Let 1 Example 1.4

Q. $T=0.1$

$$\dot{x}(t) = \begin{bmatrix} 0 & 1 \\ 0 & -1 \end{bmatrix} x(t) + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u(t)$$

$$y(t) = \begin{bmatrix} 1 & 0 \end{bmatrix} x(t)$$ 求 discrete-time state space.

Solution.

$$[sI-A]^{-1} = \begin{bmatrix} s & -1 \\ 0 & s+1 \end{bmatrix}^{-1} = \frac{1}{s(s+1)} \begin{bmatrix} s+1 & 1 \\ 0 & s \end{bmatrix}$$

$$= \begin{bmatrix} \frac{1}{s} & \frac{1}{s(s+1)} \\ 0 & \frac{1}{s+1} \end{bmatrix}$$

$$\Phi(t) = L^{-1} \begin{bmatrix} \frac{1}{s} & \frac{1}{s(s+1)} \\ 0 & \frac{1}{s+1} \end{bmatrix} = \begin{bmatrix} 1 & 1-e^{-t} \\ 0 & e^{-t} \end{bmatrix}$$

$$\Phi(T) = \begin{bmatrix} 1 & 1-e^{-T} \\ 0 & e^{-T} \end{bmatrix} = \begin{bmatrix} 1 & 1-e^{-0.1} \\ 0 & e^{-0.1} \end{bmatrix} = \begin{bmatrix} 1 & 0.0952 \\ 0 & 0.9048 \end{bmatrix}$$

$$\theta(T) = \int_0^T \Phi(y)\, dy\, B$$

$$= \begin{bmatrix} t & t+e^{-t} \\ 0 & -e^{-t} \end{bmatrix} \Bigg|_0^{0.1} \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

$$= \left\{ \begin{bmatrix} 0.1 & 0.1+e^{-0.1} \\ 0 & -e^{-0.1} \end{bmatrix} - \begin{bmatrix} 0 & 1 \\ 0 & -1 \end{bmatrix} \right\} \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} 0.1 & -0.9+e^{-0.1} \\ 0 & -e^{-0.1}+1 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} -0.9+e^{-0.1} \\ 1-e^{-0.1} \end{bmatrix} = \begin{bmatrix} 0.00484 \\ 0.0952 \end{bmatrix}$$

$$x((k+1)\cancel{T}) = \Phi(T)\, x(kT) + \Theta(T)\, u(kT)$$

$$= \begin{bmatrix} 1 & 0.0952 \\ 0 & 0.9048 \end{bmatrix} x(k\cancel{T}) + \begin{bmatrix} 0.00484 \\ 0.0952 \end{bmatrix} u(k\cancel{T})$$

$$y(k\cancel{T}) = \begin{bmatrix} 1 & 0 \end{bmatrix} x(k\cancel{T})$$

最后的离散状态模型，如果有T具体数字，要将T消除？
如果T没有给出具体数字，
请问是按照没有T的来写还是按照有T的写？

$$\mathbf{x}(k+1) = \begin{bmatrix} 1 & 0.0952 \\ 0 & 0.905 \end{bmatrix} \mathbf{x}(k) + \begin{bmatrix} 0.00484 \\ 0.0952 \end{bmatrix} u(k)$$

$$y(k) = \begin{bmatrix} 1 & 0 \end{bmatrix} \mathbf{x}(k)$$

官方解答



Example 1.4 A servomotor has a continuous-time state space representation as shown below.
Sample the system with a sampling period of $T$ = 0.1 sec and obtain a discrete-time state space model of the motor.

间模型

$$\dot{\mathbf{x}}(t)=\begin{bmatrix}0 & 1\\ 0 & -1\end{bmatrix}\mathbf{x}(t)+\begin{bmatrix}0\\ 1\end{bmatrix}u(t)$$

$$y(t)=\begin{bmatrix}1 & 0\end{bmatrix}\mathbf{x}(t)$$



态转移矩阵

$$\Rightarrow [s\mathbf{I}-\mathbf{A}]^{-1}=\begin{bmatrix}\dfrac{1}{s} & \dfrac{1}{s(s+1)}\\ 0 & \dfrac{1}{s+1}\end{bmatrix}$$

$$\Rightarrow \mathbf{\Phi}(t)=\mathcal{L}^{-1}\left\{[s\mathbf{I}-\mathbf{A}]^{-1}\right\}=\begin{bmatrix}1 & 1-e^{-t}\\ 0 & e^{-t}\end{bmatrix}$$

$$\Rightarrow \quad \mathbf{\Phi}(T)=\begin{bmatrix}1 & 1-e^{-T}\\ 0 & e^{-T}\end{bmatrix}=\begin{bmatrix}1 & 0.0952\\ 0 & 0.905\end{bmatrix}$$

$$\mathbf{\Theta}(T)=\left[\int_0^T \mathbf{\Phi}(\tau)\,\mathrm{d}\tau\right]\begin{bmatrix}0\\ 1\end{bmatrix}=\begin{bmatrix}0.00484\\ 0.0952\end{bmatrix}$$

$$\Rightarrow \mathbf{x}(k+1)=\begin{bmatrix}1 & 0.0952\\ 0 & 0.905\end{bmatrix}\mathbf{x}(k)$$

$$+\begin{bmatrix}0.00484\\ 0.0952\end{bmatrix}u(k) \qquad ...(2.11)$$

$$y(k)=\begin{bmatrix}1 & 0\end{bmatrix}\mathbf{x}(k) \qquad ...(2.12)$$