5. Consider the closed loop system in Figure 2. With a sampling period of 1 second, $G_{ZAS}(z)$ is given as

$$G_{ZAS}(z) = K\frac{0.8z + 0.2}{(z-1)(z-0.4)}$$

where K is a non-zero constant.

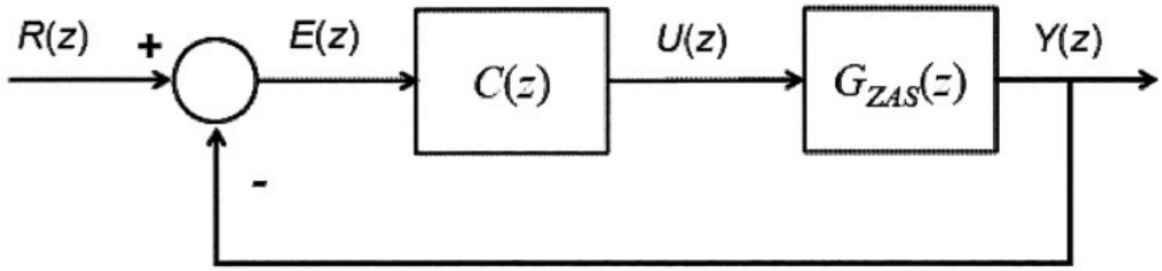


Figure 2

(a) If the controller is a proportional controller with a gain K_p, determine the range of K_pK so that the closed-loop system is stable.

(8 Marks)

(b) It is required that the output Y(z) tracks a unit-step input $R(z)$ without any steady state error. Design a ripple-free controller $C(z)$ to meet this requirement.

(8 Marks)

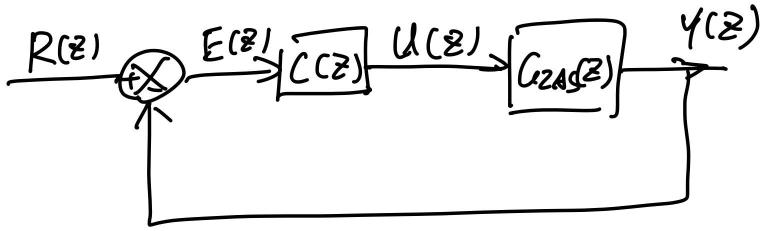
(c) Implement the controller $C(z)$ obtained in 5(b) with the standard programming approach and show the relevant block diagram.

(4 Marks)

22-S1-Q5

Q: $T=1$

$$G_{ZAS}(z) = K \frac{0.8z + 0.2}{(z-1)(z-0.4)}$$



(a) $C(s) = K_p$ range $K_pK \rightarrow$ stable ?

Solution

$$G_{cl}(z) = \frac{C(z)G_{ZAS}(z)}{1 + C(z)G_{ZAS}(z)}$$

$$= \frac{K_pK \frac{0.8z+0.2}{(z-1)(z-0.4)}}{1 + K_pK \frac{0.8z+0.2}{(z-1)(z-0.4)}}$$

$$= \frac{K_pK(0.8z+0.2)}{(z-1)(z-0.4) + K_pK(0.8z+0.2)}$$

$$= \frac{K_pK(0.8z+0.2)}{z^2 + (-1.4 + 0.8K_pK)z + 0.4 + 0.2K_pK}$$

July Test

z^0	z^1	z^2
$0.4+0.2K_pK$	$-1.4+0.8K_pK$	1

$\because$ stable

$$\therefore \begin{cases} |0.4 + 0.2 k_p K| < 1 \\ P(1) = 1 - 1.4 + 0.8 k_p K + 0.4 + 0.2 k_p K > 0 \\ P(-1) = 1 + 1.4 - 0.8 k_p K + 0.4 + 0.2 k_p K > 0 \end{cases} \quad n=2 \text{ even}$$

$$\therefore \quad -1 < 0.4 + 0.2 k_p K < 1$$

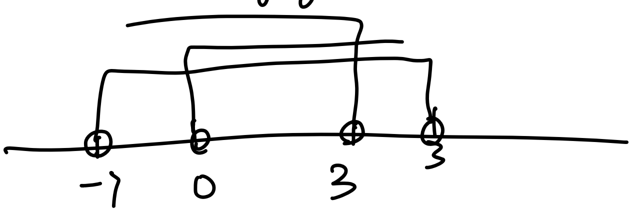
$$\frac{-1.4}{0.2} < k_p K < \frac{0.6}{0.2}$$

