

牛顿法

1. 先求一阶导数
2. 再求二阶导数
3. $x_1 = x_0 - f'(x_0) / f''(x_0)$

Example

Using Newton's method, find the **minimum** of $f(x) = x^2 + 54/x$.
 The initial value is $x_0 = 2$. Stop when $|x_{n+1} - x_n| < 10^{-5}$.

We have

$$f'(x) = 2x - 54/x^2 \quad \checkmark$$

$$f''(x) = 2 + 108/x^3 \quad \checkmark$$

Thus,

$$\begin{aligned} x_1 &= x_0 - f'(x_0) / f''(x_0) \quad \checkmark \\ &= 2 - (2(2) - 54/2^2) / (2 + 108/2^3) \\ &= 2.6129 \quad \checkmark \end{aligned}$$

The iterations are shown below

n	x_n	x_{n+1}	$ x_{n+1} - x_n $
0	2	2.612903	0.612903
1	2.612930	2.946104	0.333201
2	2.946104	2.99902	0.0529154
3	2.999020	2.999999	0.000979
4	2.99999	2.999999	0.000000

0.00001

Take $x^* = x_5 = 2.99999 \approx 3.0$