

6204 - 21 - S1 - Q1

Q: (a) LP ?

let $X = [x_1 \ x_2]$ x_1 Exterior paint tons
 x_2 Interior paint tons

$$6x_1 + 4x_2 \leq 24$$

$$x_1 + 2x_2 \leq 6$$

$$x_2 - x_1 \leq 1$$

$$x_2 \leq 2$$

$$Z = 5x_1 + 4x_2$$

So
Max $Z = 5x_1 + 4x_2$
s.t. $\begin{cases} 6x_1 + 4x_2 + x_3 = 24 \\ x_1 + 2x_2 + x_4 = 6 \\ -x_1 + x_2 + x_5 = 1 \\ x_2 + x_6 = 2 \\ x_1, x_2, x_3, x_4, x_5, x_6 \geq 0 \end{cases}$
 Z_{max}

(b) Min $Z = -4y_1 - 5y_2 + 0y_3 + 0y_4 + 0y_5 + 0y_6$

Subject to

$$\begin{cases} 4y_1 + 6y_2 + y_3 = 24 & \text{松} \\ 2y_1 + y_2 + y_4 = 6 & \text{紧} \\ y_1 - y_2 + y_5 = 1 & \text{松} \\ y_1 + y_6 = 2 & \text{紧} \\ y_1, y_2, y_3, y_4, y_5, y_6 \geq 0 \end{cases}$$

SIMPLEX Tab

	y_1	y_2	y_3	y_4	y_5	y_6	
y_3	4	6		0	0	0	24
y_4	2	1	0	1	0	0	6
y_5	1	-1	0	0	1	0	1
y_6	1	0	0	0	0	1	2
	-4	-5	0	0	0	0	0

$$A = \begin{bmatrix} 4 & 6 & 1 & 0 & 0 & 0 \\ 2 & 1 & 0 & 1 & 0 & 0 \\ 1 & -1 & 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

$$B = \begin{bmatrix} 24 \\ 6 \\ 1 \\ 2 \end{bmatrix}$$

$$Y = [y_1 y_2 y_3 y_4 y_5 y_6]^T$$

$$Y_0 = [y_3 y_4 y_5 y_6]^T$$

$$C^T = [-4 \ -5 \ 0 \ 0 \ 0 \ 0]$$

$$C_0^T = [0 \ 0 \ 0 \ 0]$$

$$C^T - C_0^T A = [-4 \ -5 \ 0 \ 0 \ 0 \ 0]$$

$$-C_0^T B = 0$$

	y_1	y_2	y_3	y_4	y_5	y_6		Ratio min
y_3	4	(6)		0	0	0	24	$24/6 = 4$
y_4	2	1	0	1	0	0	6	$6/1 = 6$
y_5	1	-1	0	0	1	0	1	
y_6	1	0	0	0	0	1	2	
	-4	(-5)	0	0	0	0	0	
		min						

	y_1	y_2	y_3	y_4	y_5	y_6	
y_3	$4\frac{2}{3}$	6	$1\frac{1}{6}$	0	0	0	24 4
y_4	2	1	0	1	0	0	6
y_5	1	-1	0	0	1	0	1
y_6	1	0	0	0	0	1	2
	-4	-5	0	0	0	0	0

	y_1	y_2	y_3	y_4	y_5	y_6	
y_3	$\frac{2}{3}$	1	$\frac{1}{6}$	0	0	0	4
y_4	$2\frac{4}{3}$	7	$\frac{-1}{6}$	1	0	0	2 6
y_5	$6\frac{5}{3}$	-7	$\frac{1}{6}$	0	1	0	5 7
y_6	1	0	0	0	0	1	2
	$-\frac{2}{3}$	-8	$\frac{5}{6}$	0	0	0	20 6

