

19-51-Q3

Q: (a) discretised?

Solution ① state transform matrix

$$\begin{aligned}[sI - A]^{-1} &= \begin{bmatrix} s & -1 \\ 2 & s+3 \end{bmatrix}^{-1} = \frac{1}{s(s+3)+2} \begin{bmatrix} s+3 & 1 \\ -2 & s \end{bmatrix} \\&= \frac{1}{s^2+3s+2} \begin{bmatrix} s+3 & 1 \\ -2 & s \end{bmatrix} \\&= \frac{1}{(s+1)(s+2)} \begin{bmatrix} s+3 & 1 \\ -2 & s \end{bmatrix} \\&= \begin{bmatrix} \frac{s+3}{(s+1)(s+2)} & \frac{1}{(s+1)(s+2)} \\ \frac{-2}{(s+1)(s+2)} & \frac{s}{(s+1)(s+2)} \end{bmatrix}\end{aligned}$$

table #9

$$\frac{b-a}{(s+a)(s+b)} = \frac{1}{(s+1)(s+2)} \quad \text{we get } e^{-t} - e^{-2t}$$

$$\frac{s}{(s+1)(s+2)} = \frac{A}{s+1} + \frac{B}{s+2} \quad \text{PFE}$$

$$A(s+2) + B(s+1) = (A+B)s + 2A+B$$

$$\begin{cases} A+B=1 \\ 2A+B=0 \end{cases} \Rightarrow \begin{cases} A=-1 \\ B=2 \end{cases}$$

$$\frac{s}{(s+1)(s+2)} = \frac{-1}{s+1} + \frac{2}{s+2}$$

4 $\frac{1}{s+a}$, we get $-e^{-t} + 2e^{-2t}$

Therefore

$$\mathcal{L}^{-1}\left\{\frac{s+3}{(s+1)(s+2)}\right\} = -e^{-t} + 2e^{-2t} + 3e^{-t} - 3e^{-2t} = 2e^{-t} - e^{-2t}$$

$$\mathcal{L}^{-1}\left\{\frac{1}{(s+1)(s+2)}\right\} = e^{-t} - e^{-2t}$$

$$\mathcal{L}^{-1}\left\{\frac{-2}{(s+1)(s+2)}\right\} = 2e^{-t} + 2e^{-2t}$$

$$\mathcal{L}^{-1}\left\{\frac{s}{(s+1)(s+2)}\right\} = -e^{-t} + 2e^{-2t}$$

$$\Phi(t) = \mathcal{L}^{-1}\left\{[sI - A]^{-1}\right\} = \begin{bmatrix} 2e^{-t} - e^{-2t} & e^{-t} - e^{-2t} \\ 2e^{-t} + 2e^{-2t} & -e^{-t} + 2e^{-2t} \end{bmatrix}$$

$$\Phi(T) = \mathcal{L}^{-1}\left\{[sI - A]^{-1}\right\} \Big|_{t=T}$$

$$= \begin{bmatrix} 2e^{-T} - e^{-2T} & e^{-T} - e^{-2T} \\ 2e^{-T} + 2e^{-2T} & -e^{-T} + 2e^{-2T} \end{bmatrix}$$

② input matrix

$$\theta(T) = \int_0^T \Phi(t) dt B$$

$$= \int_0^T \begin{bmatrix} 2e^{-t} - e^{-2t} & e^{-t} - e^{-2t} \\ 2e^{-t} + 2e^{-2t} & -e^{-t} + 2e^{-2t} \end{bmatrix} \begin{bmatrix} 0 \\ 8 \end{bmatrix} dt$$

$$= \int_0^T \begin{bmatrix} 8e^{-t} - 8e^{-2t} \\ -8e^{-t} + 16e^{-2t} \end{bmatrix} dt$$

$$\int_0^T 8e^{-t} - 8e^{-2t} dt$$

$$= 8 \int_0^T e^{-t} dt - 8 \int_0^T e^{-2t} dt$$

$$= 8(-e^{-t}) \Big|_0^T - 8\left(-\frac{1}{2}e^{-2t}\right) \Big|_0^T$$

$$= -8(e^{-T} - 1) + 4(e^{-2T} - 1)$$

$$= -8e^{-T} + 4e^{-2T} + 4$$

$$\int_0^T -8e^{-t} + 16e^{-2t} dt$$

$$= -8 \int_0^T e^{-t} + 16 \int_0^T e^{-2t} dt$$

$$= -8e^{-T} - 8e^{-2T} + 16$$

$$\theta(T) = \begin{bmatrix} -8e^{-T} + 4e^{-2T} + 4 \\ -8e^{-T} - 8e^{-2T} + 16 \end{bmatrix}$$