$$-7 < k_p K < 3 \quad (1)$$

$$k_p K > 0 \quad (2)$$

$$-0.6 k_p K + 1.8 > 0$$

$$k_p K < \frac{1.8}{0.6} = 3 \quad (3)$$



So $\quad 0 < k_p K < 3$

(b) $R(z) = \frac{1}{1-z^{-1}}$ $\quad G_{cl}(1) = 1$

$$U(z) = \frac{Y(z)}{G_{ZAS}(z)} = \frac{Y(z)}{R(z)} \frac{R(z)}{G_{ZAS}(z)} = G_{cl}(z) \frac{R(z)}{G_{ZAS}(z)}$$

$$= G_{cl}(z) \frac{\frac{1}{1-z^{-1}}}{K \frac{0.8z+0.2}{(z-1)(z-0.4)}} \xrightarrow{\text{化为}z^{-1}\text{形式}} \frac{K(0.8+0.2z^{-1})z^{-1}}{(1-z^{-1})(1-0.4z^{-1})}$$

$$= G_{cl}(z) \frac{(z-1)(z-0.4)}{K(0.8z+0.2)(1-z^{-1})} \quad \frac{1-0.4z^{-1}}{K(0.8+0.2z^{-1})z^{-1}}$$

design $G_{cl}(z) = K_0 K(0.8+0.2z^{-1})z^{-1}$

$\because G_{cl}(1) = K_0 K = 1 \quad \Rightarrow K_0 K = 1$

$$G_{cl}(z) = (0.8+0.2z^{-1})z^{-1}$$

$$C(z) = \frac{1}{G_{ZAS}(z)} \frac{G_{cl}(z)}{1-G_{cl}(z)}$$

$$= \frac{(1-z^{-1})(1-0.4z^{-1})}{K(0.8+0.2z^{-1})z^{-1}} \quad \frac{(0.8+0.2z^{-1})z^{-1}}{1-(0.8+0.2z^{-1})z^{-1}}$$

$$= \frac{(1-z^{-1})(1-0.4z^{-1})}{K[1-(0.8+0.2z^{-1})z^{-1}]}$$

约分后结果 $= \frac{(1-z^{-1})(1-0.4z^{-1})}{K(1-z^{-1})(1+0.2z^{-1})} = \frac{1-0.4z^{-1}}{K(1+0.2z^{-1})}$

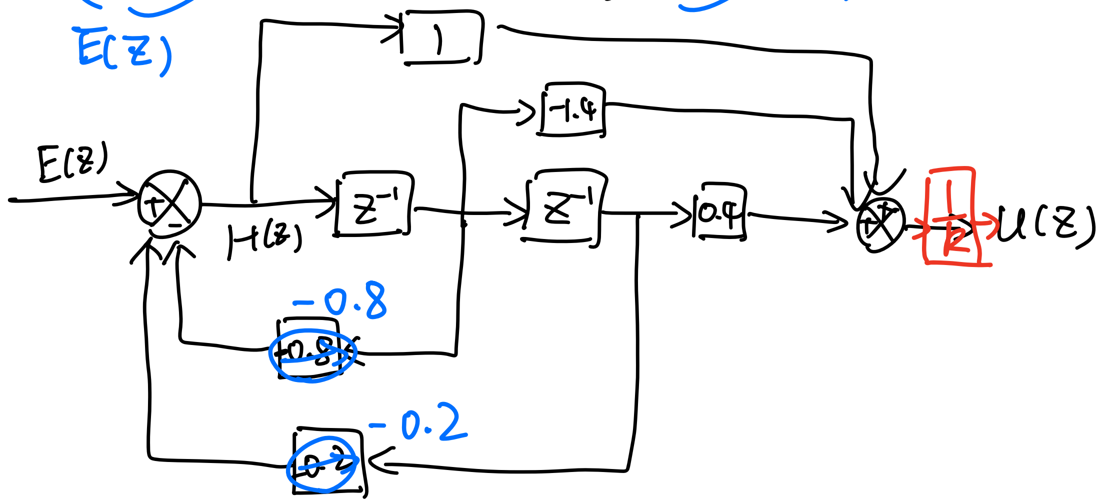
不同，

(c)不同

(c) $C(z) = \frac{U(z)}{E(z)} = \frac{1}{K}\,\frac{1-1.4z^{-1}+0.4z^{-2}}{1-0.8z^{-1}-0.2z^{-2}} = \frac{1}{K}\,\frac{U(z)}{H(z)}\,\frac{H(z)}{E(z)}$

