Example 5.3

Q : $u^*(k)$? J min

Solution $J = \frac{1}{2} x^T(N)\, S(N)\, x(N) + \frac{1}{2} \sum_{k=i}^{N-1} [x^T(k)\, Q\, x(k) + u^T(k)\, R\, u(k)]$

$$J = \frac{1}{2} s\, x^T(N)\, x(N) + \frac{1}{2} \sum_{k=0}^{N-1} (q\, x^T(N)\, x(N) + r\, u^T(k)\, u(k)) \qquad (1)$$

$$J = \sum_{k=0}^{7} (x_1^2(k) + u^2(k)) \qquad (2)$$

from (1) (2)

$N - 1 = 7 \Rightarrow N = 8$

$$\frac{1}{2} \bar{x}^T Q\, \bar{x} = [x_1 \ x_2] \underset{\uparrow Q}{\begin{bmatrix} 2 & 0 \\ 0 & 0 \end{bmatrix}} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = x_1^2$$

$$Q = \begin{bmatrix} 2 & 0 \\ 0 & 0 \end{bmatrix}$$

$\frac{1}{2} R = 1 \Rightarrow R = 2$

$S(8) = 0$

$$K(k) = (B^T S(k+1) B + R)^{-1} B^T S(k+1) A$$

$$S(k) = [A - BK(k)]^T S(k+1) [A - BK(k)] + K^T(k) R K(k) + Q$$

$$K(7) = \left[\begin{bmatrix} 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix} + 2 \right]^{-1} \begin{bmatrix} 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} 0 & 1 \\ -1 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 0 \end{bmatrix}$$

$$S(7) = \left[\begin{bmatrix} 0 & 1 \\ -1 & 1 \end{bmatrix} - \begin{bmatrix} 0 \\ 1 \end{bmatrix} \begin{bmatrix} 0 & 0 \end{bmatrix} \right]^T \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} \left[\begin{bmatrix} 0 & 1 \\ -1 & 1 \end{bmatrix} - \begin{bmatrix} 0 \\ 1 \end{bmatrix} \begin{bmatrix} 0 & 0 \end{bmatrix} \right]$$

$$+ \begin{bmatrix} 0 \\ 0 \end{bmatrix} 2 \begin{bmatrix} 0 & 0 \end{bmatrix} + \begin{bmatrix} 2 & 0 \\ 0 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 2 & 0 \\ 0 & 0 \end{bmatrix}$$

同理 $S(0) = \begin{bmatrix} 3.305 & -0.97 \\ -0.97 & 3.777 \end{bmatrix}$

$$K(0) = \begin{bmatrix} -0.6538 & 0.486 \end{bmatrix}$$

$$u^*(0) = -K(0)x(0) = \begin{bmatrix} 0.6538 & -0.486 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix} = 0.1678$$

$$x^*(1) = Ax(0) + Bu(0) = \begin{bmatrix} 0 & 1 \\ -1 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} 0.1678$$

$$= \begin{bmatrix} 1 \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ 0.1678 \end{bmatrix} = \begin{bmatrix} 1 \\ 0.1678 \end{bmatrix}$$