23-S1-Q3

Q(a) state ? $\quad$ p ?

$S = \{0, 1, 2\}$

$P_{00} = P\{B_{k+1} \geq 2\} = 1 - P\{B_{k+1} = 1\} - P\{B_{k+1} = 0\}$

$= 1 - \frac{e^{-0.1}(0.1)^1}{1!} - \frac{e^{-0.1}(0.1)^0}{0!} = 1 - 1.1e^{-0.1}$

$P_{01} = P(B_{k+1} = 1) = 0.1e^{-0.1}$

$P_{02} = P(B_{k+1} = 0) = e^{-0.1}$

$P_{10} = P(B_{k+1} \geq 2) = 1 - 1.1e^{-0.1}$

$P_{11} = P(B_{k+1} = 1) = 0.1e^{-0.1}$

$P_{12} = P(B_{k+1} = 0) = e^{-0.1}$

$P_{20} = 1 - 1.1e^{-0.1}$

$P_{21} = 0.1e^{-0.1}$

$P_{22} = e^{-0.1}$

$$P = \begin{bmatrix} 1-1.1e^{-0.1} & 0.1e^{-0.1} & e^{-0.1} \\ 1-1.1e^{-0.1} & 0.1e^{-0.1} & e^{-0.1} \\ 1-1.1e^{-0.1} & 0.1e^{-0.1} & e^{-0.1} \end{bmatrix}$$

(b) $E(T_0) = \frac{1}{1-P_{00}} = \frac{1}{1.1e^{-0.1}} = 1.0047$

$E(T_1) = \frac{1}{1-P_{11}} = \frac{1}{1-0.1e^{-0.1}} = 1.0995$

$$E(T_2) = \frac{1}{1-P_{22}} = \frac{1}{1-e^{-0.1}} = 10.5083$$

$$\text{mean} E(T_i) = \frac{1}{3}\left[E(T_0)+E(T_1)+E(T_2)\right] = 4.2042$$

State 2 stayes longer than 0 and 1

No surprize

comment: $\lambda = 0.1$ is a small number

so the usage rate is low and

the replenishment policy reset the

numbe of spare bulks to 2 whenever

there is a shortfall

(c) independent of previous state

$\pi_0 = 1 - 1.1e^{-0.1}$

$\pi_1 = 0.1e^{-0.1}$

$\pi_2 = e^{-0.1}$

(d) $P(U_{k+4}=1) = \pi_1 = 0.1e^{-0.1}$

23-S1 - Q4

Q: (i) $L = ?$ (ii) $W = ?$ (iii) $T_{\text{break-even}}$

$$L_1 = \frac{\rho}{1-\rho} = \frac{\lambda}{\mu-\lambda} = \frac{2}{3-2} = 2 \qquad W_1 = \frac{1}{\mu-\lambda} = 1$$

$$L_2 = \frac{\rho[1-\rho^N - N\rho^N(1-\rho)]}{(1-\rho)(1-\rho^{N+1})} \qquad W_2 = \frac{1-(\frac{2}{3})^3 - 3\times(\frac{2}{3})^3 \times \frac{1}{3}}{3\times\frac{1}{3}\times[1-(\frac{2}{3})^4]}$$

$$\rho = \frac{\lambda}{\mu} = \frac{2}{3} \qquad N = 3 \qquad = \frac{33}{65} = 0.5077$$

$$L_2 = \frac{\frac{2}{3}[1-(\frac{2}{3})^3 - 3\times(\frac{2}{3})^3 \frac{1}{3}]}{\frac{1}{3}(1-(\frac{2}{3})^4)} = \frac{42}{65} \quad \frac{66}{65}$$

$= 0.6462$

$$L_3 = \frac{\rho(1+b)}{2(1-\rho)} \qquad b = 3 \quad \mu = 3 \quad \lambda = \frac{2}{3}$$

$$\rho = \frac{b\lambda}{\mu} = \frac{3\times\frac{2}{3}}{3} = \frac{2}{3}$$

$$L_3 = \frac{\frac{2}{3}(1+3)}{2(1-\frac{2}{3})} = 4 \qquad W_3 = \frac{L}{\lambda b} = \frac{4}{\frac{2}{3}\times 3} = 2$$

$$L_4 = \frac{\rho(m\rho)^m \pi_0}{m!(1-\rho)^2} + \frac{\lambda}{\mu} \qquad \lambda = 2 \quad \mu = 2 \quad m = 3$$

$$\rho = \frac{\lambda}{m\mu} = \frac{2}{3\times 2} = \frac{1}{3}$$

$$\pi_0 = \left[\frac{(m\rho)^m}{m!(1-\rho)} + \sum_{k=0}^{m-1} \frac{(m\rho)^k}{k!}\right]^{-1}$$

$$= \left[\frac{(3\times\frac{1}{3})^3}{3!\times\frac{2}{3}} + \frac{(3\times\frac{1}{3})^0}{0!} + \frac{(3\times\frac{1}{3})^1}{1!} + \frac{(3\times\frac{1}{3})^2}{2!}\right]^{-1}$$

$$= \left(\frac{1}{4} + 1 + 1 + \frac{1}{2}\right)^{-1}$$

$$= \frac{4}{11}$$

$$L_4 = \frac{\rho(m\rho)^m \pi_0}{m!(1-\rho)^2} + \frac{\lambda}{\mu}$$

$$= \frac{\frac{1}{3}\times 1^3 \times \frac{4}{11}}{3!\times(\frac{2}{3})^2} + \frac{2}{2}$$

$$= \frac{23}{22} = 1.0455 \qquad W_4 = \frac{L}{\lambda} = \frac{\frac{23}{22}}{2} = \frac{23}{44} = 0.5227$$

(iii) ① Monthly profit = Revenue − operation

$= 40000 - 20000$

$= 20000$

$$\text{months} = \frac{\text{Initial renovation}}{\text{Monthly profit}} = \frac{100000}{20000} = 5$$

② $\dfrac{120000 - 90000}{30000} = 10$

③ $\dfrac{30000 - 18000}{12000} = 8.33$

④ $\dfrac{100000 - 60000}{40000} = 3.75$

best choice:

① short waiting time, only 0.5227 min

② low mean number in system

③ quick break-even time, only 3.75 months

④ highest monthly profit