耦合电感的联接及去耦等效

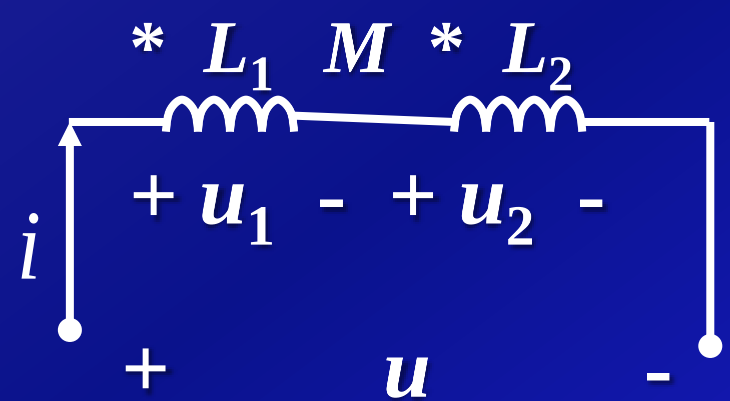
联接方式：串联，并联和三端联接

去耦等效：

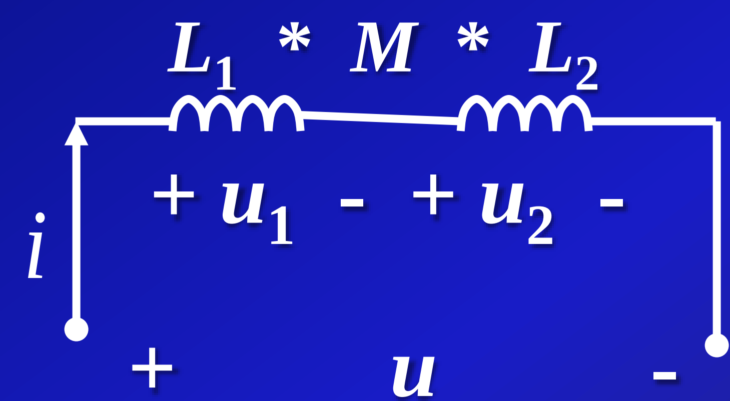
耦合电感用无耦合的等效电路去等效。

耦合电感的串联

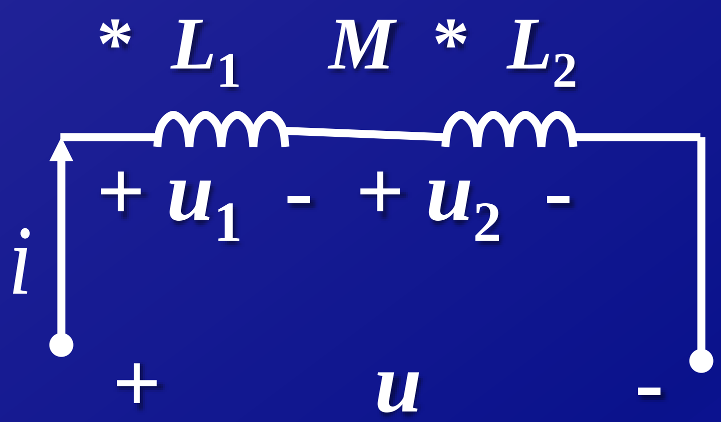
顺串：异名端相接。反串：同名端相接



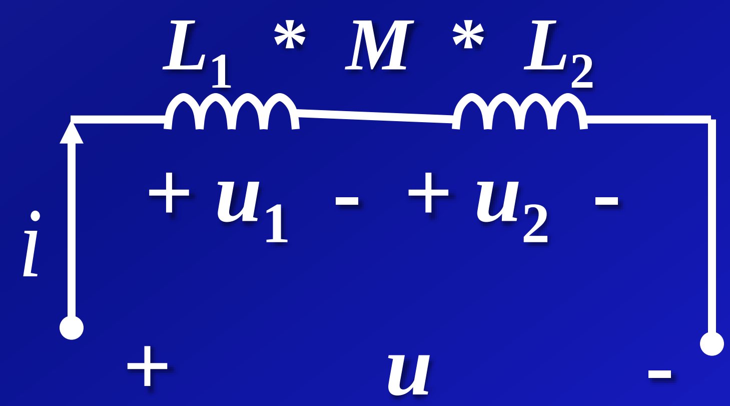
顺串



反串



顺串



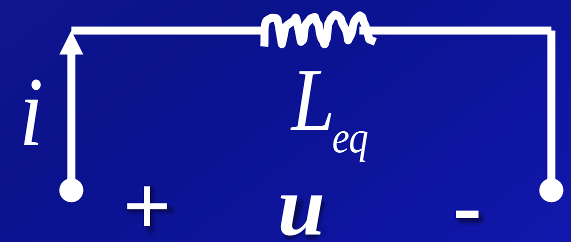
反串

在图示参考方向下，耦合电感的伏安关系为：

$$u = u_1 + u_2 = L_1 \frac{\mathrm{d}i}{\mathrm{d}t} \pm M \frac{\mathrm{d}i}{\mathrm{d}t} + L_2 \frac{\mathrm{d}i}{\mathrm{d}t} \pm M \frac{\mathrm{d}i}{\mathrm{d}t}$$

其中：顺串取+，反串取−。

$$u = u_1 + u_2 = (L_1 + L_2 \pm 2M)\frac{\mathrm{d}i}{\mathrm{d}t} = L_{eq}\frac{\mathrm{d}i}{\mathrm{d}t}$$



