6.003: Signals and Systems

Signals and Systems

September 8, 2011

6.003: Signals and Systems

Today's handouts: Single package containing
- Slides for Lecture 1
- Subject Information & Calendar

Lecturer: Denny Freeman

Instructors: Elfar Adalsteinsson
Russ Tedrake

TAs: Phillip Nadeau
Wenbang Xu

Website: mit.edu/6.003

Text: *Signals and Systems* – Oppenheim and Willsky

6.003: Homework

Doing the homework is essential for understanding the content.

- where subject matter is/isn't learned
- equivalent to "practice" in sports or music

Weekly Homework Assignments

- Conventional Homework Problems plus
- **Engineering Design Problems** (Python/Matlab)

Open Office Hours!

- Stata Basement
- Mondays and Tuesdays, afternoons and early evenings

6.003: Signals and Systems

Collaboration Policy

- **Discussion** of concepts in homework is encouraged
- **Sharing** of homework or code is not permitted and will be reported to the COD

协作政策:鼓励在作业中讨论概念，
不允许共享作业或代码，并将报告给COD

Firm Deadlines

- Homework must be submitted by the published due date
- Each student can submit **one** late homework assignment without penalty.
- Grades on other late assignments will be multiplied by 0.5 (unless excused by an Instructor, Dean, or Medical Official).

作业必须在规定的截止日期前提交，
每个学生可以迟交一次作业而不受处罚。
其他迟交作业的成绩将乘以0.5(除非有导师、教务长或医务官员的许可)。

6.003 At-A-Glance

	Tuesday	Wednesday		Thursday	Friday
Sep 6	**Registration Day:** No Classes		R1: Continuous & Discrete Systems	L1: Signals and Systems	R2: Difference Equations
Sep 13	L2: Discrete-Time Systems	HW1 due	R3: Feedback, Cycles, and Modes	L3: Feedback, Cycles, and Modes	R4: CT Systems
Sep 20	L4: CT Operator Representations	HW2 due	**Student Holiday:** No Recitation	L5: Laplace Transforms	R5: Laplace Transforms
Sep 27	L6: Z Transforms	HW3 due	R6: Z Transforms	L7: Transform Properties	R7: Transform Properties
Oct 4	L8: Convolution; Impulse Response	EX4	**Exam 1** No Recitation	L9: Frequency Response	R8: Convolution and Freq. Resp.
Oct 11	**Columbus Day:** No Lecture	HW5 due	R9: Bode Diagrams	L10: Bode Diagrams	R10: Feedback and Control
Oct 18	L11: DT Feedback and Control	HW6 due	R11: CT Feedback and Control	L12: CT Feedback and Control	R12: CT Feedback and Control
Oct 25	L13: CT Feedback and Control	HW7	**Exam 2** No Recitation	L14: CT Fourier Series	R13: CT Fourier Series
Nov 1	L15: CT Fourier Series	EX8 due	R14: CT Fourier Series	L16: CT Fourier Transform	R15: CT Fourier Transform
Nov 8	L17: CT Fourier Transform	HW9 due	R16: DT Fourier Transform	L18: DT Fourier Transform	**Veterans Day:** No Recitation
Nov 15	L19: DT Fourier Transform	HW10	**Exam 3** No Recitation	L20: Fourier Relations	R17: Fourier Relations
Nov 22	L21: Sampling	EX11 due	R18: Fourier Transforms	**Thanksgiving:** No Lecture	**Thanksgiving:** No-Recitation
Nov 29	L22: Sampling	HW12 due	R19: Modulation	L23: Modulation	R20: Modulation
Dec 6	L24: Modulation	EX13	R21: Review	L25: Applications of 6.003	Study Period
Dec 13	Breakfast with Staff	EX13	R22: Review	**Study Period:** No Lecture	**Final Exams:** No-Recitation
Dec 20	**Final Examinations: No Classes**				

Weekly meetings with **class representatives**

- help staff understand student perspective
- learn about teaching

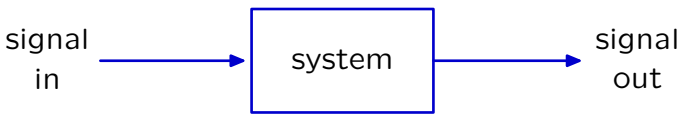
Tentatively meet on Thursday afternoon

Interested? ...

