18-52-Q3

Q car dealer — shop 1 car → order

A_k demand k^{th} week $k=1,2\cdots$ Poisson $\lambda=1$

B_k unsold end. k^{th} week

Solution (a) $\{B_k\}$ is not a Markov chain

Reason ① B_k : car unsold at the end of k^{th} week

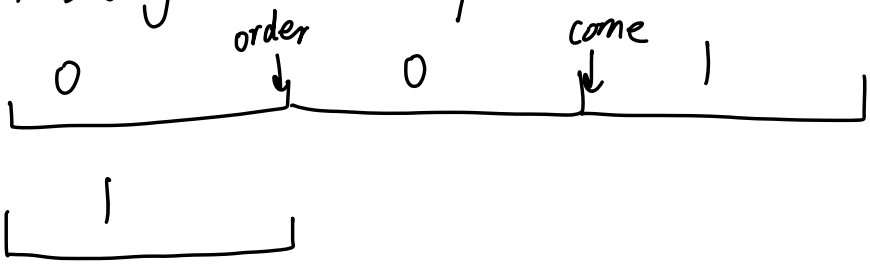
the space model of B_k is $\{0, 1\}$

In a Markov chain, the future state depends only on the current state and not on any past states.

② However, in this scenario, the number of unsold cars at the end of week $k+1$, B_{k+1}, depends not only on B_k but also on whether an order was placed in previous weeks due to two-week delivery delay.

③ The ordering and arrival of cars introduce dependencies that span more than one time period, violating the Markov property.

(b) ① define the space S as

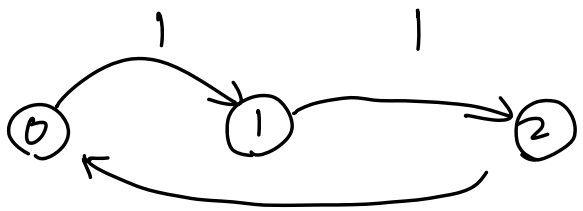


State 0 : Shop is empty, a car will arrive in the week of next week

State 1 : Shop is empty, a car will arrive in the week

State 2 : Shop has one car available for sale

(c) ①



$P_{01} = 1$ since the time to car arrival decreses by one week

$P_{12} = 1$ since the ordered car arrives

$P_{22} = P(A_k = 1) = \frac{e^{-\lambda}\lambda^k}{k!} = \frac{e^{-\lambda}\lambda^1}{1!} = e^{-1}$

$P_{21} = 0$

$P_{20} = 1 - P(A_k = 1) = 1 - e^{-1}$

So the TPM is

$$\begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1-e^{-1} & 0 & e^{-1} \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0.6321 & 0 & 0.3679 \end{bmatrix}$$

② steady-state probabilites DTMC

$$\begin{cases} Y = YP \\ \sum_{i=0}^{2} y_i = 0 \end{cases}$$

$$[y_0 \quad y_1 \quad y_2] = [y_0 \, y_1 \, y_2] \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1-e^{-1} & 0 & e^{-1} \end{bmatrix}$$

$$\begin{cases} y_0 = (1-e^{-1}) y_2 \\ y_1 = y_0 \\ y_2 = y_1 + e^{-1} y_2 \\ y_0 + y_1 + y_2 = 1 \end{cases}$$

$$\Rightarrow 2y_0 + \frac{1}{1-e^{-1}} y_0 = 1 \quad \Rightarrow \quad y_0 = 0.2792$$

$$y_1 = 0.2792$$

$$y_2 = 1 - y_0 - y_1 = 1 - 2y_0 = 0.4416$$

$$Y = [y_0 \quad y_1 \quad y_2] = [0.2792 \quad 0.2792 \quad 0.4416]$$

(d)

① comment

$0.2792 \times 2 = 0.5584 \times 100\% = 55.84\%$

The shop spends 55.84% of the time without a car available for sale due to the delivery delays. this

result in lost sales opportunities and reduce revenne.

② improve the operation

(1) reducing Delivery order delay

(2) allow for more car in the stock or in transit

(3) Implementing Pre-Order Sale