Example 2.8

Q : $x(\infty) = ?$

$$X(z) = \frac{1}{1 - z^{-1}} - \frac{1}{1 - e^{-aT} z^{-1}}, \quad a > 0$$

Solution

poles : 1 , e^{-aT}

let $1 = e^{-aT} z^{-1}$ $\quad z = e^{-aT}$

$a > 0, T > 0 \Rightarrow -aT < 0$, $e^{-aT} \in (0, 1)$

Final Value Theorem

$$\lim_{k \to \infty} x(k) = \lim_{z \to 1} (1 - z^{-1}) \left(\frac{1}{1 - z^{-1}} - \frac{1}{1 - e^{-aT} z^{-1}} \right)$$

$$= \lim_{z \to 1} \left(1 - \frac{1 - z^{-1}}{1 - e^{-aT} z^{-1}} \right)$$

$$= \lim_{z \to 1} \frac{-e^{-aT} z^{-1} + z^{-1}}{1 - e^{-aT} z^{-1}}$$

$$= \lim_{z \to 1} \frac{-e^{-aT} + 1}{z - e^{-aT}} = \frac{1 - e^{-aT}}{1 - e^{-aT}} = 1$$