

22-51-Q1

Q (a) max profit ?

Solution let profit  $Z$

product A  $x_1$  B  $x_2$  C  $x_3$  D  $x_4$

$$Z = 30x_1 - 15(800 - x_1) + 40x_2 - 20(750 - x_2) \\ + 20x_3 - 10(600 - x_3) + 10x_4 - 8(500 - x_4)$$

$$= 45x_1 + 60x_2 + 30x_3 + 18x_4 - 37000$$

subject to

$$\begin{cases} 0.3x_1 + 0.3x_2 + 0.25x_3 + 0.15x_4 \leq 1000 \\ 0.25x_1 + 0.35x_2 + 0.3x_3 + 0.1x_4 \leq 1000 \\ 0.45x_1 + 0.5x_2 + 0.4x_3 + 0.22x_4 \leq 1000 \\ 0.15x_1 + 0.15x_2 + 0.1x_3 + 0.05x_4 \leq 1000 \\ x_1 \leq 800 \\ x_2 \leq 750 \\ x_3 \leq 600 \\ x_4 \leq 500 \\ x_1, x_2, x_3, x_4 \geq 0 \end{cases}$$

(b) Max  $Z = 5x_1 + 4x_2 + 0 \cdot x_3 + 0 \cdot x_4 + 0 \cdot x_5$

Subject to

$$\begin{cases} -x_1 + x_2 + x_3 = 1 \\ x_2 + x_4 = 2 \\ 6x_1 + 4x_2 + x_5 = 24 \\ x_1, x_2, x_3, x_4, x_5 \geq 0 \end{cases}$$

$$X_0 \begin{array}{c|c} X^T & \\ \hline A & B \\ \hline -C^T + C_0^T A & C_0^T B \end{array}$$

$$X^T = [X_1 \ X_2 \ X_3 \ X_4 \ X_5]$$

$$X_0^T = [X_3 \ X_4 \ X_5]$$

$$A = \begin{bmatrix} -1 & 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 \\ 6 & 4 & 0 & 0 & 1 \end{bmatrix}$$

$$B = \begin{bmatrix} 1 \\ 2 \\ 24 \end{bmatrix}$$

$$C^T = [5 \ 4 \ 0 \ 0 \ 0]$$

$$C_0^T = [0 \ 0 \ 0]$$

$$-C^T + C_0^T A = [-5 \ -4 \ 0 \ 0 \ 0]$$

$$C_0^T B = 0$$

So

	$X_1$	$X_2$	$X_3$	$X_4$	$X_5$		Ratio
$X_3$	-1	1	1	0	0	1	
$X_4$	0	1	0	1	0	2	
$X_5$	6	4	0	0	1	24	$24/6 = 4$
	-5	-4	0	0	0	0	

↑  
work col

	$X_1$	$X_2$	$X_3$	$X_4$	$X_5$	
$X_3$	<del><math>1</math></del>	$\frac{5}{3}$	1	0	$\frac{1}{6}$	5
$X_4$	0	1	0	1	0	2
$X_5$	<del>6</del>	$\frac{2}{3}$	0	0	$\frac{1}{6}$	4
$\rightarrow X_1$	<del><math>1</math></del>	$-\frac{2}{3}$	0	0	$\frac{5}{6}$	20

Ratio

$$5 / \frac{1}{3} = 3$$

$$2 / 1 = 2$$

$$4 / \frac{2}{3} = 6$$

↑  
work column

	$X_1$	$X_2$	$X_3$	$X_4$	$X_5$	
$X_2$	$\left(\frac{1}{2}\right)$	$\left(\frac{2}{3}\right)$	0	1	0	$\left(\frac{1}{6}\right)$
$X_4$	$\left(\frac{1}{2}\right)$	$\left(\frac{1}{3}\right)$	0	0	1	$\left(\frac{1}{6}\right)$
$X_1$	$\left(\frac{3}{2}\right)$	$\left(\frac{2}{3}\right)$	1	0	0	$\left(\frac{1}{6}\right)$
$\rightarrow X_2$	$\left(\frac{1}{2}\right)$	$\left(\frac{2}{3}\right)$	0	0	0	$\left(\frac{1}{6}\right)$

$X_2 = 6$   
 $X_3 = 5$   
 $X_4 = 4$   
 $X_1 = X_5 = 0$

$$\frac{3}{2} \times \left(-\frac{5}{3}\right)$$

$$\frac{1}{4} \times \left(-\frac{5}{3}\right) + \frac{1}{6}$$

$$6 \times \left(-\frac{5}{3}\right) + 5 = -5$$

$$\frac{3}{2} \times \frac{2}{3} = 1$$

$$\frac{1}{4} \times \frac{2}{3} + \frac{5}{6}$$

$$6 \times \frac{2}{3} + 20 = 24$$

Optimal solution is  $X_2 = 6$   $X_3 = 5$   $X_4 = 4$

$$X_1 = X_5 = 0$$

$$\text{Max } Z = 24$$