



Let's break down both parts of the problem:让我们分解问题的两个部分：

Part (a): Assigning Chores to Minimize Total Pay

(a) 部分：分配家务以最小化总工资

Given the bids by each child for each chore, we need to assign one chore to each child in a way that minimizes the total pay. This is essentially a classic assignment problem, where each child can only take on one chore, and each chore is assigned to only one child.

考虑到每个孩子对每项家务的出价，我们需要以最小化总报酬的方式为每个孩子分配一项家务。这本质上是一个经典的分配问题，每个孩子只能承担一项家务，并且每项家务只分配给一个孩子。

Table 2 Restated:表 2 重述：

Children	Mow	Paint	Wash	Clean
John	\$1	\$4	\$6	\$3
Karen	\$9	\$7	\$10	\$9
Terri	\$4	\$5	\$11	\$7
Jean	\$8	\$7	\$8	\$5

To solve this, we can use the Hungarian algorithm or matrix-based optimization techniques to find the minimum-cost assignment. I'll compute the assignments for you.

为了解决这个问题，我们可以使用匈牙利算法或基于矩阵的优化技术来找到最小成本分配。我会为你计算作业。

Part (b): Quadratic Optimization Problem Using Lagrange Multipliers

(b) 部分：使用拉格朗日乘子的二次优化问题

Given the quadratic optimization problem:给定二次优化问题：

$$\text{Min } z = \frac{1}{2}x^T \begin{bmatrix} 4 & 0 \\ 0 & 8 \end{bmatrix} x + \begin{bmatrix} 1 \\ 2 \end{bmatrix}^T x + 2$$

Subject to constraints: $x_i \leq 2$ for $i = 1, 2$.受约束： $x_i \leq 2$ 为了 $i = 1, 2$ 。

- Formulate the Lagrange function:** The objective function can be rewritten as:

制定拉格朗日函数： 目标函数可以重写为：

$$z(x) = 2x_1^2 + 4x_2^2 + x_1 + 2x_2 + 2$$

Let λ_1 and λ_2 be the Lagrange multipliers for the constraints $x_1 \leq 2$ and $x_2 \leq 2$, respectively.让 λ_1 和 λ_2 是约束的拉格朗日乘子 $x_1 \leq 2$ 和 $x_2 \leq 2$ ， 分别。

The Lagrangian is then:则拉格朗日量为：

$$\mathcal{L}(x_1, x_2, \lambda_1, \lambda_2) = 2x_1^2 + 4x_2^2 + x_1 + 2x_2 + 2 + \lambda_1(2 - x_1) + \lambda_2(2 - x_2)$$

- Find the Optimal Solution:** To find the optimal solution, we differentiate the Lagrangian with respect to x_1 , x_2 , λ_1 , and λ_2 , then set these derivatives to zero to solve for the variables.

找到最优解： 为了找到最优解，我们对拉格朗日函数进行微分 x_1, x_2, λ_1 ， 和 λ_2 ， 然后将这些导数设置为零以求解变量。

I'll start by solving Part (a) for the assignments and total pay, then move on to Part (b) for the optimal values.我将首先求解 (a) 部分的任务和总工资，然后继续求解 (b) 部分的最优值。

已分析 ^

```
python

import numpy as np
from scipy.optimize import linear_sum_assignment

# Define the cost matrix based on the children's bids for each chore
cost_matrix = np.array([
    [1, 4, 6, 3],
    [9, 7, 10, 9],
    [4, 5, 11, 7],
    [8, 7, 8, 5]
])

# Use the Hungarian algorithm to find the minimum cost assignment
row_ind, col_ind = linear_sum_assignment(cost_matrix)
min_total_pay = cost_matrix[row_ind, col_ind].sum()

# Determine the assignments and corresponding total pay
assignments = list(zip(row_ind, col_ind))
min_total_pay, assignments

结果
(21, [(0, 0), (1, 2), (2, 1), (3, 3)])
```

For Part (a), the optimal assignments to minimize the total pay are as follows:

对于(a)部分，最小化总薪酬的最佳分配如下：

- John is assigned to Mow (cost: \$1)**约翰被分配给**Mow** （成本： 1 美元）
- Karen is assigned to Wash (cost: \$10)**Karen被分配到**Wash** （费用： 10 美元）
- Terri is assigned to Paint (cost: \$5)**Terri被分配到**Paint** （成本： 5 美元）
- Jean is assigned to Clean (cost: \$5)**Jean被分配到**Clean** （费用： 5 美元）

The total minimum pay for all assignments is **\$21**.所有任务的最低工资总额为**21 美元**。

Now, let's proceed with Part (b) to solve the quadratic optimization problem using the method of Lagrange multipliers.

现在，我们继续第 (b) 部分，使用拉格朗日乘子方法求解二次优化问题。 [-]

已分析 ^

```
python

from sympy import symbols, diff, Eq, solve

# Define variables
x1, x2, lambda1, lambda2 = symbols('x1 x2 lambda1 lambda2')

# Define the objective function in terms of x1 and x2
z = 2 * x1**2 + 4 * x2**2 + x1 + 2 * x2 + 2

# Define the Lagrangian with constraints x1 <= 2 and x2 <= 2
L = z + lambda1 * (2 - x1) + lambda2 * (2 - x2)

# Calculate partial derivatives with respect to x1, x2, lambda1, and lambda2
dL_dx1 = diff(L, x1)
dL_dx2 = diff(L, x2)
dL_dlambda1 = diff(L, lambda1)
dL_dlambda2 = diff(L, lambda2)

# Solve the system of equations for stationary points
solutions = solve((Eq(dL_dx1, 0), Eq(dL_dx2, 0), Eq(dL_dlambda1, 0), Eq(dL_dlambda2, 0)),
solutions

结果
{x1: 2, lambda1: 9, x2: 2, lambda2: 18}
```

For Part (b), the optimal solution to the quadratic optimization problem is:

对于(b)部分，二次优化问题的最优解为：

- $x_1 = 2$
- $x_2 = 2$
- Lagrange multipliers: $\lambda_1 = 9$ and $\lambda_2 = 18$ 拉格朗日乘子： $\lambda_1 = 9$ 和 $\lambda_2 = 18$

This solution satisfies the constraints $x_i \leq 2$ for $i = 1, 2$ and minimizes the objective function. 该解满足约束条件 $x_i \leq 2$ 为了 $i = 1, 2$ 并最小化目标函数。 [-]