Example 5.6

Q design controller

$$G(s)=\frac{1}{(s+1)(s+10)}$$ unit step setting time 4s

$$T=0.02s$$

Solution

$$G_{ZOH}(z)=(1-z^{-1})\,Z\left\{\frac{G(s)}{s}\right\}$$

$$=(1-z^{-1})Z\left\{\frac{1}{s(s+1)(s+10)}\right\}$$

$$\frac{A}{s}+\frac{Bs+C}{(s+1)(s+10)}=\frac{A(s^2+11s+10)+Bs^2+Cs}{s(s+1)(s+10)}=\frac{(A+B)s^2+(11A+C)s+10A}{s(s+1)(s+10)}$$

$$\begin{cases} A+B=0 \\ 11A+C=0 \\ 10A=1 \end{cases} \qquad \begin{cases} A=\frac{1}{10} \\ B=-\frac{1}{10} \\ C=-\frac{11}{10} \end{cases}$$

$$Z\left\{\frac{\frac{1}{10}}{s}+\frac{-\frac{1}{10}s-\frac{11}{10}}{(s+1)(s+10)}\right\}$$

$$\frac{A}{s+1}+\frac{B}{s+10}=\frac{As+10A+Bs+B}{(s+1)(s+10)}$$

$$\begin{cases} A+B=-\frac{1}{10} \\ 10A+B=-\frac{11}{10} \end{cases} \qquad \begin{cases} A=-\frac{1}{9} \\ B=\frac{1}{90} \end{cases}$$

$$9A = -\frac{11}{10} + \frac{1}{10} = -\frac{10}{10} = -1 \qquad A = \frac{-1}{9}$$

$$B = -\frac{1}{10} + \frac{1}{9} = \frac{-9+10}{90} = \frac{1}{90}$$

$$\mathcal{Z}\left\{\frac{\frac{1}{10}}{s} + \frac{-\frac{1}{9}}{s+1} + \frac{\frac{1}{90}}{s+10}\right\}$$

$$= \frac{\frac{1}{10}}{1-z^{-1}} + \frac{-\frac{1}{9}}{1-e^{-1}z^{-1}} + \frac{\frac{1}{90}}{1-e^{-10T}z^{-1}}$$

$$G_{ZOS}(z) = (1-z^{-1})\mathcal{Z}\left\{\frac{1}{s(s+1)(s+10)}\right\}$$

$$= (1-z^{-1})\left(\frac{\frac{1}{10}}{1-z^{-1}} + \frac{-\frac{1}{9}}{1-e^{-1}z^{-1}} + \frac{\frac{1}{90}}{1-e^{-10T}z^{-1}}\right)$$

$$= \frac{1}{10} - \frac{1-z^{-1}}{9(1-e^{-0.02}z^{-1})} + \frac{1-z^{-1}}{90(1-e^{-10\times 0.02}z^{-1})}$$

$$= \frac{9}{90} - \frac{10z-10}{90(z-e^{-0.02})} + \frac{z-1}{90(z-e^{-0.2})}$$

$$= \frac{9(z-e^{-0.02})(z-e^{-0.2}) - 10(z-1)(z-e^{-0.2}) + (z-1)(z-e^{-0.02})}{90(z-e^{-0.02})(z-e^{-0.2})}$$

$$= \frac{9z^2 - 9(e^{-0.2} + e^{-0.02})z + 9e^{-0.02}e^{-0.2} - 10(z^2 - e^{-0.2}z - z + e^{-0.2}) + z^2 - e^{-0.02}z - z + e^{-0.02}}{\sim}$$

= to hard to calculate.

$$= 1.8604 \times 10^{-4} \frac{z + 0.9293}{(z - 0.8187)(z - 0.9802)} \quad ?$$

$$G_{cl}(s) = \frac{1.322}{s^2 + 2.024s + 1.322} \quad ?$$

choose

$\zeta = 0.88$?

$w_n = 1.15$ rad/s ?

$\zeta w_n = 1$?

$0.88 \times 1.15 = 1.012$

$$s_1, s_2 = -1.012 \pm 0.5458i$$

$$(z - e^{s_1 T})(z - e^{s_2 T})$$

$$= z^2 - 1.9598z + 0.9603 \quad \text{([illegible])}$$

$$G_{cl}(z) = \frac{\alpha (z+1)^{2-0-1}}{z^2 - 1.9598z + 0.9603}$$

$$G_{cl}(z)\big|_{z=1} = G_{cl}(s)\big|_{s=0} = 1 \Rightarrow \alpha = 2.5 \times 10^{-4}$$

$$G_{cl}(z) = 2.5 \times 10^{-4} \frac{z+1}{z^2 - 1.9598z + 0.9603}$$

0.25921×10^{-3}

1.96

$$C(z) = \frac{1}{G_{ZAS}(z)} \frac{G_{CL}(z)}{1 - G_{ZAS}(z)}$$

$$= \frac{0.25921 \times 10^{-3}}{1.8604 \times 10^{-4}} \frac{(z - 0.8187)(z - 0.9802)}{z + 0.9293} \frac{\frac{z+1}{z^2 - 1.96z + 0.9685}}{1 - 1.8604 \times 10^{-4} \frac{z + 0.7293}{(z - 0.8187)(z - 0.9802)}}$$

$$= \frac{25921}{1.8604} \frac{(z - 0.8187)(z - 0.9802)}{z + 0.9293} \frac{\frac{z+1}{z^2 - 1.96z + 0.9685}}{\frac{(z - 0.8187)(z - 0.9802) - 1.8604 \times 10^{-4}(z + 0.7293)}{(z - 0.8187)(z - 0.9802)}}$$

$$= \frac{25921}{1.8604} \frac{(z - 0.8187)(z - 0.9802)}{z + 0.9293} \frac{(z+1)(z - 0.8187)(z - 0.9802)}{(z^2 - 1.96z + 0.9685)(z - 0.8187)(z - 0.9802) - 1.8604 \times 10^{-4}(z + 0.7293)}$$

How to calculate ?

$$= \frac{1.3932(z - 0.8187)(z - 0.9802)(z+1)}{(z-1)(z + 0.9293)(z - 0.9801)}$$

$$C(z) = \frac{1}{G_{ZAS}(z)} \cdot \frac{G_{OL}(z)}{1 + G_{OL}(z)} = \frac{(z-0.8187)(z-0.9802)}{1.8604\times10^{-4}(z+0.9293)} \cdot \frac{0.25921\times10^{-3}\cdot\frac{z+1}{z^2-1.962+0.9603}}{1-0.25921\times10^{-3}\frac{z+1}{z^2-1.962+0.9603}}$$

$$= \frac{5375.188(z-0.8187)(z-0.9802)}{(z+0.9293)} \cdot \frac{0.25921\times10^{-3}\cdot(z+1)}{z^2-1.962+0.9603 - 0.25921\times10^{-3}(z+1)}$$

$$= \frac{1.3933(z-0.8187)(z-0.9802)(z+1)}{(z+0.9293)(z^2-1.96z+0.96)} = \frac{1.3933(z-0.8187)(z-0.9802)(z+1)}{(z+0.9293)(z-1)(z-0.96)}$$