

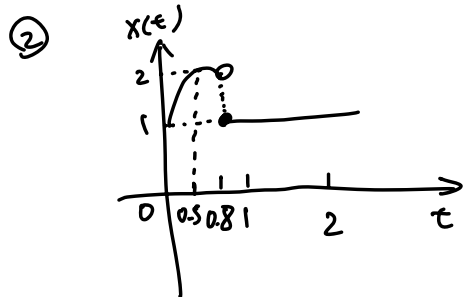
23-51-Q1

Q(a) 是? Z? 0 start $T=0.5$

Solution

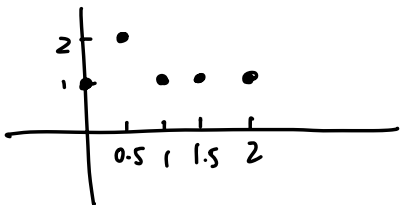
① Yes, $x(t)$ is a continuous-time signal because it is defined for all real values of t .

The function $x(t)$ provides a specific value for every time t in the real number domain, fulfilling the definition of a continuous-time signal.



$$T = \frac{2\pi}{\omega} = 2$$

↓ $T=0.5$ sampling



according to the definition of Z transform

$$X(Z) = Z[x(t)] = \sum_{k=0}^{\infty} x(kT) Z^{-k}$$

$$= x(0)Z^{-0} + x(0.5)Z^{-1} + x(1)Z^{-2} + x(1.5)Z^{-3} + \dots$$

$$= 1 + 2z^{-1} + z^{-2} + z^{-3} + \dots$$

$$\text{So } \sum_{n=2}^{\infty} z^{-n} = \frac{z^{-2}}{1 - z^{-1}}$$

$$X(z) = 1 + 2z^{-1} + \frac{z^{-2}}{1 - z^{-1}}$$

$$= \frac{(1 + 2z^{-1})(1 - z^{-1}) + z^{-2}}{1 - z^{-1}}$$

$$= \frac{1 - z^{-1} + 2z^{-1} - 2z^{-2} + z^{-2}}{1 - z^{-1}}$$

$$= \frac{1 + z^{-1} - z^{-2}}{1 - z^{-1}}$$

(b) Q: difference equation

Solution apply Z transform to

$$y(k+2) + (\beta-1)y(k+1) - \beta y(k) = \delta(k-1)$$

$$z^2 Y(z) - z^2 y(0) - z y(1) + (\beta-1)[z Y(z) - z y(0)] - \beta Y(z) = z^{-1}$$

$$\text{let } k = -2, \quad y(0) + (\beta-1)y(-1) - \beta y(-2) = \delta(-3)$$

$$\text{So } y(0) = 0$$

$$\text{let } k = -1, \quad y(1) + (\beta-1)y(0) - \beta y(-1) = \delta(-2)$$

$$\text{So } y(1) = 0$$

$$\text{So } z^2 Y(z) + (\beta-1)z Y(z) - \beta Y(z) = z^{-1}$$

$$(z^2 + \beta z - z - \beta)Y(z) = z^{-1}$$

$$Y(z) = \frac{z^{-1}}{z^2 + (\beta - 1)z - \beta}$$

$$\frac{Y(z)}{z} = \frac{1}{z^2(z-1)(z+\beta)}$$

$\times \downarrow = \frac{A}{z^2} + \frac{B}{z-1} + \frac{C}{z+\beta}$ 重根应该是 $Az + B$

$$A(z-1)(z+\beta) + Bz^2(z+\beta) + Cz^2(z-1)$$

$$A(z^2 + \beta z - z + \beta) + B(z^3 + \beta z^2) + C(z^3 - z^2)$$

$$(B+C)z^3 + (A+B\beta-C)z^2 + (A\beta-A)z + A\beta$$

$$\begin{cases} B+C=0 \\ A+B\beta-C=0 \\ A\beta-A=0 \\ A\beta=1 \end{cases}$$

$$\Rightarrow \begin{cases} A=1 \\ B=-\frac{1}{2} \\ C=\frac{1}{2} \\ \beta=1 \end{cases}$$

$$\begin{aligned} B+C &= 0 \\ 1+B-C &= 0 \\ 1+2B &= 0 \\ B &= -\frac{1}{2} \end{aligned}$$

$$\frac{Y(z)}{z} = \frac{1}{z^2} + \frac{-\frac{1}{2}}{z-1} + \frac{\frac{1}{2}}{z+1}$$

$$Y(z) = z^{-1} - \frac{1}{2} \frac{1}{1-z^{-1}} + \frac{1}{2} \frac{1}{1+z^{-1}}$$

$$y(k) = \delta_0(n-1) - \frac{1}{2} 1(k) + \frac{1}{2} \cos k\pi$$

(c) (b) 部分不全 β , so (b) 过程有误

x ↑

$$\frac{Y(z)}{z} = \frac{1}{z^2(z-1)(z+\beta)}$$

$$\frac{A}{z} + \frac{B}{z^2} = \frac{Az+B}{z^2} + \frac{C}{z-1} + \frac{D}{z+\beta}$$

