

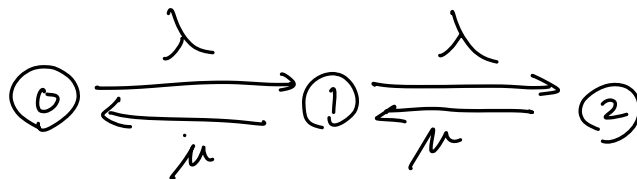
17-S2-Q4

Solution ① transition rate diagram

let state 0: The magazine is available

State 1: The magazine has been loaned out  
but nobody reserve it.

State 2: The magazine has been loaned out  
and one reservation is accept  
loan is exponentially distributed with mean  $\frac{1}{\mu}$   
$$V = \frac{1}{\frac{1}{\mu}} = \mu$$



② rate balance equation  
for state 0

$$\lambda \pi_0 = \mu \pi_1 \quad (1)$$

for state 1

$$(\lambda + \mu) \pi_1 = \lambda \pi_0 + \mu \pi_2 \quad (2)$$

for state 2.

$$\mu \pi_2 = \lambda \pi_1 \quad (3)$$

$$\pi_0 + \pi_1 + \pi_2 = 1 \quad (4)$$

$$\text{from (1)} \quad z_1 = \frac{\lambda}{\mu} z_0 \quad (5)$$

$$\text{from (3)} \quad z_2 = \frac{\lambda}{\mu} z_1 = \frac{\lambda^2}{\mu^2} z_0 \quad (6)$$

$$\text{from (4)} \quad z_0 + \frac{\lambda}{\mu} z_0 + \frac{\lambda^2}{\mu^2} z_0 = 1$$

$$\left(1 + \frac{\lambda}{\mu} + \frac{\lambda^2}{\mu^2}\right) z_0 = 1$$

$$\frac{\mu^2 + \lambda\mu + \lambda^2}{\mu^2} z_0 = 1$$

$$z_0 = \frac{\mu^2}{\mu^2 + \lambda\mu + \lambda^2}$$

$$z_1 = \frac{\lambda}{\mu} z_0 = \frac{\lambda\mu}{\mu^2 + \lambda\mu + \lambda^2}$$

$$z_2 = \frac{\lambda}{\mu} z_1 = \frac{\lambda^2}{\mu^2 + \lambda\mu + \lambda^2}$$

$$\text{So } z = [z_0 \quad z_1 \quad z_2]$$

$$= \left[ \frac{\mu^2}{\mu^2 + \lambda\mu + \lambda^2} \quad \frac{\lambda\mu}{\mu^2 + \lambda\mu + \lambda^2} \quad \frac{\lambda^2}{\mu^2 + \lambda\mu + \lambda^2} \right]$$

(b) M M 1 Queue

$$\textcircled{1} \quad \lambda = 1 \quad \mu = 4$$

$$\rho = \frac{\lambda}{\mu} = \frac{1}{4}$$

$$z_0 = 1 - \rho = \frac{3}{4} = 0.75$$

$$Q = \frac{\rho^2}{1-\rho} = \frac{\lambda_1^2}{\mu(\mu-\lambda_1)} = \frac{1}{12} = 0.083333$$

$$D = w - \frac{1}{\mu} = \frac{\lambda_1}{\mu(\mu-\lambda_1)} = \frac{1}{4 \times (4-1)} = \frac{1}{12} = 0.083333$$

$$\textcircled{2} \lambda_2 = 3 \quad \mu = 4$$

$$\rho = \frac{\lambda_2}{\mu} = \frac{3}{4}$$

$$\pi_0 = 1 - \rho = \frac{1}{4} = 0.25$$

$$Q = \frac{\rho^2}{1-\rho} = \frac{\lambda_2^2}{\mu(\mu-\lambda_2)} = \frac{3^2}{4 \times (4-3)} = \frac{9}{4} = 2.25$$

$$D = w - \frac{1}{\mu} = \frac{\lambda_2}{\mu(\mu-\lambda_2)} = \frac{3}{4 \times (4-3)} = \frac{3}{4} = 0.75$$

$$\textcircled{3} \lambda_3 = 2 \quad \mu = 4$$

$$\rho = \frac{\lambda_3}{\mu} = \frac{2}{4} = \frac{1}{2} = 0.5$$

$$\pi_0 = 1 - \rho = \frac{1}{2} = 0.5$$

$$Q = \frac{\rho^2}{1-\rho} = \frac{\lambda_3^2}{\mu(\mu-\lambda_3)} = \frac{2^2}{4 \times (4-2)} = \frac{4}{8} = \frac{1}{2} = 0.5$$

$$D = w - \frac{1}{\mu} = \frac{\lambda_3}{\mu(\mu - \lambda_3)} = \frac{2}{4 \times (4 - 2)} = \frac{2}{8} = \frac{1}{4} = 0.25$$

④ So, the answer of (b) is

	counter 1	counter 2	counter 3
(i)	0.75	0.08333	0.08333
(ii)	0.25	2.25	0.75
(iii)	0.5	0.5	0.25

c) ① Combine Queues and Servers

Merge the three  $M/M/1$  into a  $M/M/s$  queueing system, leading to shorter queues and reduced the waiting times for customer.

② Using  $M/M/1/N$  queue :

avoid the queue too long