23-S1-Q2

Q: $G_{ZAS}(z) = \dfrac{0.5z + 0.2}{(z-0.5)(z-0.4)}$

(a) PI $C(z) = 1 + \dfrac{r}{1-z^{-1}}$ stable? e_{ss}?

Solution $G_{cl}(z) = \dfrac{C(z)\,G_{ZAS}(z)}{1 + C(z)G_{ZAS}(z)}$

$$C(z) = \frac{2 - z^{-1}}{1 - z^{-1}} = \frac{2z - 1}{z - 1}$$

$$C(z)G_{ZAS}(z) = \frac{(2z-1)(0.5z+0.2)}{(z-1)(z-0.5)(z-0.4)}$$

$$= \frac{(z+0.4)}{(z-1)(z-0.4)}$$ ← 不能約分, PPT 19 (ref.)

$$G_{cl}(z) = \frac{\frac{(z+0.4)}{(z-1)(z-0.4)}}{1 + \frac{(z+0.4)}{(z-1)(z-0.4)}}$$

$$= \frac{z+0.4}{(z-1)(z-0.4) + (z+0.4)}$$

$$= \frac{z+0.4}{z^2 - 1.4z + 0.4 + z + 0.4}$$

$$= \frac{z+0.4}{z^2 - 0.4z + 0.8}$$

$G_{cl}(z) = \dfrac{G(z)}{1 + G(z)H(z)}$

CE: $1 + G(z)H(z) = 0$

$$C(z)G_{ZAS}(z) = \frac{(2z-1)(0.5z+0.2)}{(z-1)(z-0.5)(z-0.4)}$$

$$G_{CL}(z) = \frac{C(z)G_{ZAS}(z)}{1+C(z)G_{ZAS}(z)}$$

CE:

$$1 + \frac{(2z-1)(0.5z+0.2)}{(z-1)(z-0.5)(z-0.4)} = 0$$

$$(z-1)(z^2-0.9z+0.2) + (z^2+0.4z-0.5z-0.2)$$

$$= z^3 - 0.9z^2 + 0.2z - z^2 + 0.9z - 0.2$$

$$+ z^2 - 0.1z - 0.2$$

$$= z^3 - 0.9z^2 + z - 0.4 = 0$$

Jury Test

z^0	z^1	z^2	z^3
a_3			a_0
-0.4	1	-0.9	1
1	-0.9	1	-0.4
-0.84	0.5	-0.62	

- $|-0.4| < 1$ ✓
- $P(1) = 1 - 0.9 + 1 - 0.4 = 0.7 > 0$ ✓
- $P(-1) = -1 - 0.9 - 1 - 0.4 < 0$ ✓ $n = 3$ 0

$|b_2| > |b_0|$ $0.84 > 0.62$ ✓ Stable

e_{st} 稳态跟踪误差：推导在 understand－1

$$e_{ss} = \lim_{z\to 1}\left[(1-z^{-1})\frac{1}{1+C(z)G_{ZAS}(z)}R(z)\right]$$

$$R(z) = \frac{1}{1-z^{-1}}$$

$$e_{ss} = \lim_{z\to 1}\frac{1}{1+\frac{(2z-1)(0.5z+0.2)}{(z-1)(z-0.5)(z-0.4)}} = 0$$

$$e_{st} = \lim_{z\to 1}(1-z^{-1})R(z)[1-G_{CL}(z)]$$

$$= \lim_{z\to 1}(1-z^{-1})\frac{1}{1-z^{-1}}\left(1-\frac{z+0.4}{z^2-0.4z+0.8}\right) = 1-\frac{1.4}{1-0.4+0.8} = 0$$

