Lect 3 Example 3-7

Q $x(k+1) = \begin{bmatrix} 1 & T \\ 0 & 1 \end{bmatrix} x(k) + \begin{bmatrix} \frac{T^2}{2} \\ T \end{bmatrix} u(k)$

$y(k) = \begin{bmatrix} 1 & 0 \end{bmatrix} x(k)$

$C?$ $O?$ $\dfrac{Y(z)}{U(z)} = ?$

Solution

$|w_0| = \begin{vmatrix} C \\ CA \end{vmatrix} = \begin{vmatrix} 1 & 0 \\ 1 & T \end{vmatrix} = T$, $T > 0$ 时 $\overset{\cdot\cdot}{O}$

$|w_c| = \begin{bmatrix} B & AB \end{bmatrix} = \begin{vmatrix} \frac{T^2}{2} & \frac{3T^2}{2} \\ T & T \end{vmatrix} = \frac{T^3}{2} - \frac{3T^3}{2}$

$= -T^3$, $T > 0$ 时 , C

$\dfrac{Y(z)}{U(z)} = C(zI - A)^{-1} B$

$= \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} z-1 & -T \\ 0 & z-1 \end{bmatrix}^{-1} \begin{bmatrix} \frac{T^2}{2} \\ T \end{bmatrix}$

$= \dfrac{1}{(z-1)^2} \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} z-1 & T \\ 0 & z-1 \end{bmatrix} \begin{bmatrix} \frac{T^2}{2} \\ T \end{bmatrix}$

$$= \frac{1}{(z-1)^2}\begin{bmatrix} z-1 & T \end{bmatrix}\begin{bmatrix} \frac{T^2}{2} \\ T \end{bmatrix}$$

$$= \frac{1}{(z-1)^2}\left(\frac{(z-1)T^2}{2} + T^2\right)$$

$$= \frac{T^2}{2(z-1)} + \frac{T^2}{(z-1)^2}$$

$$= \frac{T^2}{2}\left(\frac{z-1}{(z-1)^2} + \frac{2}{(z-1)^2}\right)$$

$$= \frac{T^2}{2}\frac{z+1}{(z-1)^2}$$