17-S1-Q1

Q(a) $T = 0.5$ $\quad X(S) = \dfrac{1}{S^2+5S+4}$ $\quad Z(x(kT))$

Solution

$$X(S) = \frac{1}{S^2+5S+4} = \frac{1}{(S+1)(S+4)}$$

#9 $\quad \dfrac{3}{(S+1)(S+4)} \qquad \dfrac{(e^{-T} - e^{-4T})\, z^{-1}}{(1-e^{-T}z^{-1})(1-e^{-4T}z^{-1})} \qquad T=0.5$

So $$Z(x(kT)) = \frac{(e^{-0.5} - e^{-2})\, z^{-1}}{3(1-e^{-0.5}z^{-1})(1-e^{-2}z^{-1})}$$

$$= \frac{0.4712 z^{-1}}{3(1-0.6065z^{-1})(1-0.1353z^{-1})}$$

(b)Q: difference

Solution

apply z transform

$$z^2 X(z) - z^2 x(0) - z x(1) + (\alpha + 0.5)\left[z X(z) - z x(0)\right] + 0.5\alpha X(z) = 1$$

let $k=-2$, $\quad x(0) = 0$

let $k=-1$, $\quad x(1) = 0$

$$z^2 X(z) + (\alpha+0.5) z X(z) + 0.5\alpha X(z) = 1$$

$$\left[z^2 + (\alpha+0.5) z + 0.5\alpha\right] X(z) = 1$$

$$X(z) = \frac{1}{z^2+(\alpha+0.5)z+0.5\alpha}$$

$$= \frac{1}{(z+\alpha)(z+0.5)}$$

$$\frac{X(z)}{z} = \frac{z}{(z+\alpha)(z+0.5)}$$

$$= \frac{A}{z+\alpha} + \frac{B}{z+0.5}$$

$$A(z+0.5)+B(z+\alpha)$$

$$=(A+B)z+(0.5A+B\alpha) = z$$

$$\begin{cases} A+B=1 \\ 0.5A+B\alpha=0 \end{cases} \Rightarrow \begin{cases} A=-\frac{\alpha}{0.5-\alpha} \\ B=\frac{0.5}{0.5-\alpha} \end{cases}$$

$$0.5A+(1-A)\alpha=0$$

$$0.5A+\alpha-A\alpha=0$$

$$(0.5-\alpha)A+\alpha=0$$

$$A = \frac{-\alpha}{0.5-\alpha}$$

$$B = 1+\frac{\alpha}{0.5-\alpha} = \frac{0.5-\alpha+\alpha}{0.5-\alpha} = \frac{0.5}{0.5-\alpha}$$

$$X(z) = \frac{-\alpha}{0.5-\alpha}\frac{z}{z+\alpha}+\frac{0.5}{0.5-\alpha}\frac{z}{z+0.5}$$

$$X(z) = \frac{-\alpha}{0.5-\alpha}\frac{1}{1+\alpha z^{-1}}+\frac{0.5}{0.5-\alpha}\frac{1}{1+0.5z^{-1}}$$

#18 $a = -\alpha$ $\quad a = -0.5$

$$x(kT) = -\frac{\alpha}{0.5-\alpha}(-\alpha)^k + \frac{0.5}{0.5-\alpha}(-0.5)^k$$

(c) Q : convergence ?

Solution

$$X(z) = \frac{1}{(z+\alpha)(z+0.5)} = \frac{1}{(z-(-\alpha))(z-(-0.5))}$$

poles at $z_1 = -\alpha$ $\quad z_2 = -0.5$

$|z_1| = |\alpha|$ $\quad |z_2| = 0.5 < 1$

① all poles of $X(z)$ lie inside the unit circle with the possible exception of a simple pole at $z=1$, So we can use Final Value Theorem

$|z_1| = |\alpha| < 1 \Rightarrow \alpha \in (-1, 1)$

$z_1 = 1 \Rightarrow \alpha = -1 \Rightarrow \alpha \in [-1, 1)$

if $\alpha = -1$

$$\lim_{k\to\infty} x(k) = \lim_{z\to 1}(z-1)\frac{1}{(z-1)(z+0.5)}$$

$$= \frac{1}{1.5} = \frac{2}{3} = 0.6667$$

if $\alpha \in (-1, 1)$

$$\lim_{k\to\infty} x(k) = \lim_{z\to 1} \frac{z-1}{(z+\alpha)(z+0.5)}$$

$$= \frac{2}{3} \lim_{z\to 1} \frac{z-1}{z+\alpha}$$

$$= 0$$

② when $\alpha \in (-\infty, -1) \cup [1, +\infty)$

the prerequisites for FVT are not met

So we can't use Final Value Theorem

So, $X(kT)$ isn't converge