Example 5.2

Q.

Solution $J = \sum_{k=0}^{2} [x^2(k) + u^2(k)]$

$$J = \frac{1}{2} S x^T(N) x(N) + \frac{1}{2} \sum_{k=0}^{N-1} (q x^T(N) x(N) + r u^T(k) u(k))$$

$Q = 2$ $\quad R = 2$ $\quad N - 1 = 2$ $\quad N = 3$

$S(3) = 0$

$K(2) = (1 \times 0 \times 1 + 2)^{-1} \times 1 \times 0 \times 2 = 0$

$S(2) = [2 - 1 \times 0]^T \times 0 + 0 + 2 = 2$

$K(1) = (1 \times 2 \times 1 + 2)^{-1} \times 1 \times 2 \times 2$

$= \frac{4}{4} = 1$

$S(1) = [2 - 1 \times 1]^T \times 2 \times (2 - 1 \times 1)$

$+ 1 \times 2 \times 1 + 2$

$= 2 + 2 + 2 = 6$

$$K(0) = (1 \times 6 \times 1 + 2)^{-1} \times 1 \times 6 \times 2$$

$$= \frac{1}{8} \times 12 = \frac{3}{2}$$

$$S(0) = \left(2 - \frac{3}{2}\right) \times 6 \times \left(2 - \frac{3}{2}\right) + \frac{3}{2} \times 2 \times \frac{3}{2} + 2$$

$$= \frac{1}{2} \times 6 \times \frac{1}{2} + \frac{9}{2} + 2$$

$$= \frac{3}{2} + \frac{9}{2} + 2$$

$$= \frac{12}{2} + 2$$

$$= 8$$

optimal gain schedule is $\{K(0)\ K(1)\} = \{1.5, 1\}$

minimum cost is

$$J = \frac{1}{2} x(0)^T S(0)\ x(0) = 4\, x(0)^2$$