18-S2-Q4

Solution (a) ① We model the Continuous-Time Markov chain (CTMC) with the following state space

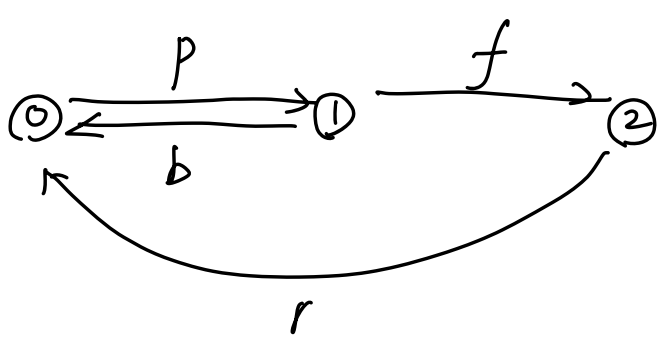
$S = \{0, 1, 2\}$

State 0: preparation: The machine is being prepared for baking (cleaning, greasing, adding ingredients)

state 1: The machine is baking a loaf of bread

state 2: The machine is under repair after a breaking during baking

(b) ① state transition diagram



② the rate balance equation

for state 0

$$p\pi_0 = b\pi_1 + r\pi_2$$

for state 1

$$b\pi_1 + f\pi_1 = p\pi_0$$

for state 2

$$r\pi_2 = f\pi_1$$

$$\begin{cases} p\pi_0 = b\pi_1 + r\pi_2 & (1) \\ b\pi_1 + f\pi_1 = p\pi_0 & (2) \\ r\pi_2 = f\pi_1 & (3) \end{cases}$$

③ steady-state probabilities

$$\begin{cases} \pi Q = 0 \\ \sum_{i=0}^{2} \pi_i = 1 \Rightarrow \pi_0 + \pi_1 + \pi_2 = 0 & (7) \end{cases}$$

from (3) $\pi_2 = \frac{f}{r}\pi_1$ (4)

from (2) $\pi_1 = \frac{p}{b+f}\pi_0$ (5)

from (4)(5) $\pi_2 = \frac{fp}{r(b+f)}\pi_0$ (6)

from (5)(6)(7) $\pi_0 + \frac{p}{b+f}\pi_0 + \frac{fp}{r(b+f)}\pi_0 = 1$

$$\left[1 + \frac{p}{b+f} + \frac{fp}{r(b+f)}\right]\pi_0 = 1$$

$$\frac{r(b+f)+rp+fp}{r(b+f)}\pi_0 = 1$$

$$\pi_0 = \frac{r(b+f)}{rb+rf+rp+fp}$$

$$\pi_1 = \frac{p}{b+f}\pi_0 = \frac{rp}{rb+rf+rp+fp}$$

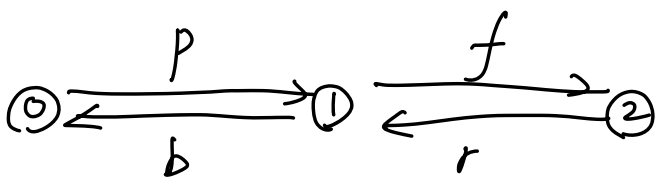
$$\pi_2 = \frac{f}{r}\pi_1 = \frac{fp}{rb+rf+rp+fp}$$

So $\pi = [\pi_0\ \pi_1\ \pi_2]$

$$= \left[\frac{r(b+f)}{rb+rf+rp+fp} \quad \frac{rp}{rb+rf+rp+fp} \quad \frac{fp}{rb+rf+rp+fp}\right]$$

(c) $R = b\pi_1 = \frac{rbp}{rb+rf+rp+fp}$

(d) ① Change: the transition from state 2 now goes directly to state 1 at rat r



② The production rate increase

Reason (1) the preparation time no longer adds to the down time after a failure

(2) reducing the total cycle time

(3) higher average production rate