23 - S1 - Q3

Q (i) Q: state - space?

Solution

$$\begin{bmatrix} x_1(k+1) \\ x_2(k+1) \end{bmatrix} = \begin{bmatrix} 2 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} x_1(k) \\ x_2(k) \end{bmatrix} + \begin{bmatrix} 1 \\ 1 \end{bmatrix} u(k)$$

$$\begin{bmatrix} y_1(k) \\ y_2(k) \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 3 \end{bmatrix} \begin{bmatrix} x_1(k) \\ x_2(k) \end{bmatrix}$$

$$y(k) = 4\, y_1(k) - 2\, y_2(k)$$

$$= \begin{bmatrix} 4 & -2 \end{bmatrix} \begin{bmatrix} y_1(k) \\ y_2(k) \end{bmatrix}$$

$$= \begin{bmatrix} 4 & -2 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 3 \end{bmatrix} \begin{bmatrix} x_1(k) \\ x_2(k) \end{bmatrix}$$

$$= \begin{bmatrix} 4 & -6 \end{bmatrix} \begin{bmatrix} x_1(k) \\ x_2(k) \end{bmatrix}$$

(ii)

$$\frac{Y(z)}{U(z)} = C[zI - A]^{-1} B + D$$

$$= \begin{bmatrix} 4 & -6 \end{bmatrix} \begin{bmatrix} z-2 & 0 \\ 0 & z-1 \end{bmatrix}^{-1} \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} 4 & -6 \end{bmatrix} \frac{1}{(z-2)(z-1)} \begin{bmatrix} z-1 & 0 \\ 0 & z-2 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

$$= [4 \quad -6] \begin{bmatrix} \frac{1}{z-2} & 0 \\ 0 & \frac{1}{z-1} \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} \frac{4}{z-2} & \frac{-6}{z-1} \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

$$= \frac{4}{z-2} - \frac{6}{z-1}$$

$$= \frac{4(z-1) - 6(z-2)}{(z-2)(z-1)}$$

$$= \frac{4z - 4 - 6z + 12}{(z-2)(z-1)}$$

$$= \frac{-2z + 8}{(z-2)(z-1)}$$

(b) @ $T = \frac{\pi}{2}$ $\quad D$?

$$A_d = \Phi(t = T) = \mathcal{L}^{-1}\{[sI - A]^{-1}\}\Big|_{t=T}$$

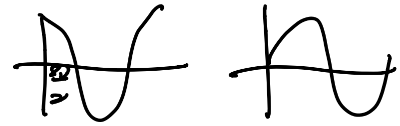
$$[sI - A]^{-1} = \begin{bmatrix} s & -1 \\ 1 & s \end{bmatrix}^{-1}$$

$$= \frac{1}{s^2+1} \begin{bmatrix} s & 1 \\ -1 & s \end{bmatrix}$$

$$= \begin{bmatrix} \frac{s}{s^2+1} & \frac{1}{s^2+1} \\ \frac{-1}{s^2+1} & \frac{s}{s^2+1} \end{bmatrix}$$

$$A_d = \begin{bmatrix} \cos t & \sin t \\ -\sin t & \cos t \end{bmatrix}\Bigg|_{t=T=\frac{\pi}{2}} = \begin{bmatrix} \cos\frac{\pi}{2} & \sin\frac{\pi}{2} \\ -\sin\frac{\pi}{2} & \cos\frac{\pi}{2} \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$$

$$B_d = \int_0^T \Phi(\tau) B \, d\tau \Big|_{T=\frac{\pi}{2}}$$



$$= \int_0^T \begin{bmatrix} \cos\tau & \sin\tau \\ -\sin\tau & \cos\tau \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix} d\tau \Bigg|_{T=\frac{\pi}{2}}$$

$$= \int_0^T \begin{bmatrix} \cos\tau + \sin\tau \\ -\sin\tau + \cos\tau \end{bmatrix} d\tau \Bigg|_{T=\frac{\pi}{2}} = \begin{bmatrix} 2 \\ 0 \end{bmatrix}$$

$$\int_0^T \cos\tau + \int_0^T \sin\tau = \sin\tau\Big|_0^{\frac{\pi}{2}} - \cos\tau\Big|_0^{\frac{\pi}{2}} = (1-0) - (0-1)$$

