22-S1-Q4

Q (a) (i) K ? → deadbeat control

$$x(k+1) = [A - BK]x(k)$$

$$[A - BK] = \begin{bmatrix} 1 & 0 \\ -1 & 1 \end{bmatrix} - \begin{bmatrix} 1 \\ 1 \end{bmatrix} K$$

$$K = [0 \ 1] W_c^{-1} \alpha_c(A)$$

$$W_c^{-1} = [B \ \ AB]^{-1}$$

$$AB = \begin{bmatrix} 1 & 0 \\ -1 & 1 \end{bmatrix}\begin{bmatrix} 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}^{-1}$$

$$= -\begin{bmatrix} 0 & -1 \\ -1 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 1 \\ 1 & -1 \end{bmatrix}$$

$$\alpha_c(z) = z^2 = A^2 = \begin{bmatrix} 1 & 0 \\ -1 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ -1 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ -2 & 1 \end{bmatrix}$$

$$K = [0 \ 1]\begin{bmatrix} 0 & 1 \\ 1 & -1 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ -2 & 1 \end{bmatrix} = [1 \ -1]\begin{bmatrix} 1 & 0 \\ -2 & 1 \end{bmatrix} = [3 \ -1]$$

$$[A - BK] = \begin{bmatrix} 1 & 0 \\ -1 & 1 \end{bmatrix} - \begin{bmatrix} 1 \\ 1 \end{bmatrix}[3 \ -1]$$

$$= \begin{bmatrix} 1 & 0 \\ -1 & 1 \end{bmatrix} - \begin{bmatrix} 3 & -1 \\ 3 & -1 \end{bmatrix}$$

$$= \begin{bmatrix} -2 & 1 \\ -4 & 2 \end{bmatrix}$$

$\exists q > 1$, $[A - BK]^q = 0$, $x(k) = [A - BK]^q x(0)$

$$[A - BK]^2 = \begin{bmatrix} -2 & 1 \\ -4 & 2 \end{bmatrix} \begin{bmatrix} -2 & 1 \\ -4 & 2 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} = 0 \quad q = 2.$$

(ii) $\hat{x}_1(k)$ 是什么? 还没学到 尝试写

$$\begin{bmatrix} \hat{x}_1(k) \\ \hat{x}_2(k) \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} x_1(k) \\ x_2(k) \end{bmatrix}$$

$$u(k) = -\hat{K} \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} x_1(k) \\ x_2(k) \end{bmatrix}$$

状态转移矩阵

定义 $x(k) = P w(k)$ ... lecture 2-5-P4

$$x(k) = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}^{-1} \hat{x}(k)$$

$$P = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}^{-1} = \begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix}$$

$$A_w = P^{-1} A P = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ -1 & 1 \end{bmatrix} \begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 1 \\ -1 & 1 \end{bmatrix} \begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 1 \\ -1 & 2 \end{bmatrix}$$

$$B_w = P^{-1} B = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$$

$$C_w = CP = [1 \;\; 1] \begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix} = [1 \;\; 0]$$

$$\hat{x}(k+1) = \begin{bmatrix} 0 & 1 \\ -1 & 2 \end{bmatrix} \hat{x}(k) + \begin{bmatrix} 2 \\ 1 \end{bmatrix} u(k)$$

$$\hat{y}(k) = [1 \;\; 0] \hat{x}(k)$$

$$\hat{K} = [0 \;\; 1] \hat{W}_c^{-1} \alpha_c(A_w)$$

$$\hat{W}_c^{-1} = [B_w \;\; A_w B_w]^{-1} = \begin{pmatrix} 2 & 1 \\ 1 & 0 \end{pmatrix}^{-1} = - \begin{bmatrix} 0 & -1 \\ -1 & 2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 1 & -2 \end{bmatrix}$$

$$A_w B_w = \begin{bmatrix} 0 & 1 \\ -1 & 2 \end{bmatrix} \begin{bmatrix} 2 \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

$$\alpha_c(z) = z^2$$

$$\alpha_c(A_w) = A_w^2 = \begin{bmatrix} 0 & 1 \\ -1 & 2 \end{bmatrix} \begin{bmatrix} 0 & 1 \\ -1 & 2 \end{bmatrix} = \begin{bmatrix} -1 & 2 \\ -2 & 3 \end{bmatrix}$$

$$\hat{K} = [0 \;\; 1] \begin{bmatrix} 0 & 1 \\ 1 & -2 \end{bmatrix} \begin{bmatrix} -1 & 2 \\ -2 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & -2 \end{bmatrix} \begin{bmatrix} -1 & 2 \\ -2 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} 3 & -4 \end{bmatrix}$$