(b) $R(z) = \frac{1}{1-z^{-1}}$

$$U(z) = G_{CL}(z)\frac{R(z)}{G_{ZAS}(z)}$$

$$G_{ZAS}(z) = \frac{0.5z+0.2}{(z-0.5)(z-0.4)}$$

$$= \frac{(0.5+0.2z^{-1})z^{-1}}{(1-0.5z^{-1})(1-0.4z^{-1})}$$

$$U(z) = G_{CL}(z)\frac{(1-0.5z^{-1})(1-0.4z^{-1})}{(1-z^{-1})(0.5+0.2z^{-1})z^{-1}}$$

$$G_{CL}(z) = Kz^{-1}(0.5+0.2z^{-1})$$

$$G_{cl}(1) = K(0.5+0.2) = 0.7K = 1$$

$$\Rightarrow K = 1.4286$$

$$C(z) = \frac{1}{G_{ZAS}(z)} \frac{G_{cl}(z)}{1 - G_{cl}(z)}$$

$$= \frac{(z-0.5)(z-0.4)}{0.5z+0.2} \frac{1.4286 z^{-1}(0.5+0.2z^{-1})}{1 - 1.4286 z^{-1}(0.5+0.2z^{-1})}$$

$$= \frac{(z-0.5)(z-0.4)}{0.5z+0.2} \frac{1.4286 \quad (0.5z+0.2)}{z^2 - 1.4286(0.5z \quad +0.2)}$$

$$= \frac{(z-0.5)(z-0.4)\,1.4286\,(0.5z+0.2)}{(0.5z+0.2)(z^2 - 0.78573z - 0.28572)}$$

$$= \frac{1.4286(z-0.5)(z-0.4)}{(z-1)(z+0.2857)}$$

(c) $$G_{ZAS}(z) = \frac{0.5z+0.2}{(z-1.5)(z-0.4)}$$

This plant has an unstable pole at $z = 1.5$

not possible, because unstable poles

cannot be canceled using a causal controller

Solution

① The set of zeros of $G_{cl}(z)$ must include all the zeros of $G_{ZAS}(z)$ that are outside the unit circle

② The zeros of $1-G_{cl}(z)$ must include all the unstable poles of $G_{ZAS}(z)$ (stability)

③ $G_{ZAS}(z)=\frac{0.5z^{-1}(1+0.6z^{-1})}{(1-1.5z^{-1})(1-0.4z^{-1})}$

纯滞后 $L=0$ 或 1

的有零 $w=1$

不稳定极(非1) $v=1$

极点为1个数 $k=0$

阶跃 $m=1$

$j=\max(k,m)=1$

$\phi=G_{cl}$阶数 $p=j+v-1=1+1-1=1$

$\phi_e=1-G_{cl}$阶数 $q=L+w-1=1+1-1=1$

$$\phi_e = 1 - G_{cl}(z) = (1 - 1.5z^{-1})(1 - z^{-1})(1 + f_1 z^{-1})$$

（1.5：特根极；1：j）

$$\phi = G_{cl}(z) = z^{-1}(\varphi_0 + \varphi_1 z^{-1})(1 + 0.4z^{-1})$$

（z^{-1}：$l=1$；$\varphi_0 + \varphi_1 z^{-1}$：$p=1$；$1+0.4z^{-1}$：分母定）

$$\phi = 1 - \phi_e \quad \text{求 } f_1 \ \varphi_0 \ \varphi_1$$

$$C(z) = \frac{1}{G_{ZAS}(z)} \frac{G_{cl}(z)}{1 - G_{cl}(z)} = \frac{\phi}{G_{ZAS}(z)\phi_e}$$

$$= \frac{z^{-1}(\varphi_0 + \varphi_1 z^{-1})(1 + 0.4z^{-1})}{G_{ZAS}(z) \quad (1 - 1.5z^{-1})(1 - z^{-1})(1 + f_1 z^{-1})}$$

$$= \frac{z^{-1}(\varphi_0 + \varphi_1 z^{-1})(1 + 0.4z^{-1})}{\dfrac{0.5z^{-1}(1 + 0.4z^{-1})}{(1 - 1.5z^{-1})(1 - 0.4z^{-1})} \quad (1 - 1.5z^{-1})(1 - z^{-1})(1 + f_1 z^{-1})}$$

$$= \frac{z^{-1}(\varphi_0 + \varphi_1 z^{-1})(1 + 0.4z^{-1})}{\dfrac{0.5z^{-1}(1 + 0.4z^{-1})}{(1 - 1.5z^{-1})(1 - 0.4z^{-1})} \quad (1 - 1.5z^{-1})(1 - z^{-1})(1 + f_1 z^{-1})}$$

$$= \frac{2(\varphi_0 + \varphi_1 z^{-1})(1 - 0.4z^{-1})}{(1 - z^{-1})(1 + f_1 z^{-1})}$$