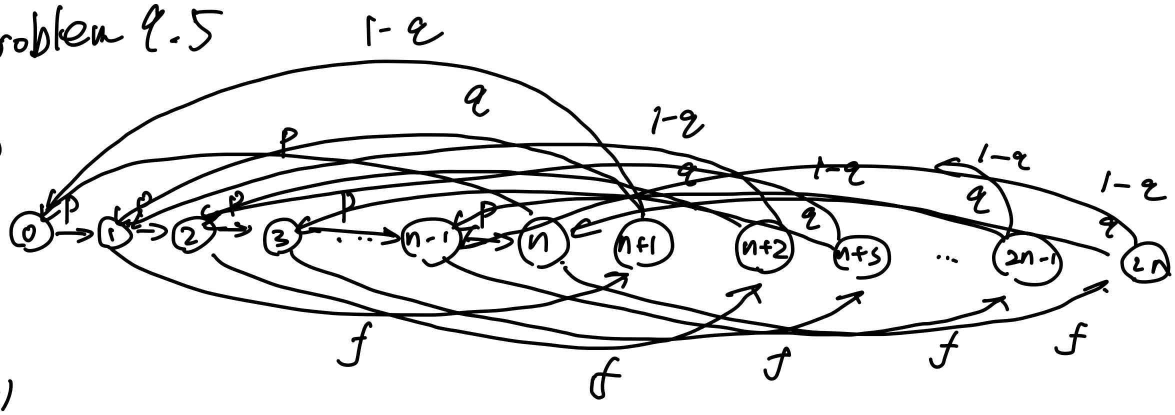
Problem 9.5

(a)



(b)

$$Q = \begin{array}{c|cccccccccccccc}
 & 0 & 1 & 2 & 3 & \cdots & n-1 & n & n+1 & n+2 & n+3 & \cdots & 2n-1 & 2n \\
\hline
0 & -p & p & & & & & & & & & & & \\
1 & & -p-f & p & & & & & f & & & & & \\
2 & & & -p-f & p & & & & & f & & & & \\
3 & & & & -p-f & p & & & & & f & & & \\
\vdots & & & & & & & & & & & & & \\
n-1 & & & & & & -p-f & p & & & & & f & \\
n & p & & & & & & -p-f & & & & & & f \\
n+1 & 1-q & q & & & & & & & & & & & \\
n+2 & & 1-q & q & & & & & & & & & & \\
n+3 & & & 1-q & q & & & & & & & & & \\
\vdots & & & & & & & & & & & & & \\
2n-1 & & & & & & 1-q & q & & & & & & \\
2n & & & & & & & 1-q & q & & & & &
\end{array}$$

(c) $p\pi(0) = p\pi(n) + (1-q)\pi(n+1)$ (d)

$$\begin{cases} p\pi(1) + f\pi(1) = q\pi(n+1) + (1-q)\pi(n+2) + p\pi(0) \end{cases} \quad (3)$$

$$\begin{cases} p\pi(2) + f\pi(2) = q\pi(n+2) + (1-q)\pi(n+3) + p\pi(1) \\ \vdots \\ p\pi(n-1) + f\pi(n-1) = q\pi(2n-1) + (1-q)\pi(2n) + p\pi(n-2) \end{cases}$$

$$p\pi(n) + f\pi(n) = q\pi(2n) + p\pi(n-1) \qquad (5)$$

$$\left.\begin{cases} q\pi(n+1) + (1-q)\pi(n+1) = f\pi(1) \\ \vdots \\ q\pi(2n-1) + (1-q)\pi(2n-1) = f\pi(n-1) \\ q\pi(2n) + (1-q)\pi(2n) = f\pi(n) \end{cases}\right\} \qquad (1)$$

$$\pi(0) + \pi(1) + \cdots + \pi(2n) = 1 \qquad (4)$$

(d)

from (1): $\begin{cases} \pi(n+1) = f\pi(1) \\ \vdots \\ \pi(2n) = f\pi(n) \end{cases}$

from (2): $p\pi(0) = p\pi(n) + (1-q)f\pi(1)$

$$\pi(0) = \pi(n) + \frac{f-qf}{p}\pi(1)$$

from (3)

$$p\pi(1) + f\pi(1) = q\pi(n+1) + (1-q)\pi(n+2) + p\pi(0)$$

$$(p+f)\pi(1) = fq\pi(1) + (1-q)f\pi(2) + p\pi(0)$$

$$(p+f-fq)\pi(1) = (f-qf)\pi(2) + p\pi(0)$$

$$= (f-qf)\pi(2) + p\pi(n) + (1-q)f\pi(1)$$

$$p\pi(1) = (f-qf)\pi(2) + p\pi(n)$$

$$\pi(1) = \frac{f-qf}{p}\pi(2) + \pi(n)$$

$$\pi(i) = \frac{f-qf}{p}\pi(i+1) + \pi(n-1+i) \quad i = 1, 2 \ldots n-1$$

$$\pi(i) = \frac{(1-q)f}{p}\pi(i+1) + f\pi(i-1)$$

from (4)

$$\pi(0) + \pi(1) + \cdots + \pi(2n) = 1$$

$$\pi(0) + (1+f)\pi(1) + \cdots + (1+f)\pi(n) = 1$$

from (5) $p\pi(n) + f\pi(n) = q\pi(2n) + p\pi(n-1)$

$$(p+f)\pi(n) = qf\pi(n) + p\pi(n-1)$$

$$(p+f-q)\pi(n) = p\pi(n-1)$$