19-S1-Q3

Q: (a) discretised?

Solution ① state transform matrix

$$[sI-A]^{-1} = \begin{bmatrix} s & -1 \\ 2 & s+3 \end{bmatrix}^{-1} = \frac{1}{s(s+3)+2}\begin{bmatrix} s+3 & 1 \\ -2 & s \end{bmatrix}$$

$$= \frac{1}{s^2+3s+2}\begin{bmatrix} s+3 & 1 \\ -2 & s \end{bmatrix}$$

$$= \frac{1}{(s+1)(s+2)}\begin{bmatrix} s+3 & 1 \\ -2 & s \end{bmatrix}$$

$$= \begin{bmatrix} \frac{s+3}{(s+1)(s+2)} & \frac{1}{(s+1)(s+2)} \\ \frac{-2}{(s+1)(s+2)} & \frac{s}{(s+1)(s+2)} \end{bmatrix}$$

table #9

$$\frac{b-a}{(s+a)(s+b)} = \frac{1}{(s+1)(s+2)}$$ we get $e^{-t} - e^{-2t}$

$$\frac{s}{(s+1)(s+2)} = \frac{A}{s+1} + \frac{B}{s+2}$$ PFE

$$A(s+2) + B(s+1) = (A+B)s + 2A + B$$

$$\begin{cases} A+B = 1 \\ 2A+B = 0 \end{cases} \Rightarrow \begin{cases} A = -1 \\ B = 2 \end{cases}$$

$$\frac{s}{(s+1)(s+2)} = \frac{-1}{s+1} + \frac{2}{s+2}$$

#4 $\frac{1}{s+a}$, we get $-e^{-t}+2e^{-2t}$

Therefore

$$L^{-1}\left\{\frac{s+3}{(s+1)(s+2)}\right\} = -e^{-t}+2e^{-2t}+3e^{-t}-3e^{-2t} = 2e^{-t}-e^{-2t}$$

$$L^{-1}\left\{\frac{1}{(s+1)(s+2)}\right\} = e^{-t}-e^{-2t}$$

$$L^{-1}\left\{\frac{-2}{(s+1)(s+2)}\right\} = -2e^{-t}+2e^{-2t}$$

$$L^{-1}\left\{\frac{s}{(s+1)(s+2)}\right\} = -e^{-t}+2e^{-2t}$$

$$\Phi(t) = L^{-1}\left\{[sI-A]^{-1}\right\} = \begin{bmatrix} 2e^{-t}-e^{-2t} & e^{-t}-e^{-2t} \\ -2e^{-t}+2e^{-2t} & -e^{-t}+2e^{-2t} \end{bmatrix}$$

$$\Phi(T) = L^{-1}\left\{[sI-A]^{-1}\right\}\Big|_{t=T}$$

$$= \begin{bmatrix} 2e^{-T}-e^{-2T} & e^{-T}-e^{-2T} \\ -2e^{-T}+2e^{-2T} & -e^{-T}+2e^{-2T} \end{bmatrix}$$

② input matrix

$$\theta(T) = \int_0^T \Phi(t)\,dt\, B$$

$$= \int_0^T \begin{bmatrix} 2e^{-t} - e^{-2t} & e^{-t} - e^{-2t} \\ -2e^{-t} + 2e^{-2t} & -e^{-t} + 2e^{-2t} \end{bmatrix} \begin{bmatrix} 0 \\ 8 \end{bmatrix} dt$$

$$= \int_0^T \begin{bmatrix} 8e^{-t} - 8e^{-2t} \\ -8e^{-t} + 16e^{-2t} \end{bmatrix} dt$$

$$\int_0^T 8e^{-t} - 8e^{-2t}\, dt$$

$$= 8\int_0^T e^{-t}\,dt - 8\int_0^T e^{-2t}\,dt$$

$$= 8(-e^{-t})\Big|_0^T - 8\left(-\tfrac{1}{2}e^{-2t}\right)\Big|_0^T$$

$$= -8(e^{-T} - 1) + 4(e^{-2T} - 1)$$

$$= -8e^{-T} + 4e^{-2T} + 4$$

$$\int_0^T -8e^{-t} + 16e^{-2t}\, dt$$

$$= -8\int_0^T e^{-t} + 16\int_0^T e^{-2t}\,dt \quad = -8(e^{-T} - 1) + 16\left[\tfrac{1}{-2} e^{-2t}\right]\Big|_0^T$$

$$= -8e^{-T} - 8e^{-2T} + 16 \quad = -8(e^{-T} - 1) - 8(e^{-2T} - 1)$$

$$= -8e^{-T} - 8e^{-2T} \cancel{+16}$$

$$\theta(T) = \begin{bmatrix} -8e^{-T} + 4e^{-2T} + 4 \\ -8e^{-T} - 8e^{-2T} + 16 \end{bmatrix} \quad 8e^{-T} - 8e^{-2T}$$

