知识点Z3.21

卷积和的性质

主要内容：

1. 卷积和的运算规则
2. 常用卷积和的公式

基本要求：

掌握卷积和的性质及其重要公式

Z3.21 卷积和的性质

1. 满足乘法的三律

(1) 交换律：$f_1(k)*f_2(k)=f_2(k)*f_1(k)$

(2) 分配律：$f_1(k)*[f_2(k)+f_3(k)]=f_1(k)*f_2(k)+f_1(k)*f_3(k)$

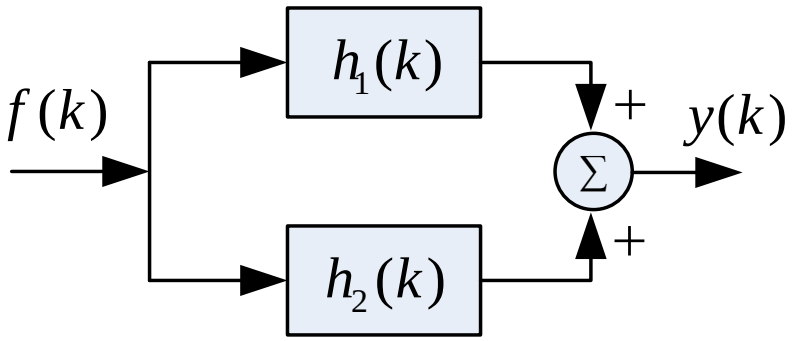
(3) 结合律：$f_1(k)*[f_2(k)*f_3(k)]=[f_1(k)*f_2(k)]*f_3(k)$

证明：(仅证明交换律，其它类似)

$$f_1(k)*f_2(k)=\sum_{i=-\infty}^{\infty}f_1(i)f_2(k-i)=\sum_{j=\infty}^{-\infty}f_1(k-j)f_2(j)$$

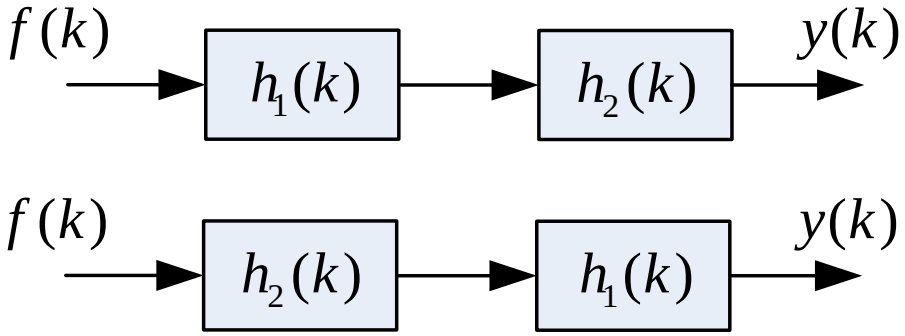
$$=\sum_{j=-\infty}^{\infty}f_2(j)f_1(k-j)=f_2(k)*f_1(k)$$

