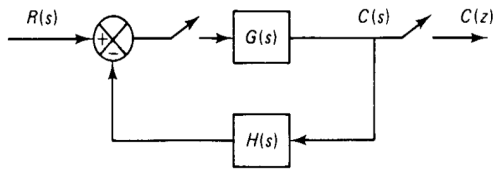
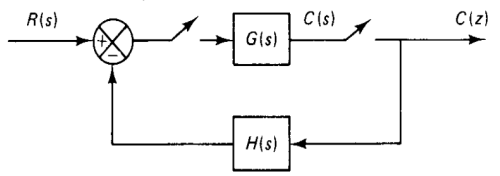
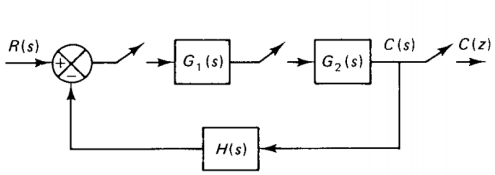
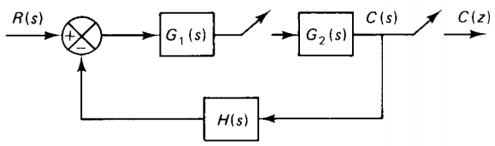
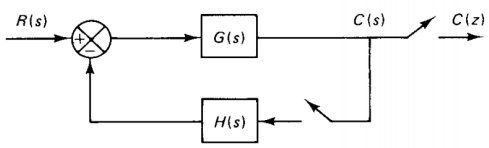
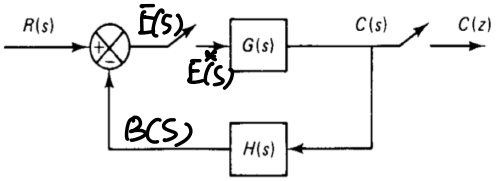
Understand 3.3.7 table 3 - 3

TABLE 3–1 FIVE TYPICAL CONFIGURATIONS FOR CLOSED-LOOP DISCRETE-TIME CONTROL SYSTEMS

	$C(z) = \frac{G(z)R(z)}{1 + GH(z)}$
	$C(z) = \frac{G(z)R(z)}{1 + G(z)H(z)}$
	$C(z) = \frac{G_1(z)G_2(z)R(z)}{1 + G_1(z)G_2H(z)}$
	$C(z) = \frac{G_2(z)G_1R(z)}{1 + G_1G_2H(z)}$
	$C(z) = \frac{GR(z)}{1 + GH(z)}$

