22- S1 - Q4

Q (a) (i) K ? → deadbeat control

$$x(k+1) = [A - BK]\, x(k)$$

$$[A - BK] = \begin{bmatrix} 1 & 0 \\ -1 & 1 \end{bmatrix} - \begin{bmatrix} 1 \\ 1 \end{bmatrix} K$$

$$K = [0\ 1]\, W_c^{-1} \alpha_c(A)$$

$$W_c^{-1} = [B\ \ AB]^{-1} \qquad AB = \begin{bmatrix} 1 & 0 \\ -1 & 1 \end{bmatrix}\begin{bmatrix} 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}^{-1}$$

$$= -\begin{bmatrix} 0 & -1 \\ -1 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 1 \\ 1 & -1 \end{bmatrix}$$

$$\alpha_c(z) = z^2 = A^2 = \begin{bmatrix} 1 & 0 \\ -1 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ -1 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ -2 & 1 \end{bmatrix}$$

$$K = [0\ 1]\begin{bmatrix} 0 & 1 \\ 1 & -1 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ -2 & 1 \end{bmatrix} = [1\ \ -1]\begin{bmatrix} 1 & 0 \\ -2 & 1 \end{bmatrix} = [3\ \ -1]$$

$$[A - BK] = \begin{bmatrix} 1 & 0 \\ -1 & 1 \end{bmatrix} - \begin{bmatrix} 1 \\ 1 \end{bmatrix}[3\ \ -1]$$

$$= \begin{bmatrix} 1 & 0 \\ -1 & 1 \end{bmatrix} - \begin{bmatrix} 3 & -1 \\ 3 & -1 \end{bmatrix}$$

$$= \begin{bmatrix} -2 & 1 \\ -4 & 2 \end{bmatrix}$$

$\exists q > 1$, $[A - BK]^q = 0$, $x(k) = [A - BK]^q x(0)$

$$[A - BK]^2 = \begin{bmatrix} -2 & 1 \\ -4 & 2 \end{bmatrix} \begin{bmatrix} -2 & 1 \\ -4 & 2 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} = 0 \qquad q = 2.$$

(ii) $\hat{x}_1(k)$ 是什么？ 还没学到 尝试写

$$\begin{bmatrix} \hat{x}_1(k) \\ \hat{x}_2(k) \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} x_1(k) \\ x_2(k) \end{bmatrix}$$

$$u(k) = -\hat{K} \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} x_1(k) \\ x_2(k) \end{bmatrix}$$

状态转移矩阵

定义 $x(k) = P w(k)$ … lecture 2-5-P4

$$x(k) = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}^{-1} \hat{x}(k)$$

$$P = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}^{-1} = \begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix}$$

$$A_w = P^{-1} A P = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ -1 & 1 \end{bmatrix} \begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 1 \\ -1 & 1 \end{bmatrix} \begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 1 \\ -1 & 2 \end{bmatrix}$$

$$B_w = P^{-1} B = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$$

$$C_w = CP = \begin{bmatrix} 1 & 1 \end{bmatrix} \begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \end{bmatrix}$$

$$\hat{x}(k+1) = \begin{bmatrix} 0 & 1 \\ -1 & 2 \end{bmatrix} \hat{x}(k) + \begin{bmatrix} 2 \\ 1 \end{bmatrix} u(k)$$

$$\hat{y}(k) = \begin{bmatrix} 1 & 0 \end{bmatrix} \hat{x}(k)$$

$$\hat{k} = \begin{bmatrix} 0 & 1 \end{bmatrix} \hat{W}_c^{-1} \alpha_c(A_w)$$

$$\hat{W}_c^{-1} = \begin{bmatrix} B_w & A_w B_w \end{bmatrix}^{-1} = \begin{pmatrix} 2 & 1 \\ 1 & 0 \end{pmatrix}^{-1} = -\begin{bmatrix} 0 & -1 \\ -1 & 2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 1 & -2 \end{bmatrix}$$

$$A_w B_w = \begin{bmatrix} 0 & 1 \\ -1 & 2 \end{bmatrix} \begin{bmatrix} 2 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

$$\alpha_c(z) = z^2$$

$$\alpha_c(A_w) = A_w^2 = \begin{bmatrix} 0 & 1 \\ -1 & 2 \end{bmatrix} \begin{bmatrix} 0 & 1 \\ -1 & 2 \end{bmatrix} = \begin{bmatrix} -1 & 2 \\ -2 & 3 \end{bmatrix}$$

$$\hat{k} = \begin{bmatrix} 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & 1 \\ 1 & -2 \end{bmatrix} \begin{bmatrix} -1 & 2 \\ -2 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & -2 \end{bmatrix} \begin{bmatrix} -1 & 2 \\ -2 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} 3 & -4 \end{bmatrix}$$

