

6204 - 21 - S1 - Q1

Q: (a) LP ?

let  $X = [x_1 \ x_2]$   $x_1$  Exterior paint tons  
 $x_2$  Interior paint tons

$$6x_1 + 4x_2 \leq 24$$

$$x_1 + 2x_2 \leq 6$$

$$x_2 - x_1 \leq 1$$

$$x_2 \leq 2$$

$$Z = 5x_1 + 4x_2$$

So  
 Max  $Z = 5x_1 + 4x_2$   
 s.t.  $\begin{cases} 6x_1 + 4x_2 + x_3 = 24 \\ x_1 + 2x_2 + x_4 = 6 \\ -x_1 + x_2 + x_5 = 1 \\ x_2 + x_6 = 2 \\ x_1, x_2, x_3, x_4, x_5, x_6 \geq 0 \end{cases}$   
 $Z_{max}$

(b) Min  $Z = -4y_1 - 5y_2 + 0y_3 + 0y_4 + 0y_5 + 0y_6$

Subject to

$$\begin{cases} 4y_1 + 6y_2 + y_3 = 24 & \text{松} \\ 2y_1 + y_2 + y_4 = 6 & \text{松} \\ y_1 - y_2 + y_5 = 1 & \text{松} \\ y_1 + y_6 = 2 & \text{松} \\ y_1, y_2, y_3, y_4, y_5, y_6 \geq 0 \end{cases}$$

SIMPLEX Tab

|       | $y_1$ | $y_2$ | $y_3$ | $y_4$ | $y_5$ | $y_6$ |    |
|-------|-------|-------|-------|-------|-------|-------|----|
| $y_3$ | 4     | 6     |       | 0     | 0     | 0     | 24 |
| $y_4$ | 2     | 1     | 0     | 1     | 0     | 0     | 6  |
| $y_5$ | 1     | -1    | 0     | 0     | 1     | 0     | 1  |
| $y_6$ | 1     | 0     | 0     | 0     | 0     | 1     | 2  |
|       | -4    | -5    | 0     | 0     | 0     | 0     | 0  |

$$A = \begin{bmatrix} 4 & 6 & 1 & 0 & 0 & 0 \\ 2 & 1 & 0 & 1 & 0 & 0 \\ 1 & -1 & 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

$$B = \begin{bmatrix} 24 \\ 6 \\ 1 \\ 2 \end{bmatrix}$$

$$Y = [y_1 y_2 y_3 y_4 y_5 y_6]^T$$

$$Y_0 = [y_3 y_4 y_5 y_6]^T$$

$$C^T = [-4 \ -5 \ 0 \ 0 \ 0 \ 0]$$

$$C_0^T = [0 \ 0 \ 0 \ 0]$$

$$C^T - C_0^T A = [-4 \ -5 \ 0 \ 0 \ 0 \ 0]$$

$$-C_0^T B = 0$$

|       | $y_1$ | $y_2$ | $y_3$ | $y_4$ | $y_5$ | $y_6$ |    | Ratio min  |
|-------|-------|-------|-------|-------|-------|-------|----|------------|
| $y_3$ | 4     | (6)   |       | 0     | 0     | 0     | 24 | $24/6 = 4$ |
| $y_4$ | 2     | 1     | 0     | 1     | 0     | 0     | 6  | $6/1 = 6$  |
| $y_5$ | 1     | -1    | 0     | 0     | 1     | 0     | 1  |            |
| $y_6$ | 1     | 0     | 0     | 0     | 0     | 1     | 2  |            |
|       | -4    | (-5)  | 0     | 0     | 0     | 0     | 0  |            |
|       |       | min   |       |       |       |       |    |            |

|       | $y_1$                                | $y_2$                      | $y_3$          | $y_4$ | $y_5$ | $y_6$ |                              |
|-------|--------------------------------------|----------------------------|----------------|-------|-------|-------|------------------------------|
| $y_3$ | <del><math>4\frac{2}{3}</math></del> | <del><math>6</math></del>  | $1\frac{1}{6}$ | 0     | 0     | 0     | <del><math>24</math></del> 4 |
| $y_4$ | 2                                    | 1                          | 0              | 1     | 0     | 0     | 6                            |
| $y_5$ | 1                                    | -1                         | 0              | 0     | 1     | 0     | 1                            |
| $y_6$ | 1                                    | 0                          | 0              | 0     | 0     | 1     | 2                            |
|       | -4                                   | <del><math>-5</math></del> | 0              | 0     | 0     | 0     | 0                            |

|                             | $y_1$                                | $y_2$                      | $y_3$                               | $y_4$ | $y_5$ | $y_6$ |                              |
|-----------------------------|--------------------------------------|----------------------------|-------------------------------------|-------|-------|-------|------------------------------|
| <del><math>y_3</math></del> | $\frac{2}{3}$                        | 1                          | $\frac{1}{6}$                       | 0     | 0     | 0     | 4                            |
| $y_4$                       | <del><math>2\frac{4}{3}</math></del> | <del><math>7</math></del>  | <del><math>\frac{7}{6}</math></del> | 1     | 0     | 0     | 2 <del><math>6</math></del>  |
| $y_5$                       | <del><math>3\frac{5}{3}</math></del> | <del><math>8</math></del>  | <del><math>\frac{8}{6}</math></del> | 0     | 1     | 0     | 5 <del><math>7</math></del>  |
| $y_6$                       | 1                                    | 0                          | 0                                   | 0     | 0     | 1     | 2                            |
|                             | <del><math>-\frac{2}{3}</math></del> | <del><math>-8</math></del> | <del><math>\frac{5}{6}</math></del> | 0     | 0     | 0     | 20 <del><math>6</math></del> |

