15-S1-1

Q: $G(s) = \frac{C(s)}{E(s)} = \frac{s+1}{s+3}$ $T=1$

forward difference method

input $e(kT)$ unit-step   output $c(kT) = ?$

(a) Q: output?

Solution ① forward ~ ref 5.2.2.2

$s = \frac{z-1}{T}$

② apply z transform

$$G(z) = \frac{C(z)}{E(z)} = \frac{\frac{z-1}{T}+1}{\frac{z-1}{T}+3} \qquad T=1$$

$$G(z) = \frac{C(z)}{E(z)} = \frac{z}{z+2}$$

③ $E(z) = \frac{1}{1-z^{-1}} = \frac{z}{z-1}$

$$C(z) = G(z)E(z) = \frac{z}{z+2}\,\frac{z}{z-1}$$

$$= \frac{z^2}{(z+2)(z-1)}$$

$$\frac{C(z)}{z} = \frac{z}{(z+2)(z-1)}$$

$$= \frac{A}{z+2} + \frac{B}{z-1}$$

$$A(z-1) + B(z+2) = (A+B)z - A + 2B$$

$$\begin{cases} A+B=1 \\ -A+2B=0 \end{cases} \Rightarrow \begin{cases} A = \frac{2}{3} \\ B = \frac{1}{3} \end{cases}$$

$$C(z) = \frac{2}{3} \frac{1}{1+2z^{-1}} + \frac{1}{3} \frac{1}{1-z^{-1}}$$

#18

$$C(kT) = \frac{2}{3}(-2)^k + \frac{1}{3} 1(k)$$

$$C(0) = \frac{2}{3} + \frac{1}{3} = 1$$

$$C(1) = \frac{2}{3} \times (-2) + \frac{1}{3} = -1$$

$$C(2) = \frac{8+1}{3} = 3$$

$$C(3) = \frac{-16+1}{3} = -5$$

(b) series programing ?

