20-S1-Q4

Q: (a) K ?

$$\alpha_c(z) = [z-(0.3+j0.6)][z-(0.3-j0.6)]$$

$$= z^2 - 0.6z + 0.3^2 + 0.6^2$$

$$= z^2 - 0.6z + 0.45$$

$$\alpha_c(A) = A^2 - 0.6A + 0.45 I_2$$

$$= \begin{bmatrix} 0 & 1 \\ -0.16 & -1 \end{bmatrix}\begin{bmatrix} 0 & 1 \\ -0.16 & -1 \end{bmatrix} - 0.6\begin{bmatrix} 0 & 1 \\ -0.16 & -1 \end{bmatrix} + 0.45\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 0.29 & -1.6 \\ 0.256 & 1.89 \end{bmatrix}$$

$$W_c = [B \;\; AB] = \begin{bmatrix} 0 & 1 \\ 1 & -1 \end{bmatrix}$$

$$AB = \begin{bmatrix} 0 & 1 \\ -0.16 & -1 \end{bmatrix}\begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$$

$$W_c^{-1} = -\begin{bmatrix} -1 & -1 \\ -1 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}$$

$$K = [0 \;\; 1]\, W_c^{-1}\, \alpha_c(A)$$

$$= [0 \;\; 1]\begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}\begin{bmatrix} 0.29 & -1.6 \\ 0.256 & 1.89 \end{bmatrix}$$

$$= [1 \;\; 0]\begin{bmatrix} 0.29 & -1.6 \\ 0.256 & 1.89 \end{bmatrix}$$

$$= [0.29 \;\; -1.6]$$

(b) first-order observer

Solution

$u(k) = -[0.29 \ \ -1.6]x(k) = [-0.29 \ \ 1.6]\, x(k)$

$$\begin{bmatrix} x_1(k+1) \\ x_2(k+1) \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -0.16 & -1 \end{bmatrix} \begin{bmatrix} x_1(k) \\ x_2(k) \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} [-0.29 \ \ 1.6] \begin{bmatrix} x_1(k) \\ x_2(k) \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 1 \\ -0.16 & -1 \end{bmatrix} \begin{bmatrix} x_1(k) \\ x_2(k) \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ -0.29 & 1.6 \end{bmatrix} \begin{bmatrix} x_1(k) \\ x_2(k) \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 1 \\ -0.45 & 0.6 \end{bmatrix} \begin{bmatrix} x_1(k) \\ x_2(k) \end{bmatrix}$$

$$y(k) = [1 \ \ 0] \begin{bmatrix} x_1(k) \\ x_2(k) \end{bmatrix} = x_1(k)$$

$$\tilde{x}_2(k+1) = u(k) - 0.16\, y(k) - y(k+1)$$

$$\tilde{x}_2(k) = u(k-1) - 0.16\, y(k-1) - y(k) \qquad (1)$$

$$u(k) = [-0.29 \ \ 1.6] \begin{bmatrix} x_1(k) \\ x_2(k) \end{bmatrix} = -0.29\, x_1(k) + 1.6\, \tilde{x}_2(k)$$

$$= -0.29\, y(k) + 1.6\, \tilde{x}_2(k) \qquad (2)$$

(1) 代入 (2)

$$u(k) = -0.29\, y(k) + 1.6\, u(k-1) - 0.256\, y(k-1) - 1.6\, y(k)$$

$$u(k) - 1.6\, u(k-1) = -1.89\, y(k) - 0.25\, y(k-1)$$

apply Z transform

$$U(z) - 1.6z^{-1}U(z) = -1.89Y(z) - 0.25z^{-1}Y(z)$$

$$(1-1.6z^{-1})U(z) = (-1.89 - 0.25z^{-1})Y(z)$$

$$\frac{U(z)}{Y(z)} = \frac{-1.89 - 0.25z^{-1}}{1-1.6z^{-1}}$$

(c) optimal control ?

Solution $A = 0.8$ $B = 1$ $N = 3$ $Q = 4$ $r = 2$

$S(3) = 0$

let $k = 2$

$K(2) = (S(3) + 2)^{-1} \times S(3) \times 0.8 = 0$

$S(2) = [0.8 - 1 \times 0] \, S(3) \, [0.8 - 1 \times 0] + 2 \times 0 \times 0 + 4 = 4$

let $k = 1$

$K(1) = (1 \times 4 \times 1 + 2)^{-1} \times 1 \times 4 \times 0.8 = \frac{8}{15} = 0.5333$

$S(1) = (0.8 - 1 \times \frac{8}{15}) \times 4 \times (0.8 - 1 \times \frac{8}{15}) + 2 \times (\frac{8}{15})^2 + 4$

$= \frac{364}{75} = 4.8533$

let $k = 0$

$K(0) = (1 \times \frac{364}{75} \times 1 + 2)^{-1} \times 1 \times \frac{364}{75} \times 0.8 = \frac{728}{1285} = 0.5665$

$S(0) = (0.8 - 1 \times \frac{728}{1285})^2 \times \frac{364}{75} + 2 \times (\frac{728}{1285})^2 + 4$

$= 4.9065$

So the optimal gain schedule is

$K(0) = 0.5665 \qquad K(1) = 0.5333$