18-S1-Q5

Q(a) G(s) $\omega_n = 5$ $\zeta = 0.5$ pole-zero? $T=1$

Solution ① $G(s) = \dfrac{25}{s^2+5s+25}$

② $\omega_d = \omega_n\sqrt{1-\zeta^2} = 5\times\sqrt{1-\frac{1}{4}} = \dfrac{5\sqrt{3}}{2} = 4.3301$

③ $s_{1,2} = \dfrac{-5 \pm 5\sqrt{3}j}{2}$ $s_1 = -2.5+4.3301j$

$s_2 = -2.5-4.3301j$

④ $G(z) = \alpha\dfrac{(z+1)^{n-m-1}}{(z-e^{s_1T})(z-e^{s_2T})}$

$n-m-1 = 2-0-1 = 1$

$(z-e^{s_1T})(z-e^{s_2T})$

$= z^2 - z(e^{s_2T}+e^{s_1T}) + e^{s_1T}e^{s_2T}$

where $e^{s_1T}+e^{s_2T} = e^{-2.5}e^{-4.3301j} + e^{-2.5}e^{4.3301j}$

$= e^{-2.5}(e^{-4.3301j}+e^{4.3301j})$

$= e^{-2.5}\,2\cos(4.3301)$

$= -0.06124$

$e^{s_1T}e^{s_2T} = e^{-5} = 0.006738$

$$z^2 + 0.06124z + 0.006738$$

$$G_D(z) = \alpha \frac{(z+1)}{z^2 + 0.06124z + 0.006738}$$

$$G_D(1) = G(0) = \frac{25}{s^2+5s+25}\Big|_{s=0} = \frac{25}{25} = 1$$

$$\alpha \frac{2}{1 + 0.06124 + 0.006738} = 1$$

$$\alpha = 0.5340$$

$$G_D(z) = 0.534 \frac{z+1}{z^2 + 0.06124z + 0.006738}$$

(b) Q: C(z)

Solution $G(z) = \frac{Y(z)}{G_{ZAS}(z)} = G_{CL}(z) \frac{R(z)}{G_{ZAS}(z)}$

? 추가