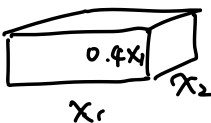23 −S1 − Q2

Q: $V = 98\ cm^3$ $L = 2H$ closed.   \$10/cm^3

(a) Q: length x_1 breadth x_2 cost $Z = ?$

Solution $Z = 10 \times [2 \times (x_1 x_2 + 0.4 x_1^2 + 0.4 x_1 x_2)]$

subject to $\begin{cases} V = x_1 \cdot x_2 \cdot \dfrac{x_1}{2.5} = 0.4 x_1^2 x_2 = 98 \\ x_1, x_2 \geqslant 0 \end{cases}$

where $Z = 20 \times (1.4 x_1 x_2 + 0.4 x_1^2)$

$\quad = 28 x_1 x_2 + 8 x_1^2$

$\quad x_1^2 x_2 = 245$

$\quad x_1, x_2 \geqslant 0$

so $\quad Z = 28 x_1 x_2 + 8 x_1^2$

subject to $\begin{cases} x_1^2 x_2 = 245 \\ \\ x_1, x_2 \geqslant 0 \end{cases}$

(b) apply Lagrange multipliers

Solution
$\quad L = 28 x_1 x_2 + 8 x_1^2 + \lambda (x_1^2 x_2 - 245)$

$$\begin{cases} \dfrac{\partial L}{\partial x_1} = 28x_2 + 16x_1 + 2\lambda x_1 x_2 = 0 & (1) \\[2em] \dfrac{\partial L}{\partial x_2} = 28x_1 + \lambda x_1^2 = 0 & (2) \\[2em] \dfrac{\partial L}{\partial \lambda} = x_1^2 x_2 - 245 = 0 & (3) \end{cases}$$

from (3) $x_1 \neq 0$

from (2) $\lambda x_1 = -28 \implies x_1 = -\dfrac{28}{\lambda} \implies \boxed{\lambda = \dfrac{-28}{x_1}}$

into (1) get

from (3) $x_2 = \dfrac{245}{x_1^2} = \dfrac{245\lambda^2}{28^2} = \dfrac{5}{16}\lambda^2$

$x_2 = \dfrac{4x_1}{7}$

from (1) $28 \times \dfrac{5}{16}\lambda^2 + 16\left(-\dfrac{28}{\lambda}\right) + 2\lambda\left(-\dfrac{28}{\lambda}\right)\dfrac{5}{16}\lambda^2 = 0$

$\dfrac{35}{4}\lambda^2 - \dfrac{448}{\lambda} - \dfrac{35}{2}\lambda^2 = 0$

from (3)

$\dfrac{4}{7}x_1^3 = 245$

$\boxed{\dfrac{35}{4}\lambda^2 - \dfrac{35}{2}\lambda - 448 = 0$

$\lambda_{1,2} = \dfrac{5 \pm 3\sqrt{145}}{5} \quad \times$

$\lambda_1 = 8.2250 \qquad \lambda_2 = -6.2250}$

$x_1 = 7.5405$

more fast!

$-8.75\lambda^2 - \dfrac{448}{\lambda} = 0$

$-8.75\lambda^3 = 448$

$\lambda^3 = -51.2$

$\lambda = -3.7133$

$$x_1 = -\frac{28}{\lambda} = 7.5405$$

$$x_2 = \frac{5}{16} \lambda^2 = 4.3089$$

$$\text{cost } Z = 28 x_1 x_2 + 8 x_1^2 = 1364.6284$$

(c) -5%, Z?

Solution

$$\Delta V = -0.05 \times 98 = -4.9 \text{ cm}^3$$

$$V_{new} = 98 - 4.9 = 93.1 \text{ cm}^3$$

new constraint $x_1^2 x_2 = \dfrac{93.1}{0.4} = 232.75$

constrain change $\Delta \beta = 232.75 - 245 = -12.25$

$$Z_{new} = Z_{old} - \Delta \beta \lambda$$

$$= 1364.6284 - (-12.25) \times (-3.7133)$$

$$= 1319.1405$$