So $x(k+1) = \begin{bmatrix} 2e^{-T} - e^{-2T} & e^{-T} - e^{-2T} \\ -2e^{-T} + 2e^{-2T} & -e^{-T} + 2e^{-2T} \end{bmatrix} x(k) + \begin{bmatrix} -8e^{-T} + 4e^{-2T} + 4 \\ \cancel{-8e^{-T} - 8e^{-2T} + 16} \end{bmatrix} u(k)$

$8e^{-T} - 8e^{-2T}$

$y(k) = \begin{bmatrix} 1 & 0 \end{bmatrix} x(k)$

(b) (i) Q C? O?

Solution ① $W_c = \begin{bmatrix} B & AB \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 1 & -6 \end{bmatrix}$

$AB = \begin{bmatrix} 0 & 1 \\ -4 & -6 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \\ -6 \end{bmatrix}$

$|W_c| = -1 \neq 0$ Controllable ✓

② $W_o = \begin{bmatrix} C \\ CA \end{bmatrix} = \begin{bmatrix} 1 & -1 \\ 4 & 7 \end{bmatrix}$

$CA = \begin{bmatrix} 1 & -1 \end{bmatrix} \begin{bmatrix} 0 & 1 \\ -4 & -6 \end{bmatrix} = \begin{bmatrix} 4 & 7 \end{bmatrix}$

$|W_o| = 7 + 4 = 13 \neq 0$ observable ✓

(ii) unobservable ?

$x(k+1) = \begin{bmatrix} 0 & 1 \\ -4 & -6 \end{bmatrix} x(k) + \begin{bmatrix} 0 \\ 1 \end{bmatrix} \begin{bmatrix} -k_1 & -k_2 \end{bmatrix} x(k) + \begin{bmatrix} 0 \\ 2 \end{bmatrix} r(k)$

$= \left(\begin{bmatrix} 0 & 1 \\ -4 & -6 \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ -k_1 & -k_2 \end{bmatrix} \right) x(k) + \begin{bmatrix} 0 \\ 2 \end{bmatrix} r(k)$

$= \begin{bmatrix} 0 & 1 \\ -4-k_1 & -6-k_2 \end{bmatrix} x(k) + \begin{bmatrix} 0 \\ 2 \end{bmatrix} r(k)$

$$W_o = \begin{bmatrix} C \\ CA_r \end{bmatrix} = \begin{bmatrix} 1 & -1 \\ 4+k_1 & 7+k_2 \end{bmatrix}$$

$$CA_r = [1 \ -1] \begin{bmatrix} 0 & 1 \\ -4-k_1 & -6-k_2 \end{bmatrix} = [4+k_1 \ \ 7+k_2]$$

$$|W_o| = 7+k_2+4+k_1 = 0$$

when $k_1+k_2=-13$, the system isn't observable

(c) optimal control ?

Solution $A=0.8$ $B=0.2$ $Q=1$ $r=6$

$$S = 0.8^2 S + 1 - 0.8\,S\,0.2\left(6+0.2^2 S\right)^{-1} 0.2\,S\,0.8$$

$$S = 0.64S - \frac{0.16^2 S^2}{6+0.04S} + 1$$

$$\frac{0.0256S^2}{6+0.04S} = -0.36S + 1$$

$$0.0256S^2 = -0.36S(6+0.04S) + 6 + 0.04S$$

$$= -2.16S - 0.0144S^2 + 6$$

$$0.04S^2 + 2.16S - 6 = 0$$

$S_1 = 2.6479$ $\quad S_2 = -50.6479$

choose the positive $S = 2.6479$

$$K = (0.2^2 \times 2.6479 + 6)^{-1} \times 0.2 \times 2.6479 \times 0.8$$

$$= 0.06939$$

$$u^*(k) = -0.06939\, x(k)$$

$$x(k+1) = [0.8 + 0.2 \times (-0.06939)]\, x(k)$$

$$= 0.7861\, x(k)$$

poles 没学过，不考