3. The inputs and outputs of a typical fully-connected layer of neural network are denoted by $x = [x_1, x_2, \ldots, x_{100}]^T$ and $y = [y_1, y_2, \ldots, y_{98}]^T$. A linear activation function is applied in this layer and the network parameters to connect the inputs and outputs are given by a matrix $W = [w_{ij}]$ of size 98×100 and a 98-dimensional vector $\theta = [\theta_1, \theta_1, \ldots, \theta_{98}]^T$.

(a) (i) Express the outputs in term of inputs in vector-matrix form and scalar form.

(ii) Compute the numbers of trainable parameters, multiplications and summations required in this layer to compute the outputs from the inputs.

(iii) What is the ratio of the number of outputs to the number of trainable parameters?
(12 Marks)

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(b) Replace this layer by a convolutional neural network layer that has 20 learnable filters of size 3 with trainable parameters, $w^k = [w_{-1}^k, w_0^k, w_1^k]^T$ and θ^k, $1 \le k \le 20$. This generates 20 output feature maps, denoted by $y^k = [y_2^k, y_3^k, \ldots, y_{99}^k]^T$, $1 \le k \le 20$. A linear activation function is applied in this layer.

(i) Express the outputs in term of inputs in scalar form.

(ii) Compute the number of trainable parameters, multiplications and summations required in this layer to compute the outputs from the inputs.

(iii) What is the ratio of the number of outputs to the number of trainable parameters?
(13 Marks)

23 – S1 – Q3

(a)(i) Q: vector – matrix form and scalar form input and output?

$x = [x_1 \ x_2 \ x_3 \cdots x_{100}]^T$

$y = [y_1 \ y_2 \cdots y_{98}]^T$

$W = [w_{ij}] \quad 98 \times 100$

$\theta = [\theta_1, \theta_2 \cdots \theta_{98}]^T$

vector – matrix form

$y = Wx + \theta$

scalar form

$y_i = \sum\limits_{j=1}^{100} w_{ij} x_j + \theta_i \qquad i = 1, 2, \cdots 98$

(ii) Q parameters? multiplications? summations?

$y_1 = w_{11} x_1 + w_{12} x_2 + \cdots + \theta_1$

Neural Networks and Deep CNN -- Neuron Model

In mathematical terms, we can describe the neuron as:

$$u_k = \sum_{j=1}^{p} w_{kj} x_j \qquad y_k = f(u_k - \theta_k)$$

Where $x_1, x_2, \ldots, x_p$ are the input signals, $w_{k1}, w_{k2}, \ldots, w_{kp}$ are the synaptic weights of neuron k, u_k is the linear combiner output, θ_k is the threshold, $f(.)$ is the activation function and y_k is the neurons output.

θ_k is an external parameter, we can consider this parameter as an input variable:

Then we have: $\quad x_0 = 1, \qquad w_{k0} = -\theta_k$

Output after FC layer

$= Wx + b$

$= \begin{bmatrix} 0 & 3 & 7 & 8 \\ 1 & 8 & 0 & 0 \\ 0 & 8 & 1 & 0 \end{bmatrix} \begin{bmatrix} 0.3 \\ 0 \\ 0.2 \\ 0.1 \end{bmatrix} + \begin{bmatrix} -1 \\ 1 \\ 2 \end{bmatrix} = \begin{bmatrix} 1.2 \\ 1.3 \\ 2.2 \end{bmatrix}$

Solution

parameters: $98 \times 100 + 98 = 9898$

multiplications: $98 \times 100 = 9800$

summations: $98 \times (99 + 1) = 9800$

(iii) Q: ratio

Here theta is the bias. Please note that in this questions theta is a 98 dimensional vector. While in the slides, theta is a scalar.
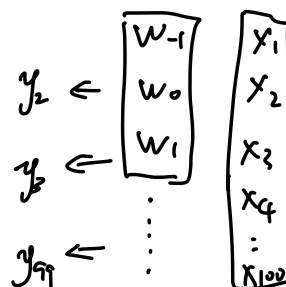
Solution Ratio $= \dfrac{98}{9898} \approx 0.0099$

100数加99个
加 1个θ

(b)(i) $y_i^{(k)} = w_{-1}^{(k)} x_{i-1} + w_0^{(k)} x_i + w_1^{(k)} x_{i+1} + \Theta^{(k)}$

$k = 1, 2, \cdots, 20$

$i = \boxed{1}, 2, \cdots 99$

$x_{i-1} = x_1$
$i \qquad 2$

$$y_2 \Leftarrow \begin{bmatrix} w_{-1} \\ w_0 \\ w_1 \\ \vdots \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ \vdots \\ x_{100} \end{bmatrix}$$

$y_3 \Leftarrow$

$y_{99} \Leftarrow$

(ii) parameters: $20 \times (3+1) = 80$

20个(3权重 + 1个 bias)

multiplications : $3 \times 98 \times 20 = 5880$

每个 y 乘 3 次, 98个 y
共 20个

summations : $(2+1) \times 98 \times 20 = 5880$

每个 y 加 2 权加 1 Θ
98个 y, 20个

(iii) Ratio $= \dfrac{98 \times 20}{80} = 24.5$