(a) Q: $C(z) = ?$



Solution

① transfer function

$$G(s) = \frac{C(s)}{E^*(s)}$$

$$H(s) = \frac{B(s)}{C(s)}$$

② ERROR, eliminate target, get $E(s)$ & $E^*(s)$

$$E(s) = R(s) - B(s)$$

$$= R(s) - H(s)\,C(s)$$

$$= R(s) - H(s)G(s)\,E^*(s)$$

③ star

$$E^*(s) = R^*(s) - HG^*(s)\,E^*(s)$$

$$E^*(s) = \frac{R^*(s)}{1 + HG^*(s)}$$

④ target

$$C(z) = C^*(s)$$

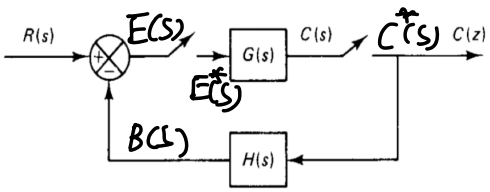
$$C(s) = G(s)\,E^*(s)$$

$$= \frac{G(s)\,R^*(s)}{1 + HG^*(s)}$$

$$C^*(s) = \frac{G^*(s)R^*(s)}{1+GH^*(s)}$$

$$C(z) = \frac{G(z)R(z)}{1+GH(z)}$$

(b) Q : $C(z) = ?$



Solution

① transform function

$$G(s) = \frac{C(s)}{E^*(s)}$$

$$H(s) = \frac{B(s)}{C^*(s)}$$

② Error

$$E(s) = R(s) - B(s)$$

$$= R(s) - H(s)C^*(s)$$

$$C(s) = G(s)E^*(s)$$

$$C^*(s) = G^*(s)E^*(s)$$

So, $E(s) = R(s) - H(s)G^*(s)E^*(s)$

$$E^*(s) = R^*(s) - H^*(s)G^*(s)E^*(s)$$

$$E^*(s) = \frac{R^*(s)}{1 + H^*(s)G^*(s)}$$

② target

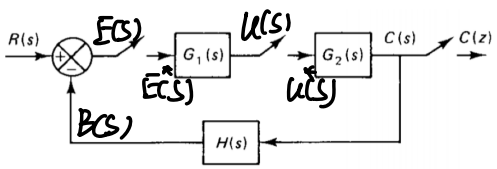
$$C(z) = C^*(s)$$

$$= G^*(s)E^*(s)$$

$$= \frac{G^*(s)R^*(s)}{1 + G^*(s)H^*(s)}$$

$$= \frac{G(z)R(z)}{1 + G(z)H(z)}$$

(c) Q: $C(z) = ?$



Solution

① transform function

$$G_1(s) = \frac{U(s)}{E^*(s)}$$

$$G_2(s) = \frac{C(s)}{U^*(s)}$$

$$H(s) = \frac{B(s)}{C(s)}$$

② ERROR

$$E(s) = R(s) - B(s)$$

$$= R(s) - H(s)C(s)$$

$$= R(s) - H(s)G_2(s)U^*(s)$$

$$U(s) = G_1(s)E^*(s)$$

$$U^*(s) = G_1^*(s)E^*(s)$$

$$E(s) = R(s) - H(s)G_2(s)G_1^*(s)E^*(s)$$

③ star

$$E^*(s) = R^*(s) - HG_2^*(s)G_1^*(s)E^*(s)$$

$$E^*(s) = \frac{R^*(s)}{1 + HG_2^*(s)G_1^*(s)}$$

④ target

$$C(z) = C^*(s)$$

$$C(s) = G_2(s)U^*(s)$$

$$U(s) = G_1(s)E^*(s)$$

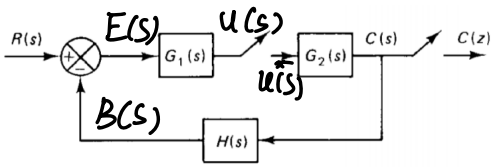
$$U^*(s) = G_1^*(s)E^*(s)$$

$$= \frac{G_1^*(s)R^*(s)}{1 + HG_2^*(s)G_1^*(s)}$$

$$C^*(s) = \frac{G_2^*(s)G_1^*(s)R^*(s)}{1 + HG_2^*(s)G_1^*(s)}$$

$$C(z) = \frac{G_1(z)G_2(z)R(z)}{1 + G_1(z)G_2H(z)}$$

cd) $C(z) = ?$



Solution

① sampling

$C(z) = C^*(s)$

② transform function

$$H(s) = \frac{B(s)}{C(s)}$$

$$E(s) = R(s) - H(s)\,C(s)$$

$$G_1(s) = \frac{U(s)}{E(s)}$$

$$G_2(s) = \frac{C(s)}{U^*(s)}$$

③ laplace star algorithm $\rightarrow U(s)$

$$U(s) = G_1(s)\,E(s)$$

$$= G_1(s)[R(s) - H(s)\,C(s)]$$

$$= G_1(s)\,R(s) - G_1(s)\,H(s)\,C(s)$$

$$= G_1(s)\,R(s) - G_1(s)\,H(s)\,G_2(s)\,\hat{U}(s)$$

$$U^*(s) = G_1R^*(s) - G_1HG_2^*(s)\,U^*(s)$$

$$U^*(s) = \frac{G_1R^*(s)}{1 + G_1HG_2^*(s)}$$

④ target . eliminate $U(s)$ $\hat{U}(s)$

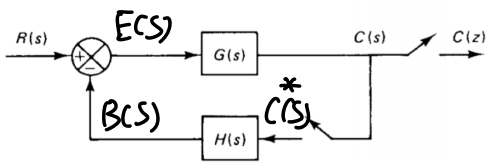
$$C(s) = G_2(s)\,U^*(s)$$

$$= \frac{G_2(s)\,G_1R^*(s)}{1+G_1HG_2^*(s)}$$

$$C^*(s) = \frac{G_2^*(s)\,G_1R^*(s)}{1+G_1HG_2^*(s)}$$

$$C(z) = \frac{G_2(z)\,G_1R(z)}{1+G_1HG_2(z)}$$

(e) $C(z) = ?$



Solution

① transform function

$$G(s) = \frac{C(s)}{E(s)}$$

$$H(s) = \frac{B(s)}{C^*(s)}$$

② ERROR

$$E(s) = R(s) - B(s)$$

$$= R(s) - H(s)C^*(s)$$

代入

$$C(s) = G(s)E(s) \quad \Rightarrow E(s) = \frac{C(s)}{G(s)}$$

$C^*(s) = GE^*(s)$ X 得不出 $E^*(s)$

所以直接換E(s)

$$E(s) = R(s) - H(s)\,G\,E^*(s)$$

$$E^*(s) = R^*(s) - H^*(s)\,G\,E^*(s)$$

③ target

$$C(z) = C^*(s)$$

$$= G\,E^*(s)$$

$$\frac{C(s)}{G(s)} = R(s) - H(s)\,C^*(s)$$

$$C(s) = R(s)G(s) - H(s)\,C^*(s)\,G(s)$$

$$C^*(s) = RG^*(s) - GH^*(s)\,C^*(s)$$

$$C^*(s) = \frac{RG^*(s)}{1+GH^*(s)}$$

$$C(z) = \frac{GR(z)}{1+GH(z)}$$