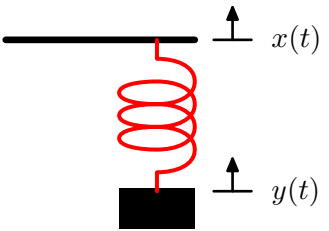
The Signals and Systems Abstraction

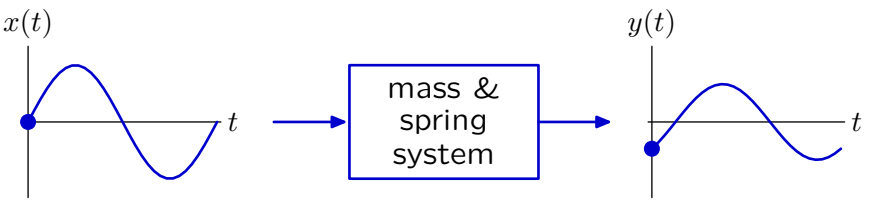
Describe a **system** (physical, mathematical, or computational) by the way it transforms an **input signal** into an **output signal**.

描述一个系统(物理的、数学的或计算的)
将输入信号转换成输出信号的方式。

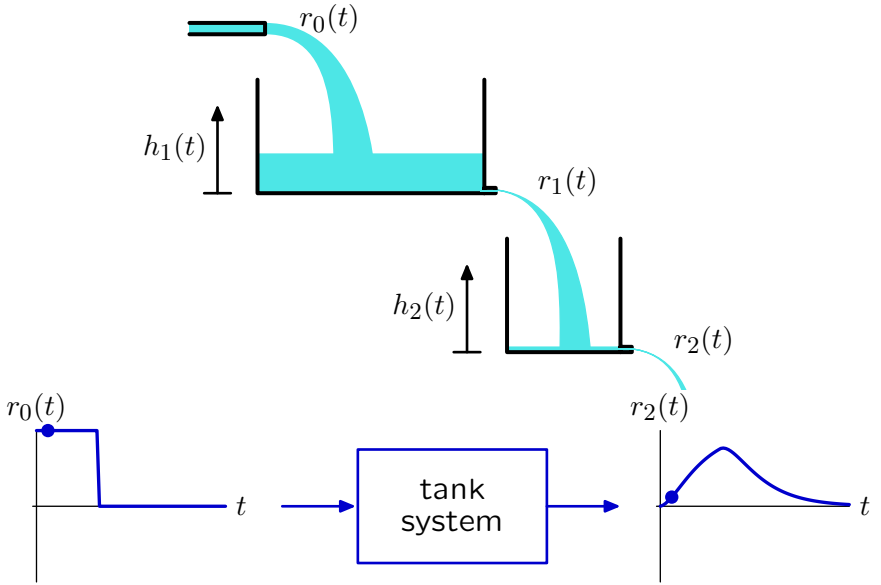


Example: Mass and Spring

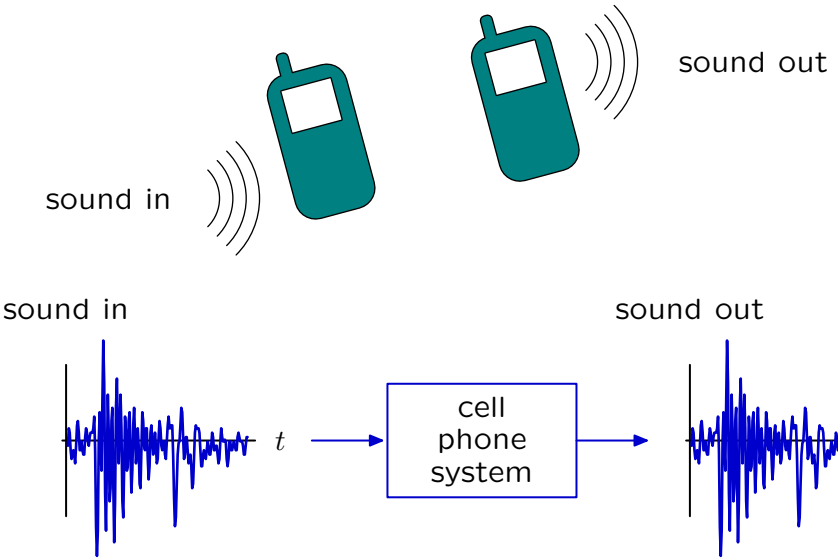




Example: Tanks

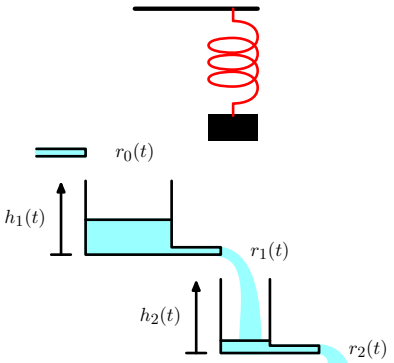