串联等效电路

顺串等效: $$L_{eq} = L_1 + L_2 + 2M$$

反串等效: $$L_{eq} = L_1 + L_2 - 2M$$

耦合电感的**储能**：

$$w(t)=\frac{1}{2}(L_1+L_2\pm 2M)i^2=\frac{1}{2}L_{eq}i^2\geq 0$$

得：

$$L_1+L_2\pm 2M\geq 0$$

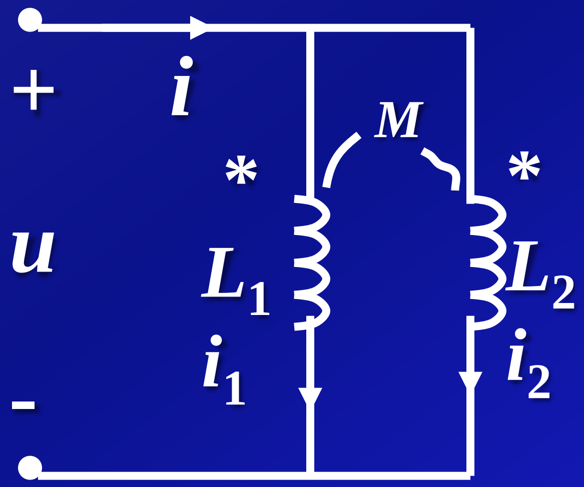
$$M\leq\frac{1}{2}(L_1+L_2)$$ 算术平均值

即：耦合电感的互感不能大于两自感的算术平均值。

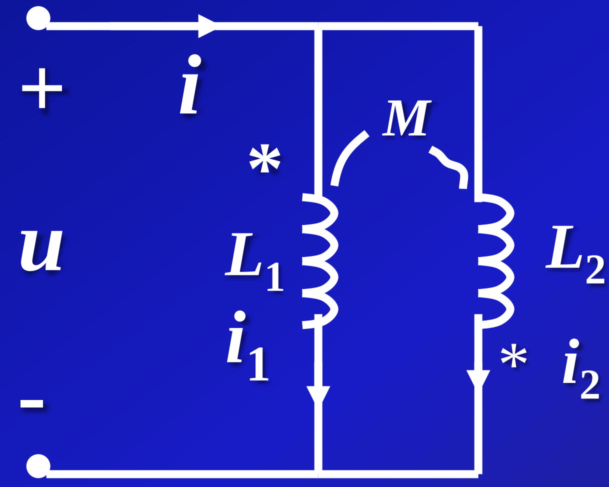
耦合电感的并联

同侧并联(顺并)：同名端两两相接。

异侧并联(反并)：异名端两两相接。



同侧并联



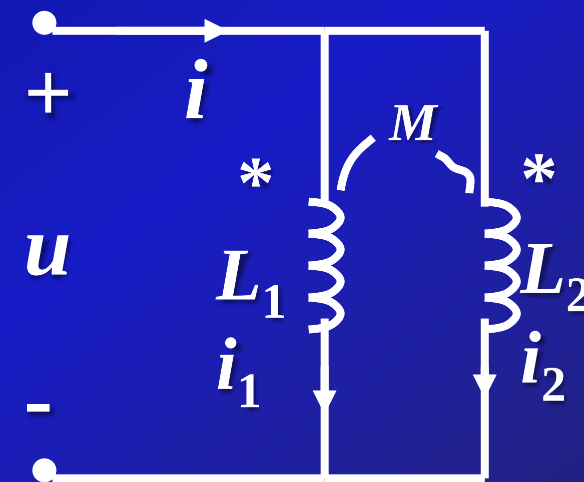
异侧并联

关联参考方向下，由耦合电感的伏安关系：

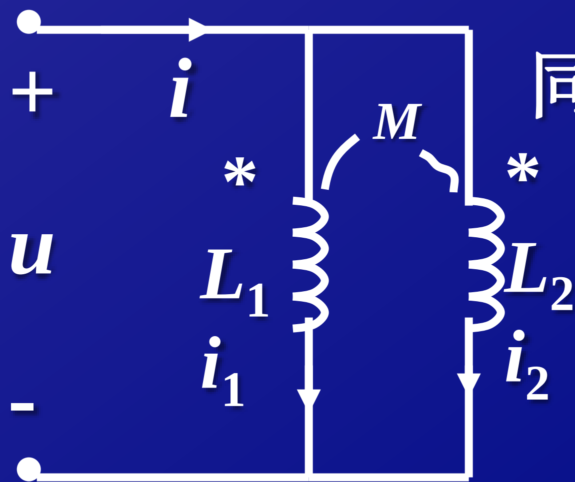
$$\left.\begin{aligned} u &= L_1\frac{\mathrm{d}i_1}{\mathrm{d}t} \pm M\frac{\mathrm{d}i_2}{\mathrm{d}t} \\ u &= \pm M\frac{\mathrm{d}i_1}{\mathrm{d}t} + L_2\frac{\mathrm{d}i_2}{\mathrm{d}t} \end{aligned}\right\} \qquad \begin{aligned} \frac{\mathrm{d}i_1}{\mathrm{d}t} &= \frac{L_2 \mp M}{L_1L_2 - M^2}u \\ \frac{\mathrm{d}i_2}{\mathrm{d}t} &= \frac{L_1 \mp M}{L_1L_2 - M^2}u \end{aligned}$$

$$\frac{\mathrm{d}\boldsymbol{i}}{\mathrm{d}t} = \frac{\mathrm{d}(\boldsymbol{i}_1 + \boldsymbol{i}_2)}{\mathrm{d}t} = \frac{\boldsymbol{L}_1 + \boldsymbol{L}_2 \mp 2\boldsymbol{M}}{\boldsymbol{L}_1\boldsymbol{L}_2 - \boldsymbol{M}^2}\boldsymbol{u}$$

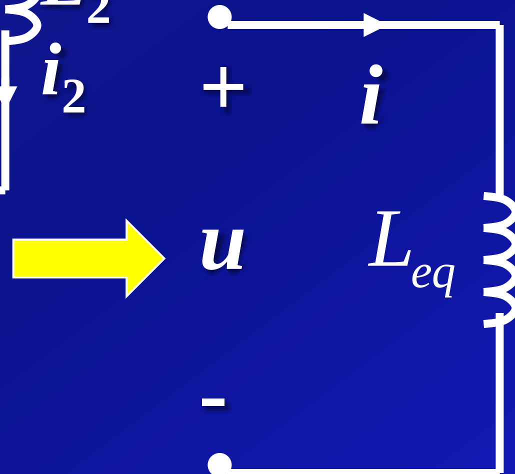
$$u = \frac{L_1L_2 - M^2}{L_1 + L_2 \mp 2M}\cdot\frac{\mathrm{d}i}{\mathrm{d}t} = L_{eq}\frac{\mathrm{d}i}{\mathrm{d}t}$$



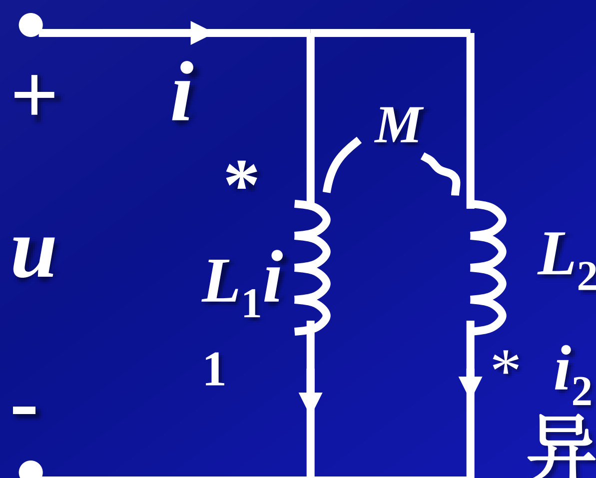
同侧并联



同侧并联



$$L_{eq同} = \frac{L_1L_2 - M^2}{L_1 + L_2 - 2M}$$



异侧并联

$$L_{eq异} = \frac{L_1L_2 - M^2}{L_1 + L_2 + 2M}$$

$$\boldsymbol{w(t)} = \frac{1}{2}\boldsymbol{L_{eq} i^2} \ge 0$$

$$\therefore \quad \frac{L_1 L_2 - M^2}{L_1 + L_2 \mp 2M} \ge 0 \quad \Rightarrow \quad L_1 L_2 \ge M^2$$

$$M \le \sqrt{L_1 L_2}$$ 几何平均值

即：耦合电感的互感也不能大于两自感的几何平均值。

$$\because \quad \sqrt{L_1 L_2} \le \frac{1}{2}(L_1 + L_2) \qquad M_{\max} = \sqrt{L_1 L_2}$$

