

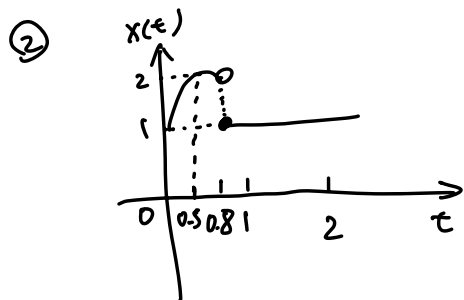
23-51-Q1

Q(a) 是? Z? 0 start  $T=0.5$

Solution

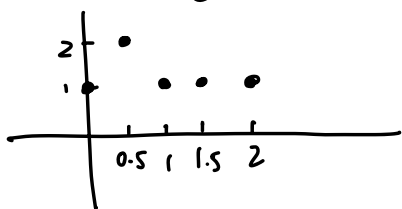
① Yes,  $x(t)$  is a continuous-time signal because it is defined for all real values of  $t$ .

The function  $x(t)$  provides a specific value for every time  $t$  in the real number domain, fulfilling the definition of a continuous-time signal.



$$T = \frac{2\pi}{\omega} = 2$$

↓  $T=0.5$  sampling



according to the definition of Z transform

$$X(Z) = Z[x(t)] = \sum_{k=0}^{\infty} x(kT) Z^{-k}$$

$$= x(0)Z^{-0} + x(0.5)Z^{-1} + x(1)Z^{-2} + x(1.5)Z^{-3} + \dots$$

$$= 1 + 2z^{-1} + z^{-2} + z^{-3} + \dots$$

$$\text{So } \sum_{n=2}^{\infty} z^{-n} = \frac{z^{-2}}{1 - z^{-1}}$$

$$X(z) = 1 + 2z^{-1} + \frac{z^{-2}}{1 - z^{-1}}$$

$$= \frac{(1 + 2z^{-1})(1 - z^{-1}) + z^{-2}}{1 - z^{-1}}$$

$$= \frac{1 - z^{-1} + 2z^{-1} - 2z^{-2} + z^{-2}}{1 - z^{-1}}$$

$$= \frac{1 + z^{-1} - z^{-2}}{1 - z^{-1}}$$

(b) Q: difference equation

Solution apply z transform to

$$y(k+2) + (\beta-1)y(k+1) - \beta y(k) = \delta(k-1)$$

$$z^2 Y(z) - z^2 y(0) - z y(1) + (\beta-1)[z Y(z) - z y(0)] - \beta Y(z) = z^{-1}$$

$$\text{let } k = -2, \quad y(0) + (\beta-1)y(-1) - \beta y(-2) = \delta(-3)$$

$$\text{So } y(0) = 0$$

$$\text{let } k = -1, \quad y(1) + (\beta-1)y(0) - \beta y(-1) = \delta(-2)$$

$$\text{So } y(1) = 0$$

$$\text{So } z^2 Y(z) + (\beta-1)z Y(z) - \beta Y(z) = z^{-1}$$

$$(z^2 + \beta z - z - \beta)Y(z) = z^{-1}$$

$$Y(z) = \frac{z^{-1}}{z^2 + (\beta - 1)z - \beta}$$

$$\frac{Y(z)}{z} = \frac{1}{z^2(z-1)(z+\beta)}$$

$\times \downarrow$  错误方法  
 $\times$  跳过  
 $\times$  去

$$= \frac{A}{z^2} + \frac{B}{z-1} + \frac{C}{z+\beta}$$

重根应该是  $Az + B$

$$A(z-1)(z+\beta) + Bz^2(z+\beta) + Cz^2(z-1)$$

$$A(z^2 + \beta z - z + \beta) + B(z^3 + \beta z^2) + C(z^3 - z^2)$$

$$(B+C)z^3 + (A+B\beta-C)z^2 + (A\beta-A)z + A\beta$$

$$\begin{cases} B+C=0 \\ A+B\beta-C=0 \\ A\beta-A=0 \\ A\beta=1 \end{cases}$$

$$\Rightarrow \begin{cases} A=1 \\ B=-\frac{1}{2} \\ C=\frac{1}{2} \\ \beta=1 \end{cases}$$

$$\begin{aligned} B+C &= 0 \\ 1+B-C &= 0 \\ 1+2B &= 0 \\ B &= -\frac{1}{2} \end{aligned}$$

$$\frac{Y(z)}{z} = \frac{1}{z^2} + \frac{-\frac{1}{2}}{z-1} + \frac{\frac{1}{2}}{z+1}$$

$$Y(z) = z^{-1} - \frac{1}{2} \frac{1}{1-z^{-1}} + \frac{1}{2} \frac{1}{1+z^{-1}}$$

$$y(k) = \delta_0(n-1) - \frac{1}{2} 1(k) + \frac{1}{2} \cos k\pi$$

(c) (b) 部分不全  $\beta$ , so (b) 过程有误

x ↑

$$\frac{Y(z)}{z} = \frac{1}{z^2(z-1)(z+\beta)}$$

$$\frac{A}{z} + \frac{B}{z^2} = \frac{Az+B}{z^2} + \frac{C}{z-1} + \frac{D}{z+\beta}$$

