19-S1-Q4

Q $x(k+1) = \begin{bmatrix} 1 & -4 \\ 1 & 1 \end{bmatrix} x(k) + \begin{bmatrix} 1 \\ 1 \end{bmatrix} u(k)$

$y(k) = [0\ 1]\ x(k)$

(i) Q: stable ?

Solution ① poles → stable

pole $\det[zI - A]$

$= |zI - A|$

$= \begin{vmatrix} z-1 & 4 \\ -1 & z-1 \end{vmatrix}$

$= (z-1)^2 - (-4)$

$= z^2 - 2z + 1 + 4$

$= z^2 - 2z + 5$

$z_1 = 1+2j \qquad z_2 = 1-2j$

$|z_{1,2}| = \sqrt{1^2 + 2^2} = \sqrt{5} > 1$

the poles are not inside the unit circle

So, the system isn't stable

(ii) Q: k ?

Solution ① Ack

$K = [0\ 1]\, W_c^{-1} \alpha_c(A)$

$\alpha_c(z) = [z-(-0.75+j0.4)][z-(-0.75-j0.4)]$

$= z^2 - z(-0.75-j0.4) - z(-0.75+j0.4)$

$+ (-0.75+j0.4)(-0.75-j0.4)$

$= z^2 + 1.5z + 0.75^2 + 0.4^2$

$= z^2 + 1.5z + 0.7225$

$\alpha_c(A) = A^2 + 1.5A + 0.7225 I_2$

$= \begin{bmatrix} -0.7775 & -14 \\ 3.5 & -0.7775 \end{bmatrix}$ Casio

$W_c^{-1} = [B\ \ AB]^{-1} = \begin{bmatrix} 1 & -3 \\ 1 & 2 \end{bmatrix}^{-1} = \frac{1}{2+3}\begin{bmatrix} 2 & 3 \\ -1 & 1 \end{bmatrix} = \begin{bmatrix} \frac{2}{5} & \frac{3}{5} \\ -\frac{1}{5} & \frac{1}{5} \end{bmatrix}$

$AB = \begin{bmatrix} 1 & -4 \\ 1 & 1 \end{bmatrix}\begin{bmatrix} 1 \\ 1 \end{bmatrix} = \begin{bmatrix} -3 \\ 2 \end{bmatrix}$

$= \begin{bmatrix} 0.4 & 0.6 \\ -0.2 & 0.2 \end{bmatrix}$

$K = [0\ 1]\begin{bmatrix} 0.4 & 0.6 \\ -0.2 & 0.2 \end{bmatrix}\begin{bmatrix} -0.7775 & -14 \\ 3.5 & -0.7775 \end{bmatrix}$

$= [0.8555 \quad 2.6445]$

(iii) L_o ?

Solution

$$L_o = \alpha_c(A)\, W_o^{-1} \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

$$\alpha_o(z) = (z-0.6)^2 = z^2 - 1.2z + 0.36$$

$$\alpha_o(A) = A^2 - 1.2A + 0.36 I_2 = \begin{bmatrix} -3.84 & -3.2 \\ 0.8 & -3.84 \end{bmatrix}$$

$$W_o^{-1} = \begin{bmatrix} C \\ CA \end{bmatrix}^{-1} = \begin{bmatrix} 0 & 1 \\ 1 & 1 \end{bmatrix}^{-1} = \frac{1}{-1}\begin{bmatrix} 1 & -1 \\ -1 & 0 \end{bmatrix} = \begin{bmatrix} -1 & 1 \\ 1 & 0 \end{bmatrix}$$

$$CA = \begin{bmatrix} 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & -4 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 1 \end{bmatrix}$$

$$L_o = \begin{bmatrix} -3.84 & -3.2 \\ 0.8 & -3.84 \end{bmatrix}\begin{bmatrix} -1 & 1 \\ 1 & 0 \end{bmatrix}\begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} -3.84 \\ 0.8 \end{bmatrix}$$

(b) A_T B_T ? C?

Solution

$$x(k+1) = ax(k) + bu(k)$$

$$\bar{x}(k+1) = a\bar{x}(k) + bu(k) + l_o y(k) - l_o c\,\bar{x}(k)$$

$$= (a - l_o c)\bar{x}(k) + bu(k) + l_o y(k)$$

$$= (a - l_o c)\bar{x}(k) + bu(k) + l_o c\, x(k)$$

$$\begin{bmatrix} x(k+1) \\ \bar{x}(k+1) \end{bmatrix} = \begin{bmatrix} a & 0 \\ l_o c & a - l_o c \end{bmatrix}\begin{bmatrix} x(k) \\ \bar{x}(k) \end{bmatrix} + \begin{bmatrix} b \\ b \end{bmatrix} u(k)$$

$$A_T = \begin{bmatrix} a & 0 \\ l_o c & a - l_o c \end{bmatrix} \qquad B_T = \begin{bmatrix} b \\ b \end{bmatrix}$$

$$W_{CT} = [B_T \;\; A_T B_T] = \begin{bmatrix} b & ab \\ b & ab \end{bmatrix} \qquad |W_{CT}| = ab^2 - ab^2 = 0$$

uncontrollable

$$A_T B_T = \begin{bmatrix} a & 0 \\ l_0 c & a - l_0 c \end{bmatrix} \begin{bmatrix} b \\ b \end{bmatrix} = \begin{bmatrix} ab \\ ab \end{bmatrix}$$

$$l_0 c b + ab - l_0 c b = ab$$