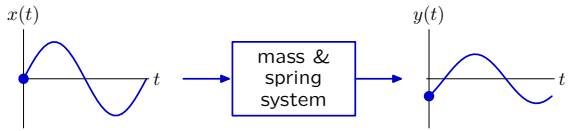
Example: Cell Phone System

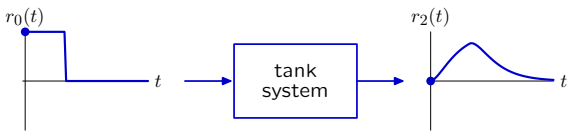


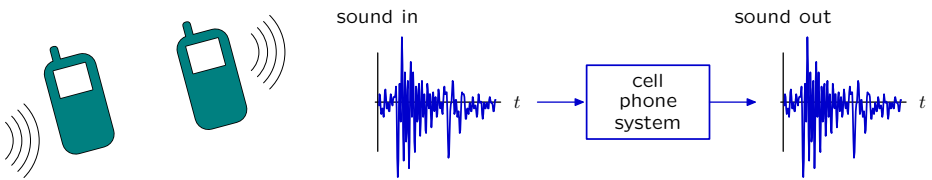
Signals and Systems: Widely Applicable

The Signals and Systems approach has broad application: electrical, mechanical, optical, acoustic, biological, financial, ...







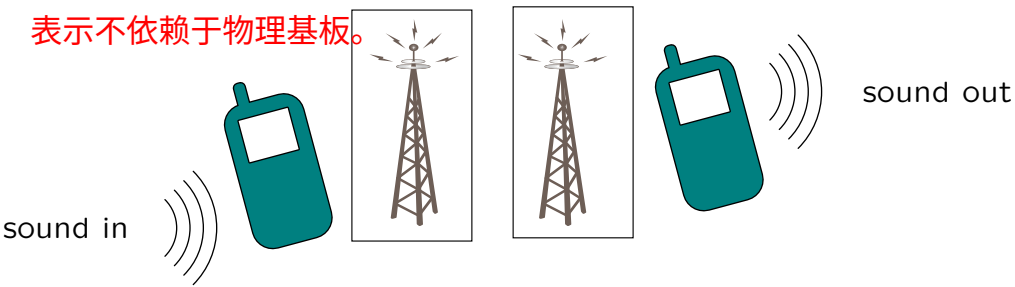


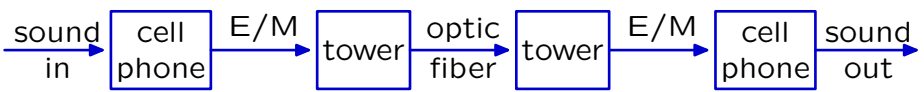
Signals and Systems: Modular

The representation does not depend upon the physical substrate.

