

22-51-Q4

Q(a)(i)  $k$  ?  $\rightarrow$  deadbeat control

$$x(k+1) = [A - BK]x(k)$$

$$[A - BK] = \begin{bmatrix} 1 & 0 \\ -1 & 1 \end{bmatrix} - \begin{bmatrix} 1 \\ 1 \end{bmatrix} K$$

$$K = [0 \ 1] W_c^{-1} \alpha_c(A)$$

$$W_c^{-1} = [B \ AB]^{-1}$$

$$= \begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}^{-1}$$

$$= - \begin{bmatrix} 0 & -1 \\ -1 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 1 \\ 1 & -1 \end{bmatrix}$$

$$AB = \begin{bmatrix} 1 & 0 \\ -1 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

$$\alpha_c(z) = z^2 = A^2 = \begin{bmatrix} 1 & 0 \\ -1 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ -1 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ -2 & 1 \end{bmatrix}$$

$$K = [0 \ 1] \begin{bmatrix} 0 & 1 \\ 1 & -1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ -2 & 1 \end{bmatrix} = [1 \ -1] \begin{bmatrix} 1 & 0 \\ -2 & 1 \end{bmatrix} = [3 \ -1]$$

$$[A - BK] = \begin{bmatrix} 1 & 0 \\ -1 & 1 \end{bmatrix} - \begin{bmatrix} 1 \\ 1 \end{bmatrix} [3 \ -1]$$

$$= \begin{bmatrix} 1 & 0 \\ -1 & 1 \end{bmatrix} - \begin{bmatrix} 3 & -1 \\ 3 & -1 \end{bmatrix}$$

$$= \begin{bmatrix} -2 & 1 \\ -4 & 2 \end{bmatrix}$$

$$\exists q > 1, [A - BK]^q = 0, x(k) = [A - BK]^q x(0)$$

$$[A - BK]^2 = \begin{bmatrix} -2 & 1 \\ -4 & 2 \end{bmatrix} \begin{bmatrix} -2 & 1 \\ -4 & 2 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} = 0 \quad q=2.$$

(ii)  $\hat{x}_1(k)$  是什么? ~~还没学到~~ ~~尝试写~~

$$\begin{bmatrix} \hat{x}_1(k) \\ \hat{x}_2(k) \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} x_1(k) \\ x_2(k) \end{bmatrix}$$

$$u(k) = -\hat{k} \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} x_1(k) \\ x_2(k) \end{bmatrix}$$

状态转移矩阵

定义  $x(k) = p w(k)$  ... lecture 2-5-p4

$$x(k) = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}^{-1} \hat{x}(k)$$

$$p = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}^{-1} = \begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix}$$

$$A_w = p^{-1} A p = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ -1 & 1 \end{bmatrix} \begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 1 \\ -1 & 1 \end{bmatrix} \begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 1 \\ -1 & 2 \end{bmatrix}$$

$$B_w = P^{-1}B = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$$

$$C_w = CP = \begin{bmatrix} 1 & 1 \end{bmatrix} \begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \end{bmatrix}$$

$$\hat{x}(k+1) = \begin{bmatrix} 0 & 1 \\ -1 & 2 \end{bmatrix} \hat{x}(k) + \begin{bmatrix} 2 \\ 1 \end{bmatrix} u(k)$$

$$\hat{y}(k) = \begin{bmatrix} 1 & 0 \end{bmatrix} \hat{x}(k)$$

$$\hat{K} = \begin{bmatrix} 0 & 1 \end{bmatrix} \hat{W}_c^{-1} \alpha_c(A_w)$$

$$\hat{W}_c^{-1} = [B_w \ A_w B_w]^{-1} = \begin{bmatrix} 2 & 1 \\ 1 & 0 \end{bmatrix}^{-1} = - \begin{bmatrix} 0 & -1 \\ -1 & 2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 1 & -2 \end{bmatrix}$$

$$A_w B_w = \begin{bmatrix} 0 & 1 \\ -1 & 2 \end{bmatrix} \begin{bmatrix} 2 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