定义：**耦合系数** $k = \dfrac{M}{\sqrt{L_1 L_2}}$ ：两线圈耦合程度；

$0 \le k \le 1$:

k=1 全耦合；

$k \approx 1$ 紧耦合；

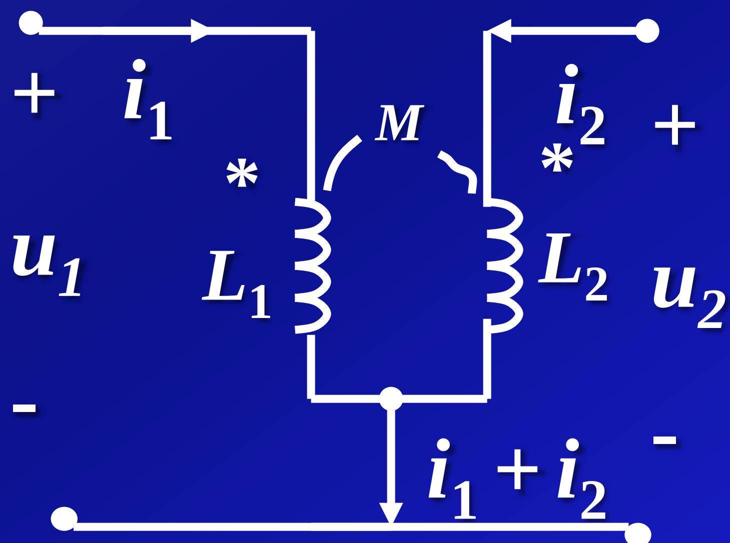
k较小，松耦合；

k=0 无耦合。

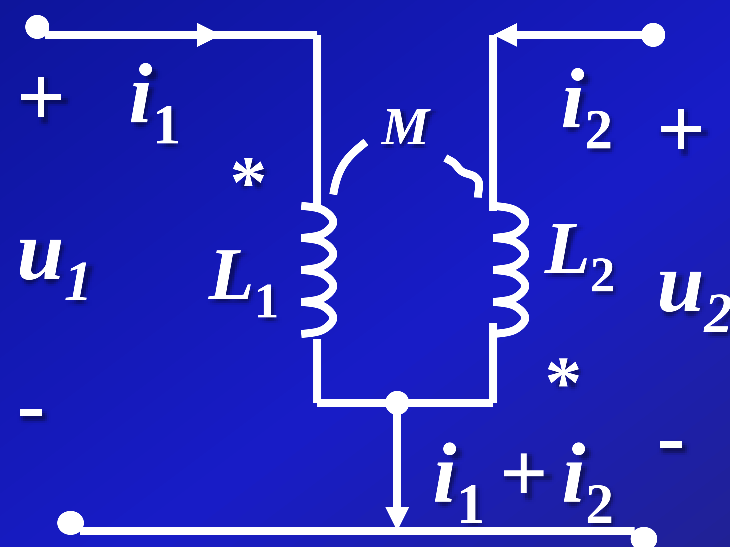
●耦合电感的三端联接

将耦合电感的两个线圈各取一端联接起来就成了耦合电感的**三端联接电路**：

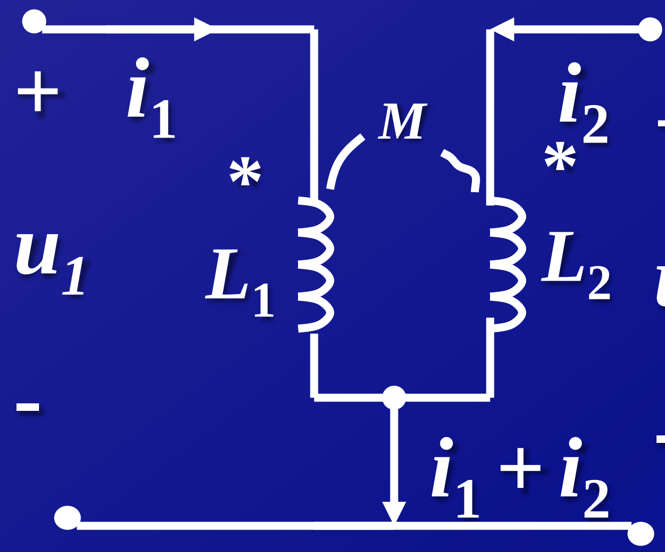
(1)**同名端相联**　　(2)**异名端相联**



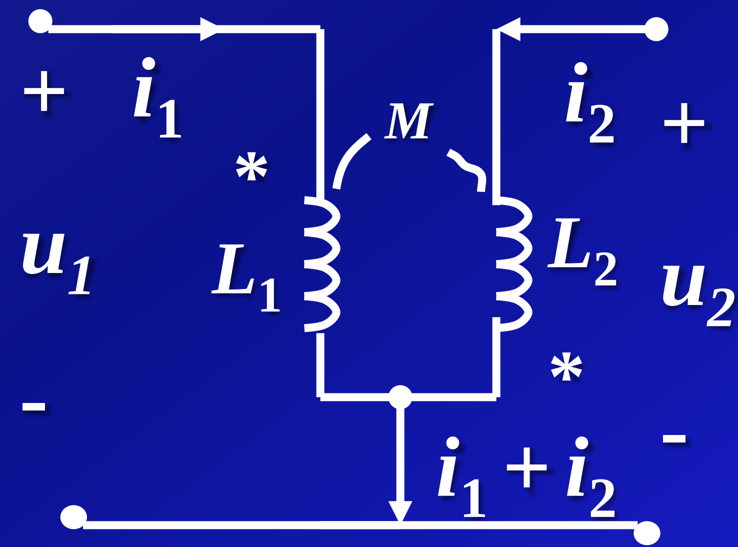
同名端相联



异名端相联



同名端相联

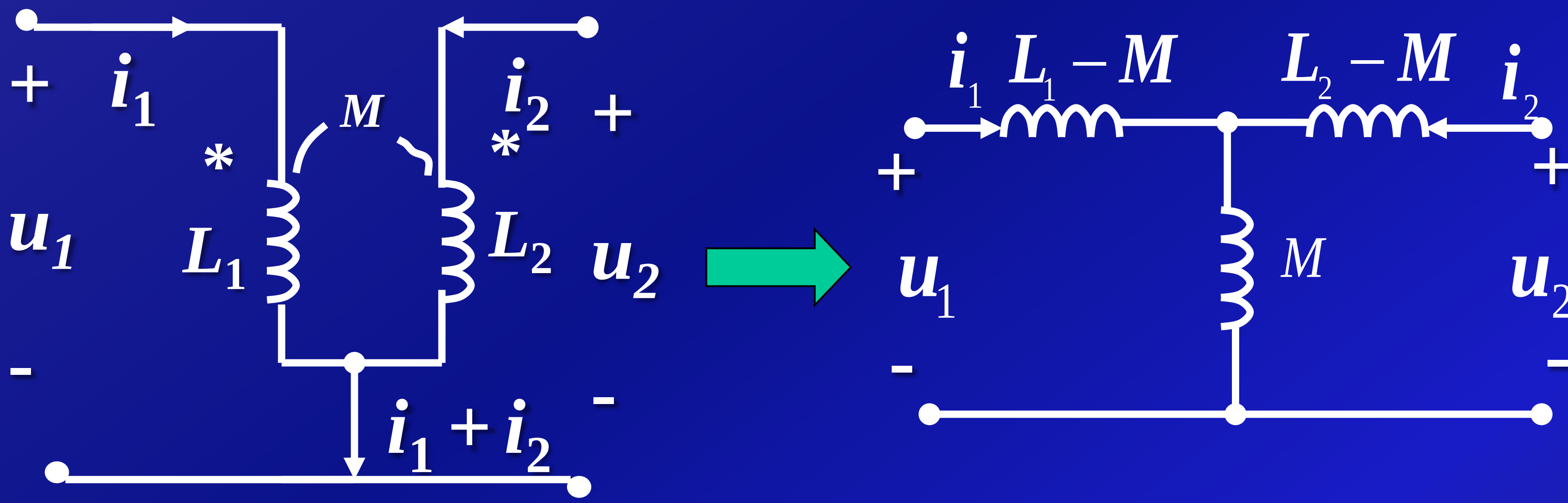


异名端相联

$$u_1 = L_1\frac{\mathrm{d}i_1}{\mathrm{d}t} \pm M\frac{\mathrm{d}i_2}{\mathrm{d}t} = (L_1 \mp M)\frac{\mathrm{d}i_1}{\mathrm{d}t} \pm M\frac{\mathrm{d}(i_1+i_2)}{\mathrm{d}t}$$

