22-S1-Q2

$50+180+280+270=780$

$100+200+180+300=780$

no virtical supply

Q: (a) transportation model

Solution ① draw

		demand period 3	4	5	6	production capacity u_i	row diff
production period	3	40 / 50	40.5	41	41.5	50	0.5
	4	42	40	40.5	41	180	0.5
	5	44	42	40	40.5	280	0.5
	6	46	44	42	40	270	2
Demand v_j		100	200	180	300		
col diff		2	0.5	0.5	0.5		

col with largest penalty

② get base solution

	3	4	5	6	supply	row diff
3	40 / 50	40.5	41	41.5	0	
4	42 / 50	40	40.5	41	130	0.5
5	44	42	40	40.5	280	0.5
6	46	44	42	40	270	2
demand	0	200	180	300		
	2	2	0.5	0.5		

max

	3	4	5	6	supply	rol diff
3	$\boxed{40}$ 50	$\boxed{40.5}$	$\boxed{41}$	$\boxed{41.5}$	0	
4	$\boxed{42}$ 50	$\boxed{40}$ 130	$\boxed{40.5}$	$\boxed{41}$	0	0.5
5	$\boxed{44}$	$\boxed{42}$	$\boxed{40}$	$\boxed{40.5}$	280	0.5
6	$\boxed{46}$	$\boxed{44}$	$\boxed{42}$	$\boxed{40}$	270	2
demand	0	70	180	300		
		2	0.5	0.5		

	3	4	5	6	supply	rol diff
3	$\boxed{40}$ 50	$\boxed{40.5}$	$\boxed{41}$	$\boxed{41.5}$	0	
4	$\boxed{42}$ 50	$\boxed{40}$ 130	$\boxed{40.5}$	$\boxed{41}$	0	
5	$\boxed{44}$	$\boxed{42}$ 70	$\boxed{40}$	$\boxed{40.5}$	210	0.5
6	$\boxed{46}$	$\boxed{44}$	$\boxed{42}$	$\boxed{40}$	270	2
demand	0	0	180	300		
		2	2	0.5		

4 max

	3	4	5	6	supply	rol diff
3	$\boxed{40}$ 50	$\boxed{40.5}$	$\boxed{41}$	$\boxed{41.5}$	0	
4	$\boxed{42}$ 50	$\boxed{40}$ 130	$\boxed{40.5}$	$\boxed{41}$	0	
5	$\boxed{44}$	$\boxed{42}$ 70	$\boxed{40}$ 180	$\boxed{40.5}$	30	0.5
6	$\boxed{46}$	$\boxed{44}$	$\boxed{42}$	$\boxed{40}$	270	2
demand	0	0	0	300		
			2	0.5		
			↑ max			

	3	4	5	6	supply
3	$\boxed{40}$ 50	$\boxed{40.5}$	$\boxed{41}$	$\boxed{41.5}$	0
4	$\boxed{42}$ 50	$\boxed{40}$ 130	$\boxed{40.5}$	$\boxed{41}$	0
5	$\boxed{44}$	$\boxed{42}$ 70	$\boxed{40}$ 180	$\boxed{40.5}$ 30	0
6	$\boxed{46}$	$\boxed{44}$	$\boxed{42}$	$\boxed{40}$ 270	0
demand	0	0	0	0	

50

	3	4	5	6	supply
3	$\boxed{40}$ 50	$\boxed{40.5}$	$\boxed{41}$	$\boxed{41.5}$	50
4	$\boxed{42}$ 50	$\boxed{40}$ 130	$\boxed{40.5}$	$\boxed{41}$	180
5	$\boxed{44}$	$\boxed{42}$ 70	$\boxed{40}$ 180	$\boxed{40.5}$ 30	280
6	$\boxed{46}$	$\boxed{44}$	$\boxed{42}$	$\boxed{40}$ 270	270
demand	100	200	180	300	

③ test optimal

	3	4	5	6	supply	u_i
3	$\boxed{40}$ 50	$\boxed{40.5}$ 2.5	$\boxed{41}$ 5	$\boxed{41.5}$ 5	50	−4
4	$\boxed{42}$ 50	$\boxed{40}$ 130	$\boxed{40.5}$ 2.5	$\boxed{41}$ 2.5	180	−2
5	$\boxed{44}$ 0	$\boxed{42}$ 70	$\boxed{40}$ 180	$\boxed{40.5}$ 30	280	0
6	$\boxed{46}$ 2.5	$\boxed{44}$ 2.5	$\boxed{42}$ 2.5	$\boxed{40}$ 270	270	−0.5
demand	100	200	180	300		
V_i	44	42	40	40.5		

So, the solution is optimal

(b) L?

