Problem 3.2

Q 证明 $D=E(d)=\frac{\lambda}{\mu(\mu-\lambda)}$

$$F_d(t)=\begin{cases}1-\rho, & t=0\\ 1-\rho e^{-\mu(1-\rho)t}, & t>0\end{cases}$$

Solution ① get $f_d(t)$

$$f_d(t)=\frac{\partial F_d(t)}{\partial t}=\begin{cases}0, & t=0\\ \mu(1-\rho)\rho e^{-\mu(1-\rho)t}, & t>0\end{cases}$$

② $E(d)=\int_0^\infty t\, f_d(t)\, dt$

$=\int_0^\infty t\mu(1-\rho)\rho e^{-\mu(1-\rho)t}\, dt$

$=\mu(1-\rho)\rho\int_0^\infty t\, e^{-\mu(1-\rho)t}\, dt$

其中 $\int_0^\infty t\, e^{-\mu(1-\rho)t}\, dt$ 令 $m=-\mu(1-\rho)$

$m<0$

$=\int_0^\infty t\, \frac{d\, e^{mt}}{m}$

$=\frac{1}{m}\left[te^{mt}\Big|_0^\infty-\int_0^\infty e^{mt}\, dt\right]=\frac{1}{m^2}$

期 $\int_0^\infty e^{mt}\,dt$

$$= \left.\frac{e^{mt}}{m}\right|_0^\infty$$

$$= -\frac{1}{m} \qquad m = -\mu(1-\rho)$$

$E(d) = \mu(1-\rho)\rho$

$$= \frac{\mu(1-\rho)\rho}{\mu^2(1-\rho)^2}$$

$$= \frac{\rho}{\mu(1-\rho)}$$

$\rho = \frac{\lambda}{\mu} \Rightarrow \quad E(d) = \frac{\frac{\lambda}{\mu}}{\mu(1-\frac{\lambda}{\mu})} = \frac{\mu\lambda}{\mu^2(\mu-\lambda)} = \frac{\lambda}{\mu(\mu-\lambda)}$