信号和系统:模块化

表示不依赖于物理基板。





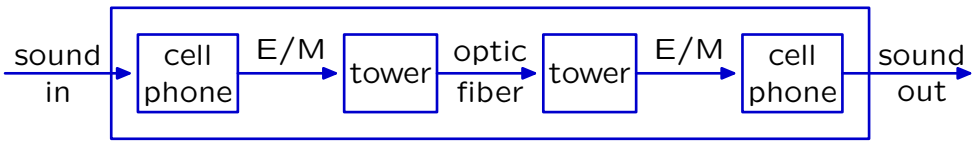
专注于信息流，抽象掉其他一切

focuses on the flow of **information**, abstracts away everything else

Signals and Systems: Hierarchical 层次

Representations of component systems are easily combined.

Example: cascade of component systems



Composite system

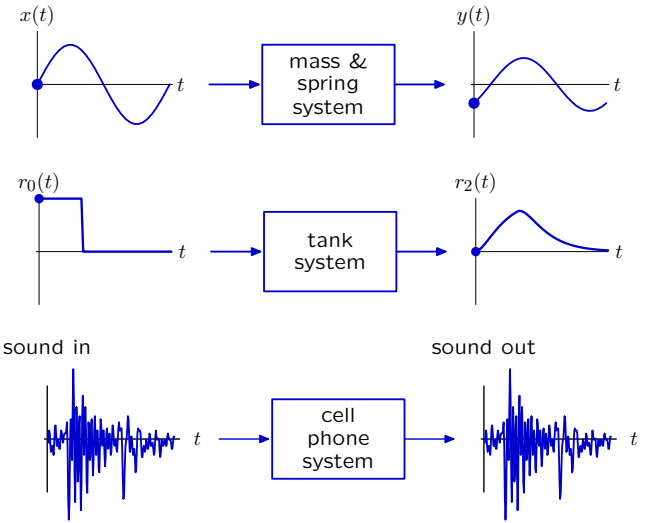
构件系统和复合系统具有相同的形式，分析方法也相同。

Component and composite systems have the same form, and are analyzed with same methods.

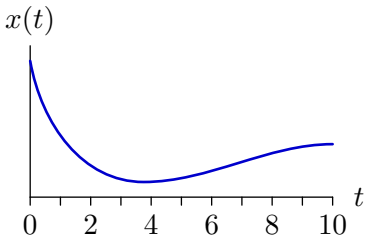
Signals and Systems

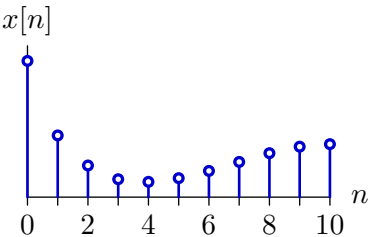
Signals are mathematical functions.

- independent variable = time
- dependent variable = voltage, flow rate, sound pressure



continuous "time" (CT) and discrete "time" (DT)





Signals from physical systems often functions of **continuous** time.

- mass and spring
- leaky tank

来自物理系统的信号通常是连续时间的函数。

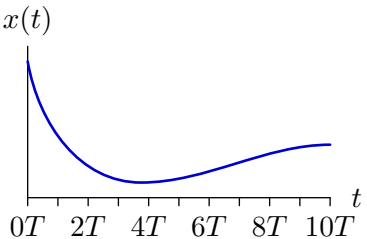
来自计算系统的信号通常是离散时间函数。

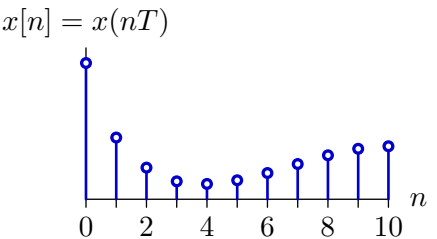
状态机:给定当前输入和当前状态，下一个输出和下一个状态是什么。

Signals from computation systems often functions of **discrete** time.

- state machines: given the current input and current state, what is the next output and next state.

