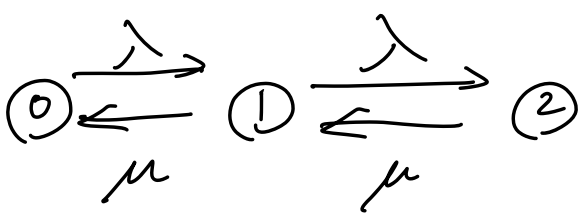
15-S2-05

Solution(a)

① state 0: machine is idle

state 1: machine is in use, chair is ~~empty~~

state 2: machine is in use, chair is occupied



② for state 0 $\lambda \pi_0 = \mu \pi_1$

for state 1 $(\lambda+\mu)\pi_1 = \lambda\pi_0 + \mu\pi_2$

for state 2 $\mu\pi_2 = \lambda\pi_1$

$\pi_0 + \pi_1 + \pi_2 = 1$

$$\pi_1 = \frac{\lambda}{\mu}\pi_0$$

$$\pi_2 = \frac{\lambda}{\mu}\pi_1 = \frac{\lambda^2}{\mu^2}\pi_0$$

$$\left(1 + \frac{\lambda}{\mu} + \frac{\lambda^2}{\mu^2}\right)\pi_0 = 1$$

$$\frac{\mu^2+\lambda\mu+\lambda^2}{\mu^2}\pi_0 = 1$$

$$\pi_0 = \frac{\mu^2}{\mu^2+\lambda\mu+\lambda^2}$$

$$\pi_1 = \frac{\lambda\mu}{\mu^2+\lambda\mu+\lambda^2}$$

$$\pi_2 = \frac{\lambda^2}{\mu^2+\lambda\mu+\lambda^2}$$

$$\pi = [\pi_0 \quad \pi_1 \quad \pi_2]$$

$$= \left[\frac{\mu^2}{\mu^2+\lambda\mu+\lambda^2} \quad \frac{\lambda\mu}{\mu^2+\lambda\mu+\lambda^2} \quad \frac{\lambda^2}{\mu^2+\lambda\mu+\lambda^2}\right]$$

(b) Revenue $= q\mu(\pi_1+\pi_2)$

Operating cost $= A\mu$

profit $= q\mu(\pi_1+\pi_2) - A\mu$

$$= q\mu\,\frac{\lambda\mu+\lambda^2}{\mu^2+\lambda\mu+\lambda^2} - A\mu$$

(c) second derivative test

$$\frac{\partial^2 \text{Profit}}{\partial \mu^2} < 0$$

$\Rightarrow$ profit function a maximum at this point