22-S1 - Q2

Q: $T = 1$

(a) show pulse transfer function

Solution let Digital controller transfer function $G_D(z)$

$$E(z) = R(z) - C(z)$$

$$G_D(z) G_h G_p(z) = \frac{C(z)}{E(z)}$$

$$\frac{C(z)}{G_D(z) G_h G_p(z)} = R(z) - C(z)$$

$$\left(\frac{1}{G_D(z) G_h G_p(z)} + 1\right) C(z) = R(z)$$

$$\frac{C(z)}{R(z)} = \frac{G_D(z) G_h G_p(z)}{1 + G_D(z) G_h G_p(z)}$$

So the system has a pulse transfer function

(b) apply z transform of the equation

$$M(z) - z^{-1} M(z) = 2.2 E(z) - 1.4 z^{-1} E(z) + 0.2 z^{-2} E(z)$$

$$\frac{M(z)}{E(z)} = \frac{2.2 - 1.4 z^{-1} + 0.2 z^{-2}}{1 - z^{-1}}$$

PID controller $G(z) = K_p + \frac{K_I}{1-z^{-1}} + K_D(1-z^{-1})$

$$\frac{M(z)}{E(z)} = \frac{A(1-z^{-1})^2 + B(1-z^{-1}) + C}{1-z^{-1}}$$

$$A(1-2z^{-1}+z^{-2}) + B - Bz^{-1} + C$$

$$= Az^{-2} + (-2A-B)z^{-1} + (A+B+C)$$

$$\begin{cases} A = 0.2 \\ -2A-B = -1.4 \\ A+B+C = 2.2 \end{cases} \qquad \begin{cases} A = 0.2 \\ B = 1 \\ C = 1 \end{cases}$$

$B = 1.4 - 2\times 0.2 = 1$

$C = 2.2 - 1.2 = 1$

$$\frac{M(z)}{E(z)} = \frac{0.2(1-z^{-1})^2 + (1-z^{-1}) + 1}{1-z^{-1}}$$

$$= 0.2(1-z^{-1}) + 1 + \frac{1}{1-z^{-1}}$$

$$= 1 + \frac{1}{1-z^{-1}} + 0.2(1-z^{-1})$$

compare standard PID controller

$G(z) = K_p + \frac{K_I}{1-z^{-1}} + K_D(1-z^{-1})$

$K_p = 1 \quad K_I = 1 \quad K_D = 0.2$ So it is a PZD controller

(c) $G_{ZAS}(z) = (1-z^{-1})\, z\left\{\frac{G_p(s)}{s}\right\}$

$= (1-z^{-1})\, z\left\{\frac{2}{s(s+2)}\right\}$

$= (1-z^{-1})\, \frac{(1-e^{-2})z^{-1}}{(1-z^{-1})(1-e^{-2}z^{-1})}$

$= \frac{0.8647\, z^{-1}}{1-0.1353\, z^{-1}}$

no use due to $G_D G_h G_p(z) \neq G_D(z)\cdot G_h G_p(z) = G_D(z) G_{ZAS}(z)$ (a) X

$$\frac{C(z)}{R(z)} = \frac{G_D^{(z)} G_h G_p(z)}{1 + G_D^{(z)} G_h G_p(z)}$$

from (b) $\quad G_D(z) = \frac{M(z)}{E(z)} = 1 + \frac{1}{1-z^{-1}} + 0.2(1-z^{-1})$

$G_D(s) = 1 + \frac{1}{s} + 0.2s$ ↓X

$G_h(s) = \frac{1-e^{-s}}{s}$

$G_p(s) = \frac{2}{s+2}$

$G_D G_h G_p(s) = \left(1 + \frac{1}{s} + 0.2s\right) \frac{1-e^{-s}}{s} \frac{2}{s+2}$

$= \left(s + 1 + 0.2s^2\right)(1-e^{-s})\frac{2}{s+2}$

$$G_D G_h G_p(z) = 2(1-z^{-1})\, Z\left\{\frac{0.2s^2+s+1}{s+2}\right\}$$

其中 $Z\left\{\frac{0.2s^2+s+1}{s+2}\right\}$

$$= 0.2\, Z\left\{\frac{s^2}{s+2}\right\} + Z\left\{\frac{s}{s+2}\right\} + Z\left\{\frac{1}{s+2}\right\}$$

