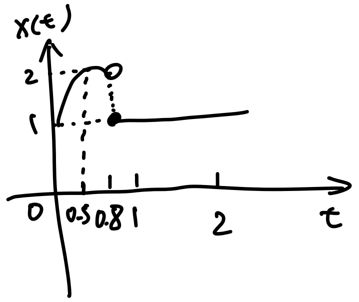
23 - S1 - Q1

Q(a) 连? Z? 0 start T = 0.5

Solution

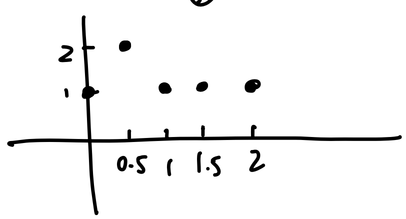
① Yes, $x(t)$ is a continuous-time signal because it is defined for all real values of t. The function $x(t)$ provides a specific value for every time t in the real number domain, fulfilling the definition of a continuous-time signal.

②



$T = \frac{2\pi}{\pi} = 2$

↓ T=0.5 sampling



according to the definition of Z transform

$$X(z) = Z[x(t)] = \sum_{k=0}^{\infty} x(kT) z^{-k}$$

$$= x(0)z^{-0} + X(0.5)z^{-1} + X(1)z^{-2} + X(1.5)z^{-3} + \cdots$$

$$= 1 + 2z^{-1} + z^{-2} + z^{-3} + \cdots$$

So $\sum_{n=2}^{\infty} z^{-n} = \dfrac{z^{-2}}{1 - z^{-1}}$

$$X(z) = 1 + 2z^{-1} + \frac{z^{-2}}{1-z^{-1}}$$

$$= \frac{(1+2z^{-1})(1-z^{-1}) + z^{-2}}{1-z^{-1}}$$

$$= \frac{1 - z^{-1} + 2z^{-1} - 2z^{-2} + z^{-2}}{1-z^{-1}}$$

$$= \frac{1 + z^{-1} - z^{-2}}{1-z^{-1}}$$

(b) Q: difference equation

Solution apply z transform to

$$y(k+2) + (\beta-1)\,y(k+1) - \beta y(k) = \delta(k-1)$$

$$z^2 Y(z) - z^2 y(0) - z y(1) + (\beta-1)\left[zY(z) - zy(0)\right] - \beta Y(z) = z^{-1}$$

let $k=-2$, $y(0) + (\beta-1)\,y(-1) - \beta y(-2) = \delta(-3)$

So $y(0) = 0$

let $k=-1$, $y(1) + (\beta-1)\,y(0) - \beta y(-1) = \delta(-2)$

So $y(1) = 0$

So $z^2 Y(z) + (\beta-1) z Y(z) - \beta Y(z) = z^{-1}$

$$(z^2+\beta z-z-\beta)Y(z)=z^{-1}$$

$$Y(z)=\frac{z^{-1}}{z^2+(\beta-1)z-\beta}$$

$$\frac{Y(z)}{z}=\frac{1}{z^2(z-1)(z+\beta)}$$

X ↓ $= \frac{A}{z^2}+\frac{B}{z-1}+\frac{C}{z+\beta}$ X 重根应该是 $Az+B$

$$A(z-1)(z+\beta)+Bz^2(z+\beta)+Cz^2(z-1)$$

$$A(z^2+\beta z-z+\beta)+B(z^3+\beta z^2)+C(z^3-z^2)$$

$$(B+C)z^3+(A+B\beta-C)z^2+(A\beta-A)z+A\beta$$

$$\begin{cases} B+C=0 \\ A+B\beta-C=0 \\ A\beta-A=0 \\ A\beta=1 \end{cases} \Rightarrow \begin{cases} A=1 \\ B=-\frac{1}{2} \\ C=\frac{1}{2} \\ \beta=1 \end{cases}$$

$B+C=0$

$1+B-C=0$

$1+2B=0$

$B=-\frac{1}{2}$

$$\frac{Y(z)}{z}=\frac{1}{z^2}+\frac{-\frac{1}{2}}{z-1}+\frac{\frac{1}{2}}{z+1}$$

