17-S1-Q1

Q(a) $T = 0.5$ $\quad X(S) = \frac{1}{S^2+5S+4}$ $\quad Z(x(kT))$

Solution

$$X(S) = \frac{1}{S^2+5S+4} = \frac{1}{(S+1)(S+4)}$$

#9 $\quad \frac{3}{(S+1)(S+4)} \quad \frac{(e^{-T}-e^{-4T})z^{-1}}{(1-e^{-T}z^{-1})(1-e^{-4T}z^{-1})}$ $\quad T=0.5$

$$\text{So } Z(x(kT)) = \frac{(e^{-0.5}-e^{-2})z^{-1}}{3(1-e^{-0.5}z^{-1})(1-e^{-2}z^{-1})}$$

$$= \frac{0.4712z^{-1}}{3(1-0.6065z^{-1})(1-0.1353z^{-1})}$$

(b)Q: difference

Solution

apply z transform

$$z^2X(z) - z^2x(0) - zx(1) + (\alpha+0.5)[zX(z) - zx(0)] + 0.5\alpha X(z) = 1$$

let $k=-2$, $\quad x(0)=0$

let $k=-1$, $\quad x(1)=0$

$$z^2X(z) + (\alpha+0.5)zX(z) + 0.5\alpha X(z) = 1$$

$$[z^2 + (\alpha+0.5)z + 0.5\alpha]X(z) = 1$$

$$X(z) = \frac{1}{z^2+(\alpha+0.5)z+0.5\alpha}$$

$$= \frac{1}{(z+\alpha)(z+0.5)}$$

$$\frac{X(z)}{z} = \frac{1}{z(z+\alpha)(z+0.5)}$$

$$= \frac{A}{z} + \frac{B}{z+\alpha} + \frac{C}{z+0.5}$$

$$A(z+\alpha)(z+0.5) + Bz(z+0.5) + Cz(z+\alpha) = 1$$

$z=0$, $A\,\alpha\,0.5 = 1 \Rightarrow A = \frac{2}{\alpha}$

$z=-\alpha$, $B(-\alpha)(-\alpha+0.5) = 1 \Rightarrow B = \frac{1}{\alpha(\alpha-0.5)}$

$z=-0.5$, $C(-0.5)(-0.5+\alpha) = 1 \Rightarrow C = \frac{1}{-\frac{1}{2}(-\frac{1}{2}+\alpha)} = \frac{4}{1-2\alpha}$

$$X(z) = \frac{2}{\alpha} + \frac{1}{\alpha(\alpha-0.5)}\frac{1}{1+\alpha z^{-1}} + \frac{4}{1-2\alpha}\frac{1}{1+0.5z^{-1}}$$

#18 $\quad a = -\alpha \qquad\qquad a = -0.5$

$$x(kT) = \frac{2}{\alpha} + \frac{1}{\alpha(\alpha-0.5)}(-\alpha)^k + \frac{4}{1-2\alpha}(-0.5)^k$$

(c) Q : convergence?

Solution

$$X(z) = \frac{1}{(z+\alpha)(z+0.5)} = \frac{1}{(z-(-\alpha))(z-(-0.5))}$$

poles at $z_1 = -\alpha$ $\quad z_2 = -0.5$

$|z_1| = |\alpha|$ $\quad |z_2| = 0.5 < 1$

① all poles of $X(z)$ lie inside the unit circle with the possible exception of a simple pole at $z=1$, So we can use Final Value Theorem

$|z_1| = |\alpha| < 1 \quad \Rightarrow \quad \alpha \in (-1,1)$

$z_1 = 1 \Rightarrow \alpha = -1 \Rightarrow \alpha \in [-1,1)$

if $\alpha = -1$

$$\lim_{k\to\infty} x(k) = \lim_{z\to 1} (z-1)\frac{1}{(z-1)(z+0.5)}$$

$$= \frac{1}{1.5} = \frac{2}{3} = 0.6667$$

if $\alpha \in (-1,1)$

$$\lim_{k\to\infty} x(k) = \lim_{z\to 1} \frac{z-1}{(z+\alpha)(z+0.5)}$$

$$= \frac{2}{3} \lim_{z \to 1} \frac{z-1}{z+\alpha}$$

$$= 0$$

② when $\alpha \in (-\infty, -1) \cup [1, +\infty)$

the prerequisites for FVT are not met

So we can't use Final Value Theorem

So, $X(kT)$ isn't converge