

23-51 - Q4

Q: (i) $L = ?$ (ii) $W = ?$ (iii) $T_{\text{break-even}}$

(i)

$$\textcircled{1} L_1 = \frac{\rho}{1-\rho} = \frac{\lambda}{\mu-\lambda} = \frac{2}{3-2} = 2$$

$$\textcircled{2} \rho = \frac{\lambda}{\mu} = \frac{2}{3}$$

$$N = 3$$

$$L_2 = \frac{\rho [1 - \rho^N - N \rho^N (1-\rho)]}{(1-\rho) (1-\rho^{N+1})}$$

$$= \frac{\frac{2}{3} [1 - (\frac{2}{3})^3 - 3 \times (\frac{2}{3})^3 \frac{1}{3}]}{\frac{1}{3} (1 - (\frac{2}{3})^4)}$$

$$= \frac{66}{65}$$

$$= 0.6462$$

$$\textcircled{3} b=3 \quad \mu=3 \quad \lambda = \frac{2}{3}$$

$$\rho = \frac{b\lambda}{\mu} = \frac{3 \times \frac{2}{3}}{3} = \frac{2}{3}$$

$$L_3 = \frac{\rho(1+b)}{2(1-\rho)}$$

$$= \frac{\frac{2}{3}(1+3)}{2(1-\frac{2}{3})}$$

$$= 4$$

$$(4) \quad \lambda = 2 \quad \mu = 2 \quad m = 3$$

$$\rho = \frac{\lambda}{m\mu} = \frac{2}{3 \times 2} = \frac{1}{3}$$

$$\pi_0 = \left[\frac{(m\rho)^m}{m!(1-\rho)} + \sum_{k=0}^{m-1} \frac{(m\rho)^k}{k!} \right]^{-1}$$

$$= \left[\frac{(3 \times \frac{1}{3})^3}{3! \times \frac{2}{3}} + \frac{(3 \times \frac{1}{3})^0}{0!} + \frac{(3 \times \frac{1}{3})^1}{1!} + \frac{(3 \times \frac{1}{3})^2}{2!} \right]^{-1}$$

$$= \left(\frac{1}{4} + 1 + 1 + \frac{1}{2} \right)^{-1}$$

$$= \frac{4}{11}$$

$$L_4 = \frac{\rho(m\rho)^m \pi_0}{m!(1-\rho)^2} + \frac{\lambda}{\mu}$$

$$= \frac{\frac{1}{3} \times 1^3 \times \frac{4}{11}}{3! \times (\frac{2}{3})^2} + \frac{2}{2}$$

$$= \frac{23}{22} = 1.0455$$

(ii)

$$\textcircled{1} w_1 = \frac{1}{\mu - \lambda} = 1$$

$$\begin{aligned}\textcircled{2} w_2 &= \frac{1 - \left(\frac{2}{3}\right)^3 - 3 \times \left(\frac{2}{3}\right)^3 \times \frac{1}{3}}{3 \times \frac{1}{3} \times \left[1 - \left(\frac{2}{3}\right)^4\right]} \\ &= \frac{33}{65} = 0.5077\end{aligned}$$

$$\textcircled{3} w_3 = \frac{L}{\lambda b} = \frac{4}{\frac{2}{3} \times 3} = 2$$

$$\textcircled{4} w_4 = \frac{L}{\lambda} = \frac{\frac{23}{22}}{2} = \frac{23}{44} = 0.5227$$

$$\begin{aligned}\text{(iii) } \textcircled{1} \text{ Monthly profit} &= \text{Revenue} - \text{operation} \\ &= 40000 - 20000 \\ &= 20000\end{aligned}$$

$$\text{months} = \frac{\text{Initial renovation}}{\text{Monthly profit}} = \frac{100000}{20000} = 5$$

$$\textcircled{2} \quad \frac{120000 - 90000}{30000} = 10$$

$$\textcircled{3} \quad \frac{30000 - 18000}{12000} = 8.33$$

$$\textcircled{4} \quad \frac{100000 - 60000}{40000} = 3.75$$

best choice: M/M/3

- ① short waiting time, only 0.5227 min
- ② low mean number in system
- ③ quick break-even time, only 3.75 months
- ④ highest monthly profit