$$u_2 = L_2\frac{\mathrm{d}i_2}{\mathrm{d}t} \pm M\frac{\mathrm{d}i_1}{\mathrm{d}t} = (L_2 \mp M)\frac{\mathrm{d}i_2}{\mathrm{d}t} \pm M\frac{\mathrm{d}(i_1+i_2)}{\mathrm{d}t}$$

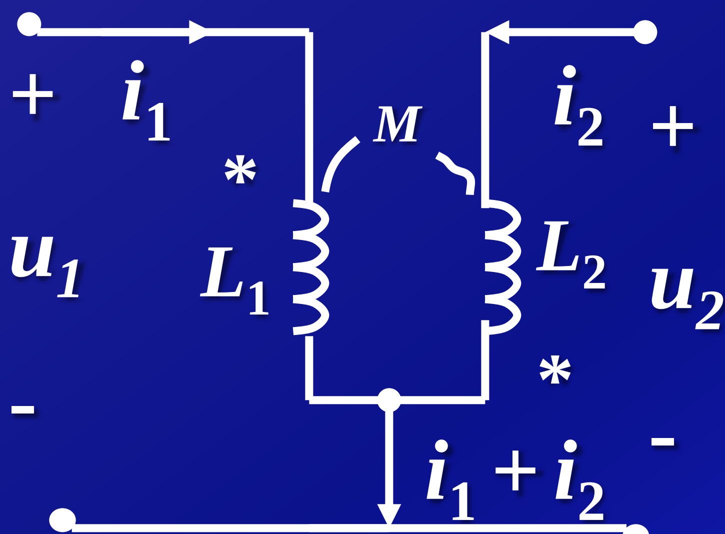
耦合电感三端联接的去耦等效



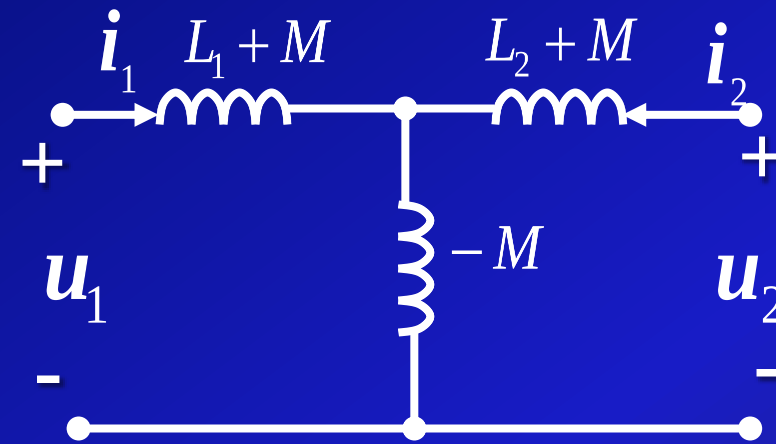
$$u_1 = L_1\frac{\mathrm{d}i_1}{\mathrm{d}t} + M\frac{\mathrm{d}i_2}{\mathrm{d}t} = (L_1 - M)\frac{\mathrm{d}i_1}{\mathrm{d}t} + M\frac{\mathrm{d}(i_1 + i_2)}{\mathrm{d}t}$$

$$u_2 = L_2\frac{\mathrm{d}i_2}{\mathrm{d}t} + M\frac{\mathrm{d}i_1}{\mathrm{d}t} = (L_2 - M)\frac{\mathrm{d}i_2}{\mathrm{d}t} + M\frac{\mathrm{d}(i_1 + i_2)}{\mathrm{d}t}$$

