Example 6.2

Q : Case 1 $\quad a = b = 0 \quad\quad Y = ?$

Solution : $P = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = I$

$$\begin{cases} Y = YP \quad (\text{not useful}) \\ \sum_j y_j = 1 \end{cases}$$

unlimited number of solutions.

$$Y = \begin{bmatrix} y_0 & 1 - y_0 \end{bmatrix}$$

Q: Case 2. $\quad a = b = 1 \quad\quad Y = ?$

Solution $P = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$

$$\begin{cases} Y = YP \\ y_0 + y_1 = 1 \end{cases}$$

$$\begin{bmatrix} y_0, y_1 \end{bmatrix} = \begin{bmatrix} y_0, y_1 \end{bmatrix} \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$

$\Rightarrow$

$$= \begin{bmatrix} y_1 & y_0 \end{bmatrix}$$

$$\Rightarrow y_0 = y_1$$

$$\Rightarrow \begin{cases} y_0 = 0.5 \\ y_1 = 0.5 \end{cases} \quad\quad Y = \begin{bmatrix} 0.5 & 0.5 \end{bmatrix}$$

Q. Case 3    $a, b \in [0, 1)$    $Y = ?$

Solution    $P = \begin{bmatrix} 1-a & a \\ b & 1-b \end{bmatrix}$

$Y = YP$    $[y_0 \quad y_1] = [y_0 \quad y_1] \begin{bmatrix} 1-a & a \\ b & 1-b \end{bmatrix}$

$\qquad\qquad\qquad\qquad = [(1-a)y_0 + by_1 \qquad ay_0 + (1-b)y_1]$

$\Rightarrow \begin{cases} y_0 = (1-a)y_0 + by_1 \\ y_1 = ay_0 + (1-b)y_1 \\ y_0 + y_1 = 1 \end{cases}$

let $y_1 = 1 - y_0$

$y_0 = (1-a)y_0 + b(1-y_0)$

$y_0 = y_0 - ay_0 + b - by_0$

$(a+b)y_0 = b$

$y_0 = \dfrac{b}{a+b}$

$y_1 = 1 - \dfrac{b}{a+b} = \dfrac{a}{a+b}$

$Y = \begin{bmatrix} \dfrac{b}{a+b} & \dfrac{a}{a+b} \end{bmatrix}$