5. Consider the closed loop system in Figure 2. With a sampling period of 1 second, $G_{ZAS}(z)$ is given as

$$G_{ZAS}(z) = K\frac{0.8z + 0.2}{(z-1)(z-0.4)}$$

where K is a non-zero constant.

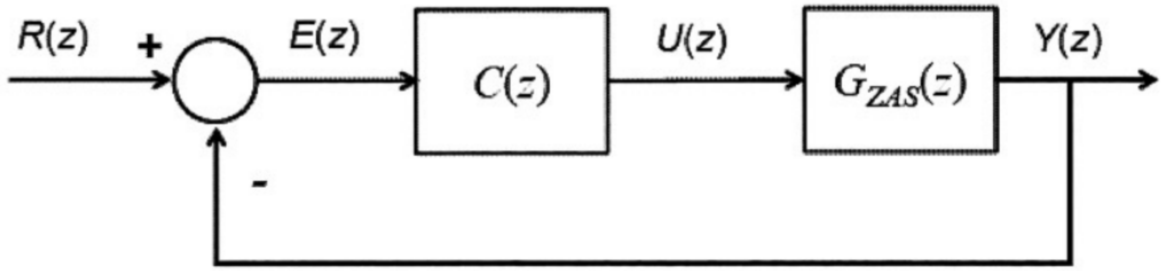

Figure 2

(a) If the controller is a proportional controller with a gain K_p, determine the range of K_pK so that the closed-loop system is stable.

(8 Marks)

(b) It is required that the output Y(z) tracks a unit-step input $R(z)$ without any steady state error. Design a ripple-free controller $C(z)$ to meet this requirement.

(8 Marks)

(c) Implement the controller $C(z)$ obtained in 5(b) with the standard programming approach and show the relevant block diagram.

(4 Marks)

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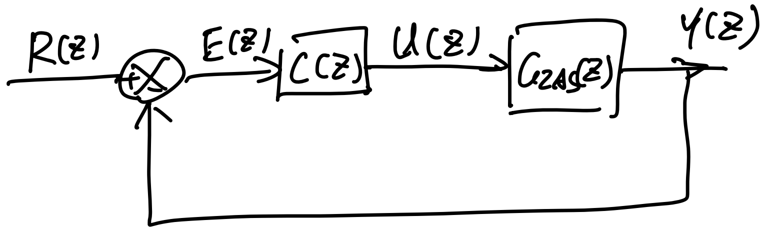
Q: $T=1$

$$G_{ZAS}(z) = K\frac{0.8z+0.2}{(z-1)(z-0.4)}$$



(a) $C(s) = K_p$ range $K_pK \to$ stable ?

Solution

$$G_{cl}(z) = \frac{C(z)G_{ZAS}(z)}{1+C(z)G_{ZAS}(z)}$$

$$= \frac{K_pK\frac{0.8z+0.2}{(z-1)(z-0.4)}}{1+K_pK\frac{0.8z+0.2}{(z-1)(z-0.4)}}$$

$$= \frac{K_pK(0.8z+0.2)}{(z-1)(z-0.4)+K_pK(0.8z+0.2)}$$

$$= \frac{K_pK(0.8z+0.2)}{z^2+(-1.4+0.8K_pK)z+0.4+0.2K_pK}$$

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z^0	z^1	z^2
$0.4+0.2K_pK$	$-1.4+0.8K_pK$	1

