21 - S1 - Q4

Q: (a)(i) diagram

$$⓪ \underset{p}{\overset{s}{\rightleftarrows}} ① \underset{r}{\overset{f}{\rightleftarrows}} ②$$

(a)(ii) ~~稳态概率 和平均产量~~

state0 $s\pi_0 = p\pi_1$

state1 $f\pi_1 + p\pi_1 = s\pi_0 + r\pi_2$

state2. $r\pi_2 = f\pi_1$

$$\pi_1 + \pi_2 + \pi_3 = 1$$

联立 $\pi_1 = \frac{s}{p}\pi_0$

$$\pi_2 = \frac{f}{r}\pi_1 = \frac{fs}{rp}\pi_0$$

$$\left(1 + \frac{s}{p} + \frac{fs}{rp}\right)\pi_0 = 1$$

$$\frac{rp + sr + fs}{rp}\pi_0 = 1$$

$$\pi_0 = \frac{rp}{rp+sr+fs} \quad \pi_1 = \frac{rs}{rp+sr+fs} \quad \pi_2 = \frac{fs}{rp+sr+fs}$$

$$\pi = [\pi_0 \quad \pi_1 \quad \pi_2]$$

$$= \left[\frac{rp}{rp+sr+fs} \quad \frac{rs}{rp+sr+fs} \quad \frac{fs}{rp+sr+fs} \right]$$

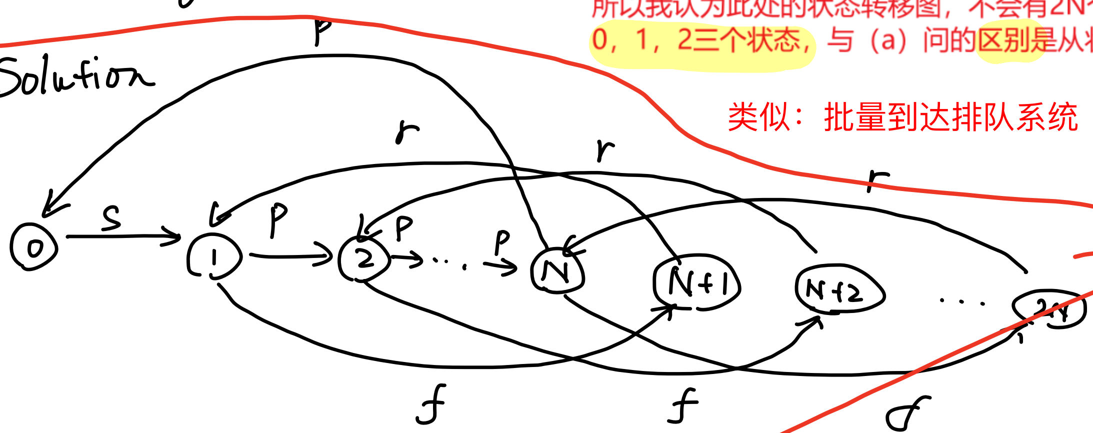
$$R_1 = \pi_1 p = \frac{rsp}{rp+sr+fs}$$

(b)(i) diagram ?

题干条件，状态1是指再生产一个零件，而不是在生产第N个零件
所以我认为此处的状态转移图，不会有2N个状态，我认为依然还是0，1，2三个状态，与（a）问的区别是从状态0~1的速率变成pN

类似：批量到达排队系统（Mb/M/1）去理解

Solution



state 0 $\quad s\pi_0 = p\pi_N$

state 1 $\quad p\pi_1 + f\pi_1 = r\pi_{N+1} + s\pi_0$

state i $\quad p\pi_i + f\pi_i = r\pi_{N+i} + p\pi_{i-1} \qquad i = 2, 3, \dots N$

state j $\quad r\pi_{N+j} = f\pi_j \qquad j = 1, 2, 3 \dots N$

如何解 π ?

(b)(i)

Solution state 0 set up

state 1 producing N parts

state 2 failed repair

$$
\begin{tikzcd}
0 \arrow[r, shift left, "s"] & 1 \arrow[l, shift left, "Np"] \arrow[r, shift left, "f"] & 2 \arrow[l, shift left, "r"]
\end{tikzcd}
$$

(ii) $\pi = \lim_{t\to\infty} \pi(t) = [\pi_0 \;\; \pi_1 \;\; \pi_2]$

$\pi_j = \lim_{t\to\infty} P_j(t) = \lim_{t\to\infty} P(X(t)=j)$

$$
\begin{cases}
\pi Q = 0 & \text{rate balance equation} \\
\pi_0 + \pi_1 + \pi_2 = 1
\end{cases}
$$

$$
\begin{cases}
s\pi_0 = Np\pi_1 \\
f\pi_1 + Np\pi_1 = s\pi_0 + r\pi_2 \\
r\pi_2 = f\pi_1 \\
\pi_0 + \pi_1 + \pi_2 = 1
\end{cases}
$$

$$\pi_0 + \frac{s}{Np}\pi_0 + \frac{f}{r}\frac{s}{Np}\pi_0 = 1$$

$$\frac{rNp + sr + fs}{rNp}\pi_0 = 1$$

$$\pi_0 = \frac{rNp}{rNp + sr + fs} \qquad \pi_1 = \frac{s}{Np}\pi_0 = \frac{rs}{rNp + sr + fs}$$

$$\pi_2 = \frac{fs}{rNp + sr + fs}$$

$$z_2 = \left[\frac{rNp}{rNp+sr+fs} \quad \frac{rs}{rNp+sr+fs} \quad \frac{fs}{rNp+sr+fs}\right]$$

$$R_2 = z_1 pN = \frac{rspN}{rNp+sr+fs}$$

(2) $$R_1 = z_1 p = \frac{rsp}{rp+sr+fs} = \frac{1\times1\times2}{1\times2+1\times1+0.1\times1}$$

$$= \frac{2}{2+1+0.1} = 0.6452$$

$$R_2 = z_1 pN = \frac{rspN}{rNp+sr+fs} = \frac{1\times1\times2\times3}{1\times3\times2+1\times1+0.1\times1}$$

$$= \frac{2\times3}{6+1+0.1} = 0.2817\times3$$

$$= 0.8452$$

$$R_1 \overset{<}{>} R_2$$

system $\overset{2}{1}$ is better than system $\overset{1}{2}$!

The efficiency improvement brought by batch production: In (b), since the machine produces multiple parts at once (N=3), compared to (a) where only one part is produced at a time, batch production reduces the proportion of time spent on "setup," thereby increasing the overall average production rate. This result is as expected because batch production reduces the overhead caused by frequent machine setups and enhances the overall efficiency of the factory.

批量生产带来的效率提升： 在 (b) 中，由于机器每次生产多个零件（N=3），相比于 (a) 中每次仅生产一个零件，批量生产的设置减少了机器的"设置时间"占比，从而提高了整体的平均生产率。这个结果符合预期，因为批量生产可以减少频繁设置机器所带来的开销，并提升工厂的总体效率。