22-S1-Q2

$50+180+280+270=780$

$100+200+180+300=780$

no virtical supply

Q: (a) transportation model

Solution ① draw

Production period \ demand period	3	4	5	6	Production capacity u_i	row diff
3	40 / 50	40.5	41	41.5	50	0.5
4	42	40	40.5	41	180	0.5
5	44	42	40	40.5	280	0.5
6	46	44	42	40	270	2
Demand v_j	100	200	180	300		
col diff	2	0.5	0.5	0.5		

col with largest penalty

② get base solution

	3	4	5	6	supply	row diff
3	40 / 50	40.5	41	41.5	0	
4	42 / 50	40	40.5	41	130	0.5
5	44	42	40	40.5	280	0.5
6	46	44	42	40	270	2
demand	0	200	180	300		
	2	2	0.5	0.5		

↑ max

	3	4	5	6	supply	rol diff
3	$\boxed{40}$ 50	$\boxed{40.5}$	$\boxed{41}$	$\boxed{41.5}$	0	
4	$\boxed{42}$ 50	$\boxed{40}$ 130	$\boxed{40.5}$	$\boxed{41}$	0	0.5
5	$\boxed{44}$	$\boxed{42}$	$\boxed{40}$	$\boxed{40.5}$	280	0.5
6	$\boxed{46}$	$\boxed{44}$	$\boxed{42}$	$\boxed{40}$	270	2
demand	0	70	180	300		
		2	0.5	0.5		

	3	4	5	6	supply	rol diff
3	$\boxed{40}$ 50	$\boxed{40.5}$	$\boxed{41}$	$\boxed{41.5}$	0	
4	$\boxed{42}$ 50	$\boxed{40}$ 130	$\boxed{40.5}$	$\boxed{41}$	0	
5	$\boxed{44}$	$\boxed{42}$ 70	$\boxed{40}$	$\boxed{40.5}$	210	0.5
6	$\boxed{46}$	$\boxed{44}$	$\boxed{42}$	$\boxed{40}$	270	2
demand	0	0	180	300		
		2	2	0.5		
		4 max				

	3	4	5	6	supply	rol diff
3	$\boxed{40}$ 50	$\boxed{40.5}$	$\boxed{41}$	$\boxed{41.5}$	0	
4	$\boxed{42}$ 50	$\boxed{40}$ 130	$\boxed{40.5}$	$\boxed{41}$	0	
5	$\boxed{44}$	$\boxed{42}$ 70	$\boxed{40}$ 180	$\boxed{40.5}$	30	0.5
6	$\boxed{46}$	$\boxed{44}$	$\boxed{42}$	$\boxed{40}$	270	2
demand	0	0	0	300		
			2	0.5		
			↑ max			

	3	4	5	6	supply
3	$\boxed{40}$ 50	$\boxed{40.5}$	$\boxed{41}$	$\boxed{41.5}$	0
4	$\boxed{42}$ 50	$\boxed{40}$ 130	$\boxed{40.5}$	$\boxed{41}$	0
5	$\boxed{44}$	$\boxed{42}$ 70	$\boxed{40}$ 180	$\boxed{40.5}$ 30	0
6	$\boxed{46}$	$\boxed{44}$	$\boxed{42}$	$\boxed{40}$ 270	0
demand	0	0	0	0	

50

	3	4	5	6	supply
3	$\boxed{40}$ 50	$\boxed{40.5}$	$\boxed{41}$	$\boxed{41.5}$	50
4	$\boxed{42}$ 50	$\boxed{40}$ 130	$\boxed{40.5}$	$\boxed{41}$	180
5	$\boxed{44}$	$\boxed{42}$ 70	$\boxed{40}$ 180	$\boxed{40.5}$ 30	280
6	$\boxed{46}$	$\boxed{44}$	$\boxed{42}$	$\boxed{40}$ 270	270
demand	100	200	180	300	

③ test optimal

	3	4	5	6	supply	u_i
3	$\boxed{40}$ 50	$\boxed{40.5}$ 2.5	$\boxed{41}$ 5	$\boxed{41.5}$ 5	50	-4
4	$\boxed{42}$ 50	$\boxed{40}$ 130	$\boxed{40.5}$ 2.5	$\boxed{41}$ 2.5	180	-2
5	$\boxed{44}$ 0	$\boxed{42}$ 70	$\boxed{40}$ 180	$\boxed{40.5}$ 30	280	0
6	$\boxed{46}$ 2.5	$\boxed{44}$ 2.5	$\boxed{42}$ 2.5	$\boxed{40}$ 270	270	-0.5
demand	100	200	180	300		
V_i	44	42	40	40.5		

So, the solution is optimal

(b) L?

