Example 3-5

Q: transfer function = ?

Solution

(a) $G(s) = \frac{U(s)}{X^*(s)}$ (1) $H(s) = \frac{Y(s)}{U^*(s)}$ (2)

from(1) $U(s) = G(s) X^*(s)$

$U^*(s) = G^*(s) X^*(s)$ (3)

from(2) $Y(s) = H(s) U^*(s)$

from(3) $Y(s) = H(s) G^*(s) X^*(s)$

$Y^*(s) = H^*(s) G^*(s) X^*(s)$

$Y(z) = H(z) G(z) X(z)$

$$\frac{Y(z)}{X(z)} = G(z) H(z) = z\left(\frac{1}{s+a}\right) z\left(\frac{1}{s+b}\right)$$

$$= \frac{1}{(1-e^{-aT}z^{-1})(1-e^{-bT}z^{-1})}$$

(b) $G(s) = \frac{U(s)}{X^*(s)}$ (1) $H(s) = \frac{Y(s)}{U(s)}$ (2)

from(1)(2) $G(s) X^*(s) = U(s) = \frac{Y(s)}{H(s)}$

$$Y(s) = G_1(s) H(s) X^*(s)$$

$$Y^*(s) = GH^*(s) X^*(s)$$

$$\frac{Y(z)}{X(z)} = GH(z)$$

$$GH(z) = Z\left(\frac{1}{s+a}\frac{1}{s+b}\right) = \frac{1}{b-a}\frac{(e^{-aT} - e^{-bT})z^{-1}}{(1-e^{-aT}z^{-1})(1-e^{-bT}z^{-1})}$$

$$\frac{Y(z)}{X(z)} = \frac{1}{b-a}\frac{(e^{-aT} - e^{-bT})z^{-1}}{(1-e^{-aT}z^{-1})(1-e^{-bT}z^{-1})}$$