Solution

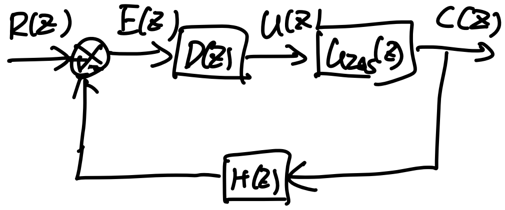
$$D(z) = \frac{z-0.5}{(z-1)(z+0.3)}$$

$$= \frac{z-0.5}{z^2-0.7z-0.3}$$

$$= \frac{(1-0.5z^{-1})z^{-1}}{1-0.7z^{-1}-0.3z^{-2}}$$

$$= \frac{z^{-1}-0.5z^{-2}}{1-0.7z^{-1}-0.3z^{-2}}$$

$$D(Z) = \frac{U(Z)}{E(Z)} = \frac{U(Z)}{H(Z)} \cdot \frac{H(Z)}{E(Z)}$$



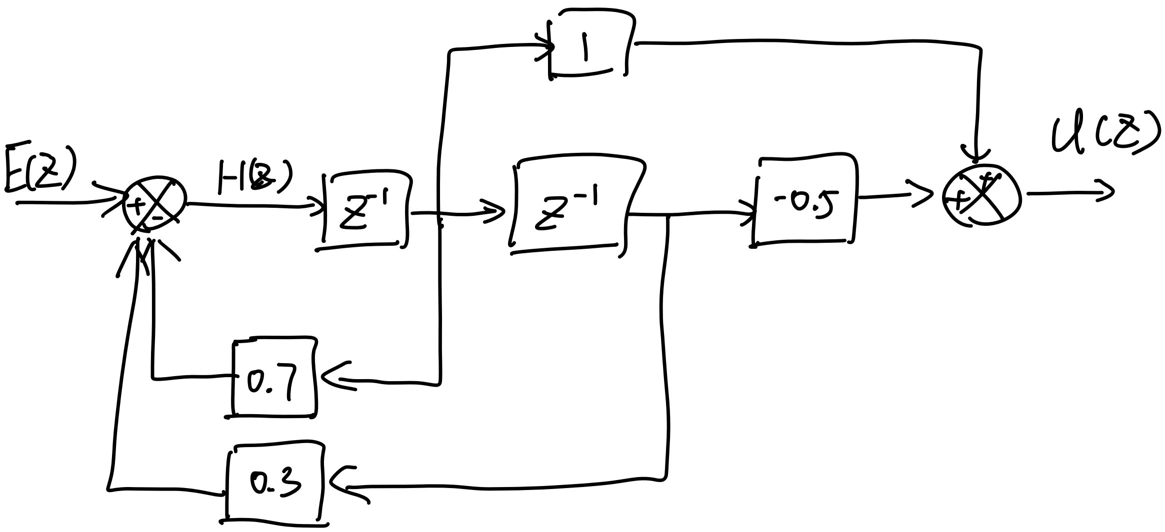
$$\frac{U(Z)}{H(Z)} = z^{-1} - 0.5\, z^{-2}$$

$$U(Z) = z^{-1} H(Z) - 0.5\, z^{-2} H(Z)$$

$$\frac{H(Z)}{E(Z)} = 1 - 0.7\, z^{-1} - 0.3\, z^{-2}$$

$$H(Z) = E(Z) - 0.7\, z^{-1} E(Z) - 0.3\, z^{-2} E(Z)$$

$$E(Z) = H(Z) + 0.7\, z^{-1} E(Z) + 0.3\, z^{-2} E(Z)$$



(c) backward?

Solution ① $s = \dfrac{z-1}{Tz}$

② $$G(Z) = \frac{C(Z)}{E(Z)} = \frac{\frac{z-1}{Tz} + 1}{\frac{z-1}{Tz} + 3}$$

$$\stackrel{T=1}{=} \frac{\frac{z-1}{z}+1}{\frac{z-1}{z}+3}$$

$$= \frac{z-1+z}{z-1+3z}$$

$$= \frac{2z-1}{4z-1}$$

③ $E(z) = \frac{1}{1-z^{-1}} = \frac{z}{z-1}$

$$C(z) = G(z)E(z) = \frac{2z-1}{4z-1} \cdot \frac{z}{z-1}$$

$$= \frac{(2z-1)z}{(4z-1)(z-1)}$$

$$\frac{C(z)}{z} = \frac{2z-1}{(4z-1)(z-1)}$$

$$= \frac{A}{4z-1} + \frac{B}{z-1}$$

$$A(z-1) + B(4z-1)$$

$$= (A+4B)z - A - B$$

$$\begin{cases} A+4B = 2 \\ -A-B = -1 \end{cases} \Rightarrow \begin{cases} A = 1 \\ B = \frac{1}{4} \end{cases}$$

$4B = 1 \qquad A = 2 - 4 \times \frac{1}{4} = 2 - 1 = 1$

$$C(z) = \frac{1}{4 - z^{-1}} + \frac{1}{4} \frac{1}{1 - z^{-1}}$$

$$= \frac{1}{4} \frac{1}{1 - \frac{1}{4} z^{-1}} + \frac{1}{4} \frac{1}{1 - z^{-1}}$$

$$C(kT) = \left(\frac{1}{4}\right)^{k+1} + \frac{1}{4} 1(k)$$

$$C(0) = \frac{1}{4} + \frac{1}{4} = \frac{1}{2} = 0.5$$

$$C(1) = \frac{1}{16} + \frac{1}{4} = \frac{1+4}{16} = \frac{5}{16} = 0.3125$$

$$C(2) = \left(\frac{1}{4}\right)^3 + \frac{1}{4} = \frac{17}{64} = 0.2656$$

$$C(3) = \left(\frac{1}{4}\right)^4 + \frac{1}{4} = \frac{65}{256} = 0.2539$$