18-S1-Q4

Q(a) T=1 deadbeat

Solution

$K = [0 \quad 1] W_c^{-1} \alpha_c(A)$

$\alpha_c(z) = z^2$

$$A = \begin{bmatrix} 0.3679 & 0 \\ 0.6321 & 1 \end{bmatrix}$$

$$\alpha_c(A) = A^2 = \begin{bmatrix} 0.1354 & 0 \\ 0.8646 & 1 \end{bmatrix}$$

$W_c = [B \quad AB]$

$$B = \begin{bmatrix} 1 - e^{-1} \\ e^{-1} \end{bmatrix} = \begin{bmatrix} 0.6321 \\ 0.3679 \end{bmatrix}$$

$$W_c = \begin{bmatrix} 0.6321 & 0.08559 \\ 0.3679 & 0.9144 \end{bmatrix}$$

$$AB = \begin{bmatrix} 0.1354 & 0 \\ 0.8646 & 1 \end{bmatrix} \begin{bmatrix} 0.6321 \\ 0.3679 \end{bmatrix} = \begin{bmatrix} 0.08559 \\ 0.9144 \end{bmatrix}$$

$$W_c^{-1} = \begin{bmatrix} 1.6732 & -0.1566 \\ -0.6732 & 1.1566 \end{bmatrix}$$

$$K = [0\ 1]\begin{bmatrix} 1.6732 & -0.1566 \\ -0.6732 & 1.1566 \end{bmatrix}\begin{bmatrix} 0.1354 & 0 \\ 0.8646 & 1 \end{bmatrix}$$

$$= [-0.6732 \quad 1.1566]\begin{bmatrix} 0.1354 & 0 \\ 0.8646 & 1 \end{bmatrix}$$

$$= [0.9088 \quad 1.1566]$$

(b) $z_{1,2} = 0$, error deadbeat verify?

Solution

$$L_o = \alpha_o(A) W_o^{-1} \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

$$\alpha_o(z) = (z-0)(z-0) = z^2$$

$$\alpha_o(A) = A^2 = \begin{bmatrix} 0.78 & 0 \\ 0.22 & 1 \end{bmatrix}^2 = \begin{bmatrix} 0.6084 & 0 \\ 0.3916 & 1 \end{bmatrix}$$

$$W_o = \begin{bmatrix} C \\ CA \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 0.22 & 1 \end{bmatrix}$$

$$CA = [0\ 1]\begin{bmatrix} 0.78 & 0 \\ 0.22 & 1 \end{bmatrix} = [0.22 \quad 1]$$

$$W_o^{-1} = \begin{bmatrix} -4.5455 & 4.5455 \\ 1 & 0 \end{bmatrix}$$

$$L_o = \begin{bmatrix} 0.6084 & 0 \\ 0.3916 & 1 \end{bmatrix}\begin{bmatrix} -4.5455 & 4.5455 \\ 1 & 0 \end{bmatrix}\begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} 2.7655 \\ 1.7800 \end{bmatrix}$$

assume $x(0) = \begin{bmatrix} a_1 \\ b_1 \end{bmatrix}$ $\bar{x}(0) = \begin{bmatrix} a_2 \\ b_2 \end{bmatrix}$

我只知道K怎么证明，假设状态x（0）=【a b】T然后算x（1）x(2) 一般x（3）的时候就是零矩阵了，所以deadbeat，但是观测器的不懂怎么搞 → 16.2

$$X_e(0) = x(0) - \bar{x}(0) = \begin{bmatrix} a_1 - a_2 \\ b_1 - b_2 \end{bmatrix} = \begin{bmatrix} a \\ b \end{bmatrix}$$

$$X_e(k+1) = [A - L_oC]x_e(k)$$

$$[A - L_oC] = \begin{bmatrix} 0.78 & 0 \\ 0.22 & 1 \end{bmatrix} - \begin{bmatrix} 2.7655 \\ 1.7800 \end{bmatrix} \begin{bmatrix} 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 0.78 & 0 \\ 0.22 & 1 \end{bmatrix} - \begin{bmatrix} 0 & 2.7655 \\ 0 & 1.7800 \end{bmatrix}$$

$$= \begin{bmatrix} 0.78 & -2.7655 \\ 0.22 & -0.78 \end{bmatrix}$$

$$X_e(1) = \begin{bmatrix} 0.78 & -2.7655 \\ 0.22 & -0.78 \end{bmatrix} X_e(0)$$

$$= \begin{bmatrix} 0.78a - 2.7655b \\ 0.22a - 0.78b \end{bmatrix}$$

$$X_e(2) = \begin{bmatrix} 0.78 & -2.7655 \\ 0.22 & -0.78 \end{bmatrix} \begin{bmatrix} 0.78a - 2.7655b \\ 0.22a - 0.78b \end{bmatrix}$$

$$= \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

So verify done

(c) $A_e = ?$

$$x(k+1) = Ax(k) + Bu(k) \quad ①$$

$$y(k) = Cx(k) \quad ②$$

$$\bar{x}(k) = A\bar{x}(k-1) + Bu(k-1) + L_o\{y(k) - C(A\bar{x}(k-1) + Bu(k-1))\} \quad ③$$

$$X_e(k) = x(k) - \bar{x}(k) \quad ④$$

$$X_e(k+1) = A_e X_e(k) \quad ⑤$$

Solution

$$X_e(k+1) = x(k+1) - \bar{x}(k+1)$$

①③

$$= Ax(k) + Bu(k) - (A\bar{x}(k) + Bu(k) + L_o\{y(k+1) - C(A\bar{x}(k) + Bu(k))\}$$

$$= A(x(k) - \bar{x}(k)) - L_o\{y(k+1) - C(A\bar{x}(k) + Bu(k))\} \quad ⑥$$

② $y(k+1) = Cx(k+1)$

$$= C(Ax(k) + Bu(k))$$

$$y(k+1) - C(A\bar{x}(k) + Bu(k))$$

$$= CA(x(k) - \bar{x}(k))$$

$$= CAX_e(k) \quad ⑦$$

⑥⑦ $X_e(k+1) = AX_e(k) - L_o CAX_e(k)$

$$= (A - L_o CA)X_e(k)$$

So $A_e = A - L_o CA$