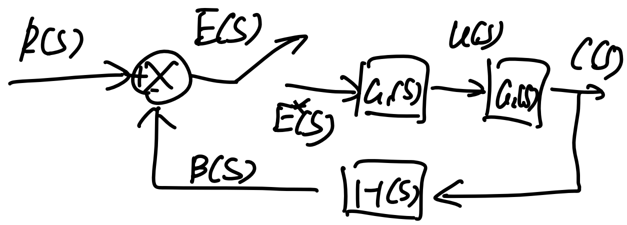
21-S1-Q2

Q (a) $G_{cl}(z)$?

Solution ① tranfer functn



$$G_1(S)=\frac{U(S)}{E^*(S)}$$

$$G_2(S)=\frac{C(S)}{U(S)}$$

$$H(S)=\frac{B(S)}{C(S)}$$

② ERROR

$$E(S)=R(S)-B(S)$$

$$=R(S)-H(S)C(S)$$

$$=R(S)-H(S)G_2(S)U(S)$$

$$=R(S)-H(S)G_2(S)G_1(S)E^*(S)$$

③ Star

$$E^*(S)=R^*(S)-G_1G_2H^*(S)E^*(S)$$

$$E^*(S)=\frac{R^*(S)}{1+G_1G_2H^*(S)}$$

④ target

$$C(S)=G_2(S)G_1(S)E^*(S)=\frac{G_1(S)G_2(S)R^*(S)}{1+G_1G_2H^*(S)}$$

$$C^*(s) = \frac{G_1G_2^*(s)\, R^*(s)}{1+G_1G_2H^*(s)}$$

$$\frac{C(z)}{R(z)} = \frac{G_1G_2(z)}{1+G_1G_2H(z)}$$

So the system has a transfer function

(b) z transfer ?

Solution

$$G_1G_2(z) = Z\left\{G_1(s)G_2(s)\right\}$$

$$= (1-z^{-1})\, Z\left\{\frac{10(0.5s+1)}{s^3}\right\}$$

where $Z\left\{\frac{5s+10}{s^3}\right\}$

$$= 5Z\left\{\frac{1}{s^2}\right\} + 5Z\left\{\frac{2}{s^3}\right\} \qquad T=0.2$$

#5 $\frac{1}{s^2}$ $\qquad \frac{0.2z^{-1}}{(1-z^{-1})^2}$

#6 $\frac{2}{s^3}$ $\qquad \frac{0.04z^{-1}(1+z^{-1})}{(1-z^{-1})^3}$

So $Z\left\{\frac{5s+10}{s^3}\right\} = \frac{z^{-1}}{(1-z^{-1})^2} + \frac{0.2z^{-1}(1+z^{-1})}{(1-z^{-1})^3}$

So $G_1G_2(z) = \dfrac{z^{-1}}{1-z^{-1}} + \dfrac{0.2z^{-1}(1+z^{-1})}{(1-z^{-1})^2}$

$$= \frac{1}{z-1} + \frac{0.2(z+1)}{(z-1)^2}$$

$$= \frac{z-1+0.2z+0.2}{(z-1)^2}$$

$$= \frac{1.2z-0.8}{(z-1)^2}$$

$H(s)=1$ $\quad G_1G_2H(z) = G_1G_2(z)$

$$G_{cl}(z) = \frac{C(z)}{R(z)} = \frac{\dfrac{1.2z-0.8}{(z-1)^2}}{1+\dfrac{1.2z-0.8}{(z-1)^2}}$$

$$= \frac{1.2z-0.8}{(z-1)^2+1.2z-0.8}$$

$$= \frac{1.2z-0.8}{z^2-2z+1+1.2z-0.8}$$

$$= \frac{1.2z-0.8}{z^2-0.8z+0.2}$$

(c) $p(z) = 1 + \frac{1.2z - 0.8}{(z-1)^2} = 0$

$p(z) = z^2 - 0.8z + 0.2$

Jury Table

z^0	z^1	z^2
a_2	a_1	a_0

z^0	z^1	z^2
0.2	-0.8	1

$$\begin{cases} ① \ |a_2| < a_0 \quad |0.2| < 1 \ ✓ \\ ② \ p(z)|_{z=1} = 1 - 0.8 + 0.2 = 0.4 > 0 \ ✓ \\ ③ \ p(z)|_{z=-1} = 1 + 0.8 + 0.2 = 2 > 0 \\ \quad n = 2, \text{ even } ✓ \end{cases}$$

So, the system is stable