Solution

① $L = x_1^2 + x_2^2 + x_3^2 + \lambda_1(x_1 + x_2 + 3x_3 - 2) + \lambda_2(5x_1 + 2x_2 + x_3 - 5)$

$$\frac{\partial L}{\partial x_1} = 2x_1 + \lambda_1 + 5\lambda_2 = 0 \qquad (1)$$

$$\frac{\partial L}{\partial x_2} = 2x_2 + \lambda_1 + 2\lambda_2 = 0 \qquad (2)$$

$$\frac{\partial L}{\partial x_3} = 2x_3 + 3\lambda_1 + \lambda_2 = 0 \qquad (3)$$

$$\frac{\partial L}{\partial \lambda_1} = x_1 + x_2 + 3x_3 - 2 = 0 \qquad (4)$$

$$\frac{\partial L}{\partial \lambda_2} = 5x_1 + 2x_2 + x_3 - 5 = 0 \qquad (5)$$

Solve the equation using $\underline{\text{casio}}$ only 4个未知数

from(3) $\lambda_2 = -(2x_3 + 3\lambda_1)$ 代入 (1) (2)

$$2x_1 + \lambda_1 - 5(2x_3 + 3\lambda_1) = 2x_1 - 10x_3 - 14\lambda_1 = 0$$

$$2x_2 + \lambda_1 - 2(2x_3 + 3\lambda_1) = 2x_2 + \lambda_1 - 4x_3 - 6\lambda_1$$

$$= 2x_2 - 4x_3 - 5\lambda_1 = 0$$

不要当成 x_1了

Casio .solve

$$\begin{bmatrix} 2 & 0 & -10 & -14 \\ 0 & 2 & -4 & -5 \\ 1 & 1 & 3 & 0 \\ 5 & 2 & 1 & 0 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 2 \\ 5 \end{bmatrix}$$

$$\begin{cases} x_1 = \frac{37}{46} \\ x_2 = \frac{8}{23} \\ x_3 = \frac{13}{46} \\ \lambda_1 = -\frac{2}{23} \\ \lambda_2 = \frac{7}{23} \end{cases}$$

$$\lambda_2 = -(2x_3 + 3\lambda_1)$$
$$= -\left[2 \times \frac{13}{46} + 3 \times \left(-\frac{2}{23}\right)\right]$$
$$= -\frac{7}{23}$$

$$x = \left[\frac{37}{46} \quad \frac{8}{23} \quad \frac{13}{46}\right]$$

$$z = x_1^2 + x_2^2 + x_3^2 = \left(\frac{37}{46}\right)^2 + \left(\frac{8}{23}\right)^2 + \left(\frac{13}{46}\right)^2$$

$$= \frac{39}{46} = 0.8478$$

② test optimal

$\nabla h_1(x) = [1 \quad 1 \quad 3]$ $\qquad$ $\nabla h_2(x) = [5 \quad 2 \quad 1]$

$\nabla h_1(x) Y = y_1 + y_2 + 3y_3 = 0$ $\qquad$ $\nabla h_2(x) Y = 5y_1 + 2y_2 + y_3 = 0$

$$\nabla^2 L_{xx}(x,\lambda) = \begin{bmatrix} \frac{\partial L}{\partial x_1 \partial x_1} & \frac{\partial L}{\partial x_1 \partial x_2} & \frac{\partial L}{\partial x_1 x_3} \\ \frac{\partial L}{\partial x_2 \partial x_1} & \frac{\partial L}{\partial x_2 \partial x_2} & \frac{\partial L}{\partial x_2 \partial x_3} \\ \frac{\partial L}{\partial x_3 \partial x_1} & \frac{\partial L}{\partial x_3 \partial x_2} & \frac{\partial L}{\partial x_3 \partial x_3} \end{bmatrix} = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix}$$

positive-definite

$$\frac{\partial L}{\partial x_1} = 2x_1 + \lambda_1 + 5\lambda_2 = 0 \qquad \frac{\partial L}{\partial x_1 \partial x_1} = 2$$

$$\frac{\partial L}{\partial x_2} = 2x_2 + \lambda_1 + 2\lambda_2 = 0 \qquad \frac{\partial L}{\partial x_2 \partial x_2} = 2$$

$$\frac{\partial L}{\partial x_3} = 2x_3 + 3\lambda_1 + \lambda_2 = 0 \qquad \frac{\partial L}{\partial x_3 \partial x_3} = 2$$

$$Y^T \nabla^2_{xx} L Y = [y_1 \; y_2 \; y_3] \begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix}$$

$$= [2y_1 \quad 2y_2 \quad 2y_3] \begin{bmatrix} y_1 \\ y_2 \\ y_2 \end{bmatrix}$$

$$= 2y_1^2 + 2y_2^2 + 2y_3^2 > 0 \qquad \forall\, y \neq 0$$

So $x = [\frac{37}{46} \; \frac{8}{23} \; \frac{13}{46}]$ is a minimum point

$z^* = x_1^2 + x_2^2 + x_3^2 = \frac{39}{46} = 0.8478$ is the minimum value