Example 4.2

Q total profit = ?

$M/M/1/N \quad \lambda \quad \mu \quad C\mu \quad q$

Solution $P = U\mu q - C\mu$

$$U = 1 - \pi_0 = 1 - \frac{1-\rho}{1-\rho^{N+1}} = \frac{1-\rho^{N+1} - 1 + \rho}{1-\rho^{N+1}}$$

$$= \frac{\rho - \rho^{N+1}}{1-\rho^{N+1}}$$

$$P = \frac{\rho - \rho^{N+1}}{1-\rho^{N+1}}\mu q - C\mu \qquad \rho = \frac{\lambda}{\mu}$$

$$= \frac{\frac{\lambda}{\mu} - \frac{\lambda^{N+1}}{\mu^{N+1}}}{1 - \frac{\lambda^{N+1}}{\mu^{N+1}}}\mu q - C\mu$$

$$= \frac{\lambda\mu(\mu^N - \lambda^N)}{\mu^{N+1} - \lambda^{N+1}} q - C\mu$$

denote $R = \mu^N - \lambda^N$

$S = \mu^{N+1} - \lambda^{N+1}$

$$p = q\lambda\mu\left(\frac{R}{S}\right) - C\mu$$

$$\frac{dp}{d\mu} = q\lambda\left(\frac{d}{d\mu}\left(\mu \cdot \frac{R}{S}\right)\right) - C$$

$$\frac{d}{d\mu}\left(\mu \cdot \frac{R}{S}\right) = \frac{R}{S} + \mu\left(\frac{R'S - RS'}{S^2}\right)$$

$$R' = N\mu^{N-1}$$

$$S' = (N+1)\mu^N$$

$$\frac{dp}{d\mu} = q\lambda\left(\frac{R}{S} + \mu\left(\frac{N\mu^{N-1}S - R(N+1)\mu^N}{S^2}\right)\right) - C$$

let $\frac{dp}{d\mu} = 0$ $\quad \hat{\mu} \quad$ Newton - Raphson

$\frac{d^2p}{d\mu^2} < 0$. $\quad$ maximum