

19-51-Q2

Q: (a) $C(z) = ?$

Solution

$$(a) G_1 G_2(z) = \frac{C(z)}{R(z) - C H(z)}$$

$$G_1 G_2(z) = (1 - z^{-1}) z \left\{ \frac{G_2(s)}{s} \right\}$$

$$= (1 - z^{-1}) z \left\{ \frac{1}{s^3} \right\}$$

$$= (1 - z^{-1}) \frac{1}{z} \frac{0.1^2 z^{-1} (1 + z^{-1})}{(1 - z^{-1})^3}$$

$$= \frac{0.005 z^{-1} (1 + z^{-1})}{(1 - z^{-1})^2}$$

$$R(z) = \frac{1}{1 - z^{-1}}$$

$$G_1 G_2(z) = \frac{C(z)}{R(z) - C H(z)}$$

$$C(z) = G_1 G_2(z) R(z) - \alpha C(z) G_1 G_2(z)$$

$$[1 + \alpha G_1 G_2(z)] C(z) = G_1 G_2(z) R(z)$$

$$C(z) = \frac{G_1 G_2(z) R(z)}{1 + \alpha G_1 G_2(z)}$$

$$\begin{aligned}
& \frac{0.005 z^{-1} (1+z^{-1})}{(1-z^{-1})} \\
&= \frac{0.005 z^{-1} (1+z^{-1})}{1+\alpha \frac{0.005 z^{-1} (1+z^{-1})}{(1-z^{-1})^2}} \\
&= \frac{0.005 z^{-1} (1+z^{-1}) (1-z^{-1})}{(1-z^{-1})^2 + \alpha 0.005 z^{-1} (1+z^{-1})} \\
&= \frac{0.005 z^{-1} - 0.005 z^{-3}}{1+(0.005\alpha-2)z^{-1}+(0.005\alpha+1)z^{-2}}
\end{aligned}$$

(b) from $G_c(z)$

$$z^2 + (0.005\alpha - 2)z + 1 + 0.005\alpha = 0$$

$$|z_{1,2}| < 1 \Rightarrow \text{stable}$$

$$|z| = \left| \frac{2 - 0.005\alpha \pm \sqrt{(0.005\alpha - 2)^2 - 4(1 + 0.005\alpha)}}{2} \right| < 1$$

$$-2 < 2 - 0.005\alpha \pm \sqrt{(0.005\alpha - 2)^2 - 4(1 + 0.005\alpha)} < 2$$

$$\alpha < 0 \text{ or } \alpha \geq 1600$$

开环脉冲传递函数 $G_c(z) = 0.005\alpha \frac{z+1}{(z-1)^2}$

有 2 个 $z=1$ 开环极点, type 2 对 step input 的输入误差为 0