23-S1-Q4

Q: (i) $L = ?$ (ii) $W = ?$ (iii) $T_{break\text{-}even}$

(i)

① $L_1 = \frac{\rho}{1-\rho} = \frac{\lambda}{\mu - \lambda} = \frac{2}{3-2} = 2$

② $\rho = \frac{\lambda}{\mu} = \frac{2}{3}$

$N = 3$

$$L_2 = \frac{\rho[1 - \rho^N - N\rho^N(1-\rho)]}{(1-\rho)(1-\rho^{N+1})}$$

$$= \frac{\frac{2}{3}[1 - (\frac{2}{3})^3 - 3 \times (\frac{2}{3})^3 \frac{1}{3}]}{\frac{1}{3}(1-(\frac{2}{3})^4)}$$

$= \frac{66}{65}$

$= 0.6462$

③ $b = 3$ $\mu = 3$ $\lambda = \frac{2}{3}$

$\rho = \frac{b\lambda}{\mu} = \frac{3 \times \frac{2}{3}}{3} = \frac{2}{3}$

$L_3 = \frac{\rho(1+b)}{2(1-\rho)}$

$$= \frac{\frac{2}{3}(1+3)}{2(1-\frac{2}{3})}$$

$$= 4$$

④ $\lambda = 2 \quad \mu = 2 \quad m = 3$

$$\rho = \frac{\lambda}{m\mu} = \frac{2}{3\times 2} = \frac{1}{3}$$

$$\pi_0 = \left[\frac{(m\rho)^m}{m!(1-\rho)} + \sum_{k=0}^{m-1}\frac{(m\rho)^k}{k!}\right]^{-1}$$

$$= \left[\frac{(3\times\frac{1}{3})^3}{3!\times\frac{2}{3}} + \frac{(3\times\frac{1}{3})^0}{0!} + \frac{(3\times\frac{1}{3})^1}{1!} + \frac{(3\times\frac{1}{3})^2}{2!}\right]^{-1}$$

$$= \left(\frac{1}{4} + 1 + 1 + \frac{1}{2}\right)^{-1}$$

$$= \frac{4}{11}$$

$$L_4 = \frac{\rho(m\rho)^m \pi_0}{m!(1-\rho)^2} + \frac{\lambda}{\mu}$$

$$= \frac{\frac{1}{3}\times 1^3 \times \frac{4}{11}}{3!\times(\frac{2}{3})^2} + \frac{2}{2}$$

$$= \frac{23}{22} = 1.0455$$

(ii)

① $W_1 = \frac{1}{\mu - \lambda} = 1$

② $W_2 = \frac{1-\left(\frac{2}{3}\right)^3 - 3\times\left(\frac{2}{3}\right)^3 \times \frac{1}{3}}{3 \times \frac{1}{3} \times \left[1-\left(\frac{2}{3}\right)^4\right]}$

$= \frac{33}{65} = 0.5077$

③ $W_3 = \frac{L}{\lambda b} = \frac{4}{\frac{2}{3} \times 3} = 2$

④ $W_4 = \frac{L}{\lambda} = \frac{\frac{23}{22}}{2} = \frac{23}{44} = 0.5227$

(iii) ① Monthly profit = Revenue − operation

= 40 − 20

= 20

$\text{months} = \frac{\text{Initial renovation}}{\text{Monthly profit}} = \frac{100}{20} = 5$

② $\frac{300}{120-90} = 10$

③ $\frac{100}{30-18} = 8.33$

④ $\frac{150}{100-60} = 3.75$

best choice: M/M/3

① short waiting time, only 0.5227 min

② low mean number in system

③ quick break-even time, only 3.75 month

④ highest monthly profit