(b) Q $\frac{\bar{X}(z)}{U(z)} = ?$

Solution ① $\bar{x}(k+1) = A\bar{x}(k) + Bu(k) + L_o(y(k) - C\bar{x}(k))$

$y(k) = C\,x(k)$

$\bar{x}(k+1) = (A - L_oC)\bar{x}(k) + Bu(k) + L_oC\,x(k)$

apply z transform

为什么zx（0）拔等于0？
他这个是prediction observer，基于上一时刻的yk，uk得到现在时刻的预测值。x（0）拔的上一时刻是-1，-1的时候输入和输出都是0。所以x（0）拔=0

$$z\bar{X}(z) = (A - L_oC)\bar{X}(z) + BU(z) + L_oC\,X(z)$$

$$(zI - A + L_oC)\bar{X}(z) = BU(z) + L_oC\,X(z)$$

② $x(k+1) = Ax(k) + Bu(k)$ apply z transform

$zX(z) = AX(z) + BU(z)$ $\therefore X(z) = (zI - A)^{-1}BU(z)$

③ $(zI - A + L_oC)\bar{X}(z) = BU(z) + L_oC(zI - A)^{-1}BU(z)$

$$\frac{\bar{X}(z)}{U(z)} = \boxed{\cancel{\frac{B + L_oC(zI - A)^{-1}B}{zI - A + L_oC}}}$$ 矩阵无除法

$$(zI - A + L_oC)^{-1}\left[B + L_oC(zI - A)^{-1}B\right]$$

(c) optimal feedback gains minimize J
obtain values of $u^*(k)$ as a function of $x(0)$
$k=0,1,2$

Solution $A=5$ $B=1$ $N=3$ $Q=8$ $r=1$

initialize $S(3)=0$

let $k=2$ $K(2)=(1\times 0\times 1+1)^{-1}\times 1\times 0\times 5 = 0$

$S(2)=[5-K(2)]^T\times 0\times[5-K(2)]+K^2(2)+8$

$=8$

let $k=1$ $K(1)=(1\times 8\times 1+1)^{-1}\times 1\times 8\times 5$

$=\frac{40}{9}=4.4444$

$S(1)=(5-1\times\frac{40}{9})\times 8\times(5-1\times\frac{40}{9})+(\frac{40}{9})^2+8$

$=\frac{272}{9}=30.2222$

let $k=0$ $K(0)=(\frac{272}{9}+1)^{-1}\times\frac{272}{9}\times 5$

$=\frac{1360}{281}=4.8399$

$S(0)=(5-\frac{1360}{281})^2\times\frac{272}{9}+(\frac{1360}{281})^2+8$

$=\frac{9048}{281}=32.1993$

$u^*(0)=-K(0)\,x(0)=-\frac{1360}{281}x(0)=-4.8399\,x(0)$

$$\overset{*}{x}(1) = Ax(0) + B\overset{*}{u}(0) = 5x(0) + \overset{*}{u}(0)$$

$$= \frac{45}{281} x(0)$$

$$= 0.1601\, x(0)$$

$$\overset{*}{u}(1) = -K(1)\,x(1) = -\frac{44}{9} \times \frac{45}{281} x(0)$$

$$= -\frac{220}{281} x(0)$$

$$= -0.7829\, x(0)$$

$$\overset{*}{x}(2) = 5x(1) + \overset{*}{u}(1)$$

$$= \left(5 \times \frac{45}{281} - \frac{220}{281}\right) x(0)$$

$$= \frac{5}{281} x(0)$$

$$= 0.01779\, x(0)$$

$$\overset{*}{u}(2) = -K(2)\,x(2) = 0$$

Summary $\overset{*}{u}(0) = -4.8399\, x(0)$

$\overset{*}{u}(1) = -0.7829\, x(0)$

$\overset{*}{u}(2) = 0$