Sampling: converting CT signals to DT





$T =$ sampling interval

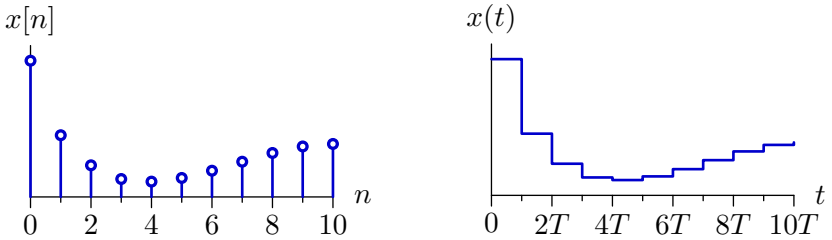
对于物理数据的计算操作很重要。
音频信号的数字表示(如MP3)图像的数字表示(如JPEG)

Important for computational manipulation of physical data.

- digital representations of audio signals (e.g., MP3)
- digital representations of images (e.g., JPEG)

Reconstruction: converting DT signals to CT

zero-order hold



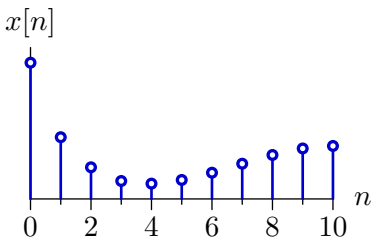
$T =$ sampling interval

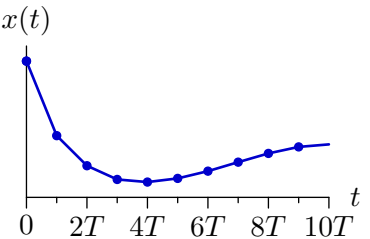
通常用于音频输出设备，如CD播放器

commonly used in audio output devices such as CD players

Reconstruction: converting DT signals to CT

piecewise linear



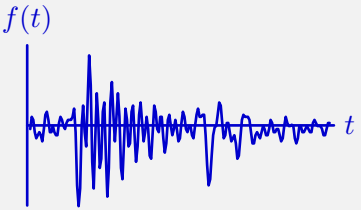


$T =$ sampling interval

commonly used in rendering images 通常用于渲染图像

Check Yourself

Computer generated speech (by Robert Donovan)



Listen to the following four manipulated signals:

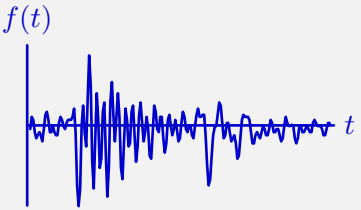
$$f_1(t),\ f_2(t),\ f_3(t),\ f_4(t).$$

How many of the following relations are true?

- $f_1(t) = f(2t)$
- $f_2(t) = -f(t)$
- $f_3(t) = f(2t)$
- $f_4(t) = \frac{1}{3}f(t)$

Check Yourself

Computer generated speech (by Robert Donovan)



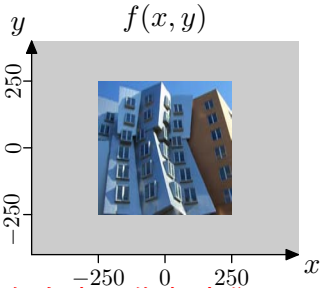
Listen to the following four manipulated signals:

$$f_1(t),\ f_2(t),\ f_3(t),\ f_4(t).$$

How many of the following relations are true? 2

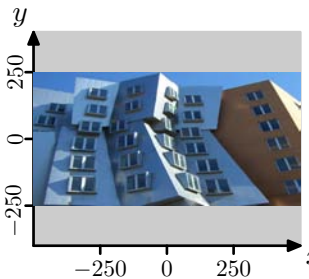
- $f_1(t) = f(2t)$ √
- $f_2(t) = -f(t)$ ×
- $f_3(t) = f(2t)$ ×
- $f_4(t) = \frac{1}{3}f(t)$ √

Check Yourself

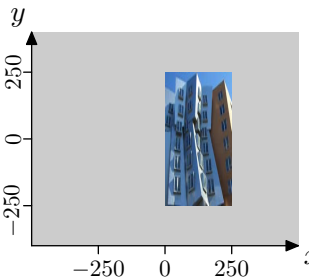


有多少图像与它们下面的表达式相匹配?

