Example 4-7

Q: k?    O?

Solution   $Z_{1,2} = 0.6 \pm j0.4$

$$\alpha_c(z) = [z - (0.6 + j0.4)][z - (0.6 - j0.4)]$$

$$= z^2 - 1.2z + 0.2$$

$$(a + bj)(a - bj) = a^2 \cancel{- abj + abj} \oplus b^2 j^2 = a^2 \ominus b^2$$

$$0.6^2 + 0.4^2 = \cancel{0.2}\; 0.52$$

$$\alpha_c(A) = \begin{bmatrix} 1 & 0.2 \\ 0 & 1 \end{bmatrix}^2 - 1.2\begin{bmatrix} 1 & 0.2 \\ 0 & 1 \end{bmatrix} + \overset{0.52}{\cancel{0.2}} I_2 = \begin{bmatrix} 0 & 0.16 \\ 0 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 0.32 & 0.16 \\ 0 & 0.32 \end{bmatrix}$$

$$W_c = [B \;\; AB] = \begin{bmatrix} 0.02 & 0.06 \\ 0.2 & 0.2 \end{bmatrix}$$

$$W_c^{-1} = \begin{bmatrix} -25 & 7.5 \\ 25 & -2.5 \end{bmatrix}$$

$$k = [0 \;\; 1]\begin{bmatrix} 25 & 7.5 \\ 25 & -25 \end{bmatrix}\begin{bmatrix} 0.32 & 0.16 \\ 0 & 0.32 \end{bmatrix}$$

$$= [25 \;\; -2.5]\begin{bmatrix} 0.32 & 0.16 \\ 0 & 0.32 \end{bmatrix}$$

$$= [\overset{8}{\cancel{0}} \;\; \overset{3.2}{\cancel{0}}]$$

$$X(k+1) = \begin{bmatrix} A_{aa} & A_{ab} \\ A_{ba} & A_{bb} \end{bmatrix} \begin{bmatrix} X_1(k) \\ X_2(k) \end{bmatrix} + \begin{bmatrix} B_a \\ B_b \end{bmatrix} u(k)$$

$$y(k) = \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} X_1(k) \\ X_2(k) \end{bmatrix}$$

$\alpha_0(z) = z$

$\alpha_0(A_{bb}) = 1$

$[A_{ab}]^{-1} = 0.2^{-1} = 5$

$L_r = \alpha_0(A_{bb})[A_{ab}]^{-1}[1] = 1 \times 5 \times 1 = 5$

$\bar{X}_b(k+1) = [1 - 5 \times 0.2]\bar{X}_b(k) + 5y(k+1) + [0 - 5 \times 1]y(k)$

$\qquad + [0.2 - 5 \times 0.02] u(k)$

$\bar{X}_b(k+1) = 5y(k+1) - 5y(k) + 0.1 u(k)$

$\qquad \bar{X}_b(k) = \bar{X}_2(k)$

$u(k+1) = \begin{bmatrix} -8 & -3.2 \end{bmatrix} \begin{bmatrix} y(k+1) \\ \bar{X}_2(k+1) \end{bmatrix}$

$\qquad = -8y(k+1) - 3.2[5y(k+1) - 5y(k) + 0.1 u(k)]$

$\qquad = -24y(k+1) + 16y(k) - 0.32 u(k)$

$z$ - transform

$\qquad z U(z) + 0.32 U(z) = -24z\, Y(z) + 16 Y(z)$

$$\frac{U(z)}{Y(z)} = \frac{-24z + 16}{z + 0.32}$$