Example

Q $m = 1000 \, kg$

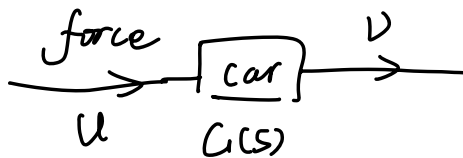$b = 100$

$8S$ $0 \to 100 \, km/h$

overshoot $< 20\%$

plant is car

Solution

力-摩擦

$$V - bv = m\dot{v} \qquad V(s) - bV(s) = msV(s)$$

$$G(s) = \frac{V(s)}{U(s)} = \frac{1}{m+bv}$$

$$e^{-\frac{\pi\zeta}{\sqrt{1-\zeta^2}}} < 0.2$$

$$-\frac{\pi\zeta}{\sqrt{1-\zeta^2}} < \ln 0.2$$

$$\frac{\pi\zeta}{\sqrt{1-\zeta^2}} > 1.6094$$

$$\pi\zeta > 1.6094\sqrt{1-\zeta^2}$$

$$\pi^2\zeta^2 > 1.6094^2(1-\zeta^2)$$

$$(\pi^2+1.6094)^2\zeta^2 > 1.6094^2$$

$$\zeta^2 > 0.2079 \qquad \zeta > 0.4559$$

choose $\xi = 0.7$

② 5% setting time   2%   1% ✓

$$t_s = \frac{3}{\xi w_n} \qquad t_s = \frac{4}{\xi w_n} \qquad t_s = \frac{4.6}{\xi w_n}$$

$$8 = \frac{4.6}{0.7 w_n} \implies w_n = 0.821$$

$$H_{desired}(S) = \frac{0.82^2}{S^2 + 2 \times 0.7 \times 0.82 S + 0.82^2}$$

$$= \frac{0.67}{S^2 + 1.15 S + 0.67}$$

PI controller $C(S) = k_p + \frac{k_i}{S}$

$$G(S) = \frac{Y(S)}{U(S)} \qquad C(S) = \frac{U(S)}{R(S) - Y(S)}$$

$$U = \frac{Y}{G} = CR - CY \implies Y = CGR - CGY \implies (1 + CG)Y = CGR$$

$$\frac{Y}{R} = \frac{CG}{1 + CG}$$

$$H(S) = \frac{Y(S)}{R(S)} = \frac{G(S) C(S)}{1 + G(S) C(S)}$$

$$G(s) = \frac{1}{ms+b} = \frac{1}{1000S+100}$$

$$H(s) = \frac{\dfrac{1}{1000S+100}\left(k_p + \dfrac{k_i}{s}\right)}{1 + \dfrac{1}{1000S+100}\left(k_p + \dfrac{k_i}{s}\right)}$$

$$= \frac{S k_p + k_i}{1000 S^2 + 100S + Sk_p + k_i}$$

$$= \frac{0.001\, k_p s + 0.001\, k_i}{S^2 + (0.1 + 0.001 k_p)S + 0.001\, k_i}$$

$$\begin{cases} 0.1 + 0.001\, k_p = 1.15 \\ 0.001\, k_i = 0.67 \end{cases} \Rightarrow \begin{cases} k_p = 1050 \\ k_i = 670 \end{cases}$$

PI controller $C(s) = 1050 + \dfrac{670}{s}$

$C(s) \longrightarrow C(z)$ 变换 T

$$H(j\omega) = \frac{1.05(j\omega) + 0.67}{(j\omega)^2 + 1.15(j\omega) + 0.67}$$

$$|H(j\omega)| = \left| \quad \sim \quad \right|$$

$$dB = 20 \log_{10} |H(j\omega)| = f(\omega)$$

let $f(\omega) = -3$

$$\Rightarrow \omega = 1.8 \, rad/s = \frac{1.8}{2\pi} = 0.3 \, HB$$

$$T \, \text{取} \, 30 \text{倍的} \, \frac{1}{30\omega} = \frac{1}{30 \times 0.3} = \frac{1}{9} \approx 0.1 \, S$$

$$S = \frac{2}{T} \left(\frac{z-1}{z+1} \right)$$

$$C(z) = \frac{1050 \frac{2}{T} \left(\frac{z-1}{z+1} \right) + 670}{\frac{2}{T} \left(\frac{z-1}{z+1} \right)} = \frac{1083.5 \, z - 1016.5}{z - 1}$$