下次写标准形式

$$(Az+B)(z-1)(z+\beta) + C z^2(z+\beta) + D z^2(z-1)$$

$$= (Az+B)(z^2+\beta z - z - \beta) + C(z^3+\beta z^2) + D(z^3-z^2)$$

$$= (Az^3 + \underbrace{A\beta z^2 - Az^2}_{\text{blue}} + \underbrace{A\beta z + Bz^2}_{\text{blue}} + \underbrace{B\beta z - Bz}_{\text{blue}} - \underbrace{B\beta}_{\text{blue}}) + C(z^3 + \beta z^2) + D(z^3 - z^2)$$

$$\begin{cases} A+C+D=0 \\ A\beta - A + B + C\beta - D = 0 \\ -A\beta + B\beta - B = 0 \\ -B\beta = 1 \end{cases} \quad \begin{matrix} (1) \\ (2) \\ (3) \\ (4) \end{matrix} \quad \begin{cases} A = \frac{1-\beta}{\beta^2} \\ B = -\frac{1}{\beta} \\ C = \frac{1}{1+\beta} \\ D = -\frac{1}{\beta^2(\beta+1)} \end{cases}$$

from (4) $B = -\frac{1}{\beta}$

from (3) $A = \frac{B - B\beta}{\beta} = \frac{B(1-\beta)}{\beta} = \frac{1-\beta}{\beta^2} = \frac{\beta-1}{\beta^2}$

from (1)

$$C + D = -A = \frac{\beta - 1}{\beta^2} \quad (5)$$

from (2) $A\beta - A + B + C\beta - D = 0$

$$\frac{1-\beta}{\beta} - \frac{1-\beta}{\beta^2} - \frac{1}{\beta} + C\beta - D = 0$$

$$C\beta - D = \frac{\beta^2 + 1 - \beta}{\beta^2} \quad (6)$$

$$(5) + (6) \quad (1+\beta)C = \frac{\beta - 1 + \beta^2 + 1 - \beta}{\beta^2} = 1 \Rightarrow C = \frac{1}{1+\beta}$$

$$\text{from (5)} \quad D = \frac{\beta - 1}{\beta^2} - C = \frac{\beta - 1}{\beta^2} - \frac{1}{1+\beta} = \frac{\beta^2 - 1 - \beta^2}{\beta^2(1+\beta)}$$

$$\text{So } D = \frac{-1}{\beta^2(1+\beta)}$$

$$\frac{Y(z)}{z} = \frac{A}{z} + \frac{B}{z^2} + \frac{C}{z-1} + \frac{D}{z+\beta}$$

$$Y(z) = \frac{1-\beta}{\beta^2} - \frac{1}{\beta} z^{-1} + \frac{1}{1+\beta} \frac{1}{1-z^{-1}} - \frac{1}{\beta^2(\beta+1)} \frac{1}{1+\beta z^{-1}}$$

$$y(k) = \frac{1-\beta}{\beta^2} \delta_0(k) - \frac{1}{\beta} \delta_0(k-1) + \frac{1}{1+\beta} 1(k) - \frac{1}{\beta^2(\beta+1)} (-\beta)^k$$

$$y(k) = \begin{cases} \frac{1-\beta}{\beta^2} + \frac{1}{1+\beta} - \frac{(-\beta)^k}{\beta^2(\beta+1)} & k=0 \\ -\frac{1}{\beta} + \frac{1}{1+\beta} - \frac{(-\beta)^k}{\beta^2(\beta+1)} & k=1 \end{cases}$$

$$k=0$$

从 (b) 可知 $y(0)=0, y(1)=0$

$$k=1$$

所以可以简化为

$$y(k) = 0, \quad k=0, 1$$

$$\left| \frac{1}{1+\beta} - \frac{(-\beta)^k}{\beta^2(\beta+1)} \right| \quad k \geq 2$$

(c) $\beta \rightarrow y(k)$ convergence?

Solution

① when $|\beta| < 1$, $k \rightarrow \infty$, $(-\beta)^k \rightarrow 0$

$$y(k) \rightarrow \frac{1}{1+\beta} \quad |z_1| = 1 \quad |z_2| = |-\beta| = |\beta| < 1$$

The Final Value Theorem applies

all poles of $Y(z)$ lie inside the unit circle, with the possible exception of a simple pole at $z=1$

Verification: $\lim_{k \rightarrow \infty} y(k) = \lim_{z \rightarrow 1} (1-z^{-1})Y(z) = \frac{1}{\beta+1}$

so confirm theorem acceptable

② when $|\beta| = 1$ $(-\beta)^k$ oscillates between ± 1

$y(k)$ doesn't converge, it oscillates

The Final value Theorem does not apply

③ when $|\beta| > 1$, $(-\beta)^k$ grows without bound

$y(k)$ diverges as $k \rightarrow \infty$

The Final value Theorem does not apply