

Simplex Two-phase method

$$\text{Min } Z = 80X_1 + 60X_2$$

$$\text{s.t. } \begin{cases} 0.2X_1 + 0.32X_2 \leq 0.25 \\ X_1 + X_2 = 1 \end{cases}$$

Solution

① Simple

$$Z = 80X_1 + 60X_2 + 0 \cdot X_3 + M\bar{X}_4$$

$$\text{s.t. } \begin{cases} 0.2X_1 + 0.32X_2 + X_3 = 0.25 \\ X_1 + X_2 + \bar{X}_4 = 1 \end{cases}$$

② Matrix

$$A = \begin{bmatrix} 0.2 & 0.32 & 1 & 0 \\ 1 & 1 & 0 & 1 \end{bmatrix} \quad B = \begin{bmatrix} 0.25 \\ 1 \end{bmatrix}$$

$$X = [X_1 \ X_2 \ X_3 \ \bar{X}_4]^T \quad X_0 = [X_3 \ \bar{X}_4]^T$$

$$C = [80 \ 60 \ 0 \ M]^T \quad C_0 = [0 \ M]^T$$

$$\begin{aligned}
 C^T - C_0^T A &= [80 \ 60 \ 0 \ M] - [0 \ M] \begin{bmatrix} 0.2 & 0.32 & 1 & 0 \\ 1 & 1 & 0 & 1 \end{bmatrix} \\
 &= [80 \ 60 \ 0 \ M] - [M \ M \ 0 \ M] \\
 &= [80-M \ 60-M \ 0 \ 0]
 \end{aligned}$$

$$C_0^T B = [0 \ M] \begin{bmatrix} 0.25 \\ 1 \end{bmatrix} = M$$

③ X^T

$X_0 \quad A \quad B$

$$C^T - C_0^T A \quad -C_0^T B$$

④ Iteration 1

	X_1	X_2	X_3	\bar{X}_4		Ratio
X_3	0.2	0.32	1	0	0.25	$0.25/0.2 = 1.25$
$\bar{X}_4 \rightarrow X_1$	1	1	0	1	1	$1/1 = 1 \text{ min}$
	80	60	0	0	0	$-80R_2 + R_3$
	-1	-1	0	0	-1	$R_2 + R_4$

↑
work
column

⑤

	x_1	x_2	x_3	x_4	
x_3	0.2^0	$0.32_{0.12}$	1	$0-0.2$	$0.25_{0.05}$
x_1	1	1	0	1	1
	80^0	60^{-20}	0	0^{-80}	0^{-80}
	-1^0	-1^0	0	0^1	-1^0

最后一行 ≥ 0
且人工变量非基
则消去最后一行

⑥ Iteration 2

	x_1	x_2	x_3		Ratio
x_3	0	0.12	1	$0.05 \times \frac{1}{0.12}$	$0.05/0.12 = 0.4167_{\min}$
x_1	1	1	0	1	$1/1 = 1$
	0	-20	0	-80	

min
↓
work column

	x_1	x_2	x_3
$x_3 \rightarrow x_2$	0	<u>0.12</u>	<u>1</u>
			8.3333
			0.4167
x_1	1	<u>1</u>	<u>0</u>
			-8.3333
			0.5833
0	0	<u>-20</u>	<u>0</u>
			166.6667
			-80
			-71.6667

work column

$-R_1 + R_2$

$20R_1 + R_3$

So optimal solution is

$$x_1^* = 0.5833 \quad x_2^* = 0.4167 \quad x_3^* = x_4^* = 0$$

with $z^* = 71.6667$ (minimization)