

22-51-Q1

Q(a) max profit ?

Solution let profit Z

product A x_1 B x_2 C x_3 D x_4

$$Z = 30x_1 - 15(800 - x_1) + 40x_2 - 20(750 - x_2) \\ + 20x_3 - 10(600 - x_3) + 10x_4 - 8(500 - x_4)$$

$$= 45x_1 + 60x_2 + 30x_3 + 18x_4 - 37000$$

Subject to

$$\begin{cases} 0.3x_1 + 0.3x_2 + 0.25x_3 + 0.15x_4 \leq 1000 \\ 0.25x_1 + 0.35x_2 + 0.3x_3 + 0.1x_4 \leq 1000 \\ 0.45x_1 + 0.5x_2 + 0.4x_3 + 0.22x_4 \leq 1000 \\ 0.15x_1 + 0.15x_2 + 0.1x_3 + 0.05x_4 \leq 1000 \\ x_1 \leq 800 \\ x_2 \leq 750 \\ x_3 \leq 600 \\ x_4 \leq 500 \\ x_1, x_2, x_3, x_4 \geq 0 \end{cases}$$

(b) Max $Z = 5x_1 + 4x_2 + 0 \cdot x_3 + 0 \cdot x_4 + 0 \cdot x_5 + 0 \cdot x_6$

Subject to

$$\begin{cases} -x_1 + x_2 + x_3 = 1 \\ x_2 + x_4 = 2 \\ x_1 + 2x_2 + x_5 = 6 \\ 6x_1 + 4x_2 + x_6 = 24 \\ x_1, x_2, x_3, x_4, x_5, x_6 \geq 0 \end{cases}$$

② list Simplex Max Table

	x^T	
x_0	A	B
	$-C^T + C_0^T A$	$C_0^T B$

$$x^T = [x_1 \ x_2 \ x_3 \ x_4 \ x_5 \ x_6] \quad x_0 = [x_3 \ x_4 \ x_5 \ x_6]^T$$

$$A = \begin{bmatrix} -1 & 1 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 & 0 \\ 1 & 2 & 0 & 0 & 1 & 0 \\ 6 & 4 & 0 & 0 & 0 & 1 \end{bmatrix} \quad B = \begin{bmatrix} 1 \\ 2 \\ 6 \\ 24 \end{bmatrix}$$

$$C^T = [5 \ 4 \ 0 \ 0 \ 0 \ 0] \quad C_0^T = [0 \ 0 \ 0 \ 0]$$

$$-C^T + C_0^T A = -C^T = [-5 \ -4 \ 0 \ 0 \ 0 \ 0]$$

$$C_0^T B = 0$$

③ draw Simplex Max Table

	x_1	x_2	x_3	x_4	x_5	x_6	
x_3	-1	1	1	0	0	0	1
x_4	0	1	0	1	0	0	2
x_5	1	2	0	0	1	0	6
x_6	6	4	0	0	0	1	24
	-5	-4	0	0	0	0	0

④ simplex Method: iteration 1

	x_1	x_2	x_3	x_4	x_5	x_6	Ratio
x_3	-1	1	1	0	0	0	1
x_4	0	1	0	1	0	0	2
x_5	1	2	0	0	1	0	6 $6/1=6$
x_6	<u>6</u>	4	0	0	0	1	24 $24/6=\textcircled{4}$ minimum
	<u>-5</u> work column minimum	-4	0	0	0	0	0

	x_1	x_2	x_3	x_4	x_5	x_6	
x_3	-1 0	1 $\frac{5}{3}$	1	0	0	0 $\frac{1}{6}$	1 5 R_3+R_1
x_4	0	1	0	1	0	0	2
x_5	1 0	2 $\frac{4}{3}$	0	0	1	0 $-\frac{1}{6}$	0 2 $-R_4+R_3$
x_6	<u>6</u>	<u>4</u> $\frac{2}{3}$	0	0	0	<u>1</u> $\frac{1}{6}$	<u>24</u> 4 $R_6 \times \frac{1}{6}$
$\rightarrow x_1$	-5 0	-4 $-\frac{2}{3}$	0	0	0	0 $\frac{5}{6}$	0 20 $5R_4+R_5$

$$2 - \frac{2}{3} = \frac{6}{3} - \frac{2}{3} = \frac{4}{3} \quad 6 - 4 = 2$$

$$\frac{10}{3} - 4 = \frac{10}{3} - \frac{12}{3} = -\frac{2}{3}$$

⑤ Simplex Iteration 2.