(2)异名端相联



异名端相联



注意：一般情况下，消去互感后的等效电路的节点数将增加。

例2（P256例8-2） 已知

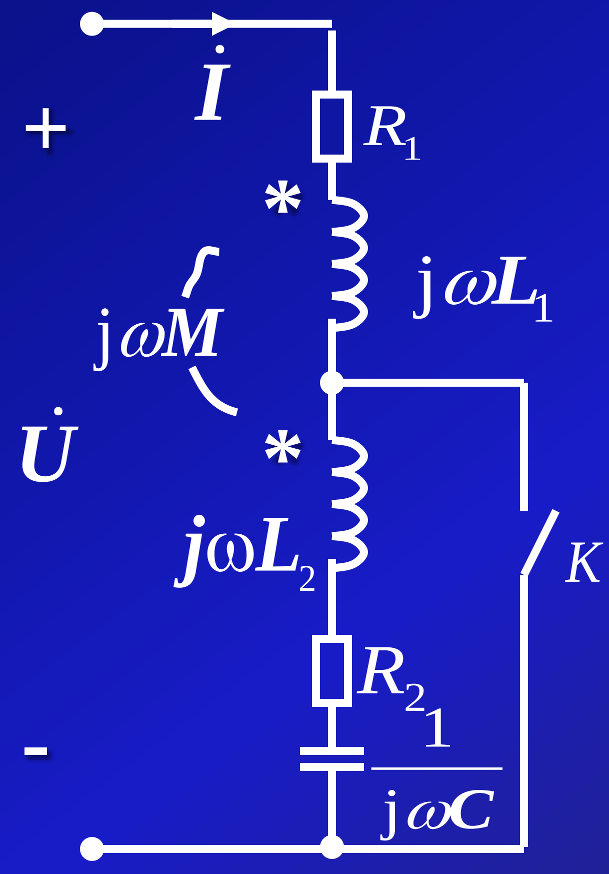
$R_1=6\Omega, R_2=6\Omega,$

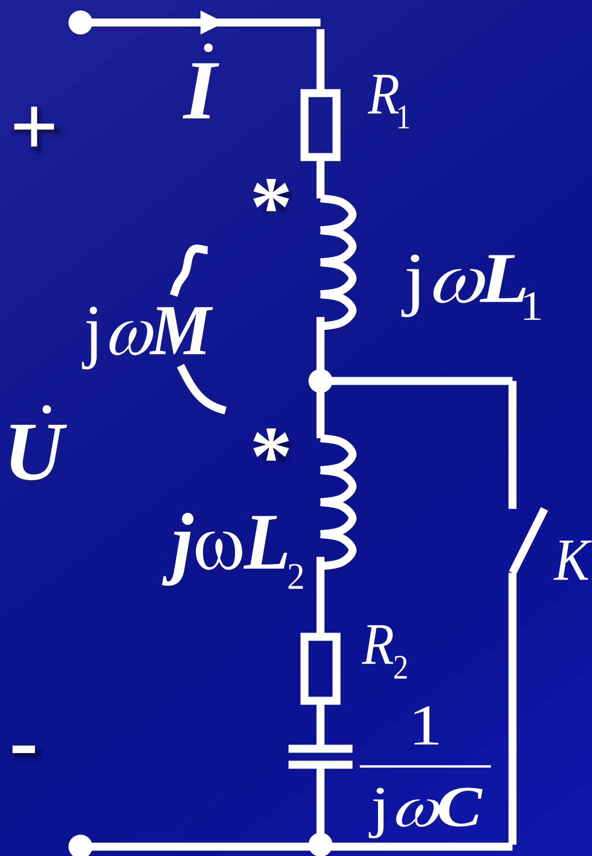
$\dfrac{1}{\omega C}=12\Omega, \omega L_1=4\Omega,$

$\omega L_2=12\Omega, \omega M=6\Omega,$

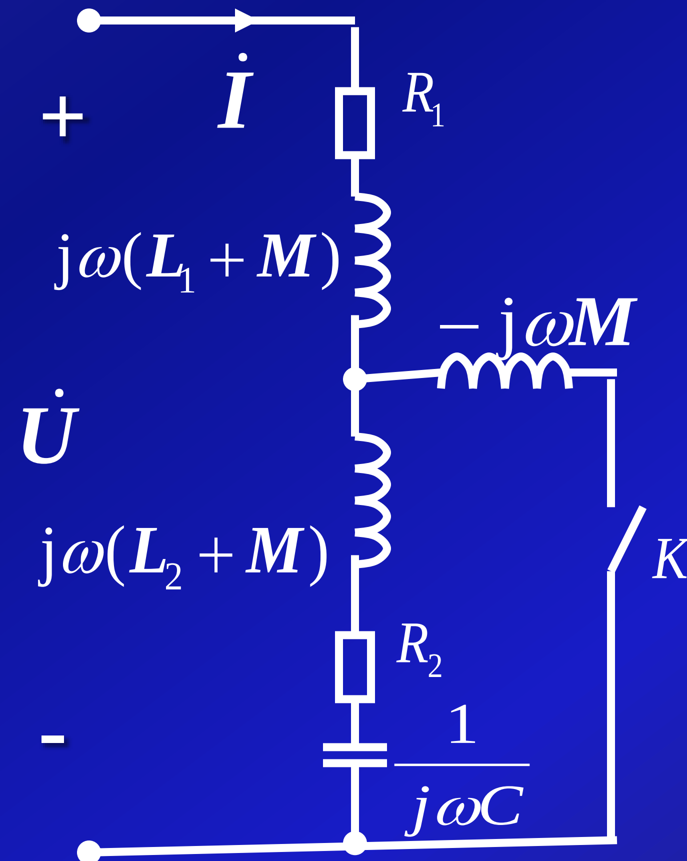
$\dot{U}=80\angle 0^\circ$

求:开关打开和闭合时的电流。





$\dot{I}$
R_1
+
*
$j\omega L_1$
$j\omega M$
$\dot{U}$
*
$j\omega L_2$
K
R_2
-
$\frac{1}{j\omega C}$



$\dot{I}$
R_1
+
$j\omega(L_1+M)$
$-j\omega M$
$\dot{U}$
$j\omega(L_2+M)$
K
R_2
-
$\frac{1}{j\omega C}$

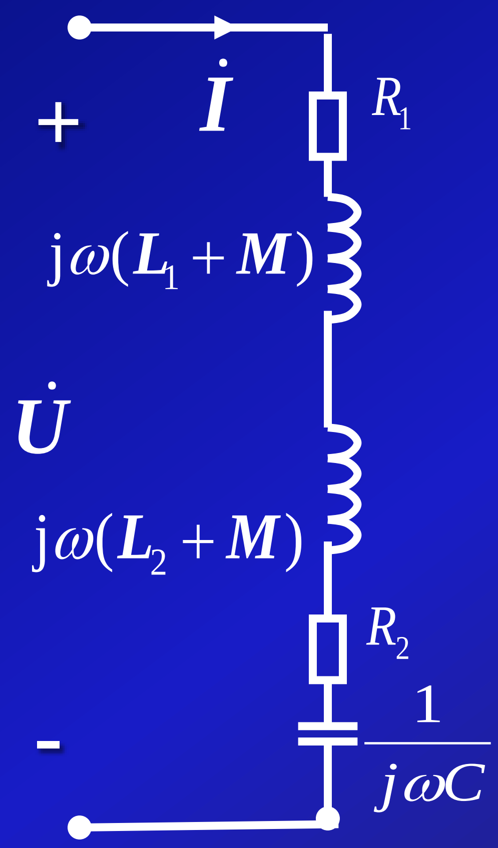
开关打开时：

$$Z = R_1 + R_2 + j\omega(L_1 + L_2 + 2M) + \frac{1}{j\omega C}$$

$$= 12 + j16\Omega$$

$$\dot{I} = \frac{\dot{U}}{Z} = \frac{80\angle 0°}{12 + j16} = \frac{80\angle 0°}{20\angle 53.1°}$$