下次写标准

改进: 试根法 ↓ 文末

$$\begin{aligned} & (Az+B)(z-1)(z+\beta) + C z^2(z+\beta) + D z^2(z-1) \\ &= (Az+B)(z^2+\beta z - z - \beta) + C(z^3+\beta z^2) + D(z^3-z^2) \\ &= (Az^3 + \underbrace{A\beta z^2 - Az^2}_{\oplus} + \underbrace{A\beta z + Bz^2}_{\oplus} + \underbrace{B\beta z - Bz}_{\oplus} - \underbrace{B\beta}_{\triangle}) \\ & \quad + C(z^3 + \beta z^2) + D(z^3 - z^2) \end{aligned}$$

$$\begin{cases} A+C+D=0 \\ A\beta - A + B + C\beta - D = 0 \\ -A\beta + B\beta - B = 0 \\ -B\beta = 1 \end{cases} \quad \begin{aligned} (1) & \quad A = \frac{1-\beta}{\beta^2} \\ (2) & \quad B = -\frac{1}{\beta} \\ (3) & \quad C = \frac{1}{1+\beta} \\ (4) & \quad D = -\frac{1}{\beta^2(\beta+1)} \end{aligned}$$

from (4)  $B = -\frac{1}{\beta}$

from (3)  $A = \frac{B - B\beta}{\beta} = \frac{B(1-\beta)}{\beta} = \frac{1-\beta}{\beta^2} = \frac{\beta-1}{\beta^2}$

from (1)

$$C + D = -A = \frac{\beta - 1}{\beta^2} \quad (5)$$

from (2)  $A\beta - A + B + C\beta - D = 0$

$$\frac{1-\beta}{\beta} - \frac{1-\beta}{\beta^2} - \frac{1}{\beta} + C\beta - D = 0$$

$$C\beta - D = \frac{\beta^2 + 1 - \beta}{\beta^2} \quad (6)$$

$$(5) + (6) \quad (1+\beta)C = \frac{\beta - 1 + \beta^2 + 1 - \beta}{\beta^2} = 1 \Rightarrow C = \frac{1}{1+\beta}$$

$$\text{from (5)} \quad D = \frac{\beta - 1}{\beta^2} - C = \frac{\beta - 1}{\beta^2} - \frac{1}{1+\beta} = \frac{\beta^2 - 1 - \beta^2}{\beta^2(1+\beta)}$$

$$\text{So } D = \frac{-1}{\beta^2(1+\beta)}$$

$$\frac{Y(z)}{z} = \frac{A}{z} + \frac{B}{z^2} + \frac{C}{z-1} + \frac{D}{z+\beta}$$

$$Y(z) = \frac{1-\beta}{\beta^2} - \frac{1}{\beta} z^{-1} + \frac{1}{1+\beta} \frac{1}{1-z^{-1}} - \frac{1}{\beta^2(\beta+1)} \frac{1}{1+\beta z^{-1}}$$

$$y(k) = \frac{1-\beta}{\beta^2} \delta_0(k) - \frac{1}{\beta} \delta_0(k-1) + \frac{1}{1+\beta} 1(k) - \frac{1}{\beta^2(\beta+1)} (-\beta)^k$$

$$y(k) = \begin{cases} \frac{1-\beta}{\beta^2} + \frac{1}{1+\beta} - \frac{(-\beta)^k}{\beta^2(\beta+1)} & k=0 \\ -\frac{1}{\beta} + \frac{1}{1+\beta} - \frac{(-\beta)^k}{\beta^2(\beta+1)} & k=1 \end{cases}$$

$$y(0) = 0$$

$$y(1) = 0$$

$$y(2) = 0$$

$$\left| \frac{1}{1+\beta} - \frac{(-\beta)^k}{\beta^2(\beta+1)} \right| \quad k \geq 2$$

$$y(k) = \begin{cases} 0 & k=0,1,2 \\ \frac{1}{1+\beta} - \frac{(-\beta)^k}{\beta^2(\beta+1)} & k=3,4,5,\dots \end{cases}$$

(c)  $\beta \rightarrow y(k)$  convergence?

