18 - S1 - Q3

Q (i) discretised

Solution ① state transfer matrix

$$[sI-A]^{-1} = \begin{bmatrix} s+10 & 0 \\ 0 & s+1 \end{bmatrix}^{-1} = \frac{1}{(s+1)(s+10)} \begin{bmatrix} s+1 & 0 \\ 0 & s+10 \end{bmatrix}$$

$$= \begin{bmatrix} \frac{1}{s+10} & 0 \\ 0 & \frac{1}{s+1} \end{bmatrix}$$

$$\Phi(t) = L^{-1}\left\{ \begin{bmatrix} \frac{1}{s+10} & 0 \\ 0 & \frac{1}{s+1} \end{bmatrix} \right\} = \begin{bmatrix} e^{-10t} & 0 \\ 0 & e^{-t} \end{bmatrix}$$

#4 $\frac{1}{s+10}$ $a=10$ e^{-10t}

$\frac{1}{s+1}$ $a=1$ e^{-t}

$$\Phi(T) = \begin{bmatrix} e^{-10T} & 0 \\ 0 & e^{-T} \end{bmatrix} \Bigg|_{T=0.1} = \begin{bmatrix} e^{-1} & 0 \\ 0 & e^{-0.1} \end{bmatrix} = \begin{bmatrix} 0.3679 & 0 \\ 0 & 0.9048 \end{bmatrix}$$

② input transfer function

$$\theta(T) = \int_0^T \Phi(t)\, dt\, B$$

$$= \int_0^T \begin{bmatrix} e^{-10t} & 0 \\ 0 & e^{-t} \end{bmatrix} \begin{bmatrix} 1 \\ 2 \end{bmatrix} dt$$

$$= \int_0^T \begin{bmatrix} e^{-10t} \\ 2e^{-t} \end{bmatrix} dt$$

$$= \begin{bmatrix} 0.06321 \\ 0.19033 \end{bmatrix}$$

③

$$x(k+1) = \begin{bmatrix} 0.3679 & 0 \\ 0 & 0.9048 \end{bmatrix} x(k) + \begin{bmatrix} 0.06321 \\ 0.19033 \end{bmatrix} u(k)$$

$$y(k) = [-2 \quad 1] \, x(k)$$

(ii) Q: poles ?

Solution $\det[zI - A]$

$$= \begin{vmatrix} z-0.3679 & 0 \\ 0 & z-0.9048 \end{vmatrix} = (z - 0.3679)(z - 0.9048)$$

$z_1 = 0.3679 \qquad z_2 = 0.9048$

poles at z_1, z_2

(b) $C = ?$

Solution $C = [c_1 \quad c_2]$

$$W_o = \begin{bmatrix} C \\ CA \end{bmatrix} = \begin{bmatrix} c_1 & c_2 \\ c_2 & c_1 \end{bmatrix}$$

$$CA = [c_1 \; c_2] \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} = [c_2 \quad c_1]$$

$$\det W_o = c_1^2 - c_2^2 \neq 0 \quad \Rightarrow \quad c_1^2 \neq c_2^2$$

$$\begin{cases} c_1 \neq c_2 \\ c_1 \neq -c_2 \end{cases}$$

for example $c_1 = 1$ $c_2 = 2$
it is acceptable

(2) ② ① unlimited time

Solution

$A = 1$ $B = 1$ $Q = q$ $r = 8$

$K = -0.5$

$$K = (s+8)^{-1} s = \frac{s}{s+8} = -\frac{1}{2}$$

$$s = -\frac{1}{2}s - 4$$

$$\frac{3}{2}s = -4$$

$$s = -\frac{8}{3}$$

$$s = s + q - s(8+s)^{-1} s$$

$$q = \frac{s^2}{8+s} = \frac{4}{3}$$

② $x(k+1) = x(k) - 0.5x(k)$

$$= 0.5x(k)$$

$$|zI - 0.5| = |z - 0.5| = (z - 0.5)$$

pole at 0.5