$\begin{matrix} \nearrow -1 \\ \nwarrow 1 \\ \searrow 5 \end{matrix}$

$$\frac{10}{3} - \frac{2}{3} = \frac{8}{3}$$

$$\frac{10}{3} - \frac{10}{3} = \frac{-2}{3}$$

	y_1	y_2	y_3	y_4	y_5	y_6	Ratio
y_2	$\frac{2}{3}$	1	$\frac{1}{6}$	0	0	0	4
y_4	$\frac{4}{3}$	0	$\frac{-1}{6}$	1	0	0	2 $\frac{2}{2}$ min
y_5	$\frac{5}{3}$	0	$\frac{1}{6}$	0	1	0	5
y_6	1	0	0	0	0	1	2
	$-\frac{2}{3}$	0	$\frac{5}{6}$	0	0	0	20

min

$$\textcircled{4} \times \frac{3}{2} = 6 \quad \textcircled{2} \times \frac{3}{\textcircled{+}2} \quad 5 \times \frac{3}{5}$$

	y_1	y_2	y_3	y_4	y_5	y_6	
y_2	$\textcircled{\frac{2}{2}} 0$	1	$\textcircled{\frac{1}{6}} \frac{1}{4}$	$\textcircled{-\frac{1}{2}} 0$	0	$\textcircled{4} 3$	$\leftarrow -\frac{2}{3}$
$\textcircled{y_4} y_1$	$\textcircled{\frac{4}{3}} 1$	0	$\textcircled{-\frac{1}{6}} -\frac{1}{8}$	$\textcircled{\frac{3}{4}} 0$	0	$\textcircled{2} \frac{3}{2}$	$R \times \frac{3}{4}$
y_5	$\textcircled{\frac{5}{3}} 0$	0	$\textcircled{\frac{1}{6}} \frac{3}{8}$	$\textcircled{-\frac{5}{4}} 1$	0	$\textcircled{5} \frac{5}{2}$	$\leftarrow \frac{5}{3}$
y_6	$\textcircled{4} 0$	0	$\textcircled{\frac{1}{6}} \frac{1}{8}$	$\textcircled{-\frac{3}{4}} 0$	1	$\textcircled{2} \frac{1}{2}$	$\leftarrow 1$
	$\textcircled{-\frac{2}{3}} 0$	0	$\textcircled{\frac{3}{6}} \frac{3}{4}$	$\textcircled{\frac{1}{2}} 0$	0	$\textcircled{20} 21$	$\leftarrow \frac{2}{3}$

$$-\frac{1}{\textcircled{6}} \times \frac{\textcircled{2}}{4} = -\frac{1}{8}$$

$$-\frac{1}{8}(-\frac{2}{3}) + \frac{1}{6} \frac{4}{2} - \frac{3}{2} = \frac{1}{2}$$

$$\frac{1}{8} \times \frac{5}{3} + \frac{1}{6}$$

$$\frac{3}{4} \times -\frac{5}{3}$$

$$-\frac{5}{2} \times -\frac{7}{3} + 5$$

$$-\frac{1}{8} \times \frac{3}{3} + \frac{5}{6}$$

$$-\frac{1}{12} + \frac{15}{12} = \frac{14}{12} = \frac{7}{6} = \frac{3}{4} \quad \frac{3}{24} \times \frac{2}{3}$$

$$\frac{3}{2} \times \frac{2}{3} + 20$$

	y_1	y_2	y_3	y_4	y_5	y_6	
y_2	0	1	$\frac{1}{4}$	$-\frac{1}{2}$	0	0	3
y_1	1	0	$-\frac{1}{8}$	$\frac{3}{4}$	0	0	$\frac{3}{2}$
y_5	0	0	$\frac{3}{8}$	$-\frac{5}{4}$	1	0	$\frac{5}{2}$
y_6	0	0	$\frac{1}{8}$	$-\frac{3}{4}$	0	1	$\frac{1}{2}$
	0	0	$\frac{3}{4}$	$\frac{1}{2}$	0	0	21

Optimal Solution

$$y_1 = \frac{3}{2} \quad y_2 = 3 \quad y_3 = y_4 = 0 \quad y_5 = \frac{5}{2}$$

$$y_6 = \frac{1}{2} \quad z = -21$$