22-S1-Q1

Q (a) max profit ?

Solution let profit 8

product A x_1 B x_2 C x_3 D x_4

$$Z = 30x_1 - 15(800 - x_1) + 40x_2 - 20(750 - x_2)$$
$$+ 20x_3 - 10(600 - x_3) + 10x_4 - 8(500 - x_4)$$
$$= 45x_1 + 60x_2 + 30x_3 + 18x_4 - 37000$$

Subject to

$$\begin{cases} 0.3x_1 + 0.3x_2 + 0.25x_3 + 0.15x_4 \le 1000 \\ 0.25x_1 + 0.35x_2 + 0.3x_3 + 0.1x_4 \le 1000 \\ 0.45x_1 + 0.5x_2 + 0.4x_3 + 0.22x_4 \le 1000 \\ 0.15x_1 + 0.15x_2 + 0.1x_3 + 0.05x_4 \le 1000 \\ x_1 \le 800 \\ x_2 \le 750 \\ x_3 \le 600 \\ x_4 \le 500 \\ x_1, x_2, x_3, x_4 \ge 0 \end{cases}$$

(b) Max $Z = 5x_1 + 4x_2 + 0 \cdot x_3 + 0 \cdot x_4 + 0 \cdot x_5$

Subject to

$$\begin{cases} -x_1 + x_2 + x_3 = 1 \\ x_2 + x_4 = 2 \\ 6x_1 + 4x_2 + x_5 = 24 \\ x_1, x_2, x_3, x_4, x_5 \ge 0 \end{cases}$$

	X^T	
X_0	A	B
	$-C^T + C_0^T A$	$C_0^T B$

$X^T = [X_1 \; X_2 \; X_3 \; X_4 \; X_5]$ $\quad$ $X_0^T = [X_3 \; X_4 \; X_5]$

$$A = \begin{bmatrix} -1 & 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 \\ 6 & 4 & 0 & 0 & 1 \end{bmatrix} \qquad B = \begin{bmatrix} 1 \\ 2 \\ 24 \end{bmatrix}$$

$C^T = [5 \; 4 \; 0 \; 0 \; 0]$ $\quad$ $C_0^T = [0 \; 0 \; 0]$

$-C^T + C_0^T A = [-5 \; -4 \; 0 \; 0 \; 0]$ $\quad$ $C_0^T B = 0$

So

	X_1	X_2	X_3	X_4	X_5		Ratio
X_3	-1	1	1	0	0	1	
X_4	0	1	0	1	0	2	
X_5	6	4	0	0	1	24	24/6 = 4
	-5	-4	0	0	0	0	

↑ work col

	X_1	X_2	X_3	X_4	X_5		Ratio
X_3	-1	$\frac{5}{3}$	1	0	$\frac{1}{6}$	5	$5/\frac{1}{3} = 3$
X_4	0	1	0	1	0	2	$2/1 = 2$
X_5	6	4	0	0	1	24	$4/\frac{2}{3} = 6$
$\rightarrow X_1$	-5	-4	0	0	0	0	
	0	$-\frac{2}{3}$	0	0	$\frac{5}{6}$	20	

1

5

work column

	X_1	X_2	X_3	X_4	X_5	
X_3	$-\frac{5}{2}$	$\frac{5}{3}$	0	1	0	$\frac{1}{6}$ $-\frac{1}{4}$ -5
X_4	$-\frac{3}{2}$	1	0	0	1	$-\frac{1}{4}$ -4
X_1 → X_2	$\frac{3}{2}$	$\frac{2}{3}$	1	0	0	$\frac{1}{6}$ $\frac{1}{4}$ 6
	0	1	0	0	0	$\frac{5}{6}$ 1 20

$-\frac{5}{3}$, -1, $\times\frac{3}{2}$, $\frac{2}{3}$

$\frac{3}{2} \times (-\frac{5}{3})$

$\frac{1}{4} \times (-\frac{5}{3}) + \frac{1}{6}$

$6 \times (-\frac{5}{3}) + 5 = 1$

$\frac{3}{2} \times \frac{2}{3} = 1$

$\frac{1}{4} \times \frac{2}{3} + \frac{5}{6}$

$6 \times \frac{2}{3} + 20 = 24$

Optical solution is $X_2 = 6$ $X_3 = 5$ $X_4 = 4$

$X_1 = X_5 = 0$

Max $Z = 24$