$$= 4\angle -53.1° \text{A}$$



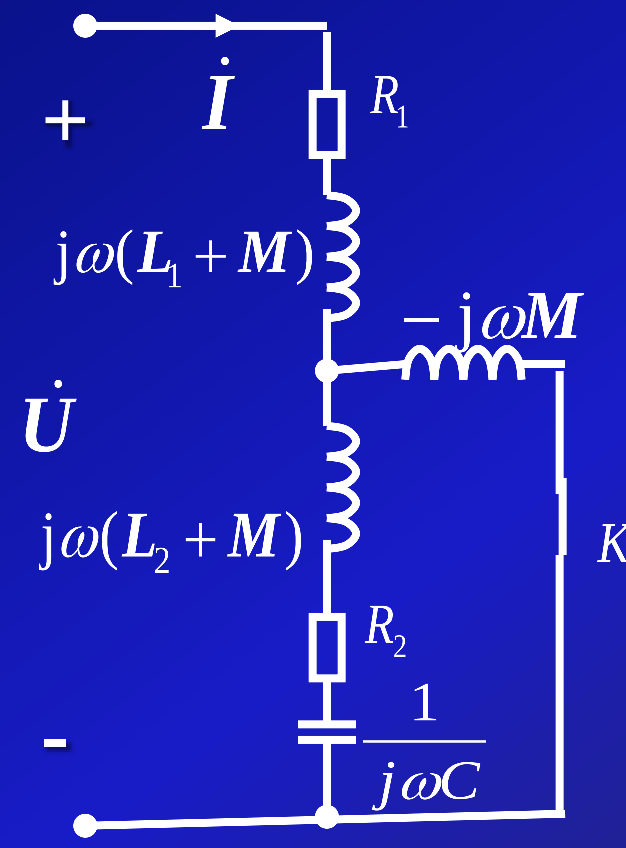
开关闭合时:

$$Z' = R_1 + \mathrm{j}\omega(L_1 + M)$$

$$+\frac{-\mathrm{j}\omega M[\mathrm{j}\omega(L_2+M)+R_2+\frac{1}{\mathrm{j}\omega C}]}{-\mathrm{j}\omega M+\mathrm{j}\omega(L_2+M)+R_2+\frac{1}{\mathrm{j}\omega C}}$$

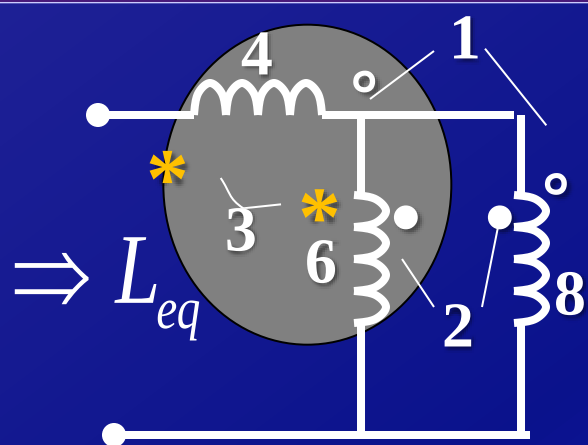
$$=12+j4\Omega$$

$$\dot{I}=\frac{\dot{U}}{Z'}=\frac{80\angle 0^\circ}{4\sqrt{10}\angle 18.4^\circ}=2\sqrt{10}\angle -18.4^\circ\mathrm{A}$$



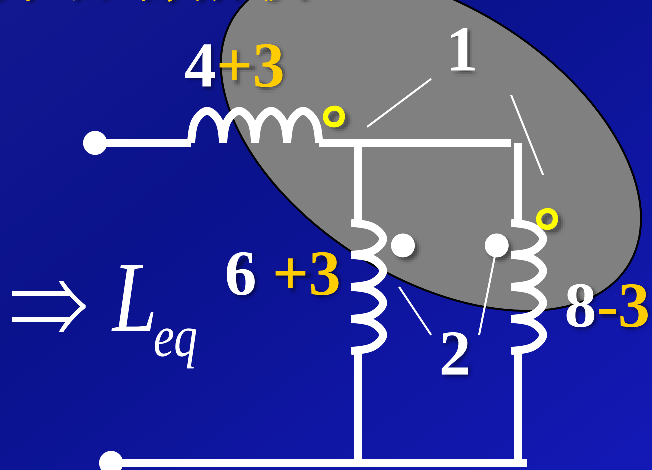
这种互感线圈常称自耦变压器。

例3：求等效电感L_{eq}。



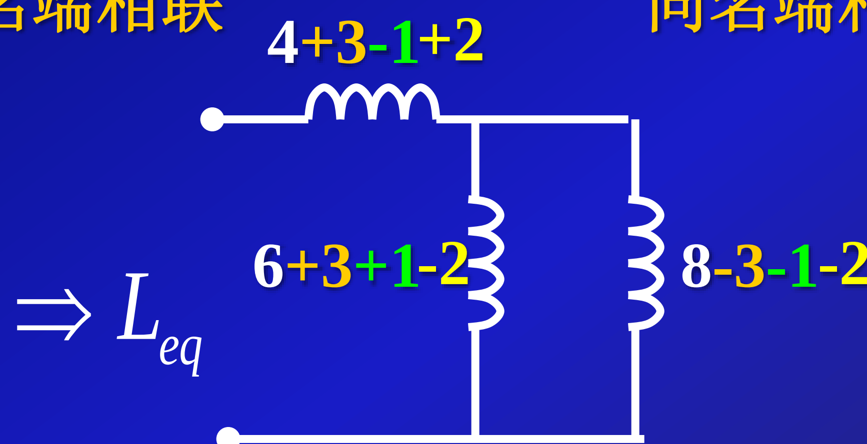
解：两两去耦

异名端相联



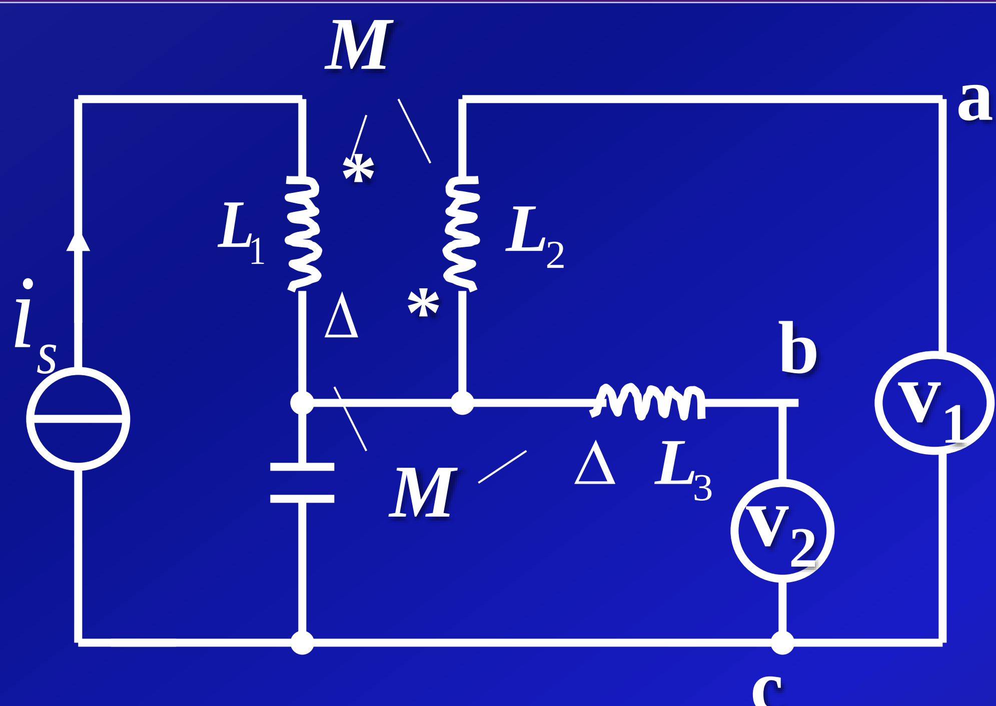
同名端相联

同名端相联



$$L_{eq}=8+8//2=9.6\ \text{H}$$

例4: 已知 $i_S(t)=\sqrt{2}\cos 5000t, M=2\text{mH}, C=2\mu\text{F}$，求电压表$V_1,V_2$的读数



