牛顿法多变量优化

1.求f梯度

2.求H

3.求H-1

4.带入x0，得到f在x0处的梯度

5.梯度下降法

x1=x0-H（x0）-1 ▽f（x0）

6.f（x1）

7.error

Example

Use the Newton method to

Maximize: $z = -(x_1 - \sqrt{5})^2 - (x_2 - \pi)^2 - 10$

to within a tolerance of $\epsilon = 0.05$. Take $X_0 = [6.597, 5.891]^T$, with $f(X_0) = -36.58$.

The gradient vector, Hessian matrix and inverse Hessian matrix are

$$\nabla f(x) = \begin{bmatrix} -2(x_1 - \sqrt{5}) \\ -2(x_2 - \pi) \end{bmatrix},\ H = \begin{bmatrix} -2 & 0 \\ 0 & -2 \end{bmatrix},\ H^{-1} = \begin{bmatrix} -0.5 & 0 \\ 0 & -0.5 \end{bmatrix}$$

for all x_1 and x_2.

$$\left[\begin{array}{cc|cc} -2 & 0 & 1 & 0 \\ 0 & -2 & 0 & 1 \end{array}\right] \to \left[\begin{array}{cc|cc} 1 & 0 & -0.5 & 0 \\ 0 & 1 & 0 & -0.5 \end{array}\right]$$

NANYANG

First iteration

$$\nabla f(x_0) = \begin{bmatrix} -2(6.597 - \sqrt{5}) \\ -2(5.891 - \pi) \end{bmatrix} = \begin{bmatrix} -8.722 \\ -5.499 \end{bmatrix}$$

$$X_1 = X_0 - H(X_0)^{-1} \nabla f(x_0)$$

$$= \begin{bmatrix} 6.597 \\ 5.891 \end{bmatrix} - \begin{bmatrix} -0.5 & 0 \\ 0 & -0.5 \end{bmatrix} \begin{bmatrix} -8.722 \\ -5.499 \end{bmatrix} = \begin{bmatrix} 2.236 \\ 3.142 \end{bmatrix}$$

with $f(x_1) = -10.00$. Since

$f(X_1) - f(X_0) = -10.00 - (-36.58)$

$= 26.58 > 0.05$ ϵ

We continue iterating

2nd iteration

$$\nabla f(X_1) = \begin{bmatrix} -2(2.236 - \sqrt{5}) \\ -2(3.142 - \pi) \end{bmatrix} = \begin{bmatrix} -0.0001 \\ 0.0008 \end{bmatrix}$$

$\nabla f(x^*) = 0$ $\nabla f(x_1)$ small

close

$$X_2 = X_1 - H(X_1)^{-1} \nabla f(x_1)$$

$$= \begin{bmatrix} 2.236 \\ 3.142 \end{bmatrix} - \begin{bmatrix} -0.5 & 0 \\ 0 & -0.5 \end{bmatrix} \begin{bmatrix} -0.0001 \\ -0.0008 \end{bmatrix} = \begin{bmatrix} 2.236 \\ 3.142 \end{bmatrix}$$

with $f(x_2) = -10.00$. Since

$f(X_2) - f(X_1) = 0 < 0.05$, we take $X^* = X_2 = [2.236, 3.142]^T$

with $z^* = f(x_2) = -10.00$.

由于在误差以内，所以接受其为X^*

代入得结果

local min