其中 $Z\left\{\frac{s^2}{s+2}\right\} = Z\left\{s-2+\frac{4}{s+2}\right\}$

$$= \text{Inz} - \frac{2}{1-z^{-1}} + \frac{4}{1-e^{-2}z^{-1}} \qquad z = e^{sT}$$

$$Z\left\{\frac{s}{s+2}\right\} = Z\left\{1-\frac{2}{s+2}\right\}$$

$$= \frac{1}{1-z^{-1}} - \frac{2}{1-e^{-2}z^{-1}}$$

$$Z\left\{\frac{1}{s+2}\right\} = \frac{1}{1-e^{-2}z^{-1}}$$

$$Z\left\{\frac{0.2s^2+s+1}{s+2}\right\} = 0.2\,\text{Inz} - \frac{0.4}{1-z^{-1}} + \frac{0.8}{1-e^{-2}z^{-1}}$$
$$+ \frac{1}{1-z^{-1}} - \frac{2}{1-e^{-2}z^{-1}}$$
$$+ \frac{1}{1-e^{-2}z^{-1}}$$

$$= \frac{-0.2}{1-0.1353z^{-1}} + \frac{0.6}{1-z^{-1}} + 0.2\,Zn\,z$$

$$= \frac{-0.2(1-z^{-1}) + 0.6(1-0.1353z^{-1}) + 0.2Zn\,z(1-0.1353z^{-1})(1-z^{-1})}{(1-0.1353z^{-1})(1-z^{-1})}$$

$$G_D G_h G_p(z) = 2(1-z^{-1})\,z\left\{\frac{0.2s^2+s+1}{s+2}\right\}$$

$$= \frac{-0.4(1-z^{-1}) + 0.12(1-0.1353z^{-1}) + 0.4Zn\,z(1-0.1353z^{-1})(1-z^{-1})}{1-0.1353z^{-1}}$$

$$\frac{C(z)}{R(z)} = \frac{G_D G_h G_p(z)}{1+G_D G_h G_p(z)}$$

$$= \frac{-0.4(1-z^{-1}) + 0.12(1-0.1353z^{-1}) + 0.4Zn\,z(1-z^{-1})(1-0.1353z^{-1})}{1-0.1353z^{-1} - 0.4(1-z^{-1}) + 0.12(1-0.1353z^{-1}) + 0.4Zn\,z(1-z^{-1})(1-0.1353z^{-1})}$$

↑ x

$$\frac{C(z)}{R(z)} = \frac{G_D(z)\,G_h G_p(z)}{1+G_D(z)\,G_h G_p(z)}$$

from (b) $\quad G_D(z) = \frac{M(z)}{E(z)} = 1 + \frac{1}{1-z^{-1}} + 0.2(1-z^{-1})$

$$G_D(z)\,G_h G_p(z)$$

$$= \left[1 + \frac{1}{1-z^{-1}} + 0.2(1-z^{-1})\right] \frac{0.8647\,z^{-1}}{1-0.1353\,z^{-1}}$$

$$= \frac{\left[2 - z^{-1} + 0.2(1-z^{-1})^2\right] 0.8647\,z^{-1}}{(1-z^{-1})(1-0.1353\,z^{-1})}$$

$$= \frac{[2z^2 - z + 0.2(z^2 - 2z + 1)]\,0.8647}{z(z-1)(z-0.1353)}$$

$$= \frac{(2z^2 - z + 0.2z^2 - 0.4z + 0.2)\,0.8647}{z(z-1)(z-0.1353)}$$

$$= \frac{(2.2z^2 - 1.4z + 0.2)\,0.8647}{z(z-1)(z-0.1353)}$$

$$= \frac{1.9023(z^2 - 0.6364z + 0.0909)}{z(z-1)(z-0.1353)}$$

$$= \frac{1.9023(z - 0.4199)(z - 0.2165)}{z(z-1)(z-0.1353)}$$

$$\frac{C(z)}{R(z)} = \frac{G_D(z)\,G_h G_p(z)}{1 + G_D(z)\,G_h G_p(z)}$$

$$= \frac{\dfrac{1.9023(z-0.4199)(z-0.2165)}{z(z-1)(z-0.1353)}}{1+\dfrac{1.9023(z-0.4199)(z-0.2165)}{z(z-1)(z-0.1353)}}$$

$$= \frac{1.9023z^2 - 1.2106z + 0.172935}{z^3 + 0.7670z^2 - 1.0753z + 0.172935}$$