

Example 5.2

Q: $G_{cl}(z)$? $\zeta = 0.88$ $\omega_n = 1.15$ $T = 0.02$

Solution $s_1 = -1.012 + 0.5458i$

卡西欧算出来的

$$s_2 = -1.012 - 0.5458i$$

$$(z - e^{s_1 T})(z - e^{s_2 T})$$

$$= z^2 - \underbrace{(e^{s_1 T} + e^{s_2 T})}_{\text{math error}} z + e^{s_1 T} \cdot e^{s_2 T}$$

math error \rightarrow Euler Theorem

$$\begin{aligned} e^{(a+bi)T} + e^{(a-bi)T} &= e^{aT} [e^{Tbi} + e^{-Tbi}] = e^{aT} 2\cos(bT) \\ \frac{e^{(a+bi)T}}{e^{(a-bi)T}} &= e^{T[2a]} = e^{2aT} \end{aligned}$$

$$= z^2 - e^{-1.012 \times 0.02} 2\cos(0.5458 \times 0.02) z + e^{2 \times (-1.012) \times 0.02}$$

$$= z^2 - 1.9598 z + 0.9603$$

$$G_{cl}(z) = \frac{\alpha (z+1)^{2-0-1}}{z^2 - 1.9598 z + 0.9603}$$

$$G_{cl}(z) \Big|_{z=1} = G_{cl}(s) \Big|_{s=0} = 1$$

$$= \frac{2\alpha}{1 - 1.9598 + 0.9603}$$

$$\alpha = 2.5 \times 10^{-4}$$

$$G_u(z) = 2.5 \times 10^{-4} \frac{z+1}{z^2 - 1.9598z + 0.9603}$$