Solution

① $L = x_1^2 + x_2^2 + x_3^2 + \lambda_1(x_1 + x_2 + 3x_3 - 2) + \lambda_2(5x_1 + 2x_2 + x_3 - 5)$

$$\frac{\partial L}{\partial x_1} = 2x_1 + \lambda_1 + 5\lambda_2 = 0 \quad (1)$$

$$\frac{\partial L}{\partial x_2} = 2x_2 + \lambda_1 + 2\lambda_2 = 0 \quad (2)$$

$$\frac{\partial L}{\partial x_3} = 2x_3 + 3\lambda_1 + \lambda_2 = 0 \quad (3)$$

$$\frac{\partial L}{\partial \lambda_1} = x_1 + x_2 + 3x_3 - 2 = 0 \quad (4)$$

$$\frac{\partial L}{\partial \lambda_2} = 5x_1 + 2x_2 + x_3 - 5 = 0 \quad (5)$$

Solve the equation using casio only 4个未知数

from(3) $\lambda_2 = 2x_3 + 3\lambda_1$ 代入(1)(2)

$2x_1 + \lambda_1 + 5(2x_3 + 3\lambda_1) = 2x_1 + 10x_3 + 16\lambda_1 = 0$

$2x_2 + \lambda_1 + 2(2x_3 + 3\lambda_1) = 2x_2 + \lambda_1 + 4x_3 + 6\lambda_1$

$= 2x_2 + 4x_3 + 7\lambda_1 = 0$

Casio .solve

$$\begin{bmatrix} 2 & 0 & 10 & 16 \\ 2 & 0 & 4 & 7 \\ 1 & 1 & 3 & 0 \\ 5 & 2 & 1 & 0 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ 2 \\ 5 \end{bmatrix}$$

$$\begin{cases} x_1 = -\frac{1}{12} = -0.08333 \\ x_2 = \frac{17}{6} = 2.8333 \\ x_3 = -\frac{1}{4} = -0.25 \\ \lambda_1 = \frac{1}{6} = 0.1667 \\ \lambda_2 = 0 \end{cases}$$

$\lambda_2 = 2x_3 + 3\lambda_1$

$= 2\times(-\frac{1}{4}) + 3\times\frac{1}{6}$

$= 0$

$X = [-\frac{1}{12} \quad \frac{17}{6} \quad -\frac{1}{4}]$

$Z = x_1^2 + x_2^2 + x_3^2 = \frac{583}{72}$

$= 8.0972$

② test optimal

$\nabla h_1(x) = [1 \quad 1 \quad 3]$

$\nabla h_1(x) Y = y_1 + y_2 + 3y_3 = 0$

$$\nabla^2 L_{xx}(x,\lambda) = \begin{bmatrix} \frac{\partial L}{\partial x_1 \partial x_1} & \frac{\partial L}{\partial x_1 \partial x_2} & \frac{\partial L}{\partial x_1 x_3} \\ \frac{\partial L}{\partial x_2 \partial x_1} & \frac{\partial L}{\partial x_2 \partial x_2} & \frac{\partial L}{\partial x_2 \partial x_3} \\ \frac{\partial L}{\partial x_3 \partial x_1} & \frac{\partial L}{\partial x_3 \partial x_2} & \frac{\partial L}{\partial x_3 \partial x_3} \end{bmatrix} = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix}$$

positive - definite

$$\frac{\partial L}{\partial x_1} = 2x_1 + \lambda_1 + 5\lambda_2 = 0 \qquad \frac{\partial L}{\partial x_1 \partial x_1} = 2$$

$$\frac{\partial L}{\partial x_2} = 2x_2 + \lambda_1 + 2\lambda_2 = 0 \qquad \frac{\partial L}{\partial x_2 \partial x_2} = 2$$

$$\frac{\partial L}{\partial x_3} = 2x_3 + 3\lambda_1 + \lambda_2 = 0 \qquad \frac{\partial L}{\partial x_3 \partial x_3} = 2$$

$$y^T \nabla_{xx}^2 L\, y = [y_1 \ y_2 \ y_3] \begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix}$$

$$= [2y_1 \ \ 2y_2 \ \ 2y_3] \begin{bmatrix} y_1 \\ y_2 \\ y_2 \end{bmatrix}$$

$$= 2y_1^2 + 2y_2^2 + 2y_3^2 > 0 \qquad \forall\, y \neq 0$$

So $x = [-\frac{1}{12} \ \ \frac{17}{6} \ \ -\frac{1}{4}]$ is a minimum point

$$z^* = x_1^2 + x_2^2 + x_3^2 = \frac{583}{72}$$

$$= 8.0972$$