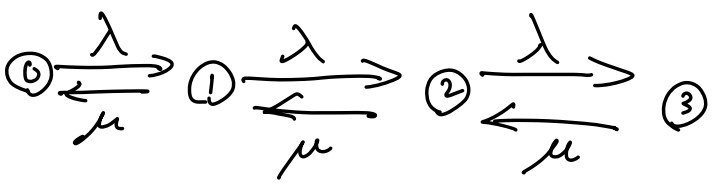
22-S1-Q4

Q(a) M/M/1/3 $\quad \lambda \quad \frac{1}{\mu}$

Solution: state

S is the number of customers in the barber shop

$S = \{0 \quad 1 \quad 2 \quad 3\}$



$$\begin{cases} \pi Q = 0 \\ \pi_0 + \pi_1 + \pi_2 + \pi_3 = 1 \end{cases}$$

$$\begin{cases} \lambda\pi_0 = \mu\pi_1 \\ \lambda\pi_1 + \mu\pi_1 = \lambda\pi_0 + \mu\pi_2 \\ \lambda\pi_2 + \mu\pi_2 = \lambda\pi_1 + \mu\pi_3 \\ \mu\pi_3 = \lambda\pi_2 \\ \pi_0 + \pi_1 + \pi_2 + \pi_3 = 1 \end{cases}$$

$$\pi_1 = \frac{\lambda}{\mu}\pi_0$$

$$\pi_3 = \frac{\lambda}{\mu}\pi_2$$

$$(\lambda+\mu)\frac{\lambda}{\mu}\pi_0 - \lambda\pi_0 = \mu\pi_2$$

$$\frac{\lambda^2}{\mu}\pi_0 = \mu\pi_2$$

$$\pi_2 = \frac{\lambda^2}{\mu^2}\pi_0$$

$$\pi_3 = \frac{\lambda^3}{\mu^2}\pi_0$$

$$\left(1+\frac{\lambda}{\mu}+\frac{\lambda^2}{\mu^2}+\frac{\lambda^3}{\mu^3}\right)\pi_0 = 1$$

$$\frac{\mu^3+\lambda\mu^2+\lambda^2\mu+\lambda^3}{\mu^3}\pi_0 = 1$$

$$\pi_0 = \frac{\mu^3}{\mu^3+\lambda\mu^2+\lambda^2\mu+\lambda^3}$$

$$\pi_1 = \frac{\lambda\mu^2}{\mu^3+\lambda\mu^2+\lambda^2\mu+\lambda^3}$$

$$\pi_2 = \frac{\lambda^2\mu}{\mu^3+\lambda\mu^2+\lambda^2\mu+\lambda^3}$$

$$\pi_3 = \frac{\lambda^3}{\mu^3+\lambda\mu^2+\lambda^2\mu+\lambda^3}$$

$$\pi = \left[\frac{\mu^3}{\mu^3+\lambda\mu^2+\lambda^2\mu+\lambda^3} \quad \frac{\lambda\mu^2}{\mu^3+\lambda\mu^2+\lambda^2\mu+\lambda^3} \quad \frac{\lambda^2\mu}{\mu^3+\lambda\mu^2+\lambda^2\mu+\lambda^3} \quad \frac{\lambda^3}{\mu^3+\lambda\mu^2+\lambda^2\mu+\lambda^3}\right]$$

let $\rho = \frac{\lambda}{\mu}$ $\quad \pi = \left[\frac{1}{1+\rho+\rho^2+\rho^3} \quad \frac{\rho}{1+\rho+\rho^2+\rho^3} \quad \frac{\rho^2}{1+\rho+\rho^2+\rho^3} \quad \frac{\rho^3}{1+\rho+\rho^2+\rho^3}\right]$

when $\rho > 1$, i.e. $\lambda > \mu$ the probabilities increase with the state number

when $\rho < 1$, i.e. $\lambda < \mu$ the probabilities decrease with the state number

(b) M/M/1

Pros
① Easy to implement and manage
② specialization, high quality due to focus
③ customer directly choose queue

Cons
① uneven queues
② inefficiency
③ long queues cause congestion

M/M/1/N

pros ① prevent excessive crowding

cons ① lost sales: when capacity is full
② management complexity: someone need to manage the queue

M/M/m

pros ① reduce variance in waiting time
② maximizes utilization of all staff
③ single queue reduce confusion

cons ① Difficult to coordinate orders across different food types

$M^b/M/1$

pros ① group efficiency
② high sales volume

cons ① Inconsistent arrival Patterns

② Resource strain

Recommendation M/NV I / N