Problem 2.4 $\quad X_1 = e^{\lambda_1} \qquad X_2 = e^{\lambda_2}$

(a) prove $p(X_1 < X_2) = \dfrac{\lambda_1}{\lambda_1 + \lambda_2}$

(b) $X = \min\{X_1, X_2\}$

prove $X = \mathrm{EXP}(\lambda_1 + \lambda_2)$

Solution (a) $p(X_1 < X_2) = 1 - e^{-\lambda_1 x_2}$

$$f_{X_1}(x) = \lambda_1 e^{-\lambda_1 x}, \quad x \geq 0$$

$$f_{X_2}(x) = \lambda_2 e^{-\lambda_2 x}, \quad x \geq 0$$

? → 佰面A

$$p(X_1 < X_2) = \int_0^\infty p(X_2 > x) f_1(x)\, dx$$

$$= \int_0^\infty e^{-\lambda_2 x} \lambda_1 e^{-\lambda_1 x}\, dx$$

$$= \lambda_1 \int_0^\infty e^{-(\lambda_1 + \lambda_2)x}\, dx$$

$$= \lambda_1 \left[\frac{e^{-(\lambda_1+\lambda_2)x}}{-(\lambda_1+\lambda_2)} \Bigg|_0^\infty \right]$$

$$= \frac{\lambda_1}{\lambda_1 + \lambda_2}$$

(b) $P(X > x) = P(X_1 > x, X_2 > x)$

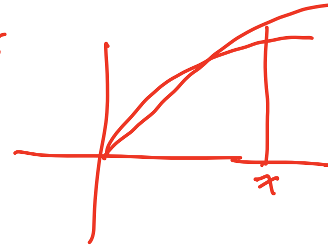
$$= P(X_1 > x)\, P(X_2 > x)$$

$$= e^{-\lambda_1 x} e^{-\lambda_2 x}$$

$$= e^{-(\lambda_1 + \lambda_2) x}$$

So $X = \mathrm{Exp}(\lambda_1 + \lambda_2)$

$$P(X_1 < X_2) = \int_{-\infty}^{\infty} P(X_1 = x, X_2 > x)\, dx$$



$$P(A|B) = \frac{P(AB)}{P(B)}$$

$$= \int_{-\infty}^{\infty} P(X_2 > x \mid x_1 = x)\, P(X_1 = x)\, dx$$

$$= \int_{-\infty}^{\infty} P(X_2 > x \mid x_1 = x)\, f_{X_1}(x)\, dx$$

X_1 X_2 独立

$$= \int_{-\infty}^{\infty} P(X_2 > x)\, f_{X_1}(x)\, dx$$

$x \geq 0$ 定义

$$= \int_{0}^{\infty} P(X_2 > x)\, f_{X_1}(x)\, dx$$