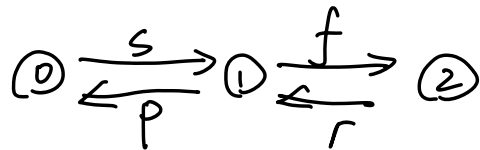


# Example 9.1

Q Case 1 resume policy.

draw? TPM = ?  $\pi$  = ?



$$TPM = \begin{bmatrix} q_{00} & q_{01} & q_{02} \\ q_{10} & q_{11} & q_{12} \\ q_{20} & q_{21} & q_{22} \end{bmatrix} = \begin{bmatrix} -s & s & 0 \\ p & 1-p-f & f \\ 0 & r & -r \end{bmatrix}$$

牢记公式

$$\begin{cases} \pi = \pi Q \\ \pi_0 + \pi_1 + \pi_2 = 1 \end{cases}$$

$$\begin{cases} -s\pi_1 + s\pi_2 = 0 \\ p\pi_1 + (1-p-f)\pi_2 + f\pi_3 = 0 \\ r\pi_2 - r\pi_3 = 0 \\ \pi_1 + \pi_2 + \pi_3 = 1 \end{cases}$$

$$\begin{cases} \pi_1 = -s\pi_1 + s\pi_2 & (1) \\ \pi_2 = p\pi_1 + (1-p-f)\pi_2 + f\pi_3 & (2) \\ \pi_3 = r\pi_2 - r\pi_3 & (3) \\ \pi_1 + \pi_2 + \pi_3 = 1 & (4) \end{cases}$$

$$(1): (1+s)\pi_1 - s\pi_2 = 0 \\ \pi_2 = \frac{1+s}{s}\pi_1$$

$$(3): (1+r)\pi_3 = r\pi_2$$

$$x_3 = \frac{r}{1+r} x_2 = \frac{r}{1+r} \frac{1+s}{s} x_1$$

$$(4) \quad \left(1 + \frac{1+s}{s} + \frac{r}{1+r} \frac{1+s}{s}\right) x_1 = 1$$

$$x_1 = \frac{1}{\frac{(1+r)s + (1+s)(1+r) + r(1+s)}{(1+r)s}}$$

$$= \frac{s(1+r)}{s + rs + 1 + r + s + sr + r + rs}$$

$$= \frac{s + sr}{3rs + 2s + 2r + 1}$$

$$\begin{bmatrix} x_0 & x_1 & x_2 \end{bmatrix} \begin{bmatrix} -s & s & 0 \\ p & -p-f & f \\ 0 & r & -r \end{bmatrix} \quad \text{不要乘错}$$

$$\begin{cases} -sx_0 + px_1 = 0 & (1) \\ sx_0 + (-p-f)x_1 + rx_2 = 0 & (2) \\ fx_1 - rx_2 = 0 & (3) \\ x_0 + x_1 + x_2 = 1 & (4) \end{cases}$$

$$(1) : x_1 = \frac{s}{p} x_0$$

$$(3): \pi_2 = \frac{f}{r} \pi_1$$

$$(4): \pi_0 + \frac{s}{p} \pi_0 + \frac{f}{r} \frac{s}{p} \pi_0 = 1$$

$$\left(1 + \frac{s}{p} + \frac{fs}{rp}\right) \pi_0 = 1$$

$$\frac{rp + rs + fs}{rp} \pi_0 = 1$$

$$\pi_0 = \frac{rp}{rp + rs + fs}$$

$$\pi_1 = \frac{s}{p} \frac{rp}{rp + rs + fs} = \frac{sr}{rp + rs + fs}$$

$$\pi_2 = \frac{f}{r} \frac{sr}{rp + rs + fs} = \frac{fs}{rp + rs + fs}$$

Q:  $s = 20/h$      $p = 4/h$      $f = 0.04/h$      $r = 1/h$

$\pi = ?$      $R = ?$     steady-state availability = ?