$U(z) = (1-1.4z^{-1}+0.4z^{-2})H(z)\,\frac{1}{K}$

$\cancel{H(z)}$ $E(z)$ $= (1-0.8z^{-1}-0.2z^{-2})\cancel{E(z)}\,H(z) = H(z) - 0.8z^{-1}H(z) - 0.2z^{-2}H(z)$



So $H(z) = E(z) + 0.8z^{-1}H(z) + 0.2z^{-2}H(z)$

$C(z) = \frac{U(z)}{E(z)} = \frac{1-0.4z^{-1}}{K(1+0.2z^{-1})} = \frac{U(z)}{H(z)}\,\frac{H(z)}{E(z)}$

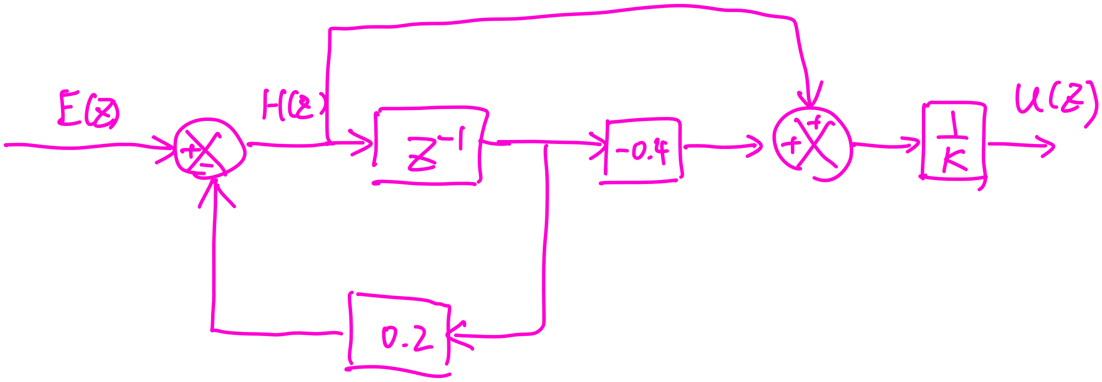
$\frac{U(z)}{H(z)} = \frac{1}{K}(1-0.4z^{-1})$

$U(z) = \frac{1}{K}\left[H(z) - 0.4z^{-1}H(z)\right]$

$\frac{H(z)}{E(z)} = \frac{1}{1+0.2z^{-1}}$

$E(z) = H(z) + 0.2z^{-1}H(z)$

$H(z) = E(z) - 0.2z^{-1}H(z)$



E(z)
H(z)
z^-1
-0.4
0.2
1/K
U(z)
+
-
+
+

Inquiry Regarding Simplification of Transfer Function in Ripple-Free Controller Design

22-S1-Q5-v7.pdf

#LI ZONGZE#

答复 全部答复 转发

收件人: Wen Changyun (Prof)

周五 2024/11/22 11:09

22-S1-Q5-v7.pdf
2 MB

Dear Professor Wen Changyun,

I hope this message finds you well.

I am writing to seek clarification regarding the simplification of the transfer function C(z) in the design of a ripple-free controller. Specifically, I would like to ask whether it is permissible to simplify the transfer function C(z) during this process.

The issue and related discussion are detailed in the attached appendix for your reference.

Initially, I did not consider simplifying C(z) when solving this problem. However, a classmate suggested that C(z) could be simplified, as shown in the pink text of the provided solution. The reasoning was that, during the calculation of U(z), the poles of R(z) and Gzas(z) are directly canceled, resulting in U(z) having no integral term. Consequently, the controller does not require an integral term. Furthermore, since the system transfer function is Y(z)/R(z), the cancellation of poles and zeros in U(z) is attributed to R(z). If we only consider Y(z)/R(z), there is only one pole; retaining this pair in the controller design would introduce an additional pole-zero pair in Y(z)/R(z).

On the other hand, some classmates argued that transfer functions should not be simplified, and that pole-zero cancellation should not occur. They pointed out that 1−z^(−1) represents a pole in the plant and therefore should not be canceled.

I have not been able to find an example in the lecture slides where C(z) is explicitly simplified. Hence, I am writing to inquire about this specific point.

Thank you very much for your time and assistance. I look forward to your response at your earliest convenience.

Best regards,

Li Zongze

Wen Changyun (Prof)

收件人: #LI ZONGZE#

周五 2024/11/22 13:30

you can do cancellation when you do calculation to obtain C(z). After C(z) is obtained, it will be implemented. You cannot have unstable pole zero cancellation between C(z) and the pulse transfer function G_{ZAS}.

Actually we had an example similar to this case in our notes. Please check.

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