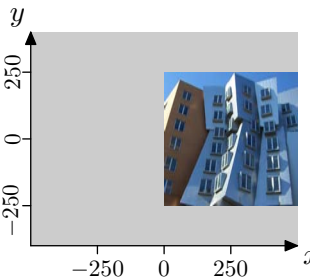
How many images match the expressions beneath them?



$f_1(x,y) = f(2x,y)$?

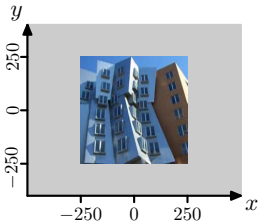


$f_2(x,y) = f(2x-250,y)$?

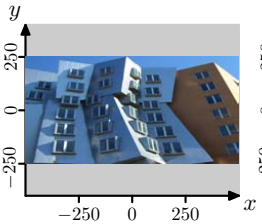


$f_3(x,y) = f(-x-250,y)$?

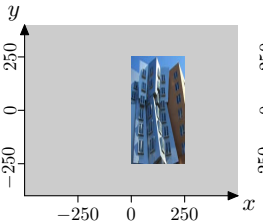
Check Yourself



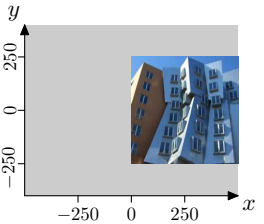
$f(x, y)$



$f_1(x, y) = f(2x, y)$?



$f_2(x, y) = f(2x-250, y)$?



$f_3(x, y) = f(-x-250, y)$?

$x = 0 \quad \rightarrow f_1(0, y) = f(0, y)$ ✓

$x = 250 \quad \rightarrow f_1(250, y) = f(500, y)$ ✗

新图像的250位置应该是原来图像的500位置，所以不对，正确的应该是f1（x,y）=f(1/2x,y)

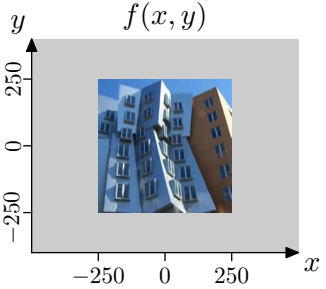
$x = 0 \quad \rightarrow f_2(0, y) = f(-250, y)$ ✓

$x = 250 \quad \rightarrow f_2(250, y) = f(250, y)$ ✓

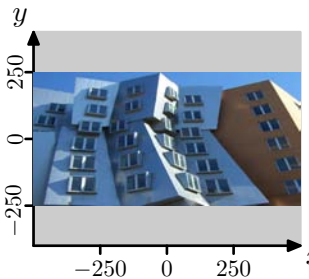
$x = 0 \quad \rightarrow f_3(0, y) = f(-250, y)$ ✗

$x = 250 \quad \rightarrow f_3(250, y) = f(-500, y)$ ✗

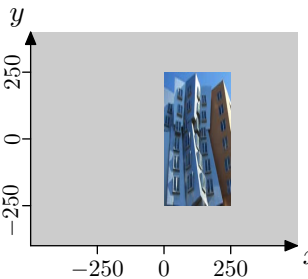
Check Yourself



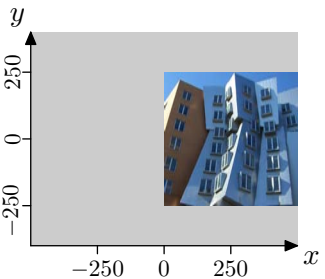
How many images match the expressions beneath them?



~~$f_1(x,y) = f(2x,y)$?~~



$f_2(x,y) = f(2x-250,y)$?



~~$f_3(x,y) = f(\quad x-250,y)$?~~

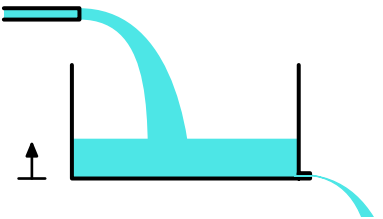
The Signals and Systems Abstraction

Describe a **system** (physical, mathematical, or computational) by the way it transforms an **input signal** into an **output signal**.

