21-S1-Q3

Q : (a) $p(a,b)=?$ $\quad p(0,b)=?$ $\quad p(a,\infty)=\infty$

Solution (a) $p(a,b) = p(a \le X \le b)$

$$= \sum_{k=a}^{b} \frac{e^{-\lambda t}(\lambda t)^k}{k!}$$

$$p(0,b) = p(0 \le X \le b)$$

$$= \sum_{k=0}^{b} \frac{e^{-\lambda t}(\lambda t)^k}{k!}$$

$$p(a,\infty) = p(X \ge a)$$

$$= 1 - p(X < a)$$

$$= 1 - \sum_{k=0}^{a-1} \frac{e^{-\lambda t}(\lambda t)^k}{k!}$$

Q(b)(i) TPM = ?

Solution $p = [p_{ij}]$

state space $\{w_0 \quad w_1 \quad w_2 \quad w_3\}$

let D_{k+1} 为下个月发的奖励数，X_{k+1} 为下个月销售总量

$P_{00} = P\{D_{k+1} = 3\}$ 已知上月底发完，下月底也发完

$= P\{X_{k+1} \ge 30\} = p(30, \infty)$

$$P_{01} = P\{D_{k+1} = 2\} = P\{20 \leq X_{k+1} \leq 29\} = P(20, 29)$$

$$P_{02} = P(10, 19)$$

$$P_{03} = P(0, 9)$$

$$P_{10} = P(10, \infty)$$

$$P_{11} = P(0, 9)$$

$$P_{12} = 0$$

$$P_{13} = 0$$

$$P_{20} = P(20, \infty)$$

$$P_{21} = P(10, 19)$$

$$P_{22} = P(0, 9)$$

$$P_{23} = 0$$

$$P_{30} = P(30, \infty)$$

$$P_{31} = P(20, 29)$$

$$P_{32} = P(10, 19)$$

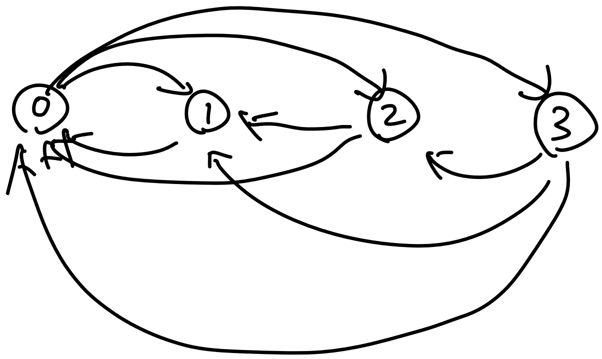
$$P_{33} = P(0, 9)$$

TPM is

$$\begin{array}{c c} & \begin{array}{cccc} 0 & 1 & 2 & 3 \end{array} \\ \begin{array}{c} 0 \\ 1 \\ 2 \\ 3 \end{array} & \begin{bmatrix} P(30,\infty) & P(20,29) & P(10,19) & P(0,9) \\ P(10,\infty) & P(0,9) & 0 & 0 \\ P(20,\infty) & P(10,19) & P(0,9) & 0 \\ P(30,\infty) & P(20,29) & P(10,19) & P(0,9) \end{bmatrix} \end{array}$$

(ii) ① steady - state probability

$$\begin{cases} Y = YP \\ \sum_{i=0}^{3} y_i = 1 \end{cases} \qquad Y = [y_0 \ y_1 \ y_2 \ y_3]$$



$$\begin{cases} [y_0 \ y_1 \ y_2 \ y_3] = [y_0 \ y_1 \ y_2 \ y_3] \begin{bmatrix} p(30,\infty) & p(20,29) & p(10,19) & p(0,9) \\ p(10,\infty) & p(0,9) & 0 & 0 \\ p(20,\infty) & p(10,19) & p(0,9) & 0 \\ p(30,\infty) & p(20,29) & p(10,19) & p(0,9) \end{bmatrix} \\ y_0 + y_1 + y_2 + y_3 = 1 \end{cases}$$

$$\begin{cases} y_0 = p(30,\infty)\, y_0 + p(20,29)\, y_1 + p(10,19)\, y_2 + p(0,9)\, y_3 \\ y_1 = p(10,\infty)\, y_0 + p(0,9)\, y_1 \\ y_2 = p(20,\infty)\, y_0 + p(10,19)\, y_1 + p(0,9)\, y_2 \\ y_3 = p(30,\infty)\, y_0 + p(20,29)\, y_1 + p(10,19)\, y_2 + p(0,9)\, y_3 \\ y_0 + y_1 + y_2 + y_3 = 1 \end{cases}$$

$$y_1 = \frac{p(10,\infty)}{1 - p(0,9)}\, y_0$$

$$y_2 = \left[\frac{p(20,\infty)}{1 - p(0,9)} + \frac{p(10,19)\, p(10,\infty)}{[1 - p(0,9)]^2} \right] y_0$$

$y_3 =$

$$\left[\frac{p(30,\infty)}{1 - p(0,9)} + \frac{p(20,29)\, p(10,\infty)}{(1 - p(0,9))^2} + \frac{p(10,19)}{1 - p(0,9)} \left[\frac{p(20,\infty)}{1 - p(0,9)} + \frac{p(10,19)\, p(10,\infty)}{[1 - p(0,9)]^2} \right] \right] y_0$$

代入求 y_0 y_1 y_2 y_3