23-S1-Q3

Q (a) state. ? P?

$S = \{0, 1, 2\}$

$P_{00} = P\{B_{k+1} \geq 2\} = 1 - P\{B_{k+1} = 1\} - P\{B_{k+1} = 0\}$

$= 1 - \frac{e^{-0.1}(0.1)^1}{1!} - \frac{e^{-0.1}(0.1)^0}{0!} = 1 - 1.1e^{-0.1}$

$P_{01} = P(B_{k+1} = 1) = 0.1e^{-0.1}$

$P_{02} = P(B_{k+1} = 0) = e^{-0.1}$

$P_{10} = P(B_{k+1} \geq 2) = 1 - 1.1e^{-0.1}$

$P_{11} = P(B_{k+1} = 1) = 0.1e^{-0.1}$

$P_{12} = P(B_{k+1} = 0) = e^{-0.1}$

$P_{20} = 1 - 1.1e^{-0.1}$

$P_{21} = 0.1e^{-0.1}$

$P_{22} = e^{-0.1}$

$$P = \begin{bmatrix} 1 - 1.1e^{-0.1} & 0.1e^{-0.1} & e^{-0.1} \\ 1 - 1.1e^{-0.1} & 0.1e^{-0.1} & e^{-0.1} \\ 1 - 1.1e^{-0.1} & 0.1e^{-0.1} & e^{-0.1} \end{bmatrix}$$

(b) $E(T_0) = \frac{1}{1 - P_{00}} = \frac{1}{1.1e^{-0.1}} = 1.0047$

$E(T_1) = \frac{1}{1 - P_{11}} = \frac{1}{1 - 0.1e^{-0.1}} = 1.0995$

$$E(T_2) = \frac{1}{1-P_{22}} = \frac{1}{1-e^{-0.1}} = 10.5083$$

$$\text{mean}\, E(T_i) = \frac{1}{3}\left[E(T_0)+E(T_1)+E(T_2)\right] = 4.2042$$

State 2 stays longer than 0 and 1

No surprize

comment: $\lambda = 0.1$ is a small number

so the usage rate is low and

the replenishment policy reset the

numbe of spare bulks to 2 whenever

there is a shortfall

(c) independant of previous state

$\pi_0 = 1 - 1.1e^{-0.1}$

$\pi_1 = 0.1e^{-0.1}$

$\pi_2 = e^{-0.1}$

(d) $P(U_{k+4} = 1) = \pi_1 = 0.1e^{-0.1}$