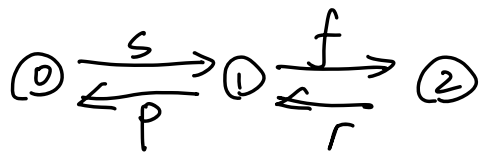
Example 9.1

Q Case 1 resume policy.

draw? TPM = ? π = ?



$$TPM = \begin{bmatrix} q_{00} & q_{01} & q_{02} \\ q_{10} & q_{11} & q_{12} \\ q_{20} & q_{21} & q_{22} \end{bmatrix} = \begin{bmatrix} -s & s & 0 \\ p & 1-p-f & f \\ 0 & r & -r \end{bmatrix}$$

平衡式

$$\begin{cases} \pi = \pi Q \quad \pi Q = 0 \\ \pi_0 + \pi_1 + \pi_2 = 1 \end{cases}$$

$$\begin{cases} -s\pi_1 + s\pi_2 = 0 \\ p\pi_1 + (1-p-f)\pi_2 + f\pi_3 = 0 \\ r\pi_2 - r\pi_3 = 0 \\ \pi_1 + \pi_2 + \pi_3 = 1 \end{cases}$$

$$\begin{cases} \pi_1 = -s\pi_1 + s\pi_2 \quad (1) \\ \pi_2 = p\pi_1 + (1-p-f)\pi_2 + f\pi_3 \quad (2) \\ \pi_3 = r\pi_2 - r\pi_3 \quad (3) \\ \pi_1 + \pi_2 + \pi_3 = 1 \quad (4) \end{cases}$$

(1): $(1+s)\pi_1 - s\pi_2 = 0$

$$\pi_2 = \frac{1+s}{s}\pi_1$$

(3) $(1+r)\pi_3 = r\pi_2$

$$z_3 = \frac{r}{1+r} z_2 = \frac{r}{1+r} \frac{1+s}{s} z_1$$

(4) $$\left(1 + \frac{1+s}{s} + \frac{r}{1+r}\frac{1+s}{s}\right) z_1 = 1$$

$$z_1 = \frac{1}{\frac{(1+r)s + (1+s)(1+r) + r(1+s)}{s(1+r)}}$$

$$= \frac{s(1+r)}{s + rs + r + 1 + s + sr + rs}$$

$$= \frac{s+sr}{1+2s+3sr}$$

$[z_0\ z_1\ z_2]$

$[z_1\ z_2\ z_3]$

$$\begin{bmatrix} -s & s & 0 \\ p & -p-f & f \\ 0 & r & -r \end{bmatrix}$$

不要乘它的

$$\begin{cases} -sz_0 + pz_1 = 0 & (1) \\ sz_0 + (-p-f)z_1 + rz_2 = 0 & (2) \\ fz_1 - rz_2 = 0 & (3) \\ z_0 + z_1 + z_2 = 1 & (4) \end{cases}$$

(1) : $z_1 = \frac{s}{p} z_0$

(3): $\pi_2 = \frac{f}{r}\pi_1$

(4): $\pi_0 + \frac{s}{p}\pi_0 + \frac{f}{r}\frac{s}{p}\pi_0 = 1$

$$\left(1+\frac{s}{p}+\frac{fs}{rp}\right)\pi_0 = 1$$

$$\frac{rp+rs+fs}{rp}\,\pi_0 = 1$$

$$\pi_0 = \frac{rp}{rp+rs+fs}$$

$$\pi_1 = \frac{s}{p}\,\frac{rp}{rp+rs+fs} = \frac{sr}{rp+rs+fs}$$

$$\pi_2 = \frac{f}{r}\,\frac{sr}{rp+rs+fs} = \frac{fs}{rp+rs+fs}$$

Q: $s = 20/h$ $p = 4/h$ $f = 0.05/h$ $r = 1/h$

$\pi_0 = ?$ $R = ?$ steady-state availability = ?

Mean completion time

Solution $\pi = [\pi_0\ \pi_1\ \pi_2] = \left[\frac{4}{4+20+1} \quad \frac{20}{25} \quad \frac{1}{25}\right]$

$= [0.16 \quad 0.8 \quad 0.04]$

Average production rate $R = \pi_1 P$

$= 0.8 \times 4$

$= 3.2$

steady-state availability $= \left(1 + \frac{f}{r}\right)^{-1}$

$= \left(1 + \frac{0.05}{1}\right)^{-1}$

$= \frac{20}{21}$

$= 0.9524$

Why it is not equal to π_1 0.8 ?

按照公式求

Mean completion time $= \left(1 + \frac{f}{r}\right) \frac{1}{P}$

$= \left(1 + \frac{0.05}{1}\right) \frac{1}{4}$

$= \frac{21}{80}$

$= 0.2625$

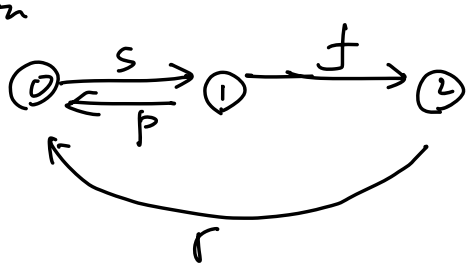
$= \frac{\text{mean processing time}}{\text{steady-state availability}}$

$\frac{21}{80} = \frac{t}{\frac{20}{21}}$ $t = \frac{1}{4}$ hour/个

Case2. discard policy

Q draw = ? TPM = ? π = ?

Solution



$$Q = \begin{bmatrix} q_{00} & q_{01} & q_{02} \\ q_{10} & q_{11} & q_{12} \\ q_{20} & q_{21} & q_{22} \end{bmatrix} = \begin{bmatrix} -s & s & 0 \\ p & -p-f & f \\ r & 0 & -r \end{bmatrix}$$

$$\begin{cases} \pi' Q = 0 \\ \pi_0' + \pi_1' + \pi_2' = 1 \end{cases} \qquad \begin{cases} -s\pi_0' + p\pi_1' + r\pi_2' = 0 \\ s\pi_0' - (p+f)\pi_1' = 0 \\ f\pi_1' - r\pi_2' = 0 \\ \pi_0' + \pi_1' + \pi_2' = 1 \end{cases}$$

$$\pi_2' = \frac{f}{r}\pi_1' \qquad \pi_1' = \frac{s}{p+f}\pi_0'$$

$$\pi_0' + \frac{s}{p+f}\pi_0' + \frac{f}{r}\frac{s}{p+f}\pi_0' = 1$$

$$\frac{rp+rf+sr+fs}{r(p+f)}\pi_0 = 1$$

$$\pi_0' = \frac{rp+rf}{rp+rf+sr+fs}$$

$$\pi_1' = \frac{s}{p+f}\pi_0 = \frac{sr}{rp+rf+sr+fs}$$

$$\pi_2' = \frac{f}{r}\pi_1 = \frac{sf}{rp+rf+sr+fs}$$

Q $s = 20/h \quad p = 4/h \quad f = 0.05/h \quad r = 1/h$

$\pi_i' = ? \quad R = ?$

Solution $rp + rf + sr + fs = 4 + 0.05 + 20 + 1 = 25.05$

$$\pi_0' = \frac{4.05}{25.05} = 0.1617$$

$$\pi_1' = \frac{20}{25.05} = 0.7984$$

$$\pi_2' = \frac{1}{25.05} = 0.03992$$

$$R = \pi_1' p = 0.7984 \times 4$$

$$= 3.1936 \quad \text{? why 3.08 ?}$$