18-S1 - Q1

Q $x(kT) = \begin{cases} kT & , k \geq 0 \\ 0 & , k < 0 \end{cases}$

$$y(kT) = \sum_{h=0}^{k} x(hT) = x(0) + x(T) + x(2T) + \cdots + x(kT)$$

(a) Q $Z \rightarrow y(kT)$? IVT?

Solution ① $y((k-1)T) = \sum_{h=0}^{k-1} x(hT)$

$$y(kT) - y((k-1)T) = x(kT) = kT$$

apply Z transform #5

$$Y(z) - z^{-1}Y(z) = \frac{Tz^{-1}}{(1-z^{-1})^2}$$

$$(1-z^{-1})Y(z) = \frac{Tz^{-1}}{(1-z^{-1})^2}$$

$$Y(z) = \frac{Tz^{-1}}{(1-z^{-1})^3}$$

② IVT $\quad y(0) = \lim_{z\to\infty} Y(z) = \lim_{z\to\infty} \frac{Tz^{-1}}{(1-z^{-1})^3} = \lim_{z\to\infty} \frac{Tz^2}{(z-1)^3} = 0$

So $\lim_{z\to\infty} X(z)$ exists, we can apply Initial Value Theorem

(b) Q $T=1$ inverse Z transform? PFE

① $\frac{X(z)}{z} = \frac{A}{z-1} + \frac{Bz}{z^2-\sqrt{2}z+1} + \frac{C}{z^2-\sqrt{2}z+1}$

$$A(z^2-\sqrt{2}z+1) + Bz(z-1) + C(z-1)$$

$$= Az^2 - \sqrt{2}Az + A + Bz^2 - Bz + Cz - C$$

$$= (A+B)z^2 + (-\sqrt{2}A - B + C)z + A - C$$

$$\begin{cases} A+B=0 \\ -\sqrt{2}A - B + C = 0 \\ A - C = 1 \end{cases} \qquad \begin{cases} A = \dfrac{2+\sqrt{2}}{2} \\ B = -\dfrac{2+\sqrt{2}}{2} \\ C = \dfrac{\sqrt{2}}{2} \end{cases}$$

$C = A - 1$

$B = -A$

$-\sqrt{2}A + A + A - 1 = 0$

$(2-\sqrt{2})A = 1$

$$A = \frac{1}{2-\sqrt{2}} = \frac{2+\sqrt{2}}{2}$$

$$\frac{X(z)}{z} = \frac{\frac{2+\sqrt{2}}{2}}{z-1} + \frac{-\frac{2+\sqrt{2}}{2}z}{z^2-\sqrt{2}z+1} + \frac{\frac{\sqrt{2}}{2}}{z^2-\sqrt{2}z+1}$$

$$X(z) = \frac{2+\sqrt{2}}{2}\,\frac{1}{1-z^{-1}} - \frac{2+\sqrt{2}}{2}\,\frac{1}{1-\sqrt{2}z^{-1}+z^{-2}} + \frac{\sqrt{2}}{2}\,\frac{z^{-1}}{1-\sqrt{2}z^{-1}+z^{-2}}$$

#18, $a = 1$

#14, $2\cos\omega T = \sqrt{2}$, $\cos\omega T = \frac{\sqrt{2}}{2}$  $\omega T = \frac{\pi}{4}$ $\sin\omega T = \frac{\sqrt{2}}{2}$

$$\frac{\frac{\sqrt{2}}{2}z^{-1}}{1-\sqrt{2}z^{-1}+z^{-2}} \xrightarrow{z^{-1}} \sin\frac{\pi}{4}k$$

#6 $$\frac{1-\frac{\sqrt{2}}{2}z^{-1}}{1-\sqrt{2}z^{-1}+z^{-2}} \xrightarrow{z^{-1}} \cos\frac{\pi}{4}k$$

$$x(kT) = \frac{2+\sqrt{2}}{2} - \frac{2+\sqrt{2}}{2}\left(\sin\frac{\pi}{4}k + \cos\frac{\pi}{4}k\right) + \sin\frac{\pi}{4}k$$

$$= \frac{2+\sqrt{2}}{2} - \frac{\sqrt{2}}{2}\sin\frac{\pi}{4}k - \frac{2+\sqrt{2}}{2}\cos\frac{\pi}{4}k$$

$= 1.7071 - 0.7071 \sin \frac{2}{4}k - 1.7071 \cos \frac{2}{4}k$

(c) FVT?

$$X(z) = \frac{z}{(z-1)(z^2 - \sqrt{2}z + 1)}$$

poles at $z_1 = 1$ $\quad z_{2,3} = \frac{\sqrt{2} \pm \sqrt{2}j}{2}$ $\quad |z_{2,3}| = \sqrt{\left(\frac{\sqrt{2}}{2}\right)^2 + \left(\frac{\sqrt{2}}{2}\right)^2} = 1$

the three poles are all in the unit circle which is not satisfy FVT. So the student is incorrect.