Example2 - class - week 13

3. A real estate company is about to launch its new condominium for sale. The weekly sales of the new condominium can be classified as "Low" and "High". The company's advertising strategy will be based on the weekly sales level. Since the company's advertising budget cannot afford advertising all the time, the company is considering the following two advertising strategies:

(A) advertise when sales are low, do not advertise when sales are high;

(B) advertise when sales are high, do not advertise when sales are low.

Running advertisements in any week will have primary positive impact on sales in the following week. So, at the beginning of each week, the company will forecast as best as it could whether sales will be low or high that week. Based on the forecast, the company will decide whether to run advertisements that week. When advertising is done during a week, the probability of having low sales the following week is $\frac{1}{5}$ or $\frac{1}{10}$, depending on whether the current week's sales are low or high. These probabilities are increased to $\frac{1}{2}$ or $\frac{1}{3}$, depending on whether the current week's sales are low or high, when advertising is not done during the current week.

Ad

next week not sale

0 1

no Advertising

P_{00} P_{10}

current low

P_{00} P_{10}

The cost of advertising for an entire week depends on the current week's sales - advertising costs \$200,000 if the current week's sales are low (more advertisements) while the cost is \$100,000 if the current week's sales are high (less advertisements). Before deducting advertising cost, the company's weekly profits are \$2,000,000 when sales are high but only \$500,000 when sales are low.

Let state 0 indicate the "Low" level of sales and state 1 indicate the "High" level of sales during a week, where each transition of the process goes from one week to the next.

(a) For each of the two advertising strategies (A) and (B), construct the (one-step) transition probability matrix.

(b) For each of the two advertising strategies (A) and (B), find the mean sojourn times for state 0 and state 1. Which strategy is better if the company wants to have short low sales period on the average? Which strategy is better if the company wants to have long high sales period on the average?

(c) Calculate the steady-state probabilities for each of the two advertising strategies (A) and (B).

(d) Find the long run expected weekly profit (including a deduction for advertising costs) for each of the two advertising strategies (A) and (B). Which strategy is better according to this measure of performance?

Solution

state 0 : low sales in the current week

state 1 : high sales in the current week

(A)

current		Do
low	$\longrightarrow$	Ad
high	$\longrightarrow$	no Ad

$P_{00} = \frac{1}{5}$ (advert) current low next low

$P_{10} = \frac{1}{3}$ (no advert) current high next low

$$P = \begin{array}{c|cc} \text{state} & 0 & 1 \\ \hline 0 & \frac{1}{5} & \frac{4}{5} \\ 1 & \frac{1}{3} & \frac{2}{3} \end{array}$$

(B)

current		Do
low	$\longrightarrow$	No Ad
high	$\longrightarrow$	Ad

$P_{00} = \frac{1}{2}$ (no advertis

$P_{10} = \frac{1}{10}$ (Advertiz)

$$P = \begin{array}{c|cc} \text{state} & 0 & 1 \\ \hline 0 & \frac{1}{2} & \frac{1}{2} \\ 1 & \frac{1}{10} & \frac{9}{10} \end{array}$$

(b) T_i = cojourn time of state i, $i=0,1$

$$E(T_i) = \frac{1}{1-p_{ii}}$$

(A): $E(T_0) = \frac{1}{1-P_{00}} = \frac{1}{1-\frac{1}{5}} = \frac{5}{4}$

$$E(T_1) = \frac{1}{1-p_{11}} = \frac{1}{1-\frac{2}{3}} = 3$$

(B) $E(T_0) = \frac{1}{1-p_{00}} = \frac{1}{1-\frac{1}{2}} = 2$

$$E(T_1) = \frac{1}{1-p_{11}} = \frac{1}{1-\frac{9}{10}} = 10$$

strategy A has the lowest $E(T_0) = \frac{1}{4}$ while strategy (B) has highest $E(T_1)=10$. Hence if the company wants to have short low sales period on the average, choose strategy A On the other hand, if the co. wants to have long high sales period on the average, choose strategy B

(c) $Y = [y_0 \ y_1]$ steady - stage probability

$$\begin{cases} Y = YP \\ y_0 + y_1 = 1 \end{cases}$$

(A) $[y_0 \ y_1] = [y_0 \ y_1] \begin{bmatrix} \frac{1}{5} & \frac{4}{5} \\ \frac{1}{3} & \frac{2}{3} \end{bmatrix}$

$\Rightarrow \frac{4}{5} y_0 = \frac{1}{3} y_1$

$y_0 = \frac{5}{17} \qquad y_1 = \frac{12}{17}$

(B) $[y_0 \ y_1] = [y_0 \ y_1] \begin{bmatrix} \frac{1}{2} & \frac{1}{2} \\ \frac{1}{10} & \frac{9}{10} \end{bmatrix}$

$\Rightarrow \frac{1}{2} y_0 = \frac{1}{10} y_1$

$y_0 = \frac{1}{6} \qquad y_1 = \frac{5}{6}$

(d) (A) long run expected profie in a week

$= (2{,}000{,}000)\, y_1 + (500{,}000 - 200{,}000)\, y_0$

$= (2{,}000{,}000) \frac{12}{17} + (300{,}000) \frac{5}{17}$

$= \$1{,}500{,}000$

(B)

long run expected profit in a week

$= (2,000,000 - 100,000)\, y_1 + (500,000)\, y_0$

$= (1,900,000)\, \frac{5}{6} + (500,000)\, \frac{\cancel{5}\; 1}{6}$

$= \$1,666,666.67$

$1,666,666.67 > 1,500000$

Hence, strategy (B) gives higher long run expected profit in a week.