18-S1-Q5

Q(a) G(s) $\omega_n = 5$ $\zeta = 0.5$ pole-zero? $T=1$

Solution ① $G(s) = \dfrac{25}{s^2+5s+25}$

② $\omega_d = \omega_n\sqrt{1-\zeta^2} = 5\times\sqrt{1-\frac{1}{4}} = \dfrac{5\sqrt{3}}{2} = 4.3301$

③ $s_{1,2} = \dfrac{-5 \pm 5\sqrt{3}j}{2}$ $s_1 = -2.5+4.3301j$

$s_2 = -2.5-4.3301j$

④ $G(z) = \alpha\dfrac{(z+1)^{n-m-1}}{(z-e^{s_1T})(z-e^{s_2T})}$

$n-m-1 = 2-0-1 = 1$

$(z-e^{s_1T})(z-e^{s_2T})$

$= z^2 - z(e^{s_2T}+e^{s_1T}) + e^{s_1T}e^{s_2T}$

where $e^{s_1T}+e^{s_2T} = e^{-2.5}e^{-4.3301j} + e^{-2.5}e^{4.3301j}$

$= e^{-2.5}(e^{-4.3301j}+e^{4.3301j})$

$= e^{-2.5}\,2\cos(4.3301)$

$= -0.06124$

$e^{s_1T}e^{s_2T} = e^{-5} = 0.006738$

$$z^2 + 0.06124z + 0.006738$$

$$G_D(z) = \alpha \frac{(z+1)}{z^2 + 0.06124z + 0.006738}$$

$$G_{D0}(1) = G(0) = \frac{25}{s^2+5s+25}\bigg|_{s=0} = \frac{25}{25} = 1$$

$$\alpha \frac{2}{1 + 0.06124 + 0.006738} = 1$$

$$\alpha = 0.5340$$

$$G_D(z) = 0.534 \frac{z+1}{z^2 + 0.06124z + 0.006738}$$

(b) Q: $C(z)$

Solution $G(z) = \frac{Y(z)}{G_{ZAS}(z)} = G_{cl}(z) \frac{R(z)}{G_{ZAS}(z)}$

$$G_{cl}(z) = \frac{0.534z + 0.534}{z^2 + 0.06124z + 0.006738}$$

$$C(z) = \frac{1}{G_{ZAS}(z)} \frac{G_{cl}(z)}{1 - G_{cl}(z)}$$

$$= \frac{(z-1)(z-0.3679)}{(0.3679z + 0.2642)} \frac{\frac{0.534z + 0.534}{z^2 + 0.06124z + 0.006738}}{\left(1 - \frac{0.534z + 0.534}{z^2 + 0.06124z + 0.006738}\right)}$$

$$= \frac{(z-1)(z-0.3679)\ (0.534z + 0.534)}{(0.3679z + 0.2642)(z^2 - 0.4728z - 0.5273)}$$

$$= \frac{(z-1)(z-0.3679)\ (0.534z + 0.534)}{(0.3679z + 0.2642)\ (z-1)\ (z + 0.5273)}$$

$$C(z) = \frac{0.534\ (z - 0.3679)(z+1)}{0.3679\ (z + 0.7176)(z+0.5273)}$$

$$= \frac{1.4515\ (z - 0.3679)(z+1)}{(z + 0.7176)(z+0.5273)}$$

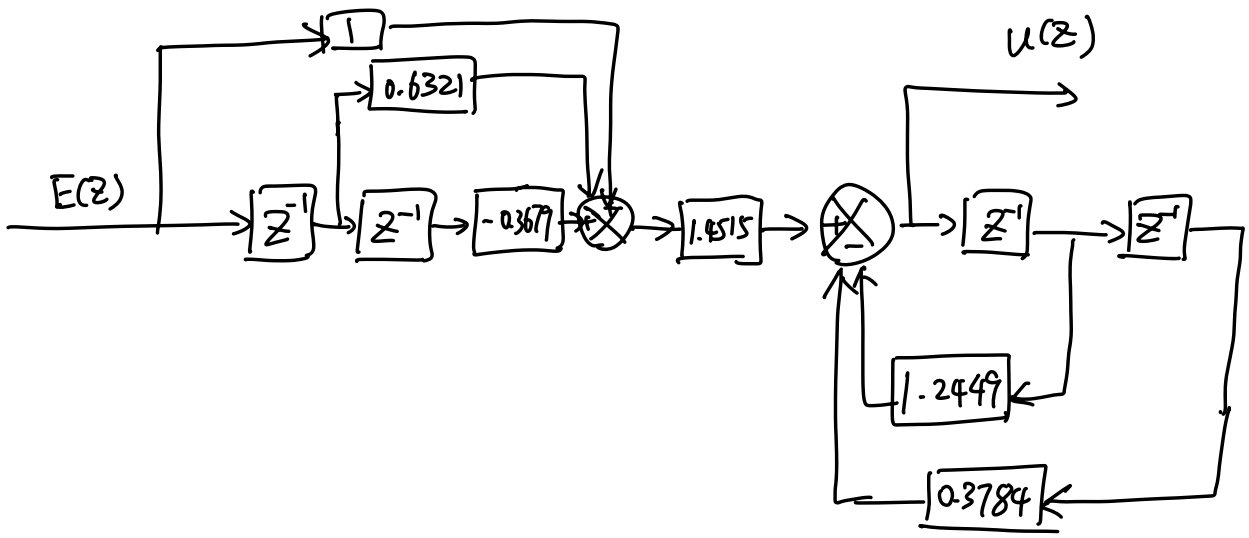
(c)

$$C(z) = \frac{1.4515\ (z^2 + 0.6321z - 0.3679)}{z^2 + 1.2449z + 0.3784}$$

$$= \frac{1.4515\ (1 + 0.6321z^{-1} - 0.3679z^{-2})}{1 + 1.2449z^{-1} + 0.3784z^{-2}}$$

$$C(z) = \frac{U(z)}{E(z)} = \frac{1.4515\ (1 + 0.6321z^{-1} - 0.3679z^{-2})}{1 + 1.2449z^{-1} + 0.3784z^{-2}}$$

$$U(z) = 1.4515\left(E(z) + 0.6321z^{-1}E(z) - 0.3679z^{-2}E(z)\right)$$
$$- 1.2449z^{-1}U(z) - 0.3784z^{-2}U(z)$$


1
0.6321
E(z)
z^{-1}
z^{-1}
-0.3679
1.8515
u(z)
z^{-1}
z^{-1}
1.2449
0.3784