Example 5.4

Q $u^*(k)$? $x^*(k)$? $k=1,2,3$ comment?

$$A=\begin{bmatrix}1 & 1\\ 1 & 0\end{bmatrix} \quad B=\begin{bmatrix}1\\ 0\end{bmatrix} \quad J=\frac{1}{2}\sum_{k=0}^{\infty}\left[x^T(k)Qx(k)+ru^2(k)\right]$$

$$Q=\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix} \quad r=1 \quad x(0)=\begin{bmatrix}1\\ 0\end{bmatrix}$$

$u(k) = -Kx(k)$ $\quad K=(B^TSB+R)^{-1}B^TSA$

$R=r=1$ $\quad S>0$

$$S=A^TSA+Q-A^TSB\left[B^TSB+R\right]^{-1}B^TSA$$

Solution

let $S=\begin{bmatrix}S_{11} & S_{12}\\ S_{12} & S_{22}\end{bmatrix}$ 代入方程 $\quad S_{12}=1$

$S_{22}=S_{11}-2$

Choose $S_{11}=3.7913$

应为要正定 $\lambda_i>0$

$$S=\begin{bmatrix}3.7913 & 1\\ 1 & 1.7913\end{bmatrix}$$

$K=[1 \quad 0.7913]$

可求 $x(1)$ $x(2)$ - - - $x(30)$ . . $x(\infty)$