

知识点Z4.18

对称性

主要内容:

傅里叶变换的对称性

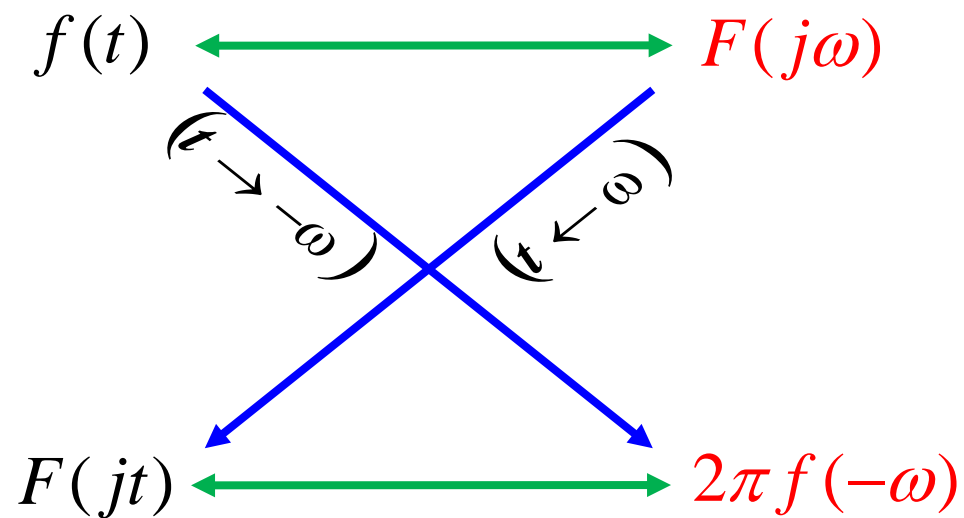
基本要求:

熟练掌握傅里叶变换时域和频域的对称性



Z4.18 对称性

若 $f(t) \leftrightarrow F(j\omega)$ 则 $F(jt) \leftrightarrow 2\pi f(-\omega)$



Z4.18对称性

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证明:
$$f(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} F(j\omega) e^{j\omega t} d\omega$$

式中, 令 $t \rightarrow \omega$, $\omega \rightarrow t$, 可得:

$$f(\omega) = \frac{1}{2\pi} \int_{-\infty}^{\infty} F(jt) e^{j\omega t} dt$$

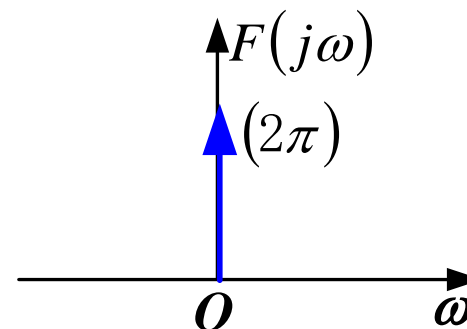
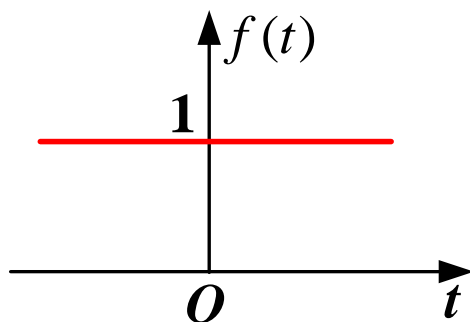
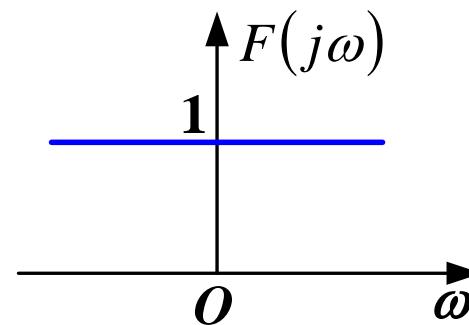
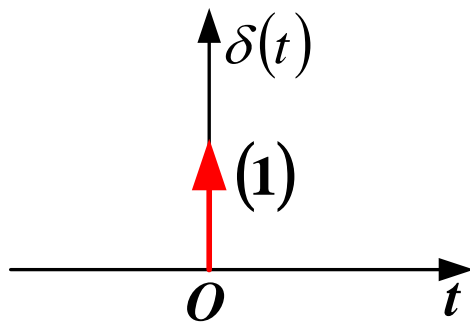
令 $\omega \rightarrow -\omega$ 可得:

