

### Lect 3 Example 3-5

Q diagram

states and output equations?

Controllable? Observable? Transfer Function?

Solution

$$-0.2x_1(k) = x_1(k+1)$$

$$-0.5x_2(k) + x_1(k) + u(k) = x_2(k+1)$$

$$y(k) = x_2(k)$$

整理  $x_1(k+1) = -0.2x_1(k)$

$$x_2(k+1) = x_1(k) - 0.5x_2(k) + u(k)$$

$$y(k) = x_2(k)$$

$$\begin{bmatrix} x_1(k+1) \\ x_2(k+1) \end{bmatrix} = \begin{bmatrix} -0.2 & 0 \\ 1 & -0.5 \end{bmatrix} \begin{bmatrix} x_1(k) \\ x_2(k) \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u(k)$$

$$y(k) = \begin{bmatrix} 0 & 1 \end{bmatrix} \begin{bmatrix} x_1(k) \\ x_2(k) \end{bmatrix}$$

---

$$\text{CCF: } |\lambda I - A| = \begin{vmatrix} \lambda + 0.2 & 0 \\ -1 & \lambda + 0.5 \end{vmatrix} = (\lambda + 0.2)(\lambda + 0.5)$$

不需要化为CCF, 直接用  $w_c = \lambda^2 + 0.7\lambda + 1$

$$A_c = \begin{bmatrix} 0 & 1 \\ -1 & -0.7 \end{bmatrix} \quad B_c = \begin{bmatrix} 0 \\ 1 \end{bmatrix} \quad C_c = CP$$

$$= [0 \ 1] \begin{bmatrix} 0 & 0 \\ 0.2 & 1 \end{bmatrix}$$

$$= [0.2 \ 1]$$

$$W_c = [B \ AB] = \begin{bmatrix} 0 & 0 \\ 1 & -0.5 \end{bmatrix}$$

$$\tilde{W}_c = [B_c \ A_c B_c] = \begin{bmatrix} 0 & 1 \\ 1 & -0.7 \end{bmatrix}$$

$$P = W_c \tilde{W}_c^{-1} = \begin{bmatrix} 0 & 0 \\ 1 & -0.5 \end{bmatrix} \begin{bmatrix} 0.7 & 1 \\ 1 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0.2 & 1 \end{bmatrix}$$

$$\frac{Y(z)}{U(z)} = C [zI - A]^{-1} B + 1$$

$$= [0 \ 1] \begin{bmatrix} z+0.2 & 0 \\ -1 & z+0.5 \end{bmatrix}^{-1} \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

$$= [0 \ 1] \begin{bmatrix} z+0.5 & 0 \\ 1 & z+0.2 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix} \frac{1}{(z+0.2)(z+0.5)}$$

$$= [1 \ z+0.2] \begin{bmatrix} 0 \\ 1 \end{bmatrix} \frac{1}{(z+0.2)(z+0.5)}$$

$$= \frac{z+0.2}{(z+0.2)(z+0.5)}$$

$$|W_c| = \begin{vmatrix} 0 & 0 \\ 1 & -0.5 \end{vmatrix} = 0 \quad \text{uncontrollable.}$$

$$|W_o| = \begin{vmatrix} C \\ C A \end{vmatrix} = \begin{vmatrix} 0 & 1 \\ 1 & -0.5 \end{vmatrix} = -1 \neq 0 \quad \text{observable}$$

there is a pole - zero cancellation