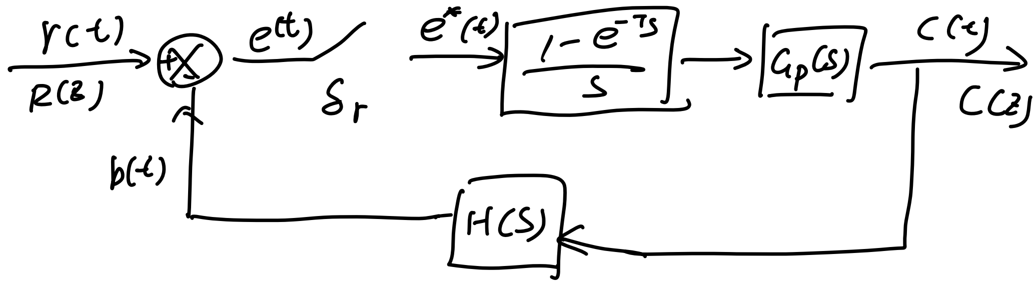
Q: est ≤ ess



Solution ① $e_{ss} = \lim_{k\to\infty} e(kT)$

$$= \lim_{z\to 1} (1-z^{-1}) E(z)$$

where $E(z) = R(z) - B(z)$

$$G_{ZOH} G_p G_H(z) = \frac{B(z)}{E(z)} = GH(z)$$

$$B(z) = GH(z)E(z)$$

$$E(z) = R(z) - GH(z)E(z)$$

$$(1+GH(z))E(z) = R(z)$$

$$E(z) = \frac{R(z)}{1+GH(z)}$$

So $e_{ss} = \lim_{z\to 1} (1-z^{-1}) E(z)$

$$= \lim_{z\to 1} (1-z^{-1}) \frac{R(z)}{1+GH(z)}$$

② $$e_{st} = \lim_{k\to\infty}\left(r(t) - c(t)\right)$$

$$= \lim_{z\to 1}(1-z^{-1})\left(R(z) - C(z)\right)$$

where $$G_{cl}(z) = \frac{C(z)}{R(z)}$$

$$C(z) = R(z)\,G_{cl}(z)$$

$$e_{st} = \lim_{z\to 1}(1-z^{-1})\left(R(z) - R(z)\,G_{cl}(z)\right)$$

$$= \lim_{z\to 1}(1-z^{-1})R(z)\left(1 - G_{cl}(z)\right)$$

稳态跟踪误差est：steady state tracking error

$$e_{st} = \lim_{k\to\infty}\left(r(k) - \overset{c}{\cancel{b}}(k)\right) = \lim_{z\to 1}\left((1-z^{-1})R(z)[1-G_{cl}(z)]\right)$$

稳态误差ess：steady-state error

$$e_{ss} = \lim_{z\to 1}\left[(1-z^{-1})\frac{1}{1+GH(z)}R(z)\right]$$

直接R-C是跟踪 R-B是驱动
一个是减开环传函，一个是减闭环传函
反馈传函为1的话，两者一样，才能共用公式

$$GH(z) = (1-z^{-1})\mathcal{Z}\left[\frac{G_P(s)H(s)}{s}\right]$$

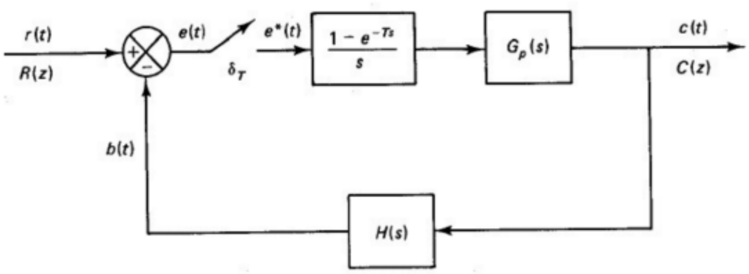


Figure 4-14: Discrete-time control system

$$E(z) = \frac{1}{1+GH(z)}R(z)$$

因为在H(s)=1的情况下，ess=est，以下有证明过程

The above analysis applies to the system in Figure 4-14.

上述分析适用于图4-14中的系统。

It is important to note that the above $E(z)$ is the actuating error $E(z) = R(z) - B(z)$. This is different from the tracking error $R(z) - C(z)$!

需要注意的是，上述E(z)是驱动误差E(z) =R(z)-B(z)。
这与跟踪误差R(z) - C(z)不同！

For other system configurations where the sampler(s) are placed differently, the results have to be modified. A few examples are given in Table 4-5.

对于放置不同采样器的其他系统配置，必须修改结果。
如表4-5所示。

Q

证明 $H(s) = 1$ 时 $e_{ss} = e_{st}$

Solution

$$e_{ss} = \lim_{z \to 1} (1 - z^{-1}) \frac{R(z)}{1 + GH(z)}$$

$$e_{st} = \lim_{z \to 1} (1 - z^{-1}) R(z) (1 - G_{cl}(z))$$

即证明 $\frac{1}{1 + GH(z)} = 1 - G_{cl}(z)$，即 $\frac{1}{1 + G_{zAS}(z)} = 1 - G_{cl}(z)$

$$G_{cl}(z) = \frac{C(z)}{R(z)}$$

$$G_{zAS}(z) = \frac{C(z)}{E(z)} = \frac{C(z)}{R(z) - B(z)} = \frac{C(z)}{R(z) - H(z)C(z)}$$

$$H(z) = \frac{B(z)}{C(z)}$$

$$C(z) = G_{ZAS}(z)R(z) - GH(z)C(z)$$

$$[1+GH(z)]C(z) = G_{ZAS}(z)\ R(z)$$

$$G_{cl}(z) = \frac{C(z)}{R(z)} = \frac{G_{ZAS}(z)}{1+GH(z)}$$

$H(s)=1$, so, $G_{cl}(z) = \dfrac{G_{ZAS}(z)}{1+G_{ZAS}(z)}$

$$1 - G_{cl}(z) = \frac{1+G_{ZAS}(z) - G_{ZAS}(z)}{1+G_{ZAS}(z)} = \frac{1}{1+G_{ZAS}(z)}$$

证明完成