19-S1-Q2

Q: (a) $C(Z) = ?$

Solution

(a) $G_1G_2(Z) = \dfrac{C(Z)}{R(Z) - CH(Z)}$

$$G_1G_2(Z) = (1-Z^{-1})\, \mathcal{Z}\left\{\frac{G_2(s)}{s}\right\}$$

$$= (1-Z^{-1})\, \mathcal{Z}\left\{\frac{1}{S^3}\right\}$$

$$= (1-Z^{-1})\frac{1}{2}\,\frac{0.1^2 Z^{-1}(1+Z^{-1})}{(1-Z^{-1})^3}$$

$$= \frac{0.005 Z^{-1}(1+Z^{-1})}{(1-Z^{-1})^2}$$

$$R(Z) = \frac{1}{1-Z^{-1}}$$

$$G_1G_2(Z) = \frac{C(Z)}{R(Z) - CH(Z)}$$

$$C(Z) = G_1G_2(Z)R(Z) - \alpha C(Z) G_1G_2(Z)$$

$$[1+\alpha G_1G_2(Z)]\, C(Z) = G_1G_2(Z)R(Z)$$

$$C(Z) = \frac{G_1G_2(Z)R(Z)}{1+\alpha G_1G_2(Z)}$$

$$= \frac{\dfrac{0.005 z^{-1}(1+z^{-1})}{(1-z^{-1})^3}}{1+\alpha\dfrac{0.005 z^{-1}(1+z^{-1})}{(1-z^{-1})^2}}$$

others answer? 别人算错了

$$\frac{0.005(z^2+1)}{z^3+(0.005\alpha+1)z^2-z-(1+0.005\alpha)}$$

$$= \frac{0.005 z^{-1}(1+z^{-1})\cancel{(1-z^{-1})}}{(1-z^{-1})^3+\alpha 0.005 z^{-1}(1+z^{-1})(1-z^{-1})}$$

$$= \frac{0.005 z^{-1} - 0.005 z^{-3}}{1+(0.005\alpha-2)z^{-1}+(0.005\alpha+1)z^{-2}} = \frac{0.005(z^2-1)}{z^3+(0.005\alpha-2)z^2+(0.005\alpha+1)z}$$

(b) from $G_{cc}(z)$

$$z^2+(0.005\alpha-2)z+1+0.005\alpha=0$$

$$|z_{1,2}|<1 \Rightarrow \text{stable}$$

$$|z| = \left|\frac{2-0.005\alpha \pm \sqrt{(0.005\alpha-2)^2-4(1+0.005\alpha)}}{2}\right| < 1$$

$$-2 < 2-0.005\alpha \pm \sqrt{(0.005\alpha-2)^2-4(1+0.005\alpha)} < 2$$

$$\alpha < 0 \quad \text{or} \quad \alpha \geq 1600$$

开环脉冲传递函数 $\alpha G(z) = 0.005\alpha \dfrac{z+1}{(z-1)^2}$

有2个 $z=1$ 开环极点，type 2 对 step input 的输入误差为 0

(b) 求根公式只算了实根，虚根在单位圆内也稳定

$$z^2 + (\alpha 0.005 - 2)z + (1+\alpha 0.005) = 0$$

$$\Delta = \sqrt{(0.005\alpha - 2)^2 - 4\times 1\times(1+0.005\alpha)}$$

$$= \sqrt{2.5\times10^{-5}\alpha^2 - 0.01\alpha + 4 - 4 - 0.02\alpha}$$

$$= \sqrt{2.5\times10^{-5}\alpha^2 - 0.03\alpha}$$

$$-4 < -5\times10^{-3}\alpha \pm \sqrt{2.5\times10^{-5}\alpha^2 - 0.03\alpha} < 0$$

很麻烦 ✗ → Jury Test

(b) try Jury test

$$\frac{C(z)}{R(z)} = \frac{G_1G_2(z)}{1+\alpha G_1G_2(z)}$$

let $1+\alpha\frac{0.005z^{-1}(1+z^{-1})}{(1-z^{-1})^2} = 0$

$$(1-z^{-1})^2 + \alpha 0.005 z^{-1} + \alpha 0.005 z^{-2} = 0$$

$$1-2z^{-1}+z^{-2}+\alpha 0.005 z^{-1} + \alpha 0.005 z^{-2} = 0$$

$$1+(\alpha 0.005-2)z^{-1}+(1+\alpha 0.005)z^{-2} = 0$$

$$z^2+(\alpha 0.005-2)z+(1+\alpha 0.005) = 0$$

Jury Test

z^0	z^1	z^2
$1+\alpha 0.005$	$\alpha 0.005-2$	1

$$\begin{cases} |1+\alpha 0.005| < 1 \\ P(1) = 1+0.005\alpha-2+1+0.005\alpha \\ \quad = 0.01\alpha > 0 \\ P(-1) = 1-0.005\alpha+2+1+0.005\alpha \\ \quad = 4 > 0 \end{cases}$$

$n=2$
even

$$-1 < 1+0.005\alpha < 1$$

$$-2 < 0.005\alpha < 0$$

$$\frac{-400}{\cancel{\frac{-2000}{0.005}}}$$

$$\begin{cases} -400 < \alpha < 0 \\ \alpha > 0 \end{cases}$$

So the system is not stable

So $\alpha \in \emptyset$ $\lim_{k \to \infty} |e(kT)|$ not exist.

别人算错了

(a) others answer

$$\frac{z}{z-1}\quad\frac{0.005(1+z)}{z^2+2z+1+\alpha 0.005+\underline{\alpha 0.005 z}}$$

$$\frac{0.005(z^2+1)}{(z-1)[z^2+(0.005\alpha+2)z+(1+0.005\alpha)]}$$

$$= z^3+(0.005\alpha+2)z^2+(1+0.005\alpha)z-z^2-(0.005\alpha+2)z-(1+0.005\alpha)$$

$$= z^3+(0.005\alpha+1)z^2-z-(1+0.005\alpha)$$

so $$\frac{0.005(z^2+1)}{z^3+(0.005\alpha+1)z^2-z-(1+0.005\alpha)}$$