22-S2-Q1

Q(a) untrained exercise

80 120

$$f(x) = 120x + (1-x)\underline{g(x)}$$

$$g(x) = 50 + 30e^{-100x}$$

$g(x) \to 80$, x small

$g(x) \to 50$, $x \to 1$

(i) Q: fill the Table with 20 value of x

optimum value for exercising

(ii) Q: fitness function $f^*(x) \to 0$ to 1

(iii) Q: Difficulty of Find the optimum

Q(b) categorize

(i) sharo price → predict

(ii) odd size boxes → de livery

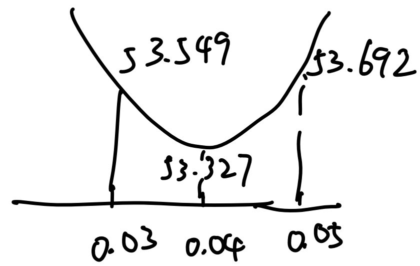
(iii) policy → making

(iv) Roster → security guards

Solution (i) ① minimize $f(x)$

$$f(x) = 120x + (1-x)g(x)$$

$$= 120x + (1-x)\left[50 + 30e^{-100x}\right]$$



②

x	$g(x)$	$f(x)$
0	80	80
0.01	61.0364	61.6266
0.02	54.0601	55.378
0.03	51.4936	53.549
0.04	50.5495	53.327
0.05	50.2021	53.6920
0.06	50.0743	54.2698
⋮	⋮	⋮
1	50	120

$$f(x) = 120x + 50 - 50x + 30e^{-100x} - 30xe^{-100x}$$

$$= 70x + 50 + 30e^{-100x} - 30xe^{-100x}$$

$$f'(x) = 70 - 3000e^{-100x} - 30\left[e^{-100x} - 100xe^{-100x}\right]$$

let $f'(x) = 0$ Using CASIO compute.

$x = 0.0373$, $f(x)_{min} = 53.3039$

about 3.7% of the day

Since the are $24 \times 60 = 1440$ minutes in a day, the optimum per day of exercise is

$$0.0373 \times 1440 = 53.712 \text{ mins/day}$$

(ii) We can use max-min normalize

$f_{max} = 120$, $f_{min} = 53.3$

So $$f^*(x) = \frac{120 - f(x)}{120 - 53.3} = \frac{120 - f(x)}{66.7}$$

when $f(x) = 53.3$ (best), $f^*(x) = 1$

when $f(x) = 120$ (worst) $f^*(x) = 0$

x	g(x)	f(x)	$f^*(x)$
0	80	80	0.60
0.01	61.0364	61.6266	0.88
0.02	54.0601	55.378	0.97
0.03	51.4936	53.549	0.997
0.04	50.5495	53.327	0.999
0.05	50.2021	53.6920	0.994
0.06	50.0743	54.2698	0.9854
⋮	⋮	⋮	
1	50	120	0

(iii) Solution

① It isn't a particularly difficult problem

② There is only one variable $x \in [0,1]$

③ The function is smooth and unimodal

④ It can be solved by elementary umerical methods, such as bisection, Newton-Raphson, simple table search

Solution (b) ① Question Type
(1) Black box model
(2) Search problem
(3) optimization vs constraint satisfaction
(4) NP problem

(i) Black box model

Reason: using historical data to forecasting or prediction regression

(ii) constraint and optimization problem (COP)
Reason: must decide how to arrange item to minimize wasted space or the number of trucks
(iii) constraint and optimization problem (COP)
Reason: multiple objectives, constraints
(iv) constraint and optimization problem (COP)

Reason: minimize cost or maximizing

coverage.