Q2

Q: queue 1

$\lambda_1 = 6/h.$

queue 2.

$\lambda_2 = 4/h.$

service time mean $= 5 = \frac{1}{\mu} \Rightarrow \mu = 0.2$

(a) $\cancel{Q}\ L = ?$ $\quad 1-\rho = ?$

(b) 合并 $\cancel{Q}\ L = ?$ $\quad 1-\rho = ?$

(c) 对比

Solution (a) There are 2. M/M/1 systems

For the 1st queue

统一单位 $\lambda_1 = \frac{1}{10}$ customer/min $= \frac{6}{60}$

$\mu_1 = \frac{1}{5}$ customer/min

$$\rho_1 = \frac{\lambda_1}{\mu_1} = \frac{\frac{1}{10}}{\frac{1}{5}} = \frac{1}{2}$$

$$L_1 = \frac{\rho_1}{1-\rho_1} = \frac{\frac{1}{2}}{1-\frac{1}{2}} = 1$$

$$\pi_0 = 1 - \rho_1 = \frac{1}{2}$$

For the 2nd queue.

$\lambda_2 = \frac{4}{60} = \frac{1}{15}$ customer/min

$\mu_2 = \frac{1}{5}$ customer/min

$\rho_2 = \frac{\lambda_2}{\mu_2} = \frac{\frac{1}{15}}{\frac{1}{5}} = \frac{1}{3}$

$L_2 = \frac{\lambda_2}{\mu_2 - \lambda_2} = \frac{\frac{1}{15}}{\frac{1}{5} - \frac{1}{15}} = \frac{\frac{1}{15}}{\frac{2}{15}} = \frac{1}{2}$

$\pi_0 = 1 - \rho_2 = \frac{2}{3}$

the average no. of customer in the system

$L_1 + L_2 = 1 + \frac{1}{2} = \frac{3}{2}$

the proportion of idle time on average server

$\frac{1}{2}\left(\frac{1}{2} + \frac{2}{3}\right) = \frac{1}{2} \times \frac{3+4}{6} = \frac{7}{12}$

(b) This is an M/M/2 system

$\lambda = \frac{10}{60} = \frac{1}{6}$ customer/min

$\mu = \frac{1}{5}$ customer/min

$\rho = \frac{\lambda}{2\mu} = \frac{5}{12}$ $\qquad \pi_0 =$

$L = \frac{\rho (m\rho)^m \pi_0}{m!(1-\rho)^2} + \frac{\lambda}{\mu} = \frac{\frac{5}{12} \times (2 \times \frac{5}{12})^2 \times \pi_0}{2!(1-\frac{5}{12})^2} + \frac{\frac{1}{6}}{\frac{1}{5}}$

$$\pi_0 = \left[\frac{(m\rho)^m}{m!(1-\rho)} + \sum_{k=0}^{m-1}\frac{(m\rho)^k}{k!}\right]^{-1} \qquad m=2$$

$$= \left[\frac{(2\times\frac{5}{12})^2}{2!(1-\frac{5}{12})} + \frac{(2\times\frac{5}{12})^0}{0!} + \frac{(2\times\frac{5}{12})^1}{1!}\right]^{-1}$$

$$= \left(\frac{(\frac{5}{6})^2}{2\times\frac{7}{12}} + 1 + \frac{5}{6}\right)^{-1}$$

$$= \left(\frac{5}{6}\times\frac{5}{6}\times\frac{6}{7} + \frac{5}{6} + 1\right)^{-1}$$

$$= \frac{7}{17} = 2.4286$$

$$L = \frac{\frac{5}{12}\times(2\times\frac{5}{12})^2\times\pi_0}{2!(1-\frac{3}{12})^2} + \frac{\frac{1}{6}}{\frac{1}{5}} = \frac{120}{119} = 1.0084$$

算π_1：因为另一台空闲

$$\pi_1 = \pi_0\frac{(m\rho)^k}{k!} = \frac{7}{17}\times 2\times\frac{5}{12} = \frac{35}{102} = 0.3431$$

idle average $(\pi_0 + \frac{1}{2}\pi_1) = \frac{7}{17} + \frac{1}{2}\times\frac{35}{102} = \frac{7}{12}$

△ 只有一台空闲

△ 与(a)一致.

(c) $L < L_1 + L_2$

$W\downarrow$