22-S1-Q1

Q (a) max profit ?

Solution let profit 8

product A x_1 B x_2 C x_3 D x_4

$$Z = 30x_1 - 15(800 - x_1) + 40x_2 - 20(750 - x_2)$$
$$+ 20x_3 - 10(600 - x_3) + 10x_4 - 8(500 - x_4)$$
$$= 45x_1 + 60x_2 + 30x_3 + 18x_4 - 37000$$

Subject to

$$\begin{cases} 0.3x_1 + 0.3x_2 + 0.25x_3 + 0.15x_4 \le 1000 \\ 0.25x_1 + 0.35x_2 + 0.3x_3 + 0.1x_4 \le 1000 \\ 0.45x_1 + 0.5x_2 + 0.4x_3 + 0.22x_4 \le 1000 \\ 0.15x_1 + 0.15x_2 + 0.1x_3 + 0.05x_4 \le 1000 \\ x_1 \le 800 \\ x_2 \le 750 \\ x_3 \le 600 \\ x_4 \le 500 \\ x_1, x_2, x_3, x_4 \ge 0 \end{cases}$$

(b) (i) Max $Z = 5x_1 + 4x_2 + 0 \cdot x_3 + 0 \cdot x_4 + 0 \cdot x_5 + 0 \cdot x_6$

Subject to

$$\begin{cases} -x_1 + x_2 + x_3 = 1 \\ x_2 + x_4 = 2 \\ x_1 + 2x_2 + x_5 = 6 \\ 6x_1 + 4x_2 + x_6 = 24 \\ x_1, x_2, x_3, x_4, x_5, x_6 \ge 0 \end{cases}$$

② list Simplex Max Table

	X^T	
X_0	A	B
	$-C^T+C_0^TA$	C_0^TB

$X^T=[X_1\ X_2\ X_3\ X_4\ X_5\ X_6]$ $X_0=[X_3\ X_4\ X_5\ X_6]^T$

$$A=\begin{bmatrix}-1&1&1&0&0&0\\0&1&0&1&0&0\\1&2&0&0&1&0\\6&4&0&0&0&1\end{bmatrix}\quad B=\begin{bmatrix}1\\2\\6\\24\end{bmatrix}$$

$C^T=[5\ 4\ 0\ 0\ 0\ 0]$ $C_0^T=[0\ 0\ 0\ 0]$

$-C^T+C_0^TA=-C^T=[-5\ -4\ 0\ 0\ 0\ 0]$

$C_0^TB=0$

③ draw Simplex Max Table

	X_1	X_2	X_3	X_4	X_5	X_6	
X_3	-1	1	1	0	0	0	1
X_4	0	1	0	1	0	0	2
X_5	1	2	0	0	1	0	6
X_6	6	4	0	0	0	1	24
	-5	-4	0	0	0	0	0

④ simplex Method: iteration 1

	x_1	x_2	x_3	x_4	x_5	x_6		Ratio
x_3	−1	1	1	0	0	0	1	
x_4	0	1	0	1	0	0	2	
x_5	1	2	0	0	1	0	6	6/1 = 6
x_6	6	4	0	0	0	1	24	24/6 = 4 minimum
	−5	−4	0	0	0	0	0	

work column minimum

	x_1	x_2	x_3	x_4	x_5	x_6	
x_3	0	$\frac{5}{3}$	1	0	0	$\frac{1}{6}$	5
x_4	0	1	0	1	0	0	2
x_5	0	$\frac{4}{3}$	0	0	1	$-\frac{1}{6}$	2
x_6	1	$\frac{2}{3}$	0	0	0	$\frac{1}{6}$	4
	0	$-\frac{2}{3}$	0	0	0	$\frac{5}{6}$	20

$R_4 + R_1$, $-R_4 + R_3$, $R_4 \times \frac{1}{6}$, $5R_4 + R_5$

x_1

$2 - \frac{2}{3} = \frac{6}{3} - \frac{2}{3} = \frac{4}{3}$ $\quad 6 - 4 = 2$

$\frac{10}{3} - 4 = \frac{10}{3} - \frac{12}{3} = -\frac{2}{3}$

⑤ Simplex Iteration 2.