	X_1	X_2	X_3	X_4	X_5	X_6		Ratio
X_3	0	$\frac{5}{3}$	1	0	0	$\frac{1}{6}$	5	$5/\frac{5}{3}=3$
X_4	0	1	0	1	0	0	2	$2/1=2$
X_5	0	$\frac{4}{3}$	0	0	1	$-\frac{1}{6}$	2	$2/\frac{4}{3}=\frac{3}{2}$ <small>min</small>
X_1	1	$\frac{2}{3}$	0	0	0	$\frac{1}{6}$	4	$4/\frac{2}{3}=6$
	0	$-\frac{2}{3}$	0	0	0	$\frac{5}{6}$	20	

work column minimum

$$20 \times (-\frac{3}{2}) = -30$$

	X_1	X_2	X_3	X_4	X_5	X_6		
X_3	0	$\frac{5}{3}$	1	0	$-\frac{5}{4}$	$\frac{1}{6}$	$\frac{3}{8}$	$\frac{5}{6}$ $\leftarrow -\frac{5}{3}R_3 + R_1$
X_4	0	1	0	1	$-\frac{3}{4}$	$\frac{1}{8}$	$\frac{1}{2}$	$\leftarrow R_3 + R_2$
X_5	0	$\frac{4}{3}$	0	0	$\frac{2}{4}$	$-\frac{1}{6}$	$-\frac{1}{8}$	$\frac{3}{2}$ $\leftarrow R_3 \times \frac{3}{4}$
X_1	1	$\frac{2}{3}$	0	0	$-\frac{1}{2}$	$\frac{1}{6}$	$\frac{1}{4}$	4 $\leftarrow \frac{2}{3}R_3 + R_4$
	0	$-\frac{2}{3}$	0	0	$\frac{1}{2}$	$\frac{5}{6}$	$\frac{3}{4}$	20 $\leftarrow \frac{2}{3}R_3 + R_5$

$\frac{5}{2}$

$$-\frac{1}{6} \times \frac{3}{4} = -\frac{1}{2} \times \frac{1}{4} = -\frac{1}{8} \quad 2 \times \frac{3}{4} = \frac{3}{2}$$

$$2 - \frac{3}{2} = \frac{4}{2} - \frac{3}{2} = \frac{1}{2} \quad \frac{3}{4} \times (-\frac{5}{3}) = -\frac{5}{4}$$

$$-\frac{1}{8} \times (-\frac{5}{3}) + \frac{1}{6} = \frac{5}{24} + \frac{4}{24} = \frac{9}{24} = \frac{3}{8}$$

$$(-\frac{5}{3}) \times (\frac{3}{2}) + 5 = -\frac{5}{2} + 5 = -\frac{5}{2} + \frac{10}{2} = \frac{5}{2}$$

$$(-\frac{2}{3}) \times \frac{3}{4} = -\frac{1}{2} \quad (-\frac{1}{8}) \times (-\frac{2}{3}) + \frac{1}{6} = \frac{1}{12} + \frac{1}{6} = \frac{3}{12} = \frac{1}{4}$$

$$\frac{3}{4} \times (\frac{2}{3}) = \frac{1}{2} \quad -\frac{1}{8} \times \frac{2}{3} + \frac{5}{6} = -\frac{1}{12} + \frac{10}{12} = \frac{9}{12} = \frac{3}{4}$$

⑥ The final row coefficients are all nonnegative

	x_1	x_2	x_3	x_4	x_5	x_6	
x_3	0	0	1	0	$-\frac{5}{4}$	$\frac{3}{8}$	$\frac{5}{2}$
x_4	0	0	0	1	$-\frac{3}{4}$	$\frac{1}{8}$	$\frac{1}{2}$
x_2	0	1	0	0	$\frac{3}{4}$	$-\frac{1}{8}$	$\frac{3}{2}$
x_1	1	0	0	0	$-\frac{1}{2}$	$\frac{1}{4}$	3
	0	0	0	0	$\frac{1}{2}$	$\frac{3}{4}$	21

the optimal result

$$x_1 = 3 \quad x_2 = \frac{3}{2} \quad x_3 = \frac{25}{6} \quad x_4 = \frac{1}{2} \quad x_5 = x_6 = 0$$

the maximum $Z = 21$