17-S2-Q4

Solution ① transition rate diagram

let state 0 : The magazine is available

state 1 : The magazine has been loaned out but nobody reserve it.

state 2 The magazine has been loaned out and one reservation is accept

loan is exponentially distributed with mean $\frac{1}{\mu}$

$$v = \frac{1}{\frac{1}{\mu}} = \mu$$

$$\begin{tikzcd} 0 \arrow[r, shift left, "\lambda"] & 1 \arrow[l, shift left, "\mu"] \arrow[r, shift left, "\lambda"] & 2 \arrow[l, shift left, "\mu"] \end{tikzcd}$$

② rate balance equation

for state 0

$\lambda \pi_0 = \mu \pi_1$ (1)

for state 1

$(\lambda + \mu)\pi_1 = \lambda \pi_0 + \mu \pi_2$ (2)

for state 2.

$\mu \pi_2 = \lambda \pi_1$ (3)

$\pi_0 + \pi_1 + \pi_2 = 1$ (4)

from (1) $\pi_1 = \frac{\lambda}{\mu}\pi_0$ (5)

from (3) $\pi_2 = \frac{\lambda}{\mu}\pi_1 = \frac{\lambda^2}{\mu^2}\pi_0$ (6)

from (4) $\pi_0 + \frac{\lambda}{\mu}\pi_0 + \frac{\lambda^2}{\mu^2}\pi_0 = 1$

$$\left(1+\frac{\lambda}{\mu}+\frac{\lambda^2}{\mu^2}\right)\pi_0 = 1$$

$$\frac{\mu^2+\lambda\mu+\lambda^2}{\mu^2}\pi_0 = 1$$

$$\pi_0 = \frac{\mu^2}{\mu^2+\lambda\mu+\lambda^2}$$

$$\pi_1 = \frac{\lambda}{\mu}\pi_0 = \frac{\lambda\mu}{\mu^2+\lambda\mu+\lambda^2}$$

$$\pi_2 = \frac{\lambda}{\mu}\pi_1 = \frac{\lambda^2}{\mu^2+\lambda\mu+\lambda^2}$$

So $\pi = [\pi_0 \quad \pi_1 \quad \pi_2]$

$$= \left[\frac{\mu^2}{\mu^2+\lambda\mu+\lambda^2} \quad \frac{\lambda\mu}{\mu^2+\lambda\mu+\lambda^2} \quad \frac{\lambda^2}{\mu^2+\lambda\mu+\lambda^2}\right]$$

(b) M M 1 Queue

① $\lambda_1 = 1 \quad \mu = 4$

$$\rho = \frac{\lambda_1}{\mu} = \frac{1}{4}$$

$$\pi_0 = 1-\rho = \frac{3}{4} = 0.75$$

$$Q = \frac{\rho^2}{1-\rho} = \frac{\lambda_1^2}{\mu(\mu-\lambda_1)} = \frac{1}{12} = 0.0833333$$

$$D = W - \frac{1}{\mu} = \frac{\lambda_1}{\mu(\mu-\lambda_1)} = \frac{1}{4\times(4-1)} = \frac{1}{12} = 0.0833333$$

② $\lambda_2 = 3 \quad \mu = 4$

$$\rho = \frac{\lambda_2}{\mu} = \frac{3}{4}$$

$$\pi_0 = 1-\rho = \frac{1}{4} = 0.25$$

$$Q = \frac{\rho^2}{1-\rho} = \frac{\lambda_2^2}{\mu(\mu-\lambda_2)} = \frac{3^2}{4\times(4-3)} = \frac{9}{4} = 2.25$$

$$D = W - \frac{1}{\mu} = \frac{\lambda_2}{\mu(\mu-\lambda_2)} = \frac{3}{4\times(4-3)} = \frac{3}{4} = 0.75$$

③ $\lambda_3 = 2 \quad \mu = 4$

$$\rho = \frac{\lambda_3}{\mu} = \frac{2}{4} = \frac{1}{2} = 0.5$$

$$\pi_0 = 1-\rho = \frac{1}{2} = 0.5$$

$$Q = \frac{\rho^2}{1-\rho} = \frac{\lambda_3^2}{\mu(\mu-\lambda_3)} = \frac{2^2}{4\times(4-2)} = \frac{4}{8} = \frac{1}{2} = 0.5$$

$$D = W - \frac{1}{\mu} = \frac{\lambda_3}{\mu(\mu-\lambda_3)} = \frac{2}{4\times(4-2)} = \frac{2}{8} = \frac{1}{4} = 0.25$$

④ So, the answer of (b) is

	counter 1	counter 2	counter 3
(i)	0.75	0.08333	0.08333
(ii)	0.25	2.25	0.75
(iii)	0.5	0.5	0.25

(c) ① Combine Queues and Servers

Merge the three M/M/1 into a M/M/s queueing system, leading to shorter queues and reduced the waiting times for customer.

② Using M/M/1/N queue: avoid the queue too long