18-S1-Q5

Q(a) $G(s)$ $w_n = 5$ $\xi = 0.5$ pole-zero? $T=1$

Solution ① $G(s) = \dfrac{25}{s^2+5s+25}$

② $w_d = w_n\sqrt{1-\xi^2} = 5 \times \sqrt{1-\frac{1}{4}} = \frac{5\sqrt{3}}{2} = 4.3301$

③ $s_{1,2} = \dfrac{-5 \pm 5\sqrt{3}j}{2}$ $s_1 = -2.5+4.3301j$

$s_2 = -2.5 - 4.3301j$

④ $G(z) = \alpha \dfrac{(z+1)^{n-m-1}}{(z-e^{s_1T})(z-e^{s_2T})}$

$n-m-1 = 2-0-1 = 1$

$(z-e^{s_1T})(z-e^{s_2T})$

$= z^2 - z(e^{s_2T}+e^{s_1T}) + e^{s_1T}e^{s_2T}$

where $e^{s_1T}+e^{s_2T} = e^{-2.5}e^{-4.3301j} + e^{-2.5}e^{4.3301j}$

$= e^{-2.5}(e^{-4.3301j}+e^{4.3301j})$

$= e^{-2.5}\,2\cos(4.3301)$

$= -0.06124$

$e^{s_1T}e^{s_2T} = e^{-5} = 0.006738$

$$z^2+0.06124z+0.006738$$

$$G_D(z)=\alpha\frac{(z+1)}{z^2+0.06124z+0.006738}$$

$$G_D(1)=G(0)=\frac{25}{s^2+5s+25}\Big|_{s=0}=\frac{25}{25}=1$$

$$\alpha\frac{2}{1+0.06124+0.006738}=1$$

$$\alpha=0.5340$$

$$G_D(z)=0.534\frac{z+1}{z^2+0.06124z+0.006738}$$

(b) Q: $C(z)$ 意思就是系统的闭环传函和上面a问算出来的离散化滤波器传函是一个

Solution $G(z)=\frac{Y(z)}{G_{ZAS}(z)}=G_{cl}(z)\frac{R(z)}{G_{ZAS}(z)}$

$$G_{cl}(z)=\frac{0.534z+0.534}{z^2+0.06124z+0.006738}$$

$$C(z)=\frac{1}{G_{ZAS}(z)}\frac{G_{cl}(z)}{1-G_{cl}(z)}$$

$$=\frac{(z-1)(z-0.3679)\dfrac{0.534z+0.534}{z^2+0.06124z+0.006738}}{(0.3679z+0.2642)\left(1-\dfrac{0.534z+0.534}{z^2+0.06124z+0.006738}\right)}$$

$$= \frac{(z-1)(z-0.3679)\;(0.534z+0.534)}{(0.3679z+0.2642)\;(z^2-0.4728z-0.5273)}$$

$$= \frac{(z-1)(z-0.3679)\;(0.534z+0.534)}{(0.3679z+0.2642)\;(z-1)\;(z+0.5273)}$$

$$C(z) = \frac{0.534\;(z-0.3679)(z+1)}{0.3679\;(z+0.7176)(z+0.5273)}$$

$$= \frac{1.4515\,(z-0.3679)(z+1)}{(z+0.7176)(z+0.5273)}$$

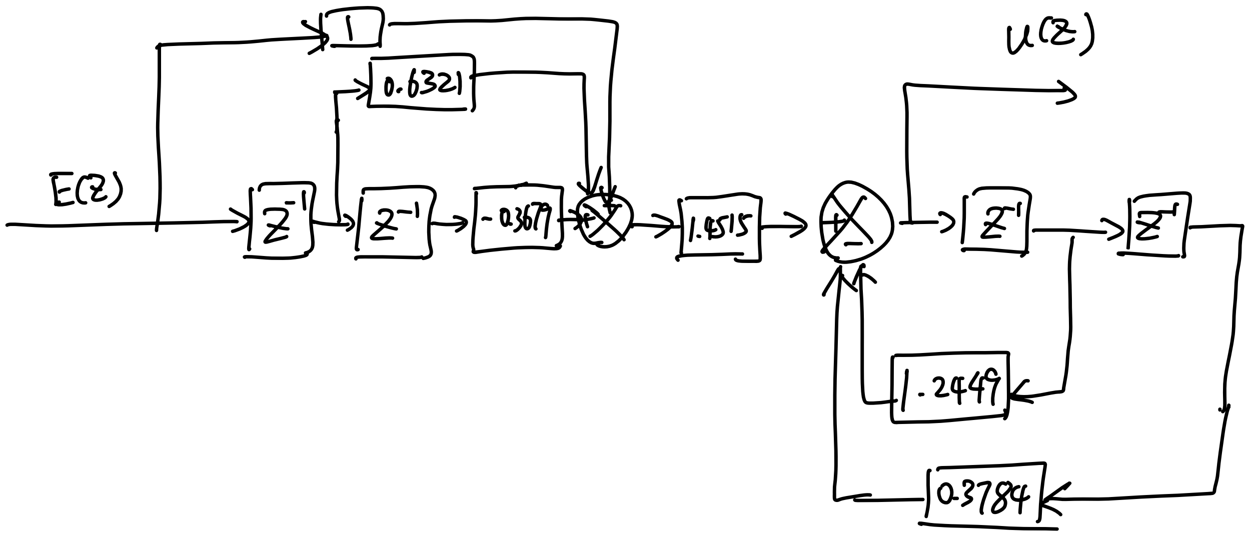
(c)

$$C(z) = \frac{1.4515\;(z^2+0.6321z-0.3679)}{z^2+1.2449z+0.3784}$$

$$= \frac{1.4515\,(1+0.6321z^{-1}-0.3679z^{-2})}{1+1.2449z^{-1}+0.3784z^{-2}}$$

$$C(z) = \frac{U(z)}{E(z)} = \frac{1.4515\,(1+0.6321z^{-1}-0.3679z^{-2})}{1+1.2449z^{-1}+0.3784z^{-2}}$$

$$U(z) = 1.4515\big(E(z)+0.6321z^{-1}E(z)-0.3679z^{-2}E(z)\big)$$
$$-1.2449z^{-1}U(z)-0.3784z^{-2}U(z)$$



E(z)
U(z)
1
0.6321
-0.3679
1.4515
1.2449
0.3784