Q: picture sampling

Solution

① sampling function

$$S(x,y) = \sum_{m=-\infty}^{\infty} \sum_{n=-\infty}^{\infty} \delta(x-m\Delta x, y-n\Delta y)$$

$$S(u,v) = \frac{1}{\Delta x \Delta y} \sum_{m=-\infty}^{\infty} \sum_{n=-\infty}^{\infty} \delta\left(u-\frac{m}{\Delta x}, v-\frac{n}{\Delta y}\right)$$

② 图x采样：c → d

$$f_d(x,y) = f_c(x,y)\, S(x,y)$$

$$= \sum_{m=-\infty}^{\infty} \sum_{n=-\infty}^{\infty} f_c(m\Delta x, n\Delta y)\, \delta(x-m\Delta x, y-n\Delta y)$$

$$F_d(u,v) = \frac{1}{\Delta x \Delta y} \sum_{m=-\infty}^{\infty} \sum_{n=-\infty}^{\infty} F_c(u-mf_{xs}, v-nf_{ys})$$

③ 恢复：d → c

$$F_c(u,v) = F_d(u,v)\, H(u,v)$$

$$f_c(x,y) = f_d(x,y) \times h(x,y)$$

$$H(u,v) = \begin{cases} \sigma_x \sigma_y & (u,v) \in R \\ 0 & \end{cases}$$

④ Sampling Theorem

sampling rates (f_{xs}, f_{ys}) > Nyquist rates

$\Rightarrow$ recovered without error

$< \sim$ $\Rightarrow$ aliasing