2. **Jack is a bachelor and he knows two nice ladies, Jane and Anne, for some time. He is seriously considering to advance his friendship with one of them to that of courtship, i.e., he is considering to pursue one of the ladies as his 'girlfriend' and eventually he would like to marry his girlfriend and start a family. Jack is not sure which lady is the better 'girlfriend' as each of them has her merits. Jack has three criteria for a perfect girlfriend -**

   (i) she is his Soul Mate (S);

   (ii) she has good Earning Power and as a wife can strengthen the family's finance (E);

   (iii) she is well liked by Jack's parents (P).

Jack has set up the following pair-wise comparison matrices.

Criterion

	S	E	P
S	1	5	6
E	1/5	1	2
P	1/6	1/2	1

Soul mate (S)

	Jane	Anne
Jane	1	4
Anne	1/4	1

Earning power (E)

	Jane	Anne
Jane	1	1/6
Anne	6	1

Liked by parents (P)

	Jane	Anne
Jane	1	1/7
Anne	7	1

(a) Draw a graphical representation of Jack's problem, stating clearly the overall goal, the criteria to be used, and the decision alternatives.

(b) Compute the priority of each criterion (S, E, P) in terms of its contribution to the overall goal. State the most important criterion, the next important criterion and the least important criterion.

(c) Compute the consistency ratio (CR) and comment on its acceptability. For three criteria, the consistency index of a randomly generated pairwise comparison matrix is 0.58. (6 marks)

(d) Determine the overall priority for each lady and give your recommendation to Jack.

AHP

b) overall goal: select the best girlfriend

Criteria: Soul mate (S) : Earning power (E) : Liked by parents (P)

Decision alternatives: Anne (A) Jane (J) | Anne (A) Jane (J) | Anne (A) Jane (J)

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a) step 1 Sum the values in each column

	S	E	P
S	1	5	6
E	$\frac{1}{5}$	1	2
P	$\frac{1}{6}$	$\frac{1}{2}$	1
sum	1.367	6.5	9

Step 2 Divide by column total, then calculate row average

	S	E	P	Priority
S	0.732	0.769	0.667	0.723
E	0.146	0.154	0.222	0.174
P	0.122	0.077	0.111	0.103

From the above table, we see that S is the most important criterion, E is the next important criterion and P is the least important criterion. #

(b) Step 1

$$\begin{bmatrix} 1 & 5 & 6 \\ \frac{1}{5} & 1 & 2 \\ \frac{1}{6} & \frac{1}{2} & 1 \end{bmatrix} \begin{bmatrix} 0.723 \\ 0.174 \\ 0.103 \end{bmatrix} = \begin{bmatrix} 2.211 \\ 0.5246 \\ 0.3105 \end{bmatrix}$$

Step 2

S : $2.211 / 0.723 = 3.058$

E : $0.5246 / 0.174 = 3.015$

P : $0.2105 / 0.103 = 3.015$

Step 3 $\lambda_{max} = \frac{1}{3}(3.052 + 2.015 + 3.015)$

$= 2.029$

Step 4 consistency index $(CI) = \frac{\lambda_{max} - 3}{3-1} = 0.0145$

Step 5 consistency ratio $(CR) = \frac{CI}{RI} = \frac{0.0145}{0.58}$

$= 0.025 < 0.1$

Since $CR < 0.1$, the consistency of the pairwise comparisons of S, E, P is acceptable. #

(c) Soul mate (S):

Step 1

	Jane	Anne
Jane	1	4
Anne	$\frac{1}{4}$	1
Sum	1.25	5

Step 2

	Jane	Anne	Priority
Jane	0.8	0.8	0.8
Anne	0.2	0.2	0.2

Earning power (E):

step 1

	Jane	Anne
Jane	1	$\frac{1}{6}$
Anne	6	1
sum	7	1.167

step 2

	Jane	Anne	Priority
Jane	0.143	0.143	0.143
Anne	0.857	0.857	0.857

Liked by parents (P):

step 1

	Jane	Anne
Jane	1	$\frac{1}{7}$
Anne	7	1
sum	8	1.143

step 2

	Jane	Anne	Priority
Jane	0.125	0.125	0.125
Anne	0.875	0.875	0.875

Combining the 3 sets of priorities gives

	S	E	P
Jane	(0.8)	0.143	0.125
Anne	0.2	(0.857)	(0.875)

We see that

Jane is the preferred choice based on S

Anne " " " " " " E

Anne " " " " " " P

The composite priority is calculated as follows

$$\begin{bmatrix} 0.8 & 0.143 & 0.125 \\ 0.2 & 0.857 & 0.875 \end{bmatrix} \begin{bmatrix} 0.723 \\ 0.174 \\ 0.103 \end{bmatrix} = \begin{bmatrix} 0.616 \\ 0.384 \end{bmatrix}$$

	Priority
Jane	0.616
Anne	0.384

Jane has a higher priority, so Jane is the preferred choice. #