斐波那契搜索方法

1. 定义

$$F_n = F_{n-1} + F_{n-2}\ ,\ n = 2, 3,\ldots$$

where $F_0 = F_1 = 1$. Thus, the Fibonacci sequence is $\{F_0, F_1, F_2, F_3, F_4, F_5, F_6, F_7, F_8,\ldots\} = \{1, 1, 2, 3, 5, 8, 13, 21, 34\}$.

<u>Example</u>

Use the Fibonacci search to approximate the location of the maximum of $f(x) = x(5\pi - x)$ on $[0, 20]$ to within $\epsilon = 1$.

<u>Initial points</u>

Find the smallest Fibonacci number that satisfies $F_N\ \epsilon \geq b\text{-}a$.
The first Fibonacci number that satisfies

$$F_N\ 1 \geq 20\text{-}0$$

is $F_7 = 21$. We set $N = 7$,

$$\epsilon' = (b\text{-}a)/F_N = (20\ \text{-}0)/21 = 0.9524$$

and then position the first two points in the search at

$$F_{N-1}\ \epsilon' = F_6(0.9524) = 12.38$$

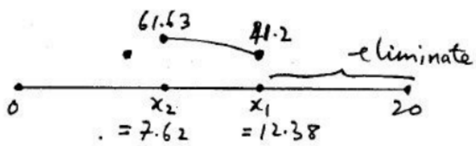
from each endpoint. Thus

$$x_1 = 0 + 12.38 = 12.38$$

$$x_2 = 20 - 12.38 = 7.62$$

$$f\ (x_1) = (12.38)(5\ \pi - 12.38) = 41.20$$

$$f\ (x_2) = (7.62)(5\ \pi - 7.62) = 61.63$$



Using the unimodal property, we conclude that the maximum must occur to the left of 12.38, and we reduce the interval of interest to $[0, 12.38]$

<u>1st iteration</u>

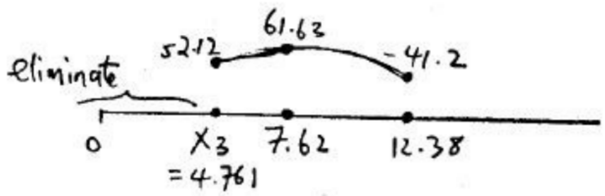
The next lower Fibonacci number (F_6 was the last one used) is $F_5 = 8$; so the next point in the search is positioned at

$$F_{N-1}\ \epsilon' = F_5(0.9524) = 7.619$$

from the newest endpoint, 12.38. Thus

$$x_3 = 12.38 - 7.619 = 4.761$$

$$f(x_3) = (4.761)(5\ \pi - 4.761) = 52.12$$



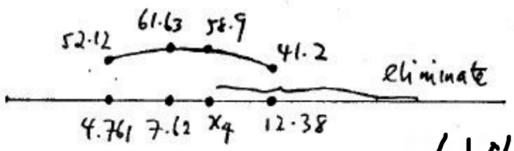
[4.761, 12.38] is the new interval of interest.

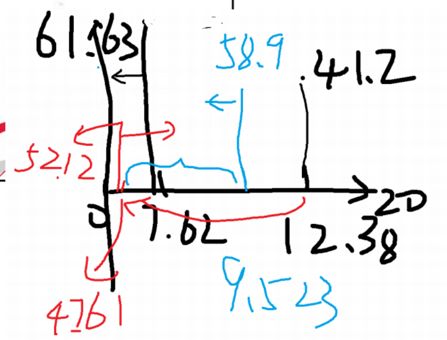
2nd iteration

The next lower Fibonacci number now is $F_4 = 5$. Thus,

$x_4 = 4.761 + F_4\,\epsilon' = 4.761 + 4(0.9524) = 9.523$

$F(x_4) = (9.523)(5\,\pi - 9.523) = 58.9$





[4.761, 9.523] is the new interval of interest.

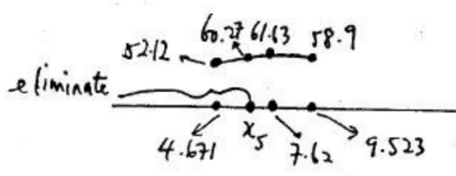
NANYANG TECHNOLOGICAL UNIVERSITY

3rd iteration

The next lower Fibonacci number now is $F_3 = 3$, Hence

$x_5 = 9.523 - F_3\,\epsilon' = 9.523 - 3(0.9524) = 6.666$

$f(x_5) = (6.666)\,(5\,\pi - 6.666) = 60.27$



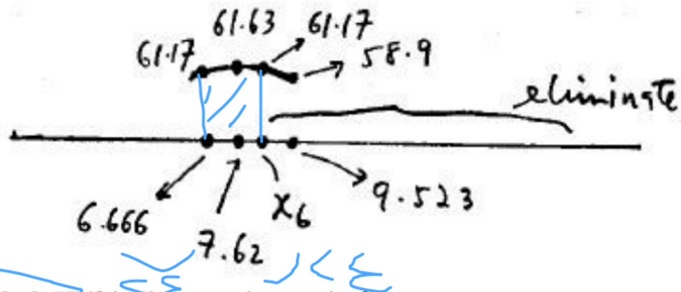
[6.666, 9.523] is the new interval of interest.

4th iteration

The next lower Fibonacci number now is $F_2 = 2$. Hence

$x_6 = 6.666 + F_2\,\epsilon' = 6.666 + 2(0.9524) = 8.571$

$F(x_6) = (8.571)(5\,\pi - 8.571) = 61.17$



[6.666, 8.571] is the new interval of interest.

Note that $x_2 = 7.62$ is within $\epsilon = 1$ of every other point of the interval. We therefore accept x_2 as the location of the maximum, i.e.

$$x^* = x_2 = 7.62 \text{ with } z^* = f(x_2) = 61.63$$