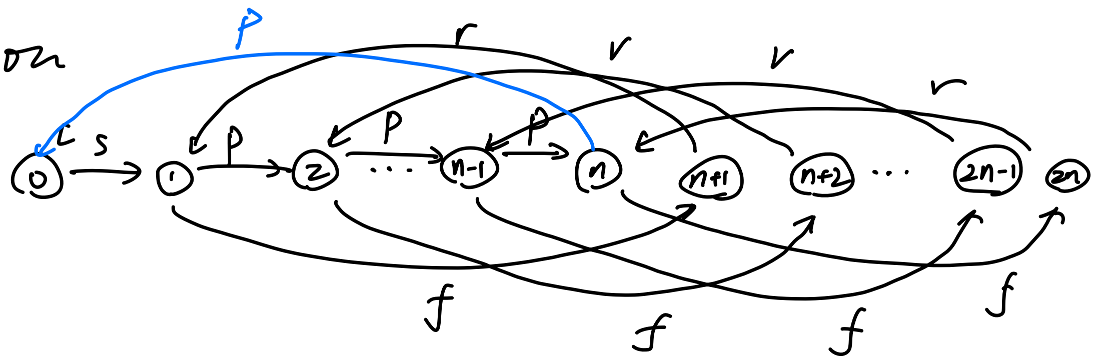
Problem 9.3

Q (a) draw state transition diagram

Solution



(b) TRM $Q = ?$

$$Q = \begin{array}{c|cccccccccccc}
 & 0 & 1 & 2 & 3 \cdots & n-1 & n & n+1 & n+2 & \cdots & 2n-1 & 2n \\
\hline
0 & -s & s & & & & & & & & & \\
1 & & -p-f & p & & & & f & & & & \\
2 & & & -p-f & p & \ddots & & & f & & & \\
n-1 & & & & & -p-f & p & & & & f & \\
n & p & & & & & -p-f & & & & & f \\
n+1 & & r & & & & & -r & & & & \\
2n & & & & & & r & & & & & -r
\end{array}$$

(c) Q : rate balance equations

$$\begin{cases} \pi Q = 0 \\ \pi(0) + \pi(1) + \cdots + \pi(2n) = 1 \qquad (6) \end{cases}$$

$\pi Q = 0$

$$\begin{cases} -s\pi(0) + p\,\pi(n) = 0 & (2) \\ s\pi(0) - (p+f)\pi(1) + r\,\pi(n+1) = 0 & (3) \\ p\pi(1) - (p+f)\pi(2) + r\,\pi(n+2) = 0 & (4) \end{cases}$$

$$\begin{cases} \quad \vdots \\ p\pi(n-1) - (p+f)\pi(n) + r\pi(2n) = 0 & (5) \\ f\pi(1) - r\pi(n+1) = 0 \\ \quad \vdots \\ f\pi(n) - r\pi(2n) = 0 \end{cases}$$

$$\left.\begin{array}{l} f\pi(1) - r\pi(n+1) = 0 \\ \vdots \\ f\pi(n) - r\pi(2n) = 0 \end{array}\right\} (1)$$

(d) Q Find π?

Solution

from (1): $\pi(n+1) = \frac{f}{r}\pi(1)$

$\pi(2n) = \frac{f}{r}\pi(n)$

求 $\pi(1) - \pi(n)$

from (2) $\cancel{\pi(0) = 0}$ $\pi(n) = \frac{s}{p}\pi(0)$

from (3) $s\pi(0) - (p+f)\pi(1) + r\,\pi(n+1) = 0$

$s\pi(0) - (p+f)\pi(1) + r\frac{f}{r}\pi(1) = 0$

$s\pi(0) - p\pi(1) - f\pi(1) + f\pi(1) = 0$

$\pi(1) = \frac{s}{p}\pi(0)$

from (4) $p\pi(1) - (p+f)\pi(2) + f\,\pi(2) = 0$

$p\pi(1) - p\pi(2) = 0$

$\pi(2) = \pi(1) = \frac{s}{p}\pi(0)$

from (5) $\pi(n) = \pi(n-1) = \cdots = \frac{s}{p}\pi(0)$

from (6) $\pi(0) + \pi(1) + \cdots + \pi(n) + \pi(n+1) + \cdots + \pi(2n) = 1$

$$\pi(0) + \frac{s}{p}\pi(0) + \cdots + \frac{s}{p}\pi(0) + \frac{f}{r}\frac{s}{p}\pi(0) + \cdots + \frac{f}{r}\frac{s}{p}\pi(0) = 1$$

$$\left(1 + \frac{s}{p}n + \frac{fs}{rp}n\right)\pi(0) = 1$$

$$\frac{rp + rsn + fsn}{rp}\pi(0) = 1$$

$$\pi(0) = \frac{rp}{rp + rsn + fsn}$$

$$\pi(i) = \frac{rs}{rp + rsn + fsn} \qquad i = 1, 2 \cdots n$$

$$\pi(j) = \frac{fs}{rp + rsn + fsn} \qquad j = n+1, \cdots 2n$$

(e) Q: productive rate R

$$R = p\left[\pi(1) + \cdots + \pi(n)\right]$$

$$= \frac{pnrs}{rp + rsn + fsn}$$