

21-51-Q4

Q (a) (i) α ?

Solution $W_c = [B \ AB] = \begin{bmatrix} 4 & 4\alpha-3 \\ 3 & -4 \end{bmatrix}$

$$AB = \begin{bmatrix} \alpha & -1 \\ -4 & 4 \end{bmatrix} \begin{bmatrix} 4 \\ 3 \end{bmatrix} = \begin{bmatrix} 4\alpha-3 \\ -4 \end{bmatrix} \quad -16+12 = -4$$

$$|W_c| = -16 - 3(4\alpha-3)$$

$$-16 - 12\alpha + 9$$

$$-7 - 12\alpha \neq 0$$

$$\alpha \neq -\frac{7}{12}$$

(ii) Q: $\alpha = 2$, $K = ?$

$$\alpha_c(z) = (z - j0.25)(z + j0.25)$$

$$= z^2 - j^2 0.25^2$$

$$= z^2 + 0.0625$$

$$\alpha_c(A) = \begin{bmatrix} 2 & -1 \\ -4 & 4 \end{bmatrix} \begin{bmatrix} 2 & -1 \\ -4 & 4 \end{bmatrix} + 0.0625 I_2 = \begin{bmatrix} 8.0625 & -6 \\ -24 & 20.0625 \end{bmatrix}$$

$$\text{from (i)} \quad W_c = \begin{bmatrix} 4 & 4\alpha-3 \\ 3 & -4 \end{bmatrix} = \begin{bmatrix} 4 & 5 \\ 3 & -4 \end{bmatrix} \quad W_c^{-1} = \begin{bmatrix} \frac{4}{31} & \frac{5}{31} \\ \frac{3}{31} & -\frac{4}{31} \end{bmatrix}$$

$$K = [0 \ 1] W_c^{-1} \alpha_c(A)$$

$$= [0 \ 1] \begin{bmatrix} \frac{4}{31} & \frac{5}{31} \\ \frac{3}{31} & -\frac{4}{31} \end{bmatrix} \begin{bmatrix} 8.0625 & -6 \\ -24 & 20.0625 \end{bmatrix}$$

$$= \begin{bmatrix} \frac{3}{31} & -\frac{4}{31} \end{bmatrix} \begin{bmatrix} 8.0625 & -6 \\ -24 & 20.0625 \end{bmatrix} = \begin{bmatrix} 3.8770 & -3.1694 \end{bmatrix}$$

(b)(i) $\alpha = 2$ estimator ?

$$\alpha_0(z) = [z - (0.4 + j0.4)][z - (0.4 - j0.4)]$$

$$= z^2 - 0.8z + (0.4^2 + 0.4^2)$$

$$= z^2 - 0.8z + 0.32$$

$$\alpha_0(A) = A^2 - 0.8A + 0.32 \quad \text{casio}$$

$$= \begin{bmatrix} 6.72 & -5.2 \\ -20.8 & 17.12 \end{bmatrix}$$

$$W_0 = \begin{bmatrix} C \\ CA \end{bmatrix} = \begin{bmatrix} 2 & 2 \\ -4 & 6 \end{bmatrix} \quad W_0^{-1} = \begin{bmatrix} 0.3 & -0.1 \\ 0.2 & 0.1 \end{bmatrix}$$

$$CA = \begin{bmatrix} 2 & 2 \end{bmatrix} \begin{bmatrix} 2 & -1 \\ -4 & 4 \end{bmatrix} = \begin{bmatrix} -4 & 6 \end{bmatrix}$$

$$\begin{aligned} 4 - 8 &= -4 \\ -2 + 8 &= 6 \end{aligned}$$

$$L_0 = \alpha_0(A) W_0^{-1} \begin{bmatrix} 0 \\ 1 \end{bmatrix} \quad \text{casio}$$

$$= \begin{bmatrix} -1.192 \\ 3.792 \end{bmatrix}$$

$$(ii) Q: Z_{er,2} \xrightarrow{?} S \quad S = ? \quad T = 0.1$$

Solution

$$\text{magnitude } |z| = \sqrt{0.4^2 + 0.4^2} = \frac{2\sqrt{2}}{5} = 0.5657$$

$$\theta = \arctan\left(\frac{0.4}{0.4}\right) = \tan^{-1}(1) = \frac{\pi}{4} \quad \text{casio}$$

$$Z = e^{sT} \quad s = \frac{1}{T} \ln z$$

$$S = \frac{Z_1 \left(\frac{2\sqrt{2}}{5} \right) + j \frac{2}{4}}{0.1} = -5.6972 + j7.8540$$

$$\zeta = \frac{-(-5.6972)}{\sqrt{5.6972^2 + 7.854^2}} = 0.5872$$

c) Q $u^*(k) = ?$ equation?

Solution $A = 1$ $B = 0.8$ $Q = 8$ $r = 1.6$

$$S = S + 8 - S \cdot 0.8 (1.6 + 0.8^2 S)^{-1} \cdot 0.8 S$$

$$S = S + 8 - \frac{0.64 S^2}{1.6 + 0.64 S}$$

$$\frac{0.64 S^2}{1.6 + 0.64 S} = 8$$

$$0.64 S^2 = 8(0.64 S + 1.6)$$

$$0.64 S^2 = 5.12 S + 12.8$$

$$0.64 S^2 - 5.12 S - 12.8 = 0 \quad \text{casio}$$

$$S_1 = 10 \quad S_2 = -2$$

So choose the positive value, $S = 10$

$$K = (0.8^2 \times 10 + 1.6)^{-1} \times 0.8 \times 10 \times 1$$

$$K = 1 \quad \text{Therefore } u^*(k) = -x(k)$$

$$\begin{aligned}
 x(k+1) &= x(k) - 0.8x(k) \\
 &= 0.2x(k)
 \end{aligned}$$

All in all, $u^*(k) = -x(k)$

$$x(k+1) = 0.2x(k)$$