部分分式展开(PFE)方法

- ***Partial Fraction Expansion (PFE) Method***:

Consider
$$X(z)=\frac{b_0z^m+b_1z^{m-1}+\cdots+b_m}{z^n+a_1z^{n-1}+\cdots+a_n}\qquad (m\le n)$$

Factorize it as
$$X(z)=\frac{b_0z^m+b_1z^{m-1}+\cdots+b_m}{(z-p_1)(z-p_2)\cdots\cdots(z-p_n)}$$

将其分解为

Then, do PFE for $X(z)/z$. If it has only simple poles, we have

然后，做PFE

只有一个极点

$$\frac{X(z)}{z}=\frac{a_0}{z}+\frac{a_1}{z-p_1}+\ldots.+\frac{a_n}{z-p_n}$$

$$X(z)=a_0+\frac{a_1z}{z-p_1}+\frac{a_2z}{z-p_2}+\cdots+\frac{a_nz}{z-p_n}$$

$$X(z)=a_0+\frac{a_1}{1-p_1z^{-1}}+\frac{a_2}{1-p_2z^{-1}}+\cdots+\frac{a_n}{1-p_nz^{-1}}$$

Hence $x(k)=a_0\delta(k)+a_1p_1^k+a_2p_2^k+\cdots+a_np_n^k$ (2-21)

例如，如果X(z)/z涉及多个极点，

If $X(z)/z$ involves multiple poles, for example,

$$\frac{X(z)}{z}=\frac{b_0z+b_1}{(z-p)^2}=\frac{\overbrace{b_0}^{c_2}(z-p)+\overbrace{b_1+b_0p}^{c_1}}{(z-p)^2}$$

then
$$\frac{X(z)}{z}=\frac{c_1}{(z-p)^2}+\frac{c_2}{z-p}$$

分母二次方

请一定拆为两项

$$X(z)=\frac{c_1z^{-1}}{(1-pz^{-1})^2}+\frac{c_2}{1-pz^{-1}}$$

$$x(k)=c_1\mathcal{Z}^{-1}\left[\frac{z^{-1}}{(1-pz^{-1})^2}\right]+c_2\mathcal{Z}^{-1}\left[\frac{1}{1-pz^{-1}}\right]$$

From Table 2-1,
$$x(k)=c_1kp^{k-1}+c_2p^k\qquad k\ge 0\qquad (2\text{-}22)$$

i.e. $x(0)=c_2,\ x(1)=c_1+c_2p, x(2)=2c_1p+c_2p^2,\ \ldots$