2. 复合系统的单位脉冲响应



$$h(k)=h_1(k)+h_2(k)$$

(a) 并联



$$h(k)=h_1(k)*h_2(k)=h_2(k)*h_1(k)$$

(b) 级联

3. $f(k)*\delta(k)=\delta(k)*f(k)=f(k)$，$f(k)*\delta(k-k_0)=f(k-k_0)$

4. $f(k)*\varepsilon(k)=\sum_{i=-\infty}^{k} f(i)$

5. $f_1(k-k_1)*f_2(k-k_2)=f_1(k-k_1-k_2)*f_2(k)$

6. $\nabla[f_1(k)*f_2(k)]=\nabla f_1(k)*f_2(k)=f_1(k)*\nabla f_2(k)$

常用卷积和公式

$$(1)\ f(k)*\delta(k)=f(k);$$

$$(2)\ f(k)*\delta(k-k_0)=f(k-k_0);$$

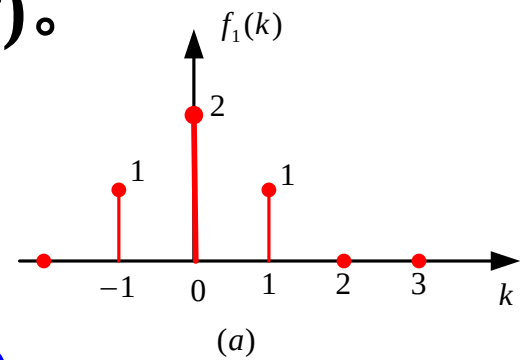
$$(3)\ \delta(k)*\delta(k)=\delta(k);$$

$$(4)\ f(k)*\varepsilon(k)=\sum_{i=-\infty}^{k}f(i);$$

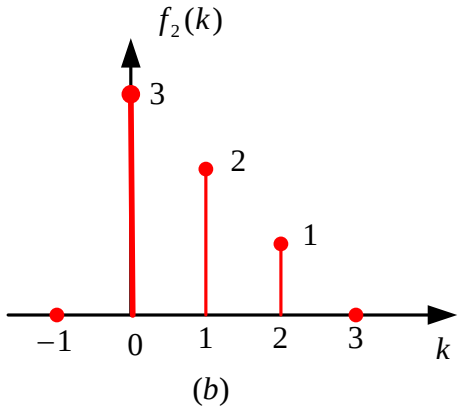
$$(5)\ f_1(k-k_1)*f_2(k-k_2)=f_1(k-k_2)*f_2(k-k_1)$$

$$=f_1(k)*f_2(k-k_1-k_2)=f_1(k-k_1-k_2)*f_2(k)$$

例1 求 $f(k)= f_1(k)* f_2(k)$。



(a)



(b)

解法I：（不进位乘法）

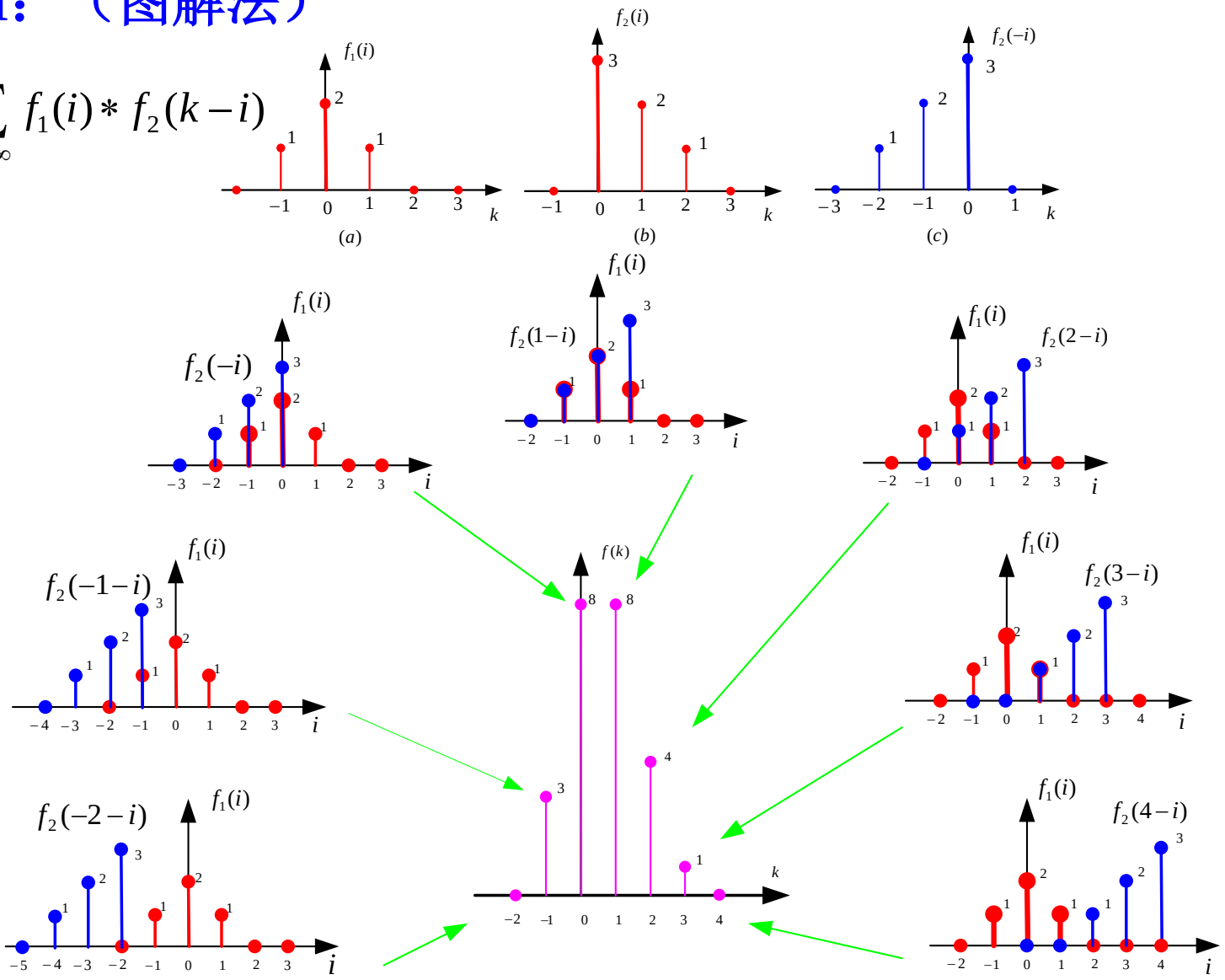
		1	2	1
		3	2	1
		1	2	1
	2	4	2	
3	6	3		
3	8	8	4	1

