6204 - 21 - S1 - Q1

Q: (a) LP ?

let $X = [x_1 \quad x_2]$ x_1 Exterior paint tons

x_2 Interior paint tons

$$6x_1 + 4x_2 \leq 24$$

$$x_1 + 2x_2 \leq 6$$

$$x_2 - x_1 \leq 1$$

$$x_2 \leq 2$$

$$Z = 5x_1 + 4x_2 \qquad Z \text{ max}$$

(b) Min $Z = -4y_1 - 5y_2 + 0y_3 + 0y_4 + 0y_5 + 0y_6$

Subject to

$$\begin{cases} 4y_1 + 6y_2 + y_3 = 24 & \text{松} \\ 2y_1 + y_2 + y_4 = 6 & \text{松} \\ y_1 - y_2 + y_5 = 1 & \text{松} \\ y_1 + y_6 = 2 & \text{松} \\ y_1, y_2, y_3, y_4, y_5, y_6 \geq 0 \end{cases}$$

SIMPLEX Tab

	y_1	y_2	y_3	y_4	y_5	y_6	
y_3	4	6	1	0	0	0	24
y_4	2	1	0	1	0	0	6
y_5	1	−1	0	0	1	0	1
y_6	1	0	0	0	0	1	2
	−4	−5	0	0	0	0	0

$$A = \begin{bmatrix} 4 & 6 & 1 & 0 & 0 & 0 \\ 2 & 1 & 0 & 1 & 0 & 0 \\ 1 & -1 & 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \quad B = \begin{bmatrix} 24 \\ 6 \\ 1 \\ 2 \end{bmatrix} \quad Y = [y_1\ y_2\ y_3\ y_4\ y_5\ y_6]^T$$

$$Y_0 = [y_3\ y_4\ y_5\ y_6]^T$$

$$C^T = [-4\ -5\ 0\ 0\ 0\ 0]$$

$$C_0^T = [0\ 0\ 0\ 0]$$

$$C^T - C_0^T A = [-4\ -5\ 0\ 0\ 0\ 0]$$

$$-C_0^T B = 0$$

work column ↓

	y_1	y_2	y_3	y_4	y_5	y_6		Ratio
y_3	4	(6)	1	0	0	0	24	24/6 = 4 (min)
y_4	2	1	0	1	0	0	6	6/1 = 6
y_5	1	−1	0	0	1	0	1	
y_6	1	0	0	0	0	1	2	
	−4	(−5)	0	0	0	0	0	

min

	y_1	y_2	y_3	y_4	y_5	y_6	
y_3	4	6	1	0	0	0	4
y_4	2	1	0	1	0	0	6
y_5	1	−1	0	0	1	0	1
y_6	1	0	0	0	0	1	2
	−4	−5	0	0	0	0	0

	y_1	y_2	y_3	y_4	y_5	y_6	
y_2	$\frac{2}{3}$	1	$\frac{1}{6}$	0	0	0	4
y_4	$\frac{4}{3}$	0	$\frac{-1}{6}$	1	0	0	2
y_5	$\frac{5}{3}$	0	$\frac{1}{6}$	0	1	0	5
y_6	1	0	0	0	0	1	2
	$-\frac{2}{3}$	0	$\frac{5}{6}$	0	0	0	20

$\frac{2}{3} = \frac{4}{3}$

$\frac{10}{3} - \frac{12}{3} = \frac{-2}{3}$

	y_1	y_2	y_3	y_4	y_5	y_6		Ratio
y_2	$\frac{2}{3}$	1	$\frac{1}{6}$	0	0	0	4	6
y_4	$\frac{4}{3}$	0	$\frac{-1}{6}$	1	0	0	2	$\frac{3}{2}$ min
y_5	$\frac{5}{3}$	0	$\frac{1}{6}$	0	1	0	5	3
y_6	1	0	0	0	0	1	2	2
	$-\frac{2}{3}$	0	$\frac{5}{6}$	0	0	0	20	

$\widetilde{min}$

$④^{2} \times \frac{3}{②} = 6$ $\quad ② \times \frac{3}{④_2}$ $\quad 5 \times \frac{3}{5}$

	y_1	y_2	y_3	y_4	y_5	y_6	
y_2	$\frac{2}{3}$	0	1	$\frac{1}{6}$ $\frac{1}{4}$	$-\frac{1}{2}$	0	0 $\quad$ 3 $\quad$ $-\frac{2}{3}$
y_4 (y_1)	$\frac{4}{3}$	1	0	$-\frac{1}{6}$ $-\frac{1}{8}$	$\frac{3}{4}$	0	0 $\quad$ $\frac{3}{2}$ $\quad R_2 \times \frac{3}{4}$
y_5	$\frac{5}{3}$	0	0	$\frac{11}{6}$ $\frac{3}{8}$	$-\frac{5}{4}$	1	0 $\quad$ $\frac{5}{2}$ $\quad \frac{5}{3}$
y_6	1	0	0	$\frac{1}{8}$	$-\frac{3}{4}$	0	1 $\quad$ $\frac{1}{2}$
	$\frac{2}{3}$	0	0	$\frac{3}{4}$	$\frac{1}{2}$	0	0 $\quad$ 20 $\quad$ 21 $\quad \frac{2}{3}$

$-\frac{1}{8} \times \frac{3}{4} = -\frac{1}{8}$ $\quad -\frac{1}{8}(-\frac{2}{3}) + \frac{1}{6}$ $\quad \frac{4}{2} - \frac{3}{2} = \frac{1}{2}$

$\frac{1}{8} \times \frac{5}{3} + \frac{1}{6}$ $\quad \frac{3}{4} \times -\frac{3}{3}$ $\quad \frac{3}{2}x - \frac{3}{3} + 5$

$-\frac{1}{8} \times \frac{8}{3} + \frac{5}{6}$ $\quad -\frac{1}{12} + \frac{15}{12} = \frac{9}{12} = \frac{3}{4}$ $\quad 10 - \frac{3}{2} = \frac{5}{2}$

$\frac{3}{24} \frac{1}{8}$

$\frac{3}{8} \times \frac{2}{3} + 20$

	y_1	y_2	y_3	y_4	y_5	y_6	
y_2	0	1	$\frac{1}{4}$	$-\frac{1}{2}$	0	0	3
y_1	1	0	$-\frac{1}{8}$	$\frac{3}{4}$	0	0	$\frac{3}{2}$
y_5	0	0	$\frac{3}{8}$	$-\frac{5}{4}$	1	0	$\frac{5}{2}$
y_6	0	0	$\frac{1}{8}$	$-\frac{3}{4}$	0	1	$\frac{1}{2}$
	0	0	$\frac{3}{4}$	$\frac{1}{2}$	0	0	21

Optimal Solution

$y_1 = \frac{3}{2}$ $y_2 = 3$ $y_3 = y_4 = 0$ $y_5 = \frac{3}{2}$

$y_6 = \frac{1}{2}$ $Z = -21$