$$Y(z)=z^{-1}-\frac{1}{2}\frac{1}{1-z^{-1}}+\frac{1}{2}\frac{1}{1+z^{-1}}$$

$$y(k)=\delta_0(n-1)-\frac{1}{2}1(k)+\frac{1}{2}\cos k\pi$$

(c) (b)部分不含β, so (b)过程有误

$$\frac{Y(z)}{z} = \frac{1}{z^2(z-1)(z+\beta)}$$

$$\frac{A}{z}+\frac{B}{z^2}$$

$$= \frac{Az+B}{z^2} + \frac{C}{z-1} + \frac{D}{z+\beta}$$

下次写符合的

$$(Az+B)(z-1)(z+\beta) + Cz^2(z+\beta) + Dz^2(z-1)$$

$$=(Az+B)(z^2+\beta z - z - \beta) + C(z^3+\beta z^2) + D(z^3-z^2)$$

$$= (Az^3 + A\beta z^2 - Az^2 + A\beta z + Bz^2 + B\beta z - Bz - B\beta)$$

$$+ C(z^3+\beta z^2) + D(z^3-z^2)$$

$$\begin{cases} A+C+D = 0 & (1) \\ A\beta - A + B + C\beta - D = 0 & (2) \\ -A\beta + B\beta - B = 0 & (3) \\ -B\beta = 1 & (4) \end{cases}
\qquad
\begin{cases} A = \frac{1-\beta}{\beta^2} \\ B = -\frac{1}{\beta} \\ C = \frac{1}{1+\beta} \\ D = -\frac{1}{\beta^2(\beta+1)} \end{cases}$$

from(4) $B = -\frac{1}{\beta}$

from(3) $A = \frac{B - B\beta}{\beta} = \frac{B(1-\beta)}{\beta} = \frac{1-\beta}{\beta^2} = \frac{\beta-1}{\beta^2}$

from (1)

$$C + D = -A = \frac{\beta - 1}{\beta^2} \qquad (5)$$

from (2) $\quad A\beta - A + B + C\beta - D = 0$

$$\frac{1-\beta}{\beta} - \frac{1-\beta}{\beta^2} - \frac{1}{\beta} + C\beta - D = 0$$

$$C\beta - D = \frac{\beta^2 + 1 - \beta}{\beta^2} \qquad (6)$$

(5)+(6) $\quad (1+\beta)\, C = \dfrac{\beta - 1 + \beta^2 + 1 - \beta}{\beta^2} = 1 \quad \Rightarrow C = \dfrac{1}{1+\beta}$

from (5) $\quad D = \dfrac{\beta - 1}{\beta^2} - C = \dfrac{\beta - 1}{\beta^2} - \dfrac{1}{1+\beta} = \dfrac{\beta^2 - 1 - \beta^2}{\beta^2 (1+\beta)}$

So $\quad D = \dfrac{-1}{\beta^2 (1+\beta)}$

$$\frac{Y(z)}{z} = \frac{A}{z} + \frac{B}{z^2} + \frac{C}{z-1} + \frac{D}{z+\beta}$$

$$Y(z) = \frac{1-\beta}{\beta^2} - \frac{1}{\beta} z^{-1} + \frac{1}{1+\beta} \frac{1}{1-z^{-1}} - \frac{1}{\beta^2(\beta+1)} \frac{1}{1+\beta z^{-1}}$$

$$y(k) = \frac{1-\beta}{\beta^2} \delta_0(k) - \frac{1}{\beta} \delta_0(k-1) + \frac{1}{1+\beta} 1(k) - \frac{1}{\beta^2(\beta+1)} (-\beta)^k .$$

$$y(k) = \begin{cases} \dfrac{1-\beta}{\beta^2} + \dfrac{1}{1+\beta} - \dfrac{(-\beta)^k}{\beta^2(\beta+1)} & k=0 \\ -\dfrac{1}{\beta} + \dfrac{1}{1+\beta} - \dfrac{(-\beta)^k}{\beta^2(\beta+1)} & k=1 \end{cases}$$

$\left| \quad \frac{1}{1+\beta} - \frac{(-\beta)^k}{\beta^2(\beta+1)} \qquad k \geq 2 \right.$

(c) $\beta \rightarrow y(k)$ convergence ?

Solution

① when $|\beta| < 1$, $k \to \infty$, $(-\beta)^k \to 0$

$$y(k) \rightarrow \frac{1}{1+\beta} \qquad |z_1| = 1 \quad |z_2| = |-\beta| = |\beta| < 1$$

The Final Value Theorem applies

all poles of $Y(z)$ lie inside the unit circlue, with the possible exception of a simple pole at $z=1$

Verification : $\lim_{k\to\infty} y(k) = \lim_{z\to 1} (1-z^{-1}) Y(z) = \frac{1}{\beta+1}$

so confirm theorem acceptable

② when $|\beta| = 1$ $\quad (-\beta)^k$ oscillates between ± 1

$y(k)$ doesn't converge, it oscillates

The Final value Theorem does not apply

③ when $|\beta| > 1$, $(-\beta)^k$ grows without bound

$y(k)$ diverges as $k \to \infty$

The Final value Theorem does not apply