$$+ 16 \times \left(-\frac{1}{2}\right)$$

$$\text{So } x(k+1) = \begin{bmatrix} 2e^{-T} - e^{-2T} & e^{-T}e^{-2T} \\ 2e^{-T} + 2e^{-2T} & -e^{-T} + 2e^{-2T} \end{bmatrix} x(k) + \begin{bmatrix} -8e^{-T} + 4e^{-2T} + 4 \\ -8e^{-T} - 8e^{-2T} + 16 \end{bmatrix} u(k)$$

$$y(k) = [1 \ 0] x(k)$$

(b) (i) Q C? O?

$$\text{Solution } \textcircled{1} W_c = [B \ AB] = \begin{bmatrix} 0 & 1 \\ 1 & -6 \end{bmatrix}$$

$$AB = \begin{bmatrix} 0 & 1 \\ -4 & -6 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \\ -6 \end{bmatrix}$$

$$|W_c| = -1 \neq 0 \quad \text{Controllable } \checkmark$$

$$\textcircled{2} W_o = \begin{bmatrix} C \\ CA \end{bmatrix} = \begin{bmatrix} 1 & -1 \\ 4 & 7 \end{bmatrix}$$

$$CA = [1 \ -1] \begin{bmatrix} 0 & 1 \\ -4 & -6 \end{bmatrix} = \begin{bmatrix} 4 & 7 \end{bmatrix}$$

$$|W_o| = 7 + 4 = 11 \neq 0 \quad \text{Observable } \checkmark$$

(ii) unobservable ?

$$x(k+1) = \begin{bmatrix} 0 & 1 \\ -4 & -6 \end{bmatrix} x(k) + \begin{bmatrix} 0 \\ 1 \end{bmatrix} [-k_1 \ -k_2] x(k) + \begin{bmatrix} 0 \\ 2 \end{bmatrix} r(k)$$

$$= \left(\begin{bmatrix} 0 & 1 \\ -4 & -6 \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ -k_1 & -k_2 \end{bmatrix} \right) x(k) + \begin{bmatrix} 0 \\ 2 \end{bmatrix} r(k)$$

$$= \begin{bmatrix} 0 & 1 \\ -4-k_1 & -6-k_2 \end{bmatrix} x(k) + \begin{bmatrix} 0 \\ 2 \end{bmatrix} r(k)$$

$$W_0 = \begin{bmatrix} C \\ C A_r \end{bmatrix} = \begin{bmatrix} 1 & -1 \\ 4+k_1 & 7+k_2 \end{bmatrix}$$

$$C A_r = [1 \ -1] \begin{bmatrix} 0 & 1 \\ -4-k_1 & -6-k_2 \end{bmatrix} = [4+k_1 \ 7+k_2]$$

$$|W_0| = 7+k_2 + 4+k_1 = 0$$

when $k_1+k_2 = -13$, the system isn't observable

(c) optimal control ?

$$\text{Solution } A = 0.8 \quad B = 0.2 \quad Q = 0.1 \quad r = 6$$

$$S = 0.8^2 S + 0.1 - 0.8 S 0.2 (6 + 0.2^2 S)^{-1} 0.2 S 0.8$$

$$S = 0.64 S - \frac{0.16^2 S^2}{6 + 0.04 S} + 1$$

$$\frac{0.0256 S^2}{6 + 0.04 S} = -0.36 S + 1$$

$$0.0256 S^2 = -0.036 S (6 + 0.04 S) + 6 + 0.04 S$$

$$= -0.216 S - 0.00144 S^2 + 6$$

$$0.02704 S^2 + 0.216 S - 6 = 0$$

$$S_1 = 10.8964 \quad S_2 = -20.3839$$

choose the positive $S = 10.8964$

$$k = (0.2^2 \times \overset{2.6479}{\cancel{0.8964}} + 6)^{-1} \times 0.2 \times \overset{2.6479}{\cancel{0.8964}} \times 0.8$$

$$= \overset{0.2709}{\cancel{0.2709}} \times 0.06939$$

$$u^*(k) = \overset{-0.06939}{\cancel{-0.2709}} \times x(k)$$

$$x(k+1) = [0.8 + 0.2 \times \overset{0.7861}{\cancel{-0.2709}}] x(k)$$

$$= \overset{0.7861}{\cancel{0.74382}} \times x(k)$$

poles 没学过, 不考