23-S1 - Q4

Q: (i) $L = ?$ (ii) $W = ?$ (iii) $T_{break\text{-}even}$

$$L_1 = \frac{\rho}{1-\rho} = \frac{\lambda}{\mu-\lambda} = \frac{2}{3-2} = 2 \checkmark \qquad W_1 = \frac{1}{\mu-\lambda} = 1$$

$$L_2 = \frac{\rho[1-\rho^N - N\rho^N(1-\rho)]}{(1-\rho)(1-\rho^{N+1})} \qquad W_2 = \frac{1-(\frac{2}{3})^3 - 3\times(\frac{2}{3})^3 \times \frac{1}{3}}{3\times\frac{1}{3}\times[1-(\frac{2}{3})^4]}$$

$$\rho = \frac{\lambda}{\mu} = \frac{2}{3} \qquad N = 3 \qquad = \frac{33}{65} \checkmark = 0.5077$$

$$L_2 = \frac{\frac{2}{3}[1-(\frac{2}{3})^3 - 3\times(\frac{2}{3})^3\frac{1}{3}]}{\frac{1}{3}(1-(\frac{2}{3})^4)} = \cancel{\frac{42}{65}} \quad \frac{66}{65}$$

$$= 0.6462$$

$$L_3 = \frac{\rho(1+b)}{2(1-\rho)} \qquad b = 3 \quad \mu = 3 \quad \lambda = \frac{2}{3}$$

$$\rho = \frac{b\lambda}{\mu} = \frac{3\times\frac{2}{3}}{3} = \frac{2}{3}$$

$$L_3 = \frac{\frac{2}{3}(1+3)}{2(1-\frac{2}{3})} = 4 \checkmark \qquad W_3 = \frac{L}{\lambda b} = \frac{4}{\frac{2}{3}\times 3} = 2 \checkmark$$

$$L_4 = \frac{\rho(m\rho)^m \pi_0}{m!(1-\rho)^2} + \frac{\lambda}{\mu} \qquad \lambda = 2 \quad \mu = 2 \quad m = 3$$

$$\rho = \frac{\lambda}{m\mu} = \frac{2}{3\times 2} = \frac{1}{3}$$

$$\pi_0 = \left[\frac{(m\rho)^m}{m!(1-\rho)} + \sum_{k=0}^{m-1}\frac{(m\rho)^k}{k!}\right]^{-1}$$

$$= \left[\frac{(3\times\frac{1}{3})^3}{3!\times\frac{2}{3}} + \frac{(3\times\frac{1}{3})^0}{0!} + \frac{(3\times\frac{1}{3})^1}{1!} + \frac{(3\times\frac{1}{3})^2}{2!}\right]^{-1}$$

$$= \left(\frac{1}{4} + 1 + 1 + \frac{1}{2}\right)^{-1}$$

$$= \frac{4}{11}$$

$$L_q = \frac{\rho(m\rho)^m\pi_0}{m!(1-\rho)^2} + \frac{\lambda}{\mu}$$

$$= \frac{\frac{1}{3}\times 1^3\times\frac{4}{11}}{3!\times(\frac{2}{3})^2} + \frac{2}{2}$$

$$= \frac{23}{22} = 1.0455$$

$$W_q = \frac{L}{\lambda} = \frac{\frac{23}{22}}{2} = \frac{23}{44} = 0.5227$$

(iii) ① Monthly profit = Revenue − operation

$= 40000 - 20000$

$= 20000$

$$\text{months} = \frac{\text{Initial renovation}}{\text{Monthly profit}} = \frac{100000}{20000} = 5$$

② $$\frac{120000-90000}{30000}=10$$

③ $$\frac{30000-18000}{12000}=8.33$$

④ $$\frac{100000-60000}{40000}=3.75$$

best choice:

① short waiting time, only 0.5227 min

② low mean number in system

③ quick break-even time, only 3.75 month

④ highest monthly profit