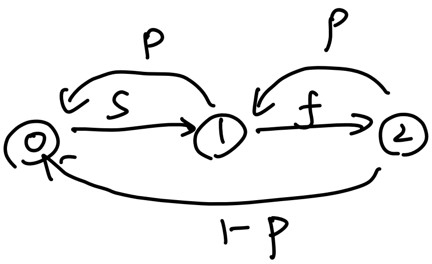
Problem 9.4

Q: (a) - (e)

Solution (a)



(b) $Q = \begin{bmatrix} -s & s & 0 \\ p & -p-f & f \\ 1-p & p & -1 \end{bmatrix}$

(c) $\begin{cases} \pi Q = 0 \\ \pi(0) + \pi(1) + \pi(2) = 1 \end{cases}$ (2)

$\pi Q = 0 \quad \begin{cases} -s\pi(0) + p\pi(1) + (1-p)\pi(2) = 0 & (4) \\ s\pi(0) - (p+f)\pi(1) + p\pi(2) = 0 \\ f\pi(1) - \pi(2) = 0 & (1) \end{cases}$

from (1) $\quad \pi(2) = f\pi(1)$

from (2) $\quad \pi(0) + \pi(1) + f\pi(1) = 1$

$$\pi(1) = \frac{1-\pi(0)}{1+f} \quad (3)$$

from (3) $\quad \pi(0) + \dfrac{1-\pi(0)}{1+f} + \dfrac{f - f\pi(0)}{1+f} = 1$

$$\pi(0) + f\pi(0) + 1 - \pi(0) + f - f\pi(0) = 1+f$$

$\pi(0) + f\pi(0) + 1(1-\pi(0)) + f(1-f\pi(0)) = 1+f$

$1+f = 1+f$ (no useful)

from (4) $-s\pi(0) + p\pi(1) + (1-p)\pi(2) = 0$

$$-s\pi(0) + \frac{p - p\pi(0)}{1+f} + \frac{(1-p)f(1-\pi(0))}{1+f} = 0$$

$$-s(1+f)\pi(0) + p - p\pi(0) + (f - pf)(1-\pi(0)) = 0$$

$$-s\pi(0) - sf\pi(0) + p - p\pi(0) + f - pf - f\pi(0) + pf\pi(0) = 0$$

$$(-s - sf - p - f + pf)\pi(0) = -p - f + pf$$

$$\pi(0) = \frac{p + f - pf}{s+p+f+sf-pf}$$

$$\pi(1) = \frac{1-\pi(0)}{1+f}$$

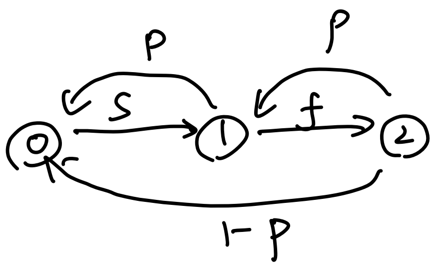
$$= \frac{s+sf}{(s+p+f+sf-pf)(1+f)}$$

$$\pi(2) = f\pi(1)$$

$$= \frac{sf+sf^2}{(s+p+f+sf-pf)(1+f)}$$

↑ method 1

rate balance equation $\rightarrow s = \epsilon$



$$\begin{cases} s\pi_0 = p\pi_1 + (1-p)\pi_2 & (2) \\ f\pi_1 + p\pi_1 = p\pi_2 + s\pi_1 & \\ p\pi_2 + (1-p)\pi_2 = f\pi_1 & (1) \\ \pi_0 + \pi_1 + \pi_2 = 1 & (3) \end{cases}$$

(d)

from (1) $\pi_2 = f\pi_1$

from (2) $s\pi_0 = p\pi_1 + (1-p)f\pi_1$

$$= (p+f-pf)\pi_1$$

$$\pi_1 = \frac{s}{p+f-pf}\pi_0$$

from (3) $\pi_0 + \frac{s}{p+f-pf}\pi_0 + \frac{sf}{p+f-pf}\pi_0 = 1$

$$\frac{p+f-pf+s+sf}{p+f-pf}\pi_0 = 1$$

$$\pi_0 = \frac{p+f-pf}{p+f-pf+s+sf}$$

$$\pi_1 = \frac{s}{p+f-pf+s+sf} \qquad \pi_2 = \frac{sf}{p+f-pf+s+sf}$$

(e) $R = p \approx 1$ r

$$= \frac{sp}{p+f-pf+s+sf}$$

Problem 9.4

Ans: $$Q = \begin{bmatrix} -s & s & 0 \\ p & -(p+f) & f \\ (1-q)r & qr & -r \end{bmatrix}$$

Rate balance equations:

$$\pi_2(1-q)r + \pi_1 p = \pi_0 s$$

$$\pi_0 s + qr\pi_2 = \pi_1(p+f)$$

$$\pi_1 f = \pi_2[qr + (1-q)r]$$

$$\pi_0 + \pi_1 + \pi_2 = 1$$

$$\pi_0 = \frac{r[p+(1-q)f)]}{rp+rf(1-q)+rs+fs}$$

$$\pi_1 = \frac{rs}{rp+rf(1-q)+rs+fs}$$

$$\pi_2 = \frac{fs}{rp+rf(1-q)+rs+fs}$$

$$R = \pi_1 p = \frac{rsp}{rp+rf(1-q)+rs+fs}$$