Solution

① when  $|\beta| < 1$ ,  $k \rightarrow \infty$ ,  $(-\beta)^k \rightarrow 0$

$$y(k) \rightarrow \frac{1}{1+\beta} \quad |z_1| = 1 \quad |z_2| = |-\beta| = |\beta| < 1$$

The Final Value Theorem applies

all poles of  $Y(z)$  lie inside the unit circle, with the possible exception of a simple pole at  $z=1$

Verification:  $\lim_{k \rightarrow \infty} y(k) = \lim_{z \rightarrow 1} (1-z^{-1})Y(z) = \frac{1}{\beta+1}$

so confirm theorem acceptable

② when  ~~$|\beta| \geq 1$~~   $\beta = 1$   $(-\beta)^k$  oscillates between  $\pm 1$

$y(k)$  doesn't converge, it oscillates,  $Y(z)$  has a pole at  $z=-1$ .

The Final value Theorem does not apply

③ when  $\beta = -1$

$$Y(z) = \frac{1}{z(z-1)(z+\beta)}$$

It has two poles at  $z=1$   
FVT doesn't apply  
 $y(k)$  doesn't converge

$$\frac{1}{1+\beta} - \frac{(-\beta)^k}{\beta^2(\beta+1)} = \frac{\beta^2 - (-\beta)^k}{\beta^2(1+\beta)}$$
~~$$\lim_{\beta \rightarrow -1} \frac{2\beta + (-1)^k k (-\beta)^{k-1}}{3\beta^2 + 2\beta} = \infty$$~~

④ ③ when  $|\beta| > 1$ ,  $(-\beta)^k$  grows without bound

$y(k)$  diverges as  $k \rightarrow \infty$   $y(z)$  has a pole  
 $|z| = |\beta| > 1$

The Final value Theorem does not apply

~~$y(k)$  does not converge~~

关于(a) z transform 的思考, 前面用定义的 z-transform  
 现在试用表求,  
 △

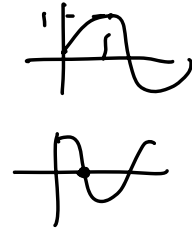
Solution

$$Z\{1 + \sin(\omega t)\} \quad \#3$$

$$= \frac{1}{1-z^{-1}} + Z\{\sin(\omega t)\} \quad \#14$$

$$= \frac{1}{1-z^{-1}} + \frac{z^{-1} \sin \omega T}{1 - 2z^{-1} \cos \omega T + z^{-2}}$$

$$T = \frac{1}{2}$$



$$= \frac{1}{1-z^{-1}} + \frac{z^{-1}}{1+z^{-2}}$$

$$= \frac{(1+z^{-2}) + z^{-1}(1-z^{-1})}{(1-z^{-1})(1+z^{-2})}$$

$$= \frac{1+z^{-2}+z^{-1}-z^{-2}}{(1-z^{-1})(1+z^{-2})}$$

$$= \frac{1+z^{-1}}{(1-z^{-1})(1+z^{-2})}$$

$$0 \leq t < 0.8$$

$$Z\{1\} = \frac{1}{1-z^{-1}} \quad t \geq 0.8$$

不会了

结论: 分段用定义

连续用查表

查不出来用定义



关于23-Q1-b 对ABCD的优化:试根法

Solution

$$\frac{Y(z)}{z} = \frac{1}{z^2(z-1)(z+\beta)}$$

$$= \frac{A}{z} + \frac{B}{z^2} + \frac{C}{z-1} + \frac{D}{z+\beta}$$

$$A z(z-1)(z+\beta) + B(z-1)(z+\beta) + C z^2(z+\beta) + D z^2(z-1) = 1$$

由于等式过于复杂,所以可以采用试根法

A B C D 对于  $z \in \mathbb{R}$  都成立, 所以代入  $z$  值求ABCD

$$\textcircled{1} z=0, \quad -\beta B = 1 \Rightarrow B = -\frac{1}{\beta}$$

$$\textcircled{2} z=1, \quad (\beta+1)C = 1 \Rightarrow C = \frac{1}{\beta+1}$$

$$\textcircled{3} z = -\beta, \quad \beta^2(\beta-1)D = 1 \Rightarrow D = \frac{-1}{\beta^2(\beta+1)}$$

④ 考虑  $z^3$  项求 A

$$A z^3 + C z^3 + D z^3 = (A+C+D) z^3$$

$$A+C+D=0$$

$$\Rightarrow A = -C - D = -\frac{1}{\beta+1} + \frac{1}{\beta^2(\beta+1)}$$

$$= \frac{1-\beta^2}{\beta^2(\beta+1)}$$

$$= \frac{(1-\beta)(1+\beta)}{\beta^2(\beta+1)}$$

$$= \frac{1-\beta}{\beta^2}$$