Q: Z平面转S平面，求damping ratio

Example 11.2 — Lect3 . P13

close-loop poles at $0.888 \pm j0.173$

Solution ① Z平面 $a \pm bj \rightarrow r \angle \theta$

method 1: $r = \sqrt{a^2 + b^2}$ $\theta = \tan^{-1}\left(\frac{b}{a}\right)$

method 2:

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菜单 → 2:复数 → $0.888 + 0.173i$

→ OPTN → ↓ → 1: $r\angle\theta$ → =

$0.905\angle +0.1924$ radians

0.193 → ppt错]

$$\zeta = -\frac{\ln r}{\sqrt{\ln^2 r + \theta^2}}$$

$$= -\frac{\ln 0.905}{\sqrt{\ln^2 0.905 + 0.193^2}}$$

$$= 0.46 \quad \checkmark$$

② Z转 S

Solution

$z = e^{sT}$ $\quad s = \frac{\ln z}{T}$ $\quad s = \sigma + j\omega$

$0.905 \angle 0.193$ $\quad T = 0.1$

$$z_{e1,2} = 0.905 e^{\pm j 0.193}$$

$$s_{1,2} = \frac{\ln 0.905 e^{\pm j 0.193}}{0.1}$$

$$= 10\left(\ln 0.905 \pm j 0.193\right)$$

$$= -0.9982 \pm j 1.93$$

方法2：方程匹配

$s_{1,2} = -\zeta\omega_n \pm j\omega_d$ $\quad \zeta = 0.46$

$\omega_d = \omega_n\sqrt{1-\zeta^2}$

$z^2 - 2e^{-\zeta\omega_n T}\cos(\omega_d T)\,z + e^{-2\zeta\omega_n T}$

$z: 0.888 \pm j\,0.173$

$[z-(0.888+j0.173)][z-(0.888-j0.173)]$

$= z^2 - z(0.888\times 2) + 0.888^2 + 0.173^2$

$= z^2 - 1.776z + 0.818473$

$e^{-2\zeta\omega_n T} = e^{-2\times 0.46\omega_n \times 0.1} = 0.818473$

$$\omega_n = \frac{\ln 0.818473}{-2\times 0.46\times 0.1} = 2.1773$$

$\omega_d = \omega_n\sqrt{1-\zeta^2}$

$= 2.1773\times\sqrt{1-0.46^2} = 1.9333$

$s_{1,2} = -\zeta\omega_n \pm j\omega_d$

$= -0.46\times(2.1773) \pm j\,1.9333$

$s_{1,2} = -1.0016 \pm j\,1.9333$

③ $S \rightarrow Z$

$S = \sigma + jw$

$z = e^{sT}$

$= e^{(\sigma + jw)T}$

$= e^{\sigma T} e^{jwT}$

$r \qquad \theta$

$z = r \angle \theta$

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$z = a + bj$