$f(k) = \{0,\ 3\ ,\ 8,\ 8,\ 4,\ 1,\ 0\}$

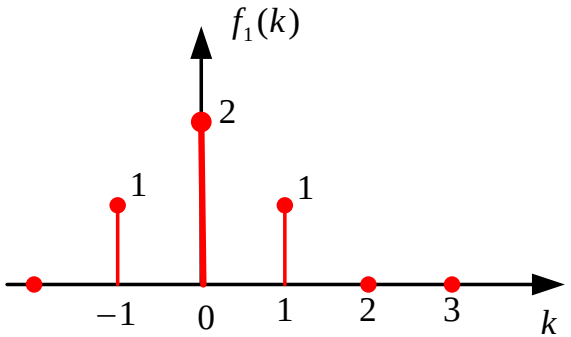
$\uparrow k=-1$

解法II：（图解法）

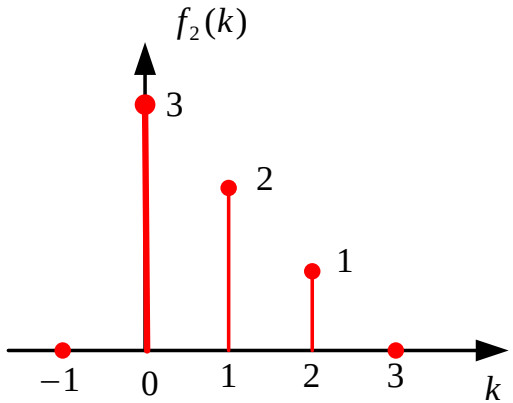
$$f(k)=\sum_{i=-\infty}^{\infty} f_1(i) * f_2(k-i)$$



解法III：（解析法）



(a)



(b)

$$f_1(k)=\delta(k+1)+2\delta(k)+\delta(k-1)$$

$$f_2(k)=3\delta(k)+2\delta(k-1)+\delta(k-2)$$

$$f_1(k)*f_2(k)$$

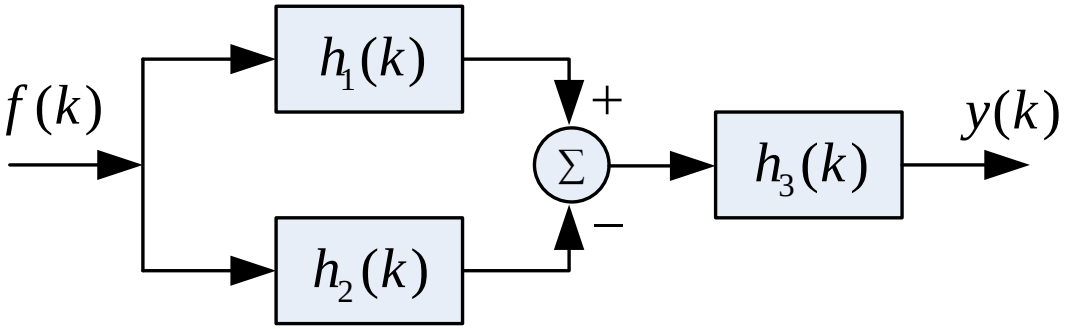
$$=[\delta(k+1)+2\delta(k)+\delta(k-1)]*[3\delta(k)+2\delta(k-1)+\delta(k-2)]$$

$$=[3\delta(k+1)+2\delta(k)+\delta(k-1)]+[6\delta(k)+4\delta(k-1)+2\delta(k-2)]$$

$$+[3\delta(k-1)+2\delta(k-2)+\delta(k-3)]$$

$$=3\delta(k+1)+8\delta(k)+8\delta(k-1)+4\delta(k-2)+\delta(k-3)$$

例2 如图所示复合系统，由3个子系统组成，它们的单位脉冲响应分别为$h_1(k)=\delta(k)$，$h_2(k)=\delta(k-N)$，N为常数，$h_3(k)=\varepsilon(k)$，求复合系统的单位脉冲响应。



解： 由复合系统的各子系统间的关系得：

$$h(k)=[h_1(k)-h_2(k)]*h_3(k)$$

$$=[\delta(k)-\delta(k-N)]*\varepsilon(k)=\delta(k)*\varepsilon(k)-\delta(k-N)*\varepsilon(k)$$

$$=\varepsilon(k)-\varepsilon(k-N)$$