

21-51 - Q3

Q: (a) $P(a, b) = ?$ $P(0, b) = ?$ $P(a, \infty) = ?$

Solution (a) $P(a, b) = P(a \leq X \leq b)$

$$= \sum_{k=a}^b \frac{e^{-\lambda t} (\lambda t)^k}{k!}$$

$$P(0, b) = P(0 \leq X \leq b)$$

$$= \sum_{k=0}^b \frac{e^{-\lambda t} (\lambda t)^k}{k!}$$

$$P(a, \infty) = P(X \geq a)$$

$$= 1 - P(X < a)$$

$$= 1 - \sum_{k=0}^{a-1} \frac{e^{-\lambda t} (\lambda t)^k}{k!}$$

Q(b) (i) TPM = ?

Solution $P = [P_{ij}]$

state space $\{w_0, w_1, w_2, w_3\}$

let D_{k+1} 为下个月发的奖品数, X_{k+1} 为下个月销售量

$$P_{00} = P\{D_{k+1} = 3\}$$

即上月底发完, 下月底也发完

$$= P\{X_{k+1} \geq 30\} = P(30, \infty)$$

$$p_{01} = P\{D_{k+1} = 2\} = P\{20 \leq X_{k+1} \leq 29\} = p(20, 29)$$

$$p_{02} = p(10, 19)$$

$$p_{03} = p(0, 9)$$

$$p_{10} = p(10, \infty)$$

$$p_{11} = p(0, 9)$$

$$p_{12} = 0$$

$$p_{13} = 0$$

$$p_{20} = p(20, \infty)$$

$$p_{21} = p(10, 19)$$

$$p_{22} = p(0, 9)$$

$$p_{23} = 0$$

$$p_{30} = p(30, \infty)$$

$$p_{31} = p(20, 29)$$

$$p_{32} = p(10, 19)$$

$$p_{33} = p(0, 9)$$

TPM is

$$\begin{bmatrix} p(30, \infty) & p(20, 29) & p(10, 19) & p(0, 9) \\ p(10, \infty) & p(0, 9) & 0 & 0 \\ p(20, \infty) & p(10, 19) & p(0, 9) & 0 \\ p(30, \infty) & p(20, 29) & p(10, 19) & p(0, 9) \end{bmatrix}$$

(ii) May 已2抽 $W_{\text{May}} = 0$ or 1 $P(\text{August 已3抽})?$

Solution

P 代表月初礼品个数

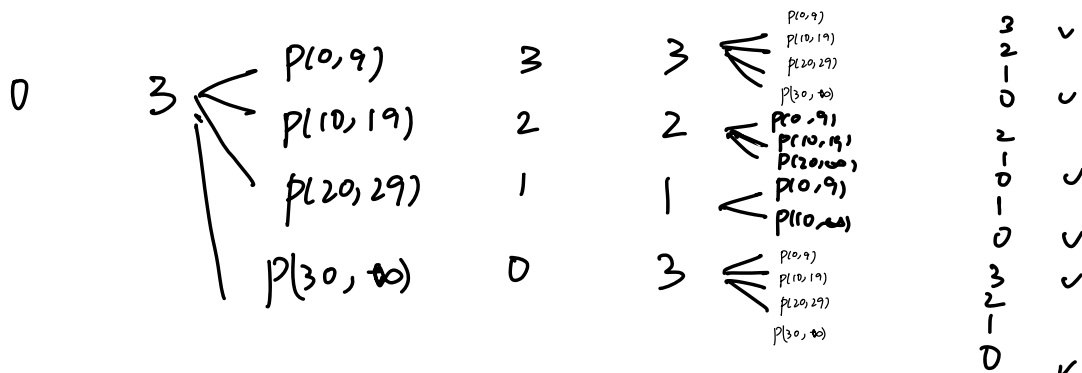
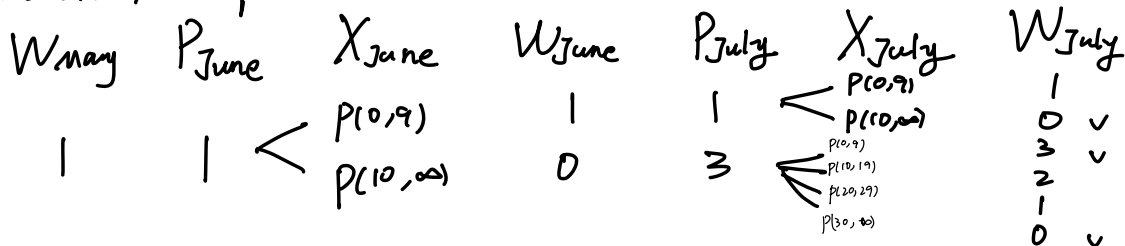
Case 1: $W_{\text{May}} = 1$

$P(\text{August giving out } 3)$

$$= P(W_{\text{July}} = 0 \text{ or } 3) P(30, \infty)$$

$$= \{ P(0, 9) P(10, \infty) + P(10, \infty) [P(0, 9) + P(30, \infty)] \} P(30, \infty)$$

Probability Map



Case 2. $W_{\text{May}} = 0$

$P(\text{August giving out } 3)$

$$= P(W_{\text{July}} = 0 \text{ or } 3) P(30, \infty)$$

$$= \left\{ P(0, 9) [P(0, 9) + P(30, \infty)] + P(10, 19) P(20, \infty) \right. \\ \left. + P(20, 29) P(10, \infty) + P(30, \infty) [P(0, 9) + P(30, \infty)] \right\} P(30, \infty)$$

胡总. 思路: 用 TPM

$$\text{case 1 } P(\text{August give out } 3) = P(30, \infty) \left[\begin{array}{l} P(W_7=0 | W_5=0) + P(W_7=3 | W_5=0) \\ + P(W_7=0 | W_5=1) + P(W_7=3 | W_5=1) \end{array} \right]$$

其中 $P(W_7=0 | W_5=0) + P(W_7=3 | W_5=0)$
 $+ P(W_7=0 | W_5=1) + P(W_7=3 | W_5=1)$

$$= P^2 |_{(0,0) + (0,3) + (1,0) + (1,3)}$$

$$= P(30, \infty) P(30, \infty) + P(20, 29) P(10, \infty) \\ + \dots$$

$W_8=0$	
$P_8=0$	give 0
$P_8=1$	give 1
$P_8=2$	give 2
$P_8=3$	give 3 ← 早期观察

$$\left[\begin{array}{cccc} P(30, \infty) & P(20, 29) & P(10, 19) & P(0, 9) \\ \hline P(10, \infty) & P(0, 9) & 0 & 0 \\ \hline P(20, \infty) & P(10, 19) & P(0, 9) & 0 \\ \hline P(30, \infty) & P(20, 29) & P(10, 19) & P(0, 9) \end{array} \right] \left[\begin{array}{cccc} P(30, \infty) & P(20, 29) & P(10, 19) & P(0, 9) \\ P(10, \infty) & P(0, 9) & 0 & 0 \\ P(20, \infty) & P(10, 19) & P(0, 9) & 0 \\ P(30, \infty) & P(20, 29) & P(10, 19) & P(0, 9) \end{array} \right]$$