23-S2-Q1

Q (a) GA $\xrightarrow{\text{tune}}$ PID

(i) Real-coded representation.

(ii) Binary ~

Solution (a)(i)

① Chromosome initialization

choose a reasonable range for each parameter

eg. $k_p \in [0, 100]$ $k_i \in [0, 10]$ $k_d \in [0, 10]$

② Randomly generate each parameter within its allowable range for the initial population

③ computing fitness proportion

(1) plug each chromosome $[k_p\ k_i\ k_d]$ into PID controller

(2) Run the close-loop simulation compute a performance matric

eg. Integral of Absolute Error

$$J = \int_0^T |e(t)|\,dt$$

(3) Convert this performance measaur into a fitness value.

eg. $\text{fitness} = \dfrac{1}{1+J}$

so, that lower J (better control) yields higher fitness

(4) selection

use roulette-wheel selection based on the fitness value

(4) Crossover

we can use whole arithmetic crossover

$$z_i = \alpha x_i + (1-\alpha) y_i \quad (0 \le \alpha \le 1)$$

z_i : child

x_i : parent 1

y_i : parent2.

⑤ Mutation

perturb each parameter slightly within its valid rang.

eg. uniform mutation $x \to x' \in [LB_i, UB_i]$

nonuniform mutation $x' = x + N(0, \sigma)$

self-Adaptive mutation $\sigma \to \sigma'$

$x' = x + N(0, \sigma')$

⑥ Replacement and Termination

form a new population

continue iterating until a stopping criterion is met

eg. fixed number of generations or no improvement

Solution (a) (ii)

① chromosome Encoding

e.g. we choose

10 bits for K_p in [0,100]

8 bits for K_i in [0,10]

8 bits for K_d in [0,10]

e.g. $\underbrace{1010\,0010\,11}_{K_p}$ $\underbrace{0011\,1001}_{K_i}$ $\underbrace{0111\,0001}_{K_d}$

② Randomly generate bit strings.

③ Decoding

Convert each bit sub-string to a decimal integer, using

$$\Gamma(a_1 \cdots a_L) = x + \frac{y-x}{2^L-1}\left(\sum_{j=0}^{L-1} a_{L-j} \cdot 2^j\right) \in [x, y]$$

get K_p K_i K_d

③ computing fitness proportions

run PID simulation

compute a performance measure

convert to fitness

use roulette wheel to obtain the mating pool

⑤ Crossover

(1) select $n/2$ pairs of parent strings for crossover

(2) randomly chosen pairs under mate.

(3) one-point crossover site are randomly chosen

(4) get new population

⑥ Mutation

choose mutation rate. p_m

Alter each genne independently with p_m

⑦ Replacement and Termination

Q(b) 15 iteams $A_1 \cdots A_{15}$

3 vouchers V_1 V_2 V_3

\$ 60 70 80

(i) Voucher : mini wastage.

(ii) genotype

(iii) fitness function

Solution (b)(i)

① Sum of cost = 12+27+3+7+15+23+9+17+4+5+30+13+22+18+6 = 211

② sum of voucher = 60+70+80 = 210

Item $A_\#$	Cost (\$)	Voucher ($V1, V2$ or $V3$)
1	12	V3
2	27	V1
3	3	V3
4	7	V3
5	15	V2
6	23	V1
7	9	V1
8	17	V3
9	4	not purchased
10	5	V3
11	30	V3
12	13	V2
13	22	V2
14	18	V2
15	6	V3

V_1 (limit \$60) $= 27 + 23 + 9 = 59$

leftover $= 1$

V_2 (limit \$70) $= 15 + 13 + 22 + 18 = 68$

leftover $= 2$

V_3 (limit \$80) $= 12 + 3 + 7 + 17 + 5 + 30 + 6 = 80$

leftover $= 0$

Total leftover $= 1 + 2 = 3$

(ii) Genotype representation

① phenotype

each solution can be a string of length 15, where each item takes a value in $\{1, 2, 3, 0\}$ $[g_1 \; g_2 \; \cdots \; g_{15}]$

where $g_i = \begin{cases} 1 & \text{, if item } i \text{ is purchased with } V_1 \\ 2 & \text{, if item } i \text{ is purchased with } V_2 \\ 3 & \text{, if item } i \text{ is purchased with } V_3 \\ 4 & \text{, if item } i \text{ is not purchased} \end{cases}$

Using the example (i)

[3 1 3 3 2 1 1 3 0 3 3 2 2 2 3]

② genotype.

represent integer value by their binary code

phenotype	genotype
0	00
1	01
2	10
3	11

Example in (i) genotype representation

11 01 11 11 10 01 01 11 00 11 11 10 10 10 11

(iii) Formulating the fitness function

① calculate the total cost

$$cost(V_1) = \sum_{i:g_i=1} cost(A_i)$$

$$cost(V_2) = \sum_{i:g_i=2} cos(A_i)$$

$$cost(V_3) = \sum_{i:g_i=3} cos(A_i)$$

② check feasibility

$cost(V_1) \le 60$ $cost(V_2) \le 70$ $cost(V_3) \le 80$

if any sum exceed its voucher limit we can give a largo penalty (P) to make the chromosome's fitness worse

③ compute Left over

$L_1 = 60 - cost(V_1)$
$L_2 = 60 - cost(V_2)$
$L_3 = 60 - cost(V_3)$

Total Left over $L = L_1 + L_2 + L_3$

④ fitness function

$$fitness = \frac{1}{1 + L + P}$$

$P = 0$ if all costs are within vocher limits

Methed 2.

max $fitness = 210 - leftover$

Method 3

$\min \quad L$