$$f(-\omega) = \frac{1}{2\pi} \int_{-\infty}^{\infty} F(jt) e^{-j\omega t} dt$$

$$\therefore F(jt) \leftrightarrow 2\pi f(-\omega)$$

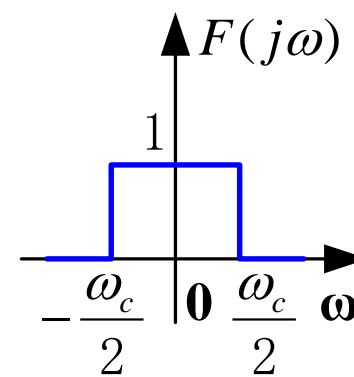
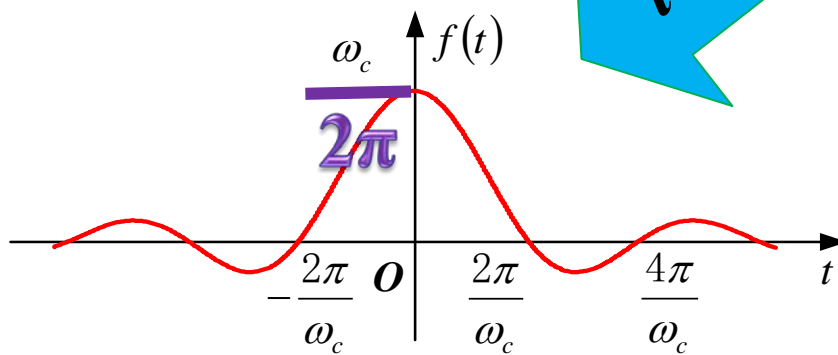
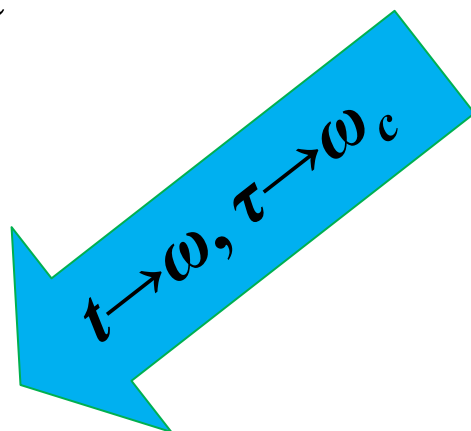
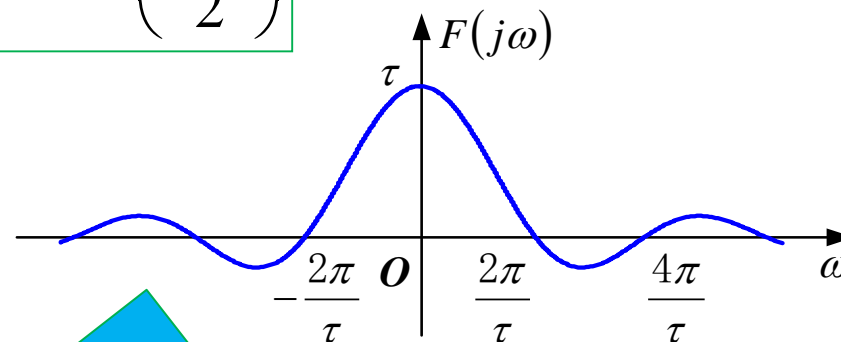
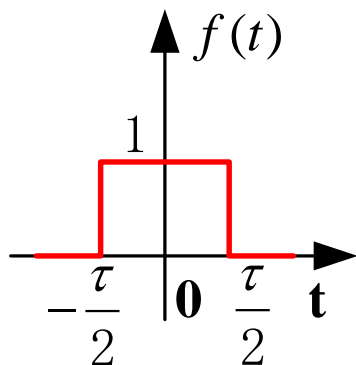


例1



例2

$$g_{\tau}(t) \leftrightarrow \tau \text{Sa}\left(\frac{\omega\tau}{2}\right)$$



例3 $f(t) = \frac{1}{1+t^2} \longleftrightarrow F(j\omega) = ?$

解:

$$e^{-\alpha|t|} \longleftrightarrow \frac{2\alpha}{\alpha^2 + \omega^2}$$

当 $\alpha=1$ 时

$$e^{-|t|} \longleftrightarrow \frac{2}{1+\omega^2}$$

根据对称性

$$\frac{2}{1+t^2} \longleftrightarrow 2\pi e^{-|\omega|}$$

所以

$$\frac{1}{1+t^2} \longleftrightarrow \pi e^{-|\omega|}$$