Mean completion time

$$\text{Solution } \pi = [\pi_0 \ \pi_1 \ \pi_2] = \left[ \frac{4}{4 + 20 + 1} \quad \frac{20}{25} \quad \frac{1}{25} \right]$$

$$= [0.16 \quad 0.8 \quad 0.04]$$

$$\begin{aligned}\text{Average production rate } R &= \lambda_1 p \\ &= 0.8 \times 4 \\ &= 3.2\end{aligned}$$

$$\text{steady-state availability} = \left(1 + \frac{f}{r}\right)^{-1}$$

$$\begin{aligned}\text{why it is not equal} &= \left(1 + \frac{0.05}{1}\right)^{-1} \\ \text{to } \pi(1) \text{ } 0.8? &= \frac{20}{21} \\ &= 0.9524\end{aligned}$$

$$\text{Mean completion time} = \left(1 + \frac{f}{r}\right) \frac{1}{p}$$

$$= \left(1 + \frac{0.05}{1}\right) \frac{1}{4}$$

$$= \frac{21}{80}$$

$$= 0.2625$$

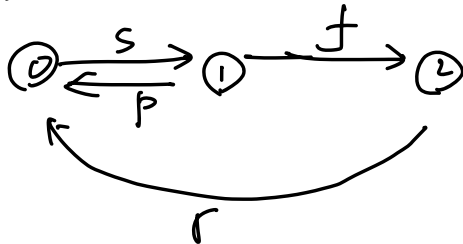
$$= \frac{\text{mean processing time}}{\text{steady-state availability}}$$

$$\frac{21}{80} = \frac{t}{\frac{20}{21}} \quad t = \frac{1}{4} \text{ hour/1}$$

Case 2. discard policy

Q draw = ? TP M = ?  $\pi = ?$

Solution



$$Q = \begin{bmatrix} q_{00} & q_{01} & q_{02} \\ q_{10} & q_{11} & q_{12} \\ q_{20} & q_{21} & q_{22} \end{bmatrix} = \begin{bmatrix} -s & s & 0 \\ p & -p-f & f \\ r & 0 & -r \end{bmatrix}$$

$$\begin{cases} \pi Q = 0 \\ \pi_0 + \pi_1 + \pi_2 = 1 \end{cases} \quad \begin{cases} -s\pi_0 + p\pi_1 + r\pi_2 = 0 \\ s\pi_0 - (p+f)\pi_1 = 0 \\ f\pi_1 - r\pi_2 = 0 \\ \pi_0 + \pi_1 + \pi_2 = 1 \end{cases}$$

$$\pi_2 = \frac{f}{r} \pi_1 \quad \pi_1 = \frac{s}{p+f} \pi_0$$

$$\pi_0 + \frac{s}{p+f} \pi_0 + \frac{f}{r} \frac{s}{p+f} \pi_0 = 1$$

$$\frac{rp+rf+sr+fs}{r(p+f)} \pi_0 = 1$$

$$\pi_0 = \frac{rp+rf}{rp+rf+sr+fs}$$

$$z_1' = \frac{s}{p+f} z_0 = \frac{sr}{rp+rf+sr+fs}$$

$$z_2' = \frac{f}{r} z_1 = \frac{sf}{rp+rf+sr+fs}$$

Q  $s = 20/h$     $p = 4/h$     $f = 0.05/h$     $r = 1/h$

$z_0' = ?$     $R = ?$

Solution  $rp+rf+sr+fs = 4 + 0.05 + 20 + 1 = 25.05$

$$z_0' = \frac{4.05}{25.05} = 0.1617$$

$$z_1' = \frac{20}{25.05} = 0.7984$$

$$z_2' = \frac{1}{25.05} = 0.03992$$

$$R = z_1' p = 0.7984 \times 4$$

$$= 3.1936 \quad ? \text{ why } 3.08 ?$$

$$\frac{1}{\frac{1}{s} + \frac{f}{sp} + \frac{1}{p} + \frac{f}{rp}} \quad \text{老师展开}$$

错了