19-S1-Q1

Q: (a) continuous-time signal? Z? t=0 T=1s

Solution

① X(t) is defined over a continuous range of time $(-\infty, \infty)$. what's more, it is a function of an independent variable t.

so X(t) is continuous-time signal

② when t=0, $x(0) = e^{\sin 0} \cos 0 = e^0 = 1$

when t=1, $x(1) = e^{\sin 2} \cos\frac{\pi}{2} = 0$

when t=k, $x(k) = 1$, $k = 2, 3, 4, \cdots$

apply z transform

$$Z(x(t)) = \sum_{k=0}^{\infty} x(kT) z^{-k} \qquad T=1$$

$$= x(0) + x(1)z^{-1} + x(2)z^{-2} + \cdots$$

$$= 1 + z^{-2} + z^{-3} + \cdots$$

$$= 1 + \frac{z^{-2}}{1 - z^{-1}}$$

$$= \frac{1 - z^{-1} + z^{-2}}{1 - z^{-1}}$$

(b) Z - difference ?

Solution: apply z transform

$$z^2 X(z) - z^2 x(0) - z x(1) - \beta^2 x(z) = 1$$

let $k=-2$, $\quad x(0) - \beta^2 x(-2) = \delta_0(-2)$

$$x(0) = 0$$

let $k=-1$, $\quad x(1) - \beta^2 x(-1) = \delta_0(-1)$

$$x(1) = 0$$

So $\quad z^2 X(z) - \beta^2 X(z) = 1$

$$X(z) = \frac{1}{z^2 - \beta^2}$$

$$\frac{X(z)}{z} = \frac{1}{z(z-\beta)(z+\beta)}$$

$$= \frac{A}{z} + \frac{B}{z-\beta} + \frac{C}{z+\beta}$$

$$A(z^2 - \beta^2) + Bz(z+\beta) + Cz(z-\beta)$$

$$= Az^2 - A\beta^2 + Bz^2 + B\beta z + Cz^2 - C\beta z$$

$$= (A+B+C)z^2 + (B\beta - C\beta)z - A\beta^2$$

$$\begin{cases} A+B+C=0 \\ B\beta - C\beta = 0 \\ -A\beta^2 = 1 \end{cases} \quad (1) \qquad \begin{cases} A = -\frac{1}{\beta^2} \\ B = \frac{1}{2\beta^2} \\ C = \frac{1}{2\beta^2} \end{cases}$$

from (1) $\quad \beta \neq 0 \quad \therefore B = C = -\frac{A}{2} = -\frac{1}{2} \times \left(-\frac{1}{\beta^2}\right) = \frac{1}{2\beta^2}$

$$\frac{X(z)}{z} = -\frac{1}{\beta^2}\frac{1}{z} + \frac{1}{2\beta^2}\frac{1}{z-\beta} + \frac{1}{2\beta^2}\frac{1}{z+\beta}$$

$$X(z) = -\frac{1}{\beta^2} + \frac{1}{2\beta^2}\frac{1}{1-\beta z^{-1}} + \frac{1}{2\beta^2}\frac{1}{1+\beta z^{-1}}$$

#18 $\quad \frac{1}{1-az^{-1}} = \frac{1}{1-\beta z^{-1}} \qquad a^k \quad \beta^k$

$$X(kT) = -\frac{1}{\beta^2}\delta_0(k) + \frac{1}{2\beta^2}\beta^k + \frac{1}{2\beta^2}(-\beta)^k$$

(c) β convergence ?

Solution $k \to \infty$, when k is odd, from (b) $\beta \neq 0$

$\lim_{k\to\infty} X(kT) = -\frac{1}{\beta^2}$, is a constant

when k is even

$\lim_{k\to\infty} X(kT) = -\frac{1}{\beta^2} + \lim_{k\to\infty}\beta^{k-2}$

① when $\beta \in (-1,0)\cup(0,1)$, $\lim_{k\to\infty}\beta^{k-2} = 0$, so, it is converge

② when $\beta \in (-\infty,-1]\cup[1,+\infty)$, $\lim_{k\to\infty}\beta^{k-2} \neq 0$, it is not converge

Method 2.

$$X(z) = -\frac{1}{\beta^2} + \frac{1}{2\beta^2}\frac{1}{1-\beta z^{-1}} + \frac{1}{2\beta^2}\frac{1}{1+\beta z^{-1}}$$

the poles of $X(z)$: $z_1 = \beta \quad z_2 = -\beta$

the poles must inside the unit circle, so $\beta \in (-1,0)\cup(0,1)$