Problem 1.5

Q: $P(a_1 < a_2) = \dfrac{p_1(1-p_2)}{p_1 + p_2 - p_1 p_2}$ prove

Solution 概率质量函数 PMF

$$P(a_1 = k) = (1-p_1)^{k-1} p_1$$

$$P(a_2 = k) = (1-p_2)^{k-1} p_2$$

$$P(a_1 < a_2) = \sum_{k=0}^{\infty} P(a_1 = k)\, P(a_2 > k)$$

$$= \sum_{k=0}^{\infty} P(a_1 = k)\,(1-p_2)^k$$

$$= \sum_{k=0}^{\infty} (1-p_1)^{k-1} p_1 (1-p_2)^k$$

$$= p_1(1-p_2) \sum_{k=0}^{\infty} \left[(1-p_1)(1-p_2)\right]^{k-1}$$

$$= p_1(1-p_2) \frac{(1-p_1)(1-p_2)}{1-(1-p_1)(1-p_2)}$$

误判 a_1

$$\sum_{k=0}^{\infty} r^k = \frac{1}{1-r}$$

$$= \frac{p_1(1-p_2)}{p_1 + p_2 - p_1 p_2}$$

$$1 + r + r^2 + \cdots + r^k$$

$$= 1 \cdot \frac{1}{1-r}$$

$$= \frac{1}{1-r}$$