解：建立电路的相量模型

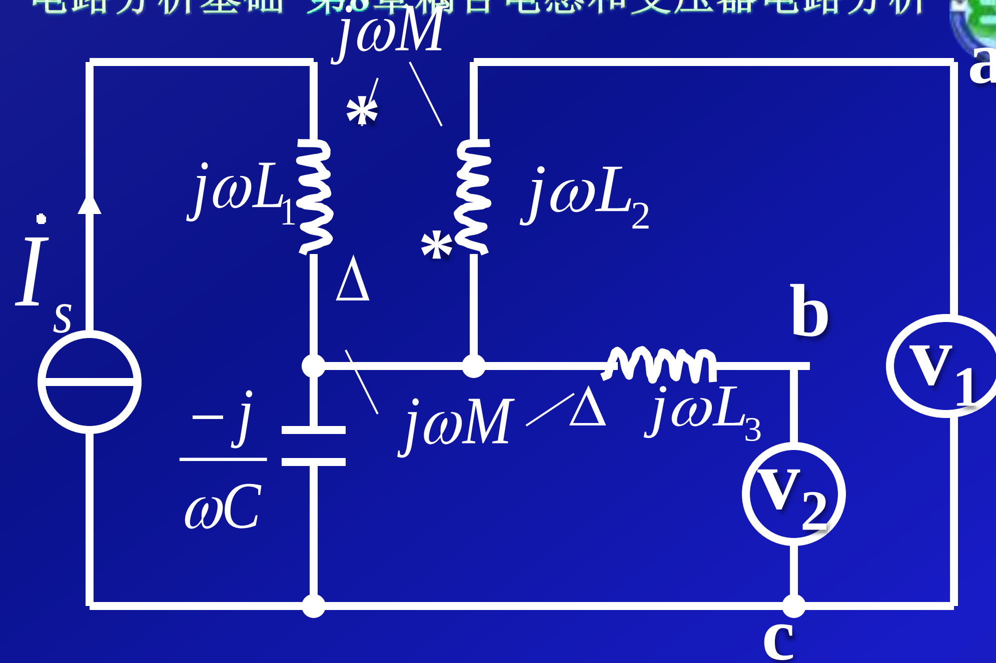
$$\dot{I}_s = 1\angle 0°, X_C = \frac{1}{\omega C} = 100\Omega, \omega M = 10\Omega$$

$\because$ $j\omega L_1$和$j\omega L_3$为同名端相连：

$$\dot{U}_{bc} = (-jX_C + j\omega M)\dot{I}_S$$

$$= -j90\ \text{V}$$

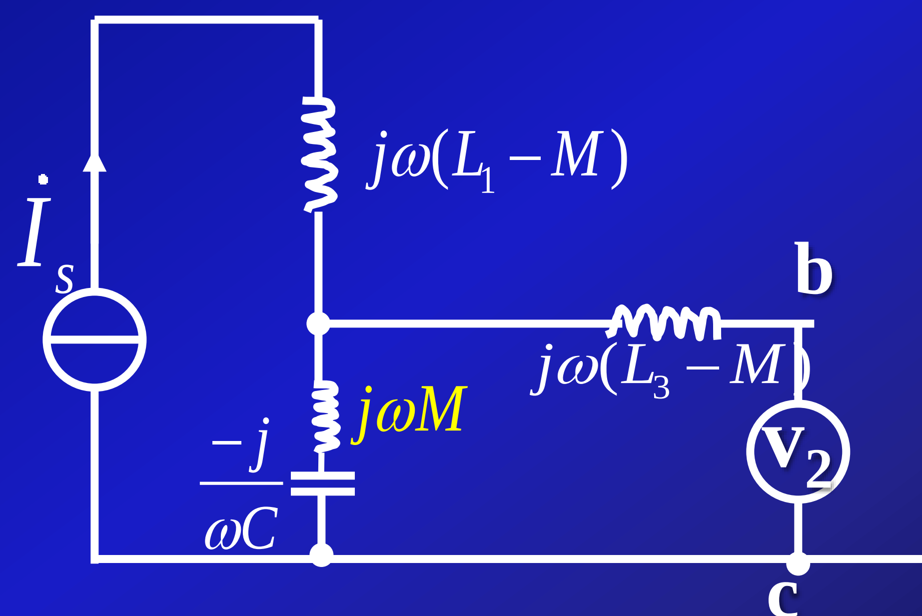
$$\therefore\ \boldsymbol{V}_2 = \boldsymbol{U}_{bc} = 90\ \text{V}$$



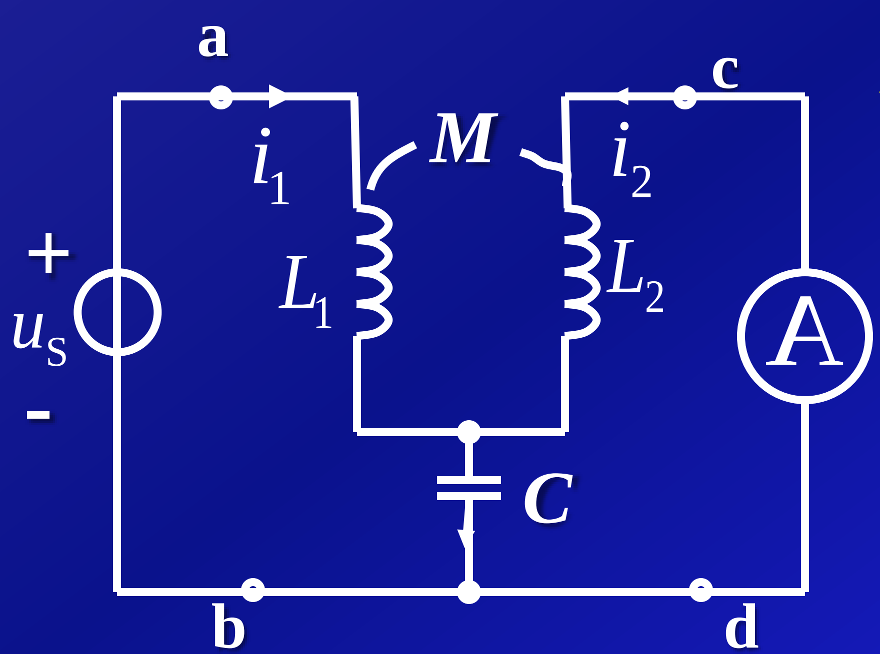
$$\dot{U}_{ac} = (-jX_C - j\omega M)\dot{I}_S$$

