Q : PCA & LDA

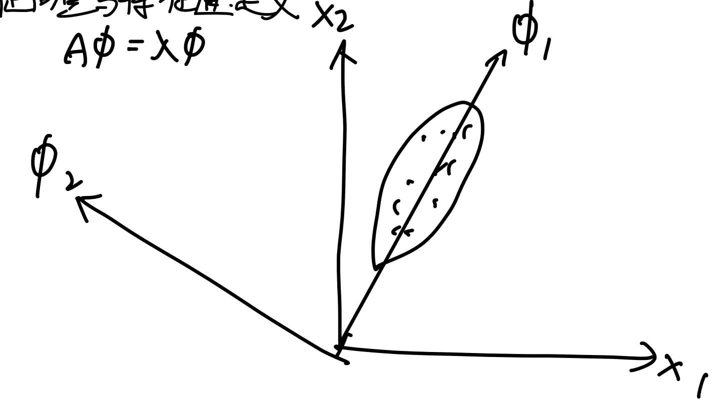
Solution ① q个样本x,每个是 n 维列向量

② 类内协方差 $n \times n$维 $\Sigma_j = \frac{1}{q_j} \sum_{x_i \in \omega_j} (x_i - \mu_j)(x_i - \mu_j)^T$

③ 求 Σ_j的特征值 λ_i 组成对角阵 $\Lambda = \begin{bmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{bmatrix}$

④ $\Sigma_j \phi = \phi \Lambda$ ⇐ 特征向量与特征值定义 $A\phi = \lambda\phi$

ϕ是主成分方向, n维 横向量



⑤ $\phi \phi^T = I = \phi \phi^{-1}$

$\Sigma_j = \phi \Lambda \phi^T$

⑥ $\max \operatorname{trac}[\phi^T \Sigma_j \phi] = \sum_{k=1}^{m} \lambda_k^t$

$= \lambda_1^t + \lambda_2^t + \cdots + \lambda_m^t$

PS ① 投影定义：点积

$\phi^T x_i$

② 方差定义

$\sigma_i^2 = \frac{1}{n}\sum_{i=1}^{n}[\phi^T x_i][\phi^T x_i]^T$

③ 正交定义：点积为0

$\phi_k^T \phi_j = 0$

④ 协方差定义

$\sigma_{kj}^2 = \frac{1}{n} y_k y_j^T$

LDA

① $$\max \ \operatorname{trace}\left[\phi^{T} \overset{\min}{\underset{\text{类内}}{S^{w}}}{}^{-1} \underset{\text{类间}}{S^{b}} \phi\right] = \sum_{k=1}^{m} \lambda_{k}^{b/w}$$