

19-51-Q2

Q: (a) $C(z) = ?$

Solution

$$(a) G_1 G_2(z) = \frac{C(z)}{R(z) - C H(z)}$$

$$G_1 G_2(z) = (1 - z^{-1}) z \left\{ \frac{G_2(s)}{s} \right\}$$

$$= (1 - z^{-1}) z \left\{ \frac{1}{s^3} \right\}$$

$$= (1 - z^{-1}) \frac{1}{z} \frac{0.1^2 z^{-1} (1 + z^{-1})}{(1 - z^{-1})^3}$$

$$= \frac{0.005 z^{-1} (1 + z^{-1})}{(1 - z^{-1})^2}$$

$$R(z) = \frac{1}{1 - z^{-1}}$$

$$G_1 G_2(z) = \frac{C(z)}{R(z) - C H(z)}$$

$$C(z) = G_1 G_2(z) R(z) - \alpha C(z) G_1 G_2(z)$$

$$[1 + \alpha G_1 G_2(z)] C(z) = G_1 G_2(z) R(z)$$

$$C(z) = \frac{G_1 G_2(z) R(z)}{1 + \alpha G_1 G_2(z)}$$

others answer?

别人算错了

$$\frac{0.005 z^{-1} (1+z^{-1})}{(1-z^{-1})}$$

$$= \frac{0.005 z^{-1} (1+z^{-1})}{1+\alpha \frac{0.005 z^{-1} (1+z^{-1})}{(1-z^{-1})^2}}$$

$$\frac{0.005 (z^2+1)}{z^3 + (0.005\alpha+1)z^2 - z - (1+0.005\alpha)}$$

$$= \frac{0.005 z^{-1} (1+z^{-1}) (1-z^{-1})}{(1-z^{-1})^2 + \alpha 0.005 z^{-1} (1+z^{-1})}$$

$$= \frac{0.005 z^{-1} - 0.005 z^{-3}}{1 + (0.005\alpha-2)z^{-1} + (0.005\alpha+1)z^{-2}} = \frac{0.005 (z^2-1)}{z^2 + (0.005\alpha-2)z^2 + (0.005\alpha+1)z}$$

(b) from $G_c(z)$

$$z^2 + (0.005\alpha-2)z + 1 + 0.005\alpha \geq 0$$

$$|z_{1,2}| < 1 \Rightarrow \text{stable}$$

$$|z| = \left| \frac{2 - 0.005\alpha \pm \sqrt{(0.005\alpha-2)^2 - 4(1+0.005\alpha)}}{2} \right| < 1$$

$$-2 < 2 - 0.005\alpha \pm \sqrt{(0.005\alpha-2)^2 - 4(1+0.005\alpha)} < 2$$

$$\alpha < 0 \text{ or } \alpha \geq 1600$$

开环脉冲传递函数 $G_c(z) = 0.005\alpha \frac{z+1}{(z-1)^2}$

有 2 个 $z=1$ 开环极点, type 2 对 step input 的输入误差为 0

(b) 求根公式只算了实根, 虚根在单位圆内也稳定

$$z^2 + (\alpha 0.005 - 2)z + (1 + \alpha 0.005) = 0$$

$$\Delta = \sqrt{(0.005\alpha - 2)^2 - 4 \times 1 \times (1 + 0.005\alpha)}$$

$$= \sqrt{2.5 \times 10^{-5} \alpha^2 - 0.01\alpha + 4 - 4 - 0.02\alpha}$$

$$= \sqrt{2.5 \times 10^{-5} \alpha^2 - 0.03\alpha}$$

$$-4 < -5 \times 10^{-3} \alpha \pm \sqrt{2.5 \times 10^{-5} \alpha^2 - 0.03\alpha} < 0$$

很麻烦 $\times \rightarrow$ Jury Test

(b) try Jury test

$$\frac{Q(z)}{R(z)} = \frac{G_1 G_2(z)}{1 + \alpha G_1 G_2(z)}$$

$$\text{let } 1 + \alpha \frac{0.005 z^{-1} (1 + z^{-1})}{(1 - z^{-1})^2} = 0$$

$$(1 - z^{-1})^2 + \alpha 0.005 z^{-1} + \alpha 0.005 z^{-2} = 0$$

$$1 - 2z^{-1} + z^{-2} + \alpha 0.005 z^{-1} + \alpha 0.005 z^{-2} = 0$$

$$1 + (\alpha 0.005 - 2) z^{-1} + (1 + \alpha 0.005) z^{-2} = 0$$

$$z^2 + (\alpha 0.005 - 2)z + (1 + \alpha 0.005) = 0$$

Jury Test

z^0	z^1	z^2
$1 + \alpha 0.005$	$\alpha 0.005 - 2$	1

$$\left\{ \begin{array}{l} |1 + \alpha 0.005| < 1 \end{array} \right.$$

$$p(1) = 1 + 0.005\alpha - 2 + 1 + 0.005\alpha$$

$$= 0.01\alpha > 0$$

$$p(-1) = 1 - 0.005\alpha + 2 + 1 + 0.005\alpha$$

$$= 4 > 0$$

$n=2$
even

$$-1 < 1 + 0.005\alpha < 1$$

$$-2 < 0.005\alpha < 0$$

$$\begin{cases} -400 < \alpha < 0 \\ \alpha > 0 \end{cases}$$

$$\begin{array}{r} -400 \\ -2000 \\ \hline 0.005 \end{array}$$

So the system is not stable

So $\alpha \in \emptyset$ $\lim_{k \rightarrow \infty} |e(kT)|$ not exist.

別人算錯了

(a) others answer

$$\frac{z}{z-1} \quad \frac{0.005(1+z)}{z^2 + 2z + 1 + \alpha 0.005 + \underline{\alpha 0.005z}}$$
$$\frac{0.005(z^2+1)}{z^2 + (0.005\alpha + 2)z + (1 + 0.005\alpha)}$$

$$(z-1)[z^2 + (0.005\alpha + 2)z + (1 + 0.005\alpha)]$$

$$= z^3 + (0.005\alpha + 2)z^2 + (1 + 0.005\alpha)z - z^2$$
$$- (0.005\alpha + 2)z - (1 + 0.005\alpha)$$

$$= z^3 + (0.005\alpha + 1)z^2 - z - (1 + 0.005\alpha)$$

so

$$\frac{0.005(z^2+1)}{z^3 + (0.005\alpha + 1)z^2 - z - (1 + 0.005\alpha)}$$