23- S1-Q1

Q:(a) LP ?

Solution

let x_1 be the number of product A produced per day

x_2 be the number of product B produced per day

x_3 be the number of product C produced per day

Z is the profit

Maximize $Z = 2.5x_1 + 2.3x_2 + 2x_3$

subject to $0.2x_1 + 0.18x_2 + 0.16x_3 \leq 10$

$(1.5+0.5)x_1 + (1.7+0.35)x_2 + (1.8+0.6)x_3 \leq 100$

$0.5x_1 + 0.35x_2 + 0.6x_3 \leq 30$

So the formulated linear programming problem is

Maximize $Z = 2.5x_1 + 2.3x_2 + 2x_3$

subject to
$$\begin{cases} 0.2x_1 + 0.18x_2 + 0.16x_3 \leq 10 \\ 2x_1 + 2.05x_2 + 2.4x_3 \leq 100 \\ 0.5x_1 + 0.35x_2 + 0.6x_3 \leq 30 \\ x_1, x_2, x_3 \geq 0 \end{cases}$$

(b) Q : solve

Solution : Reformulate

Maximize $Z = 2.5x_1 + 2.3x_2 + 2x_3 + 0 \cdot x_4 + 0 \cdot x_5 + 0 \cdot x_6$

Subject to
$$\begin{cases} 0.2x_1 + 0.18x_2 + 0.16x_3 + x_4 = 10 \\ 2x_1 + 2.05x_2 + 2.4x_3 + x_5 = 100 \\ 0.5x_1 + 0.35x_2 + 0.6x_3 + x_6 = 30 \\ x_1, x_2, x_3, x_4, x_5, x_6 \geq 0 \end{cases}$$

$Z = C^T X$ $\quad C^T = [2.5 \;\; 2.3 \;\; 2 \;\; 0 \;\; 0 \;\; 0]$

$AX = B$ $\quad A = \begin{bmatrix} 0.2 & 0.18 & 0.16 & 1 & 0 & 0 \\ 2 & 2.05 & 2.4 & 0 & 1 & 0 \\ 0.5 & 0.35 & 0.6 & 0 & 0 & 1 \end{bmatrix}$

$X_0 = [x_4, x_5 \; x_6]^T$

Matrix

	A	B
$-C^T + C_0^T A$		$C_0^T B$

$B = \begin{bmatrix} 10 \\ 100 \\ 30 \end{bmatrix}$ $\quad C_0^T = [0 \;\; 0 \;\; 0]$

$X = [x_1 \;\; x_2 \;\; x_3 \;\; x_4 \;\; x_5 \;\; x_6]^T$

	x_1	x_2	x_3	x_4	x_5	x_6		Ratio
x_4	0.2	0.18	0.16	1	0	0	10	50 ← min
x_5	2	2.05	2.4	0	1	0	100	50
x_6	0.5	0.35	0.6	0	0	1	30	60
	−2.5	−2.3	−2	0	0	0	0	

↑ work line

$R_1 \times \frac{1}{0.2}$

	x_1	x_2	x_3	x_4	x_5	x_6	
$x_4 \to x_1$	1	0.9	0.8	5	0	0	50
x_5	2	2.05	2.4	0	1	0	100
x_6	0.5	0.35	0.6	0	0	1	30
	−2.5	−2.3	−2	0	0	0	0

$\cdot 2$: $-2R_1+R_2$

-0.5 : $-0.5R_1+R_3$

2.5 : $2.5R_1+R_4$

	x_1	x_2	x_3	x_4	x_5	x_6		Ratio
x_1	1	0.9	0.8	5	0	0	50	55.556
$x_5 \to x_2$	0	(0.25)	0.8	−10	1	0	0	0 min
x_6	0	−0.1	0.2	−2.5	0	1	5	
	0	(−0.05)	0	12.5	0	0	125	

-0.9

$R_2 \times \frac{1}{0.25}$

0.1

0.05

min

	x_1	x_2	x_3	x_4	x_5	x_6	
x_1	1	0	−2.08	41	−3.6	0	50
x_2	0	1	3.2	−40	4	0	0
x_6	0	0	0.52	−6.5	0.4	1	5
	0	0	0.16	10.5	0.2	0	125

$X_1^* = 50 \quad X_2^* = 0 \quad X_6^* = 5 \quad X_3^* = X_4^* = X_5^* = 0$

$Z = 125$

(c) Q: budget packing ↓?

Solution

Since we only produce Product A

So, the total packing cose for Product A

at the optimal solution is

$0.5 \times 50 = 25$

the original budget is \$30

$30 - 25 = 5$

Therefore, the maximum cut in the packing budget is \$5.