

22-51-Q3

Q: (a) state-space model

$$m_1(t) - m_2(t) = \frac{d\phi(t)}{dt} + k_1 p(t) \quad (1)$$

$$m_2(t) = k_2 \frac{d\phi(t)}{dt} \quad (2)$$

$$k_3 p(t) = k_4 \frac{d\phi(t)}{dt} + k_5 \frac{d^2\phi(t)}{dt^2} \quad (3)$$

$$x_1(t) = \phi(t) \quad (4)$$

$$x_2(t) = \frac{d\phi(t)}{dt} \quad (5)$$

$$x_3(t) = p(t) \quad (6)$$

$$u(t) = m_1(t) \quad (7)$$

$$y(t) = \phi(t) \quad (8)$$

Solution

$$\begin{bmatrix} \dot{x}_1(t) \\ \dot{x}_2(t) \\ \dot{x}_3(t) \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & -k_1 & k_3 \\ 0 & -k_2 & -k_1 \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \\ x_3(t) \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} u(t)$$

$$y(t) = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \\ x_3(t) \end{bmatrix}$$

$$(4)(5) \quad \dot{x}_1(t) = \frac{d\phi(t)}{dt} = x_2(t)$$

$$(2) \quad m_2(t) = k_2 x_2(t)$$

$$\textcircled{1} \textcircled{7} \textcircled{6} \quad u(t) - k_2 x_2(t) = \dot{x}_3(t) + k_1 x_3(t)$$

$$\dot{x}_3(t) = -k_2 x_2(t) - k_1 x_3(t) + u(t)$$

$$\textcircled{3} \textcircled{5} \quad k_3 x_3(t) = k_1 x_2(t) + k_5 \dot{x}_2(t)$$

$$\dot{x}_2(t) = -k_1 x_2(t) + k_3 x_3(t)$$

$$\textcircled{4} \textcircled{8} \quad y = x_1(t)$$

(b) Q:  $C \rightarrow D$ ?

Solution Q  $[sI - A]^{-1}$

$$[sI - A]^{-1} = \begin{bmatrix} s+2 & 0 \\ -3 & s+3 \end{bmatrix}^{-1} = \frac{1}{(s+2)(s+3)} \begin{bmatrix} s+3 & 0 \\ 3 & s+2 \end{bmatrix}$$

$$= \begin{bmatrix} \frac{1}{s+2} & 0 \\ \frac{3}{(s+2)(s+3)} & \frac{1}{s+3} \end{bmatrix}$$

$$\# 9 \quad \frac{1}{(s+2)(s+3)} \quad e^{-2t} - e^{-3t}$$

$$\# 4 \quad \frac{1}{s+2} \quad e^{-2t}$$

$$\# 5 \quad \frac{1}{s+3} \quad e^{-3t}$$

$$\Phi(t) = L^{-1}\{[sI - A]^{-1}\} = \begin{bmatrix} e^{-2t} & 0 \\ 3e^{-2t} - 3e^{-3t} & e^{-3t} \end{bmatrix}$$

$$\Phi(T) = \begin{bmatrix} e^{-2T} & 0 \\ 3e^{-2T} - 3e^{-3T} & e^{-3T} \end{bmatrix} \quad T = 0.1$$

$$= \begin{bmatrix} 0.8187 & 0 \\ 0.2337 & 0.7408 \end{bmatrix}$$

$$\Theta(T) = \int_0^T \Phi(t) dt B$$

$$= \int_0^T \begin{bmatrix} e^{-2t} & 0 \\ 3e^{-2t} - 3e^{-3t} & e^{-3t} \end{bmatrix} \begin{bmatrix} 2 \\ 0 \end{bmatrix} dt$$

$$= \int_0^T \begin{bmatrix} 2e^{-2t} \\ 6(e^{-2t} - e^{-3t}) \end{bmatrix} dt \quad T = 0.1 \quad \text{casio}$$

$$= \begin{bmatrix} 0.1813 \\ 0.02544 \end{bmatrix}$$

$$\text{So } x(k+1) = \begin{bmatrix} 0.8187 & 0 \\ 0.2337 & 0.7408 \end{bmatrix} x(k) + \begin{bmatrix} 0.1813 \\ 0.02544 \end{bmatrix} u(k)$$

$$y(k) = [1 \ 0] x(k)$$

(ii) Q: poles  $\rightarrow$  stable? T

Solution

$$\Phi(T) = \begin{bmatrix} e^{-2T} & 0 \\ 3e^{-2T} - e^{-3T} & e^{-3T} \end{bmatrix} = A_d$$

$$|zI - A_d| = \begin{vmatrix} z - e^{-2T} & 0 \\ 3e^{-2T} - e^{-3T} & z - e^{-3T} \end{vmatrix} = (z - e^{-2T})(z - e^{-3T})$$

$$|z_1| = e^{-2T} < 1 \quad |z_2| = e^{-3T} < 1$$

for  $T > 0$   $z_{1,2} \in (0, 1)$  the system is stable

So the range of  $T$  is  $T > 0$

cc) state-mode?

Solution

$$x_1(k+1) = x_1(k) + x_2(k) + u(k) \quad ①$$

$$x_2(k+1) = x_2(k) + u(k) \quad ②$$

$$y(k) = x_1(k) \quad ③$$

$$x_3(k) = u(k) \quad ④$$

$$u(k+1) = -a u(k) + r(k) - y(k) \quad ⑤$$

$$\begin{bmatrix} x_1(k+1) \\ x_2(k+1) \\ x_3(k+1) \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ -1 & 0 & -a \end{bmatrix} \begin{bmatrix} x_1(k) \\ x_2(k) \\ x_3(k) \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} r(k)$$

$$y(k) = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1(k) \\ x_2(k) \\ x_3(k) \end{bmatrix}$$

$$\textcircled{3}\textcircled{4}\textcircled{5} \quad x_3(k+1) = -a x_3(k) + r(k) - x_1(k)$$

$$\textcircled{2}\textcircled{6} \quad x_2(k+1) = x_2(k) + x_3(k)$$

$$\textcircled{1}\textcircled{7} \quad x_1(k+1) = x_1(k) + x_2(k) + x_3(k)$$