21-S1 – Q3

Q: (a) $P(a,b)=?$ $P(0,b)=?$ $P(a,\infty)=\infty$

Solution (a) $P(a,b)=P(a\le X\le b)$

$$=\sum_{k=a}^{b}\frac{e^{-\lambda t}(\lambda t)^k}{k!}$$

$$P(0,b)=P(0\le X\le b)$$

$$=\sum_{k=0}^{b}\frac{e^{-\lambda t}(\lambda t)^k}{k!}$$

$$P(a,\infty)=P(X\ge a)$$

$$=1-P(X<a)$$

$$=1-\sum_{k=0}^{a-1}\frac{e^{-\lambda t}(\lambda t)^k}{k!}$$

Q(b)(i) TPM=?

Solution $P=[P_{ij}]$

state space $\{w_0 \quad w_1 \quad w_2 \quad w_3\}$

let D_{k+1} 为下个月发的奖品数, X_{k+1} 为下个月销售总量

$P_{00}=P\{D_{k+1}=3\}$ 即上月底发完, 下月底也发完

$=P\{X_{k+1}\ge 30\}=P(30,\infty)$

$P_{01} = P\{D_{k+1} = 2\} = P\{20 \le X_{k+1} \le 29\} = P(20, 29)$

$P_{02} = P(10, 19)$

$P_{03} = P(0, 9)$

$P_{10} = P(10, \infty)$

$P_{11} = P(0, 9)$

$P_{12} = 0$

$P_{13} = 0$

$P_{20} = P(20, \infty)$

$P_{21} = P(10, 19)$

$P_{22} = P(0, 9)$

$P_{23} = 0$

$P_{30} = P(30, \infty)$

$P_{31} = P(20, 29)$

$P_{32} = P(10, 19)$

$P_{33} = P(0, 9)$

TPM is

$$\begin{bmatrix} P(30,\infty) & P(20,29) & P(10,19) & P(0,9) \\ P(10,\infty) & P(0,9) & 0 & 0 \\ P(20,\infty) & P(10,19) & P(0,9) & 0 \\ P(30,\infty) & P(20,29) & P(10,19) & P(0,9) \end{bmatrix}$$

(ii) May已经推 $W_{May} = 0$ or 1 P(August已经推)?

Solution P代表用初礼品个数

Case 1: $W_{May} = 1$

P(August giving out 3)

$= P(W_{July} = 0 \text{ or } 3)\, P(30, \infty)$

$= \{ P(0,9)\, P(10,\infty) + P(10,\infty)[P(0,9) + P(30,\infty)] \}\, P(30,\infty)$

Probability Map

W_{May}	P_{June}	X_{June}	W_{June}	P_{July}	X_{July}	W_{July}
1	1	P(0,9)	1	1	P(0,9)	1
					P(10,∞)	0 ✓
		P(10,∞)	0	3	P(0,9)	3 ✓
					P(10,19)	2
					P(20,29)	1
					P(30,∞)	0 ✓

0	3	P(0,9)	3	3	P(0,9)	3 ✓
					P(10,19)	2
					P(20,29)	1
					P(30,∞)	0 ✓
		P(10,19)	2	2	P(0,9)	2
					P(10,19)	1
					P(20,∞)	0 ✓
		P(20,29)	1	1	P(0,9)	1
					P(10,∞)	0 ✓
		P(30,∞)	0	3	P(0,9)	3 ✓
					P(10,19)	2
					P(20,29)	1
					P(30,∞)	0 ✓

Case 2. $W_{May} = 0$

$P(\text{August giving out } 3)$

$= P(W_{\text{July}} = 0 \text{ or } 3)\, P(30,\infty)$

$= \{P(0,9)[P(0,9) + P(30,\infty)] + P(10,19)\,P(20,\infty)$

$+ P(20,29)\,P(10,\infty) + P(30,\infty)[P(0,9)+P(30,\infty)]\}\, P(30,\infty)$

胡总. 思路：用 TPM

case 1 $P(\text{August give out } 3) = P(30,\infty)\left[\begin{matrix} P(W_7=0 \mid W_5=0) + P(W_7=3 \mid W_5=0) \\ + P(W_7=0 \mid W_5=1) + P(W_7=3 \mid W_5=1)\end{matrix}\right]$

其中 $P(W_7=0 \mid W_5=0) + P(W_7=3 \mid W_5=0)$

$+ P(W_7=0 \mid W_5=1) + P(W_7=3 \mid W_5=1)$

$= P^2|_{(0,0)} + (0,3) + (1,0) + (1,3)$

$= P(30,\infty)P(30,\infty) + P(20,29)\,P(10,\infty)$

$+ \cdots$

$W_8 = 0$

$P_8 = 0$ give 0

$P_8 = 1$ give 1

$P_8 = 2$ give 2

$P_8 = 3$ give 3 ← 影响多

$$\begin{bmatrix} P(30,\infty) & P(20,29) & P(10,19) & P(0,9) \\ P(10,\infty) & P(0,9) & 0 & 0 \\ P(20,\infty) & P(10,19) & P(0,9) & 0 \\ P(30,\infty) & P(20,29) & P(10,19) & P(0,9) \end{bmatrix} \begin{bmatrix} P(30,\infty) & P(20,29) & P(10,19) & P(0,9) \\ P(10,\infty) & P(0,9) & 0 & 0 \\ P(20,\infty) & P(10,19) & P(0,9) & 0 \\ P(30,\infty) & P(20,29) & P(10,19) & P(0,9) \end{bmatrix}$$