22-S1-Q3

Q: (a) $\{L_k\}$? TPM? Sojourn time? comment?

Solution $S \in \{0, 1, 2\}$

$$P_{00} = P(N_k \geq 2) = 1 - P(N_k < 2)$$

$$= 1 - P(N_k = 0) - P(N_k = 1)$$

$$= 1 - \frac{1}{e} - \frac{1}{e} = 1 - \frac{2}{e}$$

$$P_{01} = P(N_k = 1) = \frac{1}{e}$$

$$P_{02} = P(N_k = 0) = \frac{1}{e}$$

$$P_{10} = P(N_k \geq 1) = 1 - \frac{1}{e}$$

$$P_{11} = P(N_k = 0) = \frac{1}{e}$$

$$P_{12} = 0$$

$$P_{20} = P(N_k \geq 2) = 1 - \frac{2}{e}$$

$$P_{21} = P(N_k = 1) = \frac{1}{e}$$

$$P_{22} = P(N_k = 0) = \frac{1}{e}$$

$$P = \begin{bmatrix} 1-\frac{2}{e} & \frac{1}{e} & \frac{1}{e} \\ 1-\frac{1}{e} & \frac{1}{e} & 0 \\ 1-\frac{2}{e} & \frac{1}{e} & \frac{1}{e} \end{bmatrix}$$

sojourn time.

$$E(T_0) = \frac{1}{1-p_{00}} = \frac{1}{1-1+\frac{2}{e}} = \frac{e}{2}$$

$$E(T_1) = \frac{1}{1-p_{11}} = \frac{1}{1-\frac{1}{e}} = \frac{e}{e-1}$$

$$E(T_2) = \frac{1}{1-p_{22}} = \frac{e}{e-1}$$

State 0 has a shorter mean sojourn time because she restocks upon depletion while in state 1 and 2, she more likely to remain due to the lower consumption probability

(b) $Y = [y_0 \ y_1 \ y_2]$

$$\begin{cases} Y = YP \\ y_0 + y_1 + y_2 = 1 \end{cases}$$

$$\begin{cases} y_0 = (1-\frac{2}{e})y_0 + (1-\frac{1}{e})y_1 + (1-\frac{2}{e})y_2 \\ y_1 = \frac{1}{e}y_0 + \frac{1}{e}y_1 + \frac{1}{e}y_2 \\ y_2 = \frac{1}{e}y_0 + \frac{1}{e}y_2 \\ 1 = y_0 + y_1 + y_2 \end{cases}$$

$$P = \begin{bmatrix} 1-\frac{2}{e} & \frac{1}{e} & \frac{1}{e} \\ 1-\frac{1}{e} & \frac{1}{e} & 0 \\ 1-\frac{2}{e} & \frac{1}{e} & \frac{1}{e} \end{bmatrix}$$

$$\begin{cases} -\frac{2}{e}y_0 + (1-\frac{1}{e})y_1 + (1-\frac{2}{e})y_2 = 0 \\ y_0 + (1-e)y_1 + y_2 = 0 \\ y_0 + (1-e)y_2 = 0 \\ y_0 + y_1 + y_2 = 1 \end{cases}$$

$$\begin{cases} y_0 = 0.400 \\ y_1 = 0.368 \\ y_2 = 0.233 \end{cases}$$

Comments

state 0: Highest steady-state probability, indicating the housewife often ends up with no cans due to the low usage rate and immediate restocking upon depletion

state 1 showing a significant chance of having one can left

State 2. it's less common to have two cans remaining at week end

(c) $L_k : S = \{0, 1, 2, \cdots, M\}$

$$P_{00} = P(L_k = 0 \mid L_{k-1} = 0) = P(N_k \geq M) = 1 - \sum_{k=0}^{M-1} \frac{e^{-1}}{k!}$$

$$P_{ii} = P(L_k = i \mid L_{k-1} = i) = P(N_k = 0) = \frac{1}{e}$$