(b) Q $\dfrac{\bar{x}(z)}{u(z)} = ?$

Solution ① $\bar{x}(k+1) = A\bar{x}(k) + Bu(k) + L_o(y(k) - C\bar{x}(k))$

$y(k) = C\,x(k)$

$\bar{x}(k+1) = (A - L_oC)\bar{x}(k) + Bu(k) + L_oC\,x(k)$

apply z transform

$$z\bar{X}(z) = (A - L_oC)\bar{X}(z) + BU(z) + L_oC\,X(z)$$

$$(zI - A + L_oC)\bar{X}(z) = BU(z) + L_oC\,X(z)$$

② $x(k+1) = Ax(k) + Bu(k)$ apply z transform

$zX(z) = AX(z) + BU(z)$ $\therefore X(z) = (zI - A)^{-1}BU(z)$

③ $(zI - A + L_oC)\bar{X}(z) = BU(z) + L_oC(zI - A)^{-1}BU(z)$

$$\frac{\bar{X}(z)}{U(z)} = \boxed{\cancel{\dfrac{B + L_oC(zI-A)^{-1}B}{zI - A + L_oC}}}$$ 矩阵无除法

$$(zI - A + L_oC)^{-1}\left[B + L_oC(zI - A)^{-1}B\right]$$

(c) optimal feedback gains minimize J
obtain values of $u^*(k)$ as a function of $x(0)$
$k = 0, 1, 2$

Solution $A = 5$ $B = 1$ $N = 3$ $Q = 8$ $r = 1$

initialize $S(3) = 0$

let $k=2$ $K(2) = (1 \times 0 \times 1 + 1)^{-1} \times 1 \times 0 \times 5 = 0$

$S(2) = [5 - K(2)]^T \times 0 \times [5 - K(2)] + K^2(2) + 8$
$= 8$

let $k=1$ $K(1) = (1 \times 8 \times 1 + 1)^{-1} \times 1 \times 8 \times 5$

$= \frac{40}{9} = 4.4444$

$S(1) = (5 - 1 \times \frac{40}{9}) \times 8 \times (5 - 1 \times \frac{40}{9}) + (\frac{40}{9})^2 + 8$
$= \frac{272}{9} = 30.2222$

let $k=0$ $K(0) = (\frac{272}{9} + 1)^{-1} \times \frac{272}{9} \times 5$

$= \frac{1360}{281} = 4.8399$

$S(0) = (5 - \frac{1360}{281})^2 \times \frac{272}{9} + (\frac{1360}{281})^2 + 8$

$= \frac{9048}{281} = 32.1993$

$u^*(0) = -K(0)\, X(0) = -\frac{1360}{281} X(0) = -4.8399\, X(0)$

$$x^*(1) = Ax(0) + Bu^*(0) = 5x(0) + u^*(0)$$

$$= \frac{45}{281} x(0)$$

$$= 0.1601\, x(0) \qquad -\frac{40}{9}$$

$$u^*(1) = -K(1)\, x(1) = \cancel{-\frac{44}{9}} \times \frac{45}{281} x(0)$$

$$-\frac{200}{281}$$

$$= \cancel{-\frac{220}{281}}\, x(0)$$

$$-0.7117$$

$$= \cancel{-0.7829}\, x(0)$$

$$x^*(2) = 5x(1) + u^*(1) \qquad -\frac{200}{281}$$

$$= \left(5 \times \frac{45}{281} \cancel{-\frac{220}{281}}\right) x(0)$$

$$= \frac{25\ \cancel{5}}{281} x(0)$$

$$0.08897$$

$$= \cancel{0.01779}\, x(0)$$

$$u^*(2) = -K(2)\, x(2) = 0$$

Summary

$$u^*(0) = -4.8399\, x(0)$$

$$u^*(1) = \cancel{-0.7829\, x(0)} \quad -0.7117\, x(0)$$

$$u^*(2) = 0$$