$\because$ stable

$$\therefore \begin{cases} |0.4 + 0.2 k_p K| < 1 \\ P(1) = 1 - 1.4 + 0.8 k_p K + 0.4 + 0.2 k_p K > 0 \\ P(-1) = 1 + 1.4 - 0.8 k_p K + 0.4 + 0.2 k_p K > 0 \quad n = 2 \text{ even} \end{cases}$$

$$\therefore \quad -1 < 0.4 + 0.2 k_p K < 1$$

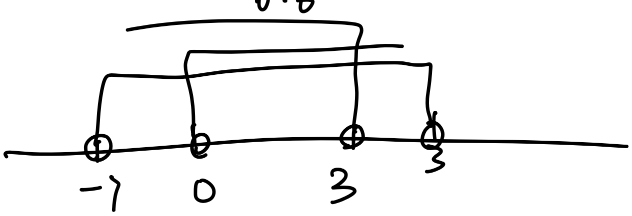
$$\frac{-1.4}{0.2} < k_p K < \frac{0.6}{0.2}$$

$$-7 < k_p K < 3 \qquad (1)$$

$$k_p K > 0 \quad (2)$$

$$-0.6 k_p K + 1.8 > 0$$

$$k_p K < \frac{1.8}{0.6} = 3 \quad (3)$$



So $\quad 0 < k_p K < 3$

(b) $R(z)=\frac{1}{1-z^{-1}}$ $\quad\quad G_{cl}(1)=1$

$$U(z)=\frac{Y(z)}{G_{ZAS}(z)}=\frac{Y(z)}{R(z)}\frac{R(z)}{G_{ZAS}(z)}=G_{cl}(z)\frac{R(z)}{G_{ZAS}(z)}$$

$$=G_{cl}(z)\frac{\frac{1}{1-z^{-1}}}{K\frac{0.8z+0.2}{(z-1)(z-0.4)}} \xrightarrow{\text{化为}z^{-1}\text{形式}} \frac{K(0.8+0.2z^{-1})z^{-1}}{(1-z^{-1})(1-0.4z^{-1})}$$

$$=G_{cl}(z)\frac{(z-1)(z-0.4)}{K(0.8z+0.2)(1-z^{-1})} \quad \frac{1-0.4z^{-1}}{K(0.8+0.2z^{-1})z^{-1}}$$

design $G_{cl}(z)=K_0K(0.8+0.2z^{-1})z^{-1}$

$\because G_{cl}(1)=K_0K=1 \quad \Rightarrow K_0K=1$

$G_{cl}(z)=(0.8+0.2z^{-1})z^{-1}$

$$C(z)=\frac{1}{G_{ZAS}(z)}\frac{G_{cl}(z)}{1-G_{cl}(z)}$$

$$=\frac{(1-z^{-1})(1-0.4z^{-1})}{K(0.8+0.2z^{-1})z^{-1}}\quad\frac{(0.8+0.2z^{-1})z^{-1}}{1-(0.8+0.2z^{-1})z^{-1}}$$

$$=\frac{(1-z^{-1})(1-0.4z^{-1})}{K[1-(0.8+0.2z^{-1})z^{-1}]}$$

约分后结果 $=\frac{(1-z^{-1})(1-0.4z^{-1})}{K(1-z^{-1})(1+0.2z^{-1})}=\frac{1-0.4z^{-1}}{K(1+0.2z^{-1})}$

