Q1

Q: PC 每周定

D₁ 一周定   D₂ 2周定

D: Poisson r.x. $\lambda = 1$

$0 \le X_0 \le 3$ 开始库存

X₁ 第一周 (结束)库存量 X_t t week end

周六晚及时送达

周一 订单 (s, S) $\begin{cases} S=0, & s=3 \\ \text{other}, & \text{不下订单} \end{cases}$

(a) X_t $X_t \sim X_{t+1}$

(b) TPM in (a)

(c) 稳态概率

Solution state space for X_t $S = \{0, 1, 2, 3\}$

if $X_t = 0$, $X_{t+1} = \max\{3 - D_{t+1}, 0\}$

if $X_t = \{1, 2, 3\}$ $X_{t+1} = \max\{X_t - D_{t+1}, 0\}$

(b) $P = \{P_{ij}\}$ $i, j \in \{0, 1, 2, 3\}$ $P(D_t = k) = \dfrac{e^{-\lambda}\lambda^k}{k!}$

$P_{ij} = Pr(X_{t+1} = j \mid X_t = i)$ $= \dfrac{1}{k! \, e}$

$P_{00} = Pr(D_{t+1} \ge 3) = \displaystyle\sum_{k=3}^{\infty} \dfrac{1}{k! \, e}$

$= 1 - Pr(D_{t+1} < 3) = 1 - \displaystyle\sum_{k=0}^{2} \dfrac{1}{k! \, e} = 1 - \left(\dfrac{1}{e} + \dfrac{1}{e} + \dfrac{1}{2e}\right)$

$= 1 - \dfrac{5}{2e} = 0.08$

$$P_{01} = Pr(D_{t+1} = 2) = \frac{1}{2! \, e} = \frac{1}{2e} = 0.184$$

$$P_{02} = Pr(D_{t+1} = 1) = \frac{1}{1! \, e} = \frac{1}{e} = 0.368$$

$$P_{03} = Pr(D_{t+1} = 0) = \frac{1}{0! \, e} = \frac{1}{e} = 0.368$$

$$P_{10} = Pr(D_{t+1} \geq 1) = 1 - Pr(D_{t+1} < 1) = 1 - \frac{1}{e} = 0.632$$

$$P_{11} = Pr(D_{t+1} = 0) = \frac{1}{e} = 0.368$$

$$P_{12} = 0$$

$$P_{13} = 0$$

$$P_{20} = Pr(D_{t+1} \geq 2) = 1 - Pr(D_{t+1} < 2) = 1 - \frac{1}{e} - \frac{1}{e} = 0.264$$

$$P_{21} = Pr(D_{t+1} = 1) = \frac{1}{e} = 0.368$$

$$P_{22} = Pr(D_{t+1} = 0) = \frac{1}{e} = 0.368$$

$$P_{23} = 0$$

$$P_{30} = Pr(D_{t+1} \geq 3) = 1 - Pr(D_{t+1} < 3) = 1 - \frac{1}{e} - \frac{1}{e} - \frac{1}{2e}$$

$$= 0.08$$

$$P_{31} = Pr(D_{t+1} = 2) = \frac{1}{2e} = 0.184$$

$$P_{32} = Pr(D_{t+1} = 1) = \frac{1}{e} = 0.368$$

$$P_{33} = P(D_{t+1} = 0) = \frac{1}{e} = 0.368$$

TPM

$$P = \begin{bmatrix} 0.08 & 0.184 & 0.368 & 0.368 \\ 0.632 & 0.368 & 0 & 0 \\ 0.264 & 0.368 & 0.368 & 0 \\ 0.08 & 0.184 & 0.368 & 0.368 \end{bmatrix}$$

(c) $\pi_j = \lim_{n \to \infty} P_j(n) = \lim_{n \to \infty} \Pr(X_n = j)$

let $Y = [\pi_0 \quad \pi_1 \quad \pi_2 \quad \pi_3]$

$$\begin{cases} Y = YP \\ \pi_0 + \pi_1 + \pi_2 + \pi_3 = 1 \\ \pi_j \geq 0 \quad j = 0, 1, 2, 3 \end{cases}$$

怎么用计算器算?

$fx - 991CN\ X$

$$Y = Y \begin{bmatrix} 0.08 & 0.184 & 0.368 & 0.368 \\ 0.632 & 0.368 & 0 & 0 \\ 0.264 & 0.368 & 0.368 & 0 \\ 0.08 & 0.184 & 0.368 & 0.368 \end{bmatrix}$$

$$\begin{cases} \pi_0 = 0.08\,\pi_0 + 0.632\,\pi_1 + 0.264\,\pi_2 + 0.08\,\pi_3 \\ \pi_1 = 0.184 \quad\quad 0.368 \quad\quad 0.368 \quad\quad 0.184 \\ \pi_2 = 0.368 \quad\quad 0 \quad\quad 0.368 \quad\quad 0.368 \\ \pi_3 = 0.368 \quad\quad 0 \quad\quad 0 \quad\quad 0.368 \end{cases}$$

$$| \quad | = \quad | \qquad | \qquad | \qquad |$$

$$\begin{cases} -0.92z_0 + 0.632z_1 + 0.264z_2 + 0.08z_3 = 0 \\ 0.184z_0 - 0.632z_1 + 0.368z_2 + 0.184z_3 = 0 \\ 0.368z_0 - 0.632z_2 + 0.368z_3 = 0 \\ 0.368z_0 - 0.632z_3 = 0 \\ z_0 + z_1 + z_2 + z_3 = 1 \end{cases}$$

冗余

(3)

换4个

进来凑

就行

(2)

(1)

(4)

from(1) $\quad z_3 = \dfrac{46}{79} z_0$

from(2) $\quad z_2 = \dfrac{46}{79}\left(1 + \dfrac{46}{79}\right) z_0 = \dfrac{5750}{6241} z_0$

from(3) $\quad z_1 = \dfrac{23}{79} z_0 + \dfrac{46}{79} z_2 + \dfrac{23}{79} z_3$

$$= \left(\dfrac{23}{79} + \dfrac{46}{79} \times \dfrac{5750}{6241} + \dfrac{23}{79} \times \dfrac{46}{79}\right) z_0$$

$$= 0.9971 z_0$$

from (4) $\quad z_0 = 0.2857$

$z_1 = 0.2848$

$z_2 = 0.2632$

$z_3 = 0.1663$