21 - S1 - Q1

Q: (1) (a) ① $Z \to y(kT)$ ② FVT ?

Solution ① #15 $\cos \omega kT = \dfrac{1 - z^{-1}\cos\omega T}{1 - 2z^{-1}\cos\omega T + z^{-2}}$

$\omega = \frac{\pi}{2}$ $T = 0.5$ $\omega T = \frac{\pi}{4}$

$$X(z) = \frac{1 - z^{-1}\cos\frac{\pi}{4}}{1 - 2z^{-1}\cos\frac{\pi}{4} + z^{-2}} = \frac{1 - \frac{\sqrt{2}}{2}z^{-1}}{1 - \sqrt{2}z^{-1} + z^{-2}}$$

$$y(kT) = \sum_{h=0}^{k} x(hT) = x(0) + x(T) + x(2T) + \cdots + x(kT)$$

$$y[(k-1)T] = x(0) + x(T) + x(2T) + \cdots + x[(k-1)T]$$

$$y(kT) - y[(k-1)T] = x(kT)$$

apply z transform

$$Y(z) - z^{-1}Y(z) = \frac{1 - \frac{\sqrt{2}}{2}z^{-1}}{1 - \sqrt{2}z^{-1} + z^{-2}}$$

$$Y(z) = \frac{(1 - \frac{\sqrt{2}}{2}z^{-1})}{(1 - \sqrt{2}z^{-1} + z^{-2})(1 - z^{-1})}$$

② $$X(z) = \frac{(z - \frac{\sqrt{2}}{2})z}{z^2 - \sqrt{2}z + 1}$$

poles at $z = \dfrac{\sqrt{2} \pm \sqrt{2}j}{2}$, 1

$|z| = \sqrt{(\frac{\sqrt{2}}{2})^2 + (\frac{\sqrt{2}}{2})^2} = 1$ it is in the unit circle

So it doesn't meet the requirement of FVT

Since FVT need all poles must within the unit circle and accept one simple pole at $z=1$

(b) Q: solve equation

apply z transform

$$z^2 X(z) - z^2 x(0) - z x(1) - z X(z) + z X(0) + X(z) = \frac{1}{1-z^{-1}}$$

$$z^2 X(z) - z X(z) + X(z) = \frac{1}{1-z^{-1}}$$

$$(z^2 - z + 1) X(z) = \frac{1}{1-z^{-1}}$$

$$X(z) = \frac{1}{(1-z^{-1})(z^2-z+1)}$$

$$\frac{X(z)}{z} = \frac{1}{(z-1)(z^2-z+1)}$$

$$= \frac{A}{z-1} + \frac{Bz+C}{z^2-z+1}$$

$$A(z^2 - z + 1) + (Bz+C)(z-1)$$

$$= \underline{Az^2} - \underline{Az} + A + \underline{Bz^2} - \underline{Bz} + \underline{Cz} - C$$

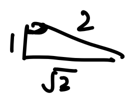
$$= (A+B)z^2 + (-A-B+C)z + (A-C)$$

$$\begin{cases} A+B=0 \\ -A-B+C=0 \\ A-C=1 \end{cases} \quad \therefore \begin{cases} A=1 \\ B=-1 \\ C=0 \end{cases}$$

$$\frac{X(z)}{z} = \frac{1}{z-1} + \frac{-z}{z^2 - z + 1}$$

$$X(z) = \frac{1}{1-z^{-1}} - \frac{1}{1-z^{-1}+z^{-2}}$$

#3 $1(k)$

#14 #15 凑系数 $-2\cos\omega T = -1$ $\cos\omega T = \frac{1}{2}$ 

$\omega T = \frac{\pi}{3}$

$\sin \omega kT$ $= \sin\frac{\pi}{3}k$ $\qquad \dfrac{z^{-1}\sin\omega T}{1-2z^{-1}\cos\omega T + z^{-2}} = \dfrac{\frac{\sqrt{3}}{2}z^{-1}}{1-z^{-1}+z^{-2}}$

$\cos\omega kT$ $= \cos\frac{\pi}{3}k$ $\qquad \dfrac{1-z^{-1}\cos\omega T}{1-2z^{-1}\cos\omega T + z^{-2}} = \dfrac{1-\frac{1}{2}z^{-1}}{1-z^{-1}+z^{-2}}$

$$-\frac{1}{2}\left[\frac{2}{\sqrt{3}}\,\frac{\frac{\sqrt{3}}{2}z^{-1}}{1-z^{-1}+z^{-2}} + 2\,\frac{1-\frac{1}{2}z^{-1}}{1-z^{-1}+z^{-2}}\right] = -\frac{1}{1-z^{-1}+z^{-2}}$$

$$= -\frac{1}{2}\left[\frac{2}{\sqrt{3}}\sin\frac{\pi}{3}k + 2\cos\frac{\pi}{3}k\right]$$

$$= -\frac{\sqrt{3}}{3}\sin\frac{\pi}{3}k - \cos\frac{\pi}{3}k$$

apply inverse z transform

$$X(kT) = 1(k) - \frac{\sqrt{3}}{3}\sin\frac{\pi}{3}k - \cos\frac{\pi}{3}k$$