23-S1-Q3

Q(a) state ? $\quad$ P ?

$S = \{0, 1, 2\}$

$P_{00} = P\{B_{k+1} \geq 2\} = 1 - P\{B_{k+1} = 1\} - P\{B_{k+1} = 0\}$

$= 1 - \frac{e^{-0.1}(0.1)^1}{1!} - \frac{e^{-0.1}(0.1)^0}{0!} = 1 - 1.1e^{-0.1}$

$P_{01} = P(B_{k+1} = 1) = 0.1e^{-0.1}$

$P_{02} = P\{B_{k+1} = 0) = e^{-0.1}$

$P_{10} = P\{B_{k+1} \geq 2\} = 1 - 1.1e^{-0.1}$

$P_{11} = 0.1e^{-0.1}$

$P_{12} = e^{-0.1}$

$P_{20} = 1 - 1.1e^{-0.1}$

$P_{21} = 0.1e^{-0.1}$

$P_{22} = e^{-0.1}$

$$P = \begin{bmatrix} 1-1.1e^{-0.1} & 0.1e^{-0.1} & e^{-0.1} \\ 1-1.1e^{-0.1} & 0.1e^{-0.1} & e^{-0.1} \\ 1-1.1e^{-0.1} & 0.1e^{-0.1} & e^{-0.1} \end{bmatrix}$$

(b) $E(T_0) = \frac{1}{1-P_{00}} = \frac{1}{1.1e^{-0.1}} = 1.0047$

$E(T_1) = \frac{1}{1-P_{11}} = \frac{1}{1-0.1e^{-0.1}} = 1.0995$

题目 (b) Compute the mean sojourn time for every state of the DTMC.

课本 **Mean sojourn time** $E(T_i) = (1 - p_{ii})^{-1}$.

没实际意义

$$E(T_2) = \frac{1}{1-p_{22}} = \frac{1}{1-e^{-0.1}} = 10.5083$$

$$\text{mean } E(T_i) = \frac{1}{3}\left[E(T_0) + E(T_1) + E(T_2)\right] = 4.2042$$

State 2 stays longer than 0 and 1

No surprize

comment: $\lambda = 0.1$ is a small number

So the usage rate is low and the replenishment policy reset the number of spare bulbs to 2 whenever there is a shortfall

(c) independent of previous state

y_0 $\pi_0 = 1 - 1.1e^{-0.1}$

DTCM要用Y=[y0 y1 y2]

y_1 $\pi_1 = 0.1e^{-0.1}$

y_2 $\pi_2 = e^{-0.1}$

(d) $P(U_{k+1} = 1) = \pi_1 = 0.1e^{-0.1}$