$\begin{matrix} \nearrow -1 \\ \nwarrow 1 \\ \searrow 5 \end{matrix}$

$$\frac{10}{3} - \frac{2}{3} = \frac{8}{3}$$

$$\frac{10}{3} - \frac{10}{3} = \frac{-2}{3}$$

|       | $y_1$                                | $y_2$ | $y_3$          | $y_4$ | $y_5$ | $y_6$ | Ratio                                     |
|-------|--------------------------------------|-------|----------------|-------|-------|-------|---|
| $y_2$ | $\frac{2}{3}$                        | 1     | $\frac{1}{6}$  | 0     | 0     | 0     | 4   |
| $y_4$ | <del><math>\frac{4}{3}</math></del>  | 0     | $\frac{-1}{6}$ | 1     | 0     | 0     | 2 <del><math>\frac{2}{2}</math></del> min |
| $y_5$ | $\frac{5}{3}$                        | 0     | $\frac{1}{6}$  | 0     | 1     | 0     | 5   |
| $y_6$ | 1                                    | 0     | 0              | 0     | 0     | 1     | 2   |
|       | <del><math>-\frac{2}{3}</math></del> | 0     | $\frac{5}{6}$  | 0     | 0     | 0     | 20  |

min

$$\textcircled{4} \times \frac{3}{2} = 6 \quad \textcircled{2} \times \frac{3}{\textcircled{4}_2} \quad 5 \times \frac{3}{5}$$

|                         | $y_1$                          | $y_2$ | $y_3$                                     | $y_4$                          | $y_5$ | $y_6$                         |                           |
|-------------------------|--------------------------------|-------|---|--------------------------------|-------|-------------------------------|---------------------------|
| $y_2$                   | $\textcircled{\frac{2}{2}} 0$  | 1     | $\textcircled{\frac{1}{6}} \frac{1}{4}$   | $\textcircled{-\frac{1}{2}} 0$ | 0     | $\textcircled{4} 3$           | $\leftarrow -\frac{2}{3}$ |
| $\textcircled{y_4} y_1$ | $\textcircled{\frac{4}{3}} 1$  | 0     | $\textcircled{-\frac{1}{6}} -\frac{1}{8}$ | $\textcircled{\frac{3}{4}} 0$  | 0     | $\textcircled{2} \frac{3}{2}$ | $R \times \frac{3}{4}$    |
| $y_5$                   | $\textcircled{\frac{5}{3}} 0$  | 0     | $\textcircled{\frac{1}{6}} \frac{3}{8}$   | $\textcircled{-\frac{5}{4}} 1$ | 0     | $\textcircled{5} \frac{5}{2}$ | $\leftarrow \frac{5}{3}$  |
| $y_6$                   | $\textcircled{4} 0$            | 0     | $\textcircled{\frac{1}{6}} \frac{1}{8}$   | $\textcircled{-\frac{3}{4}} 0$ | 1     | $\textcircled{2} \frac{1}{2}$ | $\leftarrow 1$            |
|                         | $\textcircled{-\frac{2}{3}} 0$ | 0     | $\textcircled{\frac{3}{6}} \frac{3}{4}$   | $\textcircled{\frac{1}{2}} 0$  | 0     | $\textcircled{20} 21$         | $\leftarrow \frac{2}{3}$  |

$$-\frac{1}{\textcircled{6}} \times \frac{\textcircled{2}}{4} = -\frac{1}{8}$$

$$-\frac{1}{8} \left(-\frac{2}{3}\right) + \frac{1}{6} \times \frac{3}{2} = \frac{1}{2}$$

$$\frac{1}{8} \times \frac{5}{3} + \frac{1}{6}$$

$$\frac{3}{4} \times -\frac{5}{3}$$

$$-\frac{5}{2} \times -\frac{7}{3} + 5$$

$$-\frac{1}{8} \times \frac{3}{3} + \frac{5}{6}$$

$$-\frac{1}{12} + \frac{15}{12} = \frac{14}{12} = \frac{7}{6} = \frac{3}{4} \quad \frac{3}{24} \times \frac{2}{3}$$

$$\frac{3}{2} \times \frac{2}{3} + 20$$

|       | $y_1$ | $y_2$ | $y_3$          | $y_4$          | $y_5$ | $y_6$ |               |
|-------|-------|-------|----------------|----------------|-------|-------|---------------|
| $y_2$ | 0     | 1     | $\frac{1}{4}$  | $-\frac{1}{2}$ | 0     | 0     | 3             |
| $y_1$ | 1     | 0     | $-\frac{1}{8}$ | $\frac{3}{4}$  | 0     | 0     | $\frac{3}{2}$ |
| $y_5$ | 0     | 0     | $\frac{3}{8}$  | $-\frac{5}{4}$ | 1     | 0     | $\frac{5}{2}$ |
| $y_6$ | 0     | 0     | $\frac{1}{8}$  | $-\frac{3}{4}$ | 0     | 1     | $\frac{1}{2}$ |
|       | 0     | 0     | $\frac{3}{4}$  | $\frac{1}{2}$  | 0     | 0     | 21            |

Optimal Solution

$$y_1 = \frac{3}{2} \quad y_2 = 3 \quad y_3 = y_4 = 0 \quad y_5 = \frac{3}{2}$$

$$y_6 = \frac{1}{2} \quad z = -21$$