20-S1-Q1

Q: (a) Jury Test?

Solution

	z^0	z^1	z^2	z^3
1	a_3	a_2	a_1	a_0
2	a_0	a_1	a_2	a_3
3	b_2	b_1	b_0	

	z^0	z^1	z^2	z^3
1	0.5	2K	2	1
2	1	2	2K	0.5
3	-0.75	K-2	1-2K	

$$b_0 = \begin{vmatrix} 0.5 & 2K \\ 1 & 2 \end{vmatrix} = 1-2k$$

$$b_1 = \begin{vmatrix} 0.5 & 2 \\ 1 & 2K \end{vmatrix} = K-2$$

$$b_2 = \begin{vmatrix} 0.5 & 1 \\ 1 & 0.5 \end{vmatrix} = 0.25-1 = -0.75$$

$$\begin{cases} ① \ |a_3| < a_0 & |0.5| < 1 \\ ② \ P(z)|_{z=1} > 0 & P(1) = 1+2+2K+0.5 = 2K+3.5 > 0 \quad \therefore k > -1.25 \\ ③ \ P(z)|_{z=-1} < 0 \quad n=3 \text{ odd} & \end{cases}$$

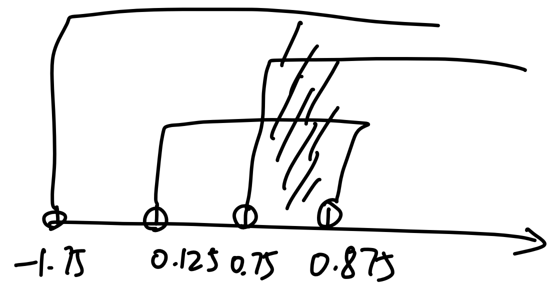
$P(-1) = -1 + 2 - 2k + 0.5 = 1.5 - 2k < 0 \quad \therefore k > 0.75$

④ $|b_2| > |b_0| \qquad 0.75 > |1-2k|$

$-0.75 < 1-2k < 0.75$

$-1.75 < -2k < -0.25$

$0.875 > k > 0.125$



So, $0.75 < k < 0.875$

(b) apply z transform

$x(k+2) - x(k+1) + x(k) = 0$

$z^2 X(z) - z^2 x(0) - z x(1) - z X(z) + z x(0) + X(z) = 0$

$z^2 X(z) - z^2 - z X(z) + z + X(z) = 0$

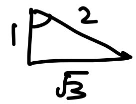
$(z^2 - z + 1) X(z) = z^2 - z$

$$X(z) = \frac{z^2 - z}{z^2 - z + 1}$$

$$X(z) = \frac{1 - z^{-1}}{1 - z^{-1} + z^{-2}}$$

#15 $$\frac{1 - z^{-1} \cos \omega T}{1 - 2z^{-1} \cos \omega T + z^{-2}}$$

let $2\cos wT = 1$ $\quad \cos wT = \frac{1}{2}$



$$wT = \frac{\pi}{3}$$

$$\frac{1-\frac{1}{2}z^{-1}}{1-z^{-1}+z^{-2}}$$

$$\#14 \quad \frac{\frac{\sqrt{3}}{2}z^{-1}}{1-z^{-1}+z^{-2}} \times \frac{2}{\sqrt{3}} \times \left(-\frac{1}{2}\right) = \frac{-\frac{1}{2}z^{-1}}{1-z^{-1}+z^{-2}}$$

So

$$X(z) = \frac{1-z^{-1}}{1-z^{-1}+z^{-2}}$$

$$= \frac{1-\frac{1}{2}z^{-1}}{1-z^{-1}+z^{-2}} + \frac{\frac{\sqrt{3}}{2}z^{-1}}{1-z^{-1}+z^{-2}} \times \frac{2}{\sqrt{3}} \times \left(-\frac{1}{2}\right)$$

$$X(kT) = \cos wkT - \frac{\sqrt{3}}{3}\sin wkT \qquad \because wT = \frac{\pi}{3}$$

$$= \cos\frac{\pi}{3}k - \frac{\sqrt{3}}{3}\sin\frac{\pi}{3}k$$

(c) $x(\infty)$?

Solution $X(z) = \dfrac{1-z^{-1}}{1-z^{-1}+z^{-2}} = \dfrac{z^2-z}{z^2-z+1}$

solve $z^2-z+1=0$

$$z_{1,2} = \frac{1 \pm \sqrt{3}j}{2} \qquad |z_{1,2}| = \sqrt{\left(\frac{1}{2}\right)^2+\left(\frac{\sqrt{3}}{2}\right)^2} = 1$$

So, the poles are in the unit circle,
which is not applicable for FVT