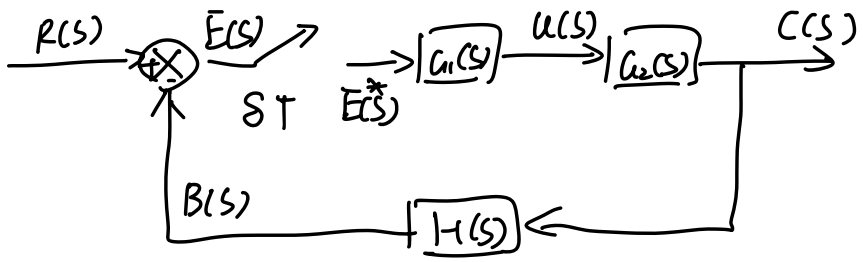
18-S1-Q2

(a) $G_1(s)$ $G_2(s)$ $H(s)$ T $r(t)$ $C(z)$?



Solution ① Transfer function

$$G_1(s) = \frac{U(s)}{E^*(s)}$$

$$G_2(s) = \frac{C(s)}{U(s)}$$

$$H(s) = \frac{B(s)}{C(s)} = 1$$

② ERROR

$$E(s) = R(s) - B(s)$$

$$= R(s) - H(s)C(s)$$

$$= R(s) - G_2(s)U(s)$$

$$= R(s) - G_2(s)G_1(s)E^*(s)$$

$$E^*(s) = R^*(s) - G_1G_2^*(s)E^*(s)$$

$$E^*(s) = \frac{R^*(s)}{1+G_1G_2^*(s)}$$

③ $$C(s) = G_1G_2(s)E^*(s)$$

$$C^*(s) = G_1G_2^*(s)E^*(s) = \frac{G_1G_2^*(s)\,R^*(s)}{1+G_1G_2^*(s)}$$

$$C(z) = \frac{G_1G_2(z)\,R(z)}{1 + G_1G_2(z)}$$

$$G_1G_2(z) = (1-z^{-1})\,Z\left\{\frac{\alpha}{s(s+1)}\right\}$$

$$= (1-z^{-1})\,\alpha\, Z\left\{\frac{1}{s(s+1)}\right\}$$

#8 $\frac{1}{s(s+1)}$ $\alpha = 1$ $\frac{(1-e^{-T})z^{-1}}{(1-z^{-1})(1-e^{-T}z^{-1})}$ $T = 0.1$

$$G_1G_2(z) = \frac{\alpha(1-e^{-T})z^{-1}}{1-e^{-T}z^{-1}}$$

$$= \frac{0.09516\,\alpha z^{-1}}{1-0.9048\,z^{-1}}$$

$$R(z) = \frac{1}{1-z^{-1}}$$

$$C(z) = \frac{\dfrac{0.09516\,\alpha z^{-1}}{1-0.9048\,z^{-1}}}{1 + \dfrac{0.09516\,\alpha z^{-1}}{1-0.9048\,z^{-1}}} \cdot \frac{1}{1-z^{-1}}$$

$$= \frac{0.09516\,\alpha\, z^{-1}}{1 - 0.9048 z^{-1} + 0.09516\,\alpha z^{-1} + (1-z^{-1})(1-0.9048\,z^{-1})}$$

(b) PE :

$$1+\frac{0.09516\alpha z^{-1}}{1-0.9048z^{-1}}=0$$

$$1+\frac{0.09516\alpha}{z-0.9048}=0$$

$$z-0.9048+0.09516\alpha=0$$

$$z=0.9048-0.09516\alpha$$

$$z\in(-1,1]$$

$$\begin{cases}0.9048-0.09516\alpha>-1 \Rightarrow \alpha<-20.0168\\ 0.9048-0.09516\alpha\le 1 \Rightarrow \alpha\ge -1.0004\end{cases}$$

So the system is not stable

(c) α ?

Solution from (b), the constant α isn't exist

$$e_{ss}=\lim_{z\to 1}\left[(1-z^{-1})\frac{1}{1+G(z)H(z)}R(z)\right] \qquad R(z)=\frac{Tz^{-1}}{(1-z^{-1})^2}$$

↓ 沒必要寫了，不能定，$\lim_{k\to\infty}|e(kT)|$ 不存在

X $$=\lim_{z\to 1}(1-z^{-1})\frac{1}{1+\frac{0.09516\alpha z^{-1}}{1-0.9048z^{-1}}}\cdot\frac{Tz^{-1}}{(1-z^{-1})^2}$$

$$=\lim_{z\to 1}\frac{0.1z^{-1}}{\left(1+\frac{0.09516\alpha z^{-1}}{1-0.9048z^{-1}}\right)(1-z^{-1})}$$

$$= \lim_{z \to 1} \frac{0.1z^{-1}(1 - 0.9048z^{-1})}{(1 + (-0.9048 + 0.09516\alpha)z^{-1})(1 - z^{-1})}$$

$$= \lim_{z \to 1} \frac{0.1z^{-1}(1 - 0.9048z^{-1})}{1 - z^{-1} + (-0.9048 + 0.09516\alpha)z^{-1} - (-0.9048 + 0.09516\alpha)z^{-2}}$$

$$= \frac{0.1 \times 0.0952}{1 - 1 + (\quad) - (\quad)} = \frac{0.00952}{0}$$