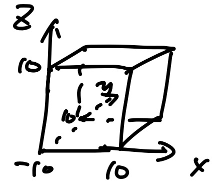
24-S1-Q1

Q(a) $f(x,y,z) = 5x^2 + 3y^2 + 2z^2 - 7x - 4y + 6z$

GA : find $f(x,y,z)$ maximum

i	0000	0000	0000
ii	0011	1100	1001
iii	1100	0111	0101
iV	0000	0000	1111
V.	1111	1111	1111



State : limit of binary representation with respect to the optimum solution

Solution (a)

① partition the 12 bits into three group of 4 bits.

i	0000	0000	0000
ii	0011	1100	1001
iii	1100	0111	0101
iV	0000	0000	1111
V.	1111	1111	1111

8 4 2 1

$2^3\ 2^2\ 2^1\ 2^0$

integer_value

i	0	0	0
ii	3	12	9
iii	12	7	5
iV	0	0	15
V	15	15	15

② spans the real interval [−10, +10] via simple linear mapping

$$\text{decoded\_value} = -10 + \left(\frac{20}{2^4-1}\right) \times \text{integer\_value}$$

$$= -10 + \frac{4}{3} \times \text{Integer}$$

decoded_value

	x	y	z	f(x,y,z)
i	−10	−10	−10	1050
ii	−6	6	2	326
iii	6	−0.6667	−3.3333	144.22
iV	−10	−10	10	1170
V	10	10	10	950

$$f(x,y,z) = 5x^2 + 3y^2 + 2z^2 - 7x - 4y + 6z$$

③ limitation of the 4-bit encoding

Because each coordinate only has 16 possible values ($2^4=16$) over [−10, +10]

There is a quantization discretization effect.

The true real-valued optimum in $\{-10 \le x, y, z \le +10\}$ need not coincide

with one of these discrete grid points
So, the GA can only approximate
the true optimum.

Q(b) $f(x,y,z) = a_0x^2 + b_0y^2 + c_0z^2 - a_1x - b_1y + c_1z$

$f(x,y,z) = 2.5x^2 + 3.88y^2 + 5.6z^2 - a_1x - b_1y + c_1z$

x	y	z	f
10	5	3	732
-10	10	5	688

candidate

	a_1	b_1	c_1
S1	7	1.6	4.8
S2	5	6.2	3.3

Using real-code GA solve a_1 b_1 c_1

calculate fitness

Solution (b)

① Typically, the fitness in regression-style
GA is inversely related to the
sum of squared errors (SSE)

So we calculate $SSE = e_1^2 + e_2^2$

$$\text{②For } S_1 = (\overset{a_1}{7}, \overset{b_1}{1.6}, \overset{c_1}{4.8})$$

$$\text{(1) point1 } (\overset{x}{10}, \overset{y}{5}, \overset{z}{5})$$

$$f(x,y,z) = 2.5x^2 + 3.88y^2 + 5.6z^2 - a_1 x - b_1 y + c_1 z$$

$$= 2.5 \times 100 + 3.88 \times 25 + 5.6 \times 25 - 7 \times 10 - 1.6 \times 5 + 48 \times 5$$

$$= 487 - 54 = 433$$

$$\text{Error: } e_1 = 433 - 732 = -299$$

$$e_1^2 = 89401$$

$$\text{(2) Point 2. } (-10, 10, 5)$$

$$f(x,y,z) = 2.5x^2 + 3.88y^2 + 5.6z^2 - a_1 x - b_1 y + c_1 z$$

$$= 2.5 \times 100 + 3.88 \times 100 + 5.6 \times 25$$

$$- 7 \times (-10) - 1.6 \times 10 + 4.8 \times 5$$

$$= 778 + 78$$

$$= 856$$

$$e_2 = 856 - 688 = 168$$

$$e_2^2 = 28224$$

$$\text{So } SSE = 89401 + 28224 = 117625$$

③ For $S_2 = (-5, 6.2, 3.3)$

(1) point 1 $(10, 5, 5)$

$f = 487 + 35.5 = 522.5$

$e_1 = 522.5 - 732 = -209.5$

$e_1^2 = 43902.25$

(2) point 2 $(-10, 10, 5)$

$f = 778 - 95.5 = 682.5$

$e_2 = 682.5 - 688 = -5.5$

$e_2^2 = 30.25$

$SSE = 43902.25 + 30.25 = 43932.5$

④ SSE $117625 > 43932.5$

fitness $S1 < S2$

So S_2 fits the two sample point data more closely