23-S1-Q2

Q: $G_{ZAS}(z) = \dfrac{0.5z+0.2}{(z-0.5)(z-0.4)}$

(a) PI $C(z) = 1 + \dfrac{1}{1-z^{-1}}$ stable? e_{ss}?

Solution $G_{cl}(z) = \dfrac{C(z)\,G_{ZAS}(z)}{1 + C(z)G_{ZAS}(z)}$

$$C(z) = \frac{2-z^{-1}}{1-z^{-1}} = \frac{2z-1}{z-1}$$

$$C(z)G_{ZAS}(z) = \frac{(2z-1)(0.5z+0.2)}{(z-1)(z-0.5)(z-0.4)}$$

$$= \frac{(z+0.4)}{(z-1)(z-0.4)}$$

x 不能约分, ref. PPT 19

$G_{cl}(z) = \dfrac{G(z)}{1+G(z)H(z)}$

CE: $1+G(z)H(z)=0$

x

$$G_{cl}(z) = \frac{\dfrac{(z+0.4)}{(z-1)(z-0.4)}}{\dfrac{(z-1)(z-0.4)+(z+0.4)}{(z-1)(z-0.4)}}$$

$$= \frac{z+0.4}{z^2-1.4z+0.4+z+0.4}$$

x

$$= \frac{z+0.4}{z^2-0.4z+0.8}$$

$$C(z)G_{ZAS}(z) = \frac{(2z-1)(0.5z+0.2)}{(z-1)(z-0.5)(z-0.4)}$$

$$G_{cl}(z) = \frac{C(z)G_{ZAS}(z)}{1+C(z)G_{ZAS}(z)}$$

CE:

$$1 + \frac{(2z-1)(0.5z+0.2)}{(z-1)(z-0.5)(z-0.4)} = 0$$

$$(z-1)(z^2-0.9z+0.2) + (z^2+0.4z-0.5z-0.2)$$

$$= z^3 - 0.9z^2 + 0.2z - z^2 + 0.9z - 0.2$$

$$+z^2 - 0.1z - 0.2$$

$$= z^3 \quad -0.9z^2 + z - 0.4 = 0$$

Jury Test

z^0 (a_3)	z^1	z^2	z^3 (a_0)
-0.4	1	-0.9	1
1	-0.9	1	-0.4
-0.84	0.5	-0.62	

$$\begin{cases} |-0.4| < 1 & \checkmark \\ P(1) = 1 - 0.9 + 1 - 0.4 = 0.7 > 0 & \checkmark \\ P(-1) = -1 - 0.9 - 1 - 0.4 < 0 & \checkmark \quad n = 3 \quad 0 \end{cases}$$

$|b_2| > |b_0|$ $\quad 0.84 > 0.62$ ✓ $\quad$ stable

e_{st} 稳态跟踪误差：推导在 understand - 1

$$e_{ss} = \lim_{z\to 1}\left[(1-z^{-1})\frac{1}{1+C(z)G_{ZAS}(z)}R(z)\right]$$

$$R(z) = \frac{1}{1-z^{-1}}$$

$$e_{ss} = \lim_{z\to 1}\frac{1}{1+\frac{(2z-1)(0.5z+0.2)}{(z-1)(z-0.5)(z-0.4)}}$$

$$= 0$$

$$e_{st} = \lim_{z\to 1}(1-z^{-1})R(z)(1-G_{cl}(z))$$

$$= \lim_{z\to 1}(1-z^{-1})\frac{1}{1-z^{-1}}\left(1-\frac{z+0.4}{z^2-0.4z+0.8}\right) = 1-\frac{1.4}{1-0.4+0.8}$$

$$= 0$$

(b) $R(z) = \dfrac{1}{1-z^{-1}}$

$$U(z) = G_{cl}(z)\frac{R(z)}{G_{ZAS}(z)}$$

$$G_{ZAS}(z) = \frac{0.5z+0.2}{(z-0.5)(z-0.4)}$$

$$= \frac{(0.5+0.2z^{-1})z^{-1}}{(1-0.5z^{-1})(1-0.4z^{-1})}$$

$$U(z) = G_{cl}(z)\frac{(1-0.5z^{-1})(1-0.4z^{-1})}{(1-z^{-1})(0.5+0.2z^{-1})z^{-1}}$$

$$G_{cl}(z) = Kz^{-1}(0.5+0.2z^{-1})$$

$$G_{cl}(1) = K(0.5+0.2) = 0.7K = 1$$

$$\Rightarrow K = 1.4286$$

$$C(z) = \frac{1}{G_{ZAS}(z)} \frac{G_{cl}(z)}{1-G_{cl}(z)}$$

$$= \frac{(z-0.5)(z-0.4)}{0.5z+0.2} \frac{1.4286z^{-1}(0.5+0.2z^{-1})}{1-1.4286z^{-1}(0.5+0.2z^{-1})}$$

$$= \frac{(z-0.5)(z-0.4)}{0.5z+0.2} \frac{1.4286\ (0.5z+0.2)}{z^2-1.4286(0.5z\ +0.2)}$$

$$= \frac{(z-0.5)(z-0.4)1.4286(0.5z+0.2)}{(0.5z+0.2)(z^2-0.78573z-0.28572)}$$

$$= \frac{1.4286(z-0.5)(z-0.4)}{(z-1)(z+0.2857)}$$

(c) $$G_{ZAS}(z) = \frac{0.5z+0.2}{(z-1.5)(z-0.4)}$$

This plant has an unstable pole at $z=1.5$

not possible, because unstable poles

cannot be canceled using a causal controller