不同，

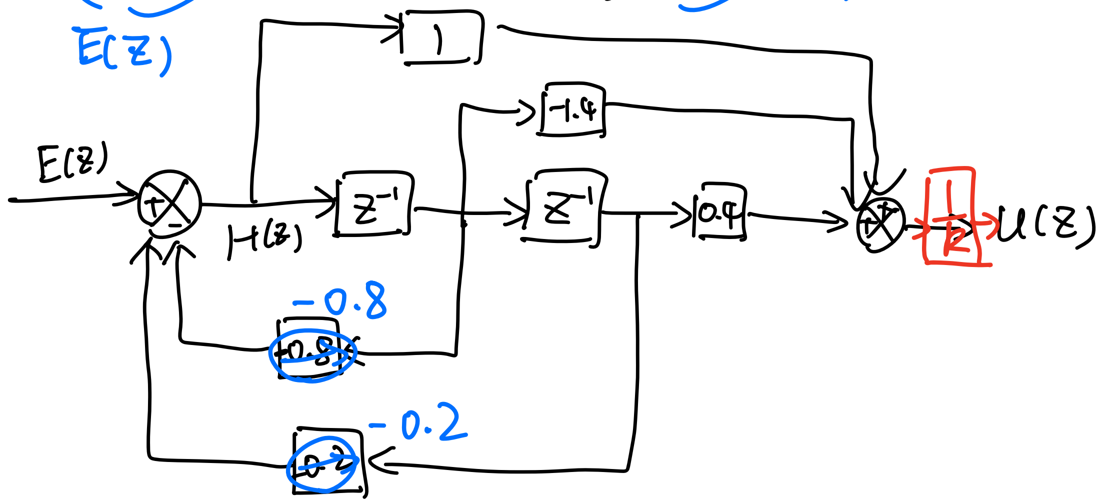
(c)不同

(c) $C(z) = \frac{U(z)}{E(z)} = \frac{1}{K}\,\frac{1-1.4z^{-1}+0.4z^{-2}}{1-0.8z^{-1}-0.2z^{-2}} = \frac{1}{K}\,\frac{U(z)}{H(z)}\,\frac{H(z)}{E(z)}$

$U(z) = (1-1.4z^{-1}+0.4z^{-2})H(z)\,\frac{1}{K}$

$\cancel{H(z)} = (1-0.8z^{-1}-0.2z^{-2})\cancel{E(z)}\; H(z) = H(z) - 0.8z^{-1}H(z) - 0.2z^{-2}H(z)$

$E(z)$



So $H(z) = E(z) + 0.8z^{-1}H(z) + 0.2z^{-2}H(z)$

$C(z) = \frac{U(z)}{E(z)} = \frac{1-0.4z^{-1}}{K(1+0.2z^{-1})} = \frac{U(z)}{H(z)}\,\frac{H(z)}{E(z)}$

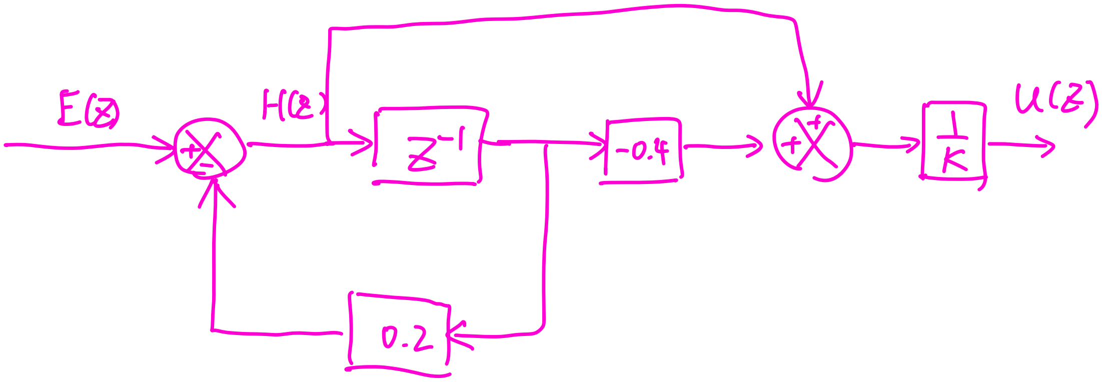
$\frac{U(z)}{H(z)} = \frac{1}{K}(1-0.4z^{-1})$

$U(z) = \frac{1}{K}\left[H(z) - 0.4z^{-1}H(z)\right]$

$\frac{H(z)}{E(z)} = \frac{1}{1+0.2z^{-1}}$

$E(z) = H(z) + 0.2z^{-1}H(z)$

$H(z) = E(z) - 0.2z^{-1}H(z)$



E(z)
H(z)
+
-
z^{-1}
-0.4
+
+
$\frac{1}{K}$
U(z)
0.2