22 - S1 - Q1

Q : (a) continuous? T=0.5 Z? $\lim_{k\to\infty} x(kT) = ?$

Solution

Justify : Yes, because it is defined for all real values of t and the function x(t) provides a specific value for every time t in the real number domain, fulfilling the definition of a continuous - time signal

① defined over a continuous range of time

② amplitude of continuous range of value or discrete values

$$Z\left\{e^{-2t} \sin\left(\frac{2}{2}t\right) + 1\right\}$$

$$= Z\left\{e^{-2t} \sin\left(\frac{2}{2}t\right)\right\} + Z\left\{1\right\}$$

$$= Z\left\{\sin\frac{2}{2}t\right\}\bigg|_{z= ze^{2T}} + \frac{1}{1-z^{-1}}$$

$$= \frac{z^{-1} \sin\frac{2}{2}T}{1-2z^{-1}\cos\frac{2}{2}T + z^{-2}}\bigg|_{z=ze^{2T}} + \frac{1}{1-z^{-1}}$$

$$= \frac{z^{-1}\frac{\sqrt{2}}{2}}{1-2z^{-1}\frac{\sqrt{2}}{2} + z^{-2}}\bigg|_{z=ez} + \frac{1}{1-z^{-1}}$$

$$= \frac{\frac{\sqrt{2}}{2e} z^{-1}}{1 - \frac{\sqrt{2}}{e} z^{-1} + \frac{1}{e^2} z^{-2}} + \frac{1}{1 - z^{-1}}$$

$$= \frac{0.2601 z^{-1}}{1 - 0.5203 z^{-1} + 0.1353 z^{-2}} + \frac{1}{1 - z^{-1}}$$

$$= \frac{0.2601 z^{-1}(1 - z^{-1}) + 1 - 0.5203 z^{-1} + 0.1353 z^{-2}}{(1 - 0.5203 z^{-1} + 0.1353 z^{-2})(1 - z^{-1})}$$

$$= \frac{1 - 0.2602 z^{-1} - 0.1248 z^{-2}}{(1 - 0.5203 z^{-1} + 0.1353 z^{-2})(1 - z^{-1})}$$

the pole at $0.2601 \pm j 0.2601$, 1 within the unit circle
and with a possible exception of a simple
pole at $z = 1$, fulfill the final Value Theorem

$$\lim_{k \to \infty} x(k) = \lim_{z \to 1} (1 - z^{-1}) X(z)$$

$$= \lim_{z \to 1} \frac{1 - 0.2602 z^{-1} - 0.1248 z^{-2}}{1 - 0.5203 z^{-1} + 0.1353 z^{-2}}$$

$$= 1$$

(b) Solve? $\lim\limits_{k\to\infty} x(kT)$?

Solution

apply z transform

$$x(k+2) - x(k) = 1(k)$$

$$z^2 X(z) - z^2 x(0) - z X(1) - X(z) = \frac{1}{1-z^{-1}}$$

let $k=-2$, $x(0) - x(-2) = 1(-2) \Rightarrow x(0) = 0$

let $k=-1$, $x(1) - x(-1) = 1(-1) \Rightarrow x(1) = 0$

So $(z^2-1) X(z) = \frac{1}{1-z^{-1}}$

$$X(z) = \frac{1}{(1-z^{-1})(z^2-1)}$$

$$= \frac{z}{(z-1)(z^2-1)}$$

$$\frac{X(z)}{z} = \frac{1}{(z-1)(z^2-1)} = \frac{1}{(z-1)(z-1)(z+1)} = \frac{1}{(z-1)^2(z+1)}$$

$$= \frac{A}{z-1} + \frac{B}{(z-1)^2} + \frac{C}{z+1}$$

$$\Rightarrow A(z-1)(z+1) + B(z+1) + C(z-1)^2$$

$$= A(z^2-1) + B(z+1) + C(z^2-2z+1)$$

$$= (A+C)z^2 + (B-2C)z + (-A+B+C)$$

$$\begin{cases} A+C = 0 \\ B-2C = 0 \\ -A+B+C = 1 \end{cases} \Rightarrow \begin{cases} A = -\frac{1}{4} \\ B = \frac{1}{2} \\ C = \frac{1}{4} \end{cases}$$

$$\frac{X(Z)}{Z} = \frac{-\frac{1}{4}}{Z-1} + \frac{\frac{1}{2}}{(Z-1)^2} + \frac{\frac{1}{4}}{Z+1}$$

$$X(Z) = -\frac{1}{4} \frac{1}{1-Z^{-1}} + \frac{1}{2} \frac{Z^{-1}}{(1-Z^{-1})^2} + \frac{1}{4} \frac{1}{1+Z^{-1}}$$

apply inverse Z transform   #3 #5 #18

$$X(kT) = -\frac{1}{4} 1(k) + \frac{1}{2}k + (-1)^k \cdot \frac{1}{4} \quad \text{:偏系数}$$

$$X(Z) = \frac{Z}{(Z-1)(Z^2-1)}$$

the pole at $Z=1$, $Z=-1$, $Z=1$

$Z=-1$ is not within the unit circle,

So it can't apply the Final value Theorem

$$X(kT) = -\frac{1}{4}1(k) + \frac{1}{2}k + (-1)^k \cdot \frac{1}{4}$$

$$\lim_{k\to\infty} \frac{1}{2}k \to \infty \text{, so } X(kT) \to \infty$$