21 - S1 - Q4

Q (a) (i) α ?

Solution $W_c = [B \quad AB] = \begin{bmatrix} 4 & 4\alpha-3 \\ 3 & -4 \end{bmatrix}$

$AB = \begin{bmatrix} \alpha & -1 \\ -4 & 4 \end{bmatrix} \begin{bmatrix} 4 \\ 3 \end{bmatrix} = \begin{bmatrix} 4\alpha-3 \\ -4 \end{bmatrix}$ $\qquad -16+12 = -4$

$|W_c| = -16 - 3(4\alpha - 3)$

$-16 - 12\alpha + 9$

$-7 - 12\alpha \neq 0$

$\alpha \neq -\frac{7}{12}$

(ii) Q: $\alpha = 2$, $K = ?$

$\alpha_c(z) = (z - j0.25)(z + j0.25)$

$= z^2 - j^2 0.25^2$

$= z^2 + 0.0625$

$\alpha_c(A) = \begin{bmatrix} 2 & -1 \\ -4 & 4 \end{bmatrix} \begin{bmatrix} 2 & -1 \\ -4 & 4 \end{bmatrix} + 0.0625 I_2 = \begin{bmatrix} 8.0625 & -6 \\ -24 & 20.0635 \end{bmatrix}$

from (i) $W_c = \begin{bmatrix} 4 & 4\alpha-3 \\ 3 & -4 \end{bmatrix} = \begin{bmatrix} 4 & 5 \\ 3 & -4 \end{bmatrix}$ $\qquad W_c^{-1} = \begin{bmatrix} \frac{4}{31} & \frac{5}{31} \\ \frac{3}{31} & -\frac{4}{31} \end{bmatrix}$

$K = [0 \, 1] W_c^{-1} \alpha_c(A)$

$= [0 \, 1] \begin{bmatrix} \frac{4}{31} & \frac{5}{31} \\ \frac{3}{31} & -\frac{4}{31} \end{bmatrix} \begin{bmatrix} 8.0625 & -6 \\ -24 & 20.0635 \end{bmatrix}$

$= \begin{bmatrix} \frac{3}{31} & -\frac{4}{31} \end{bmatrix} \begin{bmatrix} 8.0625 & -6 \\ -24 & 20.0635 \end{bmatrix} = [3.8770 \quad -3.1694]$

(b) (i) $\alpha = 2$ estimator ?

$$\alpha_o(z) = [z-(0.4+j0.4)][z-(0.4-j0.4)]$$

$$= z^2 - 0.8z + (0.4^2 + 0.4^2)$$

$$= z^2 - 0.8z + 0.32$$

$$\alpha_o(A) = A^2 - 0.8A + 0.32I$$ casio

$$= \begin{bmatrix} 6.72 & -5.2 \\ -20.8 & 17.12 \end{bmatrix}$$

$$W_o = \begin{bmatrix} C \\ CA \end{bmatrix} = \begin{bmatrix} 2 & 2 \\ -4 & 6 \end{bmatrix} \qquad W_o^{-1} = \begin{bmatrix} 0.3 & -0.1 \\ 0.2 & 0.1 \end{bmatrix}$$

$$CA = [2 \ 2]\begin{bmatrix} 2 & -1 \\ -4 & 4 \end{bmatrix} = [-4 \ \ 6]$$

$4 - 8 = -4$
$-2 + 8 = 6$

$$L_o = \alpha_o(A)\, W_o^{-1} \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$ casio

$$= \begin{bmatrix} -1.192 \\ 3.792 \end{bmatrix}$$

(ii) Q: $Z_{e1,2} \overset{?}{\rightarrow} s$ $\quad \zeta = ?$ $\quad T = 0.1$

Solution

magnitude $|z| = \sqrt{0.4^2 + 0.4^2} = \frac{2\sqrt{2}}{5} = 0.5657$

$$\theta = \arctan\left(\frac{0.4}{0.4}\right) = \tan^{-1}(1) = \frac{\pi}{4}$$ casio

$$z = e^{sT} \qquad s = \frac{1}{T} \ln z$$

$$s = \frac{\ln\left(\frac{2\sqrt{2}}{5}\right) + j\frac{\pi}{4}}{0.1} = -5.6972 + j7.8540$$

ref lect3 11.2

$$\zeta = -\frac{\ln r}{\sqrt{\ln^2 r + \theta^2}} = \frac{-\ln 0.5657}{\sqrt{\ln^2 0.5657 + (\frac{\pi}{4})^2}} = 0.5872$$

(c) Q $u^*(k) = ?$ equation?

Solution $A = 1$ $B = 0.8$ $Q = 8$ $r = 1.6$

$$S = S + 8 - S\,0.8\,(1.6 + 0.8^2 S)^{-1}\,0.8\,S$$

$$S = S + 8 - \frac{0.64S^2}{1.6 + 0.64S}$$

$$\frac{0.64S^2}{1.6 + 0.64S} = 8$$

$$0.64S^2 = 8(0.64S + 1.6)$$

$$0.64S^2 = 5.12S + 12.8$$

$$0.64S^2 - 5.12S - 12.8 = 0 \qquad \text{casio}$$

$$S_1 = 10 \qquad S_2 = -2$$

So choose the positive value, $S = 10$

$$K = (0.8^2 \times 10 + 1.6)^{-1} \times 0.8 \times 10 \times 1$$

$K = 1$ Therefor $u^*(k) = -x(k)$

$$x(k+1) = x(k) - 0.8\,x(k)$$
$$= 0.2\,x(k)$$

All in all, $u^*(k) = -x(k)$

$$x(k+1) = 0.2\,x(k)$$