$$= 1 + 1 = 2$$

$$-\int_0^T \sin\tau + \int_0^T \cos\tau = -1 + 1 = 0$$

$$C_d = C = [1 \ \ 0]$$

$$x(k+1) = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} x(k) + \begin{bmatrix} 2 \\ 0 \end{bmatrix} u(k)$$

$$y(k) = [1 \ \ 0] \, x(k)$$

(ii) Q $u(0) = ?$ $u(1) = ?$ $x(0) \to x(2)$

Solution $x(2) = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} x(1) + \begin{bmatrix} 2 \\ 0 \end{bmatrix} u(1) = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$

$$x(1) = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} x(0) + \begin{bmatrix} 2 \\ 0 \end{bmatrix} u(0) = \begin{bmatrix} 2 \\ 0 \end{bmatrix} u(0)$$

$$\begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} \begin{bmatrix} 2 \\ 0 \end{bmatrix} u(0) + \begin{bmatrix} 2 \\ 0 \end{bmatrix} u(1) = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$$

不要照抄抄错啊

-1 * 2 = -2

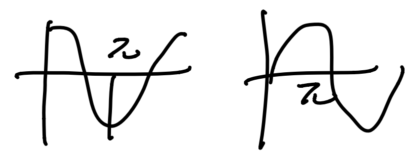
$$\begin{bmatrix} 0 \\ -1 \end{bmatrix} u(0) + \begin{bmatrix} 2 \\ 0 \end{bmatrix} u(1) = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$$

-2

$2\,u(1) = 1 \quad \Rightarrow \quad u(0) = -2 \quad -1 \qquad u(1) = \frac{1}{2}$

$-2\,u(0) = 2$

(iii) Q: $T = \pi$ C ?



Solution $\quad A_d = \begin{bmatrix} \cos t & \sin t \\ -\sin t & \cos t \end{bmatrix} \Bigg|_{t=T=\pi} = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$

$$B_d = \int_0^T \begin{bmatrix} \cos\tau + \sin\tau \\ -\sin\tau + \cos\tau \end{bmatrix} d\tau \Bigg|_{T=\pi} = \begin{bmatrix} 2 \\ -2 \end{bmatrix}$$

$$\int_0^T \cos\tau + \int_0^T \sin\tau = \sin\tau\Big|_0^\pi - \cos\tau\Big|_0^\pi = -(-1-1) = 2$$

$$-\int_0^T \sin\tau + \int_0^T \cos\tau = -2 + 0 = -2$$

$$W_c = [\,B_d \quad A_d B_d\,] = \begin{bmatrix} 2 & -2 \\ -2 & 2 \end{bmatrix} \qquad |W_c| = 4 - 4 = 0$$

$$\begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} 2 \\ -2 \end{bmatrix} = \begin{bmatrix} -2 \\ 2 \end{bmatrix}$$

uncontrollable

(c) Observability?

Solution

$$x(k+1) = Ax(k) + \beta B u(k)$$

$$y(k) = Cx(k)$$

算出来W0行列式
跟贝塔没关系就行了 可省

$$A = \begin{bmatrix} 1 & 0.2753 \\ 0 & 0.4503 \end{bmatrix} \quad B = \begin{bmatrix} 0.06233 \\ 0.2803 \end{bmatrix} \quad C = [1 \quad 0]$$

$$W_o = \begin{bmatrix} C \\ CA \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 1 & 0.2753 \end{bmatrix} \quad |W_o| = \begin{vmatrix} 1 & 0 \\ 1 & 0.2753 \end{vmatrix} \neq 0$$

So the system is observable.

the control law is $u(k) = r(k) - y(k)$

$$x(k+1) = Ax(k) + \beta B[r(k) - y(k)]$$

$$= Ax(k) + \beta B[r(k) - Cx(k)]$$

$$= (A - \beta BC)x(k) + \beta B r(k)$$

let $A_{cl} = A - \beta BC = \begin{bmatrix} 1 - 0.06233\beta & 0.2753 \\ -0.2803\beta & 0.4503 \end{bmatrix}$

$$W_o = \begin{bmatrix} C \\ CA_{cl} \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 1 - 0.06233\beta & 0.2753 \end{bmatrix}$$

$|W_o| = 0.2753 \neq 0$

So the system is observable.

Sine the determinant is non-zero

and independent of β

So the observability matrix is full rank