23-S1-Q3

Q(a) state ? $\quad P$?

$S = \{0, 1, 2\}$

$P_{00} = P\{B_k \geq 2\} = 1 - P\{B_k = 1\} - P\{B_k = 0\}$

$= 1 - \frac{e^{-0.1}(0.1)^1}{1!} - \frac{e^{-0.1}(0.1)^0}{0!} = 1 - 1.1e^{-0.1}$

$P_{01} = P(B_k = 1) = 0.1e^{-0.1}$

$P_{02} = P(B_k = 0) = e^{-0.1}$

$P_{10} = P(B_k \geq 2) = 1 - 1.1e^{-0.1}$

$P_{11} = P(B_k = 1) = 0.1e^{-0.1}$

$P_{12} = P(B_k = 0) = e^{-0.1}$

$P_{20} = 1 - 1.1e^{-0.1}$

$P_{21} = 0.1e^{-0.1}$

$P_{22} = e^{-0.1}$

$$P = \begin{bmatrix} 1 - 1.1e^{-0.1} & 0.1e^{-0.1} & e^{-0.1} \\ 1 - 1.1e^{-0.1} & 0.1e^{-0.1} & e^{-0.1} \\ 1 - 1.1e^{-0.1} & 0.1e^{-0.1} & e^{-0.1} \end{bmatrix}$$

(b) $E(T_0) = \frac{1}{1 - P_{00}} = \frac{1}{1.1e^{-0.1}} = 1.0047$

$E(T_1) = \frac{1}{1 - P_{11}} = \frac{1}{1 - 0.1e^{-0.1}} = 1.0995$

$$E(T_2) = \frac{1}{1-P_{22}} = \frac{1}{1-e^{-0.1}} = 10.5083$$

$$\text{mean} E(T_i) = \frac{1}{3}\left[E(T_0) + E(T_1) + E(T_2)\right] = 4.2042$$

State 2 stayes longer than 0 and 1

No surprize

conmment: $\lambda = 0.1$ is a small number so the usage rate is low and the replenishment polizy reset the numbe of spare bulks to 2 whenever there is a shortfall

(c) independant of previous state

$\pi_0 = 1 - 1.1 e^{-0.1}$

$\pi_1 = 0.1 e^{-0.1}$

$\pi_2 = e^{-0.1}$

(d) $P(U_{k+4} = 1) = \pi_1 = 0.1 e^{-0.1}$

23-S1 — Q4

Q: (i) $L=?$ (ii) $W=?$ (iii) $T_{\text{break-even}}$

(i)

① $L_1 = \frac{\rho}{1-\rho} = \frac{\lambda}{\mu-\lambda} = \frac{2}{3-2} = 2$

② $\rho = \frac{\lambda}{\mu} = \frac{2}{3}$

$N=3$

$$L_2 = \frac{\rho[1-\rho^N - N\rho^N(1-\rho)]}{(1-\rho)(1-\rho^{N+1})}$$

$$= \frac{\frac{2}{3}[1-(\frac{2}{3})^3 - 3\times(\frac{2}{3})^3\frac{1}{3}]}{\frac{1}{3}(1-(\frac{2}{3})^4)}$$

$= \frac{66}{65}$

$= 0.6462$

③ $b=3$ $\mu=3$ $\lambda=\frac{2}{3}$

$\rho = \frac{b\lambda}{\mu} = \frac{3\times\frac{2}{3}}{3} = \frac{2}{3}$

$L_3 = \frac{\rho(1+b)}{2(1-\rho)}$

$$= \frac{\frac{2}{3}(1+3)}{2(1-\frac{2}{3})}$$

$$= 4$$

④ $\lambda = 2 \quad \mu = 2 \quad m = 3$

$$\rho = \frac{\lambda}{m\mu} = \frac{2}{3\times 2} = \frac{1}{3}$$

$$\pi_0 = \left[\frac{(m\rho)^m}{m!(1-\rho)} + \sum_{k=0}^{m-1}\frac{(m\rho)^k}{k!}\right]^{-1}$$

$$= \left[\frac{(3\times\frac{1}{3})^3}{3!\times\frac{2}{3}} + \frac{(3\times\frac{1}{3})^0}{0!} + \frac{(3\times\frac{1}{3})^1}{1!} + \frac{(3\times\frac{1}{3})^2}{2!}\right]^{-1}$$

$$= \left(\frac{1}{4} + 1 + 1 + \frac{1}{2}\right)^{-1}$$

$$= \frac{4}{11}$$

$$L_4 = \frac{\rho(m\rho)^m\pi_0}{m!(1-\rho)^2} + \frac{\lambda}{\mu}$$

$$= \frac{\frac{1}{3}\times 1^3 \times \frac{4}{11}}{3!\times(\frac{2}{3})^2} + \frac{2}{2}$$

$$= \frac{23}{22} = 1.0455$$

(ii)

① $W_1 = \frac{1}{\mu - \lambda} = 1$

② $W_2 = \frac{1-\left(\frac{2}{3}\right)^3 - 3\times\left(\frac{2}{3}\right)^3 \times \frac{1}{3}}{3\times\frac{1}{3}\times\left[1-\left(\frac{2}{3}\right)^4\right]}$

$= \frac{33}{65} = 0.5077$

③ $W_3 = \frac{L}{\lambda b} = \frac{4}{\frac{2}{3}\times 3} = 2$

④ $W_4 = \frac{L}{\lambda} = \frac{\frac{23}{22}}{2} = \frac{23}{44} = 0.5227$

(iii) ① Monthly profit = Revenue − operation

= 40000 − 20000

= 20000

$\text{months} = \frac{\text{Initial renovation}}{\text{Monthly profit}} = \frac{100000}{20000} = 5$

② $$\frac{120000-90000}{30000} = 10$$

③ $$\frac{30000-18000}{12000} = 8.33$$

④ $$\frac{100000-60000}{40000} = 3.75$$

best choice : M/M/3

① short waiting time, only 0.5227 min

② low mean number in system

③ quick break-even time, only 3.75 months

④ highest monthly profit