$$= -j110\ \text{V}$$

$$\therefore\ \boldsymbol{V}_1 = \boldsymbol{U}_{ac} = 110\ \text{V}$$



例5：已知 $u_S(t)=U_m\cos\omega t$, C , M 也已知。求：在什么条件下，安培表读数为零，标出同名端。



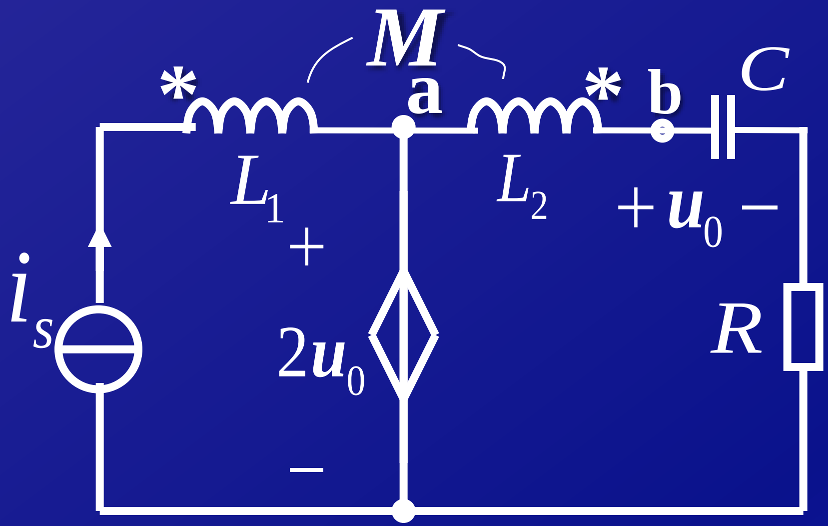
解：安培表读数为零时，cd间电压为零，即：

$$\dot{U}_{CDo}=\pm j\omega M\dot{I}_1+\frac{\dot{I}_1}{j\omega C}=0$$

$$\pm j\omega M=-\frac{1}{j\omega C}$$

$$\omega^2=\pm\frac{1}{MC}$$

显然上式只能取正号,即a,c为同名端，且 $\omega=\frac{1}{\sqrt{MC}}$



例6 已知$L_1 = 1\text{H}, L_2 = 2\text{H},$ $C = 0.5\text{F}, M = 0.5\text{H}, R = 1\Omega,$ $i_S(t) = 5\cos(2t + 30°)\ \text{A}$

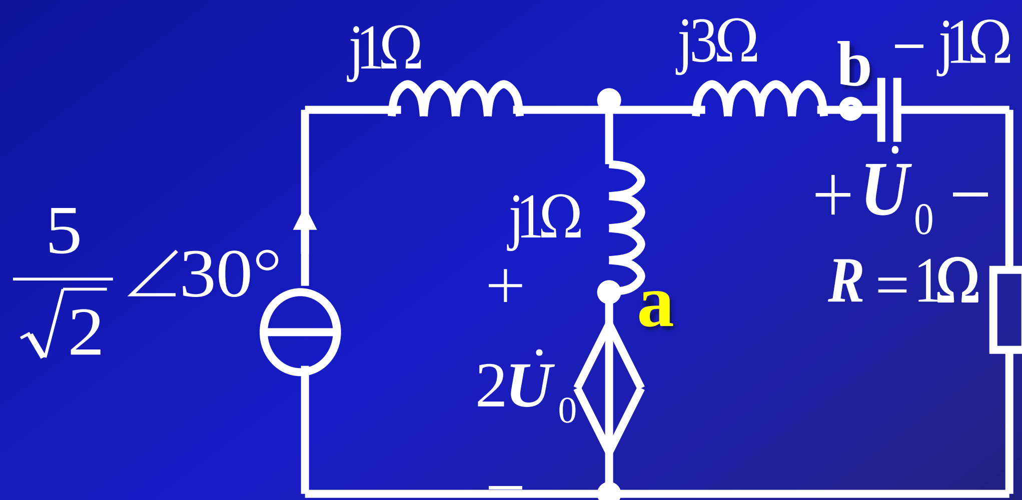
求：u_{ab}

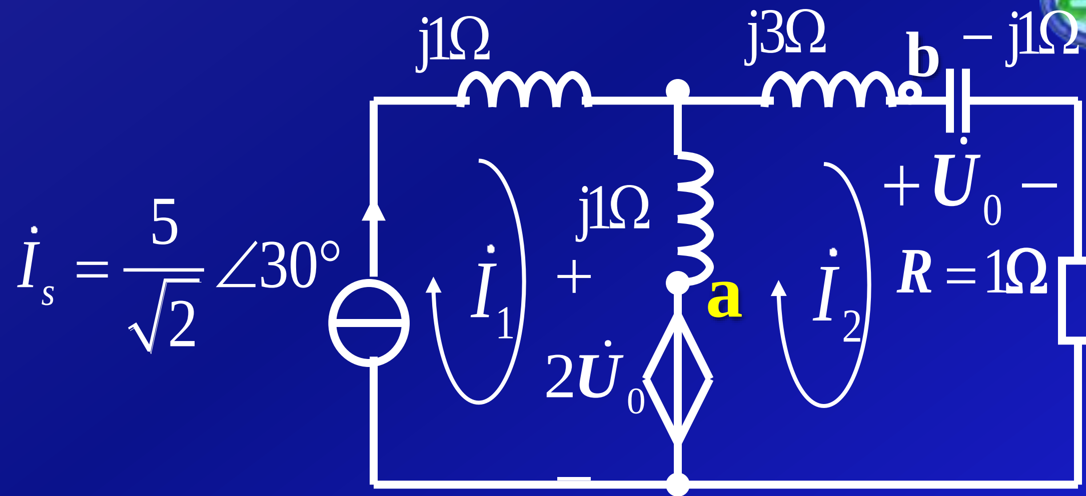
解：先作出其向量模型，并去耦等效；

$$\dot{I}_s = \frac{5}{\sqrt{2}}\angle 30°,$$

$$X_C = \frac{1}{\omega C} = 1\Omega,$$

$$\omega M = 1\Omega$$





列写网孔方程:

$$\begin{cases}\dot{I}_1=\dot{I}_s\\ -j1\dot{I}_1+(1-j1+j4)\dot{I}_2=2\dot{U}_o\\ \dot{U}_o=-j1\dot{I}_2\end{cases}$$

$$\therefore(1+j5)\dot{I}_2=j1\cdot\dot{I}_S$$

$$\dot{I}_2=0.7\angle 32.6°$$

$$\dot{U}_{ab}=j1(\dot{I}_2-\dot{I}_1)+j3\dot{I}_2=j4\dot{I}_2-j\dot{I}_S=0.95\angle-87.3°$$

$$\boldsymbol{u_{ab}}=0.95\sqrt{2}\cos(2\boldsymbol{t}-87.3^{\circ})\ \mathrm{V}$$