	X_1	X_2	X_3	X_4	X_5	X_6		Ratio
X_3	0	$\frac{5}{3}$	1	0	0	$\frac{1}{6}$	5	$5/\frac{5}{3}=3$
X_4	0	1	0	1	0	0	2	$2/1=2$
X_5	0	$\frac{4}{3}$	0	0	1	$-\frac{1}{6}$	2	$2/\frac{4}{3}=\frac{3}{2}$ min
X_1	1	$\frac{2}{3}$	0	0	0	$\frac{1}{6}$	4	$4/\frac{2}{3}=6$
	0	$-\frac{2}{3}$	0	0	0	$\frac{5}{6}$	20	

work column minimum

$20\times(-\frac{3}{2})=-30$

	X_1	X_2	X_3	X_4	X_5	X_6		
X_3	0	$\frac{5}{3}$ 0	1	0	0 $-\frac{5}{4}$	$\frac{1}{6}$ $\frac{3}{8}$	5	$-\frac{5}{3}R_3+R_1$
X_4	0	1 0	0	1	0 $-\frac{3}{4}$	0 $\frac{1}{8}$	2 $\frac{1}{2}$	$-R_3+R_2$
X_5	0	$\frac{4}{3}$ 1	0	0	1 $\frac{3}{4}$	$-\frac{1}{6}$ $-\frac{1}{8}$	2 $\frac{3}{2}$	$R_3\times\frac{3}{4}$
X_1	1	$\frac{2}{3}$ 0	0	0	0 $-\frac{1}{2}$	$\frac{1}{6}$ $\frac{1}{4}$	4 3	$-\frac{2}{3}R_3+R_4$
	0	$-\frac{2}{3}$ 0	0	0	0 $\frac{1}{2}$	$\frac{5}{6}$ $\frac{3}{4}$	20 21	$\frac{2}{3}R_3+R_5$

$\rightarrow X_2$

$\frac{5}{2}$

$-\frac{1}{6}\times\frac{3}{4}=-\frac{1}{2}\times\frac{1}{4}=-\frac{1}{8}$ $\quad 2\times\frac{3}{4}=\frac{3}{2}$

$2-\frac{3}{2}=\frac{4}{2}-\frac{3}{2}=\frac{1}{2}$ $\quad \frac{3}{4}\times(-\frac{5}{3})=-\frac{5}{4}$

$-\frac{1}{8}\times(-\frac{5}{3})+\frac{1}{6}=\frac{5}{24}+\frac{4}{24}=\frac{9}{24}=\frac{3}{8}$

$(-\frac{5}{3})\times(\frac{3}{2})+5=-\frac{5}{6}+5=-\frac{5}{6}+\frac{30}{6}=\frac{25}{6}\ \frac{5}{2}$

$(-\frac{2}{3})\frac{3}{4}=-\frac{1}{2}$ $\quad(-\frac{1}{8})(-\frac{2}{3})+\frac{1}{6}=\frac{1}{12}+\frac{1}{6}=\frac{3}{12}=\frac{1}{4}$

$\frac{3}{4}\times(\frac{2}{3})=\frac{1}{2}$ $\quad-\frac{1}{8}\times\frac{2}{3}+\frac{5}{6}=-\frac{1}{12}+\frac{10}{12}=\frac{9}{12}=\frac{3}{4}$

⑥ The final row coefficient are all nonegative

	X_1	X_2	X_3	X_4	X_5	X_6	
X_3	0	0	1	0	$-\frac{5}{4}$	$\frac{3}{8}$	$\frac{5}{2}$
X_4	0	0	0	1	$-\frac{3}{4}$	$\frac{1}{8}$	$\frac{1}{2}$
X_2	0	1	0	0	$\frac{3}{4}$	$-\frac{1}{8}$	$\frac{3}{2}$
X_1	1	0	0	0	$-\frac{1}{2}$	$\frac{1}{4}$	3
	0	0	0	0	$\frac{1}{2}$	$\frac{3}{4}$	21

the optimal result

$X_1=3$ $\quad X_2=\frac{3}{2}$ $\quad X_3=\frac{5}{2}$ $\quad X_4=\frac{1}{2}$ $\quad X_5=X_6=0$

the maximum $Z=21$