$$\alpha_c(z) = z^2$$

$$\alpha_c(A_w) = A_w^2 = \begin{bmatrix} 0 & 1 \\ -1 & 2 \end{bmatrix} \begin{bmatrix} 0 & 1 \\ -1 & 2 \end{bmatrix} = \begin{bmatrix} -1 & 2 \\ -2 & 3 \end{bmatrix}$$

$$\hat{K} = \begin{bmatrix} 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & 1 \\ 1 & -2 \end{bmatrix} \begin{bmatrix} -1 & 2 \\ -2 & 3 \end{bmatrix}$$

$$= [1 \ -2] \begin{bmatrix} -1 & 2 \\ -2 & 3 \end{bmatrix}$$

$$= [3 \ -4]$$

(b) Q  $\frac{\bar{x}(z)}{u(z)} = ?$

Solution ~ ①  $\bar{x}(k+1) = A\bar{x}(k) + Bu(k) + L_0(y(k) - C\bar{x}(k))$

$$y(k) = Cx(k)$$

$$\bar{x}(k+1) = (A - L_0C)\bar{x}(k) + Bu(k) + L_0Cx(k)$$

apply Z transform

$$z\bar{X}(z) = (A - L_0C)\bar{X}(z) + BU(z) + L_0CX(z)$$

$$(zI - A + L_0C)\bar{X}(z) = BU(z) + L_0CX(z)$$

②  $x(k+1) = Ax(k) + Bu(k)$  apply Z transform

$$zX(z) = AX(z) + BU(z) \quad \therefore X(z) = (zI - A)^{-1} BU(z)$$

③  $(zI - A + L_0C)\bar{X}(z) = BU(z) + L_0C(zI - A)^{-1} BU(z)$

$$\frac{\bar{X}(z)}{u(z)} = \frac{B + L_0C(zI - A)^{-1}B}{zI - A + L_0C}$$

矩阵求逆法

$$(zI - A + L_0C)^{-1} [B + L_0C(zI - A)^{-1}B]$$

c) optimal feedback gains minimize  $J$   
 obtain values of  $u^*(k)$  as a function of  $x(0)$   
 $k=0,1,2$

Solution  $A=5$   $B=1$   $N=3$   $Q=8$   $r=1$

initialize  $S(3)=0$

$$\text{let } k=2 \quad k(2) = (1 \times 0 \times 1 + 1)^{-1} \times 1 \times 0 \times 5 = 0$$

$$S(2) = [5 - k(2)]^T \times 0 \times [5 - k(2)] + k(2)^2 + 8 \\ = 8$$

$$\text{let } k=1 \quad k(1) = (1 \times 8 \times 1 + 1)^{-1} \times 1 \times 8 \times 5 \\ = \frac{40}{9} = 4.4444$$

$$S(1) = (5 - 1 \times \frac{40}{9}) \times 8 \times (5 - 1 \times \frac{40}{9}) + (\frac{40}{9})^2 + 8 \\ = \frac{272}{9} = 30.2222$$

$$\text{let } k=0 \quad k(0) = (\frac{272}{9} + 1)^{-1} \times \frac{272}{9} \times 5 \\ = \frac{1360}{281} = 4.8399$$

$$S(0) = (5 - \frac{1360}{281})^2 \times \frac{272}{9} + (\frac{1360}{281})^2 + 8 \\ = \frac{9048}{281} = 32.1993$$

$$u^*(0) = -k(0) x(0) = -\frac{1360}{281} x(0) = -4.8399 x(0)$$

$$x^*(1) = Ax(0) + B \hat{u}(0) = 5x(0) + \hat{u}(0)$$

$$= \frac{45}{281} x(0)$$

$$= 0.1601 x(0)$$

$$u^*(1) = -k(1) x(1) = -\frac{44}{9} \times \frac{45}{281} x(0)$$

$$= -\frac{220}{281} x(0)$$

$$= -0.7829 x(0)$$

$$x^*(2) = 5x(1) + u^*(1)$$

$$= \left( 5 \times \frac{45}{281} - \frac{220}{281} \right) x(0)$$

$$= \frac{5}{281} x(0)$$

$$= 0.01779 x(0)$$

$$u^*(2) = -k(2) x(2) = 0$$

Summary  $u^*(0) = -4.8399 x(0)$

$$u^*(1) = -0.7829 x(0)$$

$$u^*(2) = 0$$