Problem 6.3

Q: $Y = ?$

$$P = \begin{bmatrix} 0.6 & 0.2 & 0.2 \\ 0.1 & 0.8 & 0.1 \\ 0.6 & 0 & 0.4 \end{bmatrix}$$

Solution $\begin{cases} Y = YP \\ \sum_{j} y_j = 1 \end{cases}$

$$\begin{cases} y_0 = 0.6 y_0 + 0.1 y_1 + 0.6 y_2 \\ y_1 = 0.2 y_0 + 0.8 y_1 \\ y_2 = 0.2 y_0 + 0.1 y_1 + 0.4 y_2 \\ y_0 + y_1 + y_2 = 1 \end{cases}$$

$$\Rightarrow \begin{cases} -0.4 y_0 + 0.1 y_1 + 0.6 y_2 = 0 \\ 0.2 y_0 - 0.2 y_1 = 0 \\ 0.2 y_0 + 0.1 y_1 - 0.6 y_2 = 0 \\ y_0 + y_1 + y_2 = 1 \end{cases} \quad ①$$

From ① $y_0 = y_1$

$$\begin{cases} -3 y_0 = -6 y_2 \\ 3 y_0 = 6 y_2 \quad ② \\ 2 y_0 + y_2 = 1 \quad ④ \end{cases}$$

From ② $y_0 = 2 y_2$ ③

From ③④ $2 y_0 + \frac{1}{2} y_0 = 1 \Rightarrow y_0 = \frac{2}{5} \quad y_1 = \frac{2}{5} \quad y_2 = \frac{1}{5}$

$$Y = \begin{bmatrix} \frac{2}{5} & \frac{2}{5} & \frac{1}{5} \end{bmatrix}$$

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Ans: The steady-state probabilities of states 0, 1 and 2 are $0.4,\ 0.4$ **and** 0.2 **respectively.**