描述一个系统(物理的、数学的或计算的)将输入信号转换成输出信号的方式。

Example System: Leaky Tank

Formulate a mathematical description of this system.

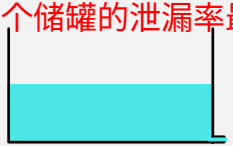


What determines the leak rate?

Check Yourself

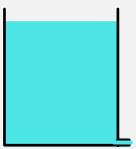
The holes in each of the following tanks have equal size. Which tank has the largest leak rate $r_1(t)$?

下面每个罐的孔大小相等。
哪个储罐的泄漏率最大?



1.

2.

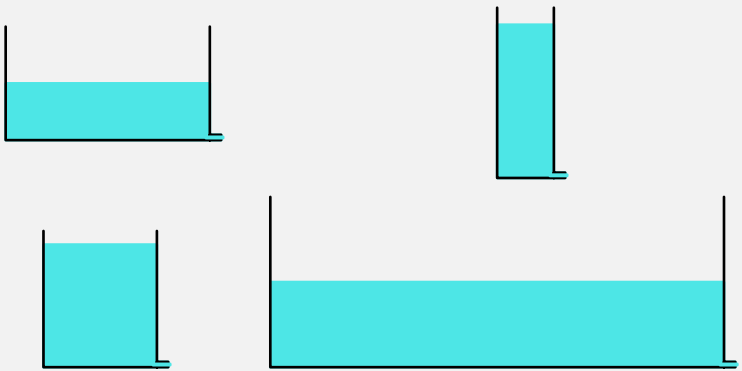


3.

4.

Check Yourself

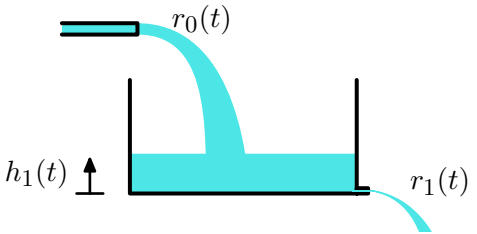
The holes in each of the following tanks have equal size. Which tank has the largest leak rate $r_1(t)$? 2



Example System: Leaky Tank

Formulate a mathematical description of this system.

用数学方法描述这个系统。

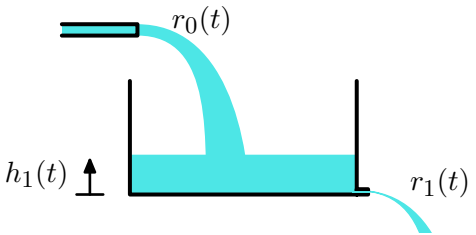


Assume linear leaking: $r_1(t) \propto h_1(t)$

What determines the height $h_1(t)$?

Example System: Leaky Tank

Formulate a mathematical description of this system.



Assume linear leaking: $r_1(t) \propto h_1(t)$

Assume water is conserved: $\dfrac{dh_1(t)}{dt} \propto r_0(t) - r_1(t)$

假设水是守恒的

Solve: $\dfrac{dr_1(t)}{dt} \propto r_0(t) - r_1(t)$

Check Yourself

What are the dimensions of constant of proportionality C?

比例常数C的维数是多少?

$$\frac{dr_1(t)}{dt} = C\Big(r_0(t) - r_1(t)\Big)$$

Check Yourself

What are the dimensions of constant of proportionality C?
inverse time (to match dimensions of dt)

比例常数C的维数是多少?
逆时间(为了匹配dt的维度)

$$\frac{dr_1(t)}{dt} = C\Big(r_0(t) - r_1(t)\Big)$$

Analysis of the Leaky Tank

Call the constant of proportionality $1/\tau$.

