

21-51-Q2

Q(a) Hungarian method?

	Mow	Paint	Wash	Clean
John	1	4	6	3
karen	9	7	10	9
Terri	4	5	11	7
Jean	8	7	8	5

Solution ① row - min row ② Col - min col

	Mow	Paint	Wash	Clean
John	1 0	4 3	6 5 2	3 2
karen	9 2	7 0	10 2 0	9 2
Terri	4 0	5 1	11 7 4	7 3
Jean	8 3	7 2	8 2 0	5 0

③ using line to cover 0

	Mow	Paint	Wash	Clean
John	0	3	2	2
karen	2	0	0	2
Terri	0	1	4	3
Jean	3	2	0	0

④ no cover — min 1 ⑤ cross position add 1
must be equal

	Mow	Paint	Wash	Clean
John	0	3 2	2 1	2 1
karen	2 3	0	0	2
Terri	0	1 0	4 3	3 2
Jean	3 4	2	0	0

⑥ assign cost 0 work, add *

	Mow	Paint	Wash	Clean
John	0*	2	1	1
karen	3	0	0*	2
Terri	0	0*	3	2
Jean	4	2	0	0*

So John do Mow, \$1

karen do wash, \$10

Terri do Paint, \$5

Jean do Clean, \$5

$$\text{Total} = 1 + 10 + 5 + 5 = \$21$$

cb) Q : Lagrange \rightarrow optimal solution

Solution

$$\textcircled{1} \text{ Min } z = \frac{1}{2} [x_1, x_2] \begin{bmatrix} 4 & 0 \\ 0 & 8 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + [1 \ 2] \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + 2$$

$$= \frac{1}{2} [4x_1 \ 8x_2] \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + x_1 + 2x_2 + 2$$

$$= \frac{1}{2} (4x_1^2 + 8x_2^2) + x_1 + 2x_2 + 2$$

$$= 2x_1^2 + 4x_2^2 + x_1 + 2x_2 + 2$$

Subject to $x_1 - 2 \leq 0$

$$x_2 - 2 \leq 0$$

$\textcircled{2}$

Lagrange multipliers

$$L = 2x_1^2 + 4x_2^2 + x_1 + 2x_2 + 2 + \mu_1(x_1 - 2) + \mu_2(x_2 - 2)$$

$$\left\{ \begin{array}{l} \frac{\partial L}{\partial x_1} = 4x_1 + 1 + \mu_1 = 0 \end{array} \right.$$

$$\frac{\partial L}{\partial x_2} = 8x_2 + 2 + \mu_2 = 0$$

$$x_1 - 2 \leq 0$$

$$x_2 - 2 \leq 0$$

$$\mu_1(x_1 - 2) = 0$$

$$\mu_2(x_2 - 2) = 0$$

$$\mu_1 \geq 0$$

$$\mu_2 \geq 0$$

③ if $\mu_1 = \mu_2 = 0$, $x_1 = -\frac{1}{4}$, $x_2 = -\frac{1}{4}$ ✓

if $\mu_1 = 0$, $\mu_2 > 0$, $x_2 = 2$, $\mu_2 = -18$, contradiction

if $\mu_1 > 0$, $\mu_2 = 0$, $x_1 = 2$, $\mu_1 = -9$, contradiction

if $\mu_1 > 0$, $\mu_2 > 0$, $x_1 = 2$, $x_2 = 2$, $\mu_1 = -9$, $\mu_2 = -18$
contradiction.

So $x_1^* = -\frac{1}{4}$, $x_2^* = -\frac{1}{4}$, $\mu_1 = \mu_2 = 0$

$$\begin{aligned} Z_{\min} &= 2x_1^2 + 4x_2^2 + x_1 + 2x_2 + 2 \\ &= 2 \times \left(-\frac{1}{4}\right)^2 + 4 \times \left(-\frac{1}{4}\right)^2 - \frac{1}{4} - \frac{1}{2} + 2 \\ &= \frac{13}{8} \\ &= 1.625 \end{aligned}$$

要 test optimal 吗? ✓ $\mu_1 = \mu_2 = 0$, 怎么 test? ✓

$$\frac{\partial L}{\partial x_1} = 4x_1 + 1 + \mu_1 = 0$$

$$\frac{\partial L}{\partial x_2} = 8x_2 + 2 + \mu_2 = 0$$

$$\frac{\partial L}{\partial x_1 \partial x_1} = 4 \quad \frac{\partial L}{\partial x_1 \partial x_2} = 0 \quad \frac{\partial L}{\partial x_2 \partial x_2} = 8$$

Don't need to
calculate $\nabla^2 g_i^T \gamma$

$$\nabla_{xx}^2 L(x, \mu) = \begin{bmatrix} 4 & 0 \\ 0 & 8 \end{bmatrix} \geq 0 \text{ on } \mathbb{R}^2 \quad \text{Since } |4| > 0 \\ |0 \ 8| > 0$$

$$Y^T \nabla_{xx}^2 L(x, \mu) Y = \begin{bmatrix} y_1 & y_2 \end{bmatrix} \begin{bmatrix} 4 & 0 \\ 0 & 8 \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \end{bmatrix}$$

$$= [4y_1 \quad 8y_2] \begin{bmatrix} y_1 \\ y_2 \end{bmatrix}$$

$$= 4y_1^2 + 8y_2^2 > 0 \text{ for all } Y \neq 0$$

So $X = (-\frac{1}{4}, -\frac{1}{4})$ is a minimum point