} l_1 - 2 = -0.8 \\ 1 - l_1 + Tl_2 = 0.32 \end{cases}$$

let $T = 0.1$ $\qquad \begin{cases} l_1 = 1.2 \\ l_2 = 5.2 \end{cases}$

方法二：Ackermman's

$$\alpha_0(A) = A^2 - 0.8A + 0.32I_2$$

$$= \begin{bmatrix} 1 & 0.1 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0.1 \\ 0 & 1 \end{bmatrix} - 0.8\begin{bmatrix} 1 & 0.1 \\ 0 & 1 \end{bmatrix} + 0.32\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 0.52 & 0.12 \\ 0 & 0.52 \end{bmatrix}$$

$$W_0 = \begin{bmatrix} C \\ CA \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 1 & 0.1 \end{bmatrix} \qquad \begin{bmatrix} 1 & 0 \end{bmatrix}\begin{bmatrix} 1 & 0.1 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0.1 \end{bmatrix}$$

$$W_0^{-1} = \frac{1}{0.1}\begin{bmatrix} 0.1 & 0 \\ -1 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 \\ -10 & 10 \end{bmatrix}$$

$$L_0 = \alpha_0(A)\, W_0^{-1}\begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 0.52 & 0.12 \\ 0 & 0.52 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ -10 & 10 \end{bmatrix}\begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1.2 \\ 5.2 \end{bmatrix}$$