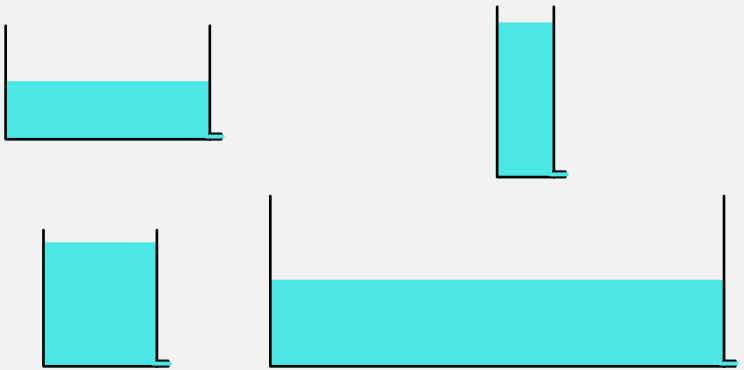
Then τ is called the **time constant** of the system.

$$\frac{dr_1(t)}{dt} = \frac{r_0(t)}{\tau} - \frac{r_1(t)}{\tau}$$

称比例常数为1/T。
称为系统的时间常数。

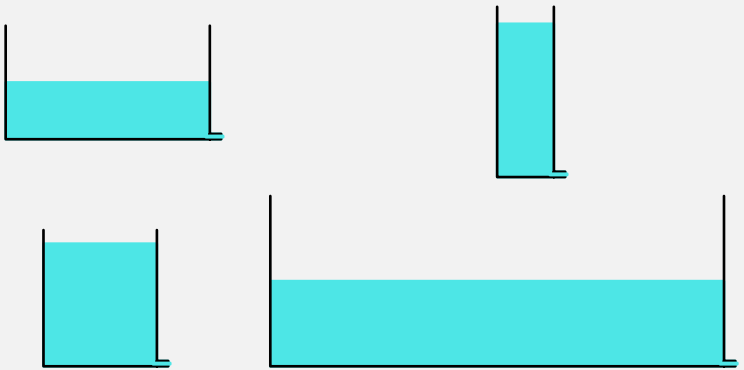
Check Yourself

Which tank has the largest time constant τ?



Check Yourself

Which tank has the largest time constant τ? 4



Analysis of the Leaky Tank

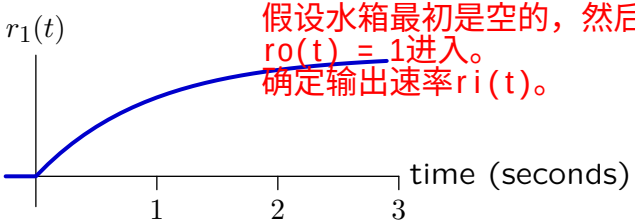
Call the constant of proportionality $1/\tau$.

Then τ is called the **time constant** of the system.

$$\frac{dr_1(t)}{dt} = \frac{r_0(t)}{\tau} - \frac{r_1(t)}{\tau}$$

称比例常数为1/ x。
称为系统的时间常数。

Assume that the tank is initially empty, and then water enters at a constant rate $r_0(t) = 1$. Determine the output rate $r_1(t)$.



假设水箱最初是空的，然后水以恒定速率ro(t) = 1进入。
确定输出速率ri(t)。

Explain the shape of this curve mathematically.

Explain the shape of this curve physically.

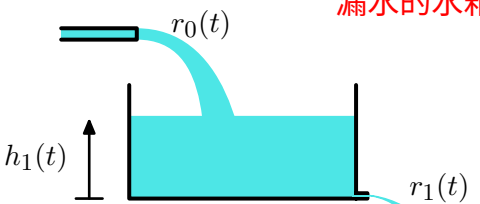
Leaky Tanks and Capacitors

Although derived for a leaky tank, this sort of model can be used to represent a variety of physical systems.

虽然这种模型是为漏槽导出的，但它可以用于表示各种物理系统。

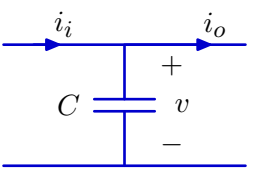
Water accumulates in a leaky tank.

漏水的水箱里积水。



Charge accumulates in a capacitor.

电荷在电容器中积累。



$$\frac{dv}{dt} = \frac{i_i - i_o}{C} \propto i_i - i_o \qquad \text{analogous to} \qquad \frac{dh}{dt} \propto r_0 - r_1$$

6.003 Signals and Systems
Fall 2011