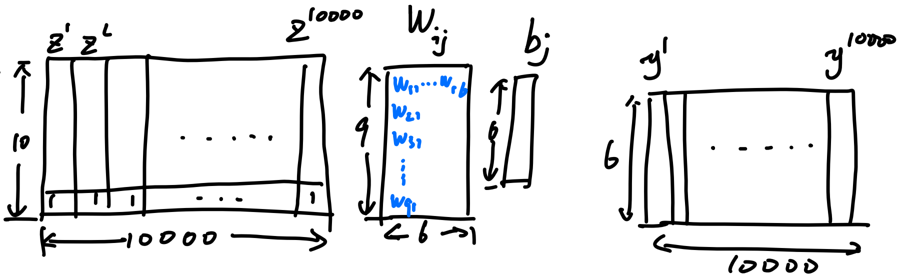
23-S2-Q2

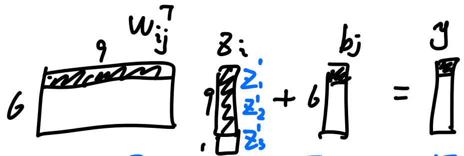
Q: $3\times3 \to 100\times100 \to 10000$ $\quad z^k = (z_1^k\ z_2^k \cdots z_9^k, 1)^T$, $k = 1\sim10000$

$y^k = (y_1^k, \cdots y_6^k)^T$ $\quad W_{ij}$ and b_j

(a) $z_i^k \to y_j^k$?

Solution





T 标量,不能转置,也不用转置,不是矩阵

$$y_j^k = \sum_{i=1}^{9} W_{ij} z_i^k + b_j \qquad (j = 1, 2, \cdots 6)$$

$$y_1^k = \sum_{i=1}^{9} W_{i1} z_i + b_1$$

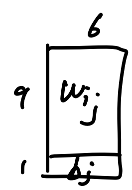
$$= W_{11} z_1 + W_{21} z_1 + \cdots W_{91} z_9 + b_1$$

(b) W? $\quad z^k \to y^k$

Solution

we can construct the weight matrix W of size 10×6 wherer each column corresponds the the weight w_{ij} and the last one is bias b_j

$$W = \begin{bmatrix} W_{11} & W_{12} & \cdots & W_{16} \\ W_{21} & W_{22} & \cdots & W_{26} \\ \vdots & \vdots & \ddots & \vdots \\ W_{91} & W_{92} & \cdots & W_{96} \\ b_1 & b_2 & \cdots & b_6 \end{bmatrix}$$



$$y^k = W^T z^k$$

(c) $W_j = \begin{bmatrix} w_{1j} & w_{2j} & w_{3j} \\ w_{4j} & w_{5j} & w_{6j} \\ w_{7j} & w_{8j} & w_{9j} \end{bmatrix}$

$$Y_j(u,v) = \sum_{s=-1}^{1} \sum_{t=-1}^{1} W_j(s+1, t+1) \cdot X(u-s, v-t) + b_j$$

$$Y_j = X * W_j + b_j$$

$*$ mean convolution operation

(d) Each of the 100 training image of size 100x100 provide 10000 Samples

Therefore,

Number of training sample = 100 images X 10000 sample/image

= 1,000,000