

典型的全连接神经网络的输入和输出用  $\mathbf{x} = [x_1, x_2, \dots, x_{100}]^T$  表示。  
 在该层中应用线性激活函数，连接输入和输出的网络参数由大小为  $98 \times 100$  的矩阵  $\mathbf{W} = [w_{ij}]$  和 98 维向量  $\boldsymbol{\theta} = [\theta_1, \theta_2, \dots, \theta_{98}]^T$  表示。

The inputs and outputs of a typical fully-connected layer of neural network are denoted by  $\mathbf{x} = [x_1, x_2, \dots, x_{100}]^T$  and  $\mathbf{y} = [y_1, y_2, \dots, y_{98}]^T$ . A linear activation function is applied in this layer and the network parameters to connect the inputs and outputs are given by a matrix  $\mathbf{W} = [w_{ij}]$  of size  $98 \times 100$  and a 98-dimensional vector  $\boldsymbol{\theta} = [\theta_1, \theta_2, \dots, \theta_{98}]^T$ .

- (a) (i) Express the outputs in term of inputs in vector-matrix form and scalar form.  
用向量矩阵形式和标量形式表示输入的输出。
- (ii) Compute the numbers of trainable parameters, multiplications and summations required in this layer to compute the outputs from the inputs.  
计算可训练参数的数量，这一层需要的乘法和求和来计算输入的输出。
- (iii) What is the ratio of the number of outputs to the number of trainable parameters?  
输出的数量与可训练参数的数量之比是多少？

(12 Marks)

将该层替换为卷积神经网络层，该层有 20 个可学习的过滤器，大小为 3，具有可训练的参数， $\mathbf{w}^k$   
 这将生成 20 个输出特征映射，用  $\mathbf{y}^k$  表示。  
 在这一层中应用线性激活函数。

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- (b) Replace this layer by a convolutional neural network layer that has 20 learnable filters of size 3 with trainable parameters,  $\mathbf{w}^k = [w_{-1}^k, w_0^k, w_1^k]^T$  and  $\theta^k$ ,  $1 \leq k \leq 20$ . This generates 20 output feature maps, denoted by  $\mathbf{y}^k = [y_2^k, y_3^k, \dots, y_{99}^k]^T$ ,  $1 \leq k \leq 20$ . A linear activation function is applied in this layer.

- (i) Express the outputs in term of inputs in scalar form. 用标量形式表示输入的输出。
- (ii) Compute the number of trainable parameters, multiplications and summations required in this layer to compute the outputs from the inputs.  
计算可训练参数的数量，这一层需要的乘法和求和来计算输入的输出。
- (iii) What is the ratio of the number of outputs to the number of trainable parameters?

输出的数量与可训练参数的数量之比是多少？

(13 Marks)

23-51-Q3

(a)(i) Q: vector-matrix form and scalar form input and output?

$$X = [x_1 \ x_2 \ x_3 \ \dots \ x_{100}]^T$$

$$y = [y_1 \ y_2 \ \dots \ y_{98}]^T$$

$$W = [w_{ij}] \quad 98 \times 100$$

$$\theta = [\theta_1 \ \theta_2 \ \dots \ \theta_{98}]^T$$

vector-matrix form

$$y = Wx + \theta$$

scalar form

$$y_i = \sum_{j=1}^{100} w_{ij} x_j + \theta_i \quad i = 1, 2, \dots, 98$$

(ii) Q parameters? multiplications? summations?

Solution

$$\text{parameters} : 98 \times 100 + 98 = 9898$$

$$\text{multiplications} : 98 \times 100 = 9800$$

$$\text{summations} : 98 \times (99+1) = 9800$$

(iii) Q: ratio

$$\text{Solution Ratio} = \frac{98}{9898} \approx 0.0099$$

$$y_i = \sum_{j=1}^{100} w_{ij} x_j + \theta_i$$

$$y_1 = w_{11}x_1 + w_{12}x_2 + \dots + \theta_1$$

#### Neural Networks and Deep CNN -- Neuron Model

In mathematical terms, we can describe the neuron as: 用数学术语，我们可以将神经元描述为：

$$u_k = \sum_{j=1}^p w_{kj} x_j \quad y_k = f(u_k - \theta_k)$$

Where  $x_1, x_2, \dots, x_p$  are the input signals,  $w_{k1}, w_{k2}, \dots, w_{kp}$  are the synaptic weights of neuron  $k$ ,  $u_k$  is the linear combiner output,  $\theta_k$  is the **threshold**,  $f(\cdot)$  is the activation function and  $y_k$  is the neurons output. 其中  $x_1, x_2, \dots, x_p$  是输入信号,  $w_{k1}, w_{k2}, \dots, w_{kp}$  是第  $k$  个神经元的突触权重,  $u_k$  是线性组合输出,  $\theta_k$  是阈值,  $f(\cdot)$  是激活函数,  $y_k$  是神经元的输出.  $\theta_k$  is an external parameter, we can consider this parameter as an input variable:  $\theta_k$  是一个外部参数, 我们可以将这个参数看作一个输入变量.

Then we have:  $x_0 = 1, \quad w_{k0} = -\theta_k$

第一题是+ 还是- ?

(a) Output after FC layer

$$= Wx + b$$

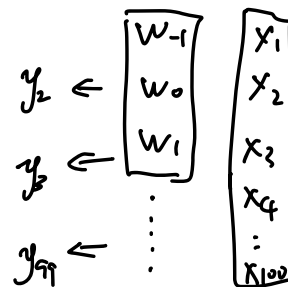
$$= \begin{bmatrix} 0 & 3 & 7 & 8 \\ 1 & 8 & 0 & 0 \\ 0 & 8 & 1 & 0 \end{bmatrix} \begin{bmatrix} 0.3 \\ 0 \\ 0.2 \\ 0.1 \end{bmatrix} + \begin{bmatrix} -1 \\ 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 1.2 \\ 1.3 \\ 2.2 \end{bmatrix}$$

100 数加 99 次  
加 1 次  $\theta$

$$(b)(i) y_i^{(k)} = w_{-1}^{(k)} x_{i-1} + w_0^{(k)} x_i + w_1^{(k)} x_{i+1} + b^{(k)}$$

$$k = 1, 2, \dots, 20$$

$$i = 1, 2, \dots, 99$$



$$(ii) \text{ parameters: } 20 \times (3 + 1) = 80$$

$$\text{multiplications: } 3 \times 98 \times 20 = 5880$$

$$\text{summations: } (2 + 1) \times 98 \times 20 = 5880$$

$$(iii) \text{ Ratio} = \frac{98 \times 20}{80} = 24.5$$

20个(3权重 + 1个bias)

每个